Redundancy in TW network and its combination with carrier phase data

- Towards a Multi-Technique-Network Time Transfer

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All the TW laboratories for the data

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13th CCTF WG TW Meeting
15-16 Nov. 2005, VSL, Delft Netherlands

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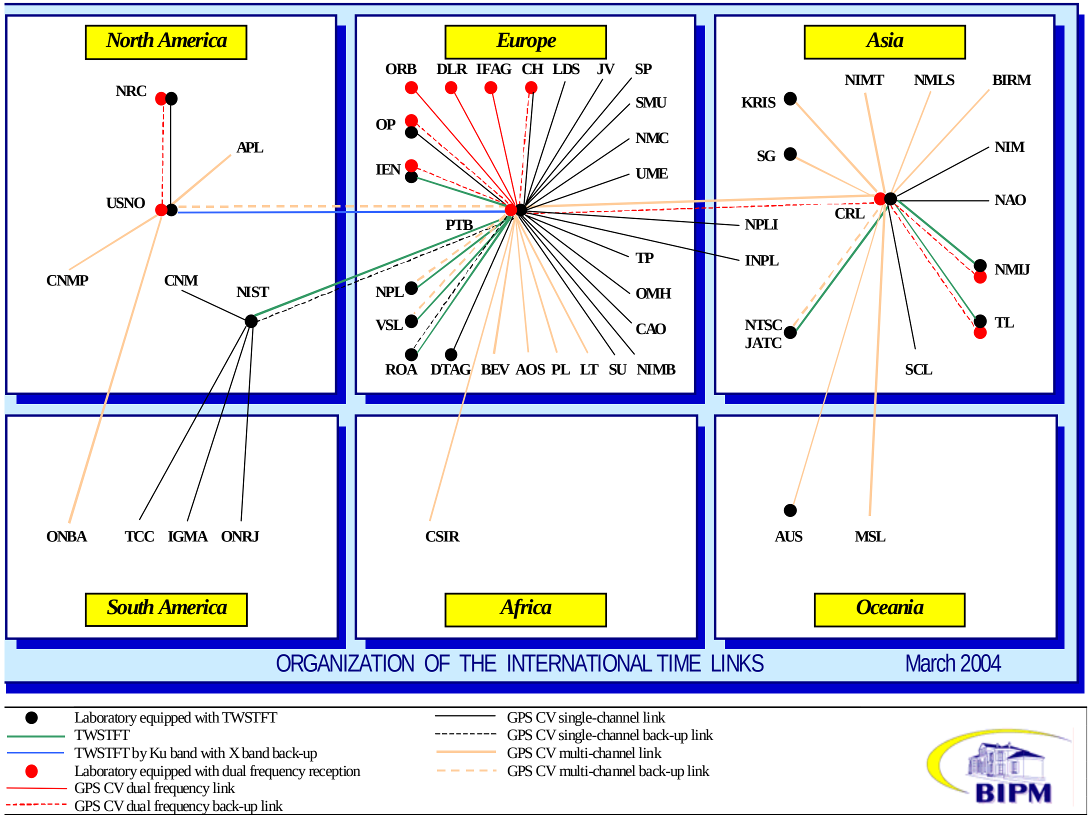
Summary

- Background
- Redundancy in the TAI TW network
- Measurement Errors issued by triangle closures
- Theory study: a combined adjustment model for
  - a pure TW network
  - combining TW and Carrier phase data
- Numerical tests



56 TAI Labs and 55 official *single baseline* links for the TAI generation

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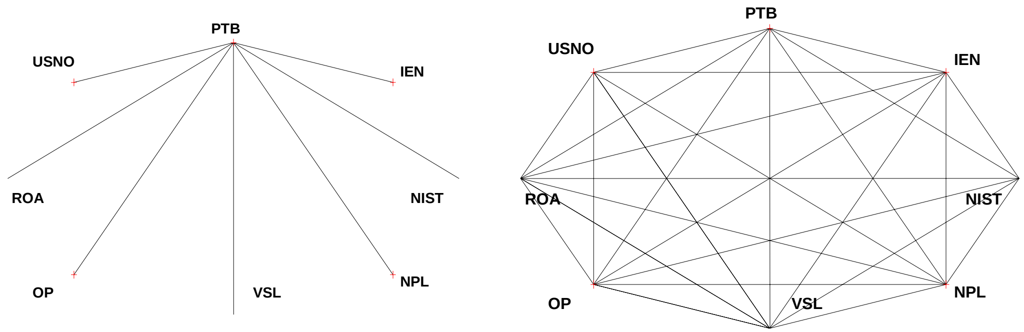
Background

- 55 TAI links with 300% redundancy of TW, GPS codes and phase …+GLN, + …
- General think: To combine the redundancy by different techniques based on the variance-covariance estimations (Petit and Jiang, FCS/PTTI 2005)
- First efforts: TW redundancy itself and combination with carrier phase information
- TW: 15% of total links but 80% total clocks

Redundancy in a TW network

In geometry: Measured as a network, used as the single baselines



In density: 2 points per 5 days used vs. more than 60-120 points measured

- Europe-America : 12 points/day (since Oct. 05)
- Asia-Pacific : 24 points/day (since Nov. 05)

Geometric redundancy in Euro-American TW network

	Lab. Number	Total Links	Necessary Links (Used for TAI)	Redundant Links (Not used for TAI)	Independent Triangles
	3	3	2	**1**	1
	7	21	6	**15**	15
0502	8	28	7	**21**	21
Available	**12**	**66**	**11**	**55**	**55**
	20	190	19	**171**	171

Total links = $N(N-1)/2$

Total independent triangles = $(N^2-3N+2)/2$

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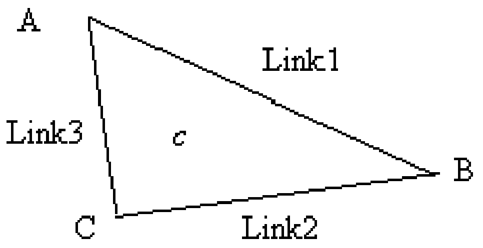
Triangle Closure *c* and baseline dependant errors *v*



