



EUROMET.M.D-K4 / EUROMET Project 702: Comparison of the calibrations of high-resolution hydrometers for liquid density determinations

S. Lorefice¹, A. Malengo¹, C. Vámosy², H. Bettin³, H. Toth³, M. do Céu Ferreira⁴, A. Gosset⁵,
T. Madec⁵, M. Heinonen⁶, C. Buchner⁷, E. Lenard⁸, R. Spurny⁹, U. Akcadag¹⁰, N. Domostroeva¹¹

¹ Istituto Nazionale di Ricerca Metrologica (INRiM) Strada delle Cacce, 91; 10135 Torino, Italy

² Országos Mérésügyi Hivatal (OMH) 1124 Budapest XII., Németvölgyi út 37-39; Budapest – Hungary

³ Physikalisch-Technische Bundesanstalt (PTB) Bundesallee 100; 38116 Braunschweig – Germany

⁴ Instituto Português da Qualidade (IPQ) Rua António Gião, 2; 2829-513 Caparica – Portugal

⁵ Laboratoire National de Métrologie et d'Essais (LNE) 1, rue Gaston Boissier ; 75724 Paris Cedex 15 – France

⁶ Centre for Metrology and Accreditation (MIKES) Lönnrotinkatu 37; 00181 Helsinki – Finland

⁷ Bundesamt für Eich- und Vermessungswesen (BEV) Arltgasse 35; 1160 Wien – Austria

⁸ Główny Urząd Miar (GUM) ul. Elektoralna 2; 00-950 Warszawa – Poland

⁹ Slovak Institute of Metrology (SMU) Karloveská 63; 842 55 Bratislava – Slovakia

¹⁰ TUBITAK- Ulusal Metroloji Enstitüsü (UME) Besevler Anibal Caddesi Tubitak-Mam Kampusu P.K. 54 41470 Gebze-Kocaeli Turkey

¹¹ VNIIM All-Russian Research Institute of Metrology (VNIIM), Moskovsky pr, 19; 190005 St. Petersburg – Russia

S.Lorefice@inrim.it

INRiM, Italy

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Abstract

The main objective of the EUROMET project 702 was to compare the extent of comparability among eleven participating European national metrology institutes: (INRIM (IT), OMH (HU), PTB (DE), BEV (AT), IPQ (PT), LNE (FR), MIKES (FI), GUM (PL), SMU (SK), UME (TR) and VNIIM (RU)) in performing calibrations of high-resolution hydrometers for liquid density determination in the range between 600 kg/m³ and 1300 kg/m³. By means of two groups of four similar transfer standards of excellent metrological characteristics, the participating laboratories were initially divided into two groups (petals) linked by the three density laboratories of INRIM, OMH and PTB.

The results of the participating laboratories have been analyzed in this report and a good agreement was found between the results provided by most of the participants. These results allowed also to determine the degrees of equivalence of each NMI participating with the EUROMET_key comparison reference values^(*) (*EU_KCRV*), they will provide a basis for the review of the Calibration Measurement Capabilities (CMC) entries on hydrometer calibration, and allowed to establish the degree of equivalence between pairs of NMIs.

The Istituto Nazionale di Ricerca Metrologica (INRIM), Italy, formerly IMGC-CNR, coordinated the project.

^(*) The term “Key comparison reference value *KCRV*” should only be used for CIPM comparisons and for comparisons that are linked to a CIPM comparison. The term “EUROMET_Key comparison reference value *EU_KCRV*” seems to be appropriate to be used for EUROMET comparison. (*EUROMET TC-M/density meeting Teddington, March 1, 2007*)

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1. INTRODUCTION

The Istituto Nazionale di Ricerca Metrologica National (INRIM), Italy, formerly IMGC-CNR, has been agreed by the Technical Committee for Mass and Related Quantities (TC-M) in the European Collaboration in Measurement Standards (EUROMET) to coordinate, as pilot institute, an interlaboratory comparison program for the calibrations of high-resolution hydrometers for liquid density determinations.

The comparison was intended to be a regional key comparison according to the Mutual Recognition Arrangement [1], it was identified on March 3, 2003 as EUROMET Project 702 and it was also registered as the EUROMET.M.D-K4 key comparison by the Consultative Committee for Mass and Related Quantities (CCM) of the International Committee for Weights and Measures (CIPM), the International Bureau of Weights and Measures (BIPM).

Due to strongly varying uncertainties initially claimed, the purpose of the comparison was to:

- Recognize and compare the different experimental setups and calibration methods applied by the participants;
- Ascertain the consistency of their calibration results;
- Determine several (4) *EU_KCRVs* in the density range between 600 kg/m³ and 1 300 kg/m³ at 20 °C
- Compare the participants' methods for the uncertainty evaluation.

The outcome of the project was that it will provide a basis for entries of the CMC tables in the density subfield and also to establish the link to the planned CIPM key CCM.D-K4 "Hydrometers".

The laboratories of OMH (HU) and PTB (DE) supported the pilot laboratory by setting up the technical protocol, which was part of this comparison [2].

Eleven European metrological laboratories (NMIs) took part in this project and each one presented a report of its own measurements before the end of the comparison according to an accompanying worksheet [2].

This report describes the organization of the actual project, the method for analysis of the calibration data and the comparison results. The results also allowed to determine the degrees of equivalence of each participating NMI with the EUROMET_key comparison reference values (*EU_KCRV*) and the degree of equivalence between pairs of NMIs.

The pilot institute prepared a report on the comparison and the analysis of data based on the results of the participants trying to apply a uniform treatment to all participants.

Draft B is intended to be a publication for the CIPM Key Comparison Data Base.

2. ORGANIZATION

2.1 Participants and schedule

Ten NMIs, plus the pilot institute, agreed to participate in the comparison ab initio. Table 1 comprises the participating NMIs and the technical contacts.

For the purpose of this project and to speed up the comparison, the participating laboratories were initially divided into two groups (petals), the three laboratories INRIM, OMH and PTB linked the two petals. Although a big effort was devoted to keep the comparison in process, there were some unforeseen difficulties relating to artefacts breakage, transportation; customs and administrative constraints forced to change the original schedule of the comparison as well.

Table 1. List of the participating NMIs and technical contacts

Institute	Country code	Responsible person
Istituto di Ricerca Metrologica (INRIM) formerly IMGC-CNR Strada delle Cacce, 91 IT-10135 Torino-ITALY	IT	Salvatore Lorefice S.lorefice@imgc.cnr.it Tel.: +39 011 3977 1 Fax: +39 011 3977437
Országos Mérésügyi Hivatal - National Office of Measures (OMH) H-1124 Budapest XII., Németvölgyi út 37-39 Budapest-HUNGARY	HU	Csilla Vámosy c.vamosy@omh.hu Tel.: +36 1 4585 947 Fax: +36 1 4585 890
Physikalisch-Technische Bundesanstalt (PTB) Bundesallee 100 D-38116 Braunschweig-GERMANY	DE	Hans Toth Hans.Toth@ptb.de Tel.: +49 531 592 3114 Fax: +49 531 592 3015
Instituto Português da Qualidade Laboratório Nacional de Metrologia (IPQ) Rua António Gião, 2 PT-2829-513 Caparica-PORTUGAL	PT	Maria do Céu Ferreira MCFerreira@mail.ipq.pt Tel.: +351 21 2948164 Fax: +351 21 2948188
Laboratoire National de Métrologie et d'Essais (LNE) 1 rue Gaston Boissier 75724 Paris Cedex 15-FRANCE	FR	Tanguy Madec tanguy.madec@lne.fr Tel.: +33 1 40 43 39 34 Fax: +33 1 40 43 37 37
Centre for Metrology and Accreditation (MIKES) Lönnrotinkatu 37 FIN-00181 Helsinki-FINLAND	FI	Martti Heinonen martti.heinonen@mikes.fi Tel.: +358 9 6167 549 Fax: +358 9 6167 467
Bundesamt für Eich- und Vermessungswesen (BEV) Gruppe Eichwesen (Metrology Service) Arltgasse 35 A-1160 Wien-AUSTRIA	AT	Christian Buchner c.buchner@metrologie.at Tel.: +43 1 49110 361 Fax: +43 1 4920875-3611
Central Office of Measures Główny Urząd Miar (GUM) Mass and Force Division, Density Laboratory, ul. Elektoralna 2 00-950 Warszawa-POLAND	PL	Elżbieta Lenard, density@gum.gov.pl Tel.: + 48 22 6200241 ext. 510/656 Fax: + 48 22 6206646
Slovak Institute of Metrology (SMU) Karloveská 63 842 55 Bratislava-SLOVAKIA	SK	Robert Spurny spurny@smu.gov.sk Tel.: + 421 2 602 94 350 Fax: + 421 2 654 29 592
TUBITAK- Ulusal Metroloji Enstitüsü (UME) Besevler Anibal Caddesi TUBITAK-MAM KAMPUSU P.K. 54 41470 GEBZE-KOCAELI-TURKEY	TR	Umit Akcadag umit.akcadag@ume.tubitak.gov.tr Tel.: + 90 (262) 679 50 00 Fax: +90 (262) 679 50 01
VNIIM All-Russian Research Institute of Metrology Moscovsky pr, 19 190005 St. Petersburg-RUSSIA	RU	Natalia Domostroeva N.G.Domostroeva@vniim.ru Tel.: + 7 812 323-96-71 Fax: + 7 812 113-01-14

Table 2. Final circulation scheme defined for each artefact

Institute (scheduled period)	Id. hydrometer Range kg/m ³		21964	21971	21958	5941	6905	0001	58431	58432
			600.0 ÷ 610.0	610.0 ÷ 620.0	810.0 ÷ 820.0	810.0 ÷ 820.0	990.0 ÷ 1 000.0	1 000.0 ÷ 1 010.0	1 290.0 ÷ 1 300.0	1 290.0 ÷ 1 300.0
INRIM (15/03 – 01/04/04)	X	X	X	X	X	X	X	X	X	X
OMH (15/04 – 12/05/04)	X	X	X	X	X	X	X	X	X	X
PTB (04/06 – 23/06/04)	X	X	X	X	X	X	X	X	X	X
IPQ (28/06 – 26/07/04)		X		X			X			
MIKES (08/08 – 27/08/04)		X		X			X			
LNE (15/03 – 25/04/05)		X	X				X	X		
BEV (29/04 – 18/05/05)		X	X				X			
SMU (01/07 – 01/08/04)	X		X			X		X		
GUM (02/08 – 03/09/04)	X		X			X		X		
UME (17/09 – 05/10/04)	X		X			X		X		
VNIIM (10/12 – 21/02/05)			X			X		X		

Table 3. Reference hydrometers used as transfer standards (TS) in this key comparison.

ID. Hydrometer	Range g/cm ³	Scale div. g/cm ³	Diameter of body mm	Length of body mm	Diameter of stem mm	Length of stem mm	Weight g
21971	0.610 0 – 0.620 0	0.000 1	28	250	5.5	150	90
5941	0.810 0 – 0.820 0				5.0		120
0001	1.000 0 – 1.010 0				4.5		150
58432	1.290 0 - 1.300 0				3.8		190
21964	0.600 0 - 0.610 0				5.5		90
21958	0.800 0 - 0.810 0				5.0		120
6905	0.990 0 - 1.000 0				4.5		150
58431	1.290 0 - 1.300 0				3.8		190

These problems delayed the circulation and the schedule was adapted each time to the comparison needs with the final circulation scheme defined for each artefact as shown in Table 2. In the end, the participating laboratories were divided in two petals in the density ranges of 600 kg/m³, 800 kg/m³ and 1 000 kg/m³, whereas they were together in the density range of 1 300 kg/m³, as one of the two artefacts was broken. Moreover, some laboratories could not calibrate the whole initially assigned group of transfer standards: The VNIIM was not able to calibrate in the range of 600 kg/m³, IPQ, MIKES and BEV were not able to in the range of 1 300 kg/m³.

2.2 Transfer standards (hydrometer samples)

After the substitution of all hydrometers broken during transport and which were initially sent to the laboratories as transfer standards, the project was able to restart using eight new commercially available high-precision hydrometers. The new hydrometers were graded in terms of density (grammes per millilitre) with a scale division of 0.000 1 g/cm³; they were intended for measurements between 600 kg/m³ and 1 300 kg/m³ (Table 3).

