# Report on the APMP key comparison of hydrometer calibrations

# APMP.M.D-K4

# Final report

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[5] NMISA, South Africa
[6] NMIA, Australia
[7] MSL, New Zealand
[8] NPLI, India
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# Abstract

This report describes the APMP key comparison on the hydrometer calibrations, APMP.M.D-K4. This comparison is an APMP regional key comparison of CIPM key comparison CCM.D-K4.

Three hydrometers in the density range from 640 kg $m^{-3}$ and 1 320 kg $m^{-3}$ were used as travelling standards. Each hydrometer has a nominal density range of 20 kg $m^{-3}$ with a scale division of 0.2 kg $m^{-3}$. Eleven laboratories participated in this comparison. All laboratories except NMISA are from APMP. NMISA from AFRIMETS is also an APMP associate member.

Each participant reported the results of hydrometer calibration using Cuckow's method. The reference temperature for the hydrometer calibration was 20 °C and the surface tension of the reference liquid was given in the protocol for this comparison. KRISS and NMIJ acted as pilot and coordinating laboratories.

The measurement was performed between July 2008 and April 2010. The report from participant was completed in February 2012. The results of this comparison were linked to CCM.D-K4 through the results of KRISS, NMIJ, and NMIA

NPLI, SNSU-BSN, and NMISA withdrew their results while preparing Draft A report of this comparison. The results of two participants, MSL and NIM, were inconsistent. However, because it has been more than 10 years since the measurements were completed, it is difficult to determine that the results reflect the current measurement capabilities of the two participants.


# 1 Introduction

This comparison is the APMP key comparison of the CIPM comparison CCM.D-K4 [1]. KRISS and NMIJ organized this key comparison as pilot and coordinating laboratories.

The transfer standards involved in the CIPM key comparison CCM.D-K4 were hydrometers in the range between 600 kg $m^{-3}$ and 2 000 kg $m^{-3}$. The measurements were performed from January 2011 to April 2012. The final report was published in 2016.

The main purpose of this APMP key comparison was to compare the experimental results and uncertainty calculations in calibrating hydrometers in the range from 640 kg $m^{-3}$ and 1 320 kg $m^{-3}$ and to link the results with the results in the CIPM key comparison CCM.D-K4.

The measurement of this comparison was performed from July 2008 to April 2010. Three laboratories KRISS, NMIJ, and NMIA participated in CCM.D-K4 acted as linking laboratories. The protocol was essentially equivalent to the protocol of CCM.D-K4.

# 2 Participants and schedule

Table 1 shows the participants in this comparison and contact persons. Eleven laboratories took part in the comparison. All laboratories except NMISA are from APMP. NMISA from AFRIMETS is an APMP associate member.

Table 2 shows the circulation scheme. Measurement was performed from July 2008 to April 2010. Final report from participant was received at February 2012.

At the beginning of this comparison, it was planned to deliver the travelling standards to the following NMI by a courier service. However, there was a problem that the travelling standards were broken twice during

delivery. In order to prevent any further delay and safely transport the travelling standards, it has been changed to deliver the travelling standards by hand-carry. The cost and time required to travel long distances was a challenge in this comparison. Considering the travelling standards were successfully circulated by a courier service in CCM.D-K4 and other regional comparisons, shipping can be chosen as a delivery method in the next APMP comparisons [1-5].

During the preparation of Draft A report of this comparison, NPLI withdrew the results in 2022, SNSU-BSN and NMISA in 2023.

**Table 1** Participants of APMP.M.D-K4.

| Institute | Contact person(s) |
|---|---|
| KRISS, Rep. of Korea | Kwang-Cheol Lee |
| | Yong Jae Lee |
| NMIJ, Japan | Futoshi Inoue |
| | Kunihiko Toda |
| | Kenichi Fujii (coordinator) |
| NMIM, Malaysia | Fazrul Mohd Nor |
| NIMT, Thailand | Chompoonoot Hirunyapruk |
| | Veera Tulasombut |
| | Tosapon Pangviwate |
| NMISA, South Africa | Ronél Steyn |
| NMIA, Australia | Kitty Fen |
| MSL, New Zealand | Yin Hsien Fung |
| | Chris Sutton |
| NPLI, India | Goutam Mandal |
| NIM, China | Jintao Wang |
| | Changhong Xu |
| NMLPHIL, Philippines | Jose Marco Latosa |
| | Marilyn C. Fos |
| SNSU-BSN, Indonesia | Renanta Hayu |

**Table 2** Circulation scheme.

| Institute | Arrival date | Departure date | Report date |
|---|---|---|---|
| KRISS-1 | - | Sep. 1, 2008 | July 22, 2008 |
| NMIJ | Sep. 1, 2008 | Oct. 16, 2008 | Oct. 27, 2008 |
| NMIM | Oct. 16, 2008 | Dec. 06, 2008 | Dec. 31, 2008 |
| NIMT | Dec. 08, 2008 | Dec. 13, 2008 | Dec. 13, 2008 |
| KRISS-2 | Dec. 13, 2008 | Mar. 02, 2009 | Feb. 26, 2009 |
| NMISA | Mar. 04, 2009 | Apr. 14, 2009 | - |
| NMIA | Apr. 14, 2009 | May 29, 2009 | July 03, 2009 |
| MSL | May 29, 2009 | Aug. 14, 2009 | Sep. 30, 2009 |
| KRISS-3 | Aug. 14, 2009 | Sep. 10, 2009 | Aug. 23, 2009 |
| NPLI | Sep. 11, 2009 | Nov. 14, 2009 | - |
| NIM | Nov. 15, 2009 | Jan. 25, 2010 | Aug. 13, 2010 |
| NMLPHIL | Jan. 26, 2010 | Feb. 24, 2010 | Mar. 15, 2010 |
| SNSU-BSN | Feb. 25, 2010 | Apr. 28, 2010 | - |
| KRISS-4 | Apr. 28, 2010 | - | Feb. 06, 2012 |

# 3 Transfer standards

Table 3 shows details of travelling standards. Three hydrometers in the density range between 640 kg $m^{-3}$ and 1 320 kg $m^{-3}$ were chosen as travelling standards for this comparison. Each hydrometer has a nominal density range of 20 kg $m^{-3}$ with a scale division of 0.2 kg $m^{-3}$. KRISS provided the three travelling standards.

**Table 3** Hydrometers.

| Range / (kg $m^{-3}$) | Serial number | Scale division / (kg $m^{-3}$) | Mass / g | Total length / mm | Stem diameter / mm |
|---|---|---|---|---|---|
| 640 — 660 | 060124 | 0.2 | 99 | 348 | 7 |
| 980 — 1 000 | 060136 | 0.2 | 151 | 346 | 6 |
| 1 300 — 1 320 | 060140 | 0.2 | 191 | 349 | 5 |

# 4 Comparison protocol

In a hydrometer calibration, the measurand is the correction $C$ evaluated at each calibration point.

$$C = \rho_x - \rho_r \qquad (1)$$

Here, $\rho_x$ is the density of the buoyant reference liquid at the reference temperature in which the hydrometer would freely float at the scale mark $\rho_r$.

