Smoothing and interpolation techniques for a TW measurement series

- in TAI calculation

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Background

- Before 2004: 3 points per week
- Since 2004 : 4 points per day
- Since Oct. 2005 : 12 points per day E-A areas
- Since Nov. 2005: 24 points per day A-P areas
- For TAI at present : simple linear interpolation

  ➔ two nearest points to the standard MJD midnights
- Is it possible to make any improvement by using the redundant points with a non-linear interpolation technique ?
- TW: 15% of total links but 80% total clocks

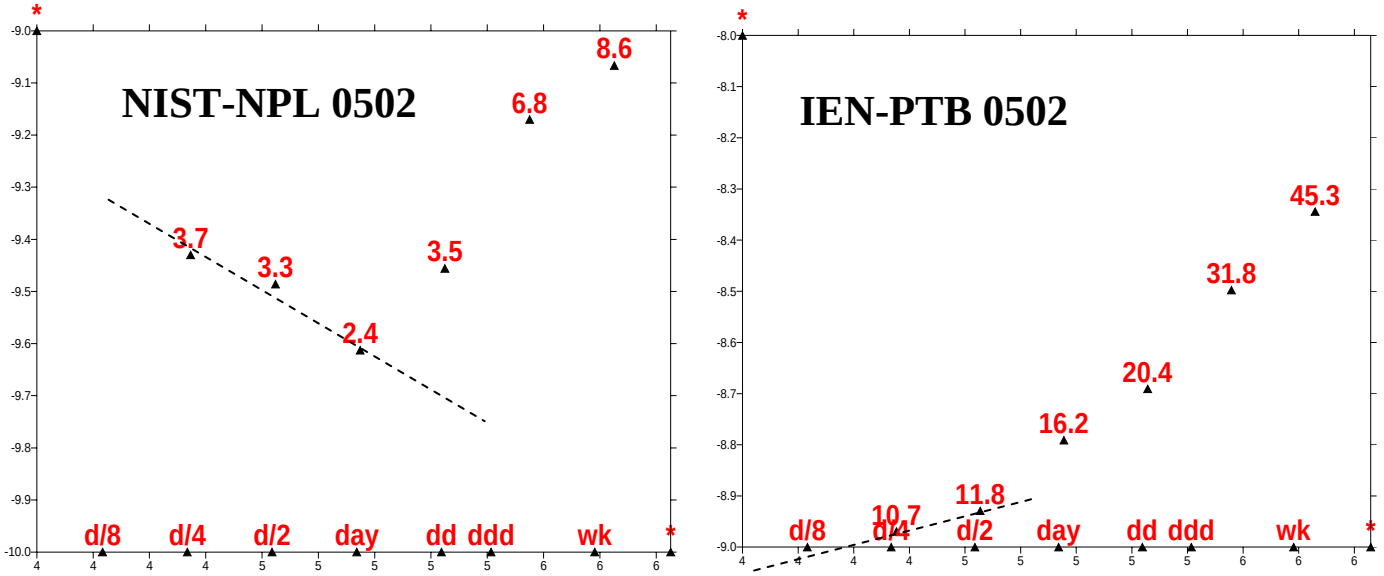
How close the measured points to the Std MJD 53664 midnight (21/Oct/2005) ?

Link	53663	53664	Minutes to 0 H
Lab2-Lab1	hhmmss	hhmmss	mm
IEN -PTB	221400	001400	14
ROA -PTB	221700	001700	17
NPL -VSL	221700	001700	17
OP -PTB	222000	002000	20
AUS -NICT	181730	000230	23
NPL -PTB	222300	002300	23
VSL -PTB	162600	002600	26
SP -PTB	222900	002900	29
USNO-NPL	223800	003800	38
NPL -NIST	224100	004100	41
USNO-PTB	224600	004600	46
NIST-PTB	224900	004900	49
AVERAGE			30mm

➔ Interpolation is necessary

Measurement error vs. Clock stability

- Interpolation method is **MORE** important when **Measurement error > Clock stability**
- Interpolation method is **LESS** important when **Measurement error < Clock stability**



