Bureau International des Poids et Mesures

Guide to
the Realization of the ITS-90

Platinum Resistance Thermometry

APPENDIX 1: *Alternative interpolating functions for special applications*

Consultative Committee for Thermometry
under the auspices of the
International Committee for Weights and Measures

Appendix 1

Alternative interpolating functions for special applications

The following subsections list alternative interpolating functions for each of the ITS-90 subranges covered by this chapter on platinum resistance thermometry. Each subsection presents the ITS-90 equation, formulae for the interpolating functions, and a graph showing the interpolating functions as a function of temperature. All of the graphs are expressed in temperature rather than resistance ratio; both axes are scaled so that the graphs plot the propagated temperature error divided by the temperature error at the specified fixed point.

Water-Gallium Subrange: 0.01 °C to 29.7646 °C

The ITS-90 equation for this subrange is

$$W_{\mathrm{r}}(W) = W - a(W-1)\,, \qquad \text{(A1.1)}$$

where the a coefficient is determined by calibration of the SPRT at the melting point of gallium. The interpolation is rewritten as

$$W_{\mathrm{r}}(W) = \frac{(W - W_{\mathrm{Ga}})}{(1 - W_{\mathrm{Ga}})} + W_{\mathrm{r,Ga}} \frac{(W-1)}{(W_{\mathrm{Ga}} - 1)}\,, \qquad \text{(A1.2)}$$

from which the sensitivity coefficients are identified as

$$f_{\mathrm{H2O}}(W) = \frac{(W - W_{\mathrm{Ga}})}{(1 - W_{\mathrm{Ga}})} \qquad \text{(A1.3a)}$$

and

$$f_{\mathrm{Ga}}(W) = \frac{(W-1)}{(W_{\mathrm{Ga}} - 1)}\,. \qquad \text{(A1.3b)}$$

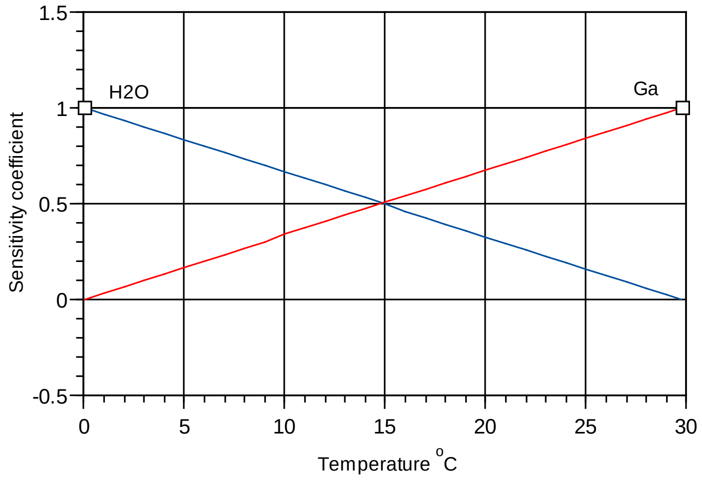

Figure A1.1: Sensitivity coefficients for the water-gallium subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Water-Indium Subrange: 0.01 °C to 156.5985 °C

The ITS-90 interpolation equation is

$$W_r(W) = W - a(W-1)\,, \tag{A1.4}$$

where the a coefficient is determined by calibration of the SPRT at the freezing point of indium. The interpolation equation can be written in the form

$$W_r(W) = \frac{(W-W_{\mathrm{In}})}{(1-W_{\mathrm{In}})} + W_{\mathrm{r,In}}\frac{(W-1)}{(W_{\mathrm{In}}-1)}\,, \tag{A1.5}$$

from which the sensitivity coefficients are identified as

$$f_{\mathrm{H2O}}(W) = \frac{(W-W_{\mathrm{In}})}{(1-W_{\mathrm{In}})} \tag{A1.6a}$$