In this comparison, the results had to be given at the reference temperature of 20 °C. The cubic expansion coefficient for all hydrometers was assumed to be $25\times10^{-6}$ °$C^{-1}$ with an uncertainty of $1\times10^{-6}$ °$C^{-1}$, rectangular distribution.

Participating laboratories should use their routine calibration procedure based on Cuckow's method.

Table 4 shows the calibration points and the reference surface tension of the liquid, in which each hydrometer was intended to be used.

**Table 4** Calibration points and the specified reference surface tension of the liquid, in which each hydrometer is intended to be used.

| Range / (kg $m^{-3}$) | Reading scale / (kg $m^{-3}$) | Reference surface tension / (mN $m^{-1}$) |
|---|---|---|
| 640 — 660 | 640 | 17 |
| | 650 | 17.5 |
| | 660 | 18 |
| 980 — 1 000 | 980 | 34 |
| | 990 | 34.5 |
| | 1 000 | 35 |
| 1 300 — 1 320 | 1 300 | 55 |
| | 1 310 | 55 |
| | 1 320 | 55 |

# 5 Calibration method and equipment used

According to the protocol, the Cuckow's method was used by all participating NMIs. For each hydrometer, at least five weighing were performed in air and in the working liquid at each calibration point. Scale readings were matched to the liquid level when the center of the line was aligned with the horizontal liquid plane and detected with the specified visual alignment. Table A.1 in appendix shows the equipment used by participating laboratories.

# 6 Results of check measurements

The pilot laboratory measured four times during the measurement period to check the stability of the travelling standards as shown in Table 5. The maximum difference $\Delta C$ between four measurements was comparable to the expanded uncertainty of the pilot laboratory. The KRISS values for the comparison have been calculated as mean values of the four measurements and used in the following evaluation.

**Table 5** KRISS results.

| Nominal value | $C_1$ | $U_{1,95}$ | $C_2$ | $U_{2,95}$ | $C_3$ | $U_{3,95}$ | $C_4$ | $U_{4,95}$ | $\Delta C$ |
|---|---|---|---|---|---|---|---|---|---|
| / (kg m$^{-3}$) | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) |
| 640 | 318 | 34 | 334 | 34 | 292 | 34 | 314 | 34 | 42 |
| 650 | 204 | 34 | 223 | 35 | 197 | 35 | 217 | 35 | 26 |
| 660 | 190 | 35 | 203 | 35 | 186 | 35 | 213 | 35 | 26 |
| 980 | -38 | 37 | -14 | 36 | -43 | 37 | -16 | 37 | 29 |
| 990 | -35 | 37 | -18 | 37 | -32 | 37 | -12 | 37 | 23 |
| 1 000 | -64 | 38 | -37 | 37 | -63 | 38 | -28 | 37 | 36 |
| 1 300 | -507 | 48 | -495 | 48 | -508 | 48 | -480 | 48 | 28 |
| 1 310 | -497 | 48 | -451 | 48 | -492 | 49 | -467 | 48 | 45 |
| 1 320 | -525 | 48 | -493 | 48 | -525 | 49 | -495 | 49 | 32 |

# 7 Results of participants

For each hydrometer, five weighing were carried out in air and in the working liquid at each calibration point. Each participant calculated the average value of the correction $C$ at 20 °C and the standard uncertainty at the calibration point. The measurement results reported by each participant are shown from Table 6 to Table 8 and from Fig. 1 to Fig. 3.

An excel sheet was provided to all participating organizations so that they could submit the measurement uncertainty for the corrections in the same format. The $t$-factor $t_{95}(\nu_{\mathrm{eff}})$ was taken from the $t$-distribution for a 95 % confidence level, where $\nu_{\mathrm{eff}}$ is the effective degrees of freedom of the combined standard uncertainty $u_c$. Each participant reported $U_{95} = t_{95}(\nu_{\mathrm{eff}}) \cdot u_c$ as the expanded uncertainty for the corrections.

The uncertainty budget for each participant is summarized from Table A.2 to Table A.4 in appendix.

**Table 6** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060124 having range between 640 kg m$^{-3}$ and 660 kg m$^{-3}$.

| Institute | $\rho_r$ = 640 kg m$^{-3}$ | | | $\rho_r$ = 650 kg m$^{-3}$ | | | $\rho_r$ = 660 kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | 314 | 17 | 34 | 210 | 18 | 35 | 198 | 18 | 35 |
| NMIJ | 310 | 23 | 55 | 221 | 23 | 55 | 205 | 24 | 56 |
| NMIM | 330 | 60 | 120 | 230 | 60 | 120 | 250 | 60 | 120 |
| NIMT | 300 | 45 | 90 | 200 | 45 | 90 | 200 | 45 | 90 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | 299 | 8.0 | 16 | 194 | 8.1 | 16 | 187 | 8.0 | 16 |
| MSL | 316 | 7.3 | 15 | 267 | 7.3 | 15 | 293 | 7.3 | 15 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | 151 | 31 | 60 | 49 | 31 | 62 | 39 | 31 | 62 |
| NMLPHIL | - | - | - | - | - | - | - | - | - |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 1** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060124 having range between 640 kg m$^{-3}$ and 660 kg m$^{-3}$.

