EUROMET.M.P-S3 Supplementary Comparison
of absolute pressure standards in the barometric range
from 80 kPa to 110 kPa

EUROMET Project N° 884

FINAL REPORT

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ABSTRACT

LNE and BIPM compared their pressure standards equipped with 20 cm^2 effective area piston-cylinder units in the absolute pressure range of 80 kPa to 110 kPa.

The pressure standards, the method for calculating the reference values and the comparison results are presented. The results of the comparison can be considered as satisfactory as the deviations from the reference value are inside the estimated combined uncertainty.

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1 Introduction

The primary standard of both BIPM and LNE is a pressure balance derived from the SI units through dimensional measurements, mass and gravity determination. The pressure delivered by the pressure balance is issued from these measurements by using the so-called effective area of the piston-cylinder assembly, and by applying different corrections.

The scope of the comparison is to demonstrate the coherency of the measurements done in the two institutes in the absolute pressure range of 80 kPa to 110 kPa by checking the validity of the calculations, and evaluating the level of uncertainty attainable in the direct comparison of two pressure balances. The present comparison covers partially the same pressure range as the CCM key comparison CCM.P-K2 [1]. It was not possible to link this comparison directly to the CCM.P-K2 comparison in absolute mode. These new results can be compared to those of the CCM.P-K1b comparison in gauge mode [2]: the same standard was used at LNE.

The pressure balance of BIPM has been moved to LNE and directly compared to the primary standard of LNE. The measurements have been performed in the period 10-12 January 2006.

This report presents a brief description of both laboratory pressure standards, the method for calculating the reference values and the comparison results.

2 Participants

The participating institutes as well as the coordinators are in table 1.

Laboratory	Address	Person responsible for the intercomparison
Laboratoire National d'Essais (LNE) Pilot institute	1, rue Gaston Boissier 75724 PARIS CEDEX 15 France	Pierre Otal Tel 33 1 40 43 39 63 Fax 33 1 40 43 37 37 e-mail: mailto:pierre.otal@lne.fr
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Table 1. *List of Participants*

3 Laboratory standards

The BIPM standard is a PG 7607 type pressure balance manufactured by DH Instruments. The LNE standard is the absolute pressure balance APX developed in cooperation with DH-Budenberg. Both standards were equipped with DH Instruments 20 cm² effective area piston-cylinder assemblies made of tungsten carbide.

The details of each pressure standard are listed in table 2. The value of the pressure distortion coefficient λ has been calculated at LNE using the Lamé equation[3]. The value of the linear thermal expansion coefficient of tungsten carbide α has been measured several times at LNE, by placing a pressure balance in a climatic chamber. This value has been confirmed by dimensional measurement on samples of the material made at NPL at the time of the CIPM comparison in the 20 - 100 MPa range [4].

Characteristics	BIPM Standard	LNE Standard
Measurement range in kPa	5 - 175	10 - 500
Material of piston	tungsten carbide	tungsten carbide
Material of cylinder	tungsten carbide	tungsten carbide
A_0, effective area at null pressure and reference temperature in mm^2	1961,0174	1961,0637
Relative standard uncertainty of A_0, in 10^{-6}	3,3	2,8
Pressure distortion coefficient λ in MPa^{-1}	7,15 x 10^{-6}	7,15 x 10^{-6}
ρ_M , density of the weights in kg·m^{-3}	8000	8000
Relative standard uncertainty of weight mass, in 10^{-6}	2,5	0,75
α_p and α_c, linear thermal expansion coefficients of the piston and the cylinder in °C^{-1}	4,5 x 10^{-6}	4,5 x 10^{-6}
t_0, reference temperature in °C	20	20

Table 2. *Characteristics of each pressure standard.*

3.1 BIPM pressure measurement

Pressure measured by the BIPM at its reference level

$$P = m\,g / [A_0\,(1+ (\alpha_p + \alpha_c)\,(t - t_0))\,(1+\lambda\;m\,g / A_0)] + \mu \qquad (1)$$

The notations not introduced in table 2 are listed in table 3.

The effective area of the piston/cylinder combination is described in an internal report of the BIPM Mass Section [5].