and

$$f_{\mathrm{In}}(W) = \frac{(W-1)}{(W_{\mathrm{In}}-1)}\,. \tag{A1.6b}$$

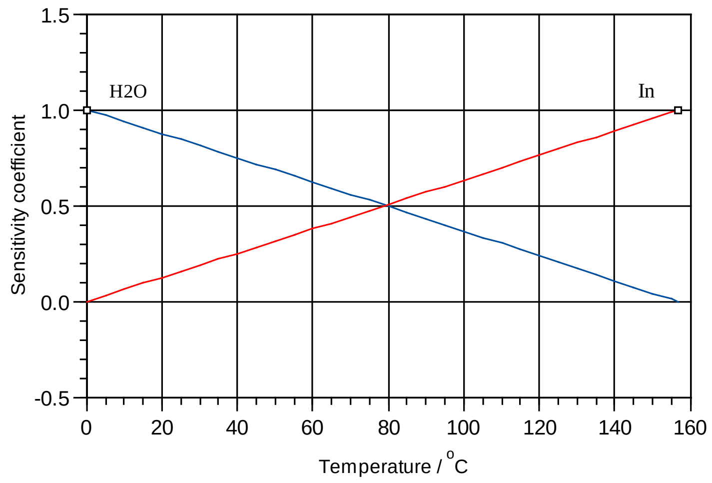

Figure A1.2: The sensitivity coefficients for the water-indium subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Mercury-Gallium Subrange: –38.8344 °C to 29.7646 °C

The ITS-90 interpolation equation is

$$W_r(W) = W - a(W-1) - b(W-1)^2\,, \tag{A1.7}$$

where the a and b coefficients are determined by calibration of the SPRT at the triple point of mercury and the melting point of gallium. The interpolation equation may be written in the form

$$W_r(W) = W_{r,Hg}\frac{(W-1)(W-W_{Ga})}{(W_{Hg}-1)(W_{Hg}-W_{Ga})} + \frac{(W-W_{Hg})(W-W_{Ga})}{(1-W_{Hg})(1-W_{Ga})} + W_{r,Ga}\frac{(W-W_{Hg})(W-1)}{(W_{Ga}-W_{Hg})(W_{Ga}-1)}\,, \tag{A1.8}$$

from which the sensitivity coefficients are identified as

$$f_{Hg} = \frac{(W-1)(W-W_{Ga})}{(W_{Hg}-1)(W_{Hg}-W_{Ga})}\,, \tag{A1.9a}$$

$$f_{H_2O} = \frac{(W-W_{Hg})(W-W_{Ga})}{(1-W_{Hg})(1-W_{Ga})}\,, \tag{A1.9b}$$

and

$$f_{Ga} = \frac{(W-W_{Hg})(W-1)}{(W_{Ga}-W_{Hg})(W_{Ga}-1)}\,. \tag{A1.9c}$$

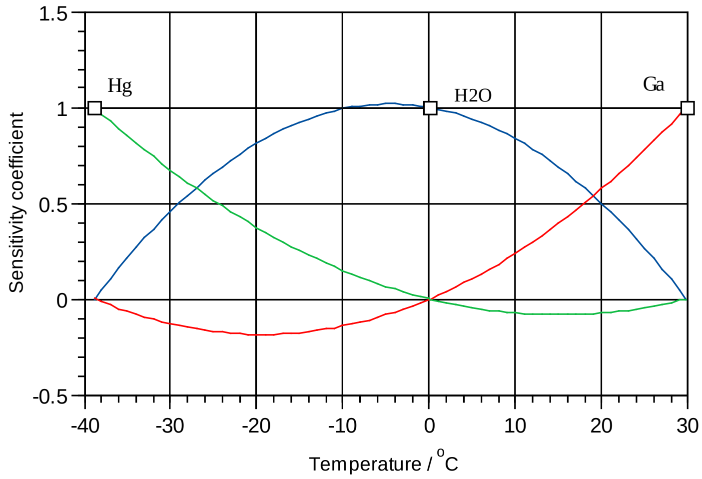

Figure A1.3: The sensitivity coefficients for the mercury-gallium subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Water-Tin Subrange: 0.01 °C to 231.928 °C

