International Comparison of Absolute Gravimeters

ICAG2009

Final Report of Key Comparison CCM.G-K1

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Introduction

The International Comparison of Absolute Gravimeters 2009 (ICAG2009) was carried out in September 2009 at the International Bureau of Weights and Measures (BIPM). The ICAG2009 and its data processing were performed in accordance with the agreed Technical Protocol (TP) [1] which was approved by the Steering Committee after discussions at two meetings, held in Sèvres (December 2008) and Prague (May 2009), and was based on previous experience [2,3,4,5].

The pilot laboratory (BIPM) evaluated the final results of the key comparison (KC) from the measurements made at the BIPM in the framework of ICAG2009. This final report is based on the **CCM.G-K1** Draft A report, as approved by the Working Group on Gravimetry (WGG) of the Consultative Committee for Mass and Related Quantities (CCM) at its meeting at the BIPM on 9 May 2011. The report also includes a short discussion of our approach to the weighted least-squares (LS) adjustments of the gravity values at the five stations and the calculation of the offset for each individual absolute gravimeter.

Notation

KC: the key comparison
PS: the pilot study
ICAG: International Comparison of Absolute Gravimeters
AG(*k*): absolute gravimeter *k*
g_{jk}: gravity value defined at station *j* and height H_{jk} after subtraction of 980 900 000 µGal
u_{jk}: uncertainty of g_{jk} reported by the participant
H_{jk}: depending on the context, reference height of AG(*k*) at station *j* (Table 1a) and/or standard height (as in table 2, 0.9 m)
w_{jk}: statistical weight of g_{jk}, $w_{jk} = 1/u_{jk}^2$
G_j: adjusted gravity value at station *j*, in µGal, after subtraction of 980 900 000 µGal
KCRV: Key Comparison Reference Value(s), the G_j values at the ICAG stations
δ_k: offset of the AG(*k*), cf. equation (1) and Note 1
$\delta_{RGk:}$ offsets of the AG(*k*) determined from the network of relative gravimeters
s_k : standard uncertainty of offset δ_k from the least-squares adjustment
u_k : standard uncertainty of offset δ_k evaluated from the uncertainties u_{jk}
SAE: self-attraction effect
SAC: self-attraction correction
SAC_b: bias in gravity due to the SAC
S_k: enlarged uncertainty of offset δ_k including the bias due to the SAC: $S_k = s_k + |SAC_b|$
u_{KCRV} : uncertainty of the key comparison reference values G_j at the five stations
$\bar{u}_{KCRV}$: enlarged uncertainty of the key comparison reference values G_j at the five stations including the bias due to the SAC: $\bar{u}_{KCRV} = u_{KCRV} + |SAC_b|$
BIPM: International Bureau of Weights and Measures, the pilot laboratory
Lab(*k*): acronyms of the 11 participating laboratories, as follows:

1. NIM : National Institute of Metrology, China
2. LNE-SYRTE: Laboratoire National de Métrologie et d'Essais – Systèmes de Référence Temps-Espace, France
3. METAS : Federal Office of Metrology, Switzerland
4. NMIJ/AIST: National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology, Japan
5. VÚGTK/RIGTC: Research Institute of Geodesy, Topography and Cartography, Czech Republic
6. BEV: Bundesamt für Eichund Vermessungswesen, Austria
7. KRISS: Korea Research Institute of Standards and Science, Republic of Korea
8. CMS/ITRI: Centre for Measurement Standards, Industrial Technology Research Institute, Chinese Taipei
9. UME/TŰBITAK: Ulusal Metroloji Enstitüsü, Turkey
10. NRC: National Research Council of Canada, Canada
11. FGI: Finnish Geodetic Institute, Finland

1. Model

Each participant communicated gravity measurements for three of the five stations used. Each measurement by absolute gravimeter k at station j, indicated by g_{jk}, is given with its uncertainty u_{jk}. A weighted LS adjustment was used to evaluate the gravity value G_j at each station (the KCRVs), as well as the offset δ_k for a particular AG(k), using the following model

$$g_{jk}=G_j - \delta_k + v_{jk} \qquad (1)$$

where v_{jk} are the residuals. Note that the offset is a negative bias.