The cubic expansion coefficient for all hydrometers was assumed to be $25 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ with an uncertainty of $2 \cdot 10^{-6} \text{ }^{\circ}\text{C}^{-1}$, rectangular distribution.

These artefacts were divided into two similar sets of four different hydrometers each to be calibrated at 20 °C.

The INRIM, as pilot laboratory, tested all artefacts both before and during the comparison except for the hydrometers 5941 and 58432, as they were broken during circulation. Through repeated measurements, before and during the comparison, the pilot laboratory did not detect any significant change in calibration of the remaining artefacts. Although at present these conditions did not allow to determine the reproducibility of each artefact, the measurements were consistent within the uncertainty evaluated by the pilot NMI (Table 4).

2.3 Conditions selected

The participating laboratories were asked to calibrate the assigned hydrometers at four graduation marks of the scale and the correction C had to be calculated for each of them at the reference temperature of 20 °C.

The test points and the surface-tension values of the liquid, in which each hydrometer was intended to be used, were stated in advance.

The participants were free to perform all measurements using their own procedure. It was, however, required that the hydrometers only stayed at the laboratory for the time necessary for calibration and not longer than the allotted time.

When the standards arrived at the participating laboratory, a visual inspection was made and each artefact was allowed to acclimate to the laboratory environment in agreement with the given instruction.

The participants took note of all information concerning the status of the transfer standards, the apparatus used during the comparison and the measurement results on the enclosed corresponding forms [2].

2.4 Procedure and method of measurement

In the following, details on how to handle the transfer standards, the test temperature and the marks to be calibrated, the minimum number of measurements, and the uncertainty analysis are described.

Table 5 summarizes the differences in the calibration procedure and in the equipments used at each NMI.

All participants carried out their task by adopting the hydrostatic weighing in a single liquid, the density of which was known for the test temperature.

At least 5 weighing sequences were carried out for each hydrometer in air and in the reference liquid at each of the four stated scale readings. The scale readings had to be adjusted to the liquid level such that the middle of the graduation mark was aligned with the horizontal plane of liquid.

Table 4. INRIM, as pilot laboratory, tested all artefacts, both before and during comparison except for the hydrometers 5941 and 58432. Although the reproducibility of each artefact was not known, no substantial drift was observed on the TSs. Both measurements of each artefact showed to be consistent within the uncertainty evaluated by the pilot laboratory.

Hydrometer		Scale mark	Date	Initial		Date	Final		ΔC (g cm ⁻³)
Range (g cm ⁻³)	S/N			C (g cm ⁻³)	U (g cm ⁻³)		C (g cm ⁻³)	U (g cm ⁻³)	
0.600 - 0.610	21964	0.600 5	March 2004	-0.000057	1.6E-05	October 2004	-0.000053	1.6E-05	-3.6E-06
		0.603 5		-0.000057			-0.000053		-4.2E-06
		0.606 5		-0.000053			-0.000048		-4.8E-06
		0.609 5		-0.000046			-0.000040		-6.4E-06
0.610 - 0.620	21971	0.610 5	March 2004	-0.000069	1.6E-05	December 2004	-0.000075	1.6E-05	6.4E-06
		0.613 5		-0.000072			-0.000076		4.4E-06
		0.616 5		-0.000077			-0.000082		5.3E-06
		0.619 5		-0.000071			-0.000077		6.2E-06
0.800 - 0.810	21958	0.800 5	March 2004	-0.000150	1.9E-05	October 2004	-0.000152	1.9E-05	1.5E-06
		0.803 5		-0.000143			-0.000145		1.7E-06
		0.806 5		-0.000141			-0.000143		2.5E-06
		0.809 5		-0.000147			-0.000149		2.0E-06
0.810 - 0.820	5941	0.810 5	March 2004	-0.000069	1.9E-05	/			
		0.813 5		-0.000083					
		0.816 5		-0.000061					
		0.819 5		-0.000004					
0.990 - 1.000	6905	0.990 5	March 2004	0.000390	2.4E-05	October 2004	0.000397	2.4E-05	-6.9E-06
		0.993 5		0.000372			0.000378		-6.3E-06
		0.996 5		0.000361			0.000371		-1.0E-05
		0.999 5		0.000381			0.000387		-6.0E-06
1.000 - 1.010	0001	1.000 5	March 2004	-0.000154	2.4E-05	December 2004	-0.000163	2.4E-05	8.8E-06
		1.003 5		-0.000147			-0.000148		8.0E-07
		1.006 5		-0.000107			-0.000104		-2.8E-06
		1.009 5		-0.000052			-0.000048		-4.2E-06
1.290 - 1.300	58431	1.290 5	March 2004	-0.000006	3.1E-05	November 2004	0.000000	3.1E-05	-5.7E-06
		1.293 5		-0.000009			-0.000009		2.3E-07
		1.296 5		-0.000011			-0.000008		-2.6E-06
		1.299 5		0.000002			0.000005		-2.7E-06
1.290 - 1.300	58432	1.290 5	March 2004	-0.000006	3.1E-05	/			
		1.293 5		-0.000009					
		1.296 5		-0.000011					
		1.299 5		0.000002					

Table 5. Summary of the experimental facilities operated at the different NMIs.

Institute	Balance		Buoyant liquid	Thermostat type, capacity	Thermometer for liquid temperature	Alignment	Surface tension method
	Max capacity [g]/readability [g] Weighing in air	Hydrostatic weighing					
INRIM	405 / 0.000 01		n-Nonane	Double-walled glass vesel, 30 litre	100 Ohm PRT, ac bridge	CCD camera automatic	Plate
OMH	1 000 / 0.00 1		n-Nonane	Tamson, 70 litre	Quartz thermometer	Magnifier hand-operated	Reference data
PTB*	410 / 0.000 1		n-Tridecane	Tamson, 70 litre	Mercury glass thermometer	CCD camera hand-operated	Plate
IPQ	405 / 0.000 1		n-Nonane	Tamson, 70 litre	100 Ohm PRT, digital thermometer	Magnifier hand-operated	Ring
MIKES	303 / 0.000 1		Ethanol	Double-walled glass vesel, 5 litre	100 Ohm PRT, digital thermometer	CCD camera hand-operated	Ring
LNE*	3 010 / 0.000 1	405 / 0.000 1	n-Tetradecane	Tamson, 70 litre	100 Ohm PRT, digital thermometer	Magnifier hand-operated	Du Nouy & Wilhelmy
BEV	300 / 0.000 1	1 109 / 0.000 1	n-Nonane	Double-walled glass vesel, 80 litre	25 Ohm PRT, digital thermometer	Magnifier hand-operated	Reference data
SMU	205 / 0.000 1		n-Nonane	Tamson, 70 litre	Mercury glass thermometer	Cathetometer hand-operated	Reference data
GUM*	1 109 / 0.000 1		n-Nonane	Tamson, 70 litre	25 Ohm PRT, ac bridge	Magnifier hand-operated	Ring
UME	405 / 0.000 1		n-Tridecane	Tamson, 70 litre	100 Ohm PRT, digital thermometer	Magnifier hand-operated	Ring
VNIIM	205 / 0.000 01		Ethanol	Tamson, 70 litre	Mercury glass thermometer	Magnifier hand-operated	Reference data

* *Weighing*: Substitution method

All laboratories except for BEV and LNE used the same balance-comparator for the weighing of the hydrometer both in air and in the reference liquid. The weighing method was usually the direct reading of the balance, however GUM, LNE and PTB used the substitution weighing by means of calibrated weights to achieve the balance readings being within a narrow electronic range.

The majority of the laboratories used the same hydrostatic apparatus to determine the density of the buoyant liquid and to check its stability before and after the hydrometer calibration. GUM and IPQ used a vibrating-tube densimeter for both activities at the same measuring conditions, INRIM and UME used a similar instrument only for monitoring the stability, PTB knew the temporal drift of the liquid density due to experience for more than ten years. All laboratories monitored the temperature of their own buoyant liquid during the hydrometer test, in particular, INRIM, MIKES, OHM, PTB, SMU and UME determined the density of the buoyant liquid during the hydrometer characterization using this value.

The surface tension of the buoyant liquids was measured in different ways. BEV, SMU, OHM, and VNIIM knew the values of their interest from reference data. MIKES and PTB used their own hydrostatic apparatus to measure the surface tension applying the ring-and-plate method, respectively. For this purpose, each of the remaining laboratories used different commercially available tensiometers.

The mean of the parameters contributing to the air-density calculation were recorded during calibration, i. e. pressure, temperature, relative humidity (or dew point); all laboratories usually assumed a constant value of 0.04% for the CO₂ content. The mean of the air-density values was calculated by the CIPM formula (CIPM81/91) [3] and reported.

Accurate calipers or suitable instruments with a resolution between 0.01 and 0.1 mm were used to measure the diameter of the stem of the hydrometer to be calibrated. PTB used an automatic measuring device by means of which the separation between graduation mark and the stem diameter throughout the whole scale of each hydrometer to be calibrated were measured.

In general, the laboratories manually aligned the liquid horizontal plane with the selected scale-mark, that is, the operator aligned the two elements with the horizontal plane by monitoring the hydrometer scale through a magnifier, or looking at a camera image on a computer monitor. Mechanical devices were used for sinking the tested hydrometer. Additionally, sinkers adjustable in height were used that the liquid level corresponds to the scale mark concerned. BEV used a wireless controlled lifting device as suspension. For hydrometer calibration, INRIM applied the method based on the image processing technique for observing the correct alignment and for allowing adjustment by moving the glass vessel to the position of intersection between the horizontal liquid surface and the stem. PTB used the same technique of alignment for checking the accuracy of the alignment.

2.4.1 Uncertainty claims of NMIs

Some laboratories presented uncertainty contributions in addition to those proposed in the worksheet by the coordinating laboratory [2], others considered several different contributions in an individual source. LNE included the temperature effects in the buoyant liquid density component and the gradient of gravitational acceleration in the gravitational acceleration components; so did INRIM.

Taking into account that hydrometers have the same scale division, the combined relative uncertainty of calibration is in most cases nearly constant to measure the density of liquids in the range between 500 kg m⁻³ and 2 000 kg m⁻³ [4].

Table 6. Evaluation of the relative uncertainty contributions to the combined standard uncertainty when calibrating hydrometers provided by participants related to hydrometer in the range of 1 000 kg/m³ and having a division of 0.1 kg/m³.

Influence quantities	INRIM %	PTB %	OMH %	IPQ %	MIKES %	LNE %	GUM %	BEV %	SMU %	UME %	VNIIM %
Weighing value of hydrometer in air	0.03	0.01	0.34	0.06	0.00	0.10	0.05	0.04	0.06	0.00	21.23
Weighing value of hydrometer in buoyant liquid	0.73	0.46	4.76	66.69	0.02	1.99	0.81	0.45	4.54	1.46	78.52
Additional weights					0.01						
Cubic thermal expansion coefficient of glass						0.44			0.01		
Diameter of stem of hydrometer	0.78	0.19	0.56	0.52	1.23	8.93	7.26			0.59	0.20
Density air	1.02	0.31	0.65	0.03	0.03	0.28	0.44	0.05	0.01	0.08	0.01
Density buoyant liquid (1)	34.01	16.88	15.72	5.77	93.25	68.94	46.86	1.03	62.04	97.20	0.03
Temperature of liquid in hydrometer	59.03	14.88	0.02	1.44			16.74	0.00	13.74		0.01
Cubic thermal expansion coefficient of liquid		0.10								0.05	
Compressibility of liquid		0.01	3.06								
Surface tension of liquid	1.65	6.76	57.46	0.06	1.44	8.29	2.28	0.45	9.07	0.05	
Gravitation acceleration (2)										0.01	
Gradient of gravitational acceleration											
Height difference of weights and hydrometer											
Error of readings	2.75	34.45	11.44	25.44	4.01	4.90	25.57	12.14		0.57	
Standard deviation of the mean of corrections or reproducibility of measurement			6.00			6.13			10.52		
Systematic uncertainty of weighing in liquid		22.84			0.01						
Influence of the temperature distribution in the bath		3.10									
Additional uncertainty due to incompletely testing device								24.37			
Drift of balance								61.47			

As example, Table 6 shows the percentage amount of the individual contribution to the uncertainty provided by the participants related to the hydrometer in the range of 1 000 kg/m³. According to this information the temperature of liquid affected directly or indirectly the major uncertainty sources which are related to: 1) buoyant liquid density, 2) readings and, finally 3) weighing in the liquid.