The ITS-90 interpolation equation is

$$W_{\mathrm{r}}(W) = W - a(W-1) - b(W-1)^2 , \tag{A1.10}$$

where the a and b coefficients are determined by calibration of the SPRT at the freezing points of indium and tin. The interpolation equation may be written in the form

$$W_{\mathrm{r}}(W) = \frac{(W-W_{\mathrm{In}})(W-W_{\mathrm{Sn}})}{(1-W_{\mathrm{In}})(1-W_{\mathrm{Sn}})} + W_{\mathrm{r,In}} \frac{(W-1)(W-W_{\mathrm{Sn}})}{(W_{\mathrm{In}}-1)(W_{\mathrm{In}}-W_{\mathrm{Sn}})} + W_{\mathrm{r,Sn}} \frac{(W-1)(W-W_{\mathrm{In}})}{(W_{\mathrm{Sn}}-1)(W_{\mathrm{Sn}}-W_{\mathrm{In}})} \tag{A1.11}$$

from which the sensitivity coefficients are identified as

$$f_{\mathrm{H2O}} = \frac{(W-W_{\mathrm{In}})(W-W_{\mathrm{Sn}})}{(1-W_{\mathrm{In}})(1-W_{\mathrm{Sn}})} , \tag{A1.12a}$$

$$f_{\mathrm{In}} = \frac{(W-1)(W-W_{\mathrm{Sn}})}{(W_{\mathrm{In}}-1)(W_{\mathrm{In}}-W_{\mathrm{Sn}})} , \tag{A1.12b}$$

and

$$f_{\mathrm{Sn}} = \frac{(W-1)(W-W_{\mathrm{In}})}{(W_{\mathrm{Sn}}-1)(W_{\mathrm{Sn}}-W_{\mathrm{In}})} . \tag{A1.12c}$$

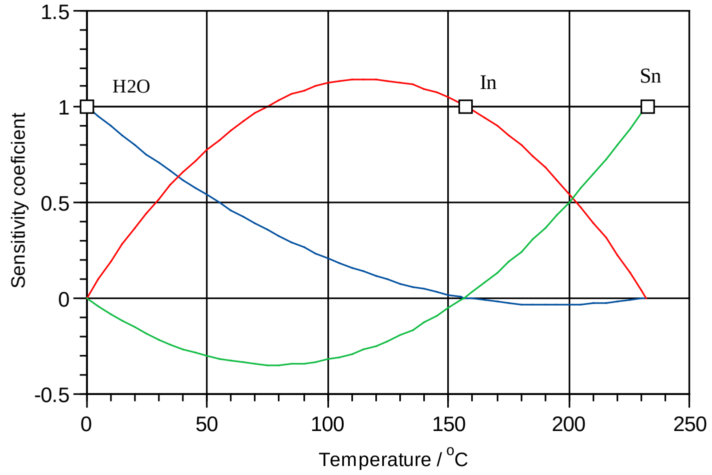

Figure A1.4: The sensitivity coefficients for the water-tin subrange. The curves show the propagated error in millikelvin for a 1 mK uncertainty at each of the fixed points.

Water-Zinc Subrange: 0.01 °C to 419.527 °C

The ITS-90 interpolation equation is

$$W_r(W) = W - a(W-1) - b(W-1)^2\,, \tag{A1.13}$$

where the a and b coefficients are determined by calibration of the SPRT at the freezing points of tin and zinc. The interpolation equation may be written in the form

$$W_r(W) = \frac{(W-W_{Sn})(W-W_{Zn})}{(1-W_{Sn})(1-W_{Zn})} + W_{r,Sn}\frac{(W-1)(W-W_{Zn})}{(W_{Sn}-1)(W_{Sn}-W_{Zn})} + W_{r,Zn}\frac{(W-1)(W-W_{Sn})}{(W_{Zn}-1)(W_{Zn}-W_{Sn})}\,, \tag{A1.14}$$

