

**Final report on the CIPM Key Comparisons for Natural Gas at High-Pressure
Conducted in November / December 2004
CCM.FF-5.a**

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**This Report has been approved by
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CCM.FF-5.a-Final Report; KCRV for natural gas at high pressure

- CCM.FF-5.a and CCM.FF-5.b; Pilot laboratories: PTB and NMI-VSL

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1 INTRODUCTION

The CIPM decided, in accordance with the CIPM Mutual Recognition Arrangement (MRA) [2], to conduct Key Comparisons (KCs) [1] among national primary standards of selected NMIs in the subject field high-pressure gases. This includes natural gas and compressed air and/or Nitrogen. The members of the responsible CCM Working Group for Fluid Flow (WGFF) elected PTB and NMI-VSL as the pilot laboratories for this Key Comparison (KC). This report comprises the results for K5.a as final Draft B. Draft A has been agreed by all participants and the CCM in April 2005.

Participants for the Natural Gas Loop:

PTB-*pigsar*TM, Dorsten, NMI

NMI-VSL, Groningen, Bergum and Westerbork, NMI

LNE-LADG, Alfortville, NMI

Since January 25th 2005, LNE is the former BNM. LNE stands for **Laboratoire National de métrologie et d'Essais**. For this reason in this paper, the previous abbreviation BNM is still used in the figures.

The invited participants: NIST-CEESI, NIST-SwRI and NRC-MC-TCC (as they are already traceable to NMI-VSL) have refrained after 4 years of negotiation from any activity in the CIPM Key Comparison for high-pressure natural gas K5.a

The K5.a has been conducted during November/December 2004 and the first draft A has been agreed by all participants of K5.a in April 2005.

2 GENERAL INFORMATION

This draft was prepared in accordance to the *Guidelines for CIMP Key Comparisons* [1]. The KCs described here, have been performed to fulfill the requirements of the CIPM MRA [2] and the requirements from the CIPM Committee Consultative for Mass and Related Quantities [3]. The aim of these KCs is to verify the claimed Calibration and Measurement Capabilities (CMCs) of the NMIs and to quantify the degree of equivalence of the national flow standards as maintained in the participating NMIs. In addition, a CIPM Key Comparison Reference Value (KCRV) should be the outcome of a key comparison. To achieve the intended quantification, these KCs are intended to produce a set of tabulated results: the first set of tables presents the measured differences between the participants and the KCRV and the second set will quantify the laboratory-to-laboratory equivalencies with the associated uncertainties of these differences, and the last set shall comprise the degree of equivalence of all laboratories to the KCRV. The KC 5.a participants will give visual presentation of the degree of equivalence E_n as recommended recently [4, 5, 6].

In these E_n tables all measured data (meter readings) will be associated with the values of degree of equivalence $E_n = |d| / U(d)$. d means the bias between KCRV and measured value and $U(d)$ is the corresponding uncertainty of this bias between KCRV and the measured value as explained in chapter 5. Following this recommendation, it will be possible to describe the degree of equivalence of a laboratory to the KCRV using a dimensionless number. E_n should be between 0 and 1 and may go up to 1.2

There are some good reasons to express the degree of equivalence in this way, as natural gas is traded across international borders frequently. The participant would like to make sure that the customers can not create a buyer and a seller market.

Organization of the high-pressure natural gas comparison loop

The high-pressure KC comprises the circulation of a tandem meter travel standard among all participants in a single loop. As the number of participants was finally very low, only one of the originally proposed four tandem packages has been used finally.

Loop: PTB-*pigsar*TM => NMi-VSL => LNE-LADG => PTB-*pigsar*TM: end of loop

As it has been decided during the KC 5.ab meetings, transfer package #2 has been used. This package comprises of a turbine and an ultrasonic meter (DN150) put in series, compare Fig. 2. For the pressure ranges and flow rate samples, see Fig. 1.

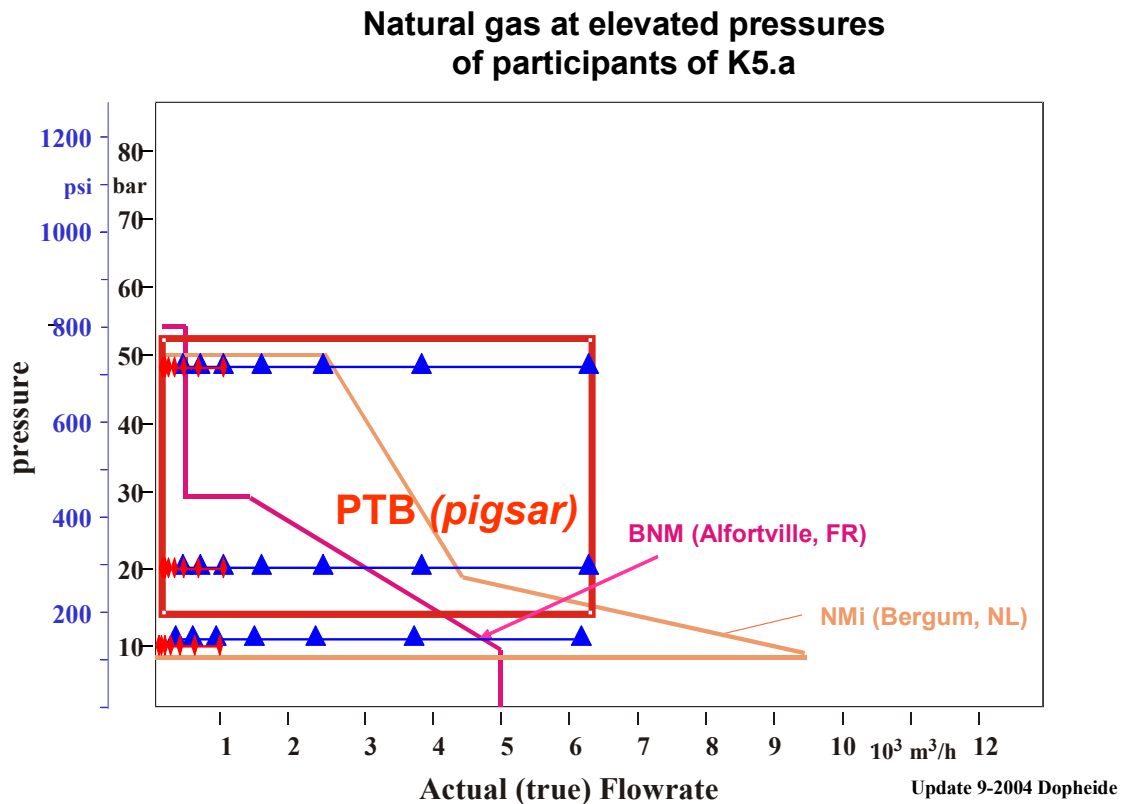


Fig.1: Visualized pressure and flow ranges of potential participants in the CIPM Key Comparison for natural gas. The suggested calibration points are indicated as triangles. The suggested G650 and G4000 could cover the flow ranges accordingly. As it turned out after 4 years of negotiation with all potential partners, only PTB (*pigsar*TM), NMI-VSL and LNE-LADG have been ready to conduct the K5.a loop. All the others like NIST and NRC and their associated laboratories have cancelled their participation. Therefore, a G650 transfer meter has been used in K5.a finally.

The selected calibration points are visualized in Figure 8.

3 THE TRANSFER PACKAGE

The transfer packages for this KC consist of two commercial G650 meters put in series. Due to generous donations from the major manufactures Elster and Instromet, an appropriate selection of a turbine and an ultrasonic meter has been made by the pilot laboratory. The meters have been optimized in the meantime by the pilots and the manufacturers to provide for meters with smooth error curves.

The basic arrangement and geometry of each transfer package can be seen below in Fig. 2

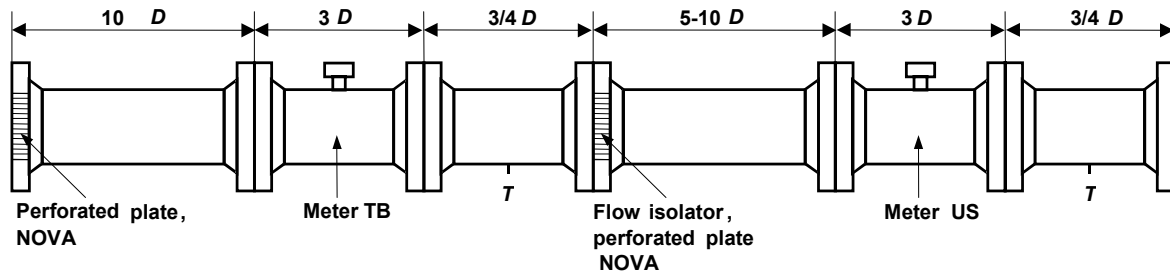


Fig. 2: Principal arrangement of the transfer packages.

The main properties on these meters:

- Flanges: ANSI 600 pressure up to 90 bar
- Temperature tabs T 1,5 D downstream of meters
- Selected Reynolds-balanced meters with flat error curves
The pilot laboratories have done this selection in close collaboration with the manufacturers.
- Output signals: high frequency pulses, NAMUR signals, open collector adapter
- Inlet lengths: 10D for ultrasonic, 5D for turbines

Each meter is provided with its own inlet and outlet sections, referred to as part #1 and part #2. Both meters are equipped with NOVA flow straighteners.

Package No. #2

Loop:	High pressure natural gases
Size (Q_{\max} , Diameter):	650 m ³ /h; DN = 150 mm (= 6 ")
Total length of package:	35 D = 5,3 m

Type of meter 1:	turbine G650
Manufacturer:	Elster
Length of part 1:	10D; 3D; 3D;
(inlet, meter 1, outlet)	

Type of meter 2:	ultrasonic G650
Manufacturer:	Instromet
Length of part 2:	10D; 3D; 3D;
(inlet, meter 2, outlet)	

A full documentation of the transfer package #2 DN 150 mm (=6") for natural gases is presented in the following Fig. 3

Package No. #2

Loop: High pressure natural gases
Size (Q_{max} , Diameter): 650 m³/h; DN = 150 mm = 6 "
total length of package: 32 D = 5,273 m

type of meter 1: turbine G650
 manufacturer: Elster

Nr. 83034939
 Length of part 1: 10D; 3D; 3D; 2403mm
 (inlet, meter 1, outlet) Flanschd. 357mm
 Weight 350kg



type of meter 2: ultrasonic G650
 manufacturer: Instromet
 Length of part 2: 10D; 3D; 3D; 2870mm
 (inlet, meter 2, outlet) Flanschd. 356mm
 Weight 430kg

Q-Sonic 4
 Nr. 2739



25.3.2004

Fig.3: The file above presents the drawing of package #2 DN 150 mm consisting of a turbine and an ultrasonic meter and provides for a full documentation.

In the following chapter, the detailed investigations on reproducibility of the transfer meter and the facilities will be summarized to demonstrate that the transfer package (and both meters) fulfill the prerequisites to compare facilities with claimed uncertainties between 0.16 % and 0.30 %.

3.1 *Reproducibility of the transfer packages and facilities*

In this chapter the impact of reproducibility/repeatability of the transfer package as well as the reproducibility of the facilities on the KC comparisons will be considered and conclusions will be drawn.

The investigation of reproducibility of the transfer meters are based on the evaluation of correlated meter readings as it is described in detail by [Pöschel], see [7].

Prerequisites for this procedure are:

- Two independent results of two meters under test (MuT, 1 and MuT, 2) simultaneously measured at the same reference (facility *fac*);
- normality of the stochastic process;
- sufficient Degree of Freedom (DOF), i.e. number of measurements;
- flow rates indicated by MuT Q_{MuT} and facility Q_{fac} are nearly the same (i.e. meter deviation is not far from zero, e.g. 1 %).

The measurand for the meter readings is the meter deviation f defined as following:

$$f = \left(\frac{Q_{\text{MuT}}}{Q_{\text{fac}}} - 1 \right) \cdot 100\% \quad (1)$$

where Q are the meter readings indicated by the MuT or the facility.

As the meter deviation is a function of flow rate, it is useful to relate the result of each measurement relative to the mean meter deviation f_m as a new zero line:

$$df = \left(\frac{Q_{\text{MuT}}}{Q_{\text{fac}}} - 1 \right) \cdot 100\% - f_m \quad (2)$$

With that, all results df determined at different flow rates are comparable.

Using the rules for propagation of uncertainty according to GUM we get for the stochastic part of uncertainty (i.e. the standard deviation of results s):

$$s_{df,abs}^2 = s_{Q_{\text{MuT}},rel}^2 + s_{Q_{\text{fac}},rel}^2 \quad (3)$$

Please note that the absolute standard deviation $s_{df,abs}$ (expressed in %) is the quadratic sum of the relative standard deviation $s_{Q_{\text{MuT}},rel}$ and $s_{Q_{\text{fac}},rel}$ (also expressed in %)!

Further on in this report the name of the variables in eq. (3) will be reduced to:

$$\mathbf{s}_{df}^2 = \mathbf{s}_{MuT}^2 + \mathbf{s}_{fac}^2 \quad (4)$$

For the evaluation of the correlated results of two MuT simultaneously measured we define two more terms:

$$\Delta_{p,i} = df_{1,i} + df_{2,i} \quad (5)$$

and

$$\Delta_{m,i} = df_{1,i} - df_{2,i} \quad (6)$$

Using again the rules of uncertainty propagation with the requirements mentioned above we get:

$$s_{\Delta_m}^2 = s_{MuT,1}^2 + s_{MuT,2}^2 \quad (7)$$

and

$$\mathbf{s}_{\Delta_p}^2 = \mathbf{s}_{MuT,1}^2 + \mathbf{s}_{MuT,2}^2 + 4 \cdot \mathbf{s}_{fac}^2 \quad (8)$$

Finally, with eq. (4), (7) and (8) we can determine:

$$s_{fac}^2 = \frac{1}{4} \cdot (s_{\Delta_p}^2 - s_{\Delta_m}^2) \quad (9)$$

$$\mathbf{s}_{MuT}^2 = \mathbf{s}_{df}^2 - \mathbf{s}_{fac}^2 \quad (10)$$

Eq. (9) and (10) are the final outcome for the estimations of reproducibility. As the estimates are based on a finite number of samples, the results of eq. (9) and (10) have also a confidence interval. It can be calculated as:

$$s_{LCL}^2 = \frac{DOF}{\text{ChiInv}\left(\frac{\alpha}{2}, DOF\right)} \cdot s^2 \quad (11)$$

$$s_{UCL}^2 = \frac{DOF}{\text{ChiInv}\left(1 - \frac{\alpha}{2}, DOF\right)} \cdot s^2 \quad (12)$$

(LCL – Lower Confidence Level; UCL - Upper Confidence Level; α – probability of error)
Because the standard deviation estimated from a number of samples follows a Chi-Square probability function, the confidence interval calculated with eq. (11) and (12) is not symmetrically!

The meaning of the standard deviation (calculated with eq. 9 and 10) depends on the sample of measurements. Roughly we can discuss three situations:

1. The sample is taken only at one flow rate and one pressure with a number of repetitions. Here we get the pure repeatability.