3. RESULTS

3.1 Reported data

After the measurements were completed by the each participating institute, all information concerning the calibration were submitted to the pilot laboratory using the sheets Report Form 1 and Report Form 2 annexed to [2]. The INRIM collected and analyzed anything related to:

- a) Details of the instrumentation used by each participant in the project, including the origin of their traceability to the SI.
- b) Details of the relevant information on the measurements and parameters used for the comparison as local gravity, mass measurements, density of working fluid and, finally, the ambient conditions including data on air density, air temperature, air pressure, humidity and CO₂ content.
- c) Calculated values of the four corrections for each transfer standard at the specified reading marks and surface tension values.
- d) Uncertainty budget of the four calculated corrections, which were estimated and combined following GUM [5] under the responsibility of each participating institute. Each laboratory also reported the uncertainty of all measured quantities as well as the effective degrees of freedom ν_{eff} of the combined standard uncertainty u_c , the t-factor $t_{95}(\nu_{\text{eff}})$ taken from the t-distribution for a 95% confidence level and the expanded uncertainty for the corrections as $U_{95} = t_{95}(\nu_{\text{eff}}) \cdot u_c$.

3.2 Analysis of reported data

3.2.1 Degrees of equivalence with respect to the *EU-KCRV*

The results from each laboratory i are characterized in terms of a ‘degree of equivalence’ representing the deviation D_i of its result x_i from the accepted EUROMET_Key Comparison Reference Value (*EU_KCRV*)

$$D_i = x_i - EU_KCRV \quad (1)$$

with the associated uncertainty

$$u_{D_i} = \left(u^2(x_i) + u^2(EU_KCRV) - 2 \text{cov}(x_i, EU_KCRV) \right)^{\frac{1}{2}} \quad (2)$$

In equation (2), the *EU_KCRV* is interpreted as an estimate of the measurand made on the basis of the measurements provided by the participating institutes, and $\text{cov}(x_i, EU_KCRV)$ is the covariance term between a result and the chosen *EU_KCRV*.

Several methods for defining the *EU_KCRV* have been proposed, among which the recommendations of the BIPM Director’s Advisory Group on Uncertainties [6] has been taken into account for the EUROMET comparison under

study. In this comparison each hydrometer had been independently circulated following one of the two possible petal patterns. A single EU_KCRV has been calculated at each calibrated mark using the results of all laboratories of each petal, in agreement with the above recommendation.

The first approach for determining the reference value was based on the calculation of the weighted mean of the institutes' measurements (Procedure A), using the inverses of the squares of the associated standard uncertainties as the weights:

$$EU_KCRV = \left(\sum_{i=1}^N \frac{x_i}{u^2(x_i)} \right) \times \left(\sum_{i=1}^N \frac{1}{u^2(x_i)} \right)^{-1} \tag{3}$$

Such EU_KCRV , however, is not applicable if some of the institutes' measurements appear to be anomalous or discrepant.

To identify inconsistent results a chi-squared test was then applied considering the consistency check as failing if

$$\Pr\{\chi^2(\nu) > \chi_{obs}^2\} < 0.05 \tag{4}$$

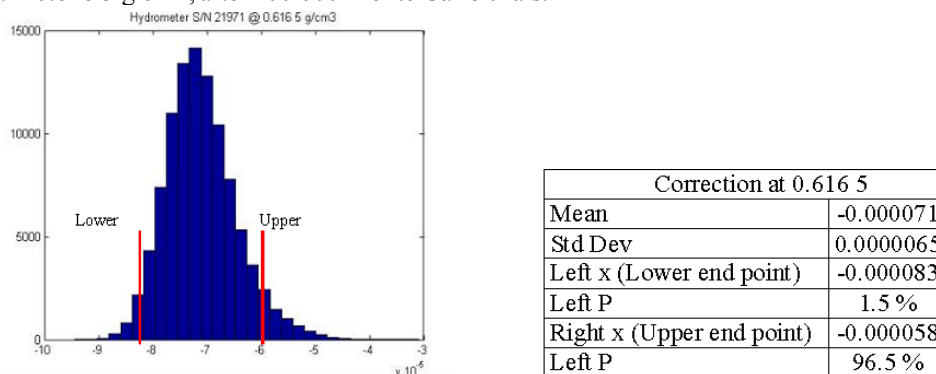
where $\nu = N - 1$ is the number of degrees of freedom and χ_{obs}^2 is the observed chi-squared value.

If the test was not satisfying, the EU_KCRV was computed by applying procedure B of [6].

By means of a Monte Carlo simulation, 100 000 random samples were generated, each made of n values drawn from the distributions representing the results from each laboratory (n , here, is the number of the laboratories of the relevant petal). In this way, 100 000 values for the median of the drawn samples were obtained. The mean of such values was taken as the EU_KCRV of the single petal. Also the corresponding simulated deviation terms of the degrees of equivalence were obtained for each laboratory and used to determine a 95% coverage interval for the laboratories' deviation from the EU_KCRV .

The use of the median as a EU_KCRV reduces the sensitivity to individual discrepant results, but usually its simulated distribution is not symmetric so that the shortest 95% coverage interval could be taken as the proper coverage intervals for the deviations (1). In Figure 1, a typical example shows the probability distribution of the EU_KCRV for the 21971 hydrometers at the mark 0.6165 g cm^{-3} after 100 000 Monte Carlo trials.

Figure 1. Approximation to the probability distribution of the EU_KCRV for the 21971 hydrometers at the relevant mark 0.6165 g cm^{-3} , after 100 000 Monte Carlo trials.



For each artefact, Appendix A shows the measurement results, standard uncertainties as reported by the participants and the calculated EU_KCRV with the related uncertainty or the lower and upper limits of the coverage interval if procedure B was applied. Moreover, for each petal Appendix A includes the table and the graph concerning the degree of equivalence of the NMIs with the EU_KCRV .

3.2.2 Degree of equivalence between pairs of NMIs

The difference between pairs of participating laboratories can be evaluated to assess the degree of compatibility between the measurement capabilities of the laboratories. By the 4 corrections provided by each participant for each measured artefact, a value x was calculated as the arithmetic average, the uncertainty u_x was determined according to the [4] and a correlation coefficient of 1 was considered between all laboratory measurements. The relative performance of participants i and j , of petals 1 and 2, respectively, in which different artefacts were circulated, were determined as the degree of equivalence between pairs of the participating NMIs d_{ij} [7] defined by

$$d_{ij} = (x_{i1} - RV_1) - (x_{j2} - RV_2) = (x_{i1} - x_{j2}) - (RV_2 - RV_1) \quad (5)$$

where x_{i1} and x_{j2} are the arithmetic averages of the four corrections of the artefacts 1 and 2 performed by laboratories i and j , respectively. The RV_1 and RV_2 represent two estimates of the artefacts derived directly from the measurements of linking laboratory/ies which are involved in both loops, they are used as a sort of “reference value” with the aim of comparing the results of laboratories participating to different loops. The reference values RV_1 and RV_2 were determined from the weighted mean of the arithmetic averages of the 4 corrections \tilde{x}_i provided by each of the three linking laboratories INRIM, PTB and OMH for each hydrometer with approximately the same range, respectively.

$$RV_{1,2} = \left(\sum_{i=1}^3 \frac{\tilde{x}_{i,2}}{u^2(\tilde{x}_{i,2})} \right) \times \left(\sum_{i=1}^3 \frac{1}{u^2(\tilde{x}_{i,2})} \right)^{-1} \quad (6)$$

and uncertainty

$$u_{RV_{1,2}} = \left(\sum_{i=1}^3 \frac{1}{u^2(\tilde{x}_{i,2})} \right)^{\frac{1}{2}} \quad (7)$$

Equation (5) shows that the difference between laboratories i and j is corrected by the difference between the reference values. In equation (5), the EU_KCRV of each loop could be used instead of the reference values RV_1 and RV_2 . This fact would not have implied large changes in the result, but would have made it necessary to evaluate the correlation between the single laboratory and the EU_KCRV for calculating the uncertainty of d_{ij} .

The uncertainty of the degree of equivalence yields:

$$u_{d_{ij}} = \left(\sum_{k=x_{i1}, x_{j2}, RV_1, RV_2} u_k^2 - 2 \text{cov}((x_{i1} - RV_1), (x_{j2} - RV_2)) \right)^{\frac{1}{2}} \quad (8)$$

$$\approx \left(u_{x_{i1}}^2 + u_{x_{j2}}^2 + 2u_{RV}^2 \right)^{\frac{1}{2}}$$

where $u_{RV1}^2 \approx u_{RV2}^2 \approx u_{RV}^2$ and any possible correlation between NMIs, i.e. such as a traceable calibration, is taken into account by the term of covariance. However, in this comparison, there is no possible correlation between the participating NMIs.

The uncertainty of the degree of equivalence at a confidence level $U_{d_{ij}}$ of 95% for a coverage factor $k = 2$ is

$$U_{d_{ij}} = ku_{d_{ij}} \quad (9)$$

In this project, three possible situations have to be considered.

- a) The laboratories are in different loops. The degree of equivalence d_{ij} and its uncertainty is given by equations (5) and (8).

A particular situation of this point is if one of the two laboratories, i.e. the j one, measures both artefacts yielding $\tilde{x}_{j1,2}$. The degree of equivalence of each petal for d_{ij} yields

$$d_{ij} = (x_{i1} - RV_1) - \frac{((\tilde{x}_{j1} - RV_1) + (\tilde{x}_{j2} - RV_2))}{2} \quad (10)$$

and uncertainty

$$u_{d_{ij}} \approx \left((u_{x_{i1}}^2 + u_{RV}^2) + \frac{3(u_{\tilde{x}_j}^2 - u_{RV}^2)}{4} \right)^{\frac{1}{2}} \quad (11)$$

where $u_{\tilde{x}_{j1}}^2 \approx u_{\tilde{x}_{j2}}^2 \approx u_{\tilde{x}_j}^2$.

- b) Both laboratories are in the same loop. The degree of equivalence of each petal for d_{ij} yields

$$d_{ij} = (x_i - x_j) \quad (12)$$

and the uncertainty

$$u_{d_{ij}} = (u_{x_i}^2 + u_{x_j}^2)^{\frac{1}{2}} \quad (13)$$

A particular situation of this point is whether both laboratories i and j participated to both petals, hence contributing to determine the reference values RV_1 and RV_2 . In this case, the degree of equivalence is given by

$$d_{ij} = \frac{(\tilde{x}_{i1} - \tilde{x}_{j1}) + (\tilde{x}_{i2} - \tilde{x}_{j2})}{2} \quad (14)$$

and the uncertainty by

$$u_{d_{ij}} \approx \left(u_{\bar{x}_i}^2 + u_{\bar{x}_j}^2 \right)^{\frac{1}{2}} \quad (15)$$

For each artefact, Appendix B presents the arithmetic average of the 4 corrections provided by participant x_{ij} and the standard uncertainty $u_{x_{ij}}$. Appendix B also shows the results for the degree of equivalence between pairs of the participating NMIs d_{ij} and the extended uncertainty $U_{d_{ij}}$ for the range between 600 kg/m³ and 1 300 kg/m³.

4. DISCUSSION and CONCLUSION

The main objective of the project was to determine the extent of comparability among participating NMIs in performing calibrations of high-resolution hydrometers for liquid density determination in the range between 600 kg/m³ and 1 300 kg/m³. After some unforeseen difficulties relating to artefact breakage, transportation, customs and administrative constraints, the project went on two independent petals in the density ranges of 600 kg/m³, 800 kg/m³ and 1 000 kg/m³, and on a single one in the density range 1 300 kg/m³.