from which the sensitivity coefficients are identified as

$$f_{H2O} = \frac{(W-W_{Sn})(W-W_{Zn})}{(1-W_{Sn})(1-W_{Zn})}\,, \tag{A1.15a}$$

$$f_{Sn} = \frac{(W-1)(W-W_{Zn})}{(W_{Sn}-1)(W_{Sn}-W_{Zn})}\,, \tag{A1.15b}$$

and

$$f_{Zn} = \frac{(W-1)(W-W_{Sn})}{(W_{Zn}-1)(W_{Zn}-W_{Sn})}\,. \tag{A1.15c}$$

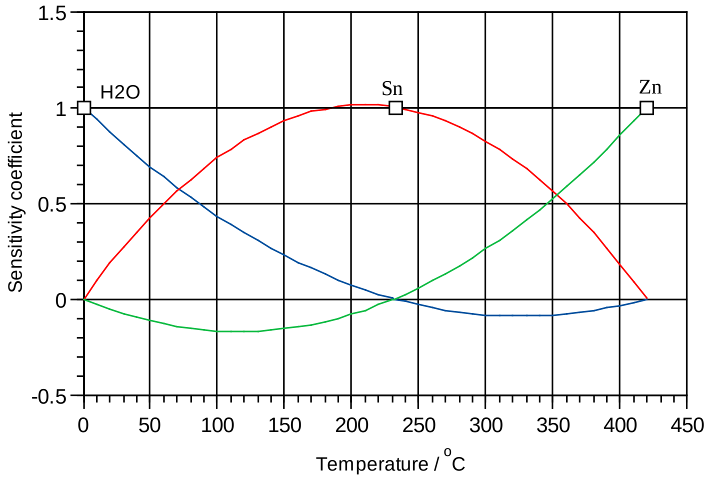

Figure A1.5: The sensitivity coefficients for the water-zinc subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Water-Aluminium Subrange: 0.01 °C to 660.323 °C

The ITS-90 interpolation equation is

$$W_{\mathrm{r}}(W) = W - a(W-1) - b(W-1)^2 - c(W-1)^3, \tag{A1.16}$$

where the a, b and c coefficients are determined by calibration of the SPRT at the freezing points of tin, zinc and aluminium. The interpolation equation may be written in the form

$$W_{\mathrm{r}}(W) = f_{\mathrm{H_2O}} + W_{\mathrm{r,Sn}} f_{\mathrm{Sn}} + W_{\mathrm{r,Zn}} f_{\mathrm{Zn}} + W_{\mathrm{r,Al}} f_{\mathrm{Al}}, \tag{A1.17}$$

where the sensitivity coefficients are

$$f_{\mathrm{H2O}} = \frac{(W-W_{\mathrm{Sn}})(W-W_{\mathrm{Zn}})(W-W_{\mathrm{Al}})}{(1-W_{\mathrm{Sn}})(1-W_{\mathrm{Zn}})(1-W_{\mathrm{Al}})}, \tag{A1.18a}$$

$$f_{\mathrm{Sn}} = \frac{(W-1)(W-W_{\mathrm{Zn}})(W-W_{\mathrm{Al}})}{(W_{\mathrm{Sn}}-1)(W_{\mathrm{Sn}}-W_{\mathrm{Zn}})(W_{\mathrm{Sn}}-W_{\mathrm{Al}})}, \tag{A1.18b}$$

$$f_{\mathrm{Zn}} = \frac{(W-1)(W-W_{\mathrm{Sn}})(W-W_{\mathrm{Al}})}{(W_{\mathrm{Zn}}-1)(W_{\mathrm{Zn}}-W_{\mathrm{Sn}})(W_{\mathrm{Zn}}-W_{\mathrm{Al}})}, \tag{A1.18c}$$

$$f_{\mathrm{Al}} = \frac{(W-1)(W-W_{\mathrm{Sn}})(W-W_{\mathrm{Zn}})}{(W_{\mathrm{Al}}-1)(W_{\mathrm{Al}}-W_{\mathrm{Sn}})(W_{\mathrm{Al}}-W_{\mathrm{Zn}})}. \tag{A1.18d}$$