Time Dev., Scale D-10 seconds

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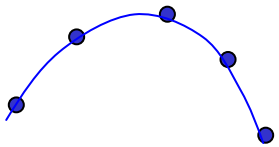
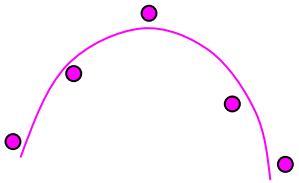
10 Smoothing-Interpolation techniques



- **Linear**
- Rational
- Polynomial
- Splint



- Least square linear fitting
- Least square polynomial fitting
- Least absolute residual fitting
- Weighted mean
- **Vondrak smoothing**
- Vondrak-Cepek TW-GPS CP combined smoothing

TW as function values
GCP as derivatives (1 pts/300s)

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Test data

- MJD 53495 – 53502:

  62 TW points (12 – 24 points/day)

  2017 GPS CP (1 point/300s)

- MJD 53509 – 53517:

  197 TW points (1 point/hour)

- Other TW Data sets for TAI 0505 ~ 0509 (closure comparisons)

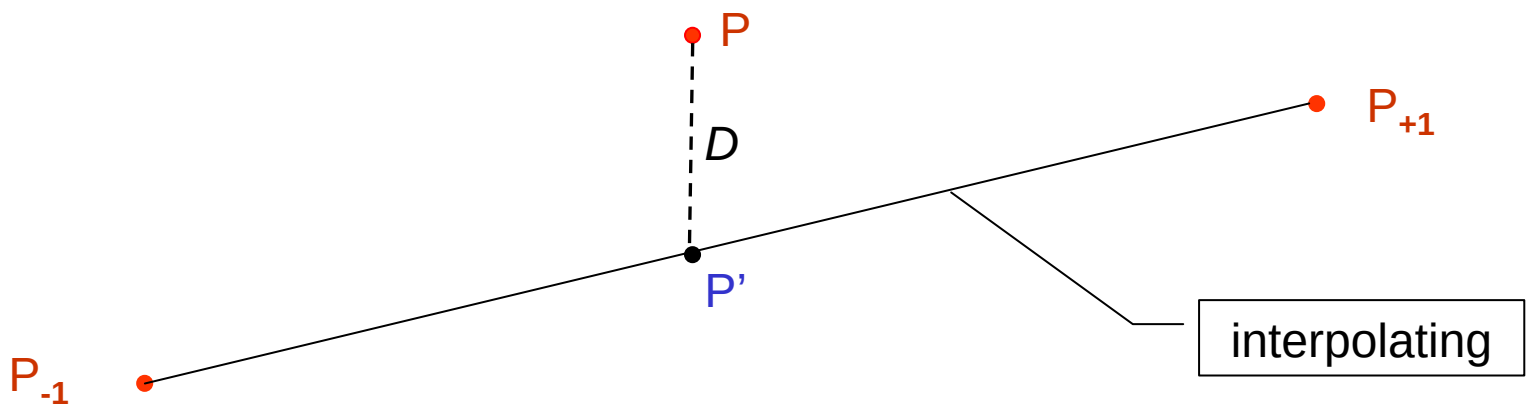
How to check the accuracy of the interpolating techniques

1. Check (1): Compared to the measured points
2. Check (2): Compare the triangle closures
3. Check (3): Compared to the GPS carrier phase data

Compared with measured point : Check(1)a

- example of linear interpolation errors

- Comparison of the Sigma of interpolation error *D* :



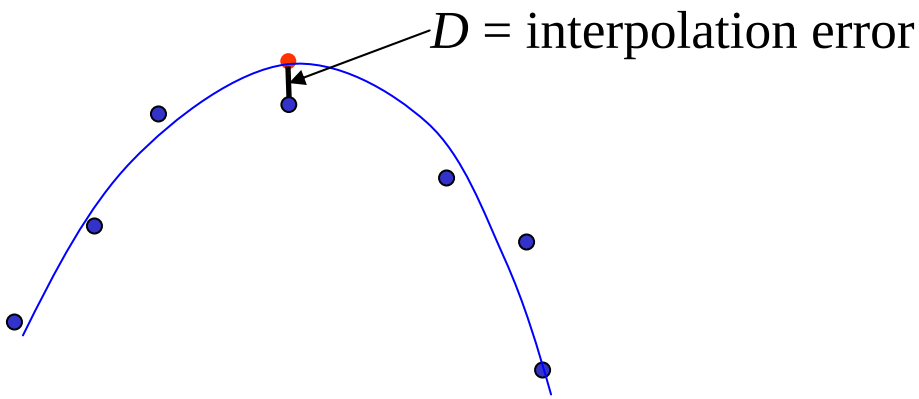
- Given three measured point (P_{-1}, P and P_{+1})
- Interpolate P' using P_{-1} and P_{+1} and compare with the measured point P :

