Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

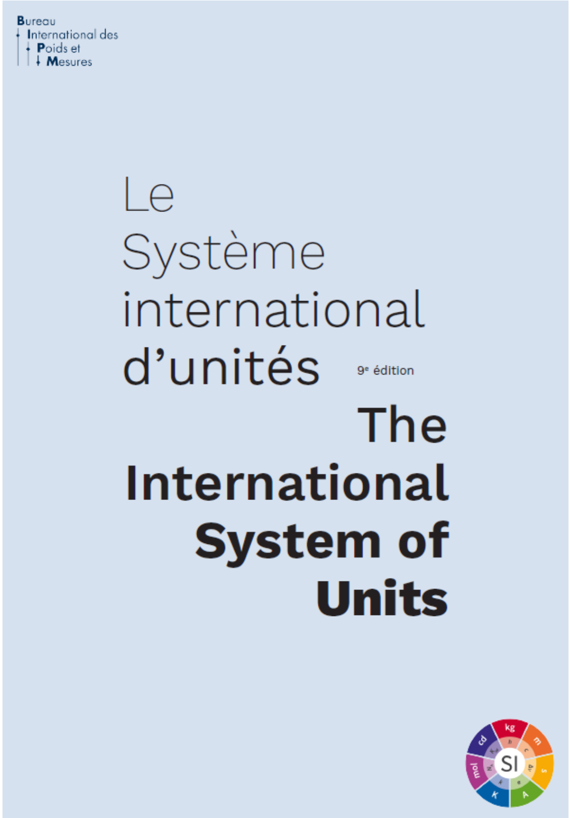
Federal Institute of Metrology METAS

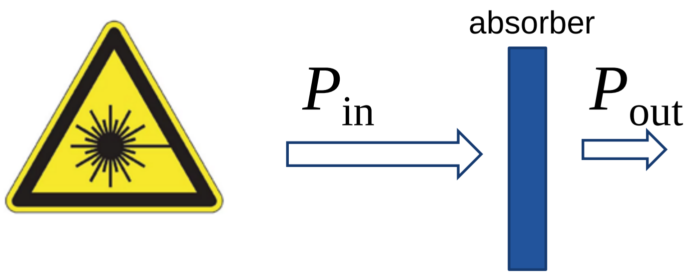
Quantities with the unit one

Peter Blattner, Chief Metrology Officer, METAS Delegate to CCU

Concept “unit one” in the SI-Brochure 9th edition (I)

2.3.3 There are quantities Q for which the defining equation is such that all of the dimensional exponents in the equation for the dimension of Q are zero. This is true in particular for any quantity that is defined as the **ratio of two quantities of the same kind**…Such quantities are simply numbers. The associated unit is the **unit one**, symbol 1, although this is rarely explicitly written.





Transmittance $\tau = \dfrac{P_{\mathrm{out}}}{P_{\mathrm{in}}}$

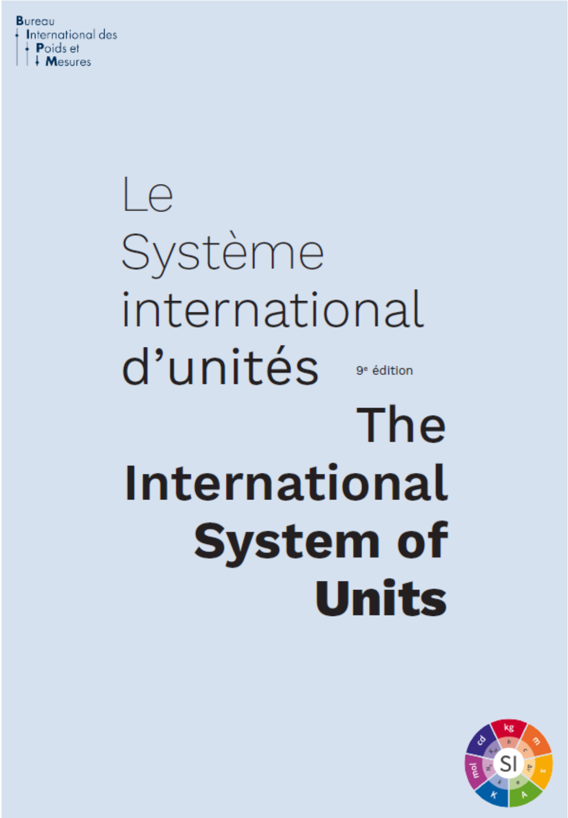
$[P_{\mathrm{in}}] = \mathrm{W}$ $[P_{\mathrm{out}}] = \mathrm{W}$ $[\tau] = 1$