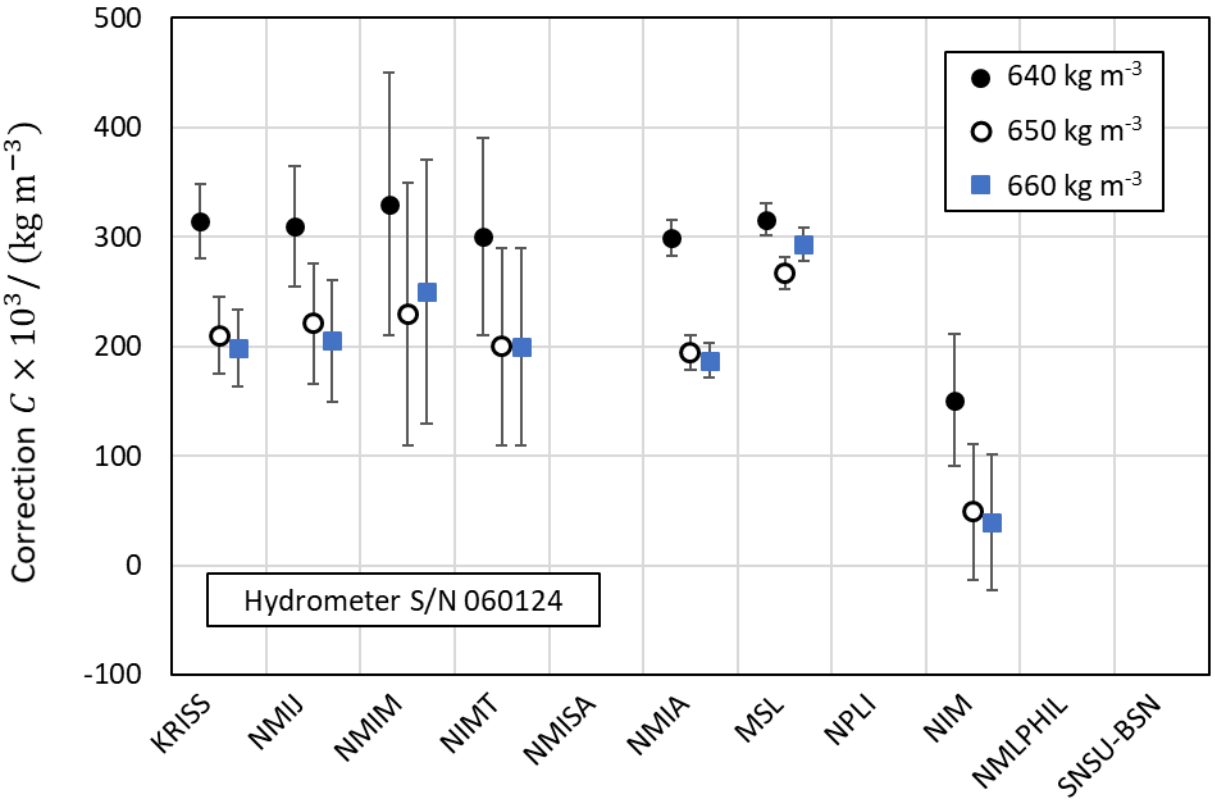


**Table 7** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060136 having range between 980 kg m$^{-3}$ and 1 000 kg m$^{-3}$.

| Institute | $\rho_r = 980$ kg m$^{-3}$ | | | $\rho_r = 990$ kg m$^{-3}$ | | | $\rho_r = 1\,000$ kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | -28 | 19 | 37 | -24 | 19 | 37 | -48 | 19 | 38 |
| NMIJ | -25 | 26 | 57 | -22 | 26 | 58 | -39 | 26 | 58 |
| NMIM | -10 | 65 | 130 | 0 | 65 | 130 | -10 | 65 | 130 |
| NIMT | 0 | 40 | 80 | 0 | 40 | 80 | 0 | 40 | 80 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | -45 | 9.3 | 18 | -45 | 9.6 | 19 | -61 | 9.8 | 19 |
| MSL | -15 | 7.6 | 15 | 36 | 7.6 | 15 | 57 | 7.6 | 15 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | -15 | 32 | 64 | -18 | 31 | 61 | -39 | 36 | 71 |
| NMLPHIL | - | - | - | - | - | - | - | - | - |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 2** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060136 having range between 980 kg m$^{-3}$ and 1 000 kg m$^{-3}$.

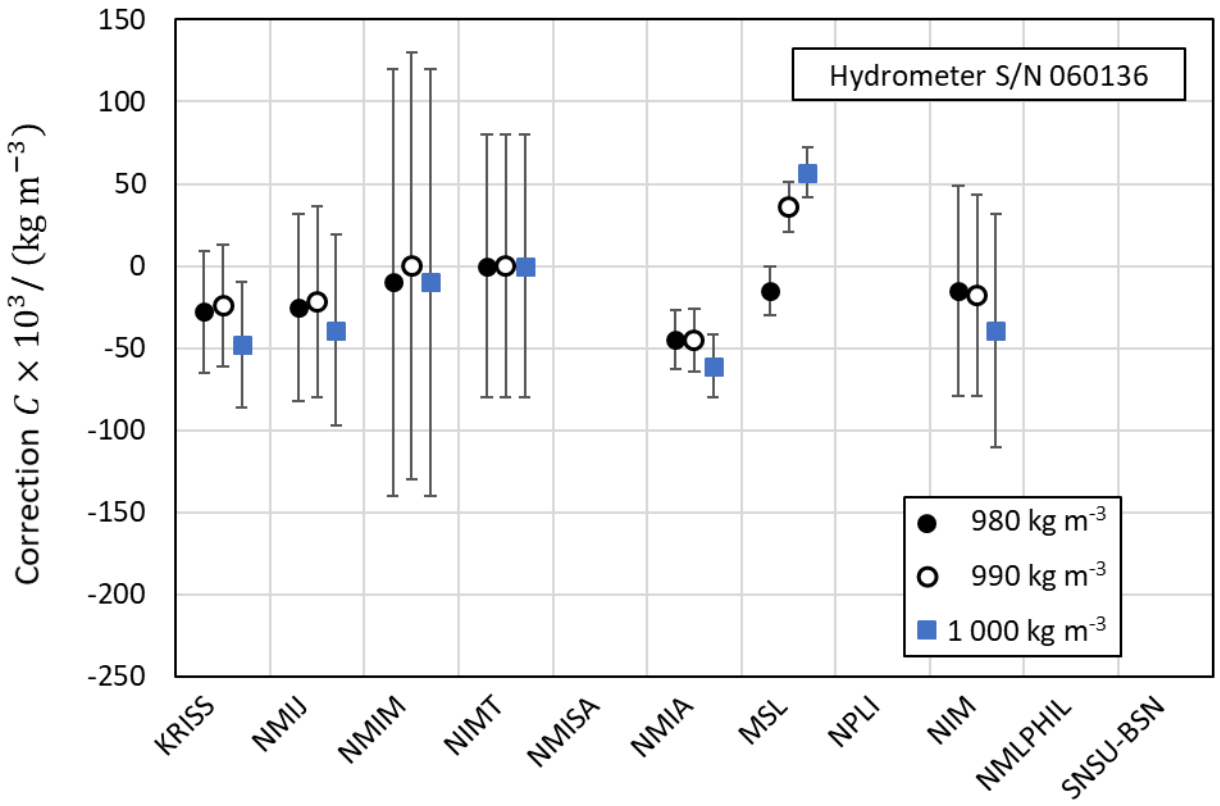
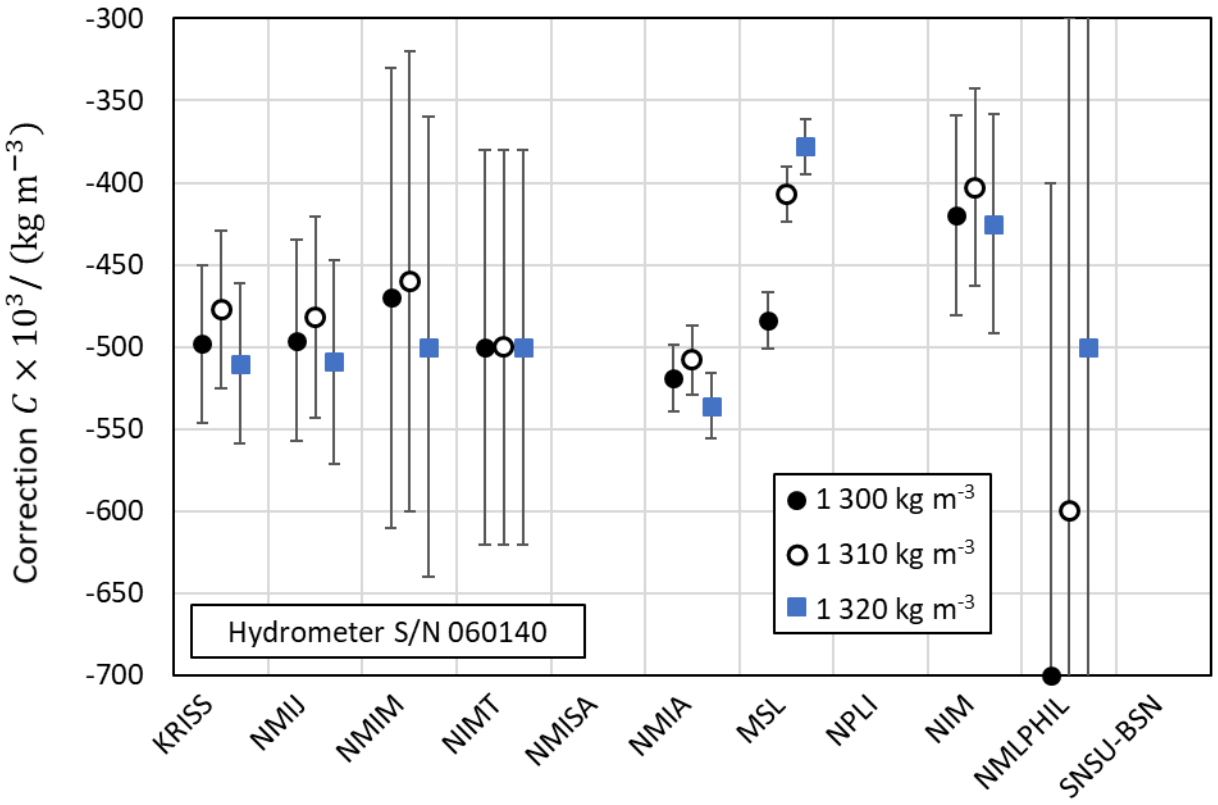