2. The sample contains measurements at different working points which were used in one experiment going up and down with the flow rate and/or pressure. Here, with the standard deviation, we get information about the repeatability and the hysteresis.
3. The measurements cover also a build-in-build-out (and a transport may be) and were done at different days. In this case the standard deviation includes information on the complete reproducibility also on the repeatability and the hysteresis.

Note that within an intercomparison, the impact of the first two parts (repeatability, hysteresis) to the average result of an experiment can easily be reduced by a higher number of individual measurements / repeats / samples.

3.2 Experiences with the transfer meters in pre-tests

To illustrate some typical results of such a reproducibility analysis of transfer meters, the outcome of the measurements in previous experiments between NMI-VSL (Netherlands) and PTB-*pigsar*TM (Germany) shall be given here. Fig. 4 shows a typical correlation plot for the residue (meter deviations df) of turbine meters on a transfer package using results from 1999 to 2003.

Please note that here we lost the information on repeatability, because we only saved the mean of repetitions at one flow rate. The correlation plot contains therefore only the information about the hysteresis and the overall reproducibility.

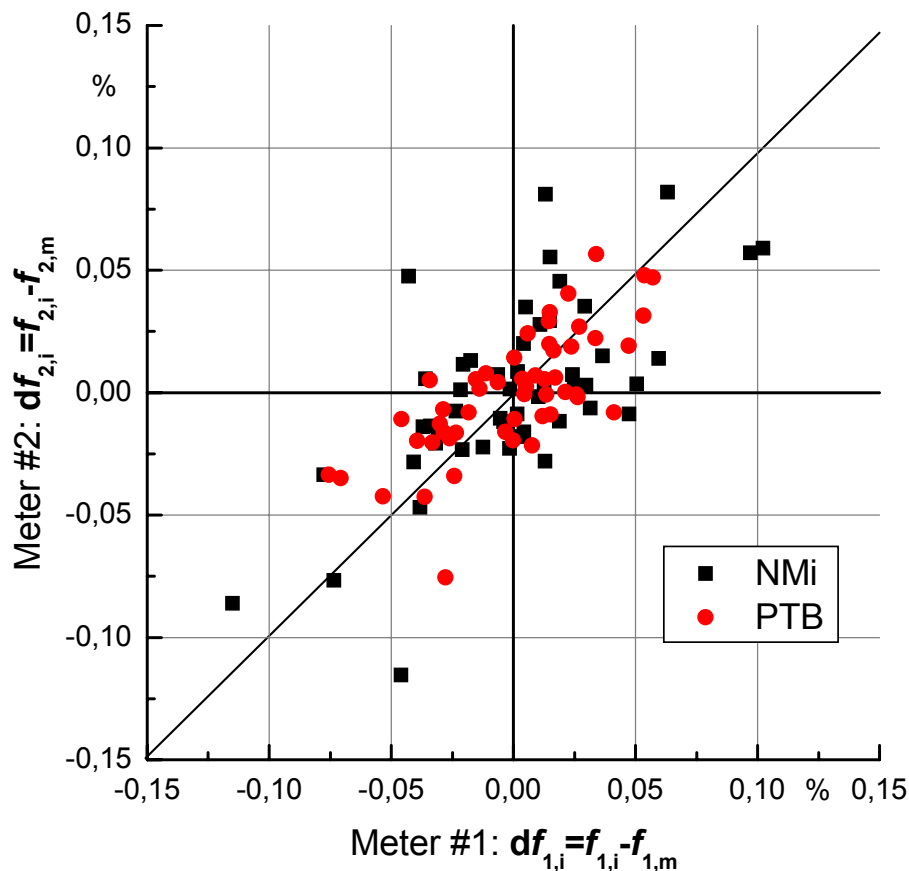


Fig. 4: Correlation plot of df for the transfer package DN250 used in previous experiments between NMI-VSL (Netherlands) and PTB *pigsar*TM (Germany).

For every transfer package used in the harmonization and for every facility (NMI-VSL and PTB *pigsar*TM) we can establish a plot according to Fig. 4 and we can calculate the related standard deviations using eq. (9) and (10). The results for the standard deviations of the transfer meters (i.e. here the reproducibility) are given in Fig. 5 at the 2σ -Level.

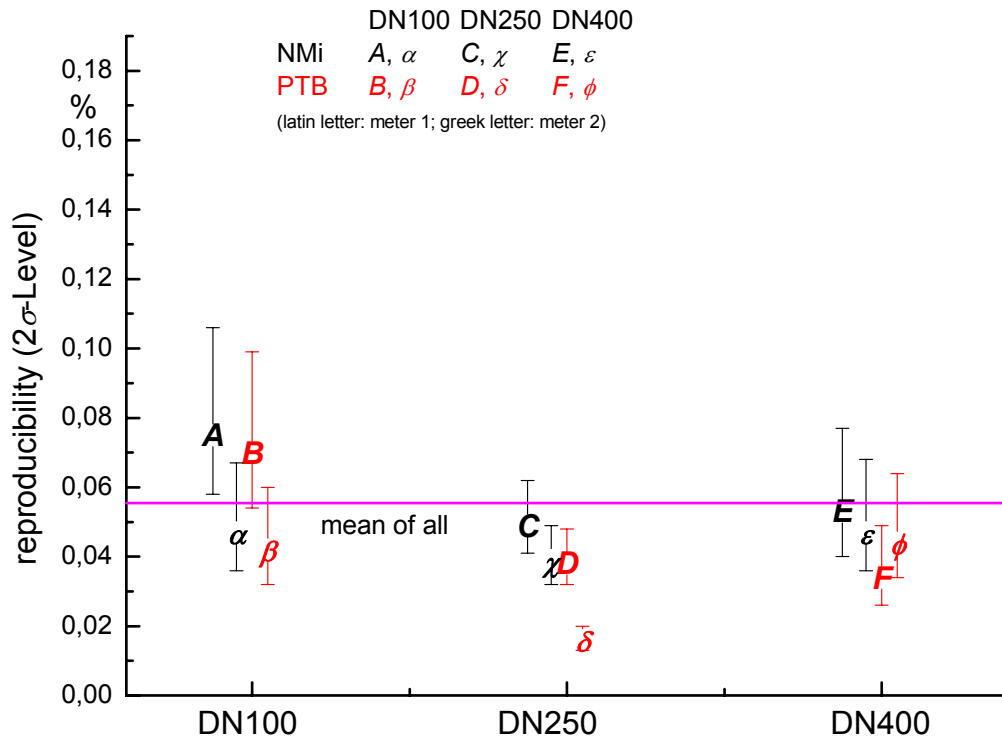


Fig. 5: Results of reproducibility calculation (eq. 9 and 10, see above) for all transfer meters used in previous experiments among NMI-VSL and PTB *pigsar*TM. The calculation is based on all measurement from 1999 to 2003.

It is to be seen in Fig. 5 that the typical value of reproducibility of a turbine meter is less than 0.06 %. Also it is very important to note, that the results do not significantly differ with the facility. This is an indicator for the reliability of such a determination process. It has to be emphasized that the turbine meters are commercial available turbine meters, which are also typically used in metering stations.

The results of reproducibility for the different test facilities are given in Fig. 6.

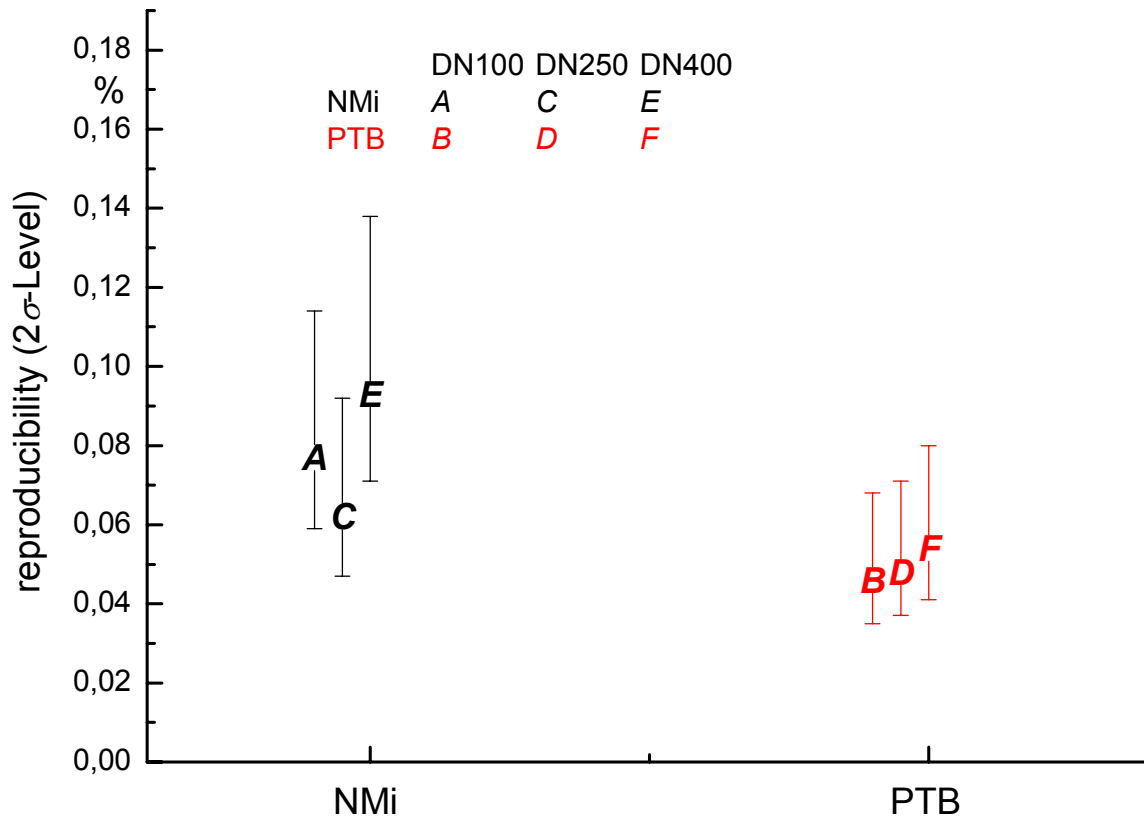


Fig. 6: Results of reproducibility calculation (eq. 9 and 10, see above) for the facilities of NMi-VSL and PTB *pigsar*TM in previous experiments. The calculation is based on all measurement from 1999 to 2003.

The reproducibility of the facilities is extremely good; much smaller than the claimed uncertainties.

3.3 Results for transfer meters used in K5.a

Using the background from gathered experience within harmonization it was the aim for the KC pilots to provide for transfer meters with a reproducibility equal or better than 0,06 %. For this purpose the chosen meters were tested at *pigsar*TM (Dorsten) several times and the corresponding values were calculated according to eq. (9) and (10). The correlation plot is shown in Fig 7 and the results of calculations are given in table 1.

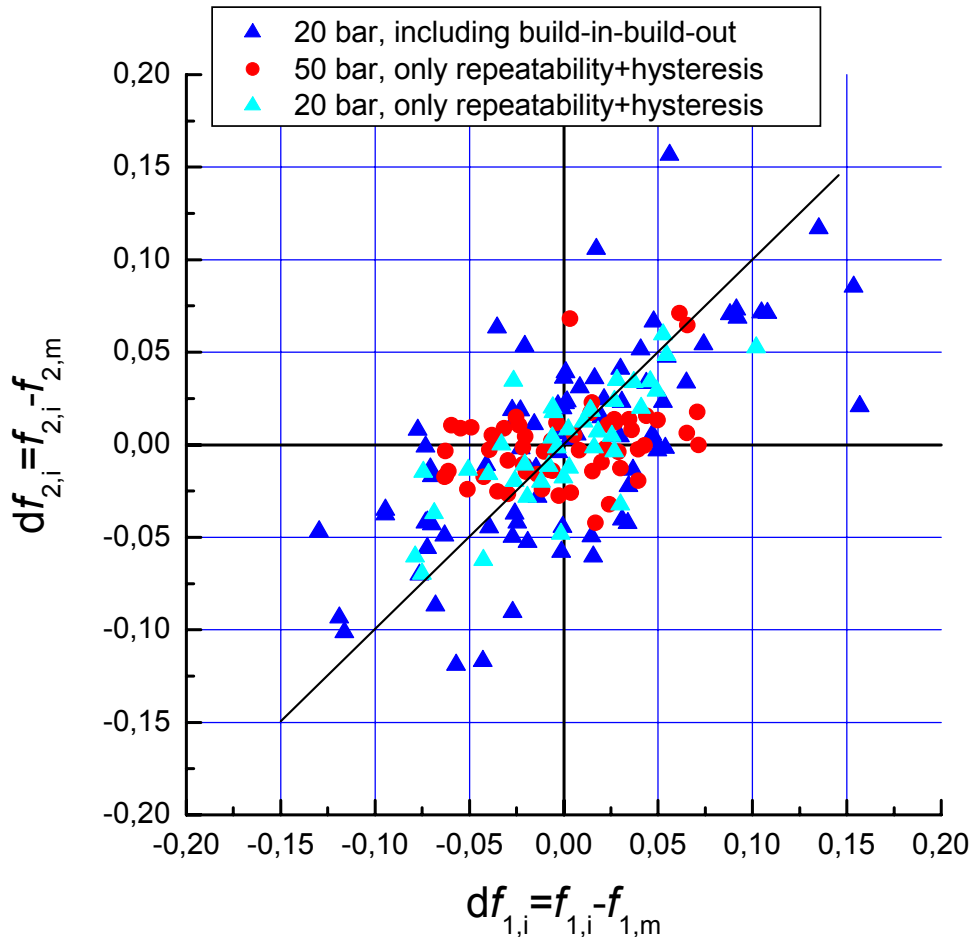


Fig. 7: Correlation plot of df for the **transfer package #2** DN150 (Meter 1: Ultrasonic meter; Meter 2: Turbine meter) designed for **K5.a** (natural gas).

The tabulated results are given in table 1 below.

	Repeatability + hysteresis only		Overall Reproducibility
	at 20 bar	at 50 bar	at 20 bar
USM	$0,051^{+0,016}_{-0,010}$	$0,067^{+0,016}_{-0,011}$	$0,077^{+0,014}_{-0,010}$
TM	$0,011^{+0,006}_{-0,002}$	$0,027^{+0,007}_{-0,004}$	$0,050^{+0,009}_{-0,007}$

Table 1: Tabulated results for repeatability, hysteresis and overall reproducibility of the transfer package #2

The turbine meter (TM) always fulfils the limit of 0,06 % but the ultrasonic meter (US) has already a repeatability of roughly 0,06 % and therefore an overall reproducibility of 0,08 %.

This does not mean that the ultrasonic meter has not the same quality! We have to consider the physics behind that.

Due to the turbulent fluid flow motion, it is necessary to integrate a certain minimum number of eddies of a turbulent pipe flow to come to a stable average of mean velocity or mean flow rate. The basic estimation of the integration time constant is $dt = D/u_m$ (D – diameter of pipe; u_m – mean velocity). This relation illustrates that the integration time necessary is proportional with the diameter and inverse proportional with the mean flow velocity.

The integration time of all measurements above was 60 seconds. A turbine meter integrates always the complete flow, whereas an ultrasonic meter integrates only a small part of the flow (area covered by its ultrasonic paths). Additionally, the flow inside a turbine meter is accelerated due to the bluff body at the location of integration (turbine meter, at least four times higher than it is natural) while the flow in the ultrasonic meter is always nearly the normal pipe flow. Therefore it is naturally, that the repeatability of an ultrasonic meter is poorer.