2. Considerations concerning the Key Comparison Reference Value (KCRV)

The purpose of a KCRV is to allow

i) the calculation of the degrees of equivalence (related to the offset and uncertainty determined here) for an AG in the KC,

ii) links to be made to the data obtained in Regional Metrology Organization (RMO) comparisons that follow a similar protocol. For ICAG2009, such a link is made to the PS comparison carried out subsequent to the KC. The KCRV is chosen as the best available estimate of the quantity in question.

It is preferable to execute the comparison at a certain number of measurement stations, normally with different gravity values. This particularity of absolute gravimetry does not always have analogues in other areas of metrology. In ICAG2009, five stations located at the BIPM were used and each gravimeter measured at least three of them. Consequently, we have calculated five KCRVs, one for each site measured by the AGs.

We make the following observations:

- according to the CIPM MRA, the KCRV is to be calculated from the results of the AGs participating in the KC part of ICAG2009;

- in principle, all the measured data and their uncertainties submitted by the laboratories participating in the KC should be used without any modification. We propose a weighting scheme for the evaluation, due to the fact that AGs have significantly different uncertainties, ranging from 2.4 μGal to 7.8 μGal (see Table 1a). Thus, in conjunction with the definition of the KCRV, the weighted average of a group of such realizations could be seen as the optimal realization and will coincide therefore with the spirit of the CIPM MRA as defined in the TP [1];
- Recent studies [5,6,7,8] have revisited the problem that the SAE of the AGs may present a non-negligible bias in the KCRV. Indeed, the SAC may be larger than the u_{KCRV} values of the site KCRVs in Table 2. However, because (1) the most recent estimate of the SAC for the FG5 [6] is almost an order of magnitude larger than its uncertainty given in the TP Annex D [1], (2) the SAC has not been evaluated for all the models of AG participating in the KC, and (3) further investigation is needed to establish the accuracy of the estimated value of the SAC, it is decided not to apply the SAC to the KC measurements but, for the sake of self-consistency in this report, to increase the final uncertainties to cover the bias ($\mathrm{SAC_b}$). Following [6] we added 1.7 μGal to the uncertainty of the KCRVs, 0.3 μGal to the uncertainty of the FG5 offsets and 1 μGal to that of the offsets of all other AGs, except for the CAG to which the $\mathrm{SAC_b}$ had already been added by the participating laboratory. The enlarged uncertainty is given in Tables 2 and 3 as $\bar{\boldsymbol{u}}_{\mathrm{KCRV}}$ and S_{k} respectively. The error bars in the Figures 1 and 2 do not include the SAC. See [5,6] for details.

In the LS adjustment the above considerations can be introduced as an additional constraint (eq. 2) where δ_k stands for the offset of the measurement of an AG from the KCRV and in this case the index k runs over the gravimeters taking part in the KC where the weight w_k is computed from the mean value of the uncertainties of the AG(k) at the three occupations (see Table 1a)

$$\sum w_k \delta_k = 0 \qquad (2)$$

Condition (2) ensures that the weighted mean of the offsets in the KC is zero. A subgroup of AGs used in the KC, assuming that their offset remains constant, can be used to provide a link to future RMO KCs using essentially the same protocol as CCM.G-K1.

3. Key comparison results

Table 1a is a compilation of the 33 measurements made by the 11 AGs that participated in the KC. Following the model presented above, a LS adjustment was made to obtain the gravity value at the 5 stations (Stn) as well as the offsets for the AGs under the constraint (2) discussed above. Note that only the AGs participating in the KC CCM.G-K1 contribute to the calculation of the KCRV.

According to [1], a relative gravity campaign was organized in conjunction with the ICAG2009 campaign, using nine high-quality gravimeters to determine the vertical gravity gradients over each station. The gravity is modeled by a second-order polynomial [1], the coefficients of which are given in Table 1b. The gradient correction between the heights H_1 and H_2 can be calculated by $\Delta g(H_2 - H_1) = g(H_2) - g(H_1) = b(H_2 - H_1) + c(H_2^2 - H_1^2)$. We then reduce the g value from the reference height given in Table 1a to the standard height 0.9 m. In most cases, the vertical gravity gradient reduction uncertainty is less than 1 μGal [4].