Parameter	BIPM Standard	LNE Standard
Mass applied to the piston	m	m'
Local acceleration due to gravity	g	g
Temperature of the unit	t	t'
Vacuum in the bell jar	μ	μ'
Density of nitrogen		ρ_{N2}
Difference in the altitude of the reference levels of the two standards		Δh

Table 3. *Additional notations.*

Input data are the diameters and the circularity of the piston and cylinder, as determined by the Length Laboratory of LNE.

The input data were limited, so interpolation was done using different regression models. For the cylinder, it was necessary to extrapolate to the ends and this was done by a linear projection of the two nearest data points.

The input data were used with the PTB/LNE formulas given in the report of EUROMET Project 740 [6], for zero pressure difference.

To see the effect of 1 atmosphere pressure difference, the equation of Dadson et al. [7] has been used. However, this equation has been criticized by Sutton [8]. So his equation has been used as well, adapted to the gas N_2.

In fact, all methods lead to very nearly the same value for the effective area. In the end, the method of Sutton has been used, and a relative standard uncertainty of $3{,}3 \times 10^{-6}$ was assigned to the effective area value.

The major uncertainty components considered are: non-circularity, calibration uncertainty of the dimensional metrology, sensitivity of the result to the engagement length, the interpolation equations used in the calculations, and the differences among the various formulas for effective area that are found in the literature.

3.2 LNE pressure measurement

The primary pressure standard of LNE has been described in [9, 10]. The pressure delivered by the balance at the reference level of the BIPM standard is calculated using a formula similar to (1), corrected for the head correction between the reference levels of the piston-cylinder units of both standards:

$$P = m' g / [A'_0 (1+ (\alpha_p + \alpha_c) (t' - t_0)) (1+\lambda \; m' g / A'_0)] + \mu' + \rho_{N2} \, g \, \Delta h \qquad (2)$$

The piston-cylinder unit used for the comparison is a 50 mm diameter one. Diameters, straightness and circularity measurements have been performed in the Length Laboratory of LNE. The standard uncertainties were 0,04 µm, 0,05 µm and 0,025 µm respectively for the 3 types of measurements.

All the measurements were combined using the method used to perform the calculation in the EUROMET Project N° 740 [6]. The standard uncertainty of the effective area was estimated to be 2,8 x 10^{-6} x A_0.

Circular comparisons have been also performed with other piston-cylinder units of 35 mm diameters the effective areas of which have been determined over the time using different methods [11]. The relative coherencies in the effective areas were within 2 x 10^{-6}.

4 Description of the comparison

The comparison has been performed for 5 cycles by increasing and decreasing pressure at the nominal pressures of:

80 90 95 100 105 110 kPa.

The pressure difference between the standards is measured using a capacitance diaphragm gauge. The pressure difference measured is typically less than 1 Pa. An uncertainty of 0,5% for this pressure difference, which is certainly very pessimistic, generates insignificant pressure measurement uncertainty.

5 Comparison procedure

For each pressure point, both standards were in equilibrium when the valve between the ports of the capacitance diaphragm gauge was closed. Each laboratory acquired the data from its own standard such as piston-cylinder temperature, residual reference pressure and eventually position of the piston or environmental conditions. The data from the differential transducer were acquired by LNE. The measuring time interval was the same for each laboratory (30 s).

The pressure has been calculated for each laboratory at the reference level of the BIPM standard.

6 Uncertainties

The uncertainty budget for both standards operating in the range from 80 kPa to 100 kPa is presented in table 4 (BIPM) and table 5 (LNE).