It yields to some recommendation for the numbers of repetitions and/or integration time. With increasing the number n of repetitions, the impact of repeatability is decreasing with square (n).

Recommendation for integration time and number of repeats for ultra sonic meters and turbine meters

From the results given in table 1 it can be derived that for turbine meters an integration time of 60 s is enough. Also the numbers of repetitions can be reduced to a minimum, e.g. three times what gives an opportunity to check the process. The participants can decide for themselves to perform a higher number of measurements if they need more repetitions to fulfill their specific requirements for the CMC.

For package #2 which includes also an ultrasonic meter, it is necessary to use at least seven repeats of 60 seconds measurement. In this case we reduce the scatter in the average result of the ultrasonic meter down to 0,025% ($0,067\% / 7^{0,5}$), see also table 1.

3.4 Impact of reproducibility to the quality of inter-comparison results

After determining the estimates for the parts of uncertainty propagated by the transfer meters to the measurement results, the question is, whether this level of quality of the transfer meters is sufficient to perform the intercomparison. To answer this, some simple calculations of virtual results of inter-comparison between two participants will be shown here. The calculation is done twice, once with the pure claimed CMC-uncertainties (i.e. with ideal transfer meters) and once with the combined uncertainties (U_{CMC} with additional uncertainties from the transfer meters).

The aim of the KC is to find out a reference value x_{ref} and the degree of equivalence of each participant to this reference as well as the degree of equivalence between the partners. The analytical definitions and the formulas are given in the appended paper of Cox [4]. For our purpose following is necessary:

- Claimed uncertainty of participant number i : $U_{CMC,i}$
We assume for our example two cases:
A) both facilities have a $U_{CMC,1} = 0,15 \%$ (0,15 % is the smallest value claimed in the CMC for high pressure gas) and
B) one facility claims $U_{CMC,1} = 0,15 \%$ and the other $U_{CMC,2} = 0,20 \%$
- Uncertainty propagated by the transfer meter $U_{TM} = \sqrt{U_{TM,repeat}^2 + U_{TM,hyst}^2 + U_{TM,repro}^2}$
With the statements above we get $U_{TM} = 0,060 \%$
- Uncertainty of the result x_i measured by participant i :
$$U_{x,i} = \sqrt{U_{CMC,i}^2 + U_{TM}^2}$$
- Uncertainty of reference value $U_{x,ref}$ (according to Cox [4]): $\frac{1}{U_{x,ref}^2} = \sum_{i=1}^n \frac{1}{U_{x,i}^2}$
(n – number of participants, $n = 2$ for our example)
- Uncertainty of the difference d_i of participant i to reference value:
$$U_{d,i,ref} = \sqrt{U_{x,i}^2 - U_{x,ref}^2}$$
- Uncertainty of the difference $d_{i,j}$ of participant i to participant j :
$$U_{d,i,j} = \sqrt{U_{x,i}^2 + U_{x,j}^2}$$

The results of calculation using the equations above are given in the following table 2 to make it transparent what is happening with the uncertainties when an ideal or real transfer meter is used.

Type of uncertainty	Example 1		Example 2	
	Ideal transfer meter	Real transfer meter	Ideal transfer meter	Real transfer meter
$U_{CMC,1}$	0,15 %		0,15 %	
$U_{CMC,2}$	0,15 %		0,20 %	
U_{TM}	0 %	0,060%	0 %	0,060 %
$U_{x,1}$	0,150 %	0,161 %	0,150 %	0,161 %
$U_{x,2}$	0,150 %	0,161 %	0,200 %	0,209 %
$U_{x,ref}$	0,106 %	0,114 %	0,120 %	0,128 %
$U_{d,1,ref}$	0,106 %	0,114 %	0,090 %	0,099 %
$U_{d,2,ref}$	0,106 %	0,114 %	0,160 %	0,165 %
$U_{d,1,2}$	0,212	0,228	0,250 %	0,264 %

Table 2: Estimated uncertainties for two typical situations, example #1 and #2 in a high-pressure flow laboratory comparison.

From the results presented in table 2 the following conclusions can be drawn:

- The use of transfer meters with level of quality (reproducibility and repeatability) mentioned in this KC increases the uncertainties of inter-comparison results (reference value, differences of participants to reference value as well between participants) in the order of only 10 % or less. (as compared with an ideal meter)
- We can determine differences in flow rate in the order of 2/3 of the claimed uncertainties of the participants, even we use ideal or real transfer meters.

4 MEASUREMENT PROGRAM in KC 5.a

4.1 Flow rate ranges in KC 5.a

The measurements should cover a wide range of flow rates and pressures according to the calibration capabilities of each participant. Hence, the aim of this protocol is to have the maximum overlapping operational range among all participants.

The following pressures and flow ranges are intended to be used in K5.a for high-pressure natural gas (using transfer packages #2):

Flow range = From 65 to 1000 m³/hr, Pressures: 10, 20 and 47 bar

The following table 3 presents the agreed test- and measurement points (loads) as well as pressures, at which the transfer standards will be calibrated.

Institute	PTB- <i>pigsar</i> TM				NMI-VSL				LNE-LADG			
	10	20	47	≥ 65	10	20	47	≥ 65	10	20	47	≥ 65
at pressure (bar)												
Flow rate												
65		X	X		X	X	X		X	X		
100		X	X		X	X	X		X	X		
160		X	X		X	X	X		X	X		
250		X	X		X	X	X		X	X		
400		X	X		X	X	X		X	X		
650		X	X		X	X	X		X	X		
1000		X	X		X	X	X		X			

Table 3. High Pressure Natural Gas Test Loop: Package No. #2: G650/ DN 150 (6”).

The following Figure 8 visualizes the applied flow rates and pressures during K5.a at PTB-*pigsar*TM, LNE-LADG(BNM) and NMI-VSL.

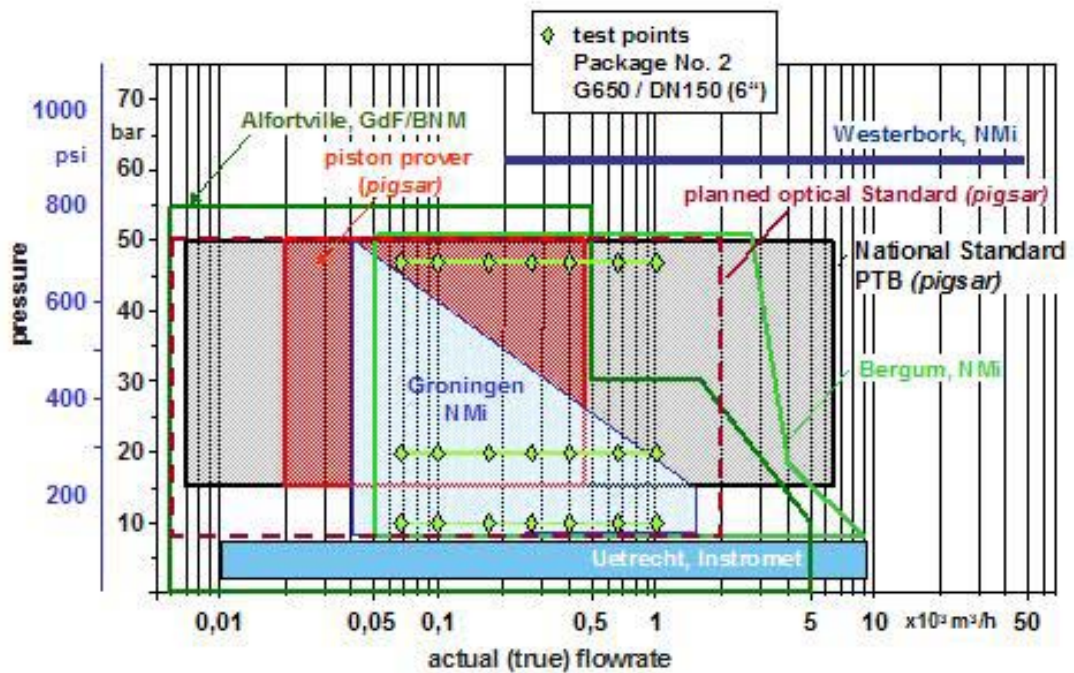


Fig.8: Visualization of selected flow rates as well as pressures during K5.a at PTB-*pigsar*TM, LNE-LADG (BNM) and NMI-VSL

4.2 Handling of results

The test results have been placed in Microsoft Excel spreadsheets. For each package one separate file shall be created.

The Excel data file for each package comprises all flow rates (loads), pressure stages, and calibration points proposed for the laboratory comparison. Each laboratory has documented the results for the flow rates and pressures, which could be realized.

The main results in the files are the meter deviation f and the corresponding total uncertainties. This total uncertainty is the quadratic sum of the uncertainty U_{CMC} of the laboratory given in the CMC-table and uncertainty caused by the transfer meter U_{TM} :

$$U_{f,total} = \sqrt{U_{CMC}^2 + U_{TM}^2} ; \text{ with } U_{TM}^2 = U_{TM,rep}^2 + U_{TM,hyst}^2 + U_{TM,repr}^2 \text{ (see also chapter 3.4)}$$

The reproducibility $U_{TM, repro}$ and the hysteresis $U_{TM, hyst}$ of transfer meters are estimated as mentioned in chapter 4.3 above. Based on the procedures in chapter 3.3 the repeatability of transfer meters $U_{TM, rep}$ will be calculated for every measured meter deviation.

Please keep in mind that the claimed uncertainties (CMCs) should cover all sources of uncertainties describing the realization of the unit at the place of metering.

5 THE OUTCOME OF THE K5.a – Degree of equivalence of the laboratories

This chapter shall explain how to get the degree of equivalence between all participants and the degree of equivalence of laboratories with respect to the KCRV.

In Fig. 9 one recognizes “measured” data at 5 fictive laboratories for a single meter and the calculated KCRV function. Using the nomenclature there, the following quantities can be obtained:

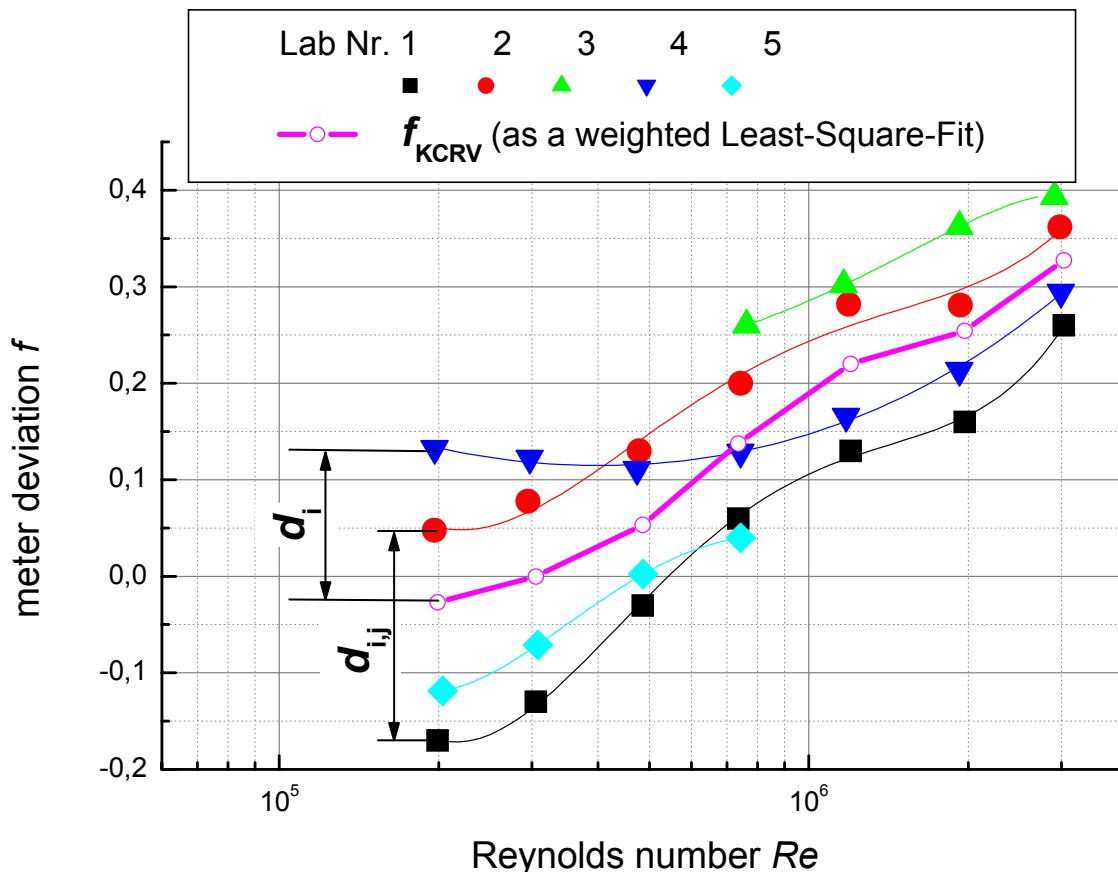


Fig. 9: Illustration of differences d_i between laboratories and KCRV and $d_{i,j}$ between the laboratories itself.

Using the results in Fig. 9, one can calculate the following quantities:

$$d_i = f_i - f_{KCRV}; \quad d_{i,j} = f_i - f_j \quad (13)$$

A so-called E_n -criterion can be calculated, which describes the degree of equivalence, see [4-6]. This E_n -criterion is widely used for accredited laboratories and which describes the degree of equivalence nicely.

$$E_{n_i} = \frac{|d_i|}{U(d_i)}; \quad U(d_i) = \sqrt{U_{MuT}^2 - U_{KCRV}^2} \quad (14)$$

with

$$U_{MuT}^2 = U_{CMC,i}^2 + U_{TM}^2$$

d_i means the bias between the KCRV and the measured value and $U(d_i)$ is the corresponding uncertainty of this bias between the KCRV and the measured value, which will be calculated according to Cox, see [4], chapter 5, section 4 equations (3) and (5). Such a non-dimensional number describes very well the degree of equivalence between the laboratories and the KCRV. Such a number E_n is in use in EAL Interlaboratory Comparisons, see e. g. [5, 6]. The meaning of “ E_n numbers” or “normalized error” has been pointed out e.g. in [5].

Following this international accepted way, it will be possible to describe the degree of equivalence of a laboratory to the KCRV using a dimensionless number. E_n should be between 0 and 1 and may go up to 1, 2.

$E_{n_i} = \frac{|d_i|}{U(d_i)}$ will be the degree of equivalence which we suggest to use here for the high-pressure KCs.

The idea behind the E_n definition has been described e. g. by Wöger in [5].

This technique has been realized in pre-test by the pilot laboratories very successfully and will be used in K5.a.

The meaning will be explained in the following chapter using measured data between the European high-pressure national standards.

6 KCRV K5.a results using transfer package #2

6.1 Calibration at 10, 20 and 47 bar

We present all calibration curves for meter 1 (turbine meter) and meter 2 (ultrasonic meter) in the transfer package #2 at all pressures separately.