  D = P – P'

Compared with measured point : Check(1)b

- Non linear interpolation errors

Moving window of 7 points



- - measured point
- - interpolated point

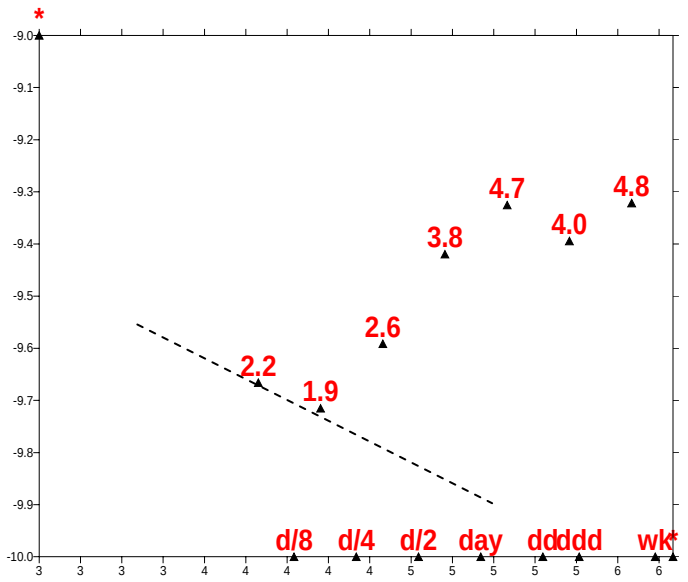
Compared with measured point : Check(1)c

USNO-PTB
53509 ~ 53517

197 TW points

~ 1 point/hour

Time Dev., Scale D-10 seconds



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Comparison of 11 different interpolation techniques with the Check Method (1) cont. USNO-PTB

Tech.	No.	R.M.S	Mean Abs.	Max
Linear	197	0.221	0.176	0.940
Vondrak power 5	197	0.215	0.169	0.940

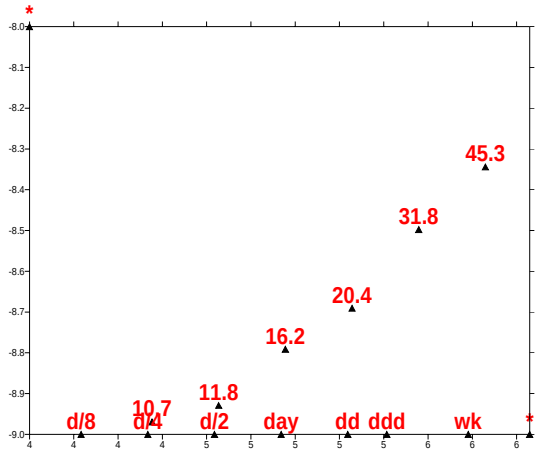
Vondrak vs. Linear: gain 3%
Improved ? Yes, but not considerable

Comparison of different interpolation techniques with the Check Method (1) cont. Other Links (Sept. 05)

Link	Tech.	No.	RMS	Max
IEN-PTB	Linear	128	1.089	2.664
	Vondrak power 5	128	1.078	2.599
NIST-PTB	Linear	133	0.352	0.970
	Vondrak power 5	133	0.356	0.964
ROA-PTB	Linear	131	0.709	2.694
	Vondrak power 5	131	0.701	2.678
USNO-PTB	Linear	133	0.366	1.018
	Vondrak power 5	133	0.363	1.023
VSL-PTB	Linear	126	1.010	2.545
	Vondrak power 5	126	0.999	2.538
OP-PTB	Linear	125	0.587	1.952
	Vondrak power 5	125	0.587	1.934
SP-PTB	Linear	125	0.926	2.721
	Vondrak power 5	125	0.926	2.672