**Table 8** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060140 having range between 1 300 kg m$^{-3}$ and 1 320 kg m$^{-3}$.

| Institute | $\rho_r = 1\,300$ kg m$^{-3}$ | | | $\rho_r = 1\,310$ kg m$^{-3}$ | | | $\rho_r = 1\,320$ kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ | $C$ | $u_c$ | $U_{95}$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | -498 | 24 | 48 | -477 | 24 | 48 | -510 | 25 | 49 |
| NMIJ | -496 | 28 | 61 | -482 | 29 | 61 | -509 | 29 | 62 |
| NMIM | -470 | 69 | 140 | -460 | 69 | 140 | -500 | 69 | 140 |
| NIMT | -500 | 60 | 120 | -500 | 60 | 120 | -500 | 60 | 120 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | -519 | 10 | 20 | -508 | 10 | 21 | -536 | 10 | 20 |
| MSL | -484 | 8.3 | 17 | -407 | 8.3 | 17 | -378 | 8.4 | 17 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | -420 | 31 | 61 | -403 | 31 | 60 | -425 | 34 | 67 |
| NMLPHIL | -700 | 150 | 300 | -600 | 150 | 300 | -500 | 150 | 300 |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 3** Corrections $C = \rho_x - \rho_r$ reported by participants for the hydrometer 060140 having range between 1 300 kg m$^{-3}$ and 1 320 kg m$^{-3}$.



# 8 Link to CCM.D-K4 and Degree of Equivalence

The degrees of equivalence for the participant in this comparison were calculated by linking to the KCRV of the CIPM key comparison CCM.D-K4.

Three laboratories, KRISS, NMIJ, and NMIA, which had participated in the CCM.D-K4 acted as linking laboratories. The results reported by the three laboratories in this comparison were consistent if the consistency was checked by applying a chi-squared test [6]. The weighted means $\bar{C}$ of corrections reported by three laboratories were used as reference values in this comparison. Table 9 shows the weighted mean $\bar{C}$ of the correction and its expanded uncertainty $U(\bar{C})$ at each calibration point.

**Table 9** Weighted means $\bar{C}$ of corrections reported by linking laboratories KRISS, NMIJ, and NMIA.

| | KRISS | | NMIJ | | NMIA | | | |
|---|---|---|---|---|---|---|---|---|
| $\rho_r$ | $C$ | $U(C)$ | $C$ | $U(C)$ | $C$ | $U(C)$ | $\bar{C}$ | $U(\bar{C})$ |
| / (kg m$^{-3}$) | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | |
| 640 | 314 | 34 | 310 | 55 | 299 | 16 | 302 | 14 |
| 650 | 210 | 35 | 221 | 55 | 194 | 16 | 199 | 14 |
| 660 | 198 | 35 | 205 | 56 | 187 | 16 | 190 | 14 |
| 980 | -28 | 37 | -25 | 57 | -45 | 18 | -40 | 16 |
| 990 | -24 | 37 | -22 | 58 | -45 | 19 | -39 | 16 |
| 1 000 | -48 | 38 | -39 | 58 | -61 | 19 | -56 | 17 |
| 1 300 | -498 | 48 | -496 | 61 | -519 | 20 | -514 | 18 |
| 1 310 | -477 | 48 | -482 | 61 | -508 | 21 | -501 | 18 |
| 1 320 | -510 | 49 | -509 | 62 | -536 | 20 | -530 | 18 |

In the final report of CCM.D-K4, degrees of equivalence for participants at nominal densities of 600 kg m$^{-3}$, 1 000 kg m$^{-3}$, 1 500 kg m$^{-3}$, and 2 000 kg m$^{-3}$ were given. The degree of equivalence in CCM.D-K4 at each nominal density for linking laboratories is reproduced in Table 10. Weighted mean $\overline{D}_C$ of degrees of equivalence for linking laboratories at each nominal density in CCM.D-K4 is also shown in Table 10.

**Table 10** Degrees of equivalence of linking laboratories KRISS, NMIJ, and NMIA in CCM key comparison on hydrometer CCM.D-K4 [1].

| | KRISS | | NMIJ | | NMIA | | | |
|---|---|---|---|---|---|---|---|---|
| $\rho_r$ | $D$ | $U(D)$ | $D$ | $U(D)$ | $D$ | $U(D)$ | $\overline{D}_C$ | $U(\overline{D}_C)$ |
| / (kg m$^{-3}$) | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | |
| 600 | 1 | 24 | 8 | 11 | -6 | 8 | -1 | 6 |
| 1 000 | 11 | 34 | -6 | 26 | -11 | 18 | -6 | 14 |
| 1 500 | 5 | 51 | -1 | 30 | -9 | 13 | -7 | 12 |
| 2 000 | 8 | 71 | -9 | 54 | -13 | 27 | -10 | 23 |

The degree of equivalence for linking laboratories at each nominal density of this comparison was calculated using a numerical method. 100 000 random samples were generated by assuming a Gaussian distribution with the mean and expanded uncertainty in Table 10. The degree of equivalence at each

nominal density was calculated by using the weighted least square method. From 100 000 degrees of equivalence, mean and standard deviation at each density was obtained. Table 11 shows the degrees of equivalence for linking laboratories at each calibration point in this comparison.

