Force Key Comparison EUROMET.M.F-K1
5 kN and 10 kN

EUROMET Project No 535

Final Report 14.10.2019

MIKES
Finland

Contents

Foreword

This comparison report “Euromet project 535” is continuing work for the Key Comparison, named CCM.F-K1.a and CCM.F-K1.b, for force with loads of 5 kN and 10 kN. After several discussion during the years the CCM force expert group proposed to complete the supplementary comparison also with one reference value for 5 kN and as well for 10 kN. The meeting was held in November 2015 in Kajaani, Finland.

0. Used symbols and abbreviations

Symbol	Description	Equation
a_{drift}	Variation width of the assumed drift for transducer	(2.9); (3.13)
$C_{\mathrm{DMP,L}}$	Correction value for DMP40 of the participating laboratory	(1.1)
D	absolute difference of deflection between laboratory and pilot	(2.1); (3.1)
d'_{L}	Relative deviation between Laboratory and Pilot based on mean values of Pilot's measured deflection in the loop	(1.8)
d_{L}	Relative measured deviation between the laboratory and the pilot	(2.4); (3.4)
$\bar{d}_{\mathrm{L}}$	The weighted mean relative deviation of the laboratory for each measured force in chapter 3	(3.9)
d_{ref}	Relative reference value of the deviation calculated as weighted mean for each transducer in chapter 2	(2.11)
$d_{\mathrm{L,ref}}$	Relative deviation between the laboratory and the reference value	(2.14); (3.19)
d_{pairs}	Degree of equivalence between pairs of laboratories	(2.16); (3.21)
$d_{\mathrm{ref,5\ kN}}$	Reference value for 5 kN	(3.14)
$d_{\mathrm{ref,10\ kN}}$	Reference value for 10 kN	(3.15)
p_{L}	Weighing factor to calculate the weighted mean, uncertainty factor	(2.12); (3.10); (3.16)
t_{total}	Total time between Pilot's two consecutive measurements, $t_1 + t_2$	(2.3); (3.3)
t_1	Time between pilot A measurement and calibration by laboratory	(2.3); (3.3)
t_2	Time between calibration by laboratory and pilot B measurement	(2.3); (3.3)
$\bar{X}$	Mean value of all measured deflections by pilot and by laboratories	(1.4)
$X_{\mathrm{DMP,L}}$	Indication of the DMP40 at the participating laboratory with the signal of BN100, calculated for each transducer from two measurements, before and after the comparison measurement	(1.1)
$X_{\mathrm{DMP,P}}$	Indication of the DMP40 at the pilot laboratory with the signal of BN100, also calculated from two measurements	(1.1)
X'_{L}	Measured deflection value by the laboratory	(1.2)
X_{L}	Used deflection value of laboratory with possible corrections (BN 100 and possible other corrections e.g. temperature, extrapolation, given by laboratory)	(1.2)
X'_{P}	Measured deflection value by pilot	(1.4)
X_{PA}	Measured deflection by pilot for A measurement	(1.5)
X_{PB}	Measured deflection by pilot for B measurement	(1.5)
$\bar{X}_{\mathrm{P}}$	Mean deflection value by pilot for loop *n*	(1.7)
X_{P}	Used deflection value for pilot, including the correction of drift	(2.2); (3.2)
w_{BN}	Relative standard uncertainty of the correction with BN100	(1.3)

w_{dL}	The relative standard uncertainty of the deviation between pilot and laboratory using the pilot as link for each transducer, Chapter 2 The relative standard uncertainty of the deviation between pilot and laboratory using the pilot as link for each measured force, Chapter 3	(2.8); (3.12)
$w_{\bar{d}L}$	The standard uncertainty of the weighted mean relative deviation for each measured force of Laboratory, Chapter 3	(3.11)
$w_{d,ref}$	Relative standard uncertainty of d_{ref}	(2.13)
$w_{dL,ref}$	Relative standard uncertainty of $d_{L,ref}$	
$W_{dL,ref}$	Relative expanded ($k = 2$) uncertainty of $d_{L,ref}$	(2.15); (3.20)
$w_{d,ref,5\ kN}$	Relative standard uncertainty of $d_{ref,5\ kN}$	(3.17)
$w_{d,ref,10\ kN}$	Relative standard uncertainty of $d_{ref,10\ kN}$	(3.18)
w_L	Relative standard uncertainty of X_L	(1.3)
W_L	Relative expanded ($k = 2$) uncertainty of X_L, $W_L = 2 \cdot w_L$	Tab. 1.7, 1.8
w'_L	Relative standard uncertainty of X'_L, without correction of BN100	(1.3)
w_{PA}	Relative standard uncertainty for measurement X_{PA}	(1.6)
w_{PB}	Relative standard uncertainty for measurement X_{PB}	(1.6)
w'_P	Relative standard uncertainty of X_P	(2.6); (3.6)
w_P	Relative standard uncertainty for Pilots FSM as link (PLM)	(2.7); (3.7)
$w_{P,corr}$	Relative standard uncertainty for measurements X_{PA} and X_{PB}, calculated as correlated	
$W_{P,corr}$	Relative expanded ($k = 2$) uncertainty for measurements X_{PA} and X_{PB}, calculated as correlated	(1.6)
w_{pairs}	Relative standard uncertainty of pairs	(2.17); (3.22)
w_{PLM}	Compound relative standard uncertainty for stability of pilots FSM	(2.5); (3.5)
$w_{\Delta,corr}$	Relative standard uncertainty for the correction of drift for the transducers between A and B measurement by pilot	(2.10); (3.14)
Δ_{corr}	Correction of the drift of transducer for the reference value X_P	(2.3); (3.3)
ΔX_t	Relative drift of transducers between A and B measurement	(1.5)

Abreviation	
PLM	Pilot Link Machine
BN 100	Calibration device for DMP40 comparator
NPL	National Physical Laboratory, UK
NIST	National Institute of Standards and Technology, USA
PTB	Physikalisch-Technische Bundesanstalt, Germany
MIKES	Mittatekniikan keskus, Finland

Chapter 1
Principle of the inter-comparison and measured data

1.1 General

The CCM force expert group made decision in October 1998 in Sydney to start the force key comparison. These were to be split into four ranges, a) 5 kN – 10 kN, b) 50 kN – 100 kN, c) 500 kN – 1000 kN, and d) 2 MN – 4 MN, with the respective pilot laboratories being a) MIKES-Raute, Finland, b) NPL, United Kingdom, c) PTB, Germany, and d) NIST, USA. As continuity for this project, after finishing the comparison between continents, the regional key comparison should give the traceability in each continent.

This report gives the results for European regional key comparison, as EUROMET project 535 and with the original key comparison transducers, 5 kN and 10 kN and measured according to scheme A or/and to scheme B.

1.1.1 Participants in the comparison

There were 7 laboratories including the pilot, listed in table 1.1.

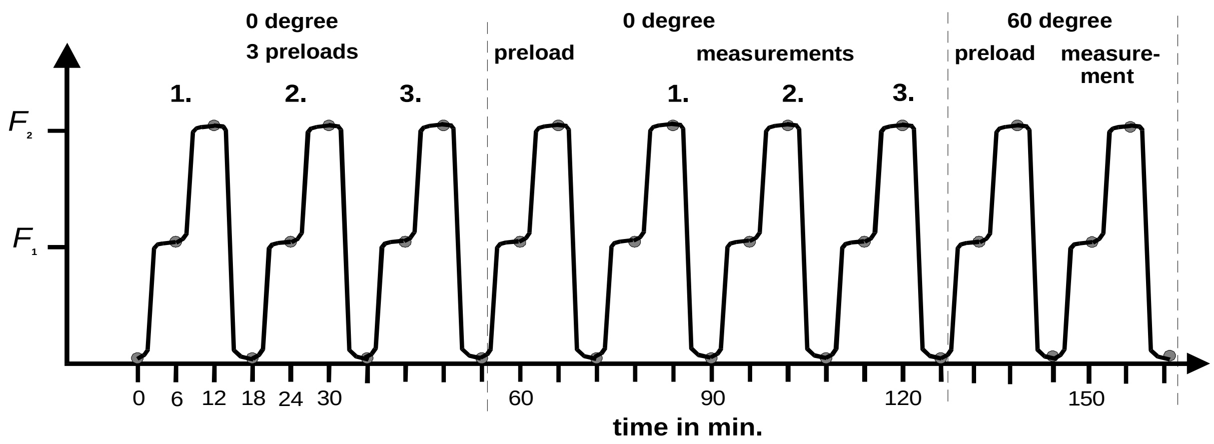
Country	Institute	Number	Country	Institute	Number
Austria	BEV	4	Poland	GUM	6
Finland	MIKES-Raute	0	Portugal	IPQ	5
Germany	PTB	2	Switzerland	METAS	1
Hungary	OMH	3			

Table 1.1 Participating countries and laboratories, including the code number used in the report

1.1.2 Principles of the comparison

The purpose of key comparisons is to compare the units of measurement as realized throughout the world. In the area of force, this is done by the using of high quality load cells subjected to similar loading profiles in national force standard machines, following a strict measurement protocol and using similar instrumentation. The CCM Force Working Group had proposed the following loading schemes:

Scheme A



Scheme B

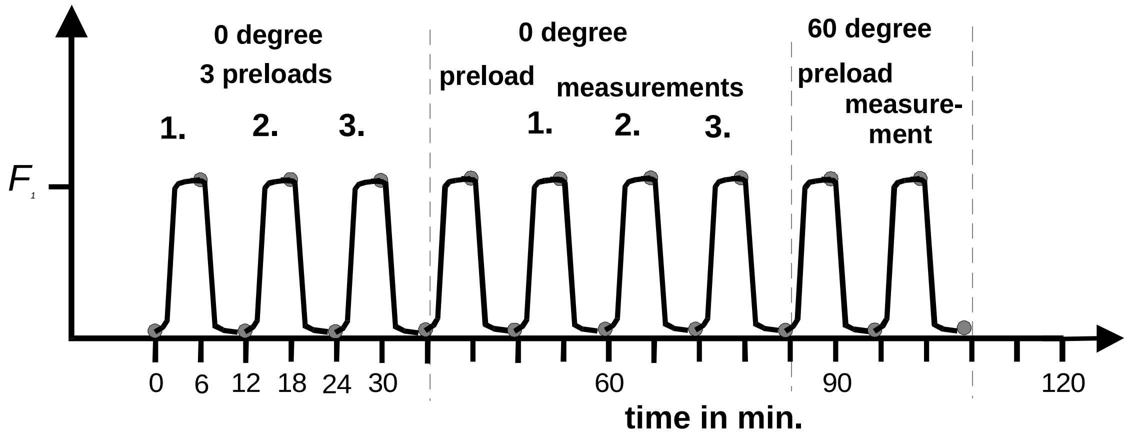


Figure 1.1 Loading scheme for both sets of transducers, forces 5 kN and 10 kN (scheme A) and force 5 kN (scheme B)

The force transducer is rotated through 720° in both schemes. One preload and one measurement (as at 60° in Figure 1.1) is carried out at 120°, 180°, 240°, 300°, 360°/0°, 60°, 120°, 180°, 240°, 300°, and 360°.

The comparison is carried out using four transducers, two with nominal capacity 10 kN for Scheme A and two with nominal capacity 5 kN for Scheme B, identified as Tr1/10 kN, Tr2/10 kN, Tr1/5 kN, and Tr2/5 kN. Both transducers starting with Tr1 are from one manufacturer and the two starting with Tr2 are from another manufacturer. The construction principles of the two transducer types are different, and they have been selected as having the best characteristics for this comparison work.

1.1.3 Realisation of the comparison

The comparison is made in a star format; the transducers come back to the pilot after each participating laboratory´s measurements. One complete measurement cycle (pilot – participating laboratory – pilot) is called a loop. The first measurement by the pilot is called the A-measurement and the second measurement by the pilot, after the participating laboratory, is called the B-measurement.