Vondrak vs. Linear: Gain 0%
Improved ? No

Time Dev., Scale D-10 seconds



- **Difference of the interpolation techniques makes sense only when the measurement errors >> than the clock stability**
- **The high order interpolation techniques have no advantages**
- **Something wrong with my computation programs**

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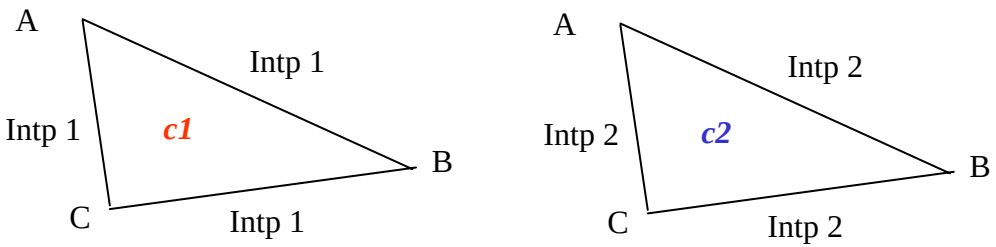
Check method (2)

- Comparison of the triangle closures interpolated on all MJD 0h : ***C1*** ***vs.*** ***C2***



➔ Linear vs. Vondrak

Comparison of different interpolation techniques with the Check Method (2) cont. All Closures (May-June 05)

Linear 0505 53509-53519 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
5 T SP VSL OP	-2.80	9.00	1.14	3.26	3.05/
6 T SP OP ROA	-2.30	2.70	-0.10	1.28	1.27/
9 T PTB IEN NIST	-34.30	-25.20	-28.76	28.87	2.55/
10 T PTB NIST NPL	29.30	30.70	30.02	30.02	0.45/
11 T PTB NPL VSL	0.20	3.00	1.17	1.34	0.64/
12 T PTB VSL OP	-2.10	8.90	1.23	3.40	3.16/
13 T PTB OP ROA	-0.60	2.30	0.06	0.65	0.65/
17 T IEN NIST NPL	-0.70	3.30	0.81	1.34	1.07/
27 T NIST OP ROA	-2.10	1.60	-0.11	0.83	0.82/
32 T NPL VSL OP	-1.50	10.20	1.81	3.86	3.41/
33 T NPL OP ROA	0.40	2.10	0.94	1.07	0.49/
38 T VSL OP ROA	-4.20	9.40	1.70	3.95	3.57/

Vondrak D5 0505 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
5 T SP VSL OP	-2.90	9.00	1.21	3.26	3.03/-
6 T SP OP ROA	-2.00	2.40	-0.11	1.08-	1.07/-
9 T PTB IEN NIST	-34.20	-25.60	-28.79	28.89+	2.39/-
10 T PTB NIST NPL	29.10	30.60	30.02	30.03+	0.48/+
11 T PTB NPL VSL	0.30	2.90	1.14	1.27-	0.56/-
12 T PTB VSL OP	-2.10	8.80	1.24	3.31-	3.06/-
13 T PTB OP ROA	-0.50	2.10	0.03	0.59-	0.59/-
17 T IEN NIST NPL	-0.90	2.50	0.75	1.17-	0.89/-
27 T NIST OP ROA	-2.20	1.60	-0.09	0.83	0.82/
32 T NPL VSL OP	-1.60	10.20	1.84	3.82-	3.35/-
33 T NPL OP ROA	-0.50	2.50	0.79	1.03-	0.65/+
38 T VSL OP ROA	-4.10	9.40	1.67	3.86-	3.48/-

Linear 0506 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
1 T SP PTB IEN	-3.00	3.30	0.63	1.34	1.18/
2 T SP IEN NIST	-3.00	3.40	0.40	1.34	1.28/
5 T SP VSL OP	-2.10	1.40	-0.33	0.85	0.78/
6 T SP OP ROA	-1.40	0.90	-0.01	0.53	0.53/
11 T PTB NPL VSL	-0.70	3.70	1.27	1.62	1.00/
12 T PTB VSL OP	-2.70	1.60	-0.28	1.03	0.99/
13 T PTB OP ROA	-1.20	0.40	-0.32	0.49	0.37/
17 T IEN NIST NPL	-5.20	1.00	-0.39	1.73	1.69/
18 T IEN NPL VSL	-0.30	7.00	1.77	2.68	2.01/
19 T IEN VSL OP	-1.90	4.20	0.82	1.49	1.25/
20 T IEN OP ROA	-4.00	2.40	0.07	1.45	1.45/
27 T NIST OP ROA	-1.40	1.50	-0.07	0.56	0.56/
32 T NPL VSL OP	-4.80	2.70	0.64	1.71	1.59/
33 T NPL OP ROA	-1.10	6.60	1.28	2.25	1.85/