As pilot laboratory, INRIM collected all results from all laboratories and determined a single *EU_KCRV* for each petal.

With the aim of including the contribution of all participating NMIs, the pilot laboratory calculated the *EU_KCRV* using either the weighted mean, or a more robust one, such as the median, as estimator. This was suggested in case some of the institutes' measurements appeared to be anomalous or discrepant and a consistency check failed.

The EUROMET project 702 (EUROMET.M.D-K4) was not only useful to each participant to determine their degrees of equivalence with the key comparison reference value, but also to gain more knowledge of their own capabilities in the calibration activities and in measuring the liquid density. Some systematic differences and either underestimated or overestimated uncertainties can be identified between the submitted results, although a particular good agreement was found among the results provided by most of the participants. Last but not least the results allowed also to determine the degree of equivalence between pairs of the participating NMIs.

In conclusion, these results could support the reduction of some uncertainty contributions of some participants, the degrees of equivalence in this comparison will serve as a basis for the new Calibration Measurement Capabilities (CMC) entries on hydrometer calibration and, finally, the experience acquired in this key comparison shall be taken into account in conducting the planned CIPM key comparisons and further intercomparisons in the future.

5. Acknowledgements

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6. References

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7. Appendix A

This section deals with the measurement results and the standard uncertainties as reported by the participants. For each artefact, the calculated EUROMET *Key-Comparison Reference Value* (EU_KCRV) at each calibrated mark, with the related uncertainty or the lower and upper limits of the confidence interval if procedure B was applied [6]. Moreover, for each petal the table and the graph concerning the degree of equivalence of the NMIs with the EU_KCRV are shown.

7.1 Hydrometer S/N 21964

The EU_KCRVs for the petal identified by the hydrometer 21964 have been calculated by applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.1**, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). **Table A.1** shows also each calculated EU_KCRVs with the expanded uncertainty.

Table A.2 and **Figure A.1** present the degree of equivalence with respect to the EU_KCRVs of the concerned NMIs.

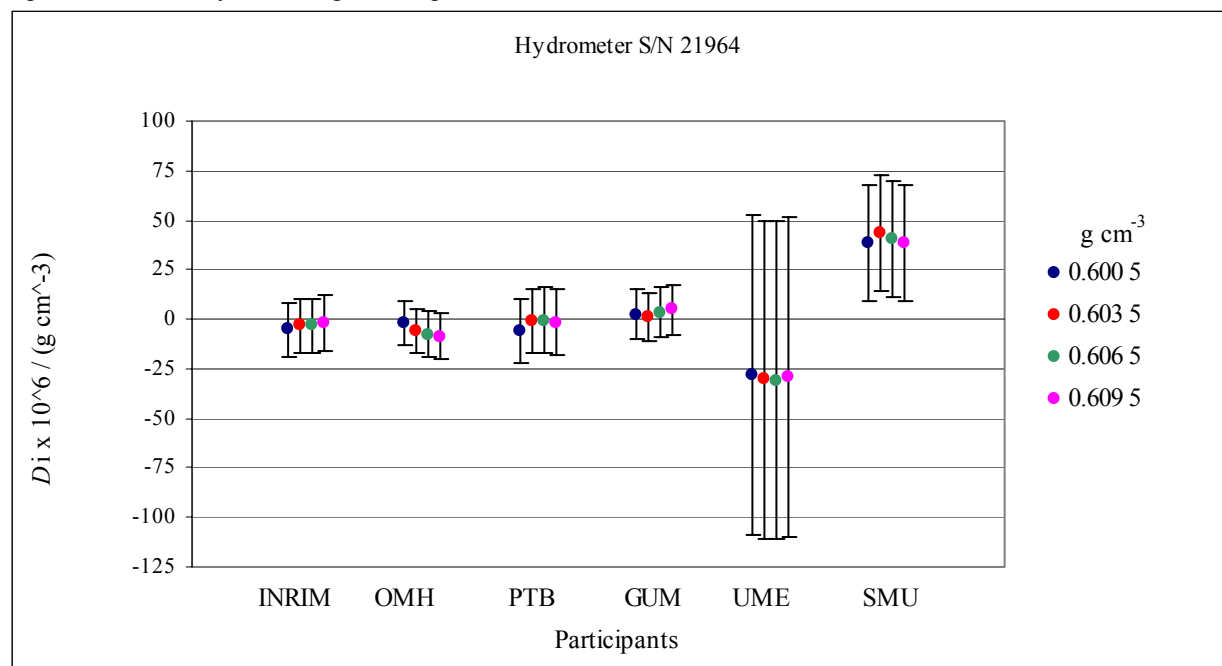
Table A.1. Measurements results as reported by the participants for the petal identified by the hydrometer 21964 and the EU_KCRV with the expanded uncertainty at each calibrated mark.

S/N. 21964 g cm ⁻³	NMI						EU_KCRV (weighted mean)	$U(EU_KCRV)$	
	INRIM	OMH	PTB	GUM	UME	SMU			
0.600 5	-57	-53	-57	-49	-80	-13	-51.2	7.3	
0.603 5	-57	-60	-55	-53	-85	-11	-54.1	7.3	
0.606 5	-53	-57	-50	-46	-80	-9	-49.5	7.3	
0.609 5	-46	-53	-46	-40	-74	-6	-44.4	7.3	
Combined standard uncertainty of corrections. u_c	7.8	6.7	9.0	7.2	40.4	15.1			
Expanded uncertainty of corrections. $U_{95} = t_{95}(n_{eff}) u_c$	16	13	18	14	81	30			
Student t-factor $t_{95}(n_{eff})$	1.98	1.97	1.98	1.98	2.01	1.97			
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$	$\chi^2(5) = 11.07 > \chi^2_{obs}$			The consistency test does not fail; procedure A					

Table A.2. Degree of equivalence with respect to the *EU_KCRV* and the expanded uncertainty at each calibrated mark of each laboratory of the relevant petal identified by the hydrometer 21964.

NMI	g cm ⁻³	0.600 5		0.603 5		0.606 5		0.609 5	
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶
INRIM	g cm ⁻³	-5	14	-3	14	-3	14	-2	14
OMH		-2	11	-6	11	-8	11	-9	11
PTB		-6	16	-1	16	-1	16	-2	16
GUM		3	12	1	12	4	12	5	12
UME		-28	80	-31	80	-31	80	-29	80
SMU		38	29	43	29	40	29	38	29

Figure A.1. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the relevant petal identified by the hydrometer 21964. The lengths of the bars show the expanded uncertainty of the degree of equivalence related to each calibrated mark of the relevant NMI.



7.2 Hydrometer S/N 21971

The *EU_KCRVs* for the petal identified by the hydrometer 21971 have been calculated by applying the “*median*” method, since the consistency check, $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$, of the measurement results and standard uncertainties of the participants reported in **Table A.3**, was not satisfying. The Instituto Português da Qualidade – IPQ data had been identified as the source of inconsistency. Due to the fact that no trivial/obvious error was found, IPQ results remain discrepant. **Table A.3** also shows each calculated *EU_KCRVs* with the lower and upper limits of the coverage interval.

Table A.4 and **Figure A.2** present the degree of equivalence with respect to the *EU_KCRV* of the concerned NMIs.

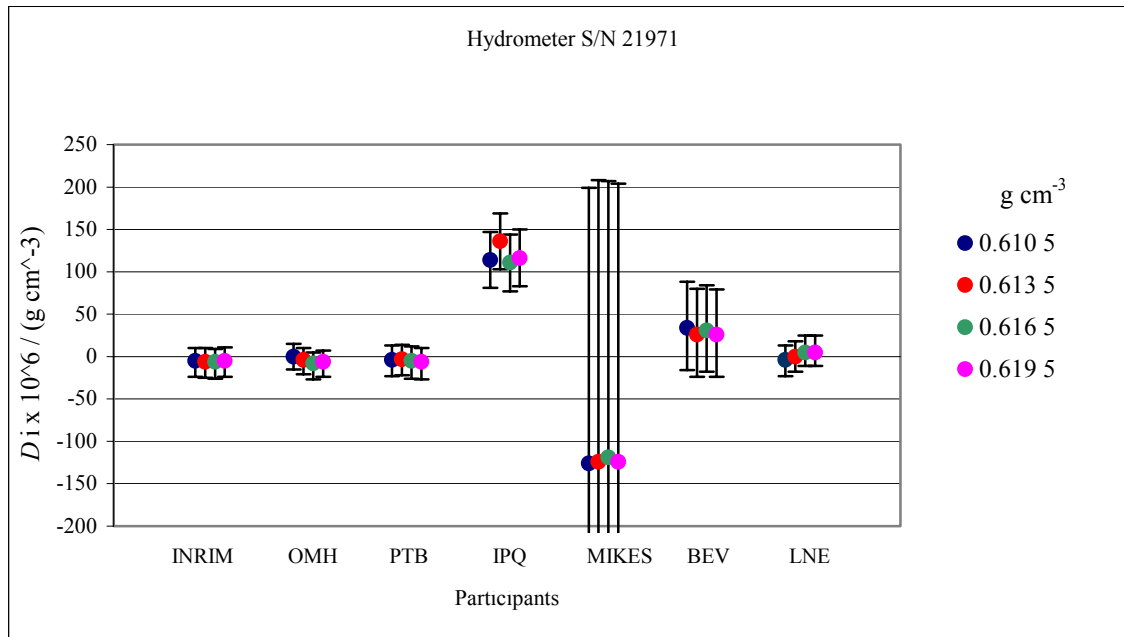
Table A.3. Measurements results as reported by the participants for the petal identified by the hydrometer 21971 and the *EU_KCRV* with the coverage interval at each calibrated mark.

S/N. 21971 g cm ⁻³	NMI							<i>EU_KCRV (median)</i>	<i>Lower</i>	<i>Upper</i>
	INRIM	OMH	PTB	IPQ	MIKES	BEV	LNE			
0.610 5	-69	-64	-68	50	-190	-30	-68	-63.8	-74.6	-52.9
0.613 5	-72	-70	-69	70	-190	-40	-66	-66.3	-77.2	-54.5
0.616 5	-77	-79	-76	40	-190	-40	-66	-71.1	-83.4	-57.7
0.619 5	-71	-72	-72	50	-190	-40	-61	-66.0	-78.1	-53.3
Combined standard uncertainty of corrections. <i>u_c</i>	8.0	6.5	8.9	15.8	168.2	27.1	9.0			
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	16	13	18	32	335	53	18			
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.96	1.98	2.00	1.99	1.96	1.99			
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(6) = 12.59 < \chi^2_{obs}$				The consistency test fails; procedure B				

Table A.4. Degree of equivalence with respect to the *EU_KCRV* and coverage interval at each calibrated mark of each laboratory of the petal identified by the hydrometer 21971.

NMI	g cm ⁻³	0.610 5			0.613 5			0.616 5			0.619 5		
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	
			Lower	Upper		Lower	Upper		Lower	Upper		Lower	Upper
INRIM		-5	19	15	-6	19	16	-6	20	15	-5	19	16
OMH		0	15	15	-4	17	14	-8	19	13	-6	18	13
PTB		-4	19	17	-3	19	17	-5	21	17	-6	21	16
IPQ		114	33	33	136	33	33	111	34	33	116	33	34
MIKES		-126	329	325	-124	323	332	-119	324	326	-124	327	328
BEV		34	50	54	26	50	54	31	49	53	26	50	53
LNE		-4	19	17	0	18	18	5	16	20	5	16	20

Figure A.2. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 21971. The lengths of the bars show the coverage interval of the degree of equivalence related to each calibrated mark of the relevant NMI.



7.3 Hydrometer S/N 5941

The *EU_KCRVs* for the petal identified by the hydrometer 5941 have been calculated applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.5** at the calibrated marks 0.810 5 g cm⁻³ and 0.813 5 g cm⁻³, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). Conversely, the *EU_KCRVs* related to the calibrated marks 0.816 5 g cm⁻³ and 0.819 5 g cm⁻³ reported in **Table A.5** have been calculated applying the “*median*” method, since the consistency check $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$, was not satisfying. The Instituto Português da Qualidade – IPQ data had been identified as the source of inconsistency at the calibrated marks 0.810 5 g cm⁻³ and 0.813 5 g cm⁻³. Due to the fact that no obvious/trivial error was found, the IPQ results remain discrepant. **Table A.5** shows also each calculated *EU_KCRVs* with the expanded uncertainties and/or the lower and upper limits of the coverage intervals.