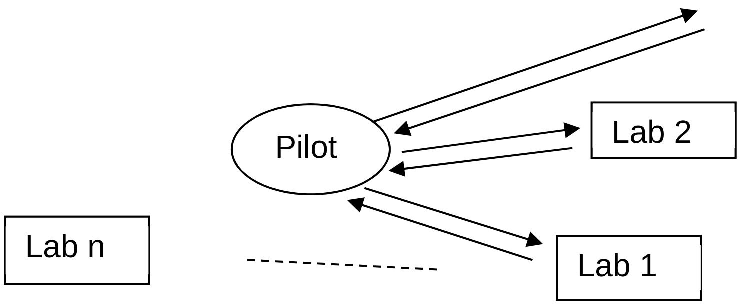


Figure 1.2 Principle of the star-type comparison

The comparison was planned with 10 laboratories but some have withdrawn their participation, therefore in following tables are more pilot measurements than needed. The tables 1.2 and 1.3 show the chronological order of the measurements.

Measuring laboratory	Chronological order	Date of calibration	
		Tr1/10 kN	Tr2/10 kN
Pilot	1	8.1.2002	7.1.2002
METAS – CH	2	14.1.2002	15.1.2002
Pilot	3	25.1.2002	28.1.2002
PTB-D	4	14.2.2002	12.2.2002
Pilot	5	21.2.2002	22.2.2002
OMH-H	6	14.3.2002	16.3.2002
Pilot	7	27.3.2002	28.3.2002
BEV-AT	8	8.4.2002	9.4.2002
Pilot	9	19.4.2002	18.4.2002
Pilot	10	22.5.2002	23.5.2002
Pilot	11	26.6.2002	27.6.2002
IPQ-PT	12	10.7.2002	9.7.2002
Pilot	13	24.7.2002	25.7.2002
Pilot	14	23.8.2002	26.8.2002
Pilot	15	17.9.2002	19.9.2002
Pilot	16	14.10.2002	15.10.2002
Pilot	17	18.11.2002	14.11.2002
Pilot	18	15.1.2003	14.1.2003
Pilot	19	11.4.2003	14.4.2003
GUM-PL	20	26.4.2003	25.4.2003
Pilot	21	14.5.2003	15.5.2003
Pilot	22	2.3.2004	1.3.2004
Pilot	23	29.3.2004	30.3.2004

Table 1.2 Chronological order of the measurements by the pilot and participating laboratories for transducers Tr1/10 kN and Tr2/10 kN

Measuring laboratory	Chronological order	Date of calibration	
		Tr1/5 kN	Tr2/5 kN
Pilot	1	4.1.2002	3.1.2002
METAS-CH	2	16.1.2002	17.1.2002
Pilot	3	30.1.2002	29.1.2002
Pilot	4	25.3.2002	26.3.2002
BEV-AT	5	11.4.2002	10.4.2002
Pilot	6	22.4.2002	23.4.2002
Pilot	7	27.5.2002	24.5.2002
Pilot	8	26.6.2002	24.6.2002
IPQ-PT	9	8.7.2002	5.7.2002
Pilot	10	23.7.2002	22.7.2002
Pilot	11	17.10.2002	16.10.2002
Pilot	12	11.11.2002	13.11.2002
Pilot	13	10.1.2003	13.1.2003
Pilot	14	16.4.2003	15.4.2003
GUM-PL	15	26.4.2003	26.4.2003
Pilot	16	12.5.2003	13.5.2003
Pilot	17	4.3.2004	3.3.2004
Pilot	18	26.3.2004	25.3.2004

Table 1.3 Chronological order of the measurements by the pilot and participating laboratories for transducers Tr1/5 kN and Tr2/5 kN

1.1.4 Limitations of the comparison

Due to the fact that there is no real reference value to circulate (as the transfer transducers do not provide constant values), the following facts should be accepted:

- every measurement loop is independent of the others,
- numerical values of different loops are not easily comparable,
- only relative deviations can be compared,
- there is no absolute numerical reference value.

1.1.5 Uniformity of the measured values

In practice, it is not possible to calibrate the DMP40 measurement instruments used (one at each laboratory), against one reference standard. The uniformity of the DMP40s used was confirmed with reference to a BN100 calibrator unit. Each participating laboratory measured the indication of their DMP40 against the signal of BN100, which is stable to better than approximately $4 \cdot 10^{-6}$. The Pilot monitored the signal of the BN 100 against two instruments in their laboratory.

The resulting correction value to be used by the participating laboratory is calculated as:

$$C_{\mathrm{DMP,L}} = X_{\mathrm{DMP,P}} - X_{\mathrm{DMP,L}} \tag{1.1}$$

where

$C_{\mathrm{DMP,L}}$ = Correction value for DMP40 of the participating laboratory, based on comparison with BN100

$X_{\mathrm{DMP,L}}$ = Indication of the DMP40 at the participating laboratory with the signal of BN100, calculated for each transducer from two measurements, before and after the comparison measurement,

$X_{\mathrm{DMP,P}}$ = Indication of the DMP40 at the pilot laboratory with the signal of BN100, also calculated from two measurements.

The corrected deflection value to be used is calculated as:

$$X_{\mathrm{L}} = X'_{\mathrm{L}} + C_{\mathrm{DMP,L}} \tag{1.2}$$

X'_{L} = Measured deflection value of the laboratory

X_{L} = Deflection value to be used (with BN 100 correction)

The standard uncertainty for deflection value w_{L} with correction of BN100 is

$$w_{\mathrm{L}} = \sqrt{w'^2_{\mathrm{L}} + w^2_{\mathrm{BN}}} \tag{1.3}$$

where

w_{L} = Standard uncertainty for the uncorrected deflection value X'_{L}

w_{BN} = Standard uncertainty for the BN100 correction.

The standard uncertainty w_{L} is used as standard uncertainty for the laboratory.

1.2 Characteristics of the transducers

1.2.1 Creep effect

To minimise the influence of creep, a relatively long reading period of 6 minutes was selected. There are two important elements of the creep:

- the creep effect should be small enough to eliminate the uncertainty of the time of reading,
- the creep effect is constant during every loading.

The aim was to have equal loading times for each laboratory, but this was not possible because the machines did not have similar capabilities. The loading times varied from 20 s to 125 s and all of transducers had constant creep after 3 min and 55 s, which was the shortest time after loading by one laboratory for the taking of readings. The pilot checked the loading time with transducer Tr2/10 kN, which has the worst creep, and the difference between loading times of 40 s and 125 s gave a difference of only $1 \cdot 10^{-6}$, which is less than any measurement uncertainty. In table 1.4 the creep effect is indicated as summary.

Transducer	Total creep value 6 min after loading the force in nV/V	Rel. change of creep between 4 min. and full time (6 min) in 1/min
Tr1/10 kN	-20	5.0×10^{-7}
Tr2/10 kN	75	1.9×10^{-6}
Tr1/5 kN	45	2.5×10^{-7}
Tr2/5 kN	-25	6.3×10^{-7}

Table 1.4 Numerical values of the creep of the transducers

The numerical values indicate that the influence of a change in the reading time by a few seconds is not significant to the uncertainty of measurement.

1.2.2 Temperature effect of the sensitivity

The effect of temperature sensitivity can be an important factor if the environmental temperature at the participating laboratory is not the same as that at the pilot laboratory. The temperature sensitivity of each transducer was determined by taking measurements at two different temperatures which differed by 15 °C. (The uniformity of the temperature scale between the pilot and participant laboratories is based on the assumption that every participant has traceability to their national temperature scale with uncertainty of less than 0,5 °C).

Transducer	Relative temperature coefficient in 1/K	Expanded uncertainty of the value ($k = 2$) in 1/K
Tr1/10 kN	$5.3 \cdot 10^{-5}$	$1.7 \cdot 10^{-6}$
Tr2/10 kN	$4.6 \cdot 10^{-7}$	$9 \cdot 10^{-7}$
Tr1/5 kN	$4.4 \cdot 10^{-6}$	$5 \cdot 10^{-7}$
Tr2/5 kN	$1.9 \cdot 10^{-6}$	$3 \cdot 10^{-7}$

Table 1.5 Temperature coefficients of each transducer

1.2.3 Stability of the transfer transducers

a) Stability of sensitivity over the complete period of the key comparison

Based on the fact that the quality of the comparison is dependent upon the three measurements during the loop, the stability of the transducers is extremely important. The following figures 1.3 to 1.6 show the stability of the transducers during the measurements made by the pilot. The pilot values are compared against the mean value calculated from all measurements.

$$\bar{X} = \frac{\sum_{i=1}^{n_1} X'_{\mathrm{P}} + \sum_{i=1}^{n_2} X_{\mathrm{L}}}{n_1 + n_2} \tag{1.4}$$

with $n_1 = 14$, $n_2 = 4$ (5 kN) and $n_1 = 17$, $n_2 = 6$ (10 kN).