The errors bars of the participants are the claimed and agreed uncertainties of the facilities accordingly to the test rigs. PTB *pigsar*TM 0,16% and LNE-LADG 0,30 % in the entire flow rate range. For NMi-VSL flow rate uncertainties have been used: 2 sigma=0,23 at 10 bar; 2sigma=0,24 at 20 bar and 2sigma=0,28 at 47 bar. Compare with later tables 7.1 and 9.1.

The KCRV function has been calculated using the weighted average of all three participants, see black line. The corresponding black error bars indicates the uncertainty of

the KCRV. Calculations have been done using the weighed mean accordingly to Cox [4], chapter 5, equation (2).

Figure 10 shows measured calibration results for meter 1 (turbine meter) calibrated by all participants at 10 bar. Meter readings are plotted as a function of Reynolds number Re .

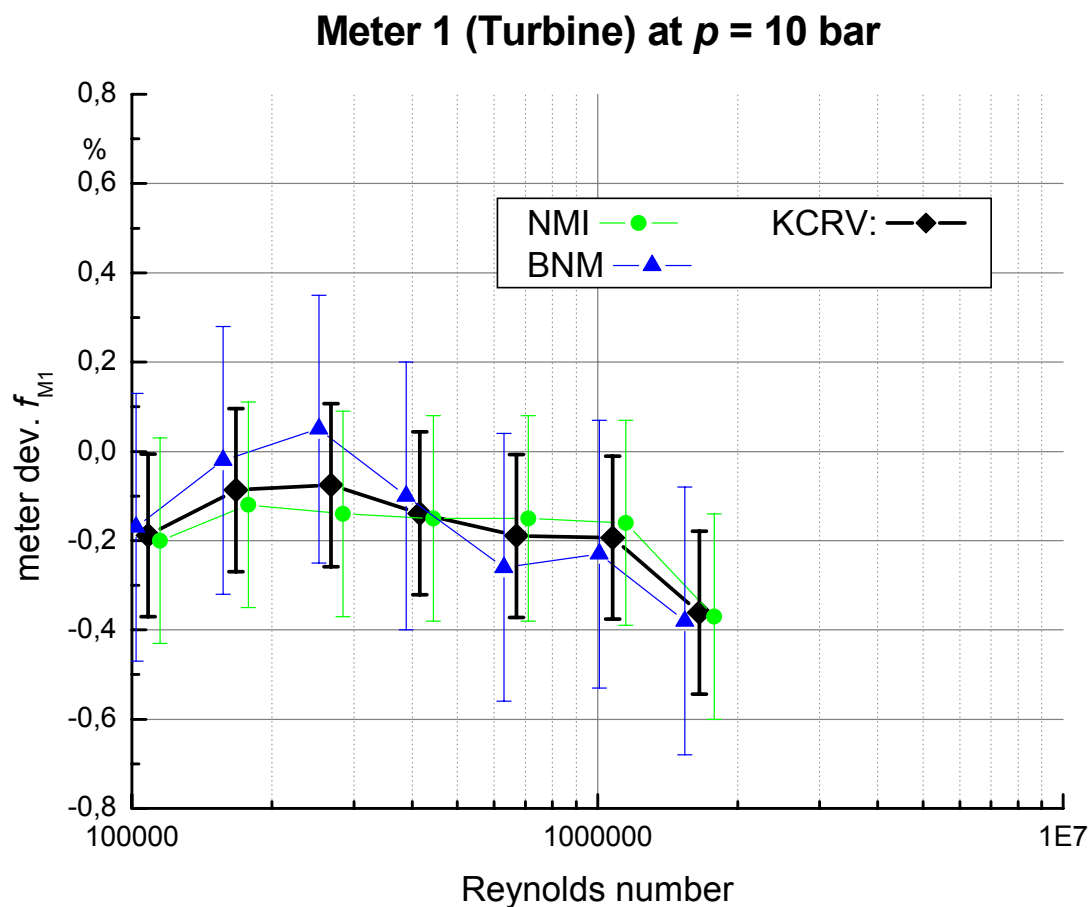


Fig.10: Measured error curves of turbine meter (meter 1) at 10 bar calibrated at LNE-LADG (BNM), and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes a quite good overlap of all institutes with the KCRV function and among each other.

Fig. 11 shows the calibration result for the turbine meter at 20 bar.

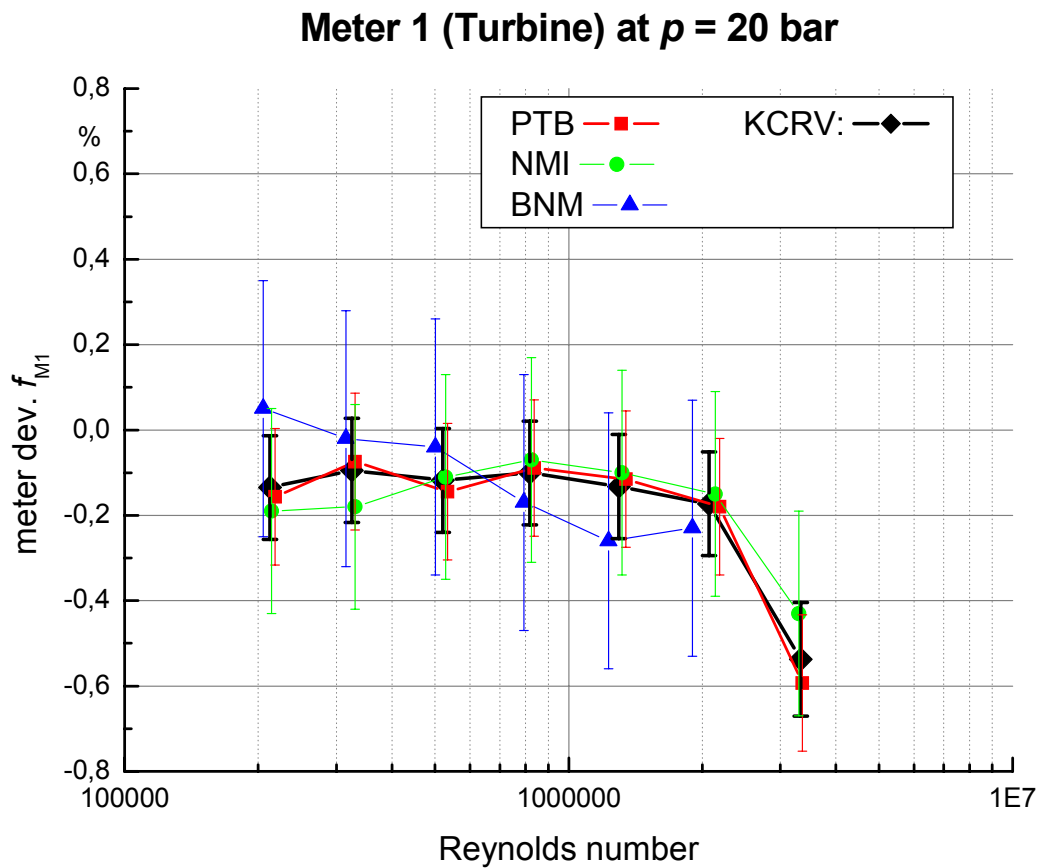


Fig. 11: Error curve of turbine meter (meter 1) at 20 bar calibrated at PTB-*pigsar*TM, LNE-LADG (BNM) and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes again a quite fine overlap of all institutes with the KCRV function and among each other.

Fig. 12 presents the calibration result for the turbine meter at 47 bar.

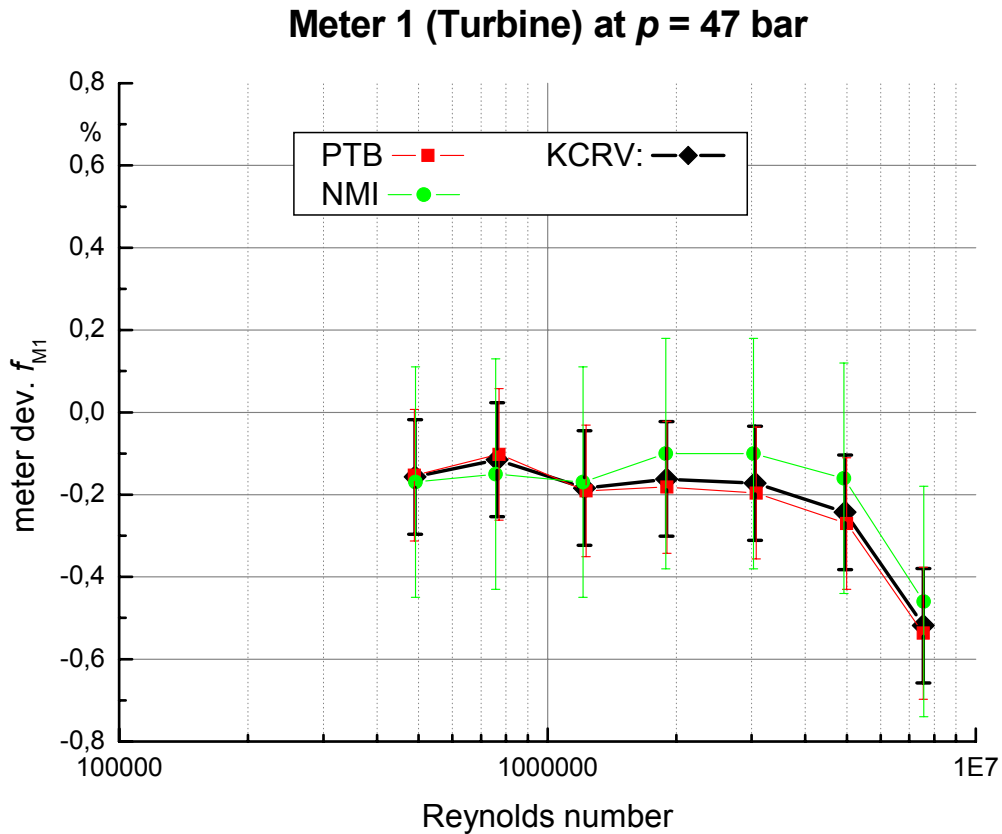


Fig. 12: Error curve of turbine meter (meter 1) at 47 bar calibrated at PTB-*pigsar*TM and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes an excellent overlap of all institutes with the KCRV function as well as among each other.

The following graphics present the calibration results for all calibrations using the results of meter 2 (ultrasonic meter).

Fig. 13 to Fig 15 present the calibration result for the ultrasonic meter from 10b to 47 bar.

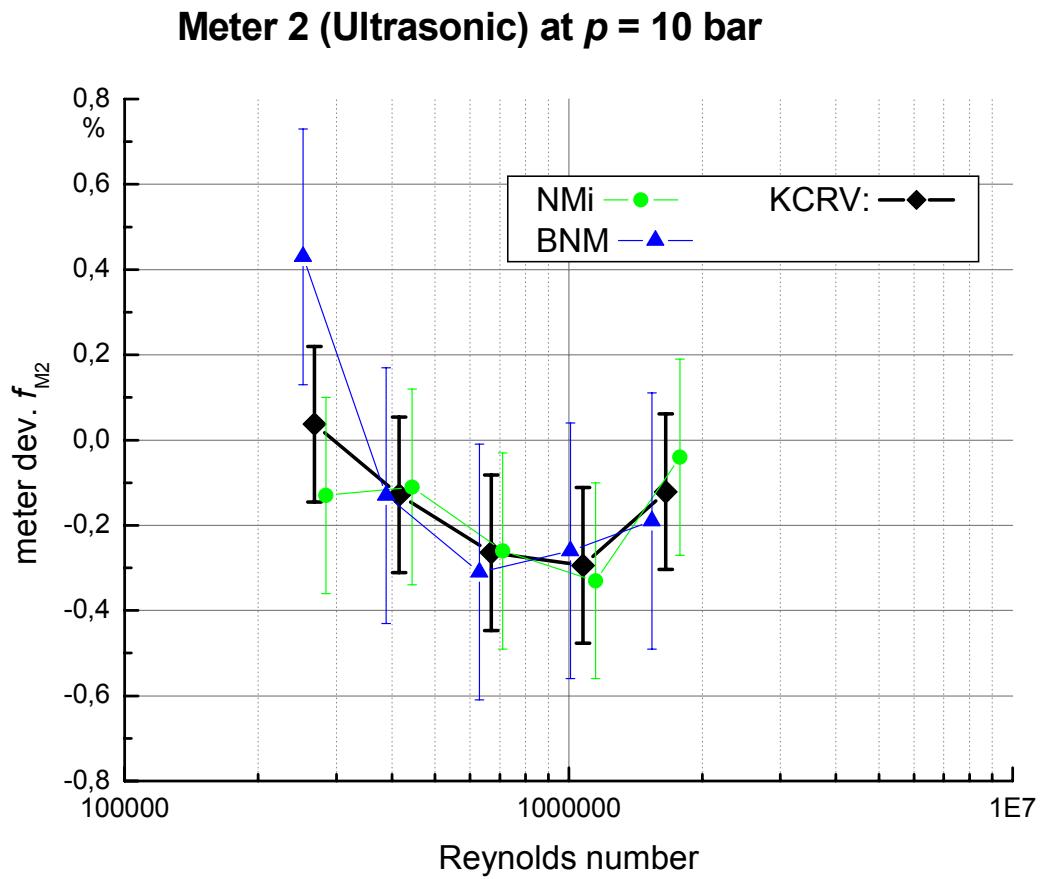


Fig. 13: Error curve of ultrasonic meter (meter 2) at 10 bar calibrated at LNE-LADG (BNM) and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes a quite good overlap of all institutes with the KCRV function.

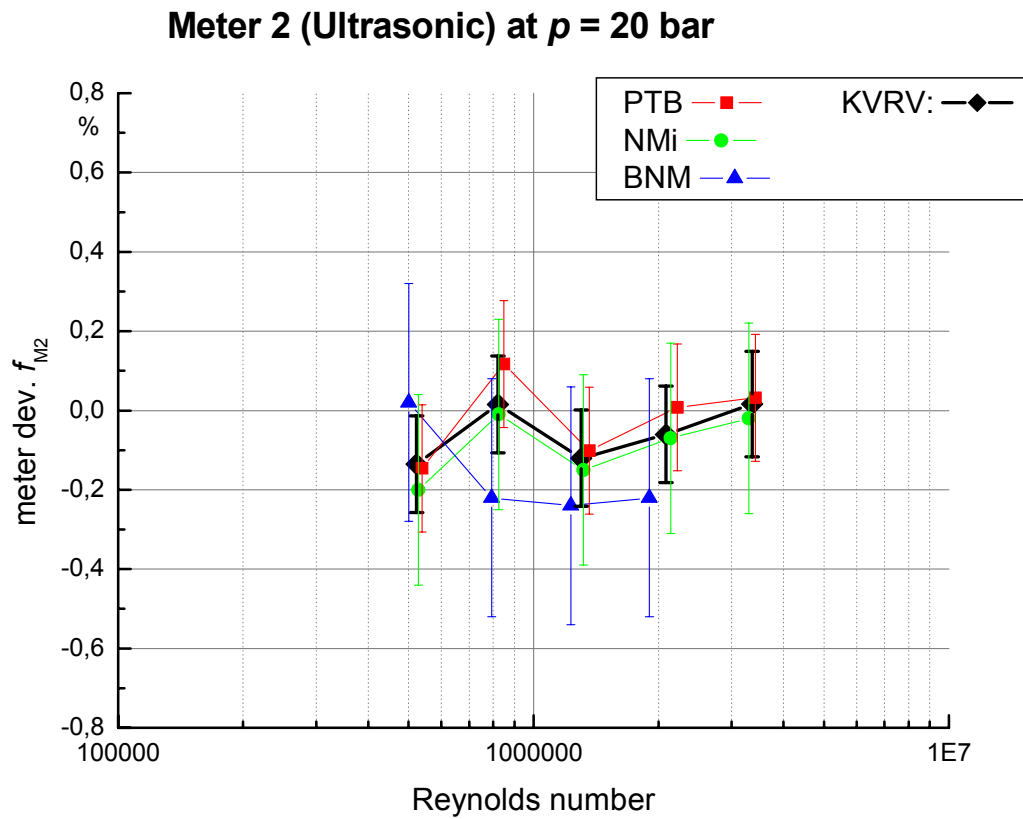


Fig. 14: Error curve of ultrasonic meter (meter 2) at 20 bar calibrated at PTB-*pigsar*TM, LNE-LADG (BNM) and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes a quite good overlap of all institutes with the KCRV function.

