Case study on implementing GUM to streamflow measurements acquired with Acoustic-Doppler Velocimeters

Marian Muste¹, Jongmin Kim², Dongsu Kim³

¹ IIHR-Hydroscience & Engineering, The University of Iowa, Iowa City, USA
² Korea Institute of Civil Engineering and Building Technology’s River Experiment Center, Andong, South Korea
³ Dankook University, Yongin, Gyeonggi, South Korea

Joint Workshop of JCGM-WG1 and WMO-ET-MU on Measurement uncertainty in meteorology and climatology
April 5-6, 2022
To illustrate implementation of GUM-based Uncertainty Analysis to a common hydrometric measurement

- Essentials of GUM framework
- Case Study
  - Methods & instrumentation
  - Study site & experimental conditions
  - Measurement protocols
  - Estimation of elemental uncertainty sources
  - Estimation of the expanded uncertainty in streamflow measurement
- Discussions
- Conclusions
GUM Essentials

GUM Implementation protocol
(JCGM100/2008)

Step 1
Step 2
Step 3
Step 4
Step 5

Measurement functional relationship

Observations using the measurement systems

Elemental & correlated error sources

Measurement result \(y\), combined \(u_e\) & expanded \(U\) uncertainties

Result estimate

\[ Y = f(X_1, X_2, \ldots, X_N) \]

\[ y = f(x_1, x_2, \ldots, x_N) \]

\[ u(x_i, x_j) \]

\[ u_e^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \]

\[ U = k u_e(y) \]

\[ Y = y \pm U \]
Velocity-area (VA) method (HUG, 2020)

\[
Q = Q_m + Q_p = F_y F_z \sum_{j=1,n}^{j} u_{jk} \Delta y_j \Delta z_{j,k} + Q_{top} + Q_{bottom} + Q_{edges}
\]
Case study: Instrumentation (main)

Acoustic Point Velocity Meter: 16 MHz MicroADV (www.sontek.com)

3-component point velocities \((u, v, w)\)

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} = G_3^{-1} \begin{bmatrix}
  f_{D1} \\
  f_{D2} \\
  f_{D3}
\end{bmatrix}
\]

- \(G_3\) - geometrical transformation matrix;
- \(c\) - speed of sound in water;
- \(f_D\) - difference in the frequency of emitted \((f_0)\) and return \((f_B)\) pulses due to Doppler shift.

<table>
<thead>
<tr>
<th>Micro-ADV Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling Rate</td>
</tr>
<tr>
<td>Sampling Volume</td>
</tr>
<tr>
<td>Distance to Sampling Volume</td>
</tr>
<tr>
<td>Resolution</td>
</tr>
<tr>
<td>Velocity Range</td>
</tr>
<tr>
<td>Resonance Pressure Transducer (RPT) - Accuracy</td>
</tr>
<tr>
<td>Compass/Tilt Sensor — Heading Accuracy</td>
</tr>
<tr>
<td>Compass/Tilt Sensor — Pitch, Roll Accuracy</td>
</tr>
<tr>
<td>Pressure Sensor Strain Gauge - Accuracy</td>
</tr>
<tr>
<td>Overall Accuracy</td>
</tr>
</tbody>
</table>
Case Study: Study site

Civil Engineering and Building Technology’s River Experiment Center (KICT-REC), Andong, Korea
(https://www.kict.re.kr/menu.es?mid=a20302030000)

Test cross section & experimental conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge</td>
<td>2.09 m³/s</td>
</tr>
<tr>
<td>Channel width</td>
<td>6.5 m</td>
</tr>
<tr>
<td>Averaged velocity</td>
<td>0.56 m</td>
</tr>
<tr>
<td>Maximum velocity</td>
<td>0.89 m</td>
</tr>
<tr>
<td>Averaged depth</td>
<td>0.61 m</td>
</tr>
<tr>
<td>Maximum depth</td>
<td>0.82 m</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>10.72</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>308,209</td>
</tr>
<tr>
<td>Froude number</td>
<td>0.23</td>
</tr>
</tbody>
</table>
Case study: Measurement protocol & Steps 1-2 of GUM