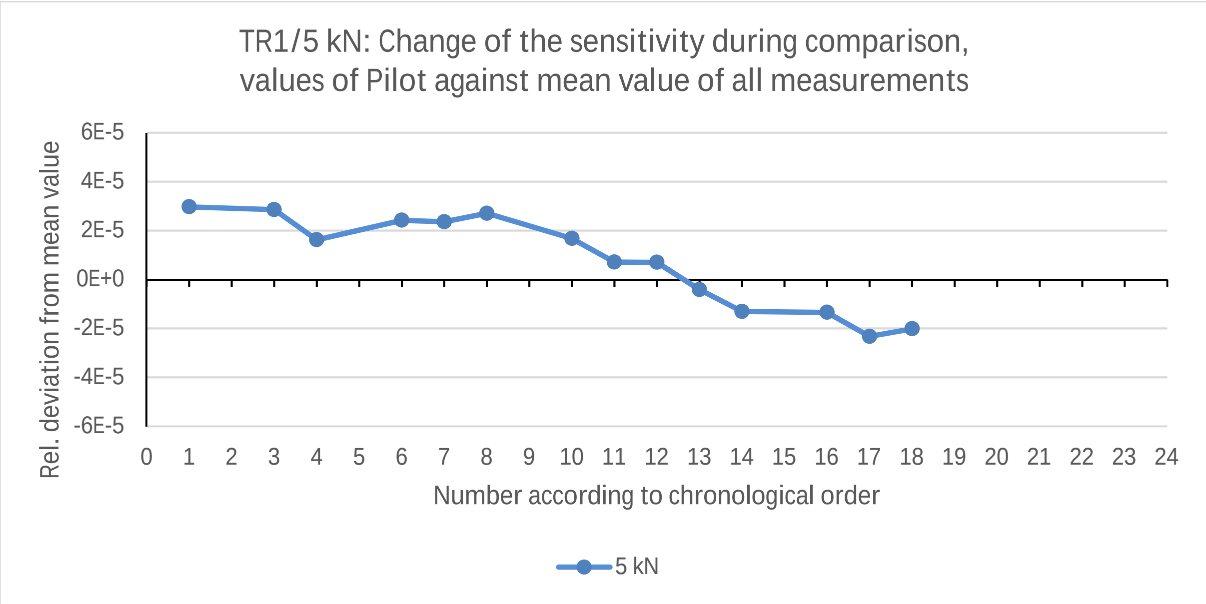


Figure 1.3 Stability of transducer Tr1/5 kN measured by the pilot

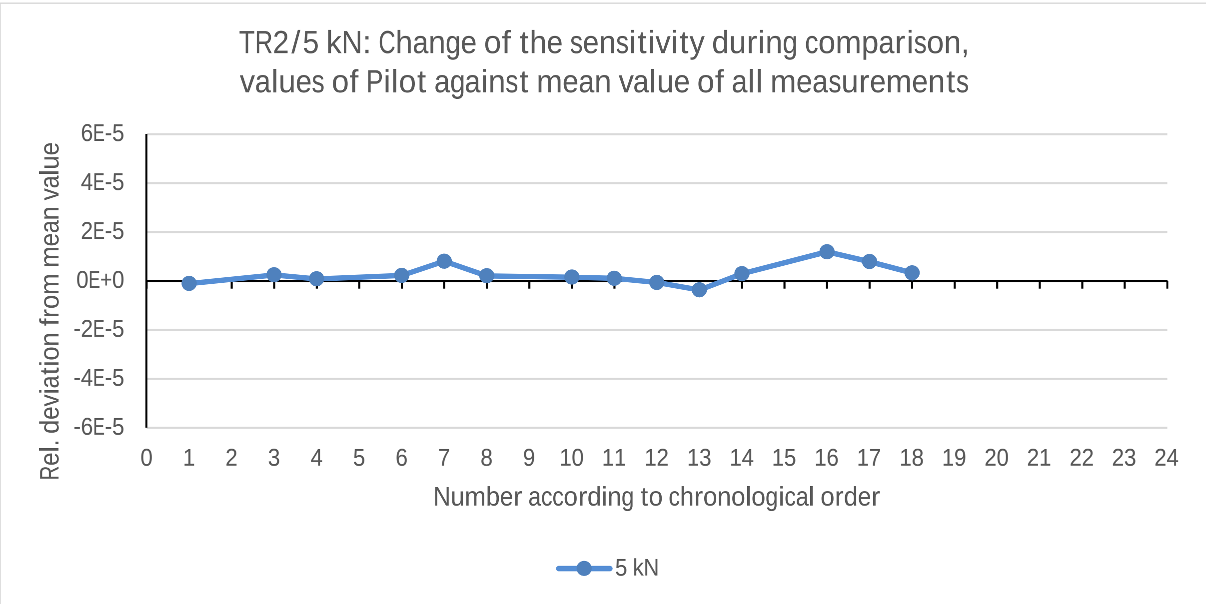


Figure 1.4 Stability of transducer Tr2/5 kN measured by the pilot

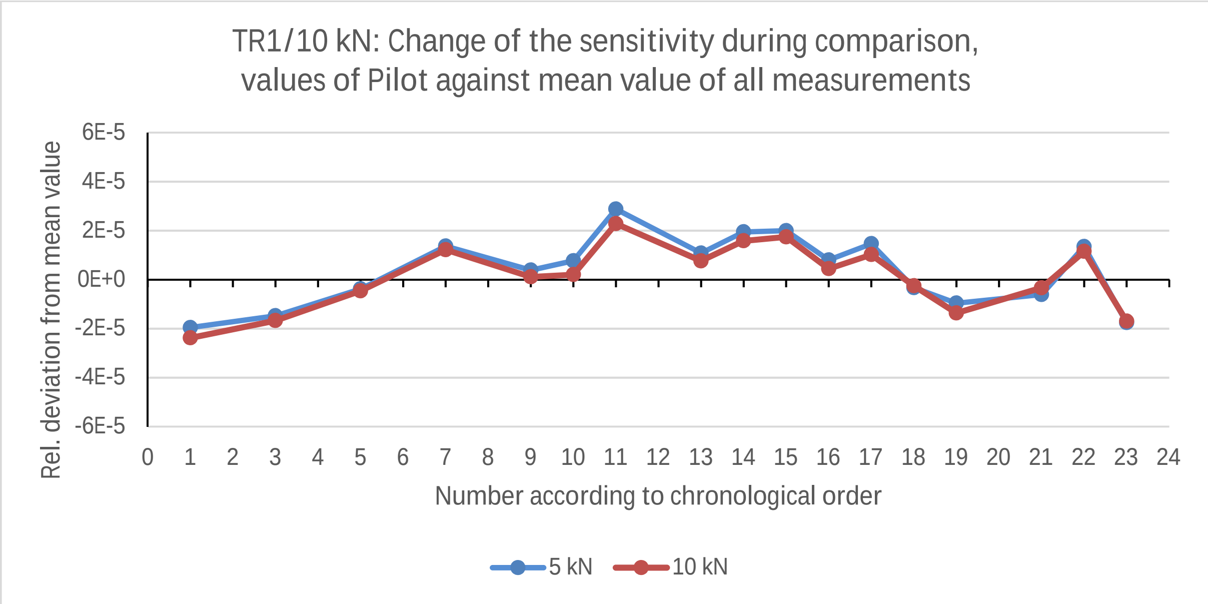


Figure 1.5 Stability of transducer Tr1/10 kN measured by the pilot

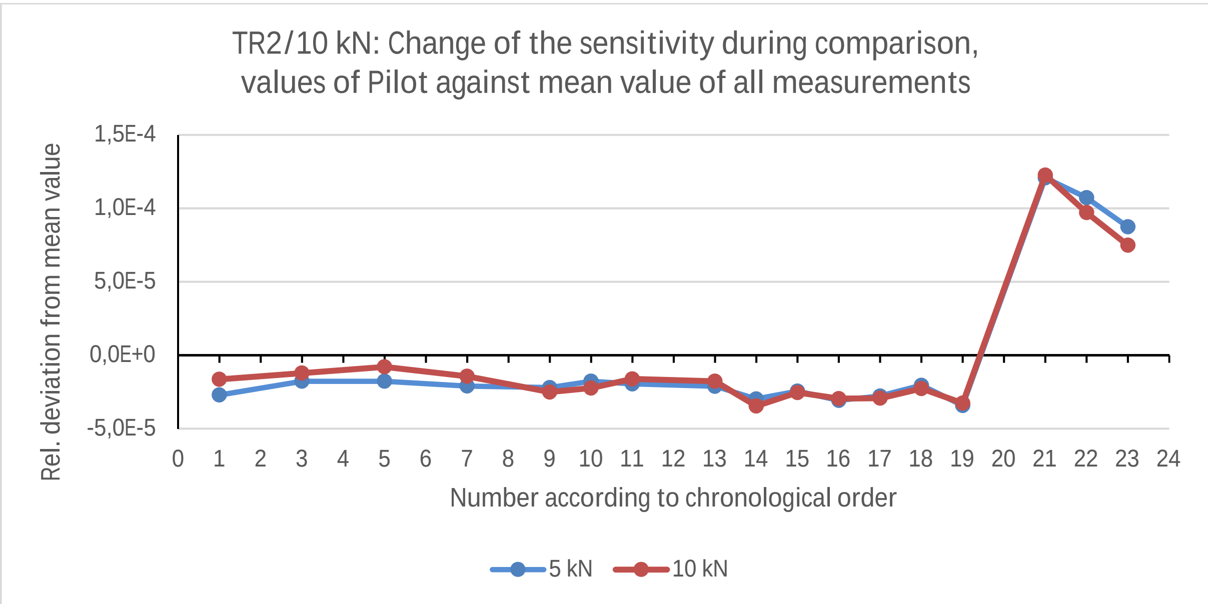


Figure 1.6 Stability of transducer Tr2/10 kN measured by the pilot

b) Stability in one loop

Figures 1.7 to 1.12 show the stability of the pilot´s measurements as relative deviations between their A and B measurements in each loop.The relative deviation between the values of the A and B measurements is called relative drift and calculated as follows:

$$\Delta X_t = \frac{X_{\mathrm{PB}} - X_{\mathrm{PA}}}{X_{\mathrm{PA}}} \tag{1.5}$$

The combined relative expanded uncertainty for the pilot's measurements A and B in each loop is calculated as a correlated uncertainty with following equation:

$$W_{\mathrm{P,\,corr}} = 2\sqrt{\frac{w_{\mathrm{PA}}^2}{4} + \frac{w_{\mathrm{PB}}^2}{4} + \frac{w_{\mathrm{PA}} \cdot w_{\mathrm{PB}}}{2}} \qquad (1.6)$$

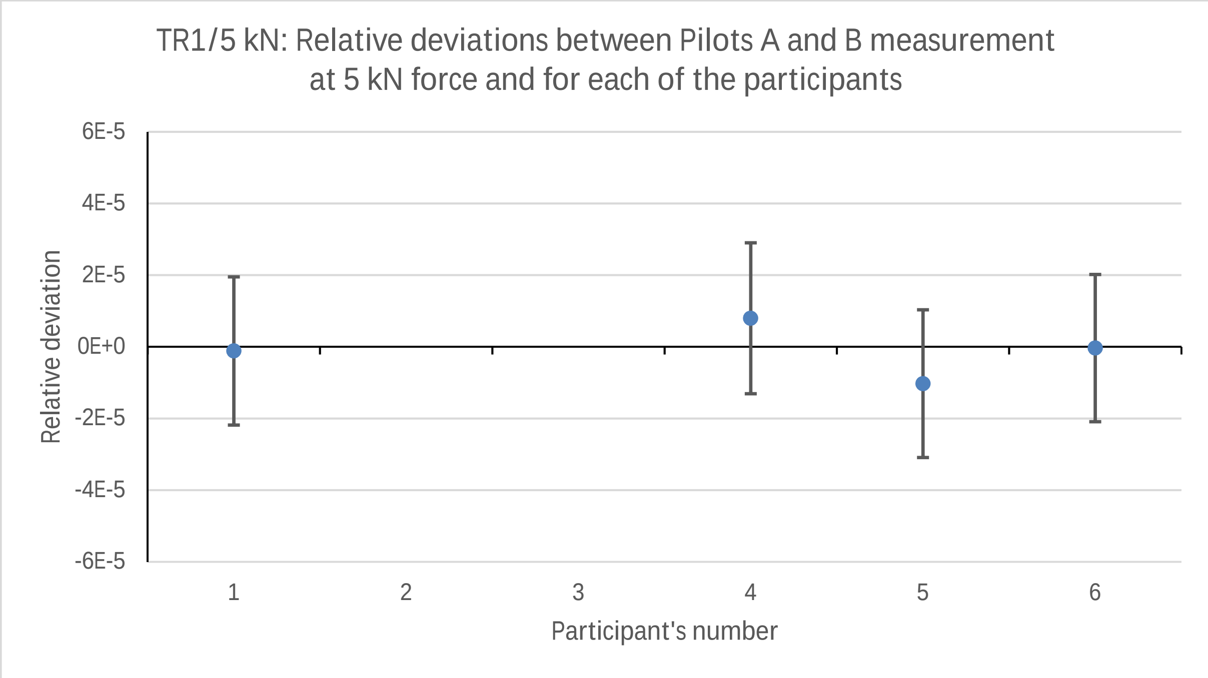


Figure 1.7 Transducer Tr1/5 kN, 5 kN load, relative deviations between pilot's A and B measurement with the relative expanded uncertainty $W_{\mathrm{P,\,corr}}$

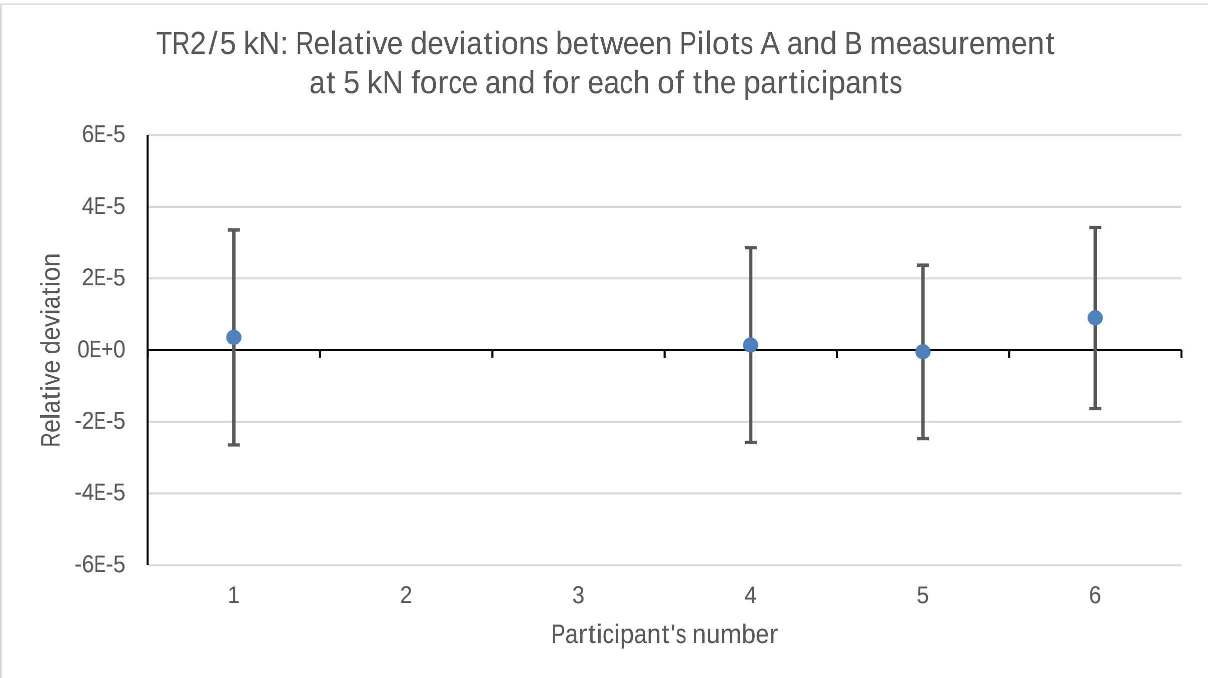


Figure 1.8 Transducer Tr2/5 kN, 5 kN load, relative deviations between pilot's A and B measurement with the relative expanded uncertainty $W_{\mathrm{P,\,corr}}$