Table A.6 and **Figure A.3** present the degree of equivalence with respect to the *EU_KCRV* of the concerned NMIs.

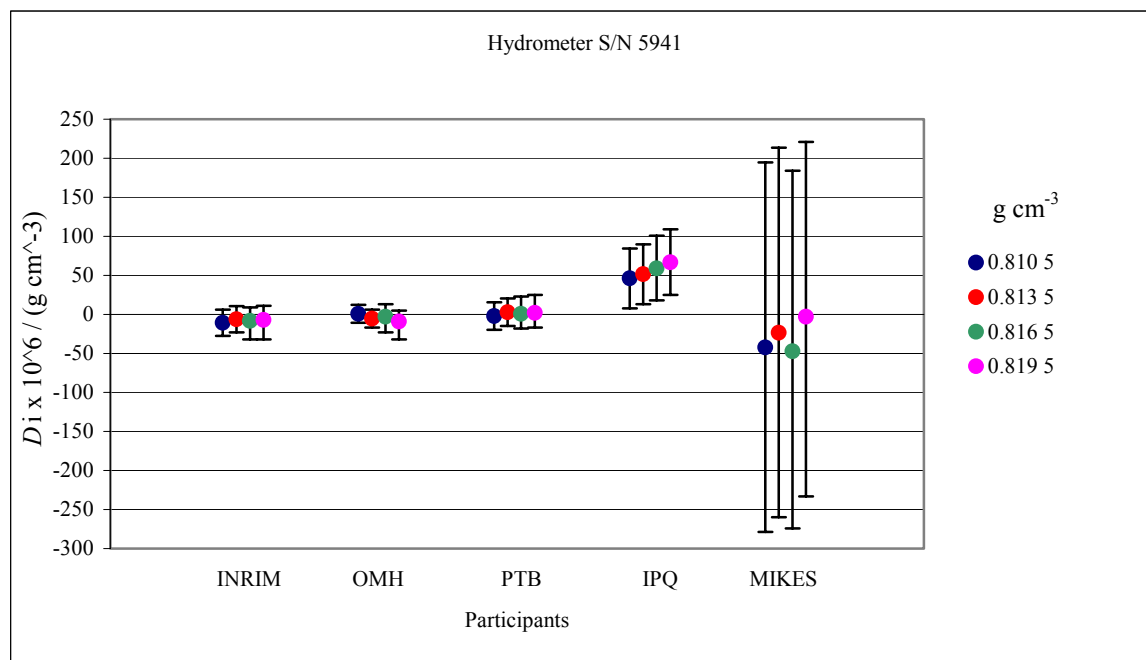
Table A.5. Measurement results as reported by the participants for the petal identified by the hydrometer 5941 and value of the *EU_KCRV* with the uncertainty or the coverage interval at each calibrated mark.

S/N. 5941 g cm ⁻³	NMI					<i>EU_KCRV</i>	<i>U(EU_KCRV)</i>	<i>Lower</i>	<i>Upper</i>
	INRIM	OMH	PTB	IPQ	MIKES				
0.810 5	-69	-57	-60	-12	-100	-58.0 (w. mean)	10.0		
0.813 5	-83	-82	-74	-25	-100	-76.7 (w.mean)	10.0		
0.816 5 (*)	-61	-56	-52	6	-100	-53.4 (median)		-67.7	-37.9
0.819 5 (*)	-4	-6	5	70	0	2.7 (median)		-13.2	20.0
Combined standard uncertainty of corrections. <i>u_c</i>	9.8	7.6	10.1	19.8	118.4				
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	19	15	20	39	237				
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.97	1.97	1.99	2.00				
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(4) = 9.49 > \chi^2_{obs}$ (*) $\chi^2(4) = 9.49 < \chi^2_{obs}$			The consistency test does not fail; procedure A The consistency test fails; procedure B				

Table A.6. Degree of equivalence with respect to the *EU_KCRV* and uncertainty of the coverage interval at each calibrated mark of each laboratory of the petal identified by the hydrometer 5941.

NMI	g cm ⁻³	0.810 5		0.813 5		0.816 5			0.819 5		
		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶	<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶	<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶	
							Lower	Upper		Lower	Upper
INRIM		-11	17	-6	17	-8	24	17	-7	25	18
OMH		1	11	-5	11	-3	20	16	-9	23	14
PTB		-2	18	3	18	1	19	22	2	19	23
IPQ		46	38	51	38	59	41	42	67	42	42
MIKES		-42	237	-23	237	-47	227	231	-3	230	224

Figure A.3. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 5941. The lengths of the bars show the uncertainty or the coverage interval of the degree of equivalence related to each calibrated mark of the concerned NMI.



7.4 Hydrometer S/N 21958

The *EU_KCRVs* for the petal identified by the hydrometer 21958 have been calculated by applying the “*median*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.7**, was not satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). The All-Russian Research Institute of Metrology - VNIIM data had been identified as the source of inconsistency. Due to the fact that no obvious/trivial errors were found, VNIIM results remain discrepant. **Table A.7** shows also each calculated *EU_KCRVs* with the lower and upper limits of the coverage intervals.

Table A.8 and **Figure A.4** present the degree of equivalence with respect to the *EU_KCRV* of the concerned NMIs.

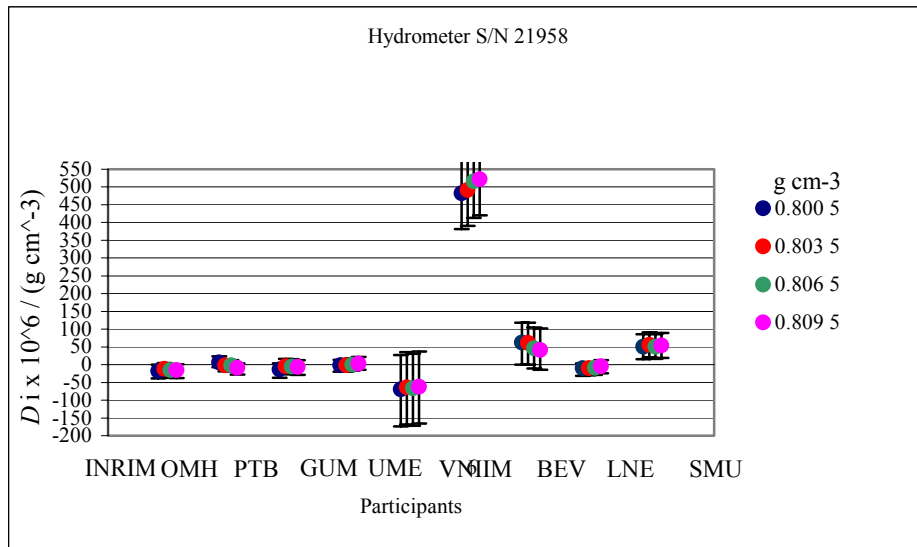
Table A.7. Measurements results as reported by the participants for the petal identified by the hydrometer 21958 and the *EU_KCRV* with the coverage interval at each calibrated mark.

S/N. 21958 g cm ⁻³	NMI										<i>EU_KCRV</i> (median)	Lower	Upper
	INRIM	OMH	PTB	GUM	UME	VNIIM	BEV	LNE	SMU				
0.800 5	-150	-126	-145	-134	-201	350	-70	-142	-81	-131.8	-144.0	-119.8	
0.803 5	-143	-133	-135	-133	-195	360	-70	-141	-75	-131.2	-142.1	-120.2	
0.806 5	-141	-128	-131	-126	-193	390	-80	-135	-75	-126.2	-137.2	-115.1	
0.809 5	-147	-141	-138	-129	-194	390	-90	-136	-78	-132.2	-143.8	-120.4	
Combined standard uncertainty of corrections. <i>u_c</i>	9.7	7.6	10.9	8.9	53.4	51.5	30.5	9.0	17.0				
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	19	15	22	18	107	132	60	18	34				
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.97	1.97	1.98	2.01	2.57	1.96	1.99	1.98				
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(8) = 15.51 < \chi^2(obs)$					The consistency test fails; procedure B						

Table A.8. Degree of equivalence with respect to the *EU_KCRV* and coverage interval at each calibrated mark of each laboratory of the petal identified by the hydrometer 21958.

NMI	g cm ⁻³	0.800 5			0.803 5			0.806 5			0.809 5		
		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶		<i>Di</i> x10 ⁶	<i>U(Di)</i> x10 ⁶	
			Lower	Upper		Lower	Upper		Lower	Upper		Lower	Upper
INRIM		-18	21	18	-12	22	17	-15	22	17	-15	23	16
OMH		6	13	18	-2	17	15	-2	17	15	-9	19	13
PTB		-13	24	17	-4	22	20	-5	22	20	-6	23	19
GUM		-2	18	16	-2	18	18	0	17	18	3	17	19
UME		-69	105	96	-64	104	99	-67	105	98	-62	104	99
VNIIM		482	101	103	491	101	101	516	103	99	522	102	100
BEV		62	62	56	61	61	57	46	57	59	42	56	60
LNE		-10	21	14	-10	20	16	-9	20	16	-4	20	17
SMU		51	36	35	56	35	35	51	35	35	54	35	35

Figure A.4. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 21958. The lengths of the bars show the coverage interval of the degree of equivalence related to each calibrated mark of the concerned NMI.



7.5 Hydrometer S/N 6905

The EU_KCRVs for the petal identified by the hydrometer 6905 have been calculated by applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.9**, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). **Table A.9** shows also each calculated EU_KCRVs with their expanded uncertainties.

Table A.10 and **Figure A.5** present the degree of equivalence with respect to the EU_KCRVs of the concerned NMIs.

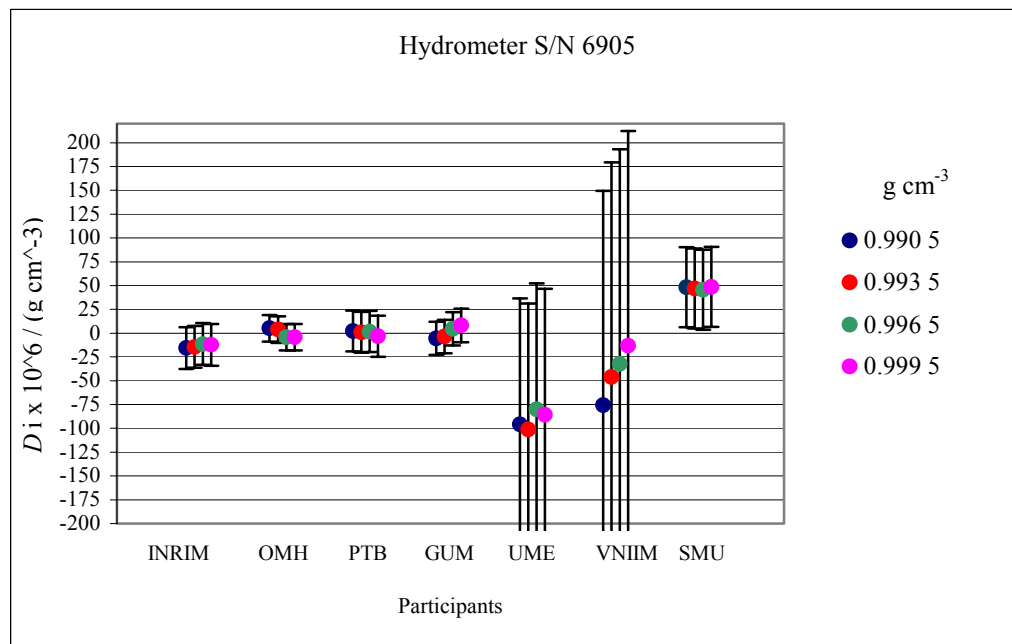
Table A.9. Measurements results as reported by the participants for the petal identified by the hydrometer 6905 and the EU_KCRV with its expanded uncertainty at each calibrated mark.

