GUM Part 6 – Developing and using measurement models. An outline

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GUM - The undisputed reference in uncertainty evaluation since 1993

Joint Committee for Guides in Metrology
JCGM, 1997

JCGM-WG1

Working Group 1 (WG1) of the Joint Committee for Guides in Metrology (JCGM) has responsibility for maintaining the Guide to the Expression of Uncertainty in Measurement, which is now used worldwide at all levels of the measurement chain, from NMIs to industry, under the name JCGM 100:2008 (GUM 1995 with minor corrections).

…and the GUM suite

In addition, the JCGM-WG1 has decided to broaden the scope of the GUM by producing a series of complementary documents to cover some topics of interest in more detail. In this new perspective, JCGM100:2008 and the entire series of complementary documents are part of the GUM, which encompasses the whole suite of documents.

The GUM suite

JCGM GUM-X:2020 Guide to the expression of uncertainty in measurement – Part X: Title

Part 1: Introduction (in preparation, based on JCGM 104:2009)
Part 2: Principles and concepts (in preparation)
Part 3: Legacy (the current JCGM 100:2008)
Part 4: Conformity assessment (the current JCGM 106:2012)
Part 5: Examples (in preparation)
Part 6: Developing and using measurement models (just published)
Part 7: Monte Carlo (the current JCGM 101:2008, revision just started)
Part 8: Multivariate (the current JCGM 102:2011, revision just started)
Part 9: Inter-Laboratory Studies (in preparation)
Part 10: Least-squared methods (planned)
Part 11: Bayesian methods (planned)
...

This way of numbering will enable harmonisation with the ISO/IEC version of the documents (a source of confusion so far)

BIPM

JCGM GUM-6:2020

Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models

Guide pour l'expression de l'incertitude de mesure — Partie 6: Élaboration et utilisation des modèles de mesure

ΜΕΤΡΩ ΧΡΩ

First edition 2020

General considerations

Although the development of a measurement model crucially depends on the nature of the measurement, some generic guidance on aspects of modelling is possible

Following the introduction in 1993 of the Guide to the expression of uncertainty in measurement, …, the practice of uncertainty evaluation has broadened to use a wider variety of models and methods. To reflect this, this Guide includes an introduction to statistical models for measurement modelling (clause 11) and additional guidance on modelling random variation in Annex C.

Scope

This document provides guidance on developing and using a measurement model and also covers the assessment of the adequacy of a measurement model. The document is of particular interest to developers of measurement procedures, working instructions and documentary standards.

Measurement model

Many measurements are modelled by a real functional relationship *f* between *N* real-valued input quantities $X_1, \ldots, X_N$ and a single real-valued output quantity (or measurand) *Y,* for example, in the case of the mass of a sphere

$$m = \frac{\pi}{6} d^3 \rho$$

Thus, an algebraic relationship among physical quantities

But also…

Measurement model

The measurement model is a mathematical relationship among quantities, and as such it is subject to the rules of quantity calculus [20].

The same symbols used for the quantities are also used for the corresponding *random variables* …, whose *probability distributions* … describe the available knowledge about the quantities.

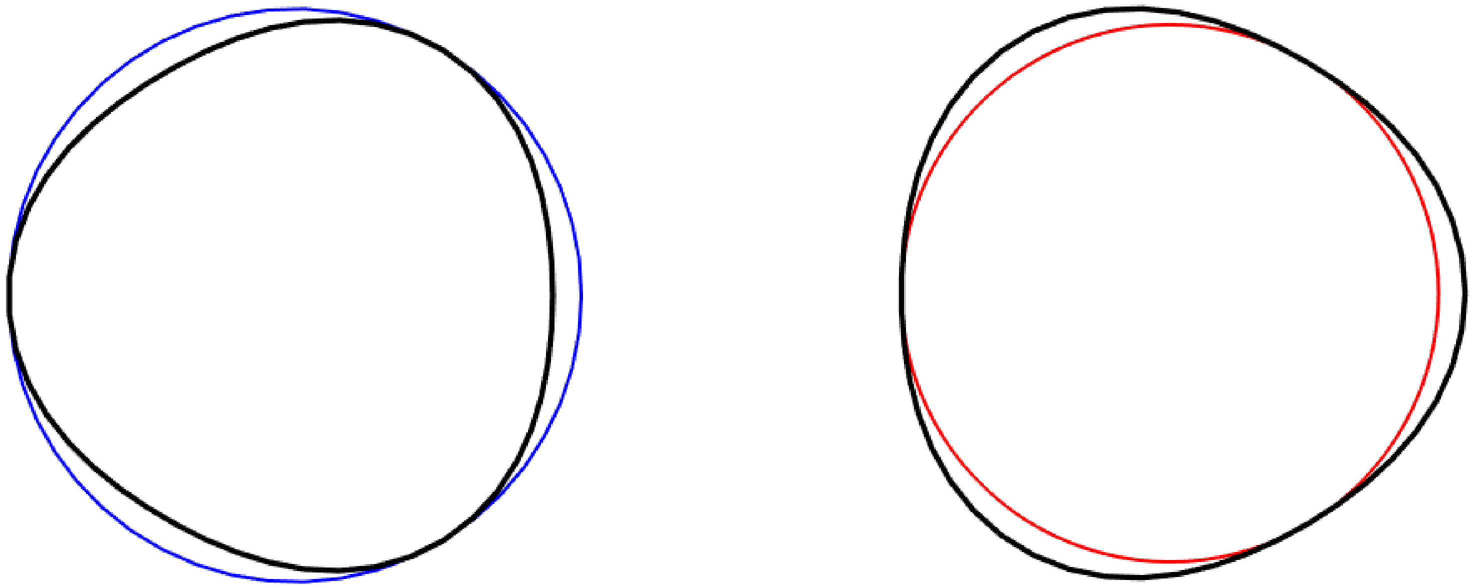
Therefore, the measurement model can also be considered to be a model involving random variables, **subject to the rules of mathematical statistics**.