<u>Closure *c* is the true error and exactly measurable:</u>

c = Link1+Link2-Link3

= [UTC(B)-UTC(A) +$v1$] - [UTC(B)-UTC(C) +$v2$] - [UTC(C)-UTC(A) +$v3$]

$c = v1 + v2 + v3$

<u>Baseline dependant error *e* or *v* are estimable</u>:

the b.d.e. : e = $RMS(v) \sim RMS(c)/\sqrt{3}$

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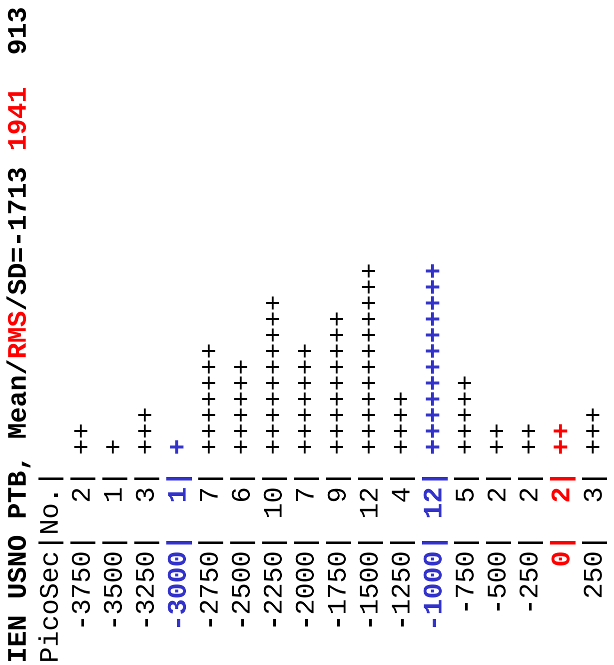
The Closures *c* and the *b.d.e.* $e \sim RMS(c)/\sqrt{3}$

RMS of closures 0502

0.0 ~ 0.5 ns : 4
0.5 ~ 1.0 ns : 14
1.0 ~ 1.5 ns : 4
1.5 ~ 2.0 ns : 1
> 2.0 ns : 1
Total 24-1: *e* ~ 0.5ns

IEN-USNO-PTB: RMS= 1.94 ns

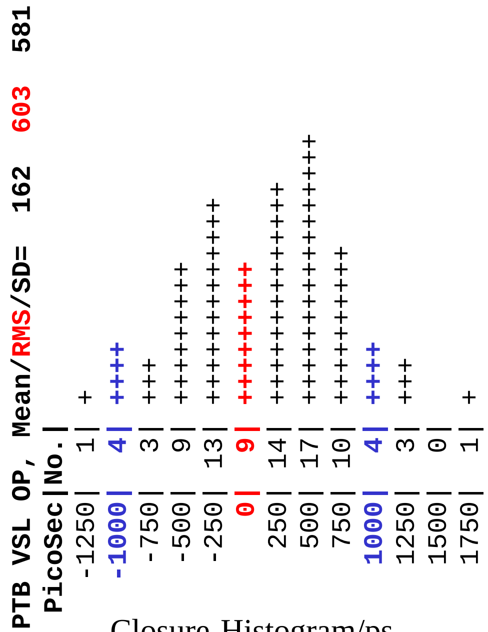
$e \sim$ 1.1ns



Closure-Histogram/ps

PTB-VSL-OP: RMS= 0.63 ns

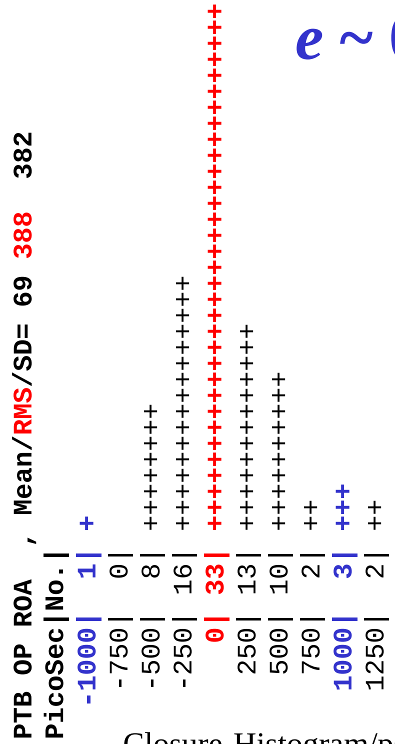
$e \sim$ 0.4ns



Closure-Histogram/ps

PTB-OP-ROA: RMS= 0.39 ns

$e \sim$ 0.2ns



Closure-Histogram/ps

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Closure condition and least square condition

- ➔ Baseline dependant errors vary: ***e ~ 0.2 - 1.1 ns***
- ➔ *Is it possible to use the "good" baselines to improve the "poor" ones ?* **YES !**

- Closure condition: ***Closure = v1 + v2 + v3 = 0***
- Least square condition: $\sum W_i \times v_i^2 = Minimun$

➔The good ones contribute more than their gains

➔The poor ones gain more than their contributions

The *Network* adjustment

- least square unequal-weight indirect parameter adjustment

- Function and random models :

$$\hat{L} = B\hat{X} + d$$
$$D = \sigma_0^2 Q = \sigma_0^2 P^{-1}$$

- Correction (*b.d.e.*) equation:

$$V = B\hat{x} - l$$

$$l = L - L^0 = L - (BX^0 + d)$$

- Weight and Normal equation:

$$P_i = \frac{\sigma_0^2}{M_i^2} \qquad B^T PB\hat{x} - B^T Pl = 0$$

- Adjusted unknown corrections:

$$\hat{x} = (B^T PB)^{-1} B^T Pl = N_{BB}^{-1} W$$

- Adjusted observations and unknowns:

$$\hat{L} = L + V$$
$$\hat{X} = X^0 + \hat{x}$$

- Unit mean square error:

$$\hat{\sigma}_0 = \sqrt{\frac{V^T PV}{r}} = \sqrt{\frac{V^T PV}{n-t}}$$

- Variance-covariance of the unknowns:

$$D_{\hat{X}\hat{X}} = \sigma_0^2 Q_{\hat{X}\hat{X}} = \hat{\sigma}_0^2 N_{BB}^{-1}$$

- Function of unknowns and its variance:

$$d\varphi = F^T \hat{x}$$

$$D_{\hat{\varphi}\hat{\varphi}} = F^T Q_{\hat{X}\hat{X}} F = F^T N_{BB}^{-1} F$$

Output of the *Network* adjustment ?