The degrees of equivalence for linking laboratories were consistent when the consistency was checked by using a chi-squared test. The weighted mean $\overline{D}_W$ of the degrees of equivalence for linking laboratories was used as the degree of equivalence for this comparison. The last two columns in Table 11 show the weighted mean $\overline{D}_W$ of the degrees of equivalence for linking laboratories and its expanded uncertainty.

**Table 11** Weighted means $\overline{D}_W$ of degrees of equivalence for linking laboratories, KRISS, NMIJ, and NMIA, at several nominal densities. Degree of equivalence for each linking laboratory at nominal density was calculated using the weighted least square method.

| | KRISS | | NMIJ | | NMIA | | | |
|---|---|---|---|---|---|---|---|---|
| $\rho_r$ | $D$ | $U(D)$ | $D$ | $U(D)$ | $D$ | $U(D)$ | $\overline{D}_W$ | $U(\overline{D}_W)$ |
| / (kg m$^{-3}$) | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | | × 10$^3$ / (kg m$^{-3}$) | |
| 600 | 3 | 22 | 7 | 11 | -6 | 8 | -1 | 6 |
| 640 | 3 | 21 | 7 | 10 | -6 | 7 | -2 | 6 |
| 650 | 3 | 21 | 7 | 10 | -6 | 7 | -2 | 6 |
| 660 | 3 | 20 | 6 | 10 | -7 | 7 | -2 | 6 |
| 980 | 5 | 18 | 2 | 11 | -8 | 6 | -5 | 5 |
| 990 | 5 | 18 | 2 | 11 | -8 | 6 | -5 | 5 |
| 1 000 | 5 | 18 | 2 | 11 | -8 | 6 | -5 | 5 |
| 1 300 | 7 | 24 | -2 | 17 | -9 | 8 | -7 | 7 |
| 1 310 | 7 | 25 | -2 | 17 | -9 | 8 | -7 | 7 |
| 1 320 | 7 | 25 | -2 | 17 | -9 | 8 | -7 | 7 |
| 1 500 | 8 | 31 | -5 | 21 | -10 | 10 | -8 | 9 |
| 2 000 | 11 | 50 | -11 | 34 | -12 | 16 | -10 | 14 |

The degree of equivalence $D_i$ for each participant in this comparison is the difference of the correction reported by the participant from reference values calculated using results from linking laboratories as following.

$$D_i = C_i - \bar{C} + \overline{D}_W \quad (2)$$

The expanded uncertainty $U(D_i) = 2u(D_i)$ and the standard uncertainty $u(D_i)$ is calculated as

$$u(D_i) = \sqrt{u^2(C_i) + u^2(\bar{C}) + u^2(\overline{D}_W)} \quad (3)$$

The degree of equivalence $D_i$, the expanded uncertainty of the degree of equivalence $U(D_i)$, and normalized error $E_n = D_i / U(D_i)$ for each participant are shown from Table 12 to Table 14 and from Fig. 4 to Fig. 6.

**Table 12** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060124 having range between 640 kg m$^{-3}$ and 660 kg m$^{-3}$.

| Institute | $\rho_r$ = 640 kg m$^{-3}$ | | | $\rho_r$ = 650 kg m$^{-3}$ | | | $\rho_r$ = 660 kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | 10 | 37 | 0.3 | 9 | 38 | 0.2 | 6 | 38 | 0.2 |
| NMIJ | 6 | 57 | 0.1 | 20 | 57 | 0.4 | 13 | 58 | 0.2 |
| NMIM | 26 | 121 | 0.2 | 29 | 121 | 0.2 | 58 | 121 | 0.5 |
| NIMT | -4 | 91 | 0.0 | -1 | 91 | 0.0 | 8 | 91 | 0.1 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | -5 | 22 | -0.2 | -7 | 22 | -0.3 | -5 | 22 | -0.2 |
| MSL | 12 | 21 | 0.5 | 66 | 21 | 3.1 | 101 | 21 | 4.7 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | -153 | 62 | -2.5 | -152 | 64 | -2.4 | -153 | 64 | -2.4 |
| NMLPHIL | - | - | - | - | - | - | - | - | - |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 4** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060124 having range between 640 kg m$^{-3}$ and 660 kg m$^{-3}$.