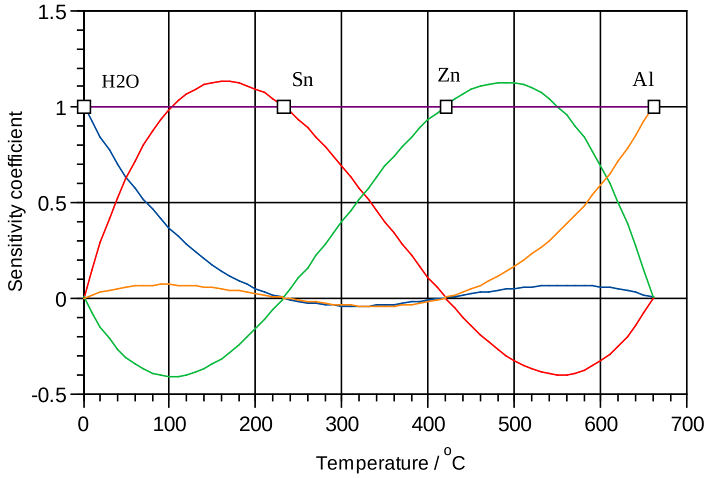

Figure A1.6: The sensitivity coefficients for the water-aluminium subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Water-Silver Subrange: 0.01 °C to 961.78 °C

The ITS-90 interpolation equation takes the form

$$W_{\mathrm{r}}(W) = W - a(W-1) - b(W-1)^2 - c(W-1)^3 - d(W - W_{\mathrm{Al}})^2 , \tag{A1.19}$$

where the *a*, *b* and *c* coefficients are the same as determined from the water–aluminium subrange, and the *d* coefficient is determined by calibration of the SPRT at the freezing point of silver. The equation is applicable only over the temperature range between the aluminium and silver points (660.323 °C to 961.78 °C). The interpolation equation may be written in the form

$$W_{\mathrm{r}}(W) = f_{\mathrm{H_2O}} + W_{\mathrm{r,Sn}} f_{\mathrm{Sn}} + W_{\mathrm{r,Zn}} f_{\mathrm{Zn}} + W_{\mathrm{r,Al}} f_{\mathrm{Al}} + W_{\mathrm{r,Ag}} f_{\mathrm{Ag}} , \tag{A1.20}$$

where the sensitivity coefficients (applicable only above 660.323 °C) are

$$f_{\mathrm{H_2O}} = f_{\mathrm{H_2O}}^{\mathrm{Al}}(W) - f_{\mathrm{H_2O}}^{\mathrm{Al}}(W_{\mathrm{Ag}}) \left(\frac{W - W_{\mathrm{Al}}}{W_{\mathrm{Ag}} - W_{\mathrm{Al}}} \right)^2 , \tag{A1.21a}$$

$$f_{\mathrm{Sn}} = f_{\mathrm{Sn}}^{\mathrm{Al}}(W) - f_{\mathrm{Sn}}^{\mathrm{Al}}(W_{\mathrm{Ag}}) \left(\frac{W - W_{\mathrm{Al}}}{W_{\mathrm{Ag}} - W_{\mathrm{Al}}} \right)^2 , \tag{A1.21b}$$

$$f_{\mathrm{Zn}} = f_{\mathrm{Zn}}^{\mathrm{Al}}(W) - f_{\mathrm{Zn}}^{\mathrm{Al}}(W_{\mathrm{Ag}}) \left(\frac{W - W_{\mathrm{Al}}}{W_{\mathrm{Ag}} - W_{\mathrm{Al}}} \right)^2 , \tag{A1.21c}$$

$$f_{\mathrm{Al}} = f_{\mathrm{Al}}^{\mathrm{Al}}(W) - f_{\mathrm{Al}}^{\mathrm{Al}}(W_{\mathrm{Ag}}) \left(\frac{W - W_{\mathrm{Al}}}{W_{\mathrm{Ag}} - W_{\mathrm{Al}}} \right)^2 , \tag{A1.21d}$$

$$f_{\mathrm{Ag}} = \left(\frac{W - W_{\mathrm{Al}}}{W_{\mathrm{Ag}} - W_{\mathrm{Al}}} \right)^2 , \tag{A1.21e}$$

where the f_i^{Al} are the sensitivity coefficients for the water-aluminium subrange, as given by (A1.18a-d). For temperatures below 660.323 °C, Equations (A1.18a-d) apply.