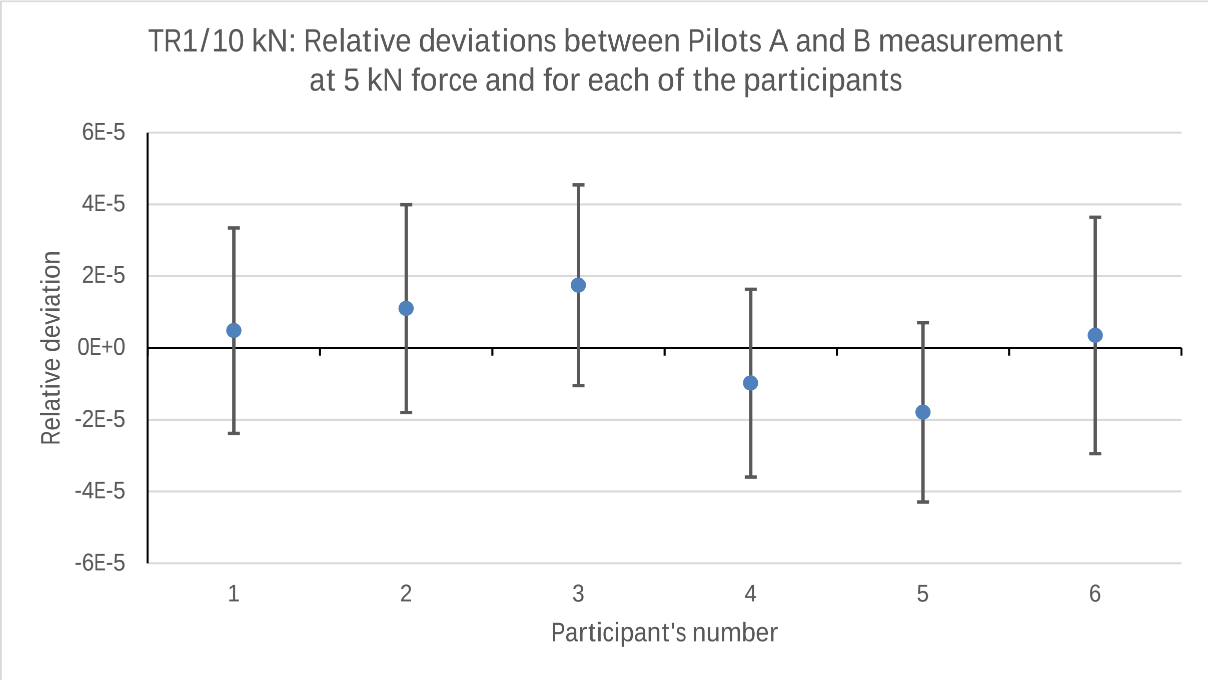


Figure 1.9 Transducer Tr1/10 kN, 5 kN load, relative deviations between pilot's A and B measurements with the relative expanded uncertainty $W_{P,\,corr}$

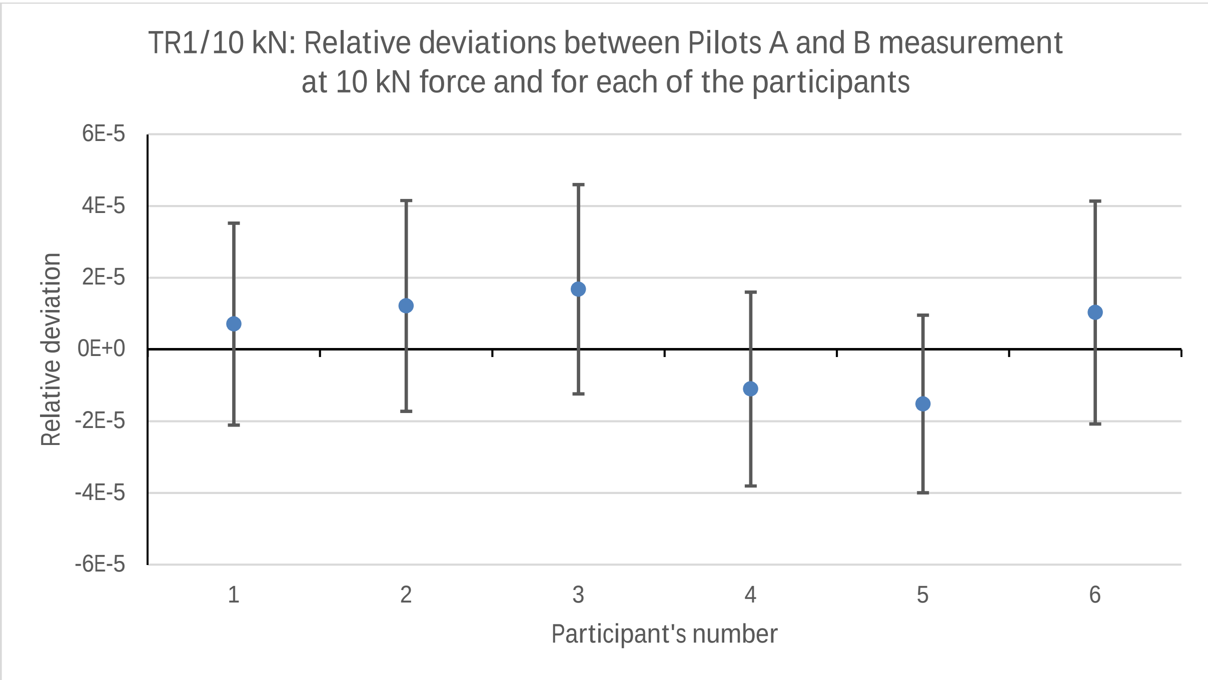


Figure 1.10 Transducer Tr1/10 kN, 10 kN load, relative deviations between pilot's A and B measurement with the relative expanded uncertainty $W_{P,\,corr}$

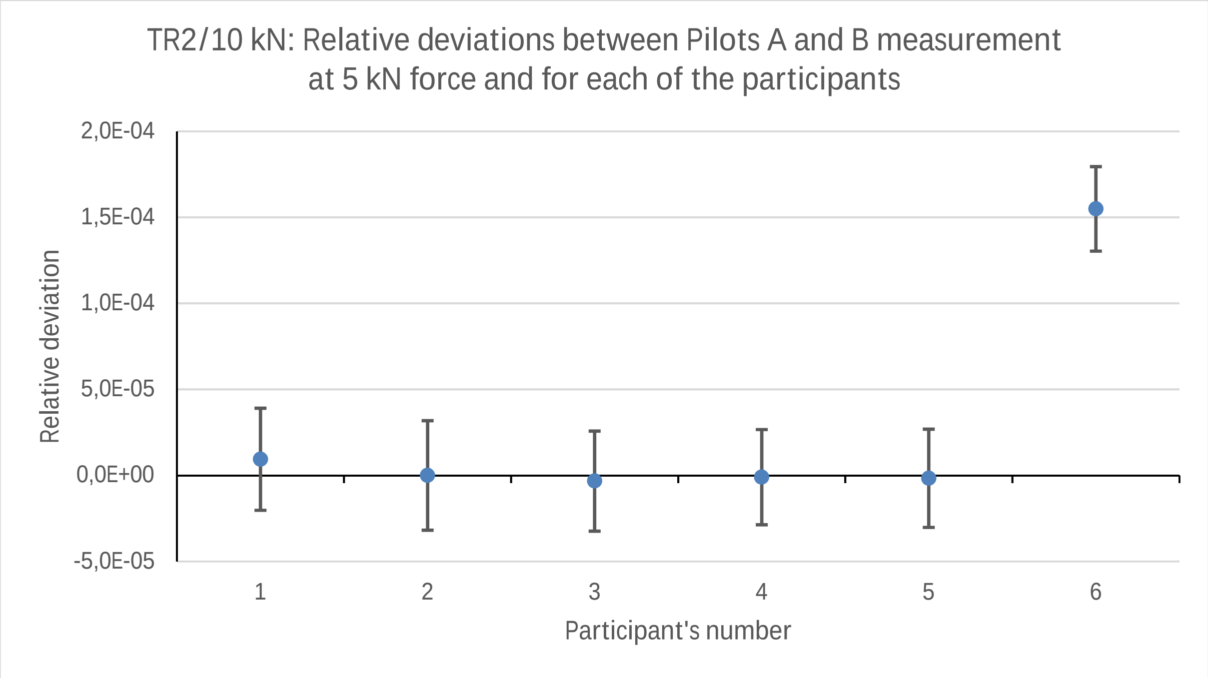


Figure 1.11 Transducer Tr2/10 kN, 5 kN load, relative deviations between pilot's A and B measurement with the relative expanded uncertainty $W_{P,\,corr}$

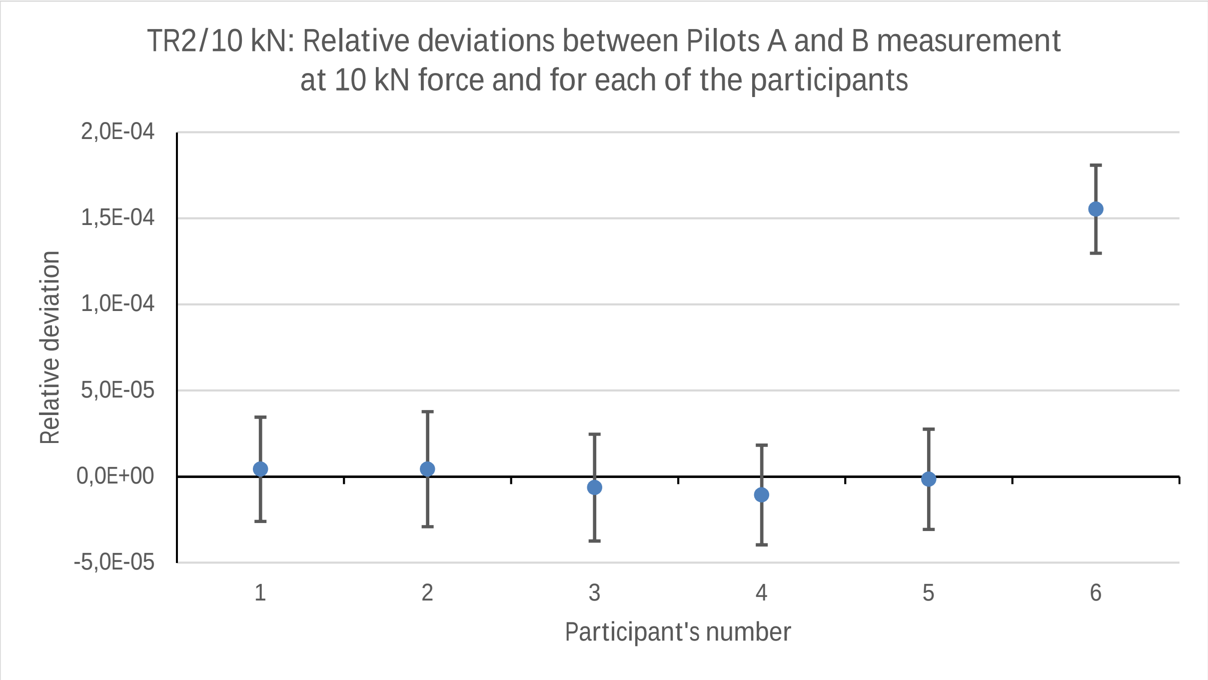


Figure 1.12 Transducer Tr2/10 kN, 10 kN load, relative deviations between pilot's A and B measurement with the relative expanded uncertainty $W_{P,\,corr}$

The figures demonstrate that the stability is relatively good with a small random change of sensitivity. Only for transducer Tr2/10 kN there is significant change of sensitivity in the results of last laboratory, figures 1.11 and 1.12. The reason for this change is not known. Table 1.6 shows the stability of the transducers at the pilot laboratory as numerical values. The mean value is calculated as the mean of the relative deviations between the pilot´s A and B measurements from all measurement loops with a participant laboratory L in between. The standard deviation has been calculated from all these relative deviations.