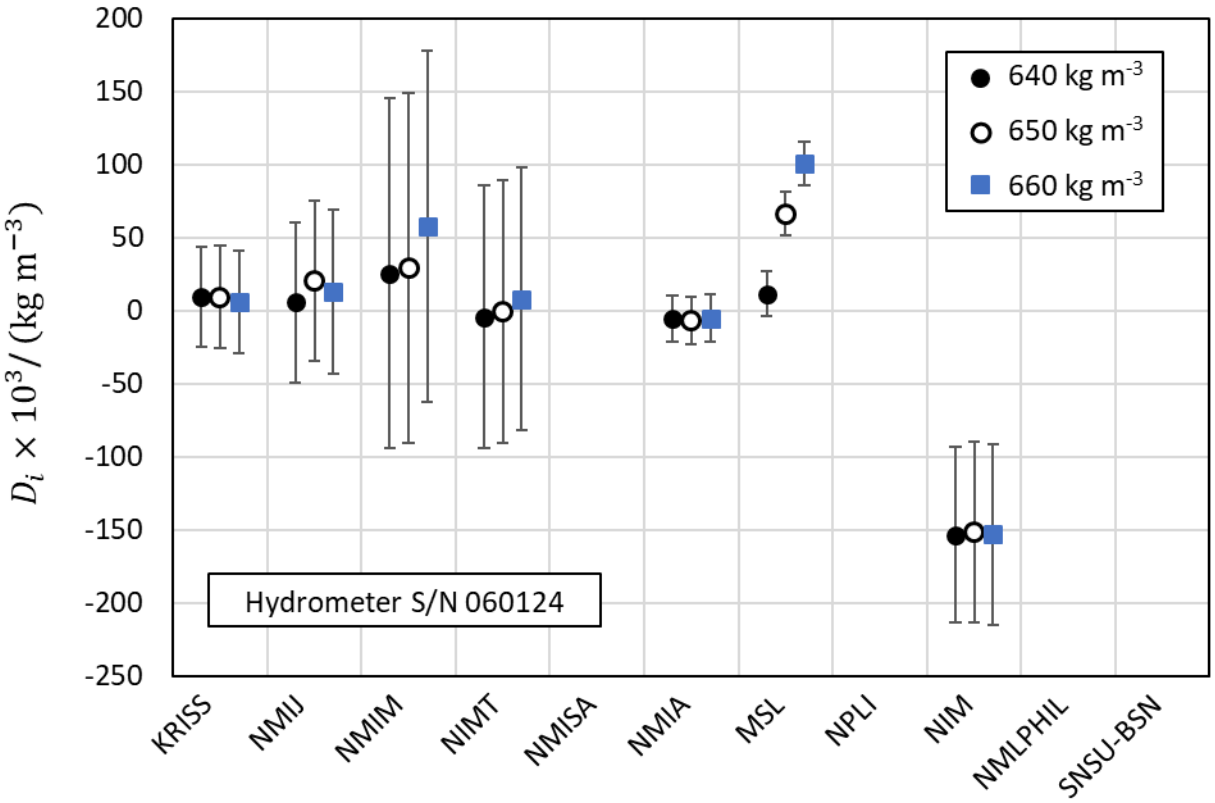


**Table 13** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060136 having range between 980 kg m$^{-3}$ and 1 000 kg m$^{-3}$.

| Institute | $\rho_r$ = 980 kg m$^{-3}$ | | | $\rho_r$ = 990 kg m$^{-3}$ | | | $\rho_r$ = 1 000 kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | 7 | 41 | 0.2 | 10 | 41 | 0.2 | 3 | 42 | 0.1 |
| NMIJ | 10 | 59 | 0.2 | 12 | 60 | 0.2 | 12 | 61 | 0.2 |
| NMIM | 25 | 131 | 0.2 | 34 | 131 | 0.3 | 41 | 131 | 0.3 |
| NIMT | 35 | 82 | 0.4 | 34 | 82 | 0.4 | 51 | 82 | 0.6 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | -10 | 25 | -0.4 | -11 | 26 | -0.4 | -10 | 26 | -0.4 |
| MSL | 20 | 22 | 0.9 | 70 | 23 | 3.1 | 108 | 23 | 4.7 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | 20 | 66 | 0.3 | 16 | 63 | 0.3 | 12 | 73 | 0.2 |
| NMLPHIL | - | - | - | - | - | - | - | - | - |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 5** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060136 having range between 980 kg m$^{-3}$ and 1 000 kg m$^{-3}$.

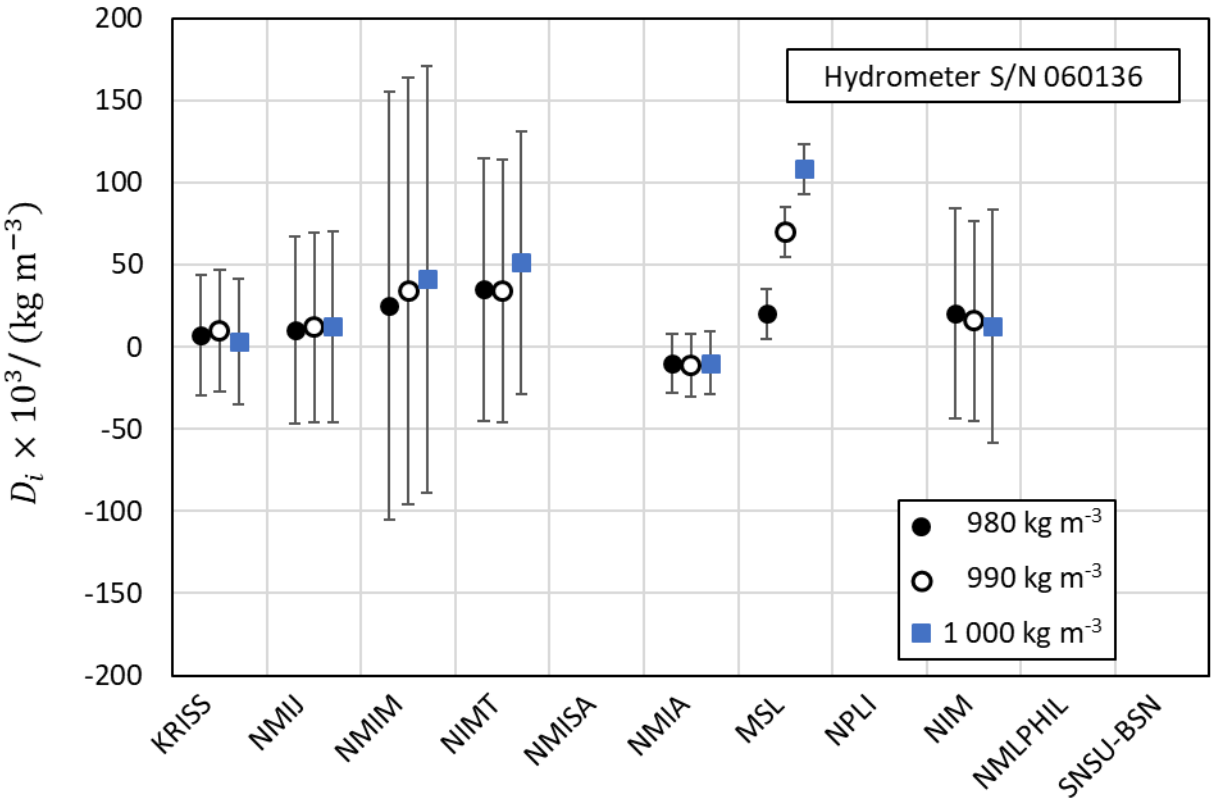
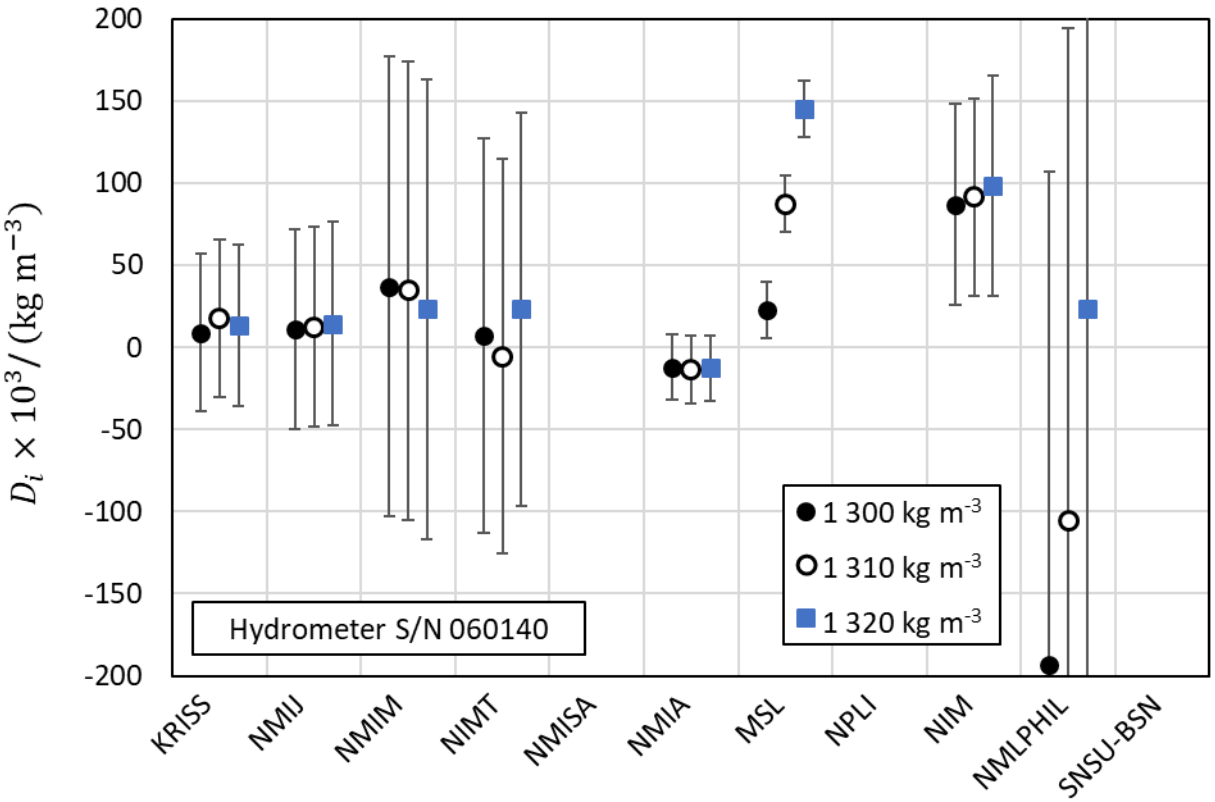