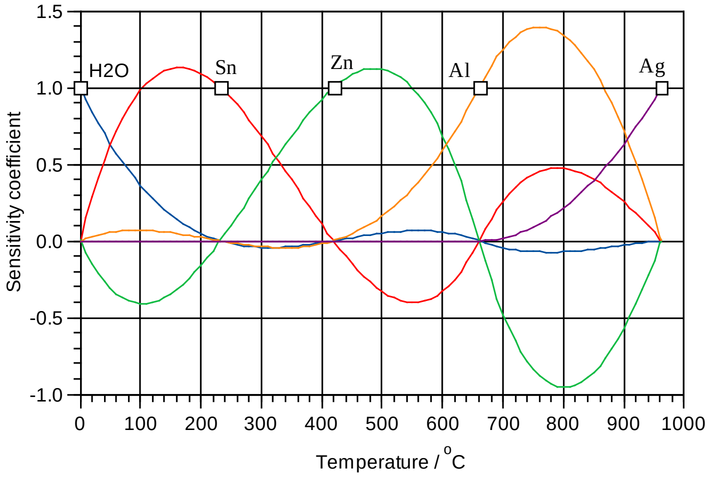

Figure A1.7: The sensitivity coefficients for the water-silver subrange. The curves plot the propagated error in millikelvin for a 1 mK error at each of the fixed points. Below 660.323 °C (the aluminium point) the curves are identical to those in Figure A1.6.

Argon-Water Subrange: 83.8058 K to 273.16 K

The ITS-90 interpolation equation is

$$W_{\mathrm{r}} = W - a(W-1) - b(W-1)\ln(W)\,, \tag{A1.22}$$

where the a and b coefficients are determined by calibration of the SPRT at the triple points of argon and mercury. The interpolation may be written in the form

$$W_{\mathrm{r}}(W) = f_{\mathrm{H_2O}} + W_{\mathrm{r,Hg}} f_{\mathrm{Hg}} + W_{\mathrm{r,Ar}} f_{\mathrm{Ar}}\,, \tag{A1.23}$$

where the sensitivity coefficients are

$$f_{\mathrm{H_2O}} = \frac{(W-1)(W_{\mathrm{Hg}} - W_{\mathrm{Ar}})\ln W + (W_{\mathrm{Hg}}-1)(W_{\mathrm{Ar}} - W)\ln W_{\mathrm{Hg}} - (W_{\mathrm{Hg}} - W)(W_{\mathrm{Ar}}-1)\ln W_{\mathrm{Ar}}}{(W_{\mathrm{Hg}}-1)(W_{\mathrm{Ar}}-1)(\ln W_{\mathrm{Hg}} - \ln W_{\mathrm{Ar}})}, \tag{A1.24a}$$

$$f_{\mathrm{Hg}} = \frac{(W-1)(\ln W - \ln W_{\mathrm{Ar}})}{(W_{\mathrm{Hg}}-1)(\ln W_{\mathrm{Hg}} - \ln W_{\mathrm{Ar}})}, \tag{A1.24b}$$

$$f_{\mathrm{Ar}} = \frac{(W-1)(\ln W - \ln W_{\mathrm{Hg}})}{(W_{\mathrm{Ar}}-1)(\ln W_{\mathrm{Ar}} - \ln W_{\mathrm{Hg}})}. \tag{A1.24c}$$

The sensitivity coefficient for the triple point of water is most easily calculated as $f_{\mathrm{H_2O}} = 1 - f_{\mathrm{Hg}} - f_{\mathrm{Ar}}$.

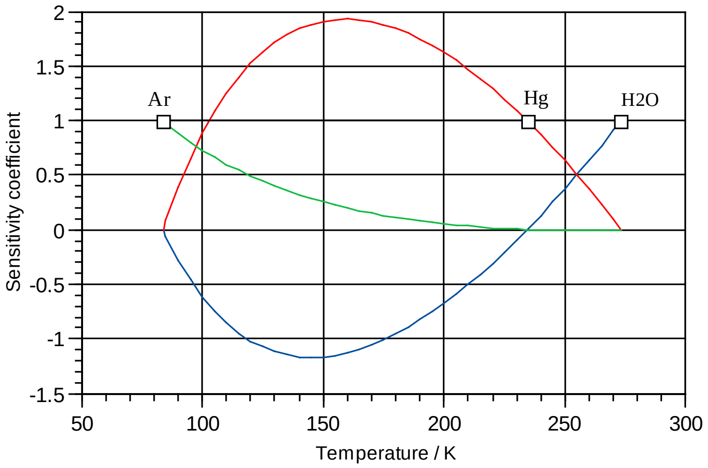

Figure A1.8: The sensitivity coefficients for the argon-water subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Oxygen-Water Subrange: 54.3584 K to 273.16 K