Mid-Section VA method (102 point measurements)

\[ Q_t = U_1 \times d_1 \times \left( \frac{b_2-b_1}{2} \right) + \sum_{j=2}^{22} \left( U_j \times d_j \times \left( \frac{b_{j+1}-b_{j-1}}{2} \right) \right) + U_{23} \times d_{23} \times \left( \frac{b_{23}-b_{22}}{2} \right) + Q_{Re} + Q_{Le} \]  
Eq. (1)
Case Study: Reference for UA (benchmark dataset)

KICT-REC Andong offers ideal conditions for obtaining a high-quality reference (i.e., natural stream flow & boundary roughness, calibrated data acquisition equipment, trained operators, and controlled, stable, and repeatable experiments)

Cross-section distributions: mean streamwise velocity and normalized streamwise turbulence intensity
Case Study: Elemental uncertainty source

The elemental sources of uncertainty are grouped around the variables in the data reduction Equation (1):
- mean depth-averaged velocity, $U_j$
- depth of the verticals $d_j$
- distance between verticals, $(b_{j+1} - b_{j-1})/2$
- model for discharge estimation (includes measured & unmeasured areas), $Q_{MO}$

Estimates for all elemental uncertainty sources (17) are needed:
- 12 Type A (determined from own measurements & judgements)
- 5 Type B (other information sources)

Notes:
- 9 uncertainty sources Type A directly estimated in KICT-REC
- No uncertainty estimates for “Correlated bias errors” available
- Given the ideal measurement conditions for the KICT-REC facility, the “Operational conditions” sources are assumed negligible.

### Table: Elemental uncertainty

<table>
<thead>
<tr>
<th>Elemental uncertainty</th>
<th>ID</th>
<th>Type</th>
<th>Standard uncertainty*</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Mean velocity in verticals, $u(U_j)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Instrument accuracy</td>
<td>$u(U_{ac})$</td>
<td>B</td>
<td>1.0 %</td>
</tr>
<tr>
<td>Sampling duration</td>
<td>$u(U_{sd})$</td>
<td>A</td>
<td>0.7 %</td>
</tr>
<tr>
<td>Vertical sampling model</td>
<td>$u(U_{vd})$</td>
<td>A</td>
<td>1.6 %</td>
</tr>
<tr>
<td>Vertical velocity model</td>
<td>$u(U_{vm})$</td>
<td>A</td>
<td>0.3 %</td>
</tr>
<tr>
<td>Correlated bias errors</td>
<td>$r(U_j, U_{j+1})$</td>
<td>-</td>
<td>N/A</td>
</tr>
<tr>
<td>Operational conditions</td>
<td>$u(U_{op})$</td>
<td>A</td>
<td>Negligible</td>
</tr>
</tbody>
</table>

| ii. Depth in verticals, $u(d_j)$      |      |      |                       |
| Instrument accuracy                    | $u(d_{ac})$ | B | 0.0005 m              |
| Correlated bias errors                | $r(d_j, d_{j+1})$ | - | N/A                   |
| Operational conditions                | $u(d_{op})$ | A | Negligible            |

| iii. Distance between verticals, $u(b_j)$ |      |      |                       |
| Instrument accuracy                    | $u(b_{ac})$ | B | 0.0005 m              |
| Correlated bias errors                | $r(b_j, b_{j+1})$ | - | N/A                   |
| Operational conditions                | $u(b_{op})$ | A | Negligible            |

| iv. Discharge model, $u(Q_{MO})$       |      |      |                       |
| Discharge model                        | $u(Q_{mo})$ | B | 0.5%                  |
| Number of verticals                    | $u(Q_{nv})$ | A | 1.5%                  |
| Edge discharge model                   | $u(Q_{ea})$ | A | 1.7%                  |
| Near-bed effects                       | $u(U_{bd})$ | A | 2.1%                  |
| Operational conditions                 | $u(Q_{op})$ | B | Negligible            |
Case Study: Step 3 of GUM

Step 3 of GUM is the most complex and expensive aspect of GUM implementation

Sample of Type A uncertainty estimation: Velocity model for obtaining the depth average velocity, $u(U_{vm})$