Components	Distribution	Standard uncertainty	Sensitivity coefficients	Contribution to pressure uncertainty
Effective area at *t* and nominal pressure				
Effective area at reference temperature and null pressure	Normal	$3{,}3 \times 10^{-6} \times A_0$	$-P/ A_0$	$3{,}3 \times 10^{-6} \times P$
Effective area drift	Normal	$0{,}5 \times 10^{-6} \times A_0$ per year	$-P/ A_0$	$0{,}5 \times 10^{-6} \times P$
Thermal expansion	Normal	$2{,}5\ \% \times (\alpha_p+\alpha_c)$	$-P \times (t\text{-}20)$	$0{,}7 \times 10^{-6} \times P$
Piston-cylinder temperature	Normal	0,06 °C	$-P \times (\alpha_p+\alpha_c)$	$0{,}5 \times 10^{-6} \times P$
Pressure distortion coefficient	Normal	$5\% \times \lambda$	$-P^2/(1+\lambda P)$	$4 \times 10^{-8} \times P$
Mass				
Applied mass	Normal	$2{,}5 \times 10^{-6} \times M$	P/M	$2{,}5 \times 10^{-6} \times P$
Bell mass	Normal	5 mg	$g/A(T,P)$	0,03
Cylinder mass	Normal	1,7 mg	$g/A(T,P)$	0,01
Local gravity	Normal	$1 \times 10^{-7} \times g$	P/g	$1 \times 10^{-7} \times P$
Balance				
Linearity	Normal	$0{,}5 \times 10^{-6} \times P$	1	$0{,}5 \times 10^{-6} \times P$
Verticality	Rectangular	$0{,}29/(2\sqrt{3})$ mrad	$P \times \sin\theta$	$2 \times 10^{-8} \times P$
Vacuum				
Calibration	Normal	0,015 Pa	1	0,02
Repeatability	Normal	0,004 Pa	1	0,00
Resolution	Rectangular	$0{,}01/(2\sqrt{3})$ Pa	1	0,00
Combined standard uncertainty			**$4{,}3 \times 10^{-6} \times P + 0{,}03$ Pa**	

Table 4: *Uncertainty budget for BIPM standard operating from 80 kPa to 100 kPa*

Components	Distribution	Standard uncertainty	Sensitivity coefficients	Contribution to pressure uncertainty
Effective area at reference temperature and null pressure	Normal	$2{,}8 \times 10^{-6} \times A_0$	$-P/ A_0$	$2{,}8 \times 10^{-6} \times P$
Stability of the effective area	Normal	$1{,}5 \times 10^{-6} \times A_0$	$- P/ A_0$	$1{,}5 \times 10^{-6} \times P$
Pressure distortion coefficient (max value)	Normal	$5\% \times \lambda$	$-P^2/ (1+\lambda.P)^2$	$4 \times 10^{-8} \times P$
Temperature	Normal	0,025 °C	$- (\alpha_p+\alpha_c) \times P$	$2{,}3 \times 10^{-7} \times P$
Thermal expansion	Normal	$2.5\% \times (\alpha_p+\alpha_c)$	$- (t\text{-}20) \times P$	$5 \times 10^{-7} \times P$
Mass, including cylinder and bell, and stability	Normal	$7{,}5 \times 10^{-7} \times M$	P/M	$7{,}5 \times 10^{-7} \times P$
Head correction	Rectangular	0,0018 m	$(\rho_{N2}\ g/ P\ ) \times P$	$2 \times 10^{-7} \times P$
Verticality	Rectangular	0,00058 rad	$P \times \sin\theta$	$4 \times 10^{-8} \times P$
Vacuum measurement	Normal	0,010 Pa	1	0,010 Pa
Local gravity	Normal	$9{,}8 \times 10^{-7}$ m.s^{-2}	P/g	$1{,}0 \times 10^{-7} \times P$
Combined standard uncertainty			**$3{,}3 \times 10^{-6} \times P$**	

Table 5: *Uncertainty budget for LNE standard operating from 80 kPa to 100 kPa*

Since the dimensional measurements of both piston-cylinder assemblies have been performed at LNE, a correlation coefficient of 0,7 was considered between both pressure standards. It corresponds to the contribution of the effective area uncertainty to the overall uncertainty approximately.

7 Results

7.1 *Calculation of the reference values*

A reference pressure P_{ref} has been calculated for each pressure point as the average of the pressure determined by each laboratory:

$$P_{ref} = \frac{P_{BIPM} + P_{LNE}}{2}$$

The uncertainty of P_{ref} is calculated from the uncertainty estimated by each laboratory and the correlation coefficient between both pressure standards.

$$u_{P\,ref} = 1/2 \times \sqrt{u_{P_{BIPM}}^2 + u_{P_{LNE}}^2 + 1{,}4 \times u_{P_{BIPM}} \times u_{P_{LNE}}}$$

Table 6 presents, for each pressure point, the mean measured pressure by each laboratory, the reference values, and their associated uncertainties.