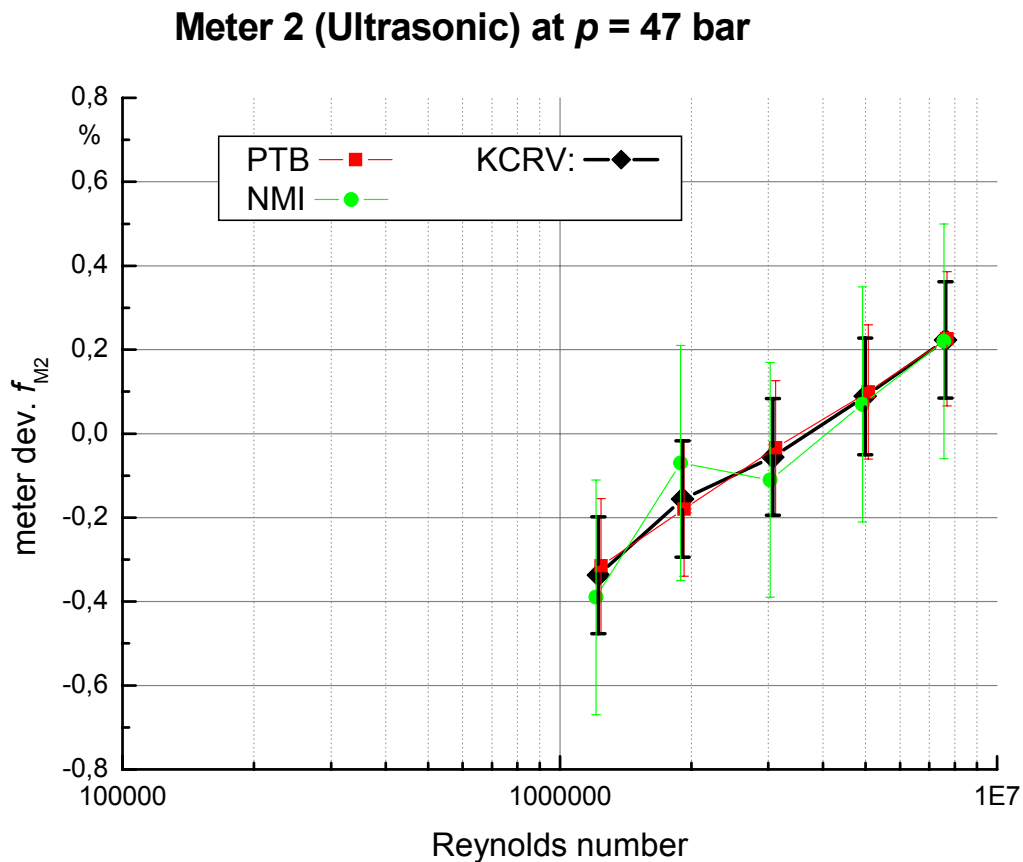


Fig. 15: Error curve of ultrasonic meter (meter 2) at 47 bar calibrated at PTB-*pigsar*TM and NMI-VSL. The KCRV is the weighted means of all single results.

One recognizes from Figures 10 - 15 a very good overlap of all institutes with the KCRV function.

Obviously, the reproducibility of the ultrasonic meter is a little bit lower compared to turbine meter, but good enough to fulfill the requirements for the comparison, see chapter 3.4

In the following chapter the degree of equivalence E_n between all laboratories and the KCRV will be presented, as this is the most important result of the KC.

6.2 Degree of Equivalence E_n with the KCRV

In order to quantify the degree of equivalence between the KVRV function and the participants as well as the degree of equivalence among the participants, we have used the relationship in equation (14) for E_n .

Fig. 16 to 18 will present the degrees of equivalence of PTB, NMI-VSL and LNE-LADG referred to the KCRV. In order to calculate this degree, all measured data for turbine and ultrasonic meters calibrated at all pressures (10 bar, 20 bar and 47 bar) have been used.

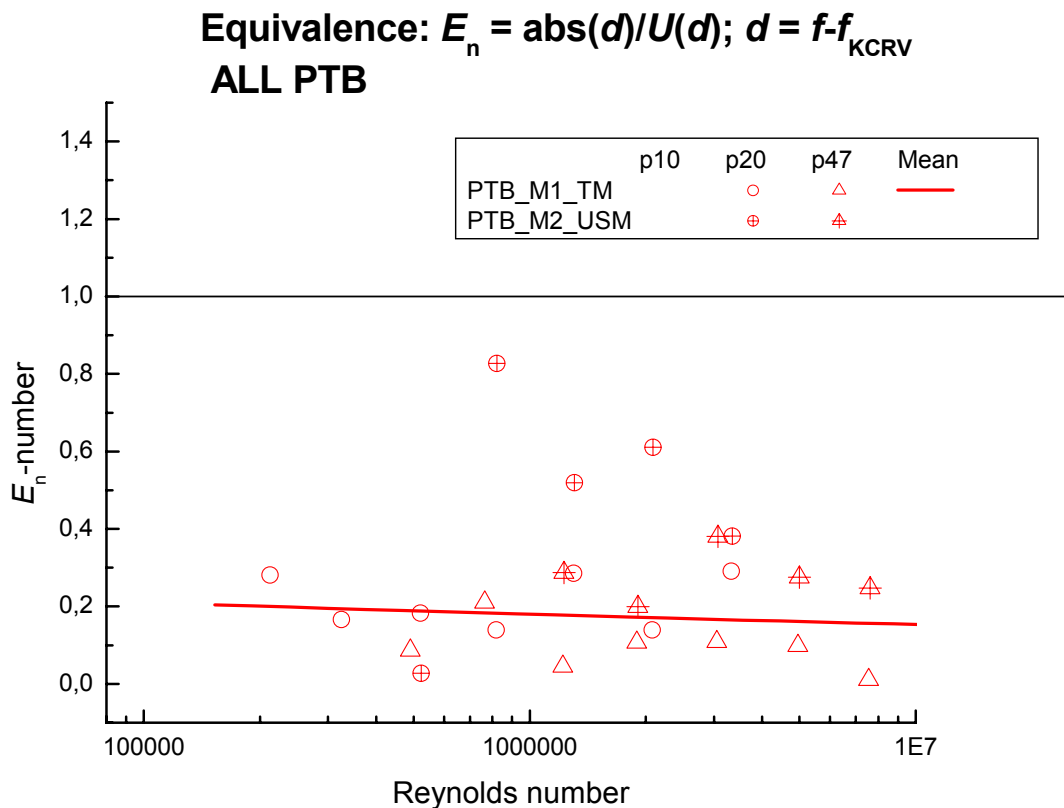


Fig. 16: Calculated degree of equivalence E_n of PTB with respect to the KCRV as determined from all calibration results, according to Fig. 10 to Fig. 11. PTB_M1_TM: turbine meter and PTB_M2_USM: ultrasonic meter

E_n should be as close as possible to “0”. $E_n=0$ means no deviation between the KCRV and the laboratory.

$E_n=0$ means complete agreement. $E_n=1$ means, that the error bars do overlap nicely.

The average of all results are plotted in red and is close to 0,2 for the entire flow rate range which means a very good degree of equivalence with the KCRV.

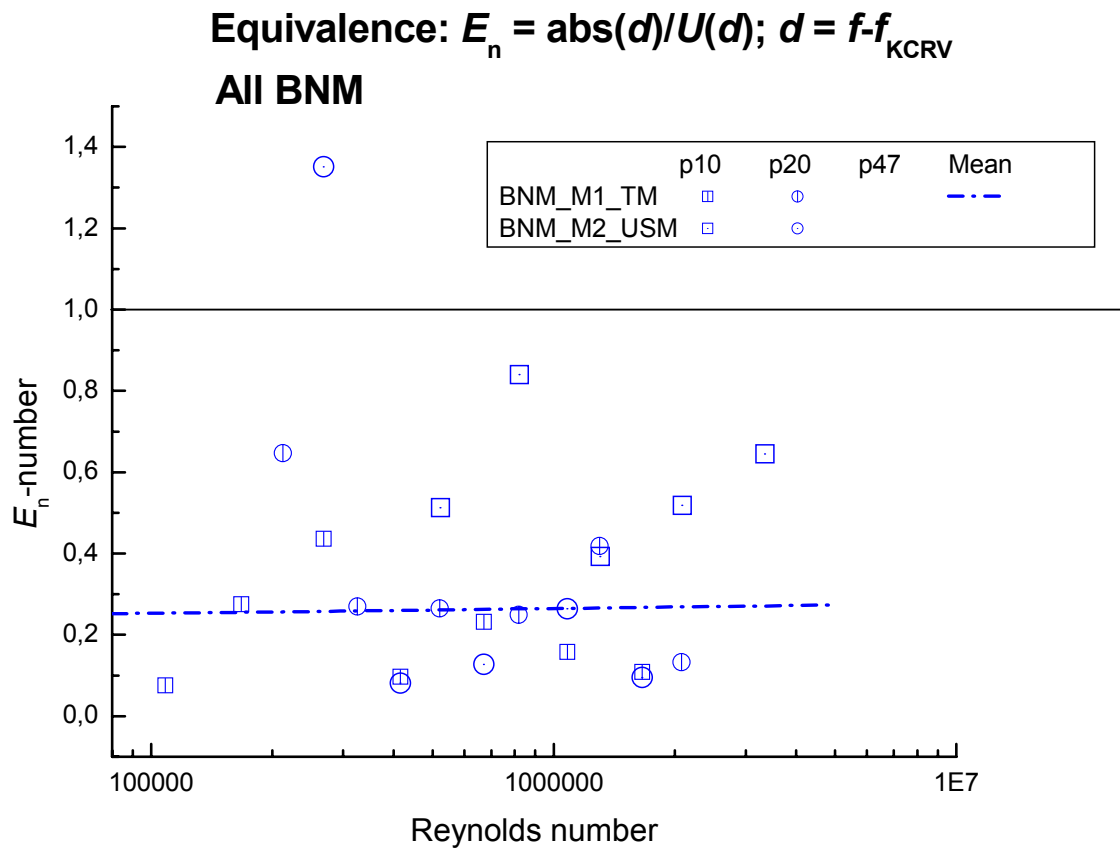


Fig. 17: Calculated degree of equivalence E_n of LNE-LADG (BNM) with respect to the KCRV as determined from all calibration results, according to Fig. 10 to Fig. 14. BNM_M1_TM: turbine meter and BNM_M2_USM: ultrasonic meter

The average of all results are plotted in blue and is close to 0,25 for the entire flow rate range which means a very good degree of equivalence with the KCRV.

$E_n=0$ means complete agreement. $E_n=1$ means, that the error bars do overlap quite well.

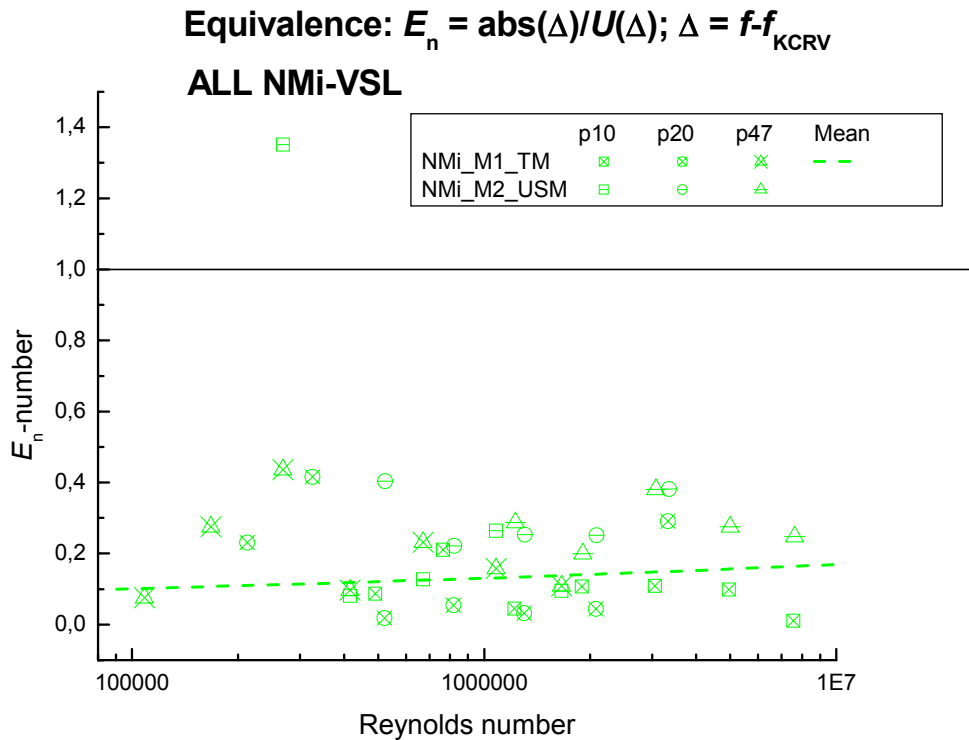


Fig 18: Calculated degree of equivalence E_n of NMI-VSL with respect to the KCRV as determined from all calibration results, according to Fig. 10 to Fig. 15. NMI_M1_TM: turbine meter and NMI_M2_USM: ultrasonic meter

The average of all results is plotted in green and is close to 0,15 for the entire flow rate range which means a very good degree of equivalence with the KCRV. $E_n=0$ means complete agreement. $E_n=1$ means, that the error bars do still overlap.

As we get a lot of single values of E_n for each measured result, it is helpful to define an overall value as a characteristic criteria for each laboratory taking part in the KC. Based on the fact that the degree of equivalence is a random variable with a log-normal probability density, it is the simplest approach to use the geometric mean as the characteristic value $E_{n_{\text{total}}}$:

$$E_{n_{\text{total}}} = \left(\prod_{i=1}^n E_{n_i} \right)^{\frac{1}{n}} = \exp \left\{ \frac{1}{n} \sum_{i=1}^n \ln(E_{n_i}) \right\} \quad (15)$$

Using the data obtained from Figures 15 – 18, one gets the following E_n -numbers, which characterize the degree of equivalence of PTB, NMI-VSL and LNE-LADG with respect to the KCRV.