All verticals & various distribution models for velocity profiles

<table>
<thead>
<tr>
<th></th>
<th>Log law</th>
<th>Power law</th>
<th>1/6 Power law</th>
<th>10pt. Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge (m$^3$/s)</td>
<td>2.050</td>
<td>2.128</td>
<td>2.044</td>
<td>2.024</td>
</tr>
<tr>
<td>Discharge difference (m$^3$/s)</td>
<td>-</td>
<td>0.078</td>
<td>-0.005</td>
<td>-0.026</td>
</tr>
<tr>
<td>Uncertainty [%]</td>
<td>-</td>
<td>3.82</td>
<td>-0.26</td>
<td>-1.25</td>
</tr>
</tbody>
</table>

Case Study: Steps 4-5 GUM

Propagation of elemental uncertainties to the final result (combined standard uncertainty)

\[ u_c(Q_t) = \sum_{j=1}^{23} u(U_j)^2 \left( \frac{\partial Q_t}{\partial U_j} \right)^2 + \sum_{j=1}^{23} u(d_j)^2 \left( \frac{\partial Q_t}{\partial d_j} \right)^2 + \sum_{j=1}^{23} u(b_j)^2 \left( \frac{\partial Q_t}{\partial b_j} \right)^2 + 2 \sum_{j=1}^{22} \sum_{j+1}^{23} \left( \frac{\partial q_{n,j}}{\partial U_j} \right) \left( \frac{\partial q_{n,j+1}}{\partial U_{j+1}} \right) u(U_j)u(U_{j+1}) \left( \frac{\partial U_j}{\partial U_{j+1}} \right) + \] 

\[ +2 \sum_{j=1}^{22} \sum_{j+1}^{23} \left( \frac{\partial q_{n,j}}{\partial d_j} \right) \left( \frac{\partial q_{n,j+1}}{\partial d_{j+1}} \right) u(d_j)u(d_{j+1}) \left( \frac{\partial d_j}{\partial d_{j+1}} \right) + 2 \sum_{j=1}^{22} \sum_{j+1}^{23} \left( \frac{\partial q_{n,j}}{\partial b_j} \right) \left( \frac{\partial q_{n,j+1}}{\partial b_{j+1}} \right) u(b_j)u(b_{j+1}) \left( \frac{\partial b_j}{\partial b_{j+1}} \right) \] 

\[ u(U_j) = \sqrt{u(U_{ac})^2 + u(U_{sa})^2 + u(U_{va})^2 + u(U_{vm})^2 + u(U_{op})^2} \] 

\[ u(d_j) = \sqrt{u(d_{ac})^2 + (d_{op})^2} \] 

\[ u(b_j) = \sqrt{u(b_{ac})^2 + u(b_{op})^2} \] 

\[ u(Q_{MO}) = \sqrt{u(Q_{mo})^2 + u(Q_{nv})^2 + u(Q_{eg})^2 + u(U_{bd})^2 + u(U_{op})^2} \] 

The expanded uncertainty:

\[ U(Q_t) = k u_c(Q_t) \] 95% confidence level

Calculations executed with QMSys GUM software (equipped with an interface for hydrometry developed through WMO funding)
Case Study: GUM execution

QMSys interface for user input: measurement definition and elemental uncertainty sources (Steps 1-2 GUM)

\[
\text{index } n = (1:23)
\]

\[
Q_p[n] = v[n] \cdot d[n] \cdot (b[n+1] - b[n-1])/2
\]

\[
Q_m = \text{SUM}(Q_p[n])
\]

\[
v[n] = v_m[n] + v_{\text{ac}}[n] + v_{\text{sd}}[n] + v_{\text{sd}}[n] + v_{\text{vm}}[n] + v_{\text{op}}[n]
\]

\[
d[n] = d_m[n] + d_{\text{ac}}[n] + d_{\text{op}}[n]
\]

\[
b[n] = b_m[n] + b_{\text{ac}}[n] + b_{\text{op}}[n]
\]

\[
Q_{\text{Le}} = 0
\]

\[
Q_{\text{Re}} = 0
\]

\[
Q_t = Q_m + Q_{\text{bo}} + Q_{\text{bd}} + Q_{\text{nv}} + Q_{\text{eg}} + U_{\text{bd}} + Q_{\text{op}} + Q_{\text{Re}} + Q_{\text{Le}}
\]
Case Study: GUM execution