The ITS-90 interpolation equation takes the form

$$W_r(W) = W - a(W-1) - b(W-1)^2 - c\left(\ln W\right)^2, \tag{A1.25}$$

where the a, b and c coefficients are determined by calibration of the SPRT at the triple points of oxygen, argon and mercury. The interpolation may be written in the form

$$W_r = f_{H20} + W_{r,Hg} f_{Hg} + W_{r,Ar} f_{Ar} + W_{r,O2} f_{O2}, \tag{A1.26}$$

where the sensitivity coefficient for the oxygen point is

$$f_{O2}(W) = \frac{(\ln W)^2 (W_{Hg}-1)(W_{Ar}-1)(W_{Ar}-W_{Hg}) - (\ln W_{Ar})^2 (W_{Hg}-1)(W-1)(W-W_{Hg}) + (\ln W_{Hg})^2 (W-1)(W_{Ar}-1)(W-W_{Ar})}{(\ln W_{O2})^2 (W_{Hg}-1)(W_{Ar}-1)(W_{Ar}-W_{Hg}) - (\ln W_{Ar})^2 (W_{Hg}-1)(W_{O2}-1)(W_{O2}-W_{Hg}) + (\ln W_{Hg})^2 (W_{O2}-1)(W_{Ar}-1)(W_{O2}-W_{Ar})} \tag{A1.27}$$

The equations for the argon and mercury functions can be found by permuting the indices (e.g., swap the Ar and O2 subscripts to obtain the $f_{Ar}(W)$ function), and the water function found from $f_{H2O} = 1 - f_{O2} - f_{Ar} - f_{Hg}$. Alternatively, the following numerical approximations may be used:

$$f_{\mathrm{O2}} = -1.598425679 + 3.403991489W - 1.805565809W^2 + 0.4032254894(\ln W)^2, \qquad \text{(A1.28a)}$$

$$f_{\mathrm{Ar}} = +3.930746333 - 8.465193581W + 4.534447249W^2 - 0.5593488293(\ln W)^2, \qquad \text{to}$$

$$f_{\mathrm{Hg}} = -4.044936086 + 16.33613127W - 12.29119519W^2 + 0.4643043265(\ln W)^2,$$

$$f_{\mathrm{H2O}} = +2.712615432 - 11.27492918W + 9.562313751W^2 - 0.3081809866(\ln W)^2. \qquad \text{(A1.28d)}$$

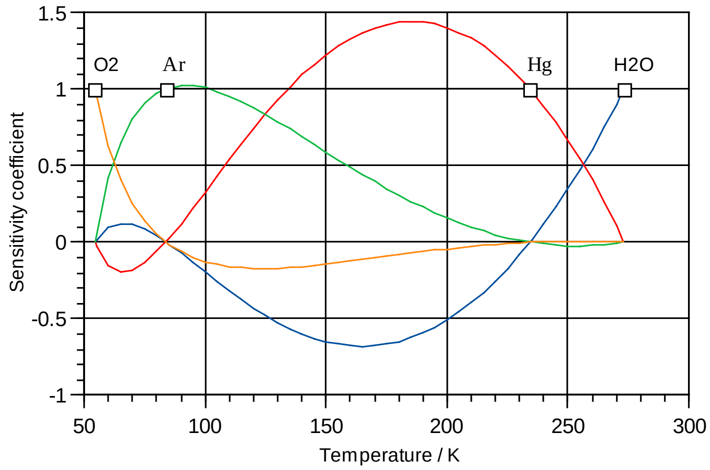

Figure A1.9: The sensitivity coefficients for the oxygen-water subrange. The curves show the propagated error in millikelvin for a 1 mK error at each of the fixed points.