PTB: $E_{n_{\text{total}}} = 0,15$;
 NMI-VSL: $E_{n_{\text{total}}} = 0,16$;
 LNE-LADG: $E_{n_{\text{total}}} = 0,30$;

The visualization of all En numbers is given in Fig. 18

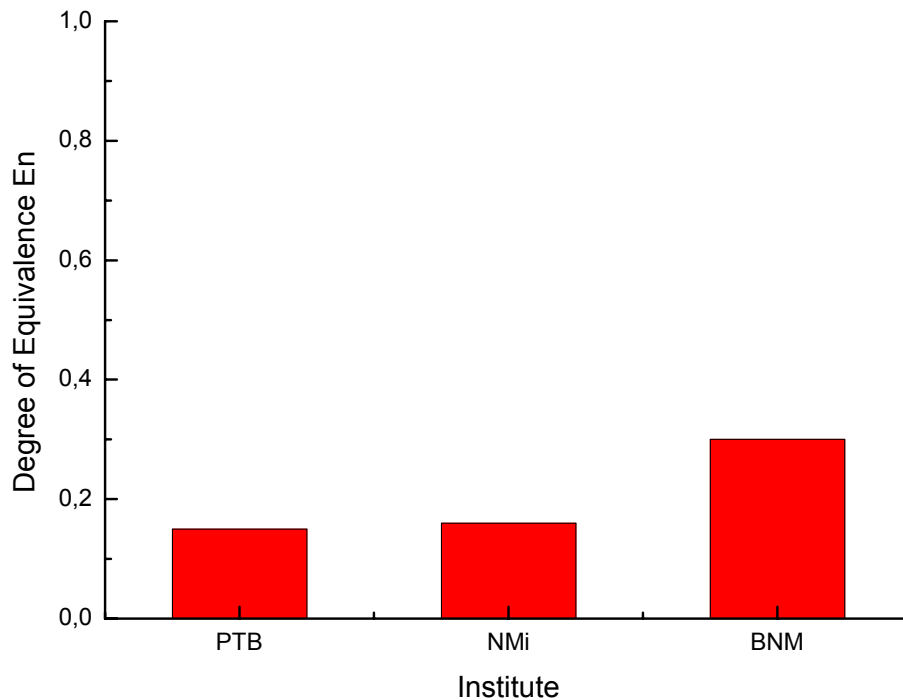


Fig. 19: Characteristic degree of equivalence En for all institutes PTB, NMI-VSL and LNE-LADG (BNM) based on the geometric mean (eq. 15) using all results in Fig. 15 – 18.

These data should be finally presented on the BIPM web site.

6.3 Tabulation of results: labs-to-KCRV, lab-to-lab and En

In this chapter, the tabulation of all deviations between the KCRV and the participating labs $d_{lab, KCRV}$ as well as associated uncertainty $U(d_{BNM, KCRV})$ and the degree of equivalence $En_{lab, KCRV}$ of the individual labs with the KCRV have to be presented.

Table 6.1 shows presents the results for LNE-LADG (France), Table 6.2 is for NMI-VSL and Table 6.3 is for PTB-pigsarTM.

Tables 6.1 to 6.3 present the following quantities:

$d_{lab, KCRV} = f_{lab} - f_{KCRV}$ (in %) and $U(d_{LNE-LADG, KCRV})$ ($k = 2$) for $d_{lab, KCRV}$ as well as

$En_{lab, KCRV} = \text{abs}(d_{lab, KCRV}) / U(d_{lab, KCRV})$

Table 6.1 shows the results for LNE-LADG (France).

Table 6.1 LNE-LADG (France) - KCRV

$$d_{\text{LNE-LADG,KCRV}} = f_{\text{LNE-LADG}} - f_{\text{KCRV}} \quad (\text{in } \%)$$

$$U(d_{\text{LNE-LADG,KCRV}}) (k = 2)$$

$$En_{\text{BNM,KCRV}} = \frac{\text{abs}(d_{\text{LNE-LADG,KCRV}})}{U(d_{\text{LNE-LADG,KCRV}})}$$

Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar		Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar		Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar		En Flow			
	M1	M2	M1	M2	M1	M2		M1	M2	M1	M2	M1	M2		M1	M2	M1	M2	M1	M2				
65	0,04	*	0,18	*	--	--	65	0,24	*	0,27	*	--	--	65	0,16	*	0,66	*	--	--	0,32			
100	0,08	*	0,07	*	--	--	100	0,24	*	0,27	*	--	--	100	0,32	*	0,27	*	--	--	0,29			
160	0,10	0,32	0,07	0,14	--	--	160	0,24	0,24	0,27	0,27	--	--	160	0,44	1,32	0,25	0,50	--	--	0,52			
250	0,02	-0,03	-0,07	-0,24	--	--	250	0,24	0,24	0,27	0,27	--	--	250	0,07	0,11	0,27	0,86	--	--	0,21			
400	-0,07	-0,04	-0,12	-0,12	--	--	400	0,24	0,24	0,27	0,27	--	--	400	0,28	0,17	0,45	0,43	--	--	0,31			
650	-0,06	0,04	-0,05	-0,15	--	--	650	0,24	0,24	0,27	0,27	--	--	650	0,25	0,19	0,18	0,56	--	--	0,26			
1000	-0,04	-0,06	--	--	--	--	1000	0,24	0,24	--	--	--	--	1000	0,19	0,24	--	--	--	--	0,21			
															<i>En(p)</i>		0,23		0,40		--			

The degree of equivalence for all measurements of LNE-LADG with the KCRV is $En_{\text{all}} = 0,3$

Table 6.1 shows the differences d (%) between the KCRV and the French LNE-LADG at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers.

Table 6.2 shows the results for NMi-VSL (Netherlands).

Table 6.2 NMi (Netherland) - KCRV

$d_{\text{NMi,KCRV}} = f_{\text{NMi}} - f_{\text{KCRV}}$ (in %)

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar	
	M1	M2	M1	M2	M1	M2
65	-0,02	*	-0,06	*	-0,01	*
100	-0,04	*	-0,09	*	-0,04	*
160	-0,06	-0,19	0,01	-0,07	0,02	-0,05
250	-0,01	0,02	0,03	-0,03	0,06	0,08
400	0,04	0,02	0,03	-0,03	0,07	-0,05
650	0,03	-0,03	0,03	-0,02	0,08	-0,01
1000	0,03	0,03	0,10	-0,03	0,06	0,00

$U(d_{\text{NMi,KCRV}})$ ($k = 2$)

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar	
	M1	M2	M1	M2	M1	M2
65	0,14	*	0,21	*	0,24	*
100	0,14	*	0,21	*	0,24	*
160	0,14	0,14	0,21	0,21	0,24	0,24
250	0,14	0,14	0,21	0,21	0,24	0,24
400	0,14	0,14	0,21	0,21	0,24	0,24
650	0,14	0,14	0,21	0,21	0,24	0,24
1000	0,14	0,14	0,20	0,20	0,24	0,24

$En_{\text{NMi,KCRV}} = \text{abs}(d_{\text{NMi,KCRV}}) / U(d_{\text{NMi,KCRV}})$

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar		En Flow
	M1	M2	M1	M2	M1	M2	
65	0,16	*	0,27	*	0,05	*	0,13
100	0,32	*	0,41	*	0,15	*	0,27
160	0,44	1,32	0,03	0,36	0,06	0,19	0,20
250	0,07	0,11	0,14	0,13	0,25	0,35	0,15
400	0,28	0,17	0,16	0,12	0,29	0,21	0,20
650	0,25	0,19	0,13	0,07	0,32	0,06	0,14
1000	0,19	0,24	0,50	0,17	0,24	0,00	0,12
$En(p)$	0,23		0,14		0,12		

The degree of equivalence for all measurements of NMi with the KCRV is $En_{\text{all}} = 0,16$

Table 6.2 shows the differences d (%) between the KCRV and the Dutch NMI at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers.

Table 6.3 presents the results of PTB-*pigsar*TM.

Table 6.3 PTB-*pigsar*TM (Germany) - KCRV

$d_{PTB,KCRV} = f_{PTB} - f_{KCRV}$ (in %)

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar	
	M1	M2	M1	M2	M1	M2
	65	--	--	-0,03	*	0,00
100	--	--	0,02	*	0,01	*
160	--	--	-0,02	-0,01	-0,00	0,01
250	--	--	0,01	0,08	-0,02	-0,03
400	--	--	0,02	0,04	-0,02	0,02
650	--	--	0,00	0,05	-0,03	0,00
1000	--	--	-0,04	0,01	-0,02	-0,00

$U(d_{PTB,KCRV})$ ($k = 2$)

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar	
	M1	M2	M1	M2	M1	M2
	65	--	--	0,10	*	0,08
100	--	--	0,10	*	0,08	*
160	--	--	0,10	0,10	0,08	0,08
250	--	--	0,10	0,10	0,08	0,08
400	--	--	0,10	0,10	0,08	0,08
650	--	--	0,10	0,10	0,08	0,08
1000	--	--	0,09	0,09	0,08	0,08

$En_{PTB,KCRV} = \text{abs}(d_{PTB,KCRV}) / U(d_{PTB,KCRV})$

Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar		En Flow
	M1	M2	M1	M2	M1	M2	
	65	--	--	0,26	*	0,05	
100	--	--	0,16	*	0,15	*	0,15
160	--	--	0,21	0,06	0,06	0,19	0,11
250	--	--	0,08	0,77	0,25	0,35	0,27
400	--	--	0,19	0,43	0,29	0,21	0,27
650	--	--	0,02	0,49	0,32	0,06	0,12
1000	--	--	0,50	0,17	0,24	0,00	0,09
$En(p)$	--		0,17		0,12		

The degree of equivalence for all measurements of PTB-*pigsar*TM with the KCRV is
En all = 0,15

Table 6.3 shows the differences d (%) between the KCRV and the German PTB-*pigsar*TM at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers.

From Tables 6.1 to 6.3 it can be concluded, that the deviations between the KCRV and the measuring results of the participants are much smaller than the claimed uncertainties of the participants. All uncertainties of all participants in the KCRV are overlap very well with each other and therefore it can be concluded, that the claimed uncertainties are realistic and verified by this KC.

In the following, the tabulation of all deviations $d_{lab\#n, lab\#m} = f_{lab\#n} - f_{lab\#m}$ (in %) among the KCRV participants as well as associated uncertainty $U(d_{lab\#n, lab\#m})$ ($k = 2$) for these differences and the degree of equivalence $En_{lab\#n, lab\#m}$ of the individual labs have to be presented.

Table 6.4 shows presents the results for LNE-LADG (France), Table 6.5 is for NMI-VSL and Table 6.6 is for PTB-pigsarTM.

Tables 6.4 to 6.6 present the following quantities among lab #n and lab #m:

$d_{lab\#n, lab\#m} = f_{lab\#n} - f_{lab\#m}$ (in %) and $U(d_{lab\#n, lab\#m})$ ($k = 2$) for $d_{lab, KCRV}$ as well as

$En_{lab\#n, lab\#m} = \text{abs}(d_{lab\#n, lab\#m}) / U(d_{lab\#n, lab\#m})$

The following tables 6.4 to 6.6 show the equivalence among the participants

Table 6.4 LNE-LADG (France) – NMI (Netherland)

$d_{LNE-LADG, NMI} = f_{LNE-LADG} - f_{NMI}$ (in %)

$U(d_{LNE-LADG, NMI})$ ($k = 2$)

$En_{LNE-LADG, NMI} = \text{abs}(d_{LNE-LADG, NMI}) / U(d_{LNE-LADG, NMI})$

Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar		Flow m ³ /h	p = 10 bar	p = 20 bar	p = 47 bar	En Flow			
	M1	M2	M1	M2	M1	M2						M1	M2	M1
65	0,06	*	0,24	*	--	--	65	0,38	*	0,38	*	--	--	0,31
100	0,12	*	0,16	*	--	--	100	0,38	*	0,38	*	--	--	0,36
160	0,17	0,50	0,06	0,21	--	--	160	0,38	0,38	0,38	0,38	--	--	0,48
250	0,03	-0,04	-0,10	-0,21	--	--	250	0,38	0,38	0,38	0,38	--	--	0,18
400	-0,11	-0,06	-0,16	-0,09	--	--	400	0,38	0,38	0,38	0,38	--	--	0,26
650	-0,09	0,07	-0,08	-0,14	--	--	650	0,38	0,38	0,38	0,38	--	--	0,24
1000	-0,07	-0,09	--	--	--	--	1000	0,38	0,38	--	--	--	--	0,21
		0,23		0,35		--								

The degree of equivalence for all measurements between LNE-LADG and NMI is En all = 0,26

Table 6.4 shows the differences d (%) between the French LNE-LADG and the Dutch NMI at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers

Table 6.5 shows the equivalence among France and Germany

Table 6.5 LNE-LADG (France) – PTB-pigsarTM (Germany)

$$d_{\text{LNE-LADG,PTB}} = f_{\text{LNE-LADG}} - f_{\text{PTB}} \text{ (in \%)}$$

Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar	
	M1	M2	M1	M2	M1	M2
65	--	--	0,21	*	--	--
100	--	--	0,06	*	--	--
160	--	--	0,09	0,14	--	--
250	--	--	-0,08	-0,32	--	--
400	--	--	-0,14	-0,16	--	--
650	--	--	-0,05	-0,21	--	--
1000	--	--	--	--	--	--

$$U(d_{\text{LNE-LADG,PTB}}) (k = 2)$$

Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar	
	M1	M2	M1	M2	M1	M2
65	--	--	0,34	*	--	--
100	--	--	0,34	*	--	--
160	--	--	0,34	0,34	--	--
250	--	--	0,34	0,34	--	--
400	--	--	0,34	0,34	--	--
650	--	--	0,34	0,34	--	--
1000	--	--	--	--	--	--

$$En_{\text{LNE-LADG,PTB}} = \text{abs}(d_{\text{LNE-LADG,PTB}}) /$$

$$U(d_{\text{LNE-LADG,PTB}})$$

Flow rate m ³ /h	p = 10 bar		p = 20 bar		p = 47 bar		En Flow
	M1	M2	M1	M2	M1	M2	
65	--	--	0,61	*	--	--	0,61
100	--	--	0,17	*	--	--	0,17
160	--	--	0,27	0,42	--	--	0,34
250	--	--	0,24	0,93	--	--	0,47
400	--	--	0,42	0,48	--	--	0,45
650	--	--	0,15	0,60	--	--	0,30
1000	--	--	--	--	--	--	
En(p)	--	--	0,37	--	--	--	

The degree of equivalence for all measurements between LNE-LADG and PTB is $En_{\text{all}} = 0,37$

Table 6.5 shows the differences d (%) between the French LNE-LADG and the German PTB-pigsarTM at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers.