To determine the *baseline dependent error* ***v*** for each epoch of each baseline and the *variance* matrix:

$$V = B\hat{x} - l$$

V = v1,v2, … vN = *Correction vector* (the *b.d.e*. ***v***)

l = *measurement vector*

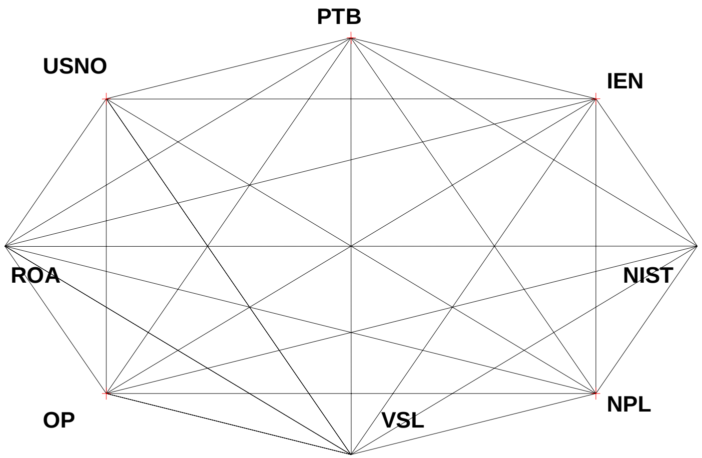
x = *parameter vector* (pseudo-clock values by setting a set of clock == *K*)