Building a model

a) Select and specify the measurand
b) Model the measurement principle, thus providing a basic model for this purpose, choosing an appropriate mathematical form
c) Identify effects involved in the measurement
d) Extend the basic model as necessary to include terms accounting for these effects
e) Assess the resulting measurement model for adequacy

For each stage a) to e), a specific clause

Measurand



Minimum circumscribed circle (left, blue) and maximum inscribed circle (right, red)

Modelling the measurement principle

Theoretical, empirical and hybrid models

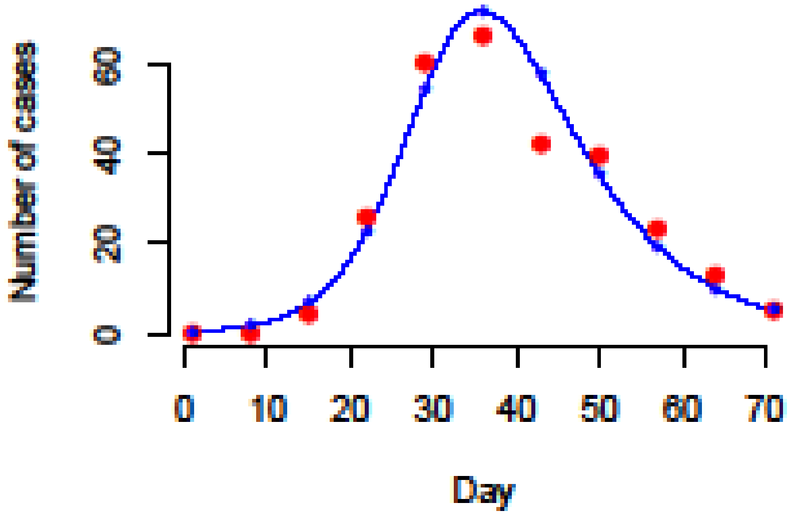
Avogadro constant determined by XRCD: $N_\mathrm{A} = \frac{8V}{a_0^3}\frac{M}{m}$

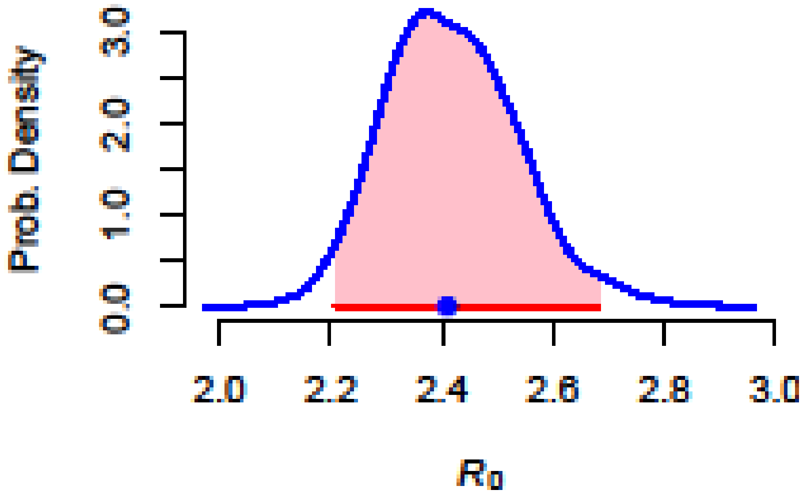
Callendar-Van Dusen eq. for PRTs: $R(t) = R_0(1 + At + Bt^2)$ $\quad 0\ °\mathrm{C} \leq t \leq 800\ °\mathrm{C}$

Spectral irradiance of a lamp

Differential equations

SIR models for influenza viruses (**S**usceptible, **I**nfectious, **R**ecovered)





Choosing the form of the measurement model

The same measurement principle can give rise to different models. Therefore, the experimenter is often faced with the need to choose a suitable form for the model from various possible forms.