Concept "unit one" in the SI-Brochure 9th edition (II)

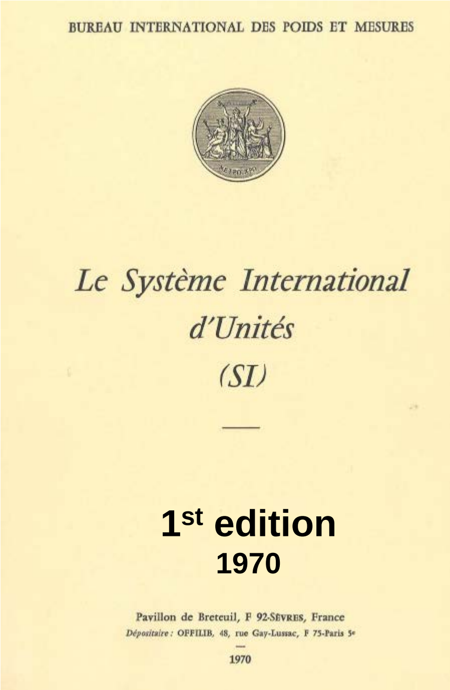
2.3.3…There are also some quantities that cannot be described in terms of the seven base quantities of the SI, but **have the nature of a count**. Counting quantities are also quantities with the associated **unit one**.

Count of red cars C_{rc}

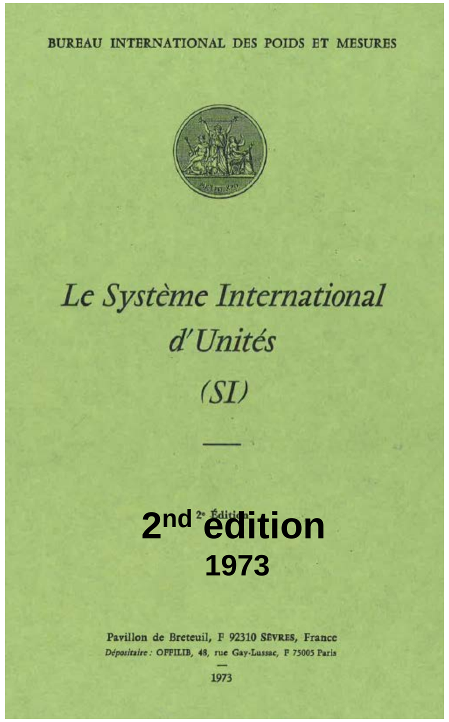
$$[C_{\mathrm{rc}}] = 1$$



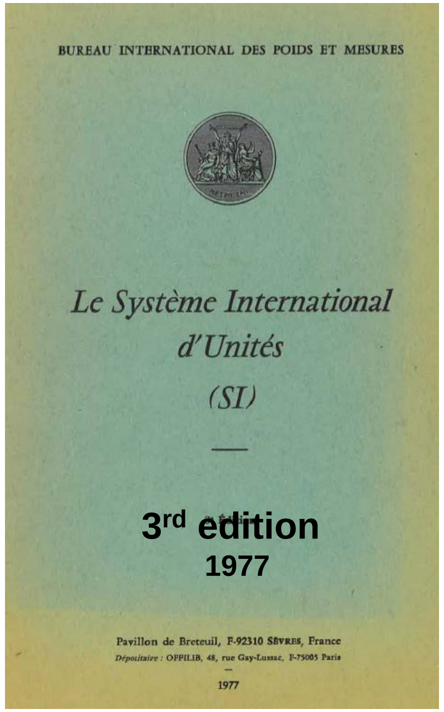
History of the concept “unit one” in the SI-Brochure (I)