Table 6.6 shows the equivalence among Netherlands and Germany

Table 6.6 NMI (Netherland) – PTB-pigsarTM (Germany)

$d_{NMI,PTB} = f_{NMI} - f_{PTB}$ (in %)							$U(d_{NMI,PTB})$ ($k = 2$)						$En_{NMI,PTB} = \text{abs}(d_{NMI,PTB}) / U(d_{NMI,PTB})$								
Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar		Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar		Flow rate m ³ /h	$p = 10$ bar		$p = 20$ bar		$p = 47$ bar		En Flow
	M1	M2	M1	M2	M1	M2		M1	M2	M1	M2	M1	M2		M1	M2	M1	M2	M1	M2	
65	--	--	-0,03	*	-0,02	*	65	--	--	0,29	*	0,32	*	65	--	--	0,10	*	0,05	*	0,07
100	--	--	-0,10	*	-0,05	*	100	--	--	0,29	*	0,32	*	100	--	--	0,36	*	0,15	*	0,23
160	--	--	0,03	-0,07	0,02	-0,06	160	--	--	0,29	0,29	0,32	0,32	160	--	--	0,10	0,24	0,06	0,19	0,13
250	--	--	0,02	-0,11	0,08	0,11	250	--	--	0,29	0,29	0,32	0,32	250	--	--	0,07	0,37	0,25	0,35	0,22
400	--	--	0,01	-0,07	0,10	-0,07	400	--	--	0,29	0,29	0,32	0,32	400	--	--	0,05	0,24	0,29	0,21	0,16
650	--	--	0,03	-0,07	0,10	-0,02	650	--	--	0,29	0,29	0,32	0,32	650	--	--	0,09	0,23	0,32	0,06	0,14
1000	--	--	0,15	-0,05	0,08	0,00	1000	--	--	0,29	0,29	0,32	0,32	1000	--	--	0,50	0,17	0,24	0,00	0,09
													$En(p)$								
													--		0,15		0,12				

The degree of equivalence for all measurements between NMI and PTB is $En_{all} = 0,14$

Table 6.6 shows the differences d (%) between the Dutch NMI and the German PTB-pigsarTM at all measured flow rates and pressures, the uncertainty U of the differences d and the corresponding En -numbers.

7 SUMMARY, FINAL REMARKS and OUTLOOK

All comparisons showed complete agreement of the participants with the KCRV as well as among each other. The degrees of equivalence are about 0,15 to 0,3, which means an excellent equivalence to each other and of course with the KCRV.

The claimed uncertainties have been fully verified for all the participants PTB-*pigsar*TM, LNE-LADG and NMi-VSL.

Table 7.1 shows the original uncertainties U_{origin} of the participant's facilities as well as the indicated uncertainties after KCRV-corrected calibration curves U_{CMC} . PTB have not applied the improvement in the uncertainty due to the application of KC results, but NMi-VSL made use of this, which leads to an improved uncertainty. In addition table 7.1 shows that the claimed uncertainties U_{CMC} for all participants are larger than the observed deviations during the KCs, see Fig. 10 to 15.

Table 7.1: Summary of uncertainty results during the KCs for all participants

Lab	pressure (bar)	U_{origin}	U_{random}	Korr. r	$U[x^{(-d)}]$	U_{CMC}
LNE	8	0,30	0,14	0,78	0,24	0,24^{*)}
	20	0,30	0,14	0,78	0,22	0,22^{*)}
	50	0,30	0,14	0,78	0,22	0,22^{*)}
NMi	8	0,23	0,09	0,85	0,20	0,21
	20	0,24	0,09	0,86	0,16	0,18
	50	0,28	0,09	0,90	0,18	0,18
PTB	8	--	--	--	--	--
	20	0,16	0,07	0,81	0,14	0,16
	50	0,16	0,07	0,81	0,15	0,16

U_{origin} - original expanded uncertainty of the laboratories

U_{random} - stochastic (random part) of uncertainty budget (reproducibility), result of investigations on reproducibility experiments as describes e. g. in chapter 3.3 and 3.4 of this protocol draft A

Korr. r - correlation coefficient according Eq. 9.5

$U[x^{(-d)}]$ - expanded uncertainty of the corrected measuring results after KCs according to chapter 4, see also chapter 9.4.

U_{CMC} - claimed uncertainties in the participants' CMCs.;

^{*)} The actual CMC-tables describe the situation of 2003, the claimed uncertainties of LNE-LADG are determined for a future updated version

The theoretical background of this statistical procedure is given in the annex chapter 9.

8 REFERENCES

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9 Annex A: Propagation of uncertainties disseminating the KCRV

9.1 The characterisation of a measured quantity

The measuring result and its uncertainty qualify the measuring process. The total uncertainty comprises of a stochastic part (u_{random}) and a static part (u_{common}).

The stochastic part can be evaluated by investigations regarding reproducibility. In contrast to that, the static part remains the same during each measurement. Both parts together lead to the total uncertainty of the measurement. Fig. 1 visualizes this situation. From many repeats of the measurement one gets the mean value (blue line) and statistical scatter (u_{random}) around this line within a limit of 2 standard deviations (2sigma, indicated by the blue dashed line). u_{random} is actually the statistical contribution to the measurements. However, the mean value will differ by a certain amount, from the so-called “true value”, the SI-quantity. This is due to a certain offset in the measuring result, which is unknown at first and will be estimated as uncertainty type B in the uncertainty budget.

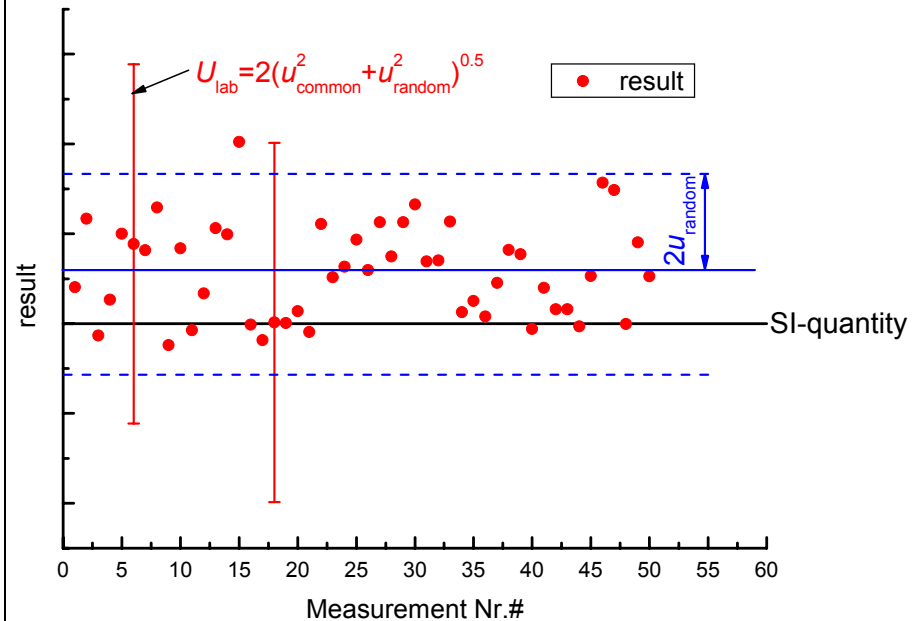


Figure 9.1: Visualization of stochastic and statistical data for measuring results

In the following, we will use the phrase variance instead of uncertainty, which are associated according to equation 1 to each other. Error or uncertainty propagations are always associated with variances using the exponent “2”. Using variances allows, avoiding continuously the exponent “2” during calculation of uncertainty propagation.

Eq. 2 describes the relation between total uncertainty, the stochastic part (u_{random}) and the statistical part (u_{common}). The corresponding quantities for the variances are presented in eq. 3.

The existence of an offset results in a covariance between the results according to eq. 4. In this equation x is the measuring result at t and x^* is the measuring result at time $t^* = t + dt$. The covariance can also be described by the correlation coefficient r_i . If the correlation coefficient r_i is close to 0 this means according to eq. 4 and eq. 3, that the total uncertainty of a measuring result of laboratory # i will be determined mainly by stochastic contributions and that this laboratory as a very low offset. On the other hand, a correlation coefficient r_i close to 1 means a measuring result, which has a scatter much less than the total uncertainty.

If the reproducibility (u_{random}) at the laboratory has been experimentally evaluated, see e.g. chapter 3.2 on reproducibility, the correlation coefficient can be determined according to equation 5.

For later explanations is shall be mentioned here, that the individual parts of uncertainties and the correlation coefficients r_i for laboratory # i may not change over time very much.

$$\text{Eq. 1} \quad U_i = 2u_i = 2\sqrt{v_i}$$

$$\text{Eq. 2} \quad u_i^2 = u_{i,\text{common}}^2 + u_{i,\text{random}}^2$$

$$\text{Eq. 3} \quad v_i = v_{i,\text{common}} + v_{i,\text{random}}$$

$$\text{Eq. 4} \quad \text{COV}(x_i, x_i^*) = v_{i,\text{common}} = r_i \cdot v_i$$

$$\text{Eq. 5} \quad r_i = \frac{v_{i,\text{common}}}{v_i} = 1 - \frac{v_{i,\text{random}}}{v_i}$$

9.2 General formulation of the propagation of uncertainty (variances)

In order to calculate the propagation of uncertainties according to the Guidelines of Uncertainty Measurements (GUM) effectively, the vector and matrix technique is a very effective tool for description. The equations 6 up to eq. 11 are the basis for the calculation of uncertainty propagation starting with a data file \mathbf{x} (vector \mathbf{x} of the input data eq. 6) and ending with a resulting data file (vector) \mathbf{y} (vector \mathbf{y} of the resulting output data in Eq. 7). The functional relationship between vector \mathbf{y} and \mathbf{x} is given.

The vector \mathbf{x} is associated with a Variance-Covariance-Matrix $\mathbf{V}(\mathbf{x})$, which gives the uncertainty of the input variables (Eq. 8). If all input data are independent of each other, as it is usually the case for independent laboratories, one realizes that only the variables along the diagonal axis are different from zero (Eq. 9).

Important for the calculation of the Variance-Covariance-Matrix $\mathbf{V}(\mathbf{y})$ of the output vector \mathbf{y} is the calculation of the so-called Jacobi matrix $\mathbf{J}(\mathbf{y}, \mathbf{x})$. The elements $j_{i,k}$ of this Jacobi matrix are the first derivatives $\partial y_i / \partial x_k$ of the vector elements y_i towards the vector elements x_k , see eq. 10. With the help of this Jacobi matrix the calculation of $\mathbf{V}(\mathbf{y})$ can be done formally in a very short way using eq. 11.

$$\text{Eq. 6 } \mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\text{Eq. 7 } \mathbf{y} = \begin{pmatrix} y_1 = f_1(\mathbf{x}) \\ \vdots \\ y_m = f_m(\mathbf{x}) \end{pmatrix}$$

$$\text{Eq. 8 } \mathbf{V}(\mathbf{x}) = \begin{pmatrix} v_1 & \text{COV}_{1,2} & \cdots & \text{COV}_{1,n} \\ \text{COV}_{2,1} & v_2 & \cdots & \text{COV}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \text{COV}_{n,1} & \text{COV}_{n,2} & \cdots & v_n \end{pmatrix}$$

$$\text{Eq. 9 } \mathbf{V}(\mathbf{x}_{\text{noncorr}}) = \begin{pmatrix} v_1 & 0 & \cdots & 0 \\ 0 & v_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_n \end{pmatrix}$$

$$\text{Eq. 10 } \mathbf{J}(\mathbf{y}, \mathbf{x}) = \begin{pmatrix} j_{1,1} = \frac{\partial y_1}{\partial x_1} & \cdots & j_{1,n} = \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ j_{n,1} = \frac{\partial y_m}{\partial x_1} & \cdots & j_{n,n} = \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

$$\text{Eq. 11 } \mathbf{V}(\mathbf{y}) = \mathbf{J}(\mathbf{y}, \mathbf{x}) \cdot \mathbf{V}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{y}, \mathbf{x})^T$$

9.3 Results of a KC with mutually independent partners

Rules and procedures for calculating a Key Comparison Reference Value (KCRV), in particular the parameter reference value x_{ref} have been proposed and summarized by Cox at the request of the Director of the BIPM, see [8]. Below we refer to these rules and procedures as BIPM recommendations. They have been presented in Eq. 12 to Eq. 15. Eq. 15 and Eq. 16 are equations, which follow directly using the weighing factors w_i (Eq. 13). They will be referred to later.

In addition, a central result of any Key Comparison is the difference $d_{i,ref}$ (Eq. 17), between a measuring value x_i of a laboratory #i and the reference value x_{ref} . The entirety of these differences of the individual laboratory #i can be defined as the vector of result \mathbf{d}_{ref} (Eq. 18, analogue to Eq. 7). The relation between the elements x_i of the input vector \mathbf{x} and the elements of the resulting output vector \mathbf{d}_{ref} is given by Eq. 17 .

Following the rules for the application of Jacobi matrixes $\mathbf{J}(\mathbf{d}_{ref}, \mathbf{x})$, leads to a matrix as given by Eq. 19 :

- Line #i stands for laboratory #i; all derivatives of Eq. 17 towards the elements with $i \neq k$ result in an element $-w_k$; all derivatives of Eq. 17 towards the elements with $i = k$ result in an element $1-w_i$.
- The same procedure in line #j for laboratory #j.

$$\text{Eq. 12 } x_{ref} = \sum_{i=1}^n w_i x_i$$

$$\text{Eq. 13 } w_i = \frac{1}{v_i \sum_{k=1}^n \frac{1}{v_k}}$$

$$\text{Eq. 14 } \sum_{i=1}^n w_i = 1$$

$$\text{Eq. 15 } v_{ref} = \sum_{i=1}^n w_i^2 v_i = \frac{1}{\sum_{k=1}^n \frac{1}{v_k}}$$

$$\text{Eq. 16 } v_{ref} = w_i v_i$$

$$\text{Eq. 17 } d_{i,ref} = x_i - \sum_{k=1}^n w_k x_k$$

$$\text{Eq. 18 } \mathbf{d}_{ref} = \begin{pmatrix} d_{1,ref} \\ \vdots \\ d_{n,ref} \end{pmatrix}$$

$$\text{Eq. 19 } \mathbf{J}(\mathbf{d}_{ref}) = \begin{pmatrix} \vdots \\ -w_1 & \cdots & -w_{i-1} & 1-w_i & -w_{i+1} & \cdots & -w_j & \cdots & -w_n \\ \vdots \\ -w_1 & \cdots & -w_{i-1} & -w_i & -w_{i+1} & \cdots & 1-w_j & \cdots & -w_n \\ \vdots \end{pmatrix}$$

Using the procedures above, one gets the Jacobi matrix as given in Eq. 19.

Applying matrix multiplication according to eq. 20, one gets the matrix $\mathbf{V}(\mathbf{d}_{ref})$ according to eq. 21. The individual elements in the main diagonal comprises of the variances of the individual $d_{i,ref}$. The other elements are the co variances. The result for a single matrix element looks to be quite complex at first, but it can be presented using eq. 15 and 16. The element $v(d_{i,ref})$ in the main diagonal of matrix $\mathbf{V}(\mathbf{d}_{ref})$ may be presented as an example in detail in eq. 22.