**Table 14** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060140 having range between 1 300 kg m$^{-3}$ and 1 320 kg m$^{-3}$.

| Institute | $\rho_r$ = 1 300 kg m$^{-3}$ | | | $\rho_r$ = 1 310 kg m$^{-3}$ | | | $\rho_r$ = 1 320 kg m$^{-3}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ | $D$ | $U(D)$ | $E_n$ |
| | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | | × 10$^3$ / (kg m$^{-3}$) | | |
| KRISS | 9 | 52 | 0.2 | 17 | 52 | 0.3 | 13 | 53 | 0.3 |
| NMIJ | 11 | 64 | 0.2 | 12 | 64 | 0.2 | 14 | 65 | 0.2 |
| NMIM | 37 | 141 | 0.3 | 34 | 141 | 0.2 | 23 | 141 | 0.2 |
| NIMT | 7 | 121 | 0.1 | -6 | 121 | 0.0 | 23 | 121 | 0.2 |
| NMISA | - | - | - | - | - | - | - | - | - |
| NMIA | -12 | 28 | -0.4 | -14 | 28 | -0.5 | -13 | 28 | -0.5 |
| MSL | 23 | 25 | 0.9 | 87 | 25 | 3.4 | 145 | 26 | 5.7 |
| NPLI | - | - | - | - | - | - | - | - | - |
| NIM | 87 | 64 | 1.4 | 91 | 63 | 1.5 | 98 | 70 | 1.4 |
| NMLPHIL | -193 | 301 | -0.6 | -106 | 301 | -0.4 | 23 | 301 | 0.1 |
| SNSU-BSN | - | - | - | - | - | - | - | - | - |

**Fig. 6** Degrees of equivalence for all participants in the APMP.M.D-K4 identified by the hydrometer 060140 having range between 1 300 kg m$^{-3}$ and 1 320 kg m$^{-3}$.



# 9 Conclusions

This comparison on calibration of high-resolution hydrometer is the APMP regional key comparison of the CIPM comparison CCM.D-K4. The three hydrometers with a density range between 640 kg $m^{-3}$ and 1 320 kg $m^{-3}$ was used as travelling standards. Each hydrometer has a nominal range of 20 kg $m^{-3}$ with a scale division of 0.2 kg $m^{-3}$.

The measurement was performed from July 2008 to April 2010. Final report from participant was in February 2012. Eleven NMIs participated in this comparison. Three laboratories, NPLI, SNSU-BSN, and NMISA, withdrew their results during preparation of the first draft, Draft A.

The results of the comparison were linked to the results of CCM.D-K4 through results of three linking laboratories, KRISS, NMIJ, and NMIA. The results of three linking laboratories were consistent when the consistency was checked by applying a chi-squared test.

The results from two laboratories, MSL and NIM, disagreed at least one calibration point. However, because the measurement was performed more than ten years ago, it is difficult to judge that the result of this comparison shows the current measurement capability of the institution if it has performed efforts to improve the equipment and measurement method.

# 10 Acknowledgements

The authors would like to acknowledge the kind assistance of all the colleagues in the participating laboratories for helping this comparison to complete successfully. We thank our colleagues who delivered the transfer standards voluntarily to the next participant.

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# Appendix

**Table A.1** Equipment.

| Item | KRISS | NMIJ | NMIM | NIMT | NMIA | MSL | NIM | NMLPHIL |
|---|---|---|---|---|---|---|---|---|
| Balance used for weighing in air | | | | | | | | |
| capacity/g | 405 | 520 | 310 | 500 | 1 000/160 | 205 | 210 | 5 100 |
| Resolution/mg | 0.1 | 0.1 | 0.1 | 0.1 | 0.1/0.01 | 0.1 | 0.01 | 1 |
| Balance used for weighing in liquid | | | | | | | | |
| capacity/g | 230 | 2 300 | 310 | 500 | 205 | 205 | 210 | 200 |
| Resolution/mg | 0.01 | 0.1 | 0.1 | 0.1 | 0.01 | 0.1 | 0.01 | 0.1 |
| Thermometer for liquid temperature | | | | | | | | |
| Resolution/mK | 1 | 1 | 10 | 1 | 2.6 | 1 | 1 | 100 |
| Instruments | Pt 100 Ω<br>ASL F300 | Pt 100 Ω<br>ASL F200 | ASL F250 | ASL F200 | Pt 100 Ω<br>HP3458A | Pt 100 Ω<br>ASL F250 | | Hart |
| Buoyant liquid | n-Tridecane | n-Tridecane | n-Tridecane | n-Tridecane | n-Nonane | Water-triton | Ethanol | Distilled water |
| Alignment | CCD camera manual | Magnifier manual | Eye manual | CCD camera manual | Optical lens manual | Eye manual | | |

T**able A.2** Uncertainty budget reported by participants for the hydrometer 060124 having range between 640 kg $m^{-3}$ and 660 kg $m^{-3}$.

