Correlations taken into account in the KCDB

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In this document, the correlations taken into account in the calculation of the degrees of equivalence used in the BIPM ongoing comparison and linked comparison reports are presented.

Firstly, the correlations between the NMIs that use the same method of standardization or the same solution should normally be estimated case by case. With the new Working Group on Uncertainties set up by the CCRI(II) in 2001, a more complete approach may be considered in the future. To date these correlations have been neglected except when the correlation between two NMIs is clearly identified (e.g. use of an IC calibrated by another NMI). This is described in the part A of this document.

Secondly, in the uncertainty evaluation of the degree of equivalence $D_i = x_i - x_R$ where x_R is the key comparison reference value (KCRV), the correlation between x_i and x_R is taken into account if relevant (if x_i contributes to the KCRV). This has been fully described in the Appendix 2 of each Final Report of the SIR.

Thirdly, correlations occur when a CCRI(II) or RMO key comparison is linked to the SIR through the measurement at the BIPM of one or more ampoules of the comparison. The correlation coming from the linking SIR measurement is taken into account as shown in part B of this document.

Finally, for all SIR results in the KCDB, the measurements carried out at the BIPM may be, in principle, a source of correlation between all the data. This is discussed in the final part C of this document.

A. Correlation between one laboratory traceable to another laboratory

When a laboratory 1 makes an activity measurement using an instrument calibrated by a laboratory 2, then:

$$A_1 = m_1 \times C \qquad \text{and} \qquad A_2 = k \times C \quad , \tag{1}$$

where A_i is the activity in an ampoule i submitted to the SIR by laboratory i
m_1 is the measurement reading at laboratory 1
C is the calibration factor of the instrument used by laboratory 1
k is the factor describing the traceability at laboratory 2 of the measurement in the calibrated instrument used by laboratory 1 to the activity measurement A_2.

At the BIPM, the measurement S_i in the SIR of the ampoule standardized by each laboratory i gives an equivalent activity A_{ei}, that is proportional to A_i :

$$A_{ei} = S_i \times A_i \tag{2}$$

giving $A_{e1} = S_1 \times m_1 \times C$ and $A_{e2} = S_2 \times k \times C$ (3)

The degree of equivalence between the two laboratories is

$$D_{12} = A_{e1} - A_{e2}\,, \tag{4}$$

$$u_c^2(D_{12}) = u^2(A_{e1}) + u^2(A_{e2}) - 2 \times u(A_{e1}, A_{e2}) \qquad \text{GUM 5.2.2 [1],} \tag{5}$$

where $u(A_{e1}, A_{e2})$ is the estimated covariance associated with A_{e1} and A_{e2}.

If A_{e1} and A_{e2} are correlated through the common parameter C, then the covariance can be estimated by

$$\begin{aligned} u(A_{e1}, A_{e2}) &= \frac{\partial A_{e1}}{\partial C}\frac{\partial A_{e2}}{\partial C}u^2(C) \\ &= \frac{A_{e1}}{C}\frac{A_{e2}}{C}u^2(C) \qquad \text{GUM F.1.2.3 [1],} \\ &= A_{e1}\,A_{e2}\left(\frac{u(C)}{C}\right)^2 \end{aligned} \tag{6}$$

The relative uncertainty of the calibration factor C is normally given in the uncertainty budget of laboratory 1.

B. Correlation between participants of a K2 comparison linked to the SIR

When a CCRI(II) or RMO K2 comparison is linked to the SIR through the measurement in the SIR of one or more ampoules of the comparison, the linking factor introduces a correlation between all the participants in this K2 comparison. The link may be described by:

$$A_{ei} = (A/m)_i \times L \tag{7}$$

where $(A/m)_i$ is the result of laboratory i in the K2 comparison and L is the linking factor (see Final Reports of the SIR with linked comparisons, e.g. [2]).

The degree of equivalence between the participants in the K2 comparison is then

$$D_{ij} = A_{ei} - A_{ej} \tag{8}$$

$$u_c^2(D_{ij}) = u^2(A_{ei}) + u^2(A_{ej}) - 2 \times u(A_{ei}, A_{ej}) \tag{9}$$

with

$$
\begin{aligned}
u\left(A_{\mathrm{e}i}, A_{\mathrm{e}j}\right) &= \frac{\partial A_{\mathrm{e}i}}{\partial L}\frac{\partial A_{\mathrm{e}j}}{\partial L}u^2(L) \\
&= \frac{A_{\mathrm{e}i}}{L}\frac{A_{\mathrm{e}j}}{L}u^2(L) \\
&= A_{\mathrm{e}i}\,A_{\mathrm{e}j}\left(\frac{u(L)}{L}\right)^2
\end{aligned}
\qquad (10)
$$

The relative uncertainty of the linking factor L is given by the uncertainty of the SIR measurement of the ampoule(s) of the K2 comparison.

C. Correlations due to the SIR measurement

A correlation is expected to have an influence on the degrees of equivalence only if the uncertainty of the correlated quantity contributes significantly to the combined uncertainty. In practice, the SIR measurement generally has an uncertainty that is negligible in terms of the uncertainty of the NMI standardization measurement. Exceptions may occur when the solution contains an impurity that has not been quantified precisely by the NMI, but this is related to a given ampoule and does not result in a correlation with other NMIs.

As an example, a common factor between the SIR measurements is the ionization current ratio between the different Ra reference sources used. However, this ratio has a small uncertainty ($< 10^{-3}$ in relative terms) so that the correlation produced is negligible (see the $^{110}Ag^{m}$ Final Report [3]).