S/N. 6905 g cm ⁻³	NMI							EU_KCRV (weighted mean)	$U(EU_KCRV)$
	INRIM	OMH	PTB	GUM	UME	VNIIM	SMU		
0.990 5	390	411	408	400	310	330	454	405.8	10.0
0.993 5	372	390	387	382	285	340	433	386.1	10.0
0.996 5	361	368	374	377	292	340	418	372.3	10.0
0.999 5	381	389	390	401	308	380	442	393.2	10.0
Combined standard uncertainty of corrections. u_c	12.1	8.6	11.9	10.1	66.4	112.8	21.6		
Expanded uncertainty of corrections. $U_{95} = t_{95}(n_{eff}) u_c$	24	17	23	20	133	276	43		
Student t-factor $t_{95}(n_{eff})$	1.98	1.97	1.97	1.97	2.01	2.45	1.98		
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(6) = 12.59 > \chi^2(obs)$			The consistency test does not fail; procedure A				

Table A.10. Degree of equivalence with respect to the *EU_KCRV* and expanded uncertainty at each calibrated mark of each laboratory of the petal identified by the hydrometer 6905.

NMI	g cm ⁻³	0.990 5		0.993 5		0.996 5		0.999 5	
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶
INRIM		-16	22	-14	22	-11	22	-12	22
OMH		5	14	4	14	-4	14	-4	14
PTB		2	22	1	22	2	22	-3	22
GUM		-6	18	-4	18	5	18	8	18
UME		-96	132	-101	132	-80	132	-86	132
VNIIM		-76	225	-46	225	-32	225	-13	225
SMU		48	42	47	42	46	42	49	42

Figure A.5. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 6905. The lengths of the bars show the expanded uncertainty of the degree of equivalence related to each calibrated mark of the concerned NMI.



7.6 Hydrometer S/N 0001

The *EU_KCRVs* for the petal identified by the hydrometer 0001 have been calculated by applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.11**, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). **Table A.11** shows also each calculated *EU_KCRVs* with the expanded uncertainties.

Table A.12 and **Figure A.6** present the degree of equivalence with respect to the *EU_KCRVs* of the concerned NMIs.

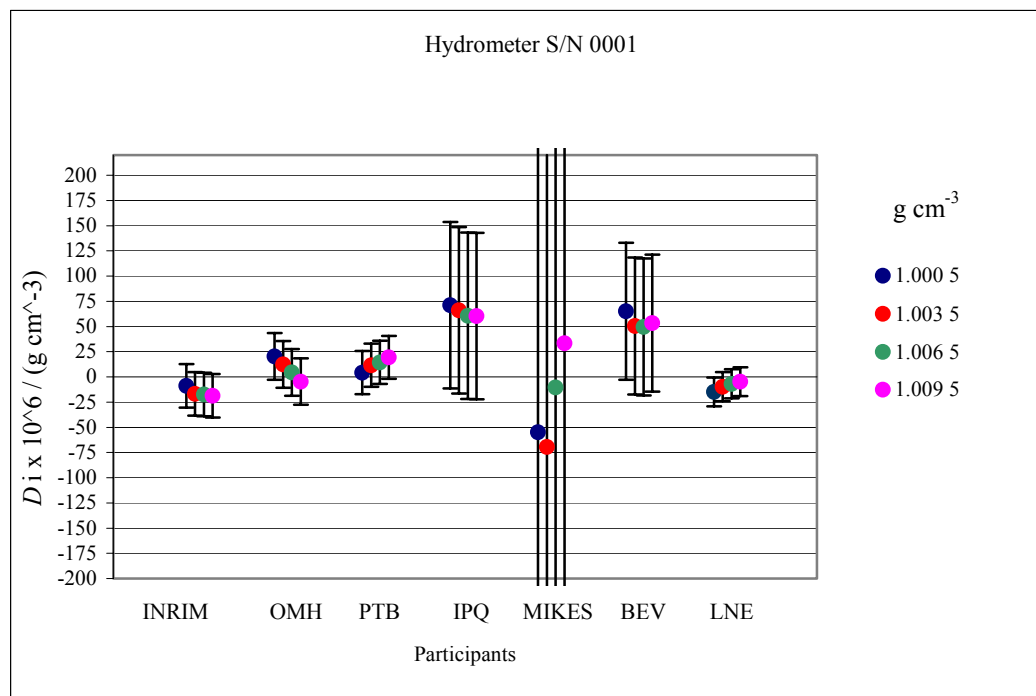
Table A.11. Measurement results as reported by the participants for the petal identified by the hydrometer 0001 and the *EU_KCRV* with its expanded uncertainty at each calibrated mark.

S/N. 0001 g cm ⁻³	NMI							<i>EU_KCRV (weighted mean)</i>	<i>U(EU_KCRV)</i>
	INRIM	OMH	PTB	IPQ	MIKES	BEV	LNE		
1.000 5	-154	-125	-141	-74	-200	-80	-160	-145.2	10.9
1.003 5	-147	-118	-119	-65	-200	-80	-140	-130.5	10.9
1.006 5	-107	-85	-75	-29	-100	-40	-96	-89.4	10.9
1.009 5	-52	-38	-14	27	0	20	-38	-33.4	10.9
Combined standard uncertainty of corrections. <i>u_c</i>	12.1	12.8	12.0	41.6	144.9	34.4	9.0		
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	24	25	24	96	290	67	18		
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.96	1.97	2.31	2.00	1.96	1.99		
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(6) = 12.59 > \chi^2_{obs}$				The consistency test does not fail; procedure A			

Table A.12. Degree of equivalence with respect to the *EU_KCRV* and expanded uncertainty at each calibrated mark of each laboratory of the petal identified by the hydrometer 0001.

NMI	g cm ⁻³	1.000 5		1.003 5		1.006 5		1.009 5	
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶
INRIM		-9	22	-17	22	-17	22	-19	22
OMH		20	23	13	23	4	23	-5	23
PTB		4	21	12	21	14	21	19	21
IPQ		71	82	66	82	61	82	60	82
MIKES		-55	290	-69	290	-11	290	33	290
BEV		65	68	51	68	49	68	53	68
LNE		-15	14	-10	14	-7	14	-5	14

Figure A.6. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 0001. The lengths of the bars show the expanded uncertainty of the degree of equivalence related to each calibrated mark of the concerned NMI.



7.7 Hydrometer S/N 58431

The *EU_KCRVs* for the petal identified by the hydrometer 58431 have been calculated by applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants at the calibrated marks 1.293 5 g cm⁻³, 1.296 5 g cm⁻³ and 1.299 5 g cm⁻³ reported in **Table A.13**, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). Conversely, the *EU_KCRVs* related at the calibrated mark 1.290 5 g cm⁻³ reported in **Table A.13** have been calculated by applying the “*median*” method, since the consistency check as $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$, was not satisfying. The Országos Mérésügyi Hivatal - OMH data had been identified as the source of inconsistency at the calibrated marks 1.290 5 g cm⁻³. Due to the fact that no obvious/trivial error was found, OMH result remains discrepant. **Table A.13** shows also each calculated *EU_KCRVs* with the expanded uncertainties and/or the lower and upper limits of the coverage intervals.

Table A.14 and **Figure A.7** present the degree of equivalence with respect to the *EU_KCRV* of the concerned NMIs.

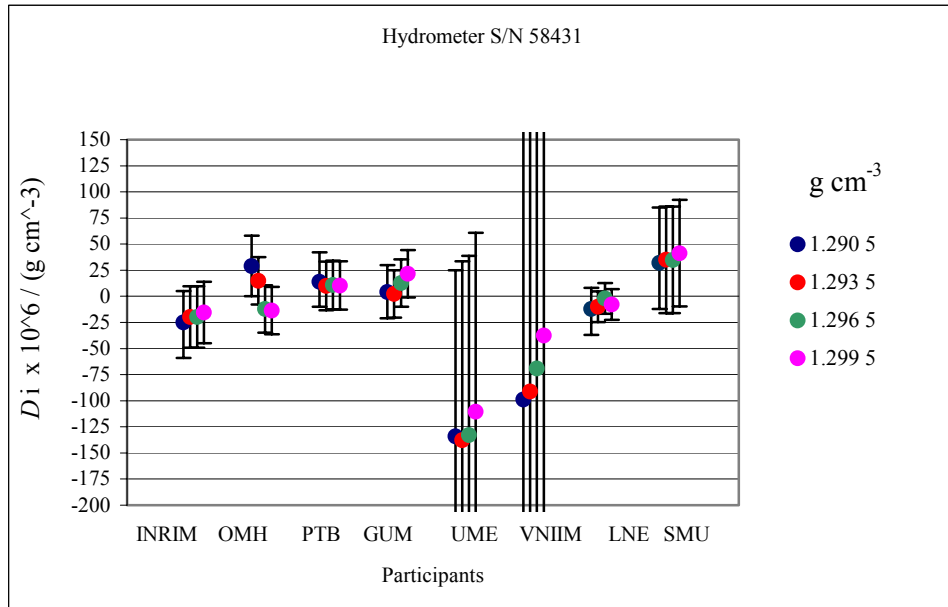
Table A.13. Measurements results as reported by the participants for the petal identified by the hydrometer 58431 and the *EU_KCRV* with its uncertainty or the coverage interval at each calibrated mark.

S/N. 58431 g cm ⁻³	NMI								<i>EU_KCRV</i>	<i>U(EU_KCRV)</i>	<i>Lower</i>	<i>Upper</i>
	INRIM	OMH	PTB	GUM	UME	VNIIM	LNE	SMU				
1.290 5 (*)	-6	48	33	23	-115	-80	7	51	19.2 (<i>median</i>)		0.1	38.3
1.293 5	-9	26	21	13	-127	-80	1	46	11.1 (<i>w. mean</i>)	10.4		
1.296 5	-11	-3	20	22	-124	-60	7	44	9.1 (<i>w. mean</i>)	10.4		
1.299 5	2	4	28	39	-93	-20	10	59	17.7 (<i>w. mean</i>)	10.4		
Combined standard uncertainty of corrections. <i>u_c</i>	15.6	12.5	12.7	12.5	85.9	380.2	9.0	26.0				
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	31	25	25	25	173	977	18	52				
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.96	1.97	1.98	2.01	2.57	1.99	1.99				
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$		$\chi^2(7) = 14.07 > \chi^2_{obs}$ (*) $\chi^2(7) = 14.07 < \chi^2_{obs}$						The consistency test does not fail; procedure A The consistency test fails; procedure B				

Table A.14. Degree of equivalence with respect to the *EU-KCRV* and the uncertainty or the coverage interval at each calibrated mark of each laboratory of the petal identified by the hydrometer 58431.

NMI	g cm ⁻³	1.290 5			1.293 5		1.296 5		1.299 5	
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶
			<i>Lower</i>	<i>Upper</i>						
INRIM		-25	34	30	-20	29	-20	29	-15	29
OMH		29	29	29	15	23	-12	23	-14	23
PTB		14	24	28	10	23	11	23	10	23
GUM		4	25	26	2	23	13	23	22	23
UME		-134	169	159	-138	171	-133	171	-111	171
VNIIM		-99	751	727	-91	760	-69	760	-38	760
LNE		-12	25	20	-10	15	-2	15	-8	15
SMU		32	44	53	35	51	35	51	41	51

Figure A.7. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 58431. The lengths of the bars show the uncertainty of the confidence interval of the degree of equivalence related at each calibrated mark of the concerned NMI.



7.8 Hydrometer S/N 58432

The *EU_KCRVs* for the petal identified by the hydrometer 0001 have been calculated by applying the “*weighted mean*” method, since the consistency check of the measurement results and standard uncertainties of the participants reported in **Table A.15**, was satisfying (namely: $\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$). **Table A.15** also shows each calculated *EU_KCRVs* with the expanded uncertainties.

Table A.16 and **Figure A.8** present the degree of equivalence with respect to the *EU_KCRVs* of the concerned NMIs.