Table 1a. Raw AG data submitted to the BIPM by the KC participants.
g_{jk} is the gravity measured at the height H_{jk} and u_{jk} is its combined standard uncertainty. Both g_{jk} and u_{jk} are reported by the participants as requested in the Annex C of the TP; the estimate of u_{jk} was supported by a complete uncertainty budget.
See Notation for quantity definitions.

k	AG(*k*)	Lab(*k*)	Stn(*j*)	u_{jk} /μGal	g_{jk} /μGal	H_{jk} /μGal
1	NIM-2	NIM	B2	6.6	27928.1	1.1870
			B6	7.4	27915.7	1.1870
			B	6	27949.3	1.1870
2	CAG-1	LNE-SYRTE	B1	6.1	28026.5	0.8160
			B6	6.7	28027.3	0.8178
			B	6	28050.9	0.8165
3	FG5-209	METAS	B5	2.9	27907.9	1.2983
			B	2.9	27904.1	1.2980
			B2	2.9	27891.0	1.3020
4	FG5-213	NMIJ /AIST	B5	2.5	27909.8	1.2772
			B	2.5	27908.5	1.2781
			B1	2.5	27904.4	1.2779
5	FG5-215	VUGTK /RIGTC	B6	2.4	27910.2	1.2119
			B1	2.4	27923.6	1.2124
			B5	2.4	27928.5	1.2115
6	JILAg-6	BEV	B2	7.8	28027.9	0.8400
			B5	7.3	28042.8	0.8400
			B1	7.3	28035.5	0.8400
7	FGL-103	KRISS	B2	4.5	28020.0	0.8090
			B1	4.5	28040.0	0.8090
			B6	4.5	28020.0	0.8250
8	FG5-224	CMS/ ITRT	B6	2.8	27886.0	1.2822
			B5	2.9	27902.4	1.2832
			B2	2.8	27886.3	1.2852
9	A10-5	UME	B1	5.9	28008.7	0.9000
			B6	4.8	27999.1	0.9000
			B	4.9	28012.6	0.9000
10	FG5-105	NRC	B	2.7	27898.0	1.3110
			B1	2.7	27898.7	1.3110
			B6	2.7	27883.8	1.3110
11	FG5-221	FGI	B5	2.7	27935.9	1.2000
			B2	2.7	27915.8	1.2000
			B1	2.7	27930.4	1.2000

Table 1b. Linear and second-order coefficients of the gravity model [9]

Stn	*b*/ (μGal/m)	*c*/ (μGal/m²)
B	-301.37	2.667
B1	-295.57	4.917
B2	-290.77	4.667
B5	-302.57	3.667
B6	-296.73	4.083

4. Gravity values at stations

The KCRVs and their uncertainties for the five stations are given in Table 2, where in each case the KCRV is defined at the height of 0.9 m above the ground benchmarks. See Notation for quantity definitions and units.

Table 2. The Key Comparison Reference Values (KCRV) at height 0.9 m

No.	Stn	KCRV/μGal	u_{KCRV}/μGal	$\bar{u}_{KCRV}$/μGal
1	B	28019.8	1.3	3.0
2	B1	28013.3	1.0	2.7
3	B2	27999.2	1.3	3.0
4	B5	28021.3	1.0	2.7
5	B6	28001.0	1.2	2.9

It is interesting to note that stations B and B2 have the largest uncertainties, while the smallest are those of B1 and B5. This is because the occupations were not completely homogeneous with respect either to the number of the AGs at each site or to the uncertainties of the AGs. We have ignored the small correlation between the KCRV value and *g* values from those AGs that contributed to the KCRV.

5. Offsets

The offsets[1] δ_k for the AGs(*k*) with respect to the KCRV are presented in Table 3; the standard uncertainties s_k given in the table are those of the offsets. We have also calculated offsets δ_{RGk} determined from the network of relative gravimeters (RG) that made measurements during the ICAG2009 at the BIPM [9]. This network has been fixed by imposing the gravity value of 28018.8 μGal (after subtraction of the usual 980 900 000 μGal) obtained in ICAG2005 at site B. The difference of the two offsets is given in the last column of Table 3. The mean value of $\delta_{RGk} - \delta_k$ is -1.0 μGal and the standard deviation and root mean square are 0.4 μGal and 1.1 μGal respectively. Note that the δ_k and δ_{RGk} are uncorrelated. The δ_{RGk} have been calculated to give confidence in the AG results. We conclude that there is agreement between the results of the AGs in the KC and the relative measurements, which confirms the validity of the solution obtained from those absolute gravimeters participating in the KC.