B = *coefficient matrix*

All these are possible thanks to the

TW network geometric redundancy

Adjustment of TW network 0502



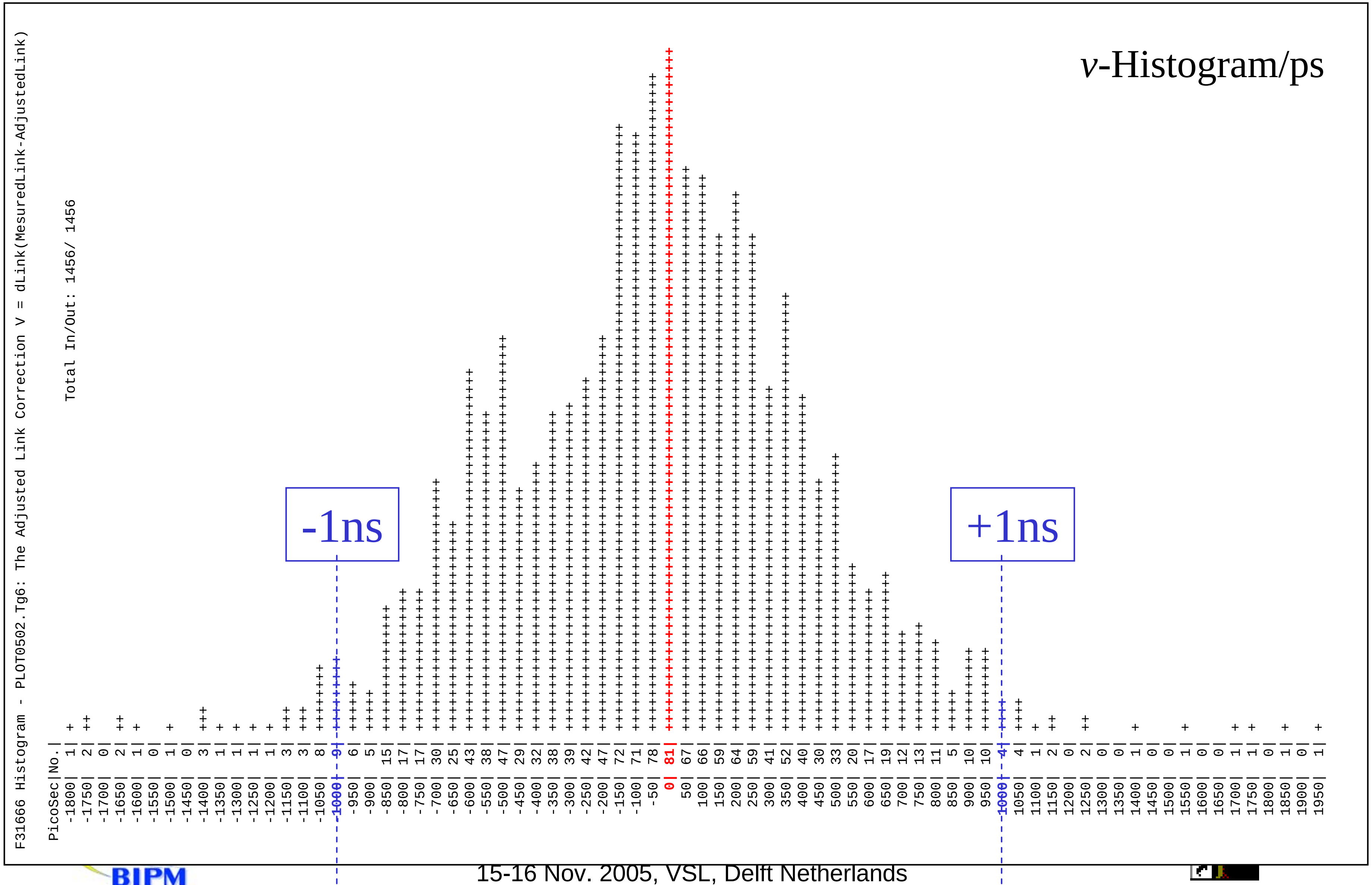
53404-53429

1456 links

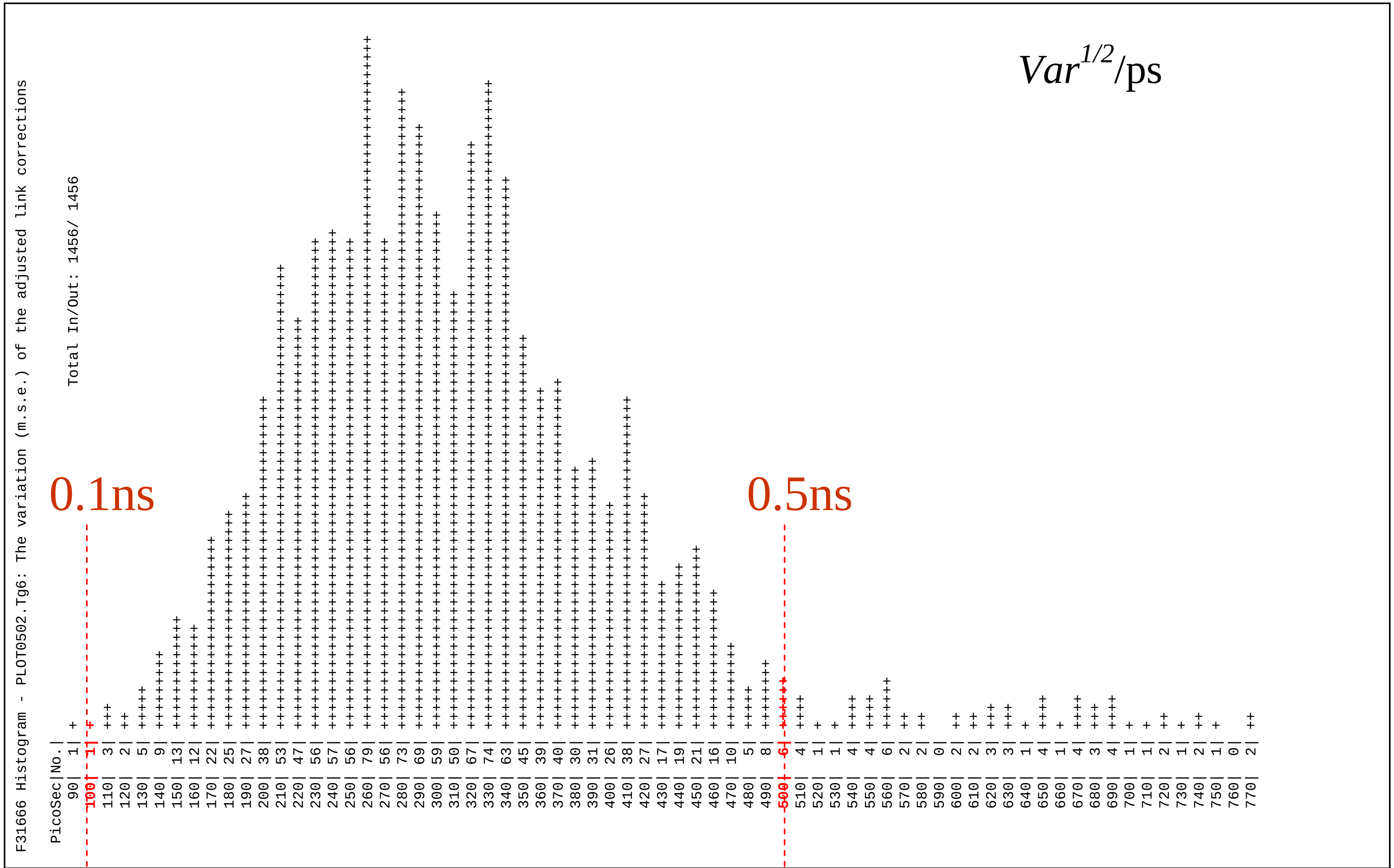
with no IEN, USNO-NIST and some missing data

- Determine the link corrections (*Vij*) and their variances
- *Testing Example* : Simple adjustment without constrain between the measuring epochs ➔ Use Time Deviations to check the improvement in stability of the adjusted result
- *Remark*: Only the link measurement error adjustment: Remove-Restore the CLAR/ESDVAR from the TAI links

The adjusted link correction or *-b.d.e. v*



Variances$^{1/2}$ of the correction *V or -b.d.e.*



Improvement in Time Stability

USNO-NPL

```
N/Mean/RMS= 88 20 362
  -1100|  1| +
  -1000|  0|
   -900|  0|
   -800|  0|
   -700|  0|
   -600|  4| ++++
   -500|  5| +++++
   -400|  3| +++
   -300| 10| ++++++++++
   -200|  5| +++++
   -100| 10| ++++++++++
      0|  4| ++++
    100| 11| +++++++++++
    200|  9| +++++++++
    300|  8| ++++++++
    400|  9| +++++++++
    500|  2| ++
    600|  4| ++++
    700|  3| +++
```