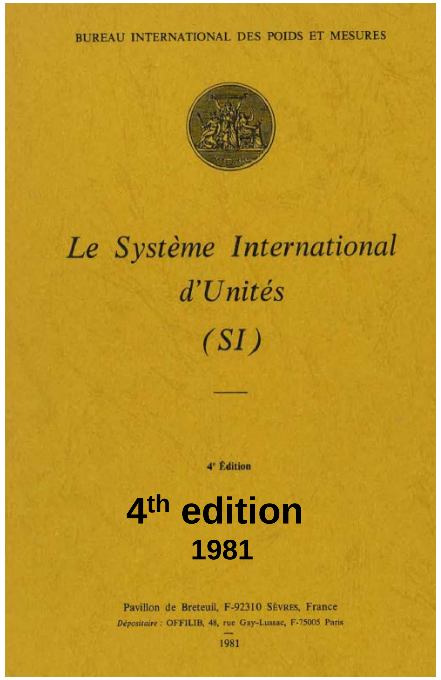
1st edition
1970



2nd edition
1973



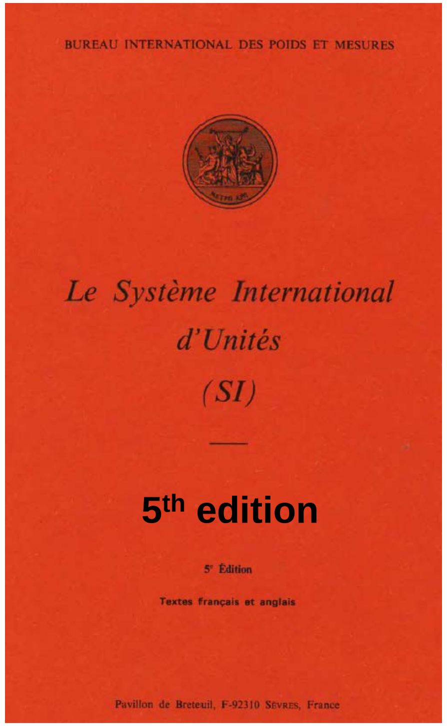
3rd edition
1977



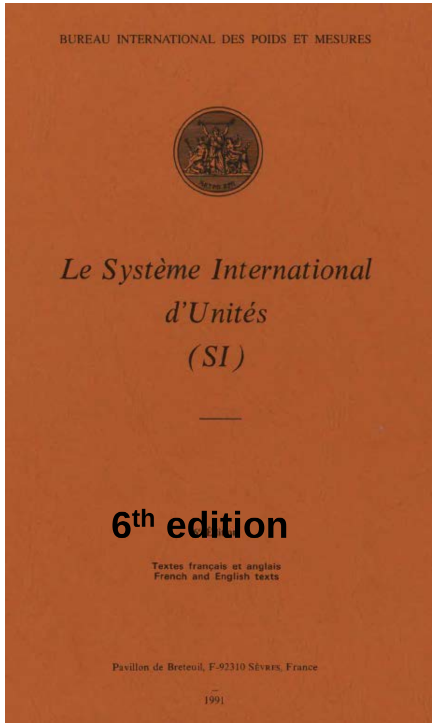
4th edition
1981

Les valeurs de certaines grandeurs, dites sans dimensions, comme par exemple l'indice de réfraction, la perméabilité relative ou la permittivité relative, sont exprimées par des nombres purs. L’unité SI correspondante est, dans ce cas, le rapport de deux unités SI égales et peut être exprimée par le nombre 1.

History of the concept "unit one" in the SI-Brochure (II)

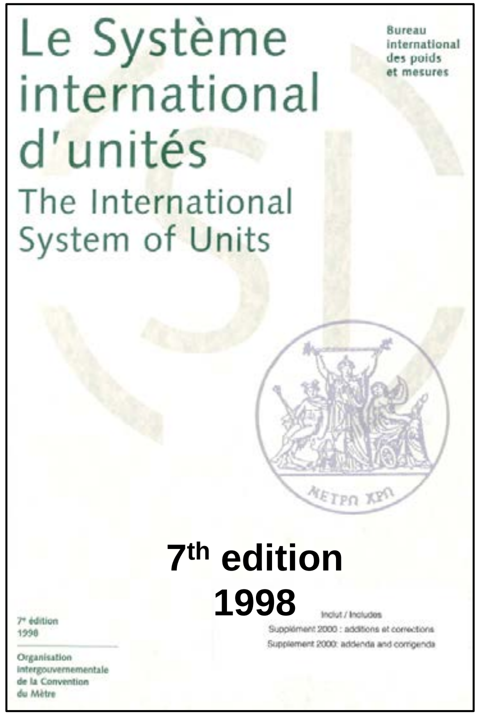


Dimensionless quantities. - The values of certain so-called dimensionless quantities, …, are defined as the ratio of two comparable quantities. These dimensionless quantities are expressed by pure numbers. The *coherent* SI unit is then the ratio of two identical SI units and may be expressed by the number 1.



Quantities expressed as pure numbers. - Certain so-called dimensionless quantities, …, are defined as the ratio of two comparable quantities. Such quantities have a dimensional product - or dimension - equal to 1 and are therefore expressed by pure numbers. The coherent SI unit is then the ratio of two identical SI units and may be expressed by the number 1.