[1] Note : Although the term "offset" has been used traditionally to report ICAG results, it is not a common term in metrology. Refer to eq. (1) for the definition of "offset" and note the sign.

Table 3. Offsets of the Absolute Gravimeters.
See Notation for the elements in the table.

k	AG(*k*)	δ_k/μGal	s_k/μGal	S_k/μGal	(δ_{RGk}-δ_k)/μGal
1	NIM-2	-8.3	3.8	4.8	-0.1
2	CAG-1	0.9	3.5	3.5	-1.2
3	FG5-209	-3.5	1.6	1.9	-1.1
4	FG5-213	0.4	1.3	1.6	-0.9
5	FG5-215	0.8	1.3	1.6	-1.2
6	JILAg-6	-6.5	4.2	5.2	-1.3
7	FGL-103	2.4	2.5	2.8	0.3
8	FG5-224	5.3	1.5	1.8	-1.1
9	A10-5	4.5	2.9	3.9	-1.1
10	FG5-105	-1.0	1.4	1.7	-0.7
11	FG5-221	-2.2	1.4	1.7	-0.8

Figure 1 shows the offsets of the participant AGs with respect to the KCRV, with their standard uncertainties.

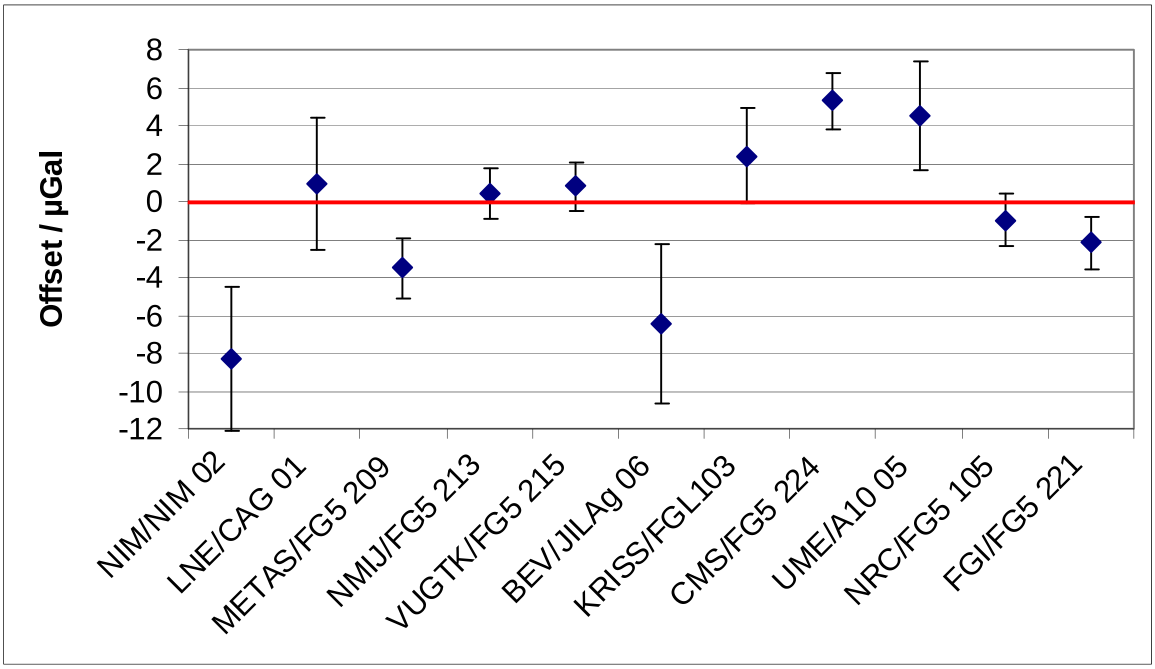

Figure 1. Offsets of the AGs participating in the KC with respect to the KCRV. The bars represent the respective standard uncertainties (s_k) as listed in Table 3.