Mean measured pressure		Combined standard uncertainty of the measured pressure		$\langle P_{ref} \rangle$	Expanded uncertainty of $\langle P_{ref} \rangle$ (k=2)
<LNE>	<BIPM>	LNE	BIPM		
/ Pa	/ Pa	/ Pa	/ Pa	/ Pa	/ Pa
80 033,77	80 033,55	0,26	0,37	80 033,660	0,59
90 038,11	90 037,88	0,30	0,42	90 037,995	0,66
95 040,33	95 040,05	0,31	0,44	95 040,190	0,70
100 042,36	100 042,08	0,33	0,46	100 042,220	0,73
105 044,45	105 044,13	0,35	0,48	105 044,290	0,77
110 046,51	110 046,20	0,36	0,50	110 046,355	0,80
110 046,14	110 045,80	0,36	0,50	110 045,970	0,80
105 044,17	105 043,83	0,35	0,48	105 044,000	0,77
100 042,15	100 041,88	0,33	0,46	100 042,015	0,73
95 039,94	95 039,67	0,31	0,44	95 039,805	0,70
90 037,91	90 037,68	0,30	0,42	90 037,795	0,66
80 033,77	80 033,54	0,26	0,37	80 033,655	0,59

Table 6: *Mean pressure measured by each laboratory, reference values and their associated uncertainties*

7.2 Degree of equivalence

7.2.1 Deviations from the reference values

The deviation D_{lab} of each laboratory from the reference value is determined as:

$$D_{lab} = P_{lab} - P_{ref}$$

Note that in this case: $D_{BIPM} = \frac{P_{BIPM} - P_{LNE}}{2} = -D_{LNE}$

Due to the correlation coefficient of 0,7 between both pressure standards, the standard uncertainty of D_{lab} is defined by:

$$\boldsymbol{u}_{D\,lab} = 1/2 \times \sqrt{\boldsymbol{u}_{P_{BIPM}}^2 + \boldsymbol{u}_{P_{LNE}}^2 - 1{,}4 \times \boldsymbol{u}_{P_{BIPM}} \times \boldsymbol{u}_{P_{LNE}}}$$

Table 7 gives for each pressure point and each laboratory:

- the mean of the five deviations from the reference values $<D_{lab}>$,
- the standard deviation of $<D_{lab}>$,
- the expanded uncertainty of $<D_{lab}>$, U_{lab}
- the ratio $<D_{lab}>/U_{lab}$

Lab	Nominal pressure	$<D_{lab}>$	Standard deviation of $<D_{lab}>$	U_{lab}, Expanded uncertainty of $<D_{lab}>$ (k=2)	$<D_{lab}>$ /U_{lab}
	/Pa	/Pa	/Pa	/Pa	-
BIPM	80 000	- 0,110	0,017	0,27	- 0,41
	90 000	- 0,115	0,023	0,30	- 0,39
	95 000	- 0,140	0,031	0,31	- 0,45
	100 000	- 0,140	0,028	0,33	- 0,43
	105 000	- 0,160	0,013	0,34	- 0,46
	110 000	- 0,155	0,021	0,36	- 0,43
	110 000	- 0,170	0,026	0,36	- 0,47
	105 000	- 0,170	0,028	0,34	- 0,49
	100 000	- 0,135	0,022	0,33	- 0,41
	95 000	- 0,135	0,014	0,31	- 0,43
	90 000	- 0,115	0,016	0,30	- 0,39
	80 000	- 0,115	0,018	0,27	- 0,43
LNE	80 000	+ 0,110	0,017	0,27	+ 0,41
	90 000	+ 0,115	0,023	0,30	+ 0,39
	95 000	+ 0,140	0,031	0,31	+ 0,45
	100 000	+ 0,140	0,028	0,33	+ 0,43
	105 000	+ 0,160	0,013	0,34	+ 0,46
	110 000	+ 0,155	0,021	0,36	+ 0,43
	110 000	+ 0,170	0,026	0,36	+ 0,47
	105 000	+ 0,170	0,028	0,34	+ 0,49
	100 000	+ 0,135	0,022	0,33	+ 0,41
	95 000	+ 0,135	0,014	0,31	+ 0,43
	90 000	+ 0,115	0,016	0,30	+ 0,39
	80 000	+ 0,115	0,018	0,27	+ 0,43

Table 7: *Mean deviation from the reference values $<D_{lab}>$, the standard deviation and the expanded uncertainty (k=2) of $<D_{lab}>$, and $<D_{lab}>$ /U_{lab}*

Whatever the pressure, the condition |$<D_{lab}>$ /U_{lab}|<1 is met.