v-Histogram/ps

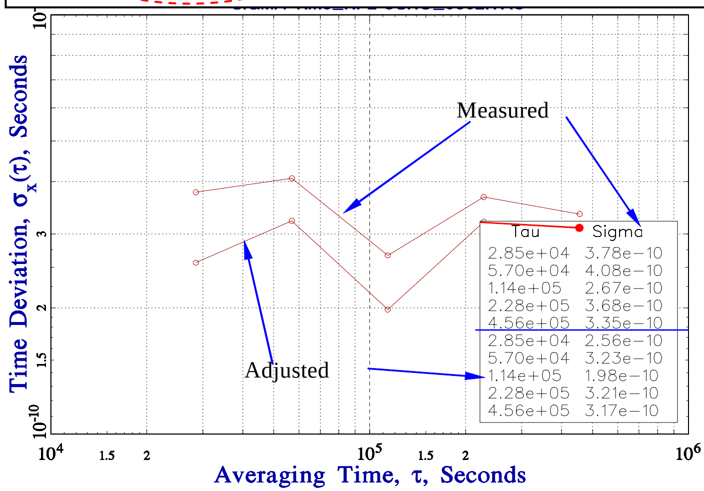
b.d.e. v

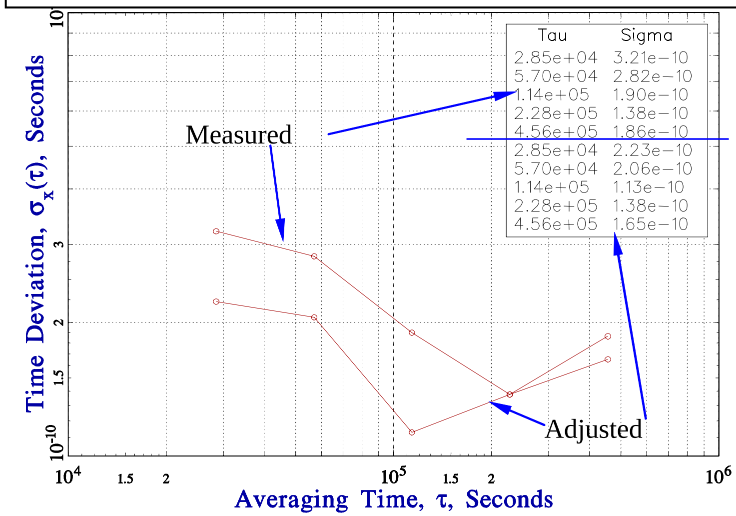
NIST-NPL

```
N/Mean/RMS= 88 28 299
   -600|  2| ++
   -500|  4| ++++
   -400|  4| ++++
   -300|  6| ++++++
   -200|  8| ++++++++
   -100| 13| +++++++++++++
      0| 13| +++++++++++++
    100|  7| +++++++
    200| 12| ++++++++++++
    300|  6| ++++++
    400|  3| +++
    500|  8| ++++++++
    600|  0|
    700|  2| ++
```

v-Histogram/ps

b.d.e. v





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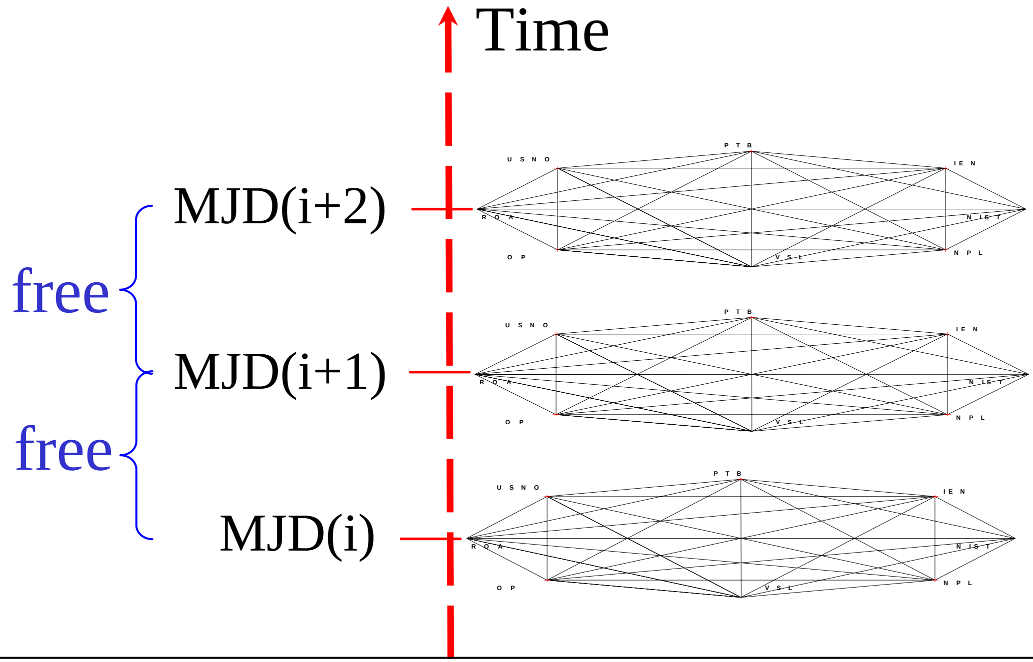
Remark: the *network* adjustment is made independently on each measuring epoch MJD(i)



➔ Time Deviations can be used to estimate the improvement

➔ Can there epoch intervals be measured ? **YES !**

Constrain between the TW epochs by using carrier phase measurement

Basic relation of the *double differences* :

$$\mathbf{TW}_{i+1} - \mathbf{TW}_{i} = \mathbf{CP}_{i+1} - \mathbf{CP}_{i}$$

General : $$\int_{Mjd1}^{Mjd2} f(TW)dt = \int_{Mjd1}^{Mjd2} F(CP)dt$$

- **TW phase not yet available**
- **GPS phase available**

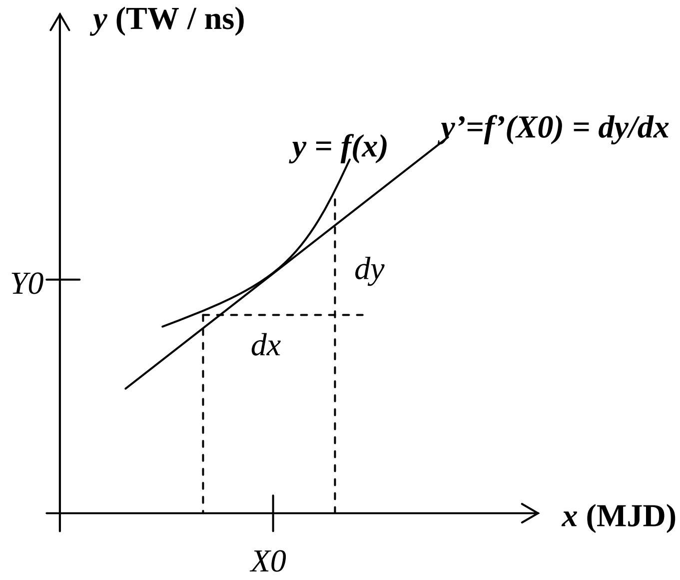
Combination of TW and GPS carrier phase (GCP)