Figure 2 shows the offsets of the participating AGs in the KC with respect to the KCRV with uncertainties u_k estimated from those submitted by the participants as listed in Table 1a, but expanded by a factor of 2 (k = 2), according to the CIPM MRA Guidelines.

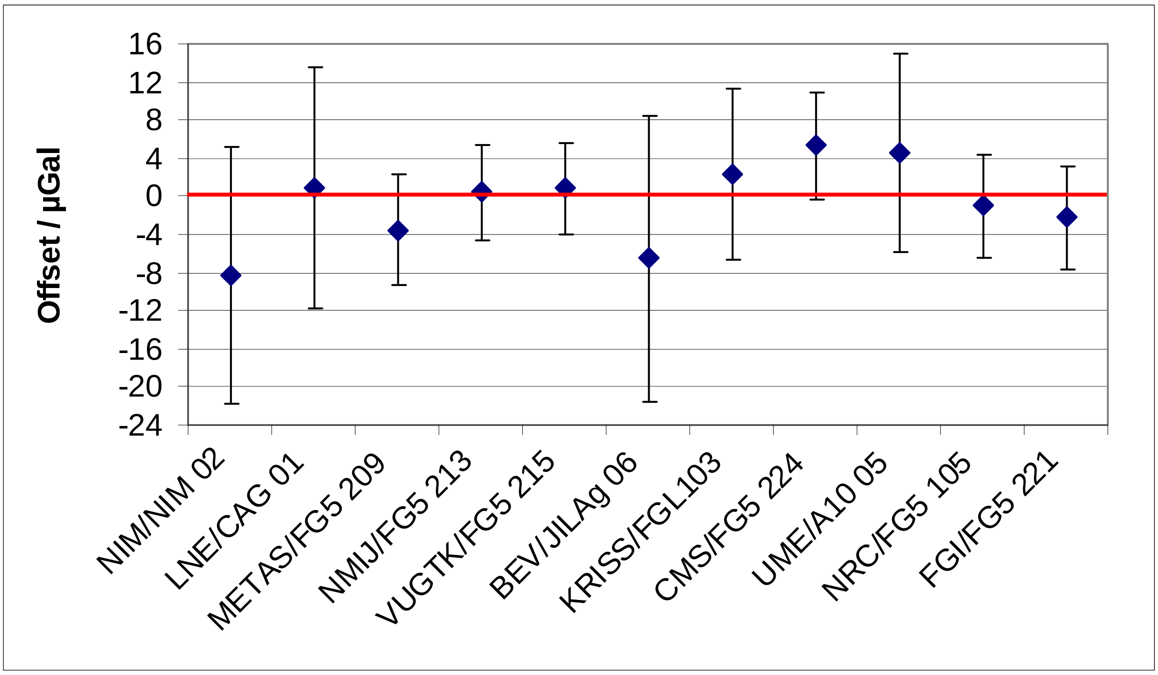

Figure 2. Offsets of the AGs participating in the KC with respect to the KCRV. The bars represent the uncertainties (u_k) submitted by the participants as listed in Table 1a, but expanded by a factor of 2 (k = 2).

6. Discussion

Within the framework of [1] we tested several methods of evaluation of the ICAG2009 CCM.G-K1 results. We found that the approach applied here allows for a clear and optimal evaluation of the AG measurements.

As compared to previous ICAGs the gravity at the BIPM's site stations seems to be stable. For station B, for example, the KCRV and the standard uncertainty out of the LS adjustment is (28019.8±1.3) µGal, which is in good agreement, within the level of uncertainty, with previous values obtained from the AG only solutions: (28018.8±0.5) µGal in the ICAG2005 campaign [3] and (28019.6±1.3) µGal in the ICAG2001 [2]. The 1 µGal difference between the gravity values in 2005 and 2009 is reflected in the values of the offsets ($\delta_{\mathrm{RG}k}$-δ_k) in the final column of Table 3.

The results indicate that all participating gravimeters are consistent within their expanded uncertainties (k = 2).

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