Transducer	Load 5 kN		Load 10 kN	
	Mean value	Standard deviation	Mean value	Standard deviation
Tr1/10 kN	0.15×10^{-5}	1.32×10^{-5}	0.33×10^{-5}	1.32×10^{-5}
Tr2/10 kN	2.64×10^{-5}	6.31×10^{-5}	2.42×10^{-5}	6.45×10^{-5}
Tr1/5 kN	-0.10×10^{-5}	0.75×10^{-5}	-	-
Tr2/5 kN	0.34×10^{-5}	0.41×10^{-5}	-	-

Table 1.6 Relative deviations to mean value of all A and B measurements and associated standard deviations between the pilot´s A and B measurements

1.3 Results of the measurements

1.3.1 Measured deflections and uncertainties of the measurements

The following table contains the measured absolute deflection D with uncertainties for each laboratory and the measured absolute deflections by pilot for A and B measurement. The table shows also the measured correction $C_{\mathrm{DMP,L}}$ from BN100 (deviation between the DMP 40 of pilot and the laboratory's DMP 40) and with this correction calculated final absolute deflection X_{L}. Every laboratory has given their own standard uncertainty w'_{L} for the measurement. This uncertainty was combined with the standard uncertainty of the BN100 using equation (1.3) and resulting in the standard uncertainty w_{L} for each laboratory. From this value, the expanded (k = 2) uncertainty W_{L} was calculated.

One laboratory has given the uncertainty with the temperature correction based on information from sensitivity of transducers and it is included in the uncertainty w_{L} of this laboratory.

Country	No.	Transducer Identification	Load in kN	X'_{L} in mV/V	$C_{DMP,L}$ in 10^{-6} mV/V	X_{L} in mV/V	W_{L} in 10^{-5}	X_{PA} in mV/V	W_{PA} in 10^{-5}	X_{PB} in mV/V	W_{PB} in 10^{-5}
Switzerland	1	Tr1/10 kN	5	1.0028383	-11.0	1.0028273	2.41	1.0028653	2.73	1.0028702	2.99
		Tr1/10 kN	10	2.0060001	-26.0	2.0059741	2.24	2.0060377	2.76	2.0060518	2.87
		Tr2/10 kN	5	1.0099440	-11.0	1.0099330	2.19	1.0099472	2.51	1.0099567	3.42
		Tr2/10 kN	10	2.0199100	-25.0	2.0198850	2.07	2.0199268	2.59	2.0199353	3.47
		Tr1/5 kN	5	1.8820376	-25.0	1.8820126	2.09	1.8820182	2.05	1.8820160	2.09
		Tr2/5 kN	5	2.0277627	-23.0	2.0277397	2.21	2.0277747	3.16	2.0277818	2.84
Austria	4	Tr1/10 kN	5	1.0029874	-4.5	1.0028822*	25.66	1.0028987	2.80	1.0028888	2.44
		Tr1/10 kN	10	2.0062469	-9.0	2.0061319*	19.45	2.0061098	2.83	2.0060877	2.57
		Tr2/10 kN	5	1.0099752	-5.5	1.0099709*	20.15	1.0099533	2.88	1.0099523	2.65
		Tr2/10 kN	10	2.0199163	-7.5	2.0199076*	10.27	2.0199310	2.99	2.0199093	2.80
		Tr1/5 kN	5	1.8819194	-7.5	1.8819014*	20.06	1.8819928	2.11	1.8820078	2.10
		Tr2/5 kN	5	2.0277813	-1.5	2.0277753*	20.07	2.0277785	3.03	2.0277813	2.40
Portugal	5	Tr1/10 kN	5	1.0029010	-3.0	1.0028880	2.47	1.0029138	2.59	1.0028958	2.41
		Tr1/10 kN	10	2.0061025	-3.0	2.0060995	2.50	2.0061312	2.60	2.0061007	2.35
		Tr2/10 kN	5	1.0099432	1.0	1.0099442	2.97	1.0099548	2.68	1.0099532	3.03
		Tr2/10 kN	10	2.0199062	0.0	2.0199062	2.69	2.0199273	2.90	2.0199242	2.92
		Tr1/5 kN	5	1.8819775	-1.0	1.8819765	2.19	1.8820132	2.08	1.8819938	2.03
		Tr2/5 kN	5	2.0277618	-2.0	2.0277598	3.07	2.0277810	2.39	2.0277800	2.45
Poland	6	Tr1/10 kN	5	1.0029008	-6.5	1.0028943	1.48	1.0028753	3.51	1.0028788	3.08
		Tr1/10 kN	10	2.0061057	-10.5	2.0060952	1.34	2.0060580	3.05	2.0060787	3.16
		Tr2/10 kN	5	1.0101023	-4.5	1.0100978	1.67	1.0099400	2.45	1.0100965	2.47
		Tr2/10 kN	10	2.0202093	-12.5	2.0201968	1.55	2.0198940	2.56	2.0202077	2.55
		Tr1/5 kN	5	1.8817748	-16.0	1.8817588	1.61	1.8819377	2.05	1.8819370	2.06
		Tr2/5 kN	5	2.0277600	-7.5	2.0277525	1.70	2.0277828	2.25	2.0278010	2.81
Germany	2	Tr1/10 kN	5	1.0028643	-0.5	1.0028638	2.09	1.0028702	2.99	1.0028812	2.80
		Tr1/10 kN	10	2.0060338	-1.5	2.0060323	2.08	2.0060518	2.87	2.0060762	3.01
		Tr2/10 kN	5	1.0099433	0.5	1.0099438	2.07	1.0099567	3.42	1.0099567	2.94
		Tr2/10 kN	10	2.0199102	-1.3	2.0199089	2.05	2.0199353	3.47	2.0199440	3.21
Hungary	3	Tr1/10 kN	5	1.0028842	4.5	1.0028887	3.26	1.0028812	2.80	1.0028987	2.80
		Tr1/10 kN	10	2.0061258	7.0	2.0061328	3.51	2.0060762	3.01	2.0061098	2.83
		Tr2/10 kN	5	1.0099692	4.5	1.0099737	3.66	1.0099567	2.94	1.0099533	2.88
		Tr2/10 kN	10	2.0199688	9.5	2.0199783	3.21	2.0199440	3.21	2.0199310	2.99

Table 1.7 Summary of the measured data. * Values include as well the temperature correction from laboratory

1.3.2 Relative deviation of the measured deflections between the participant laboratories and the pilot

The following figures give, for each transducer, the relative deviations between each participating laboratory and the pilot, compared against the pure mean value of pilot's measurements (A and B). The deflection value for each loop is defined as the mean value of the A and B measurements;

$$\bar{X}_{\mathrm{P}} = \frac{X_{\mathrm{PA}} + X_{\mathrm{PB}}}{2} \tag{1.7}$$

The relative deviation is calculated for each individual loop using the following equation;

$$d'_{\mathrm{L}} = \frac{X_{\mathrm{L}} - \bar{X}_{\mathrm{P}}}{\bar{X}_{\mathrm{P}}} \tag{1.8}$$

The participating laboratory value incorporates the BN100 correction. The used uncertainty value is given by the participating laboratory and includes the uncertainty of correction with BN100, the equation (1.8) shows the calculation. The mean value of all measurement uncertainties of the pilot's A and B measurements is used as uncertainty for pilot.

Lab	Values in 10^{-5}											
	TR1/10 kN				TR2/10 kN				Tr1/5 kN		Tr2/5 kN	
	5 kN		10 kN		5 kN		10 kN		5 kN		5 kN	
	d'_{L}	W_{L}	d'_{L}	W_{L}	d'_{L}	W_{L}	d'_{L}	W_{L}	d'_{L}	W_{L}	d'_{L}	W_{L}
1	-4.04	2.41	-3.52	2.24	-1.87	2.19	-2.28	2.07	-0.24	2.09	-1.90	2.21
2	-1.18	2.09	-1.58	2.08	-1.27	2.07	-1.52	2.05	-	-	-	-
3	-0.12	3.26	1.99	3.51	1.85	3.66	2.02	3.21	-	-	-	-
4	-1.15	25.66	1.65	19.45	1.79	20.15	-0.62	10.27	-5.26	20.06	-0.23	20.07
5	-1.68	2.47	-0.82	2.50	-0.97	2.97	-0.97	2.69	-1.43	2.19	-1.02	3.07
6	1.72	1.48	1.34	1.34	7.88	1.67	7.23	1.55	-9.48	1.61	-1.94	1.70

Table 1.8 Relative differences between pilot and participating laboratories for each transducer with the relative expanded uncertainty of the measurements by the laboratories

Transducer	Mean value $\bar{W}_{\mathrm{P,mean}}$ of uncertainty at 5 kN load	Standard deviation s of the mean value	Mean value $\bar{W}_{\mathrm{P,mean}}$ of uncertainty at 10 kN load	Standard deviation s of the mean value
TR1/10 kN	$2.89 \cdot 10^{-5}$	$3.3 \cdot 10^{-6}$	$2.83 \cdot 10^{-5}$	$2.3 \cdot 10^{-6}$
TR2/10 kN	$2.94 \cdot 10^{-5}$	$7.8 \cdot 10^{-6}$	$3.03 \cdot 10^{-5}$	$6.2 \cdot 10^{-6}$
TR1/5 kN	$2.07 \cdot 10^{-5}$	$0.2 \cdot 10^{-6}$	-	-
TR2/5 kN	$2.62 \cdot 10^{-5}$	$3.3 \cdot 10^{-6}$	-	-

Table 1.9 Mean values of the measurement uncertainties of all A and B measurements by pilot for each transducers and for each load and the standard deviation for mean values

TR1/10 kN	5 kN		10 kN	
Laboratory	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L
1	$-4.04 \cdot 10^{-5}$	$2.41 \cdot 10^{-5}$	$-3.52 \cdot 10^{-5}$	$2.24 \cdot 10^{-5}$
2	$-1.18 \cdot 10^{-5}$	$2.09 \cdot 10^{-5}$	$-1.58 \cdot 10^{-5}$	$2.08 \cdot 10^{-5}$
3	$-0.12 \cdot 10^{-5}$	$3.26 \cdot 10^{-5}$	$1.99 \cdot 10^{-5}$	$3.51 \cdot 10^{-5}$
4	$-1.15 \cdot 10^{-5}$	$25.66 \cdot 10^{-5}$	$1.65 \cdot 10^{-5}$	$19.45 \cdot 10^{-5}$
5	$-1.68 \cdot 10^{-5}$	$2.47 \cdot 10^{-5}$	$-0.82 \cdot 10^{-5}$	$2.50 \cdot 10^{-5}$
6	$1.72 \cdot 10^{-5}$	$1.48 \cdot 10^{-5}$	$1.34 \cdot 10^{-5}$	$1.34 \cdot 10^{-5}$

Table 1.10 Relative differences between pilot and participating laboratories in each loop and the relative expanded uncertainties of the measurements by the laboratories for transducer TR1/10 kN