- GPS carrier phases (Dach this meeting)
  - ➔ Closures == 0 (global solutions)
  - ➔ More precise than TWFT (Fountain comp., Bauch et al. 2005)
  - ➔ Improve TWTT
- Several Possibilities (Joint study BIPM-AIUB)
  - By network ➔ … ...
  - By baseline ➔ Combined Smoothing

TW-Phase combination - baseline case

- **The measurements to combine:**
  - TW ➔ Absolute function value
  - Phase ➔ Relative (derivative) value

  ➔ $dY_{TW} = y'_{GCP} * dX$

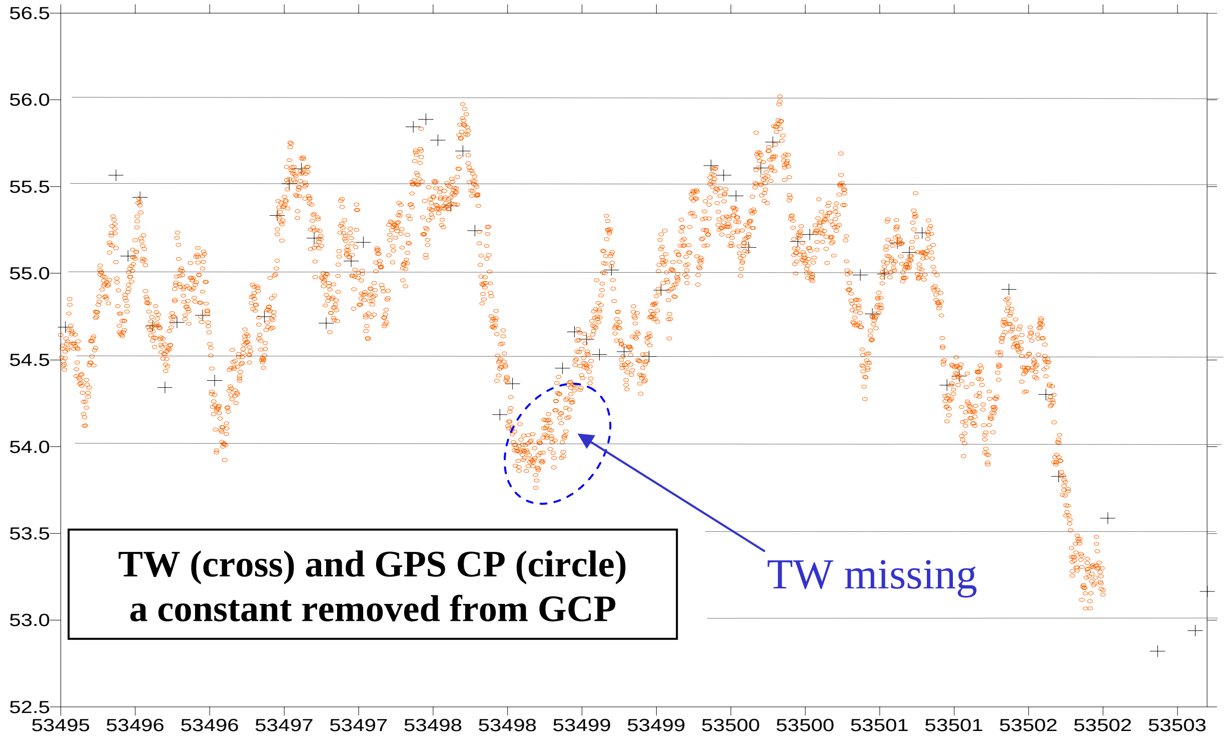


- **With balanced considerations:**
  - Fidelity to the TW values
  - Fidelity to the GCP rate values
  - Smoothness of the combined values

One of the many solutions:

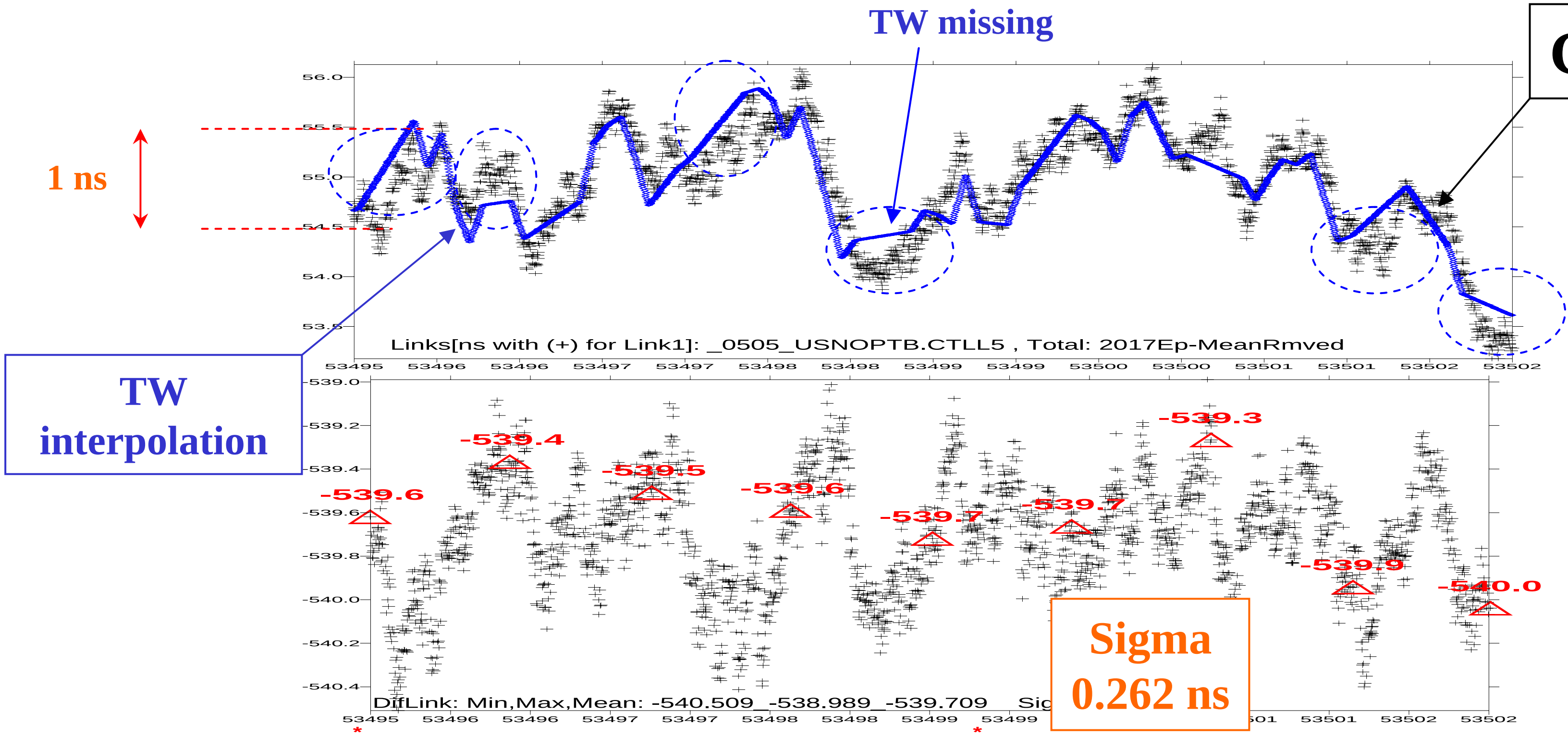
- ➔ The *Vondrak-Cepek* Combined Smoothing

The TW and GPS CP data



USNO-PTB: MJD 53496-53503
TW : 62 points (Av. 8 points/day)
GCP : 2017 points (every 300s)

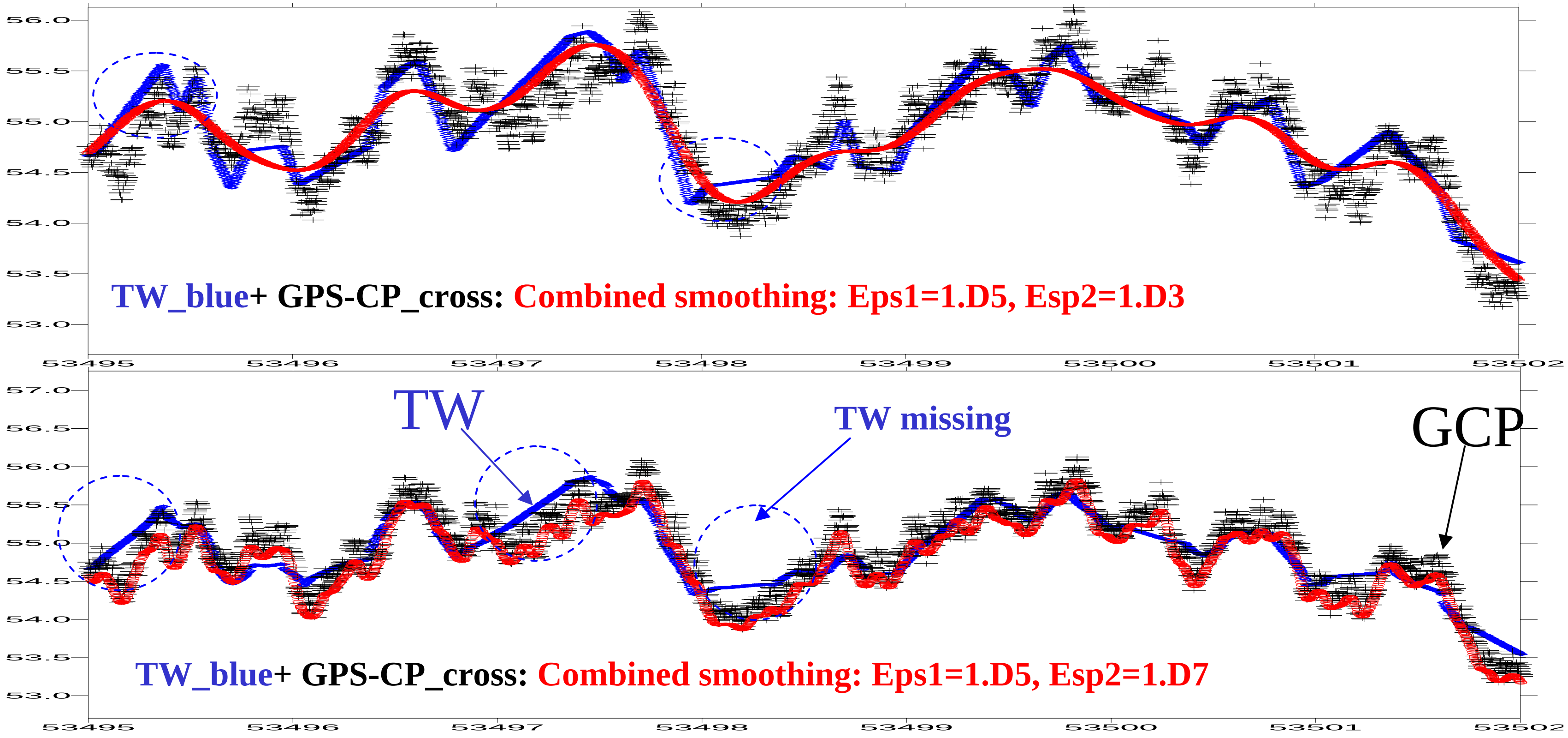
Differences of TW and GPS CP



➔TW or GPC: Noise < 0.2 ns

🡐 TW u*A* = 0.5 ns !!