History of the concept “unit one” in the SI-Brochure (III)



7th edition 1998

Certain quantities are defined as the ratios of two quantities of the same kind, and thus have a dimension which may be expressed by the number one. The unit of such quantities is necessarily a derived unit coherent with the other units of the SI and, since it is formed as the ratio of two identical SI units, the unit also may be expressed by the number one. Thus the SI unit of all quantities having the dimensional product one is the number one.

Other quantities having the unit 1 include “ characteristic numbers ” like the Prandtl number $\eta c_p / \lambda$ and numbers which represent a count, such as a number of molecules, degeneracy (number of energy levels) and partition function in statistical thermodynamics. All of these quantities are described as being dimensionless, or of dimension one, and have the coherent SI unit 1. Their values are simply expressed as numbers and, in general , the unit 1 is not explicitly shown.

What is a unit?

JCGM 200:2012 *International Vocabulary of Metrology* (VIM 3)

1.9 measurement unit

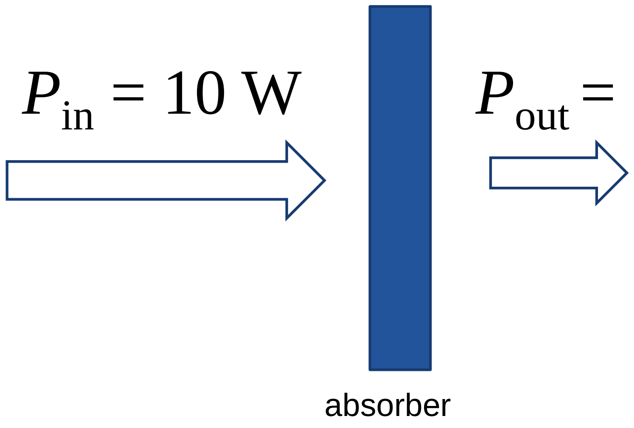
real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number

-> The ratio of two quantities of the same kind is a number

- Result: pure number

Transmittance τ

$P_{\text{in}} = 10\text{ W}$ $\quad$ $P_{\text{out}} = 5\text{ W}$



$$\tau = \frac{5\text{W}}{10\text{W}} = 0.5$$

What is a quantity?

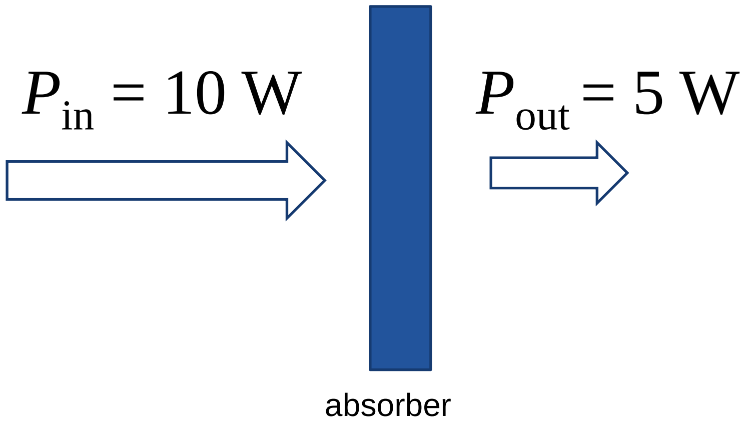
JCGM 200:2012 *International Vocabulary of Metrology* (VIM 3)

1.1 property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference

→ Observation: The definition is quite general (no need for multiplication) however we expect that any quantity has number and a reference.

- Result: pure number
- Where is the reference?
- Is "transmittance" not a quantity according VIM3?

Transmittance τ



$$\tau = \frac{5\mathrm{W}}{10\mathrm{W}} = 0.5$$

Observation in SI Broschure 9th edition:
The value of a quantity is *generally* expressed as the product of a number and a unit.

What is a quantity?

JCGM 200:2012 *International Vocabulary of Metrology* (VIM 3)

1.1 property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference

- Can “red car” serve as a reference?
- Reference = units?
- Is “red car” a unit?

VIM 3: Quantity of dimension one (I)

(1.8) quantity of dimension one

quantity for which all the exponents of the factors corresponding to the base quantities in its quantity dimension are zero”.

$$\dim C_{\mathrm{rc}} = \mathrm{T}^0\mathrm{L}^0\mathrm{M}^0\mathrm{I}^0\Theta^0\mathrm{N}^0\mathrm{J}^0$$

NOTE 2 The measurement units and values of quantities of dimension one are numbers, but such quantities convey more information than a number

$$\tau = 0.5$$

Ratio of which quantities?