Neon-Water Subrange: 24.5561 K to 273.16 K

The ITS-90 interpolation equation is

$$W_{\mathrm{r}} = W - a(W-1) - b(W-1)^2 - c\ln W - d(\ln W)^2 - e(\ln W)^3, \qquad \text{(A1.29)}$$

where the a, b, c, d, and e coefficients are determined by calibration of the SPRT at the triple points of equilibrium hydrogen, neon, oxygen, argon and mercury. The interpolation may be written in the form

$$W_{\mathrm{r}} = f_{\mathrm{H2O}} + W_{\mathrm{r,Hg}} f_{\mathrm{Hg}} + W_{\mathrm{r,Ar}} f_{\mathrm{Ar}} + W_{\mathrm{r,O2}} f_{\mathrm{O2}} + W_{\mathrm{r,Ne}} f_{\mathrm{Ne}} + W_{\mathrm{r,e\text{-}H2}} f_{\mathrm{e\text{-}H2}}. \qquad \text{(A1.30)}$$

The sensitivity coefficients in this case are too complex to write down in algebraic form. The following numerical approximations may be used:

$$f_{\mathrm{Hg}} = -24.30265740 + 42.99793720W - 18.69528095W^2 - 14.20610018\ln(W) - 2.778961721\ln^2(W) - 0.1788181624\ln^3(W), \quad \text{(A1.31a)}$$
$$f_{\mathrm{Ar}} = +33.02979668 - 46.64306946W + 13.61327293W^2 + 20.51522598\ln(W) + 4.195086350\ln^2(W) + 0.2785479955\ln^3(W),$$
$$f_{\mathrm{O_2}} = -29.45087407 + 39.79347654W - 10.34260259W^2 - 19.77485453\ln(W) - 4.269113231\ln^2(W) - 0.2941747479\ln^3(\mathrm{W}),$$
$$f_{\mathrm{Ne}} = +7.413452875 - 9.508661062W + 2.095208241W^2 + 5.423905534\ln(W) + 1.384488984\ln^2(W) + 0.1102290468\ln^3(W), \quad \text{to}$$
$$f_{\mathrm{H}} = -1.715380724 + 2.160321157W - 0.4449404100W^2 - 1.291145588\ln(W) - 0.3520630280\ln^2(W) - 0.03269335415\ln^3(W),$$
$$f_{\mathrm{H2O}} = +16.02566264 - 28.80000438W + 13.77434278W^2 + 9.33296879\ln(W) + 1.820562646\ln^2(W) + 0.1169092222\ln^3(W). \quad \text{(A1.31f)}$$

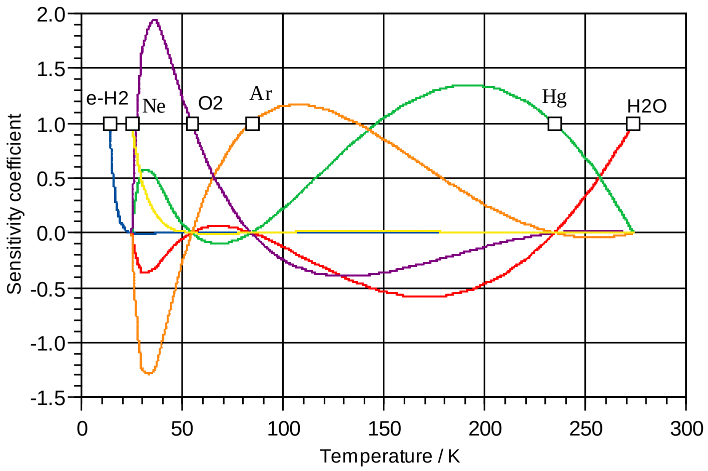

Figure A1.10: The sensitivity coefficients for the neon-water subrange. The curves plot the propagated error in millikelvin for a 1 mK error at each of the fixed points. Note that the subrange extends only down to the Neon point (24.5561 K), although the hydrogen point is below this temperature.

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