Vondrak D5 0506 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
1 T SP PTB IEN	-2.70	3.50	0.60-	1.28-	1.13/-
2 T SP IEN NIST	-3.00	3.20	0.39-	1.32-	1.26/-
5 T SP VSL OP	-2.20	1.40	-0.33	0.82-	0.75/-
6 T SP OP ROA	-0.90	1.00	-0.03+	0.47-	0.47/-
11 T PTB NPL VSL	-0.50	3.40	1.28+	1.56-	0.90/-
12 T PTB VSL OP	-2.40	1.60	-0.17-	0.91-	0.90/-
13 T PTB OP ROA	-1.10	0.40	-0.31-	0.47-	0.36/-
17 T IEN NIST NPL	-5.40	0.90	-0.42-	1.76-	1.71/+
18 T IEN NPL VSL	-0.10	7.00	1.80+	2.71+	2.03/+
19 T IEN VSL OP	-1.60	4.10	0.84+	1.42-	1.14/-
20 T IEN OP ROA	-3.60	2.20	0.10+	1.30-	1.30/-
27 T NIST OP ROA	-1.20	1.10	-0.07	0.46-	0.45/-
32 T NPL VSL OP	-3.90	2.80	0.77+	1.65-	1.46/-
33 T NPL OP ROA	-0.90	5.80	1.23-	2.10-	1.71/-

Linear better than Vondrak

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Comparison of different interpolation techniques with the Check Method (2) cont. All Closures (July 05)

Liner 0507 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
1 T SP PTB IEN :	-1.20	2.60	0.37	0.78	0.68/
5 T SP VSL OP :	-2.90	5.30	0.13	1.46	1.45/
6 T SP OP ROA :	-5.90	1.70	-0.44	1.60	1.53/
10 T PTB NIST NPL :	29.90	31.10	30.45	30.45	0.29/
11 T PTB NPL VSL :	-0.10	2.30	1.41	1.53	0.58/
12 T PTB VSL OP :	-3.10	0.60	-0.37	0.81	0.72/
13 T PTB OP ROA :	-9.60	1.40	-0.80	2.05	1.89/
20 T IEN OP ROA :	-8.00	2.90	-0.18	2.45	2.44/
32 T NPL VSL OP :	0.10	5.50	0.88	1.35	1.03/
33 T NPL OP ROA :	-3.70	2.90	0.25	1.14	1.11/
38 T VSL OP ROA :	-2.50	1.10	0.13	0.81	0.80/

Vondrak D5 0507 1 day

Triangle/Stat./Mjd:	Min	Max	Mean	RMS	StdD
1 T SP PTB IEN :	-1.00	2.50	0.35-	0.70-	0.60/-
5 T SP VSL OP :	-2.90	5.50	0.18+	1.48+	1.47/+
6 T SP OP ROA :	-5.80	2.00	-0.42-	1.56-	1.50/-
10 T PTB NIST NPL :	30.00	31.00	30.44-	30.44-	0.26/-
11 T PTB NPL VSL :	0.00	2.10	1.32-	1.43-	0.54/-
12 T PTB VSL OP :	-2.90	0.80	-0.18-	0.73-	0.71/-
13 T PTB OP ROA :	-8.80	1.50	-0.77-	1.89-	1.73/-
20 T IEN OP ROA :	-6.90	2.50	-0.15-	2.15-	2.15/-
32 T NPL VSL OP :	0.10	5.30	0.92+	1.33-	0.97/-
33 T NPL OP ROA :	-3.00	2.90	0.27+	1.01-	0.97/-
38 T VSL OP ROA :	-2.20	1.10	0.20+	0.80-	0.77/-