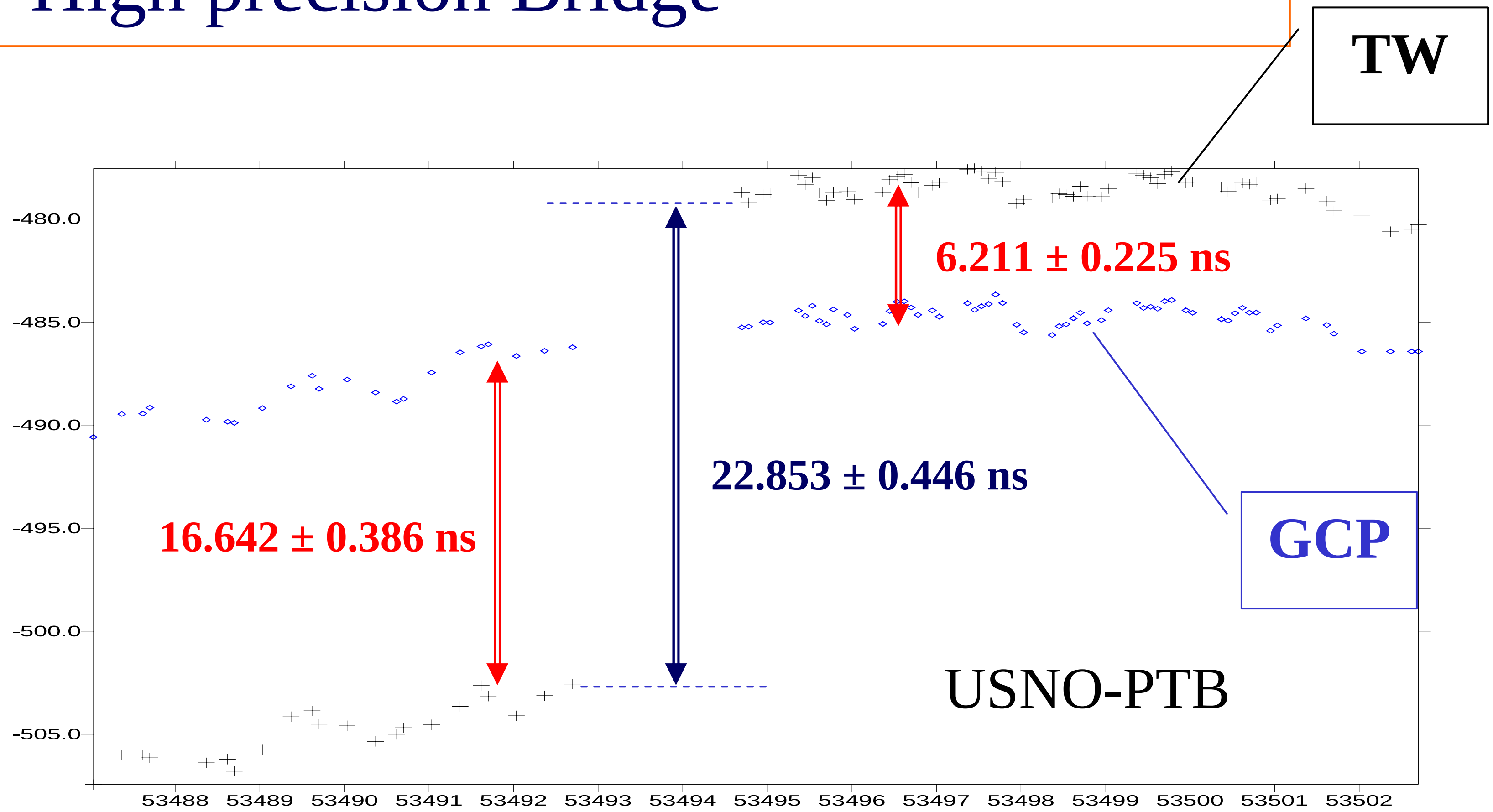
Combination TW and GPS CP



Remark: optimal setting of the smoothing powers Eps1 and Eps2 is being studied

Other Applications of GPS CP (1)

- High precision Bridge



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Other Applications of CP (2)

- High precision interpolation and smoothing

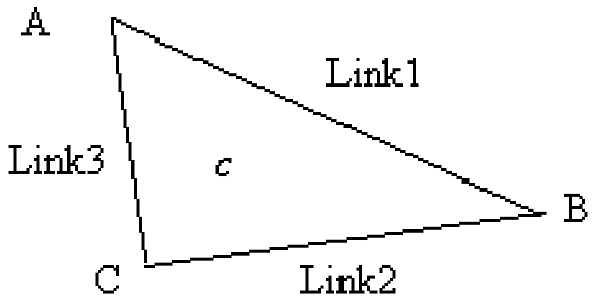
- Next talk

Conclusion

- TW network is highly redundant in geometry and in desity
- The closure analysis and network adjustment show: TW baseline dependent errors vary between about 0.2 ~ 1.1 ns.
- Global *network* adjustment is theoretically serious, fully profits the total redundancy of a network, allowed to combine different techniques and gives variance-covariance information. Obvious improvement in stability and in solution is expected. The disadvantage is that it is practically rather complex
- For a first step, baseline based combination smoothing is easy and can considerably reduce the measurement errors without new more measurements required
- GPS carrier phase information has a prospective potential
- Future works: More robust mathematical models and more effective software
- We are facing an epoch of *multi-technique-network* time transfer

Other Applications of GPS CP (3)

- Alignment/Calibration TW links with P3-CP combination



- Align P3-CP*i* to link*i*, *i=1*, *calibrated*
- Align link*i to* P3-CP*i*, *i=2,3*, *not calibrated*
- GPS P3-CP triangle closures = 0

  ➔ Link2 and Link3 are 'calibrated' with the precision of CP (~ 0.2 ns?)

Future study: Can we align/calibrate all the links in a network by a global adjustment ?