Various situations are discussed and examples given (change of coordinates, change of variables, different representation of polynomial functions…)

Great attention is given to the potential loss of numerical accuracy due to using formulae or algorithms that are not suitable to the task

Re-parametrisation

Re-parametrization of a measurement model is the use of a different set of parameters that depend in some manner on the original parameters. For unchanged values of the input quantities the values of the output quantities are unchanged following re-parametrization.

However:

Impact on uncertainties

In general, it is not recommended to perform uncertainty calculations using logarithmic quantities in, for example, dB. Logarithmic quantities should be converted to the equivalent linear format before performing uncertainty calculations.

For uncertainty propagation, not all algebraically equivalent models behave necessarily in the same way because they are not equivalent statistically. An algebraic transformation should be accompanied by a corresponding statistical transformation to ensure the integrity of the results. An example is given.

Multi-stage models

- Directly connected with the concepts of dissemination and traceability
- Large number of cases in metrology
- Key case of calibration
- Often, correlations « propagating » through the various stages
- Example given

Extending the basic model

The basic model describing the measurement principle holds in ideal conditions. That model should usually be extended to cover effects arising in the practical implementation of the measurement.

Well-understood effects: corrections or correction factors included in the basic model
Poorly understood effects: values typically unknown, contribution to the uncertainty often determined using top-down techniques and in general prior knowledge
Shared effects: induce correlations, multi-stage models often used
Time-dependent effects: drift
Examples given

Statistical models

Calibration and analysis (errors-in-variables regression)
Homogeneity studies (ANOVA, Snedecor-Fisher…)
Adjustment of redundant observations (least squares…)
Time series (ARIMA)
Bayesian models (Bayes-Laplace rule, MCMC)
Model selection, model uncertainty and model averaging (AIC, BIC, QQ Plots…)

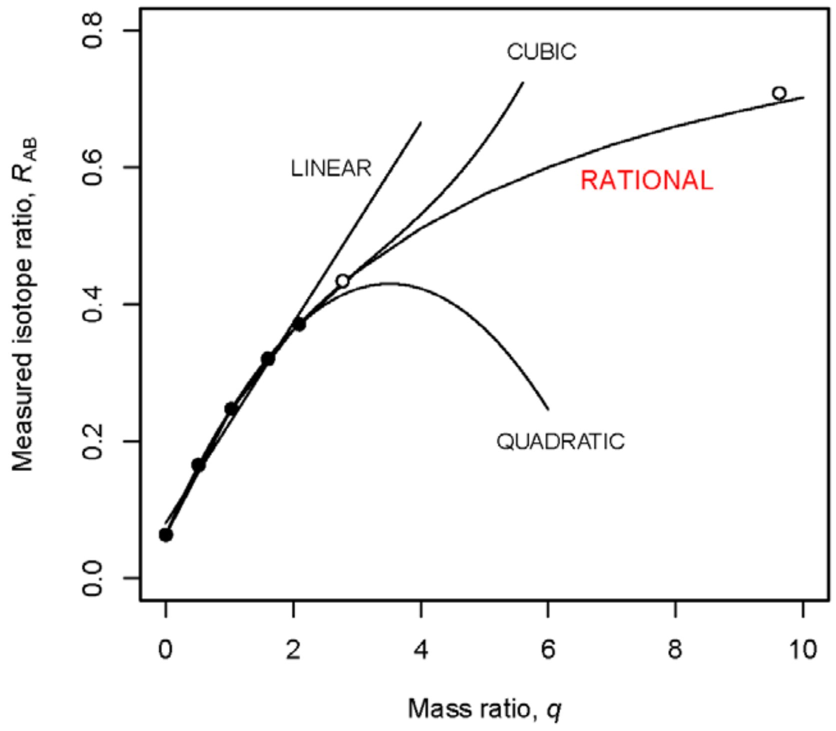
Assessing the model accuracy

Inter-laboratory comparisons
Reference data
Thorough inspection
Scientific literature

Using the model

Care in:
using the Gauss formula for highly nonlinear models, or highly asimmetric distributions, or large input uncertainties
Using a model beyond the range for which it has been validated (extrapolation)

Using the model



Isotope dilution theoretical (rational) model and empirical models fitted to calibration data and used for prediction

Annexes

- Glossary
- Dynamic measurements
- Modelling random variation
- Representing polynomials
- Cause-and-effect analysis (fishbone diagram)
- Linearising a model and checking its adequacy

Freely available in electronic (PDF) form from the websites of the BIPM and OIML, and published in paper and PDF forms by ISO under the name "ISO/IEC Guide 98-6:2021".