Vondrak vs. Linear:
Av. Gain ~ 7 %
Improved ? Yes clearly

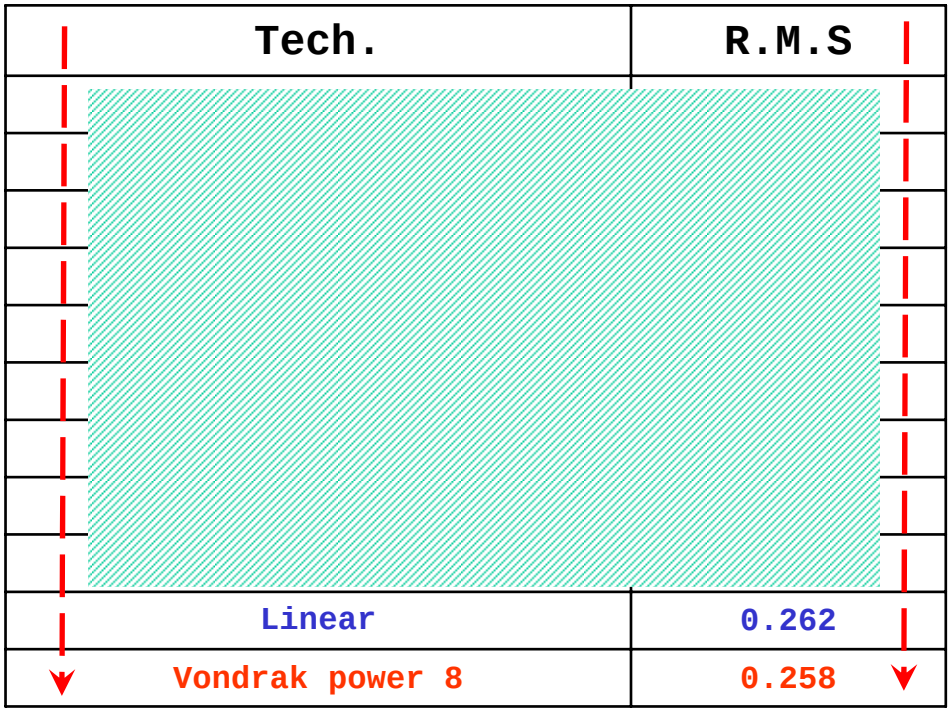
Check method (3)

- Comparison with the GPS carrier phase:

USNO – PTB, 53495 – 53502, May 2005

TW : 81 points
CP : 2017 points (0.05 ~ 0.1 ns)