QMSys interface for user input: values for elemental uncertainty sources (Step 3 GUM)
Case Study: GUM execution

QMSys output: total uncertainty in discharge using (Step 4 GUM) obtained with:
- GUM framework (*GUF in QMSys)
- Monte Carlo method

<table>
<thead>
<tr>
<th>Assessment Method</th>
<th>Number of trials (M)</th>
<th>Estimated mean ($Q_w$) (m$^3$/s)</th>
<th>Combined Standard Uncertainty ($u_c(Q_w)$) (m$^3$/s)</th>
<th>Expanded standard uncertainty (at 95% confidence interval)</th>
<th>$u_{exp}$</th>
<th>$u_{exp}$</th>
<th>Numerical tolerance $\delta$</th>
<th>GUF** Validated?</th>
</tr>
</thead>
<tbody>
<tr>
<td>*GUF</td>
<td>n/a</td>
<td>2.048</td>
<td>± 0.054</td>
<td>± 0.109</td>
<td>± 5.30</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>MCM</td>
<td>10°</td>
<td>2.048</td>
<td>± 0.054</td>
<td>± 0.109</td>
<td>± 5.30</td>
<td>1.939</td>
<td>2.157</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:
- The relatively low values for the total discharge uncertainty are indicative for the favorable measurement environment, instrumentation, and measurement protocols used in the KICT-REC case study.
- It can be stated that the uncertainties in discharge provided by this study represent the minimum expected uncertainty (baseline) compared with other measurement sites.
Case Study: GUM execution

QMSys output: uncertainty budget

Elemental uncertainty sources

- Near-bed effects for Q: 61.79%
- Discharge model for Q: 31.52%
- Number of verticals for Q: 3.5%
- Vertical sampling model for v: 1.93%
- Instrument accuracy for v: 0.75%
- Sampling duration for v: 0.35%
- Edge discharge model for Q: 0.09%
- Vertical velocity model for v: 0.03%
- Instrument accuracy for d: 0%
- Instrument accuracy for b: 0%
Discussions: GUM in Hydrometry

Unsettled Issues

• Is the use of relative uncertainties lumped as in ISO 748 for estimation of the total uncertainty equivalent to the GUM propagation using absolute values? Use of relative uncertainties eliminates the sensitivity coefficients prescribed in GUM, preclude probability distributions other than normal distribution for estimation of the uncertainty sources (e.g., ISO 1088)

• How to determine the “difficult-to-estimate” uncertainties? Often time these sources dominate the uncertainty budget.

Challenges

• The case specific vs. generalized UA requires conceptualization for uniform application

• Alternative approaches (more or less conform with GUM framework) continue to be used. The various approaches provide quite different uncertainties. How do we mediate the differences?

• Given the multiple approaches used for conducting UA in hydrosience, specialized institutions (such as WMO, ISO) have to agree and prescribe convergent, interoperable methodologies

• Overall there is still a considerable resistance to UA adoption (hydrometry is a special case as the measurement environment is complex)
Conclusions: Lessons learned from GUM implementation

• **UA protocols converge toward common ground** (compared with 50 years ago); e.g., JCGM works toward unifying and grouping standards (GUM-based) rather than expanding them (the ISO approach)

• **WMO proposed the adoption of the GUM (1993)** for UA for measurements and modeling of hydrologic processes (WMO Report No. 1097, 2017)

• **GUM framework implementation is doable** (irrespective of instrument and measurement protocol)

• **It is possible to automate the laborious UA calculations with generic software** (e.g., QMSys)

• **Uncertainty analysis brings along a suite of benefits***
  • Provide confidence that the measured data can stand scientific and legal scrutinies
  • Minimize the measurement cost for a given output accuracy
  • Improve the measurement process
  • Inform field operators on optimal measurement strategy for a specific site

Questions?