| Uncertainty sources | Value / ($10^{-3}$ kg $m^{-3}$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | KRISS | NMIJ | NMIM | NIMT | NMISA | NMIA | MSL | NPLI | NIM | NMLPHIL | SNSU-BSN |
| Weighing in air | 0.04 | 0.11 | 0.40 | 2.23 | | 0.03 | 0.32 | | 0.22 | | |
| Weighing in liquid | 2.75 | 1.72 | 0.40 | 4.04 | | 0.52 | 0.28 | | 4.48 | | |
| Additional weights | 0.69 | 0.65 | 3.30 | 2.57 | | 0.52 | 0.19 | | 2.31 | | |
| Volumetric thermal expansion coefficient of hydrometer | 0.01 | 0.02 | 0.00 | 1.52 | | 0.03 | 0.15 | | 0.00 | | |
| Stem diameter | 0.14 | 0.09 | 0.06 | 1.79 | | 0.11 | 0.12 | | 0.03 | | |
| Air density | 0.13 | 0.62 | 0.48 | 3.42 | | 0.01 | 0.49 | | 0.23 | | |
| Liquid density | 8.67 | 5.14 | 18.02 | 14.91 | | 2.60 | 3.18 | | 6.81 | | |
| Liquid temperature | 0.36 | 4.05 | 0.16 | 5.21 | | 0.32 | 0.61 | | 0.04 | | |
| Surface tension of liquid | 6.19 | 3.73 | 2.70 | 10.39 | | 2.50 | 0.94 | | 0.60 | | |
| Gravitational acceleration | 0.00 | 0.07 | | 1.82 | | 0.00 | 0.00 | | 0.00 | | |
| Reading error | 8.00 | 7.04 | 58.00 | 2.98 | | 7.00 | | | 28.90 | | |
| Repeatability | 10.00 | 20.00 | 1.20 | 1.82 | | 1.10 | 6.40 | | 2.21 | | |
| Weighing value of suspension | | 5.50 | | 0.74 | | | | | | | |
| Combined standard uncertainty ($k$=1) | 17 | 23 | 61 | 45 | | 8.0 | 7.3 | | 31 | | |
| Expanded uncertainty $U_{95}$ | 34 | 55 | 120 | 90 | | 16 | 15 | | 60 | | |

T**able A.3** Uncertainty budget reported by participants for the hydrometer 060136 having range between 980 kg $m^{-3}$ and 1 000 kg $m^{-3}$.

| Uncertainty sources | Value / ($10^{-3}$ kg $m^{-3}$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | KRISS | NMIJ | NMIM | NIMT | NMISA | NMIA | MSL | NPLI | NIM | NMLPHIL | SNSU-BSN |
| Weighing in air | 0.40 | 0.21 | 0.78 | 1.91 | | 0.17 | 0.01 | | 0.26 | | |
| Weighing in liquid | 4.68 | 2.65 | 0.40 | 2.32 | | 0.77 | 0.48 | | 7.76 | | |
| Additional weights | | | | 1.11 | | | 0.28 | | | | |
| Volumetric thermal expansion coefficient of hydrometer | 0.02 | 0.05 | 0.00 | 1.56 | | 0.03 | 0.10 | | 0.00 | | |
| Stem diameter | 0.11 | 0.00 | 0.01 | 1.10 | | 0.04 | 0.02 | | 0.13 | | |
| Air density | 0.40 | 1.18 | 0.91 | 2.22 | | 0.04 | 0.02 | | 0.29 | | |
| Liquid density | 10.32 | 7.87 | 27.99 | 18.91 | | 3.80 | 4.65 | | 9.43 | | |
| Liquid temperature | 0.57 | 6.21 | 0.24 | 11.87 | | 0.49 | 1.04 | | 0.05 | | |
| Surface tension of liquid | 8.65 | 4.72 | 3.46 | 6.22 | | 4.20 | 1.19 | | 0.75 | | |
| Gravitational acceleration | 0.00 | 0.00 | | 1.00 | | 0.00 | 0.00 | | 0.00 | | |
| Reading error | 8.00 | 7.32 | 57.74 | 0.27 | | 7.00 | | | 28.90 | | |
| Repeatability | 10.00 | 20.00 | 6.32 | 1.56 | | 2.05 | 5.70 | | 5.59 | | |
| Weighing value of suspension | | 8.53 | | 3.28 | | | | | | | |
| Combined standard uncertainty ($k$=1) | 19 | 26 | 65 | 40 | | 9.3 | 7.6 | | 32 | | |
| Expanded uncertainty $U_{95}$ | 38 | 57 | 130 | 80 | | 18 | 15 | | 64 | | |

T**able A.4** Uncertainty budget reported by participants for the hydrometer 060140 having range between 1 300 kg $m^{-3}$ and 1 320 kg $m^{-3}$.

| Uncertainty sources | Value / ($10^{-3}$ kg $m^{-3}$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | KRISS | NMIJ | NMIM | NIMT | NMISA | NMIA | MSL | NPLI | NIM | NMLPHIL | SNSU-BSN |
| Weighing in air | 0.80 | 1.13 | 1.92 | 3.44 | | 0.76 | 0.28 | | 0.66 | 3.29 | |
| Weighing in liquid | 6.35 | 3.53 | 0.40 | 3.15 | | 1.00 | 0.72 | | 7.92 | 1.98 | |
| Additional weights | | | | 1.47 | | | | | | 1.49 | |
| Volumetric thermal expansion coefficient of hydrometer | 0.03 | 0.05 | 0.00 | 2.08 | | 0.03 | 0.33 | | 0.01 | | |
| Stem diameter | 0.05 | 0.68 | 0.10 | 0.87 | | 0.39 | 0.15 | | 0.38 | 0.05 | |
| Air density | 0.44 | 2.88 | 2.22 | 4.18 | | 0.08 | 0.41 | | 0.78 | 0.00 | |
| Liquid density | 16.15 | 10.44 | 37.12 | 26.33 | | 5.10 | 6.12 | | 0.65 | 57.74 | |
| Liquid temperature | 0.76 | 8.23 | 0.32 | 14.45 | | 0.65 | 1.78 | | 0.13 | 0.00 | |
| Surface tension of liquid | 9.89 | 5.36 | 3.93 | 5.10 | | 4.70 | 1.36 | | 0.86 | | |
| Gravitational acceleration | 0.00 | 0.11 | | 1.52 | | 0.00 | 0.00 | | 0.00 | 0.01 | |
| Reading error | 8.00 | 7.00 | 57.74 | 0.34 | | 7.00 | | | 28.90 | | |
| Repeatability | 10.00 | 20.00 | 3.25 | 2.07 | | 2.40 | 5.10 | | 6.52 | | |
| Weighing value of suspension | | 11.51 | | 6.77 | | | | | | | |
| Combined standard uncertainty ($k$=1) | 24 | 28 | 69 | 59 | | 10 | 8.3 | | 31 | 150 | |
| Expanded uncertainty $U_{95}$ | 48 | 61 | 140 | 120 | | 20 | 17 | | 61 | 300 | |