Figure 1 presents graphically the difference $<D_{lab}>$ from the reference value versus the nominal pressure. Figure 2 presents the ratio of the laboratory difference to its uncertainty $<D_{lab}>/U_{lab}$.

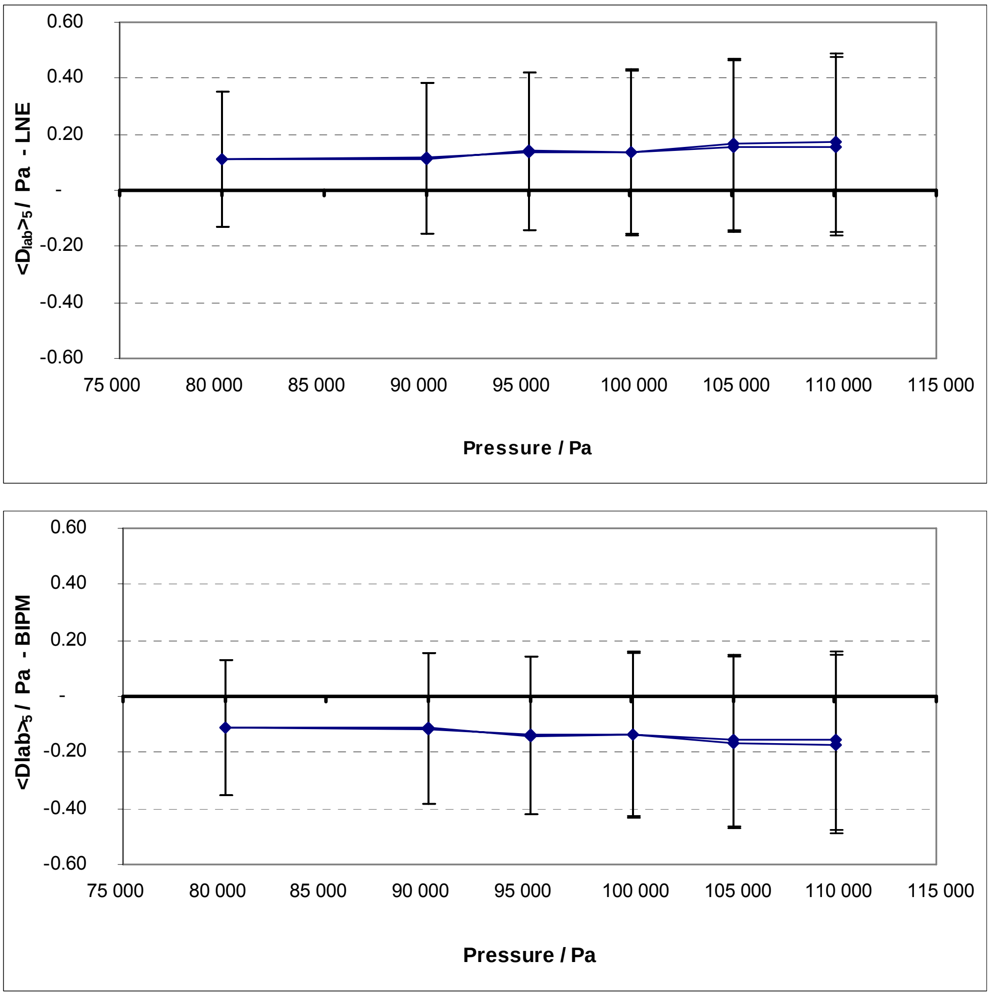

Figure 1: *Deviations from the reference values $<D_{lab}>$. The bars represent the expanded uncertainty of $<D_{lab}>$,*