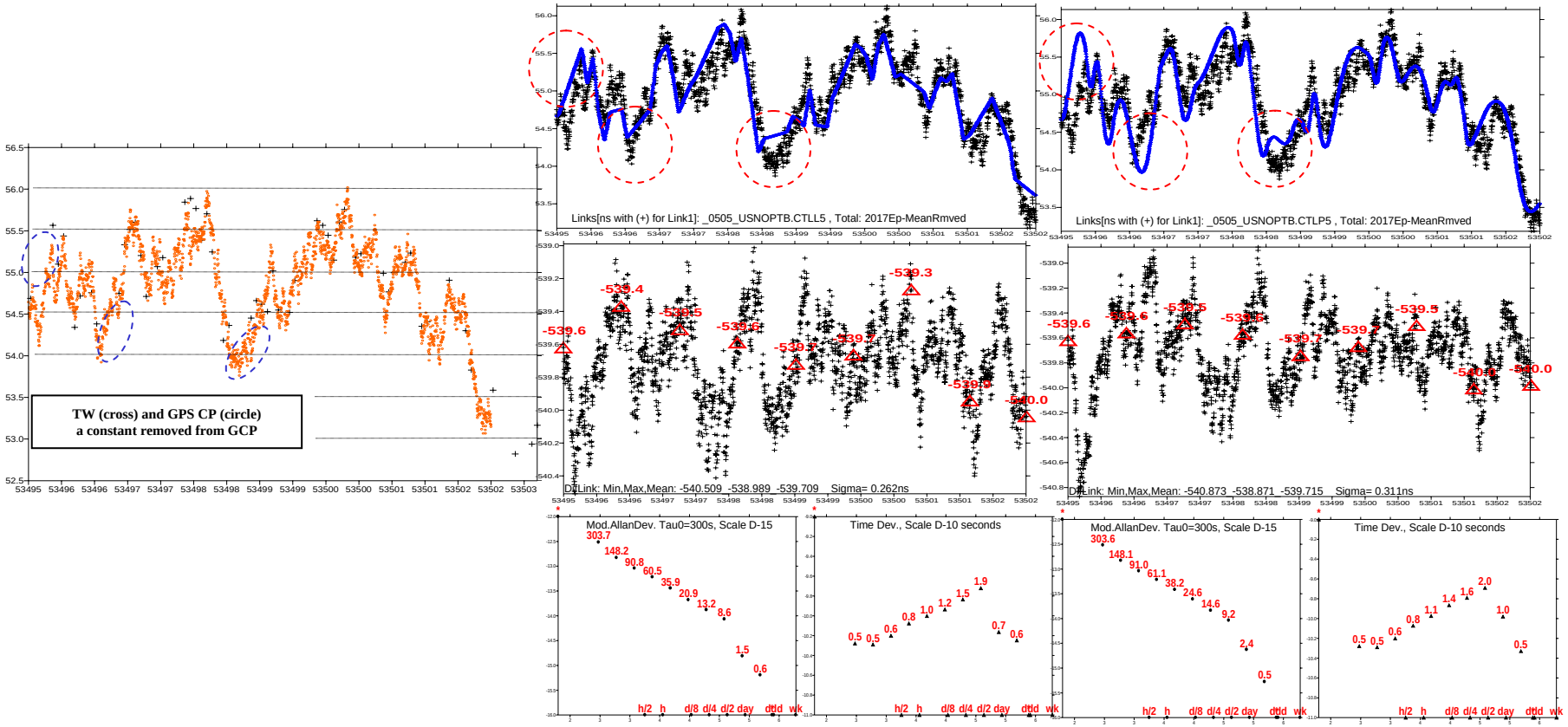
Comparison of different interpolation techniques with the Check Method (3) cont. USNO-PTB: TW vs. GPS CP



Tech.	R.M.S
Linear	0.262
Vondrak power 8	0.258

Linear and vondrak are the best
Vondrak vs. Linear: gain 2%
Improved ? Not clear

Comparison of different interpolation techniques with the Check Method (3) cont. USNO-PTB: TW vs. GPS CP



Linear

Polynomial

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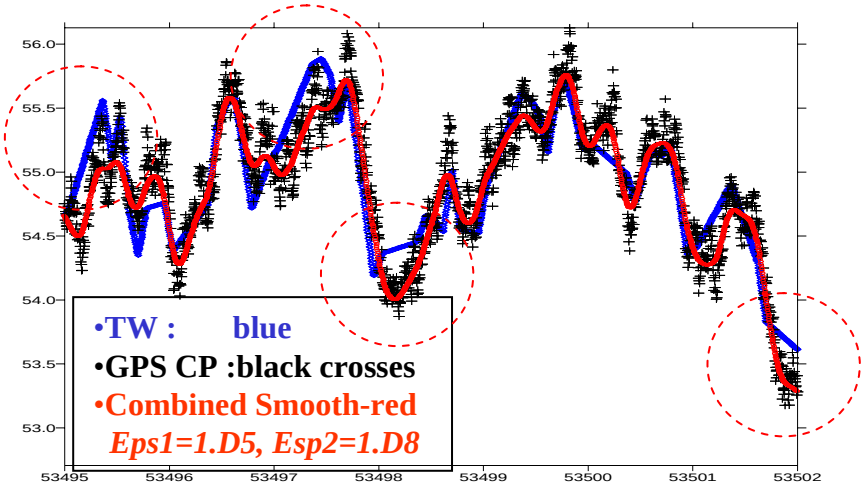
Combination of the TW and Carrier Phases

- USNO-PTB: TW + GPS CP



Combined Smoothing of TW+GCP:

- High precision
- High density: Greatly shorten interpolation intervals from 2 hours to 300s

Summary

- 10 different interpolation techniques were tested with 3 different checking methods
- The simplest linear and the Vondrak interpolations seem the best
- By comparing to the measured points and GPS CP, the linear and Vondrak techniques seem no considerable differences
- By comparing the triangle closures, the Vondrak technique seem better than the linear technique
- The combined TW-CP smoothing-interpolating technique should be the more accurate