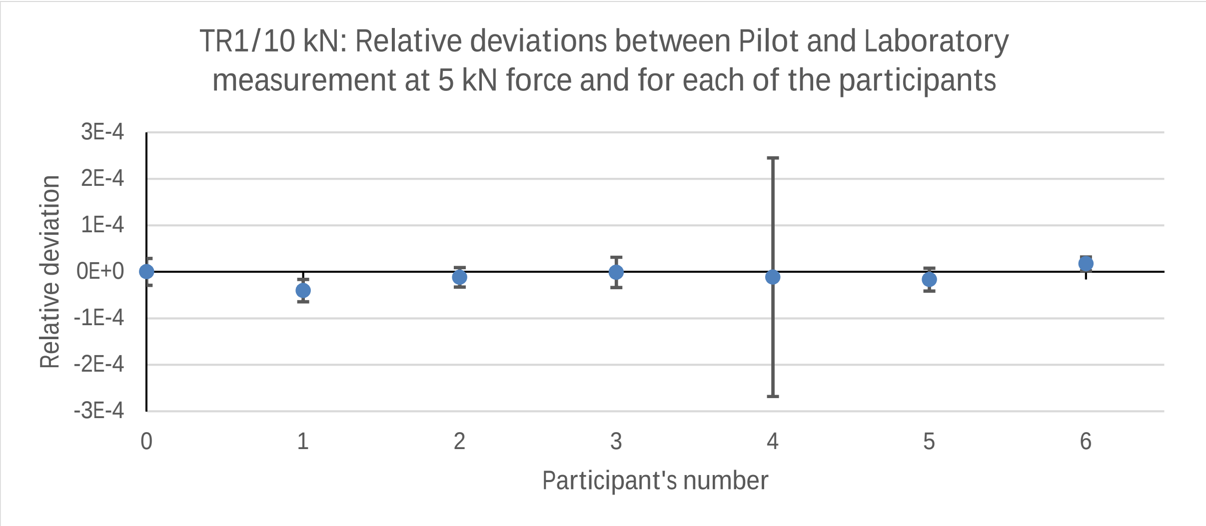


Figure 1.13 Transducer Tr1/10 kN, 5 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

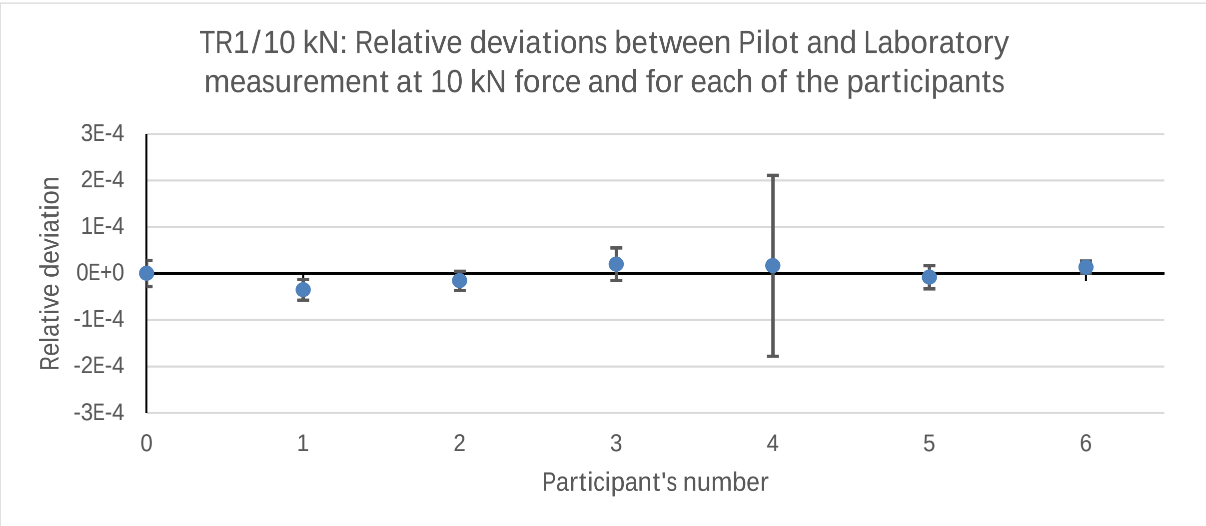


Figure 1.14 Transducer Tr1/10 kN, 10 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

TR2/10 kN	5 kN		10 kN	
Laboratory	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L
1	$-1.87 \cdot 10^{-5}$	$2.19 \cdot 10^{-5}$	$-2.28 \cdot 10^{-5}$	$2.07 \cdot 10^{-5}$
2	$-1.27 \cdot 10^{-5}$	$2.07 \cdot 10^{-5}$	$-1.52 \cdot 10^{-5}$	$2.05 \cdot 10^{-5}$
3	$1.85 \cdot 10^{-5}$	$3.66 \cdot 10^{-5}$	$2.02 \cdot 10^{-5}$	$3.21 \cdot 10^{-5}$
4	$1.79 \cdot 10^{-5}$	$20.15 \cdot 10^{-5}$	$-0.62 \cdot 10^{-5}$	$10.27 \cdot 10^{-5}$
5	$-0.97 \cdot 10^{-5}$	$2.97 \cdot 10^{-5}$	$-0.97 \cdot 10^{-5}$	$2.69 \cdot 10^{-5}$
6	$7.88 \cdot 10^{-5}$	$1.67 \cdot 10^{-5}$	$7.23 \cdot 10^{-5}$	$1.55 \cdot 10^{-5}$

Table 1.11 Relative differences between pilot and participating laboratories in each loop and the relative expanded uncertainties of the measurements by the laboratories for transducer TR2/10 kN

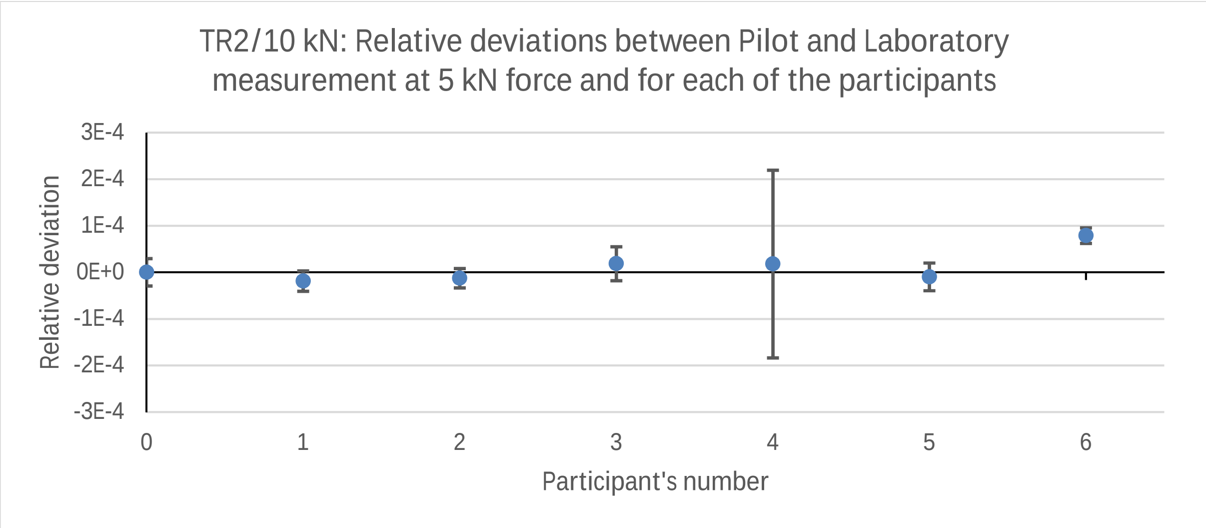


Figure 1.15 Transducer Tr2/10 kN, 5 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

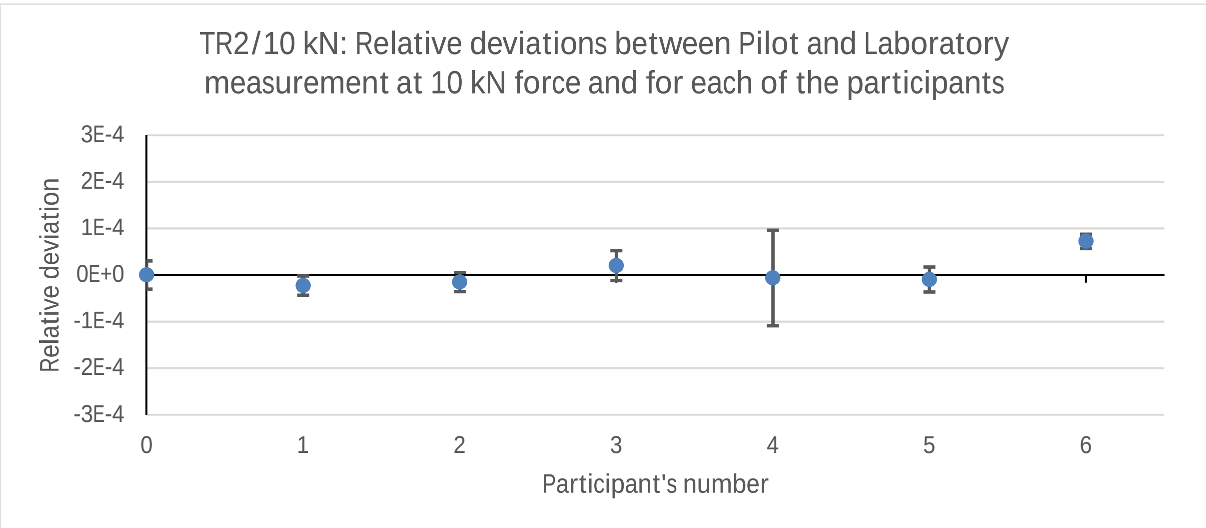


Figure 1.16 Transducer Tr2/10 kN, 10 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

Laboratory	TR1/5 kN		TR2/5 kN	
	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L	Rel. deviation between laboratory and pilot d'_L	Exp. (k = 2) rel. uncertainty of the laboratory W_L
1	$-0.24 \cdot 10^{-5}$	$2.09 \cdot 10^{-5}$	$-1.90 \cdot 10^{-5}$	$2.21 \cdot 10^{-5}$
2	-	-	-	-
3	-	-	-	-
4	$-5.26 \cdot 10^{-5}$	$20.06 \cdot 10^{-5}$	$-0.23 \cdot 10^{-5}$	$20.07 \cdot 10^{-5}$
5	$-1.43 \cdot 10^{-5}$	$2.19 \cdot 10^{-5}$	$-1.02 \cdot 10^{-5}$	$3.07 \cdot 10^{-5}$
6	$-9.48 \cdot 10^{-5}$	$1.61 \cdot 10^{-5}$	$-1.94 \cdot 10^{-5}$	$1.70 \cdot 10^{-5}$

Table 1.12 Relative differences between pilot and participating laboratories in each loop and the relative expanded uncertainties of the measurements by the laboratories for transducers TR1/5 kN and TR2/5 kN

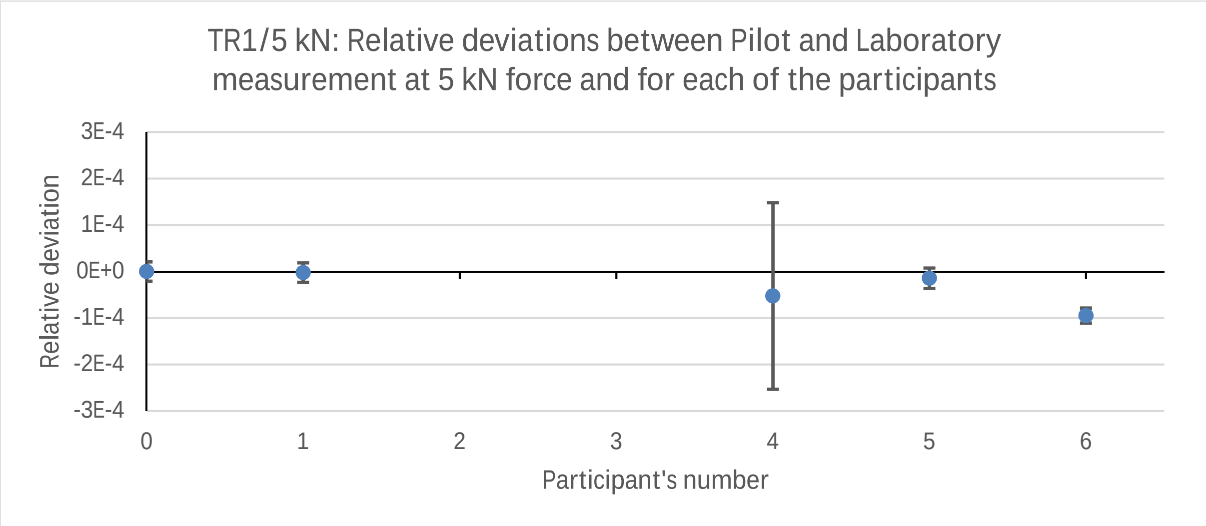


Figure 1.17 Transducer Tr1/5 kN, 5 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