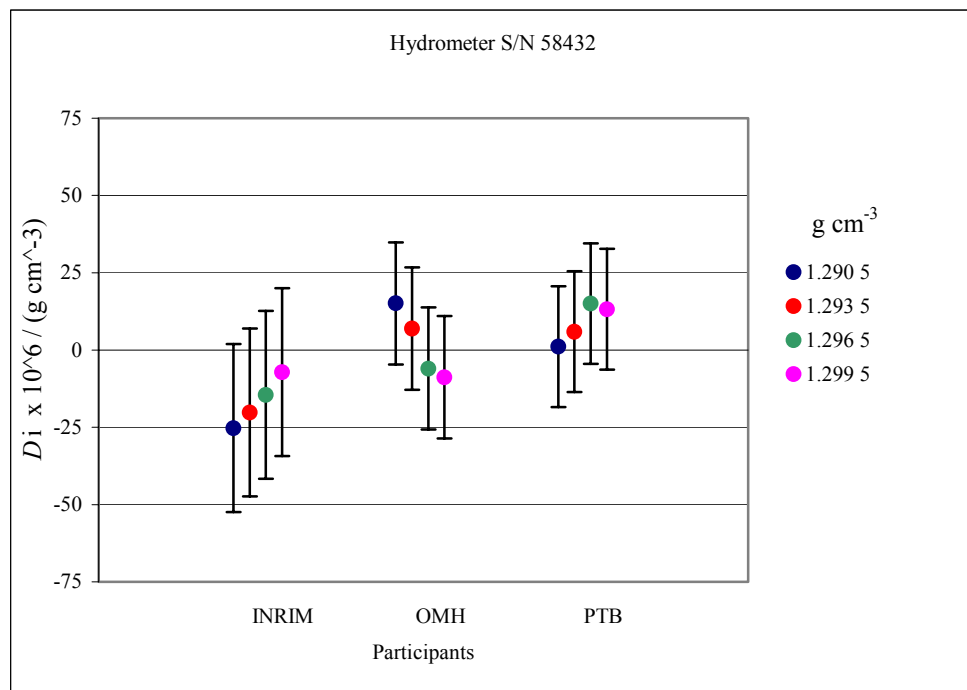
Table A.15. Measurement results as reported by the participants for the petal identified by the hydrometer 58432 and the *EU_KCRV* with its expanded uncertainty at each calibrated mark.

S/N. 58432 g cm ⁻³	NMI			<i>EU_KCRV (weighted mean)</i>	<i>U(EU_KCRV)</i>
	INRIM	OMH	PTB		
1.290 5	21	61	47	45.9	15.3
1.293 5	19	46	45	39.1	15.3
1.296 5	13	22	43	28.0	15.3
1.299 5	18	16	38	24.8	15.3
Combined standard uncertainty of corrections. <i>u_c</i>	15.6	12.5	12.4		
Expanded uncertainty of corrections. <i>U₉₅ = t₉₅(n_{eff}) u_c</i>	31	25	24		
Student t-factor <i>t₉₅(n_{eff})</i>	1.98	1.96	1.97		
$\Pr\{\chi^2(\nu) > \chi^2_{obs}\} < 0.05$	$\chi^2(2) = 5.99 > \chi^2(obs)$		The consistency test does not fail; procedure A		

Table A.16. Degree of equivalence with respect to the *EU-KCRV* and the uncertainty at each calibrated mark of each laboratory of the petal identified by the hydrometer 58432.

NMI	g cm ⁻³	1.290 5		1.293 5		1.296 5		1.299 5	
		<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶	<i>Di</i> x 10 ⁶	<i>U(Di)</i> x 10 ⁶
INRIM		-25	27	-20	27	-15	27	-7	27
OMH		15	20	7	20	-6	20	-9	20
PTB		1	19	6	19	15	19	13	19

Figure A.8. Degree of equivalence with respect to the *EU_KCRV* of each laboratory of the petal identified by the hydrometer 58432. The lengths of the bars show the uncertainty of the degree of equivalence related to each calibrated mark of the concerned NMI.



8. Appendix B

For each artefact, this section presents the arithmetic average of the 4 corrections and the standard uncertainty u , provided by each participant in which a correlation coefficient of 1 was considered. Moreover, the results for the degree of equivalence between pairs of the participating NMIs d_{ij} and the expanded uncertainties $U_{d_{ij}}$ in the range between 600 kg/m³ and 1 300 kg/m³ are also shown.

8.1 Degree of equivalence between pairs of the NMIs in the range of 600 kg/m³

Table B.1. shows the arithmetic average of the 4 correction values and the standard uncertainty u , provided by each participant regarding the artefacts 21964 and 21971.

The degrees of equivalence d_{ij} between pairs of NMIs i and j , and the associated uncertainty at the 95 % confidence level, resulting in the range of 600 kg/m³ are presented in Table B.2.

Table B.1. Arithmetic average of the 4 correction values and the standard uncertainty provided by each participant regarding the artefacts 21964 and 21971.

Hydrometer 21964		INRIM	OMH	PTB	GUM	UME	SMU	
0.600 5 - 0.609 5	Average / g cm ⁻³	-0.000053	-0.000056	-0.000052	-0.000047	-0.000080	-0.000010	
	u / g cm ⁻³	0.000008	0.000007	0.000009	0.000007	0.000040	0.000015	
Hydrometer 21971		INRIM	OMH	PTB	IPQ	MIKES	BEV	LNE
0.610 5 - 0.619 5	Average / g cm ⁻³	-0.000072	-0.000071	-0.000071	0.000052	-0.000190	-0.000038	-0.000065
	u / g cm ⁻³	0.000008	0.000007	0.000009	0.000016	0.000168	0.000027	0.000009

Table B.2. Calculation of the degrees of equivalence d_{ij} between pairs of NMIs i and j, in the range of 600 kg/m³. The table also shows the uncertainty of each value at the 95% confidence level.

i \ j	INRIM		OMH		PTB		GUM		UME	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM			9.2E-7	1.5E-5	-9.6E-7	1.7E-5	-7.1E-6	2.0E-5	2.6E-5	8.2E-5
OMH	-9.2E-7	1.5E-5			-1.9E-6	1.6E-5	-8.1E-6	1.9E-5	2.5E-5	8.2E-5
PTB	9.6E-7	1.7E-5	1.9E-6	1.6E-5			-6.2E-6	2.2E-5	2.7E-5	8.2E-5
GUM	7.1E-6	2.0E-5	8.1E-6	1.9E-5	6.2E-6	2.2E-5			3.3E-5	8.2E-5
UME	-2.6E-5	8.2E-5	-2.5E-5	8.2E-5	-2.7E-5	8.2E-5	-3.3E-5	8.2E-5		
SMU	4.4E-5	3.3E-5	4.5E-5	3.3E-5	4.3E-5	3.4E-5	3.7E-5	3.3E-5	7.0E-5	8.6E-5
IPQ	1.2E-4	3.5E-5	1.2E-4	3.4E-5	-1.2E-4	3.5E-5	1.2E-4	3.7E-5	1.5E-4	8.8E-5
MIKES	-1.2E-4	3.4E-4	-1.2E-4	3.4E-4	-1.2E-4	3.4E-4	-1.3E-4	3.4E-4	-9.3E-5	3.5E-4
BEV	3.4E-5	5.6E-5	3.5E-5	5.5E-5	3.3E-5	5.6E-5	2.7E-5	5.7E-5	5.9E-5	9.8E-5
LNE	6.3E-6	2.3E-5	7.2E-6	2.2E-5	5.3E-6	2.4E-5	-8.9E-7	2.6E-5	3.2E-5	8.4E-5

i \ j	SMU		IPQ		MIKES		BEV		LNE	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM	-4.4E-5	3.3E-5	-1.2E-4	3.5E-5	1.2E-4	3.4E-4	-3.4E-5	5.6E-5	-6.3E-6	2.3E-5
OMH	-4.5E-5	3.3E-5	-1.2E-4	3.4E-5	1.2E-4	3.4E-4	-3.5E-5	5.5E-5	-7.2E-6	2.2E-5
PTB	-4.3E-5	3.4E-5	-1.2E-4	3.5E-5	1.2E-4	3.4E-4	-3.3E-5	5.6E-5	-5.3E-6	2.4E-5
GUM	-3.7E-5	3.3E-5	-1.2E-4	3.7E-5	1.3E-4	3.4E-4	-2.7E-5	5.7E-5	8.9E-7	2.6E-5
UME	-7.0E-5	8.6E-5	-1.5E-4	8.8E-5	9.3E-5	3.5E-4	-5.9E-5	9.8E-5	-3.2E-5	8.4E-5
SMU			-8.0E-5	4.5E-5	1.6E-4	3.4E-4	1.0E-5	6.3E-5	3.8E-5	3.7E-5
IPQ	8.0E-5	4.5E-5			2.4E-4	3.4E-4	9.0E-5	6.3E-5	1.2E-4	3.6E-5
MIKES	-1.6E-4	3.4E-4	-2.4E-4	3.4E-4			-1.5E-4	3.4E-4	-1.2E-4	3.4E-4
BEV	-1.0E-5	6.3E-5	-9.0E-5	6.3E-5	1.5E-4	3.4E-4			2.8E-5	5.7E-5
LNE	-3.8E-5	3.7E-5	-1.2E-4	3.6E-5	1.2E-4	3.4E-4	-2.8E-5	5.7E-5		

8.2 Degree of equivalence between pairs of the NMIs in the range of 800 kg/m³

Table B.3. shows the arithmetic average of the 4 correction values and the standard uncertainty u , provided by each participant regarding the artefacts 21958 and 5941. The degrees of equivalence $d_{i,j}$ between pairs the of NMIs i and j , and the associated uncertainty at the 95% confidence level, resulting in the range 800 kg/m³ are presented in Table B.4.

Table B.3. Arithmetic average of the 4 correction values and the standard uncertainty provided by each participant regarding the artefacts 21958 and 5941.

Hydrometer 21958		INRIM	OMH	PTB	GUM	UME	VNIIM	BEV	LNE	SMU
0.800 5 - 0.809 5	Average / g/cm ³	-0.000145	-0.000132	-0.000137	-0.000130	-0.000195	0.000373	-0.000078	-0.000139	-0.000077
	u / g/cm ³	0.000010	0.000008	0.000011	0.000009	0.000053	0.000052	0.000031	0.000009	0.000017
Hydrometer 5941		INRIM	OMH	PTB	IPQ	MIKES				
0.810 5 - 0.819 5	Average / g/cm ³	-0.000054	-0.000050	-0.000045	0.000010	-0.000075				
	u / g/cm ³	0.000010	0.000008	0.000010	0.000020	0.000118				

Table B.4. Calculation of the degrees of equivalence d_{ij} between pairs of the NMIs i and j , in the range of 800 kg/m^3 . The table also shows the uncertainty of each value at the 95% confidence level.