The result is exactly identical with the procedure described by Cox.

$$\text{Eq. 20 } \mathbf{V}(\mathbf{d}_{ref}) = \mathbf{J}(\mathbf{d}_{ref}, \mathbf{x}) \cdot \mathbf{V}(\mathbf{x}) \cdot \mathbf{J}(\mathbf{d}_{ref}, \mathbf{x})^T$$

$$\text{Eq. 21 } \mathbf{V}(\mathbf{d}_{ref}) = \begin{pmatrix} V_1 - V_{ref} & \cdots & V_{ref} & \cdots & V_{ref} & \cdots & V_{ref} \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ V_{ref} & \cdots & V_i - V_{ref} & \cdots & V_{ref} & \cdots & V_{ref} \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ V_{ref} & \cdots & V_{ref} & \cdots & V_j - V_{ref} & \cdots & V_{ref} \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ V_{ref} & \cdots & V_{ref} & \cdots & V_{ref} & \cdots & V_n - V_{ref} \end{pmatrix}$$

$$\begin{aligned} \text{Eq. 22 } v(d_{i,ref}) &= (1 - w_i)^2 v_i + \sum_{k=1, k \neq i}^n w_k^2 v_k \\ &= v_i - 2w_i v_i + w_i^2 v_i + \sum_{k=1, k \neq i}^n w_k^2 v_k = v_i - 2w_i v_i + \sum_{k=1}^n w_k^2 v_k = v_i - V_{ref} \end{aligned}$$

9.4 The benefit of a key comparison: applying the recognized differences

In the following we indicate values with equations with $x^{(KC)}$, which have been obtained in a KC and the upper index $(-d)$ stands for results as obtained a short interval after a KC and which have been corrected.

Every participant #i of a KC has the chance to apply a correction to his own results, by using the above obtained difference $d_{i,ref}$ between his laboratory #i result and the reference value x_{ref} assuming this value to be the best estimation of the offset of his facility. Therefore, it is a consequent procedure to apply a correction factor to all measuring results x of laboratory #i after the Key Comparison has been done. This correction factor is the difference $d_{i,ref}$ between the measuring result of the laboratory #i and the KCRV result, reference value x_{ref} . At the end the laboratory can disseminate a corrected measuring result $x^{(-d)}$ to his client (customer), see **Eq. 23**.

As shown in **Eq. 23**, such a corrected measuring result $x^{(-d)}$ of laboratory #i comprises not only of the actual results of laboratory #i, but also of all measuring results $x^{(KC)}_k$, which have been used during a Key Comparison in order to get the KC reference value. Calculation the Jacobi matrix requires to use the actual measuring results \mathbf{x} as well as the results of the Key Comparison $\mathbf{x}^{(KC)}$ according to Eq. 24.

The Jacobi matrix (**Eq. 25**) comprises accordingly to different parts. The left part comprises of derivatives towards the $\mathbf{x}^{(KC)}$ (**Eq. 23**) and the right part derivatives towards the actual \mathbf{x} .

$$\text{Eq. 23 } \mathbf{x}^{(-d)} = \mathbf{x} - \mathbf{d}_{ref} = \begin{pmatrix} x_1 - d_{1,ref}^{(KC)} \\ x_2 - d_{2,ref}^{(KC)} \\ \vdots \\ x_n - d_{n,ref}^{(KC)} \end{pmatrix} = \begin{pmatrix} x_1 - x_1^{(KC)} + \sum_{k=1}^n w_k^{(KC)} x_k^{(KC)} \\ x_2 - x_2^{(KC)} + \sum_{k=1}^n w_k^{(KC)} x_k^{(KC)} \\ \vdots \\ x_n - x_n^{(KC)} + \sum_{k=1}^n w_k^{(KC)} x_k^{(KC)} \end{pmatrix}$$

$$\text{Eq. 24 } (\mathbf{x}^{(KC)}, \mathbf{x})^T = \begin{pmatrix} \mathbf{x}^{(KC)} \\ \mathbf{x} \end{pmatrix} = \begin{pmatrix} x_1^{(KC)} \\ \vdots \\ x_n^{(KC)} \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\text{Eq. 25 } \mathbf{J}(\mathbf{x}^{(-d)}, (\mathbf{x}^{(KC)}, \mathbf{x})^T) = \left(\begin{array}{cccc|ccc} w_1-1 & w_2 & \cdots & w_n & 1 & 0 & \cdots & 0 \\ w_1 & w_2-1 & \cdots & w_n & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ w_1 & w_2 & \cdots & w_n-1 & 0 & 0 & \cdots & 1 \end{array} \right)$$

Important is the correct structure of the Variance-Covariance Matrix $\mathbf{V}(\mathbf{x}^{(KC)}, \mathbf{x})$. In order to get this matrix, it has to be considered that:

- the measuring results x_i and $x^{(KC)}_i$ of laboratory #i are correlated to each other according to chapter 1, Eq. 4. and that
- the variances as well as the correlation coefficients r_i have to be stable in time.

The north-west corner of the variance-covariance matrix $\mathbf{V}(\mathbf{x}^{(KC)}, \mathbf{x})$ comprises as well as the south-east-corner in its main diagonal of the variances of the measuring results of the laboratories, while on the other hand one finds in the south-west- and the north-east corner the covariance of x_i and $x^{(KC)}_i$.

Using these matrices, the formal application of multiplication rules for matrices, the final result for the Variance-covariance matrix of the corrected measuring results can be obtained, see (Eq. 27). Again Eq. 15 and Eq. 16 have to be applied in return, to get a simplification. Eq. 28 presents again the variance for a single laboratory #i after application of the correction.

In order to understand the meaning and assessment of the results, Eq. 28 may be used to consider two extreme situations for a laboratory:

- $r_i = 1$ (laboratory #i shows as compared with its total uncertainty an extrem good reproducibility) => the resulting uncertainty (variance) is equal to the variance of the reference value (!).
- $r_i = 0,5$ (the reproducibility of laboratory #i is about $1/2^{0.5}$ of its total uncertainty) => the resulting uncertainty (variance) remains unchanged and is still the same of the laboratory's #i original uncertainty.
- $r_i = 0$ (the reproducibility of laboratory #i determines /dominates the total uncertainty, no offset) => the resulting uncertainty (variance) will be larger as compared with the original uncertainty of laboratory #i by means of the variance of the difference $d_{i,ref}$ with the value of $v_i - v^{(KC)}_{ref}$.

$$\text{Eq. 26 } \mathbf{V}(\mathbf{x}^{(KC)}, \mathbf{x}) = \begin{pmatrix} v_1 & 0 & \dots & 0 & r_1 v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 & 0 & r_2 v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_n & 0 & 0 & \dots & r_n v_n \\ r_1 v_1 & 0 & \dots & 0 & v_1 & 0 & \dots & 0 \\ 0 & r_2 v_2 & \dots & 0 & 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & r_n v_n & 0 & 0 & \dots & v_n \end{pmatrix}$$

Eq. 27

$$\mathbf{V}(\mathbf{x}^{(-d)}) = \begin{pmatrix} \ddots & & & \vdots & & & & \\ & 2(1-r_i)v_i + (2r_i-1)v_{ref}^{(KC)} & \dots & (r_j+r_i-1)v_{ref}^{(KC)} & & & & \\ \dots & \vdots & \ddots & \vdots & & & & \\ & (r_j+r_i-1)v_{ref}^{(KC)} & \dots & 2(1-r_j)v_j + (2r_j-1)v_{ref}^{(KC)} & & & & \\ & & & \vdots & & & & \ddots \end{pmatrix}$$

$$\text{Eq. 28 } v_i^{(-d)} = 2(1-r_i)v_i + (2r_i-1)v_{ref}^{(KC)}$$

9.5 The resulting uncertainty claims of PTB, NMi und LNE (BNM) before and after Key Comparisons.

The contract between PTB, NMi-VSL and LNE (BNM) regarding the dissemination of harmonized reference values, requires regular intercomparisons between all three partners on a half year basis in order to get every half year updated comparison results. These comparisons have been performed since its very beginning in 1998 following all above mentioned BIPM recommendations for Key Comparisons. Since 1999 PTB and NMi-VSL were disseminating common reference values and recently in May 4th, the LNE (BNM) has joined the contract in order to disseminate the "Harmonized European Reference Value for the Natural Gas Cubic Meter at High Pressure". In order to maintain very stable and reliable reference values, the partners conduct intercomparisons in order get reference values x_{ref} as well as the deviations $d_{i,ref}$ of the individual laboratories. According to the above described techniques, the participating laboratories practise actually to apply their individual measuring results by means of the correction $-d_{i,ref}$ (Chapter 4).

Table 9.1 presents a comparison between the uncertainty of the participants of the CIPM KCs before (U_{origin}) conducting the KCs. In addition, we present the correlation coefficients $Korr. r_i$ according to Eq. 5 as well as the uncertainty claims (U_{CMC}) of the laboratories in their Calibration and Measuring Capabilities (CMCs).

Table 9.1: Summary of uncertainty results during the KCs for all participants

Lab	pressure (bar)	U_{origin}	U_{random}	Korr. r	$U[x^{(-d)}]$	U_{CMC}
LNE	8	0,30	0,14	0,78	0,24	0,24^{*)}
	20	0,30	0,14	0,78	0,22	0,22^{*)}
	50	0,30	0,14	0,78	0,22	0,22^{*)}
NMi	8	0,23	0,09	0,85	0,20	0,21
	20	0,24	0,09	0,86	0,16	0,18
	50	0,28	0,09	0,90	0,18	0,18
PTB	8	--	--	--	--	--
	20	0,16	0,07	0,81	0,14	0,16
	50	0,16	0,07	0,81	0,15	0,16

U_{origin} - original expanded uncertainty of the laboratories

U_{random} - stochastic (random part) of uncertainty budget (reproducibility), result of investigations on reproducibility experiments as describes e. g. in chapter 3.3 and 3.4 of this protocol draft A

Korr. r - correlation coefficient according Eq. 5

$U[x^{(-d)}]$ - expanded uncertainty of the corrected measuring results after KCs according to chapter 4.

U_{CMC} - claimed uncertainties in the participants CMCs.;

*) The actual CMC-tables describe the situation of 2003, the claimed uncertainties of LNE (BNM) are determined for a future updated version

The assessment of table 9.1 shall take into account, that the calculations according to chapter 9.4 in this annex are based on a single reference value. In contrast to that, PTB, NMI-VSL as well as LNE (BNM) get their reference values from multiple measurements at many flow rates at different pressures. This means in detail, in the harmonization procedure the participants get for each calibration point (pressure and flow rate) at least two differences $d_{i, \text{ref}}$ for each laboratory (minimum two transfer meters).

In table 9.2 all applied packages for harmonization and CIPM KCs are listed and all technical details like diameters and pressures are summarized.

In Fig. 9.2 all deviations $d_{i, \text{ref}}$ of the laboratories obtained during all in 2004 performed intercomparisons and the actual CIPM KC 5.a (see also Fig.9.2 with results and visualization) are presented. The deviations are presented as function of Reynolds number * diameter. This number is proportional to the mass flow rate. The presentation has been grouped for laboratories, results of harmonization obtained in 2004 and results of K5.a.

It shall be pointed out, that all comparisons have been done for all transfer meters within 3 months. The reproducibility of the differences $d_{i, \text{ref}}$ in Figure 2 shows, that the data for the reproducibility of the participating test / calibration facilities and transfer standards are realistic and reliable. In addition, it indicates that the total uncertainty of the corrected measuring data $x^{(-d)}_i$ may probably be much smaller than calculated in chapter 4, if one takes into account all comparisons and uses accordingly a corresponding mean $d^{(\text{ave})}_{i, \text{ref}}$ from all comparison results in order to get the correction factor. Table 1 is therefore a very conservative upper limit for the uncertainties.

Table 9.2: Summary of applied transfer packages during all comparisons in the past. (pressure, meter type, nominal diameters for harmonization as well as KCs)

	Harmonised European Cubic meter LNE- LADG/NMi/PTB				CIPM KC 5.a
	100 (4")	150 (6")	250 (10")	400 (16")	150 (6")
nominal diam. DN/mm (inch)	100 (4")	150 (6")	250 (10")	400 (16")	150 (6")
pressures (bar)	8, 20, 50	8, 20, 50	20, 50	20, 50	10, 20, 47
No. of meters	2	2	2	2	2
type of meter	turbine/ turbine	turbine/ turbine	turbine/ turbine	turbine/ turbine	turbine/ ultrasonic

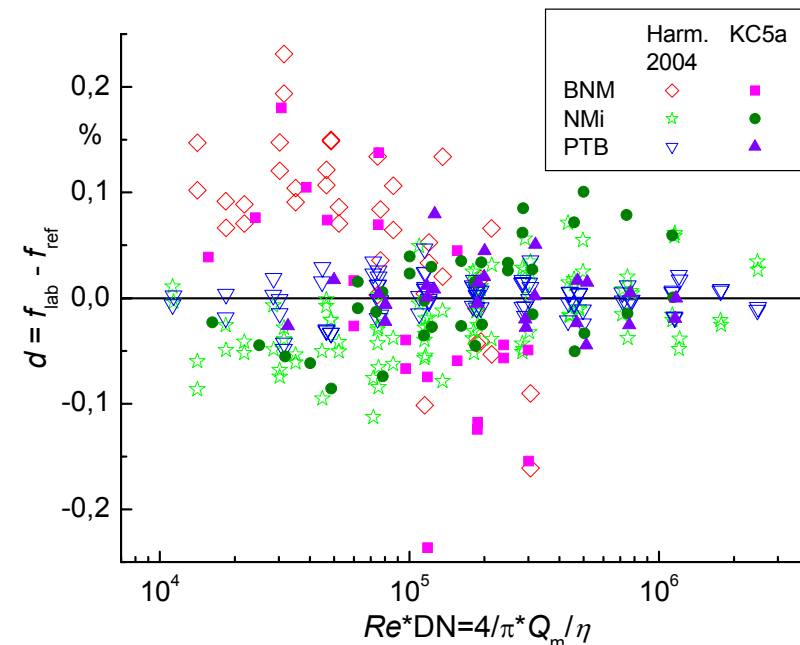


Fig. 9.2: Presentation of differences between all laboratories and the reference values.

9.6 Outlook and closing remarks

In chapter 9.4 the technique for the calculation of uncertainties of facilities after intercomparisons and KCs has been demonstrated in order to show the advantageous benefit which can be obtained if the partners follow the BIPM recommendations for conducting comparisons in order to get reliable reference values. The method to apply corrections to the individual results in order to disseminate the intercomparison reference value has been demonstrated in detail. It has been pointed out, that the well known "European Harmonized Reference Value" strictly follows BIPM recommendation and actually practices the BIPM recommendation as they are disseminating intercomparison reference values.

The stability and the power of this correction technique using the differences $d_{i,ref}$ between reference value in an intercomparison and the participating facility #i has been explained in detail.

For future intercomparisons (mainly KCs) among partners, who have already conducted separate own intercomparisons and are already applying corrections, the evaluation of comparisons can be done too of course, but the presentation will be more complicated. In this case of non-independent partners, the calculation of KCRV and its uncertainty has to take into account that the partners are not independent any more and will be correlated to each other according to Eq. 27.

The authors will show in a separate publication that the KCRV and its uncertainty will not be affected, if the correlation among the partners is taken into account properly. This procedure is also applicable for laboratories which show traceability (e.g. secondary laboratory) to a participant of a KC.

9.7 Reference

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