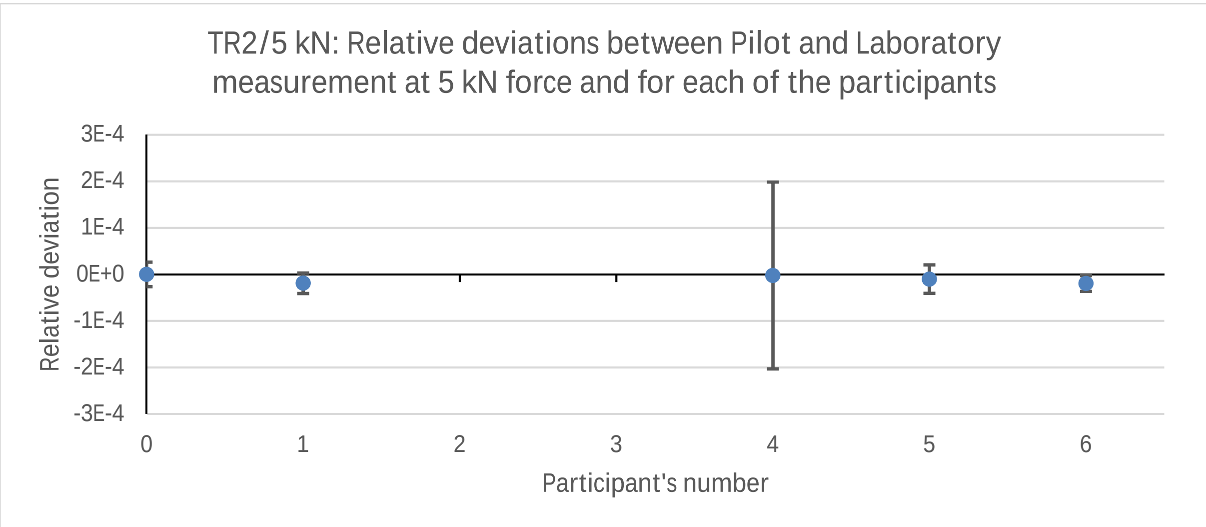


Figure 1.18 Transducer Tr2/5 kN, 5 kN load, relative deviations d_L between laboratory and pilot and the relative expanded uncertainty W_L of the measurement by the participating laboratory

Chapter 2
Reference values for single transducers

2.1 The deviation between pilot and laboratory

For each laboratory and each transducer a single value will be calculated as deviation. This value is based on results of differences between pilot and laboratories. The model of deviation is:

$$D = X_{\mathrm{L}} - X_{\mathrm{P}} \tag{2.1}$$

with

$$X_{\mathrm{P}} = X_{\mathrm{PA}} + \Delta_{\mathrm{corr}} \tag{2.2}$$

The correction Δ_{corr} is used for the pilot measurements to get the best estimated value for transducers sensitivity by participant. It is assumed that the drift is linear.

2.1.1 The used deflection value by pilot for loop *n*

The used deflection for loop *n* is calculated from A and B measurements by pilot with the correction of the drift. The assumption for the drift is a linear drift between A and B measurements, which are made as close as possible before and after the measurement of the participating laboratory. The needed time for transport in one direction has not been equal for all participants, variation from four days up to two weeks. The correction of the measured deflection due the drift is made as function of the time.

$$\Delta_{\mathrm{corr}} = \frac{X_{\mathrm{PB}} - X_{\mathrm{PA}}}{t_{\mathrm{total}}} \cdot t_1 \tag{2.3}$$

where:

$t_{\mathrm{total}} = t_1 + t_2$
t_1 = time between pilot A measurement and calibration by laboratory
t_2 = time between calibration by laboratory and pilot B measurement.

The value X_{P} has been used as reference deflection for loop *n*.

Used reference values for each laboratory with correction of the non-symmetry in time						
Transducer	TR1/10 kN	TR1/10 kN	TR2/10 kN	TR2/10 kN	TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
Laboratory	in mV/V	in mV/V	in mV/V	in mV/V	in mV/V	in mV/V
1	1.0028670	2.0060427	1.0099508	2.0199301	1.8820172	2.0277785
2	1.0028783	2.0060699	1.0099567	2.0199405	-	-
3	1.0028920	2.0060970	1.0099545	2.0199356	-	-
4	1.0028935	2.0060983	1.0099528	2.0199186	1.8820019	2.0277800
5	1.0029048	2.0061159	1.0099541	2.0199260	1.8820046	2.0277806
6	1.0028769	2.0060674	1.0099955	2.0200053	1.8819374	2.0277900

Table 2.1 Used pilot's reference values X_{P} for each loop after the correction of non-symmetric timing of measurement by participating laboratories

The correction Δ_{corr} is in most of the cases relatively small; the maximum for correction has been $+2.1 \cdot 10^{-5}$ mV/V and the minimum $-1.5 \cdot 10^{-5}$ mV/V for all transducers except the Tr2/10 kN. For the latter the maximum correction was $+11.1 \cdot 10^{-5}$ mV/V for participant 6 due to the sensitivity change of the transducer shown in Figures 1.5 and 1.6. By using the mean value $\overline{X}_{\mathrm{P}}$ (1.7) as value X_{P} the difference is between $-1.5 \cdot 10^{-5}$ mV/V and $+1.7 \cdot 10^{-5}$ mV/V except transducer Tr2/10 kN. In practice, the average value $\overline{X}_{\mathrm{P}}$ of the A and B measurements could also be used instead of X_{P} in most of the cases.

2.2 Principle to use the pilot FSM as link in the comparison

The comparison has been made as a star form; the transducers came back to the pilot after every measurement by the participating laboratory. One complete measurement, pilot – participating laboratory – pilot is called a loop.

Pilot's measurements have been made always on the same deadweight machine for the whole key comparison. This pilot's machine is a link between all participants. The pilot's link machine is marked as PLM on this paper.

The pilot laboratory is also a participating laboratory (Lab 0). Pilot laboratory, as a participant, makes also a comparison between PLM and its own reference calibration machine (PM).

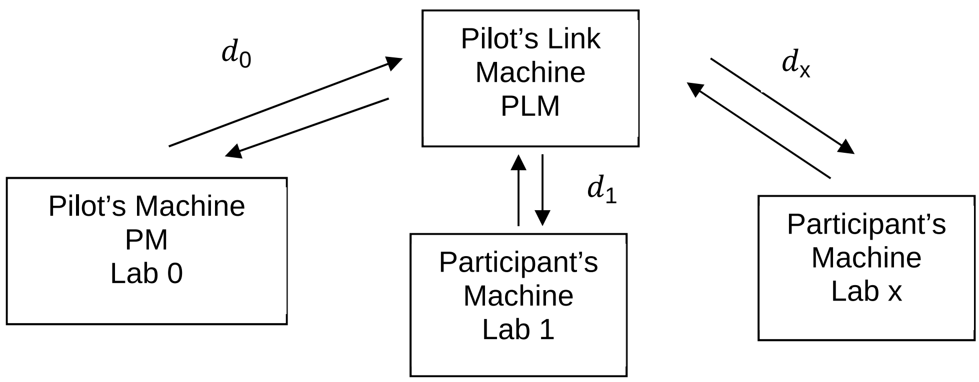
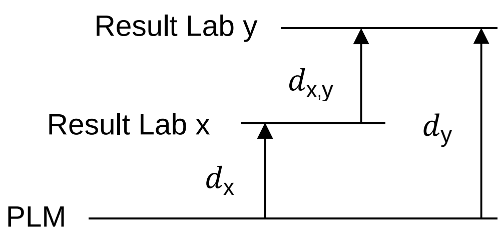


Figure 2.1 Principle of the star-type comparison with the link machine.

This graph can be applied even if the PLM and the PM are the same machine.



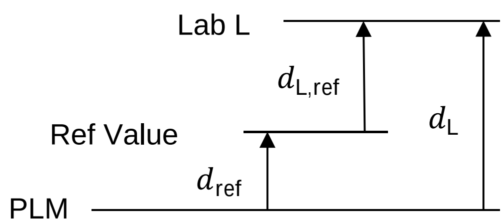


Figure 2.2 Using the pilot as a link, the deviation between two laboratories is equal to $d_{x,y} = d_x - d_y$. So to compute the deviation between 2 laboratories, it is not necessary to use a traceable PLM but a machine stable enough between the 2 loops.

Figure 2.3 The same approach is made for the deviation from the reference value and a laboratory. Using the pilot as a link, the deviation between a laboratory and the reference value is equal to $d_{L,ref} = d_L - d_{ref}$. To compute the deviation between a laboratory and the reference value, it is not necessary to use a traceable PLM but a machine stable enough during all the comparisons.

For each loop, i.e. for each laboratory, the relative deviation d_L from the PLM is computed according to:

$$d_L = \frac{X_L - X_P}{X_P} \tag{2.4}$$

2.2.1 Relative standard uncertainty due to the stability of the pilot link machine w_{PLM}

For the two reasons given above, the uncertainty of the PLM is computed taking into account only components of stability. Following components are given in relative values with $k = 1$

- § stability of masses: $1.0 \cdot 10^{-6}$
- § stability of gravity: $0.2 \cdot 10^{-6}$
- § stability of the air buoyancy: $3.5 \cdot 10^{-6}$

The combined relative standard uncertainty due to the stability of pilot FSM w_{PLM} can be estimated equal to:

$$w_{PLM} = 0.35 \cdot 10^{-5} \tag{2.5}$$

2.2.2 Relative standard uncertainty of deflection obtained at pilot laboratory w_P

The reference value X_P for each loop is based on A and B measurements according to equation (2.2). The pilot had a variation of relative reproducibility with rotation $w_{RP} = 0.2 \cdot 10^{-5}$ ... $1.0 \cdot 10^{-5}$, based on the information from pilots A and B measurements. The standard uncertainty w_{RP} contains the stability of the FSM as well the instability of the force transducers. By omitting the effect of the transducer, the value of the pure relative stability of the FSM has value of $0.6 \cdot 10^{-5}$. This value can be assumed as maximum value of the stability of the FSM by pilot. The uncertainty of the deflection by pilot (w_{PA} or w_{PB}) includes the pilot reproducibility uncertainty (w_{RP}) and the stability of PLM (w_{PLM}):

$$w'_P = \sqrt{w_{PR}^2 + w_{PLM}^2} = 0.6 \cdot 10^{-5} \tag{2.6}$$

However, to get the consistency for all measurements, which are influenced by the uncertainty of the pilot's FSM we have to use a value for standard uncertainty of

$$w_P = 0.75 \cdot 10^{-5} \;. \tag{2.7}$$

2.3 Calculation of the reference value for each transducer

2.3.1 Relative deviation between the laboratory and pilot

The relative deviation between laboratory and pilot value has been calculated using equation (2.4) for each transducer and for each laboratory.

2.3.2 The relative standard uncertainty w_{dL} of the deviation between pilot and laboratory by using the pilot as link

The standard uncertainty w_{dL} of the relative deviation between laboratory and pilot is calculated using the equation (2.9).

Relative uncertainty of the deviation is taking into account:

- the relative uncertainty of the calibration laboratory,
- the relative uncertainty of the pilot
- the relative uncertainty of the drift.

$$w_{dL} = \sqrt{w_L^2 + w_P^2 + w_{\Delta,corr}^2} \tag{2.8}$$

Whereas the relative standard uncertainty of the pilot machine has been used the value $w_{\mathrm{P}} = 0.75 \cdot 10^{-5}$ from equation (2.7).The drift has been assumed as a linear function between A and B measurements by pilot. The time between these measurements is t_{total} and the participating laboratory has measured it in time $t_{\mathrm{total}}/2$ with deviation ± *n* days. By comparing the calculated value X_{P} based on the drift against the value $\overline{X}_{\mathrm{P}}$, which is the mean value of measurements A and B, the difference between these two values is not greater than $5 \cdot 10^{-6}$. However, the calculation is based on the method used for drift.