The other possible source of correlation through the SIR measurement is the half life used to correct for the decay. Indeed, when the uncertainty on the half life is significant and the SIR measurement is made some time after the reference date, then the uncertainty on the SIR result may become non negligible. If λ is the decay constant of the radionuclide measured, then:

$$
A_{\mathrm{e}i} \propto \exp(-\lambda\,\Delta t_i) \quad , \qquad \lambda = \ln 2 / T \qquad (11)
$$

where Δt_i is the time difference between the SIR measurement and the reference date and T the half life. The degree of equivalence is again given by equations (8) and (9), with the covariance given by

$$
\begin{aligned}
u\left(A_{\mathrm{e}i}, A_{\mathrm{e}j}\right) &= \frac{\partial A_{\mathrm{e}i}}{\partial \lambda}\frac{\partial A_{\mathrm{e}j}}{\partial \lambda}u^2(\lambda) \\
&= \left(-\Delta t_i A_{\mathrm{e}i}\right)\left(-\Delta t_j A_{\mathrm{e}j}\right)u^2(\lambda)
\end{aligned}
\qquad (12)
$$

As an example, the matrix of degrees of equivalence for BIPM.RI(II)-K1.Fe-59 with and without the correlation due to half life is given overleaf.

References

[1] Guide to the expression of uncertainty in measurement (GUM) – BIPM, IEC, IFCC, ISO, IUPAC, IUPAP and OIML, 1995, ISO, Geneva.

[2] Ratel G, Michotte C, BIPM comparison BIPM.RI(II)-K1.Ba-133 of the activity measurements of the radionuclide ^{133}Ba and the links for the 1984 international comparison CCRI(II)-K2.Ba-133, *Metrologia*, 2003, **40**, *Tech. Suppl.*, in press.

[3] Ratel G, Michotte C, BIPM comparison BIPM.RI(II)-K1.Ag-110m of the activity measurements of the radionuclide ^{110}Ag^{m}, *Metrologia*, 2002, **39**, *Tech. Suppl.*, 06001.

[4] Ratel G, Michotte C, BIPM comparison BIPM.RI(II)-K1.Fe-59 of the activity measurements of the radionuclide ^{59}Fe, *Metrologia*, , 2003, **40**, *Tech. Suppl.*, in press.

Part of the matrix of degrees of equivalence for BIPM.RI(II)-K1.Fe-59, as published in the KCDB [4]:

Lab *j*

Lab *i*			NPL		ANSTO		CMI-IIR		BNM-LNHB		PTB		NMIJ		BARC		KRISS	
	D_i / MBq	U_i	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}
NPL	0.02	0.11			0.13	0.17	-0.03	0.15	0.07	0.14	-0.03	0.15	-0.01	0.14	0.17	0.18	-0.05	0.30
ANSTO	-0.11	0.12	-0.13	0.17			-0.16	0.16	-0.06	0.16	-0.16	0.17	-0.13	0.15	0.04	0.19	-0.18	0.31
CMI-IIR	0.05	0.10	0.03	0.15	0.16	0.16			0.10	0.13	0.00	0.14	0.03	0.13	0.20	0.17	-0.02	0.30
BNM-LNHB	-0.06	0.09	-0.07	0.14	0.06	0.16	-0.10	0.13			-0.11	0.14	-0.08	0.12	0.09	0.16	-0.12	0.29
PTB	0.05	0.11	0.03	0.15	0.16	0.17	0.00	0.14	0.11	0.14			0.03	0.13	0.20	0.17	-0.02	0.30
NMIJ	0.02	0.08	0.01	0.14	0.13	0.15	-0.03	0.13	0.08	0.12	-0.03	0.13			0.17	0.16	-0.05	0.29
BARC	-0.15	0.13	-0.17	0.18	-0.04	0.19	-0.20	0.17	-0.09	0.16	-0.20	0.17	-0.17	0.16			-0.22	0.31
KRISS	0.07	0.26	0.05	0.30	0.18	0.31	0.02	0.30	0.12	0.29	0.02	0.30	0.05	0.29	0.22	0.31		

Same matrix as above but taking into account the correlation due to the uncertainty of the half life of ^{59}Fe used in the SIR. The values that differ from the published matrix are shown in red.

Lab *i*			NPL		ANSTO		CMI-IIR		BNM-LNHB		PTB		NMIJ		BARC		KRISS	
	D_i / MBq	U_i	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}	D_{ij} / MBq	U_{ij}
NPL	0.02	0.11			0.13	0.17	-0.03	0.15	0.07	0.14	-0.03	0.15	-0.01	0.14	0.17	0.18	-0.05	0.30
ANSTO	-0.11	0.12	-0.13	0.17			-0.16	0.13	-0.06	0.12	-0.16	0.13	-0.13	0.15	0.04	0.13	-0.18	0.24
CMI-IIR	0.05	0.10	0.03	0.15	0.16	0.13			0.10	0.13	0.00	0.10	0.03	0.13	0.20	0.11	-0.02	0.23
BNM-LNHB	-0.06	0.09	-0.07	0.14	0.06	0.12	-0.10	0.13			-0.11	0.08	-0.08	0.12	0.09	0.09	-0.12	0.22
PTB	0.05	0.11	0.03	0.15	0.16	0.13	0.00	0.10	0.11	0.08			0.03	0.13	0.20	0.10	-0.02	0.22
NMIJ	0.02	0.08	0.01	0.14	0.13	0.15	-0.03	0.13	0.08	0.12	-0.03	0.13			0.17	0.16	-0.05	0.29
BARC	-0.15	0.13	-0.17	0.18	-0.04	0.13	-0.20	0.11	-0.09	0.09	-0.20	0.10	-0.17	0.16			-0.22	0.18
KRISS	0.07	0.26	0.05	0.30	0.18	0.24	0.02	0.23	0.12	0.22	0.02	0.22	0.05	0.29	0.22	0.18		