VIM 3: Quantity of dimension one (II)

NOTE 3

Some quantities of dimension one are defined as the ratios of two quantities of the same kind.

NOTE 4

Numbers of entities are quantities of dimension one.

But where is the unit one?

Harmonization is necessary:

- CCU Task Group on angle and dimensionless quantities in the SI Brochure (CCU-TG-ADQSIB) , Convenor: Richard Brown
- Working Group on Core Metrological Terms (CCU-WG-CMT), Convenor: Pavel Neyezhmakov
- JCGM Working Group on the International Vocabulary of Metrology (JCGM-WG2:VIM), Convenor: Charles Ehrlich

Metrological concepts applied to quantities with unit one

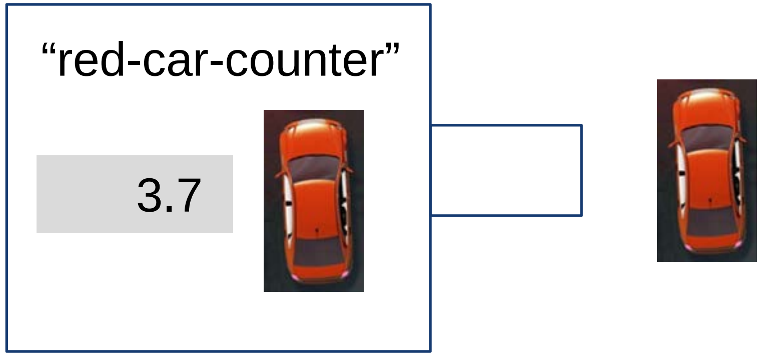
Calibration of quantities with the unit one?

VIM3

2.39 calibration

operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication

Example :



Is it sufficient to compare to the measurement standard “red car”?
What are the *calibration conditions* and *measurement uncertainty* for a “red-car-counter”?

Metrological traceability of quantities with the unit one?

VIM3

2.41 metrological traceability

property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty.

What is the reference for a quantity with unit one?

Example 1:

$$\tau = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}$$

N.B. there is no need that the quantities P_{in} and P_{out} are traceable to the SI, it is sufficient that the powermeter is linear -> need for characterization only

What is the role of National Metrology Institutes for quantities with unit one if characterization is sufficient?

Quantity calculus / dimensional analysis for quantities with unit one

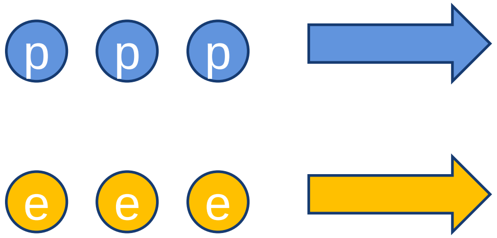
For quantities with unit one all the exponents are zero $\dim C_{\mathrm{rc}} = \mathrm{T}^0\mathrm{L}^0\mathrm{M}^0\mathrm{I}^0\Theta^0\mathrm{N}^0\mathrm{J}^0$

My claim: For problems that includes quantities of unit one the concept of dimensional analysis (*as defined presently*) gives little (or no) added value

Example

Photo flux Φ_p

Electron flux Φ_e



$$[\Phi_\mathrm{p}] = [\Phi_\mathrm{e}] = \mathrm{s}^{-1}$$

quantity calculus “allows” adding and dividing photon flux and electron flux

$$\cancel{\Phi_\mathrm{p} + \Phi_\mathrm{e}} \qquad \frac{\Phi_\mathrm{e}}{\Phi_\mathrm{p}}$$

No physical meaning

Quantum efficieny of a optical detector

Unit of «Photosynthetic photon flux» : μmol (*photons*) s^{-1}

How to improve the situation?

- CIPM (through CCU) in collaboration with stakeholders shall improve the definition of terms and their usage. (-> CCU WG CMT, CCU TG ADQSIB, JCGM WG2)

- CIPM (through the CC's) in collaboration with stakeholders shall define "primary" realization procedures (*mise-en-pratiques*), characterization, validation and calibration procedures/conditions for quantities with unit one. (-> work in progress)

- NMIs should offer lowest possible uncertainties enabling the validation of realizations by the costumers (-> work in progress)

- All stakeholders of the SI shall well describe the quantities under consideration avoiding misunderstanding ("ratio of optical power", "count of red car")