The uncertainty $w_{\Delta,\mathrm{corr}}$ of the drift is based on variation of the drift during four days. By this way the mean value of the drift is $2.0 \cdot 10^{-6}$ and standard deviation is $1.7 \cdot 10^{-6}$. Only for transducer TR2/5 kN the drift is closely one decade lower. The background for that is to see on figures 1.3 and 1.4, the original phenomenon of change is not known.

The relative uncertainty of the drift a_{drift} is based on the variation width during four days and assumed to have rectangular distribution. By this way the uncertainty is connected individually to the drift of the transducer.

$$a_{\mathrm{drift}} = \frac{\Delta X_t}{t_{\mathrm{total}}} \cdot 4 \tag{2.9}$$

The relative uncertainty of the drift is:

$$w_{\Delta,\mathrm{corr}} = \frac{a_{\mathrm{drift}}}{2 \cdot \sqrt{3} \cdot X_{\mathrm{P}}} \tag{2.10}$$

2.4 Reference value as weighted mean

The calculated reference value is based on the weighted mean and it has been calculated for each transducer separately, TR1/5 kN, TR2/5 kN,TR1/10 kN and TR2/10 kN. The uncertainty of the relative deviation for the measurement by each participating laboratory has been used as weighing factor. For the calculation the uncertainties of the calibration laboratory and the relative measured deviation between the laboratory and the pilot have been used.

Reference value as relative deviation for each transducer:

$$d_{\mathrm{ref}} = \sum_{L=1}^{n} p_L \cdot d_L \bigg/ \sum_{L=1}^{n} p_L \tag{2.11}$$

with n = 7 or 5, depending on the number of participants and with the weighing factor

$$p_L = \frac{1}{w_{\mathrm{dL}}^2} \tag{2.12}$$

The standard uncertainty of the reference value, weighted by its uncertainties is:

$$w_{\mathrm{d,ref}} = \sqrt{1 \bigg/ \sum_{L=1}^{n} \frac{1}{w_{\mathrm{dL}}^2}} \tag{2.13}$$

where n = 7 or 5.

2.5 The degree of equivalence

The degree of equivalence of each participating laboratory is expressed by (according to M. G. Cox, Metrologia 2002,39):

- its deviation from the key comparison reference value, equation (2.14)
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (2.15).

$$d_{L,\mathrm{ref}} = d_L - d_{\mathrm{ref}} \tag{2.14}$$

The uncertainty of the relative deviation between laboratory and reference value is calculated using the equation (2.15) and standard uncertainty of the deviation is taken into account:

- the uncertainty between the calibration laboratory and the pilot, caused by the deviation and including the uncertainty of laboratory,
- the uncertainty of the reference value.

$$W_{\mathrm{dL,ref}} = 2 \cdot \sqrt{w_{\mathrm{dL}}^2 - w_{\mathrm{d,ref}}^2} \tag{2.15}$$

The values are given in table 2.11.

The degree of equivalence between pairs of laboratories is expressed by:

- its difference of their deviations from each other, equation (2.16),
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (2.17).

$$d_{\mathrm{pairs}} = d_{L,n} - d_{L,n+1} \tag{2.16}$$

$$W_{\mathrm{pairs}} = 2 \cdot \sqrt{w_{\mathrm{d}L,n}^2 + w_{\mathrm{d}L,n+1}^2} \tag{2.17}$$

The values of pairs and their uncertainties are given in tables 2.12 … 2.23.

2.6 Consistency check

According to the proposal of M. G. Cox “The evaluation of key comparison data” the consistency should be checked with χ^2 test. The results are following:

Transducer	TR1/10 kN		TR2/10 kN		TR1/5 kN	TR2/5 kN
Force	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
χ^2_{obs}	12.13	11.18	63.26	62.51	42.39	2.71
$\chi^2(n)$ (n = 6 or 4)	12.59	12.59	12.59	12.59	9.49	9.49

Table 2.2 Result of the consistency check

According to this result the comparison does not have the required consistency. The main reasons are the transducers TR2/10 kN and TR1/5 kN, which give too high deviations to reference values by one laboratory. The number of measurements for each transducer is low, which is influencing also reliability of the comparison. By increasing the relative standard uncertainty w_{P} of pilot up to $2.8 \cdot 10^{-5}$ instead the used value of $0.75 \cdot 10^{-5}$ the comparison shows the consistency.

2.7 Tables and diagrams for transducers

The reference values and deviations are given in tables 2.3 to 2.10 and figures 2.4 to 2.9.

Transducer TR1/10 kN

Reference values for transducer TR1/10 kN, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Reference value	Uncertainty $W_{d,ref}$ (k = 2)	Reference value	Uncertainty $W_{d,ref}$ (k = 2)
-0.48	0.93	-0.33	0.92

Table 2.3 Reference values for transducer TR1/10 kN with corresponding relative uncertainty

Laboratory	Transducer TR1/10 kN, values in 10^{-5}			
	Rel. deviation to reference value	Expanded uncertainty	Rel. deviation to reference value	Expanded uncertainty
	5 kN	$W_{dL,ref}$ (k = 2)	10 kN	$W_{dL,ref}$ (k = 2)
0	0.48	1.77	0.33	1.18
1	-3.49	2.68	-3.09	2.53
2	-0.97	2.40	-1.54	2.39
3	0.15	3.47	2.12	3.70
4	-0.65	25.69	2.01	19.48
5	-1.20	2.74	-0.49	2.76
6	2.21	1.89	1.71	1.79

Table 2.4 The deviations to reference value of transducer TR1/10 kN for each laboratory with corresponding expanded relative uncertainties

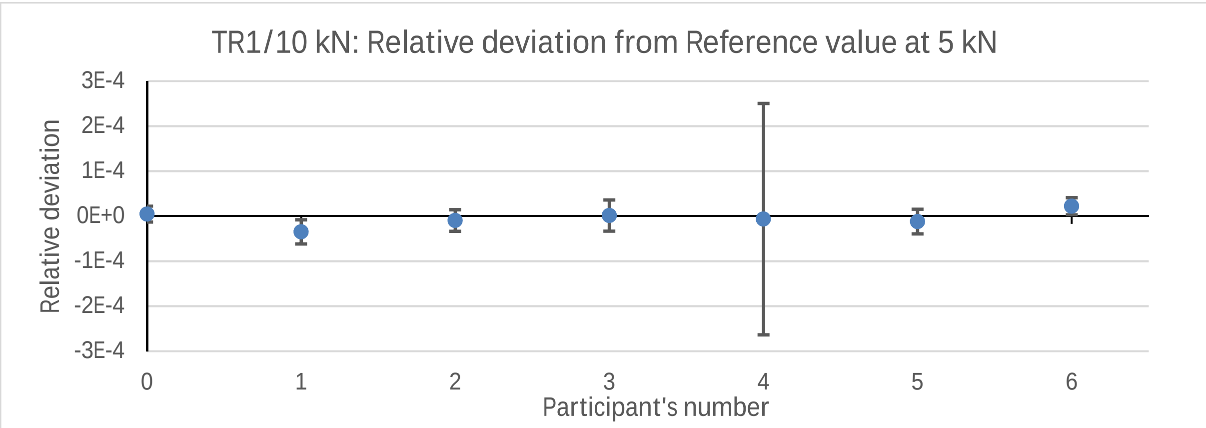


Figure 2.4 TR1/10 kN, load 5 kN, deviations from reference value with the expanded uncertainties, k = 2

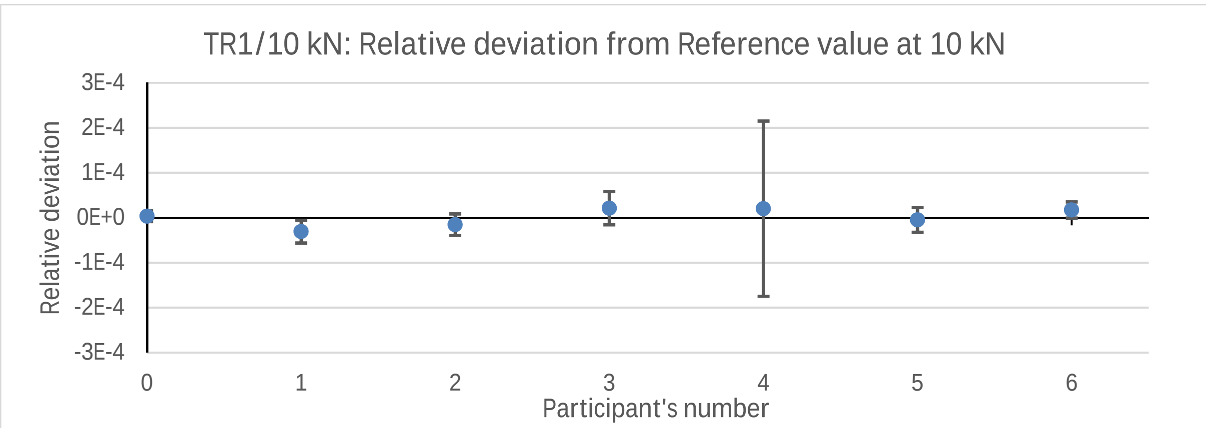


Figure 2.5 TR1/10 kN, load 10 kN, deviations from reference value with the expanded uncertainties, k = 2

Transducer TR2/10 kN

Reference values for transducer TR1/10 kN, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Reference value	Uncertainty $W_{d,ref}$ (k = 2)	Reference value	Uncertainty $W_{d,ref}$ (k = 2)
1.13	0.97	0.95	0.95

Table 2.5 Reference values for transducer TR2/10 kN with corresponding relative uncertainty

Laboratory	Transducer TR2/10 kN, values in 10^{-5}			
	Rel. deviation to reference value	Expanded uncertainty	Rel. deviation to reference value	Expanded uncertainty
	5 kN	$W_{dL,ref}$ (k = 2)	10 kN	$W_{dL,ref}$ (k = 2)
0	-1.13	1.14	-0,95	1.16
1	-2.89	2.47	-3.18	2.37
2	-2.40	2.37	-2.52	2.36
3	0.77	3.83	1.16	3.42
4	0.67	20.19	-1.50	10.33
5	-2.11	3.18	-1.93	2.93
6	9.00	2.33	8.53	2.25

Table 2.6 The deviations to reference value of transducer TR2/10 kN for each laboratory with corresponding expanded relative uncertainties

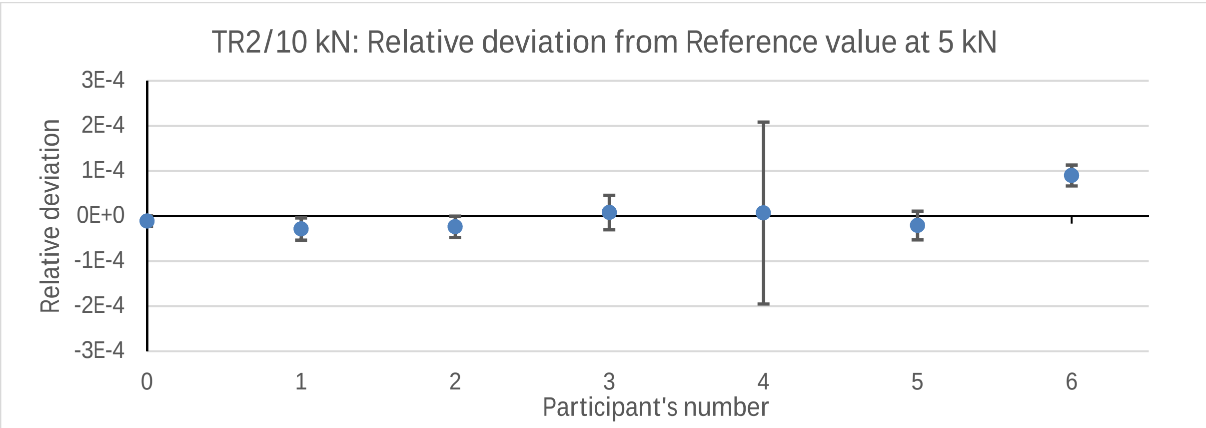


Figure 2.6 TR2/10 kN, load 5 kN, deviations from reference value with expanded uncertainties, k = 2