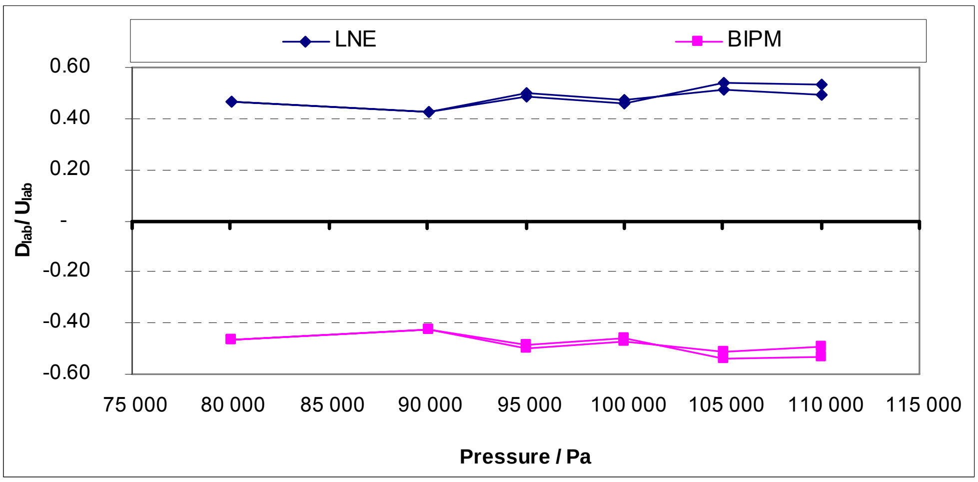

Figure 2: *Ratio $<D_{lab}>$ /U_{lab}*

7.2.2 Difference between BIPM and LNE

The difference between the laboratories lab*i* and lab*j* is calculated as:

$$D_{ij}=D_i-D_j = P_i-P_j$$

Since a correlation coefficient of 0,7 has been considered between both standard pressures, the uncertainty of the difference is estimated as:

$$u_{ij} = \sqrt{u_{Pi}^2 + u_{Pj}^2 - 1{,}4 \times u_{Pi} \times u_{Pj}}$$

The degree of equivalence and its expanded uncertainty are expressed by $\langle D_{ij} \rangle$ and U_{ij} respectively. $\langle D_{ij} \rangle$ is the mean of five differences D_{ij}. The results are reported in table 8.

	j	**LNE**			**BIPM**		
i	**Reference pressure / Pa**	$\langle D_{ij} \rangle$ **/ Pa**	U_{ij} **/ Pa**	$\langle D_{ij} \rangle / U_{ij}$	$\langle D_{ij} \rangle$ **/ Pa**	U_{ij} **/ Pa**	$\langle D_{ij} \rangle / U_{ij}$
LNE	80 033,660				0,22	0,53	0,41
	90 037,995				0,23	0,60	0,39
	95 040,190				0,28	0,63	0,45
	100 042,220				0,28	0,66	0,43
	105 044,290				0,32	0,69	0,46
	110 046,355				0,31	0,72	0,43
	110 045,970				0,34	0,72	0,47
	105 044,000				0,34	0,69	0,49
	100 042,015				0,27	0,66	0,41
	95 039,805				0,27	0,63	0,43
	90 037,795				0,23	0,60	0,39
	80 033,655				0,23	0,53	0,43
BIPM	80 033,660	- 0,22	0,53	- 0,41			
	90 037,995	- 0,23	0,60	- 0,39			
	95 040,190	- 0,28	0,63	- 0,45			
	100 042,220	- 0,28	0,66	- 0,43			
	105 044,290	- 0,32	0,69	- 0,46			
	110 046,355	- 0,31	0,72	- 0,43			
	110 045,970	- 0,34	0,72	- 0,47			
	105 044,000	- 0,34	0,69	- 0,49			
	100 042,015	- 0,27	0,66	- 0,41			
	95 039,805	- 0,27	0,63	- 0,43			
	90 037,795	- 0,23	0,60	- 0,39			
	80 033,655	- 0,23	0,53	- 0,43			

Table 8: *Differences between the laboratories* $\langle D_{ij} \rangle$**,** *expanded uncertainty (k=2)* U_{ij}*,* $\langle D_{ij} \rangle / U_{ij}$

8 CONCLUSION

The results of the comparison can be considered as satisfactory as the deviations from the reference value are inside the estimated uncertainty. The ratio $<D_{ij}>/U_{ij}$ between both laboratories is approximately 0,5.

The standard deviation of the mean deviation $<D_{lab}>$ of each laboratory from the reference value is less than 0,031 Pa, representing 3,3 x 10^{-7} x P in the worst case. Over the whole pressure range, the mean differences $<D_{ij}>$ were between 0,22 Pa and 0,34 Pa, which is equivalent to relative differences between 2,8 x 10^{-6} and 3,2 x 10^{-6}. These results support the hypothesis that a pressure balance equipped with a 20 cm^2 effective area piston-cylinder unit allows a transfer of pressure within 1 x 10^{-6}, relative value, in absolute pressure mode.

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