i \ j	INRIM		OMH		PTB		GUM		UME		SMU	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM			-8.6E-6	1.7E-5	-8.5E-6	2.0E-5	-1.3E-5	2.5E-5	5.2E-5	1.1E-4	-6.6E-5	3.8E-5
OMH	8.6E-6	1.7E-5			1.2E-7	1.8E-5	-4.4E-6	2.3E-5	6.1E-5	1.1E-4	-5.7E-5	3.7E-5
PTB	8.5E-6	2.0E-5	-1.2E-7	1.8E-5			-4.5E-6	2.6E-5	6.1E-5	1.1E-4	-5.8E-5	3.9E-5
GUM	1.3E-5	2.5E-5	4.4E-6	2.3E-5	4.5E-6	2.6E-5			6.5E-5	1.1E-4	-5.3E-5	3.8E-5
UME	-5.2E-5	1.1E-4	-6.1E-5	1.1E-4	-6.1E-5	1.1E-4	-6.5E-5	1.1E-4			-1.2E-4	1.1E-4
SMU	6.6E-5	3.8E-5	5.7E-5	3.7E-5	5.8E-5	3.9E-5	5.3E-5	3.8E-5	1.2E-4	1.1E-4		
VNIM	5.2E-4	1.0E-4	5.1E-4	1.0E-4	5.1E-4	1.0E-4	5.0E-4	1.0E-4	5.7E-4	1.5E-4	4.5E-4	3.4E-5
IPQ	6.6E-5	4.3E-5	5.7E-5	4.2E-5	5.8E-5	4.4E-5	5.3E-5	4.6E-5	1.2E-4	1.1E-4	6.8E-8	5.4E-5
MIKES	-1.9E-5	2.4E-4	-2.7E-5	2.4E-4	-2.7E-5	2.4E-4	-3.2E-5	2.4E-4	3.3E-5	2.6E-4	-8.5E-5	2.4E-4
BEV	6.6E-5	6.4E-5	5.7E-5	6.3E-5	5.7E-5	6.4E-5	5.3E-5	6.4E-5	1.2E-4	1.2E-4	-2.5E-7	7.0E-5
LNE	4.6E-6	2.5E-5	-4.0E-6	2.3E-5	-3.9E-6	2.6E-5	-8.4E-6	2.5E-5	5.7E-5	1.1E-4	-6.1E-5	3.8E-5

i \ j	VNIM		IPQ		MIKES		BEV		LNE	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM	-5.2E-4	1.0E-4	-6.6E-5	4.3E-5	1.9E-5	2.4E-4	-6.6E-5	6.4E-5	-4.6E-6	2.5E-5
OMH	-5.1E-4	1.0E-4	-5.7E-5	4.2E-5	2.7E-5	2.4E-4	-5.7E-5	6.3E-5	4.0E-6	2.3E-5
PTB	-5.1E-4	1.0E-4	-5.8E-5	4.4E-5	2.7E-5	2.4E-4	-5.7E-5	6.4E-5	3.9E-6	2.6E-5
GUM	-5.0E-4	1.0E-4	-5.3E-5	4.6E-5	3.2E-5	2.4E-4	-5.3E-5	6.4E-5	8.4E-6	2.5E-5
UME	-5.7E-4	1.5E-4	-1.2E-4	1.1E-4	-3.3E-5	2.6E-4	-1.2E-4	1.2E-4	-5.7E-5	1.1E-4
SMU	-4.5E-4	3.4E-5	-6.8E-8	5.4E-5	8.5E-5	2.4E-4	2.5E-7	7.0E-5	6.1E-5	3.8E-5
VNIM			4.5E-4	1.1E-4	5.3E-4	2.6E-4	4.5E-4	1.2E-4	5.1E-4	1.0E-4
IPQ	-4.5E-4	1.1E-4			8.5E-5	2.4E-4	3.2E-7	7.4E-5	6.1E-5	4.6E-5
MIKES	-5.3E-4	2.6E-4	-8.5E-5	2.4E-4			-8.5E-5	2.4E-4	-2.3E-5	2.4E-4
BEV	-4.5E-4	1.2E-4	-3.2E-7	7.4E-5	8.5E-5	2.4E-4			6.1E-5	6.4E-5
LNE	-5.1E-4	1.0E-4	-6.1E-5	4.6E-5	2.3E-5	2.4E-4	-6.1E-5	6.4E-5		

8.3 Degree of equivalence between pairs of the NMIs in the range of 1 000 kg/m³

Table B.5. shows the arithmetic average of the 4 correction values and the standard uncertainty u , provided by each participant regarding the artefacts 6905 and 0001.

The degrees of equivalence $d_{i,j}$ between pairs of NMIs i and j , and the associated uncertainty at the 95% confidence level, resulting in the range 1 000 kg/m³ are presented in Table B.6.

Table B.5. Arithmetic average of the 4 correction values and the standard uncertainty provided by each participant regarding the artefacts 6905 and 0001.

Hydrometer 6905		INRIM	OMH	PTB	GUM	UME	VNIIM	SMU
0.990 5 - 0.999 5	Average / g/cm ³	0.000376	0.000390	0.000390	0.000390	0.000299	0.000348	0.000437
	u / g/cm ³	0.000012	0.000009	0.000012	0.000010	0.000066	0.000113	0.000022
Hydrometer 0001		INRIM	OMH	PTB	IPQ	MIKES	BEV	LNE
1.000 5 - 1.009 5	Average / g/cm ³	-0.000115	-0.000092	-0.000087	-0.000035	-0.000125	-0.000045	-0.000109
	u / g/cm ³	0.000012	0.000013	0.000012	0.000042	0.000145	0.000034	0.000009

Table.B.6. Calculation of the degrees of equivalence d_{ij} between pairs of the NMIs i and j, in the range of 1 000 kg/m³. The table also shows the uncertainty of each value at the 95% confidence level.

i \ j	INRIM		OMH		PTB		GUM		UME		VNIIM		SMU	
	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$
INRIM			-1.9E-5	2.3E-5	-2.1E-5	2.4E-5	-1.8E-5	2.9E-5	7.4E-5	1.3E-4	2.5E-5	2.3E-4	-6.4E-5	4.8E-5
OMH	1.9E-5	2.3E-5			-2.2E-6	2.3E-5	8.8E-7	2.9E-5	9.2E-5	1.3E-4	4.4E-5	2.3E-4	-4.6E-5	4.8E-5
PTB	2.1E-5	2.4E-5	2.2E-6	2.3E-5			3.1E-6	2.9E-5	9.5E-5	1.3E-4	4.6E-5	2.3E-4	-4.3E-5	4.8E-5
GUM	1.8E-5	2.9E-5	-8.8E-7	2.9E-5	-3.1E-6	2.9E-5			9.2E-5	1.3E-4	4.3E-5	2.3E-4	-4.6E-5	4.8E-5
UME	-7.4E-5	1.3E-4	-9.2E-5	1.3E-4	-9.5E-5	1.3E-4	-9.2E-5	1.3E-4			-4.9E-5	2.6E-4	-1.4E-4	1.4E-4
VNIIM	-2.5E-5	2.3E-4	-4.4E-5	2.3E-4	-4.6E-5	2.3E-4	-4.3E-5	2.3E-4	4.9E-5	2.6E-4			-8.9E-5	2.3E-4
SMU	6.4E-5	4.8E-5	4.6E-5	4.8E-5	4.3E-5	4.8E-5	4.6E-5	4.8E-5	1.4E-4	1.4E-4	8.9E-5	2.3E-4		
IPQ	7.7E-5	8.6E-5	5.8E-5	8.6E-5	-5.6E-5	8.6E-5	5.9E-5	8.8E-5	1.5E-4	1.6E-4	1.0E-4	2.4E-4	1.2E-5	9.6E-5
MIKES	-1.3E-5	2.9E-4	-3.2E-5	2.9E-4	-3.4E-5	2.9E-4	-3.1E-5	2.9E-4	6.1E-5	3.2E-4	1.2E-5	3.7E-4	-7.7E-5	2.9E-4
BEV	6.7E-5	7.2E-5	4.8E-5	7.2E-5	4.6E-5	7.2E-5	4.9E-5	7.4E-5	1.4E-4	1.5E-4	9.2E-5	2.4E-4	2.5E-6	8.3E-5
LNE	3.0E-6	2.9E-5	-1.6E-5	2.8E-5	-1.8E-5	2.9E-5	-1.5E-5	3.3E-5	7.7E-5	1.4E-4	2.8E-5	2.3E-4	-6.1E-5	5.0E-5

i \ j	IPQ		MIKES		BEV		LNE	
	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$	$d_{ij} / g/cm^3$	$Ud_{ij} / g/cm^3$
INRIM	-7.7E-5	8.6E-5	1.3E-5	2.9E-4	-6.7E-5	7.2E-5	-3.0E-6	2.9E-5
OMH	-5.8E-5	8.6E-5	3.2E-5	2.9E-4	-4.8E-5	7.2E-5	1.6E-5	2.8E-5
PTB	-5.6E-5	8.6E-5	3.4E-5	2.9E-4	-4.6E-5	7.2E-5	1.8E-5	2.9E-5
GUM	-5.9E-5	8.8E-5	3.1E-5	2.9E-4	-4.9E-5	7.4E-5	1.5E-5	3.3E-5
UME	-1.5E-4	1.6E-4	-6.1E-5	3.2E-4	-1.4E-4	1.5E-4	-7.7E-5	1.4E-4
VNIIM	-1.0E-4	2.4E-4	-1.2E-5	3.7E-4	-9.2E-5	2.4E-4	-2.8E-5	2.3E-4
SMU	-1.2E-5	9.6E-5	7.7E-5	2.9E-4	-2.5E-6	8.3E-5	6.1E-5	5.0E-5
IPQ			9.0E-5	3.0E-4	9.9E-6	1.1E-4	7.4E-5	8.5E-5
MIKES	-9.0E-5	3.0E-4			-8.0E-5	3.0E-4	-1.6E-5	2.9E-4
BEV	-9.9E-6	1.1E-4	8.0E-5	3.0E-4			6.4E-5	7.1E-5
LNE	-7.4E-5	8.5E-5	1.6E-5	2.9E-4	-6.4E-5	7.1E-5		

8.4 Degree of equivalence between pairs of the NMIs in the range of 1 300 kg/m³

Table B.7. shows the arithmetic average of the 4 correction values and the standard uncertainty u , provided by each participant regarding the artefact 58431.

The degrees of equivalence $d_{i,j}$ between pairs of the NMIs i and j , and the associated uncertainty at the 95% confidence level, resulting in the range 1 300 kg/m³ are presented in Table B.8.

TableB.7. Arithmetic average of the 4 correction values and the standard uncertainty provided by each participant regarding the artefact 58431.

Hydrometer 58431		INRIM	OMH	PTB	GUM	UME	VNIM	SMU	LNE
1.290 5 - 1.299 5	Average / g/cm ³	-0.000006	0.000019	0.000026	0.000024	-0.000114	-0.000060	0.000050	0.000006
	u / g/cm ³	0.000016	0.000013	0.000013	0.000013	0.000086	0.000380	0.000026	0.000009

Table B.8. Calculation of the degrees of equivalence d_{ij} between pairs of the NMIs i and j , in the range 1 300 kg/m³. The table also shows the uncertainty of each value at the 95% confidence level.

i \ j	INRIM		OMH		PTB		GUM		UME	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM			-2.4E-5	4.0E-5	-3.1E-5	4.0E-5	-3.0E-5	4.0E-5	1.1E-4	1.7E-4
OMH	2.4E-5	4.0E-5			-6.8E-6	3.6E-5	-5.7E-6	3.5E-5	1.3E-4	1.7E-4
PTB	3.1E-5	4.0E-5	6.8E-6	3.6E-5			1.0E-6	3.6E-5	1.4E-4	1.7E-4
GUM	3.0E-5	4.0E-5	5.7E-6	3.5E-5	-1.0E-6	3.6E-5			1.4E-4	1.7E-4
UME	-1.1E-4	1.7E-4	-1.3E-4	1.7E-4	-1.4E-4	1.7E-4	-1.4E-4	1.7E-4		
SMU	5.6E-5	6.1E-5	3.1E-5	5.8E-5	2.5E-5	5.8E-5	2.6E-5	5.8E-5	1.6E-4	1.8E-4
VNIIM	-5.4E-5	7.6E-4	-7.9E-5	7.6E-4	8.6E-5	7.6E-4	-8.4E-5	7.6E-4	5.4E-5	7.8E-4
LNE	1.2E-5	3.6E-5	-1.3E-5	3.1E-5	-1.9E-5	3.1E-5	-1.8E-5	3.1E-5	1.2E-4	1.7E-4

i \ j	SMU		VNIIM		LNE	
	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$	$d_{ij} / \text{g/cm}^3$	$Ud_{ij} / \text{g/cm}^3$
INRIM	-5.6E-5	6.1E-5	5.4E-5	7.6E-4	-1.2E-5	3.6E-5
OMH	-3.1E-5	5.8E-5	7.9E-5	7.6E-4	1.3E-5	3.1E-5
PTB	-2.5E-5	5.8E-5	8.6E-5	7.6E-4	1.9E-5	3.1E-5
GUM	-2.6E-5	5.8E-5	8.4E-5	7.6E-4	1.8E-5	3.1E-5
UME	-1.6E-4	1.8E-4	-5.4E-5	7.8E-4	-1.2E-4	1.7E-4
SMU			1.1E-4	7.6E-4	4.4E-5	5.5E-5
VNIIM	-1.1E-4	7.6E-4			-6.6E-5	7.6E-4
LNE	-4.4E-5	5.5E-5	6.6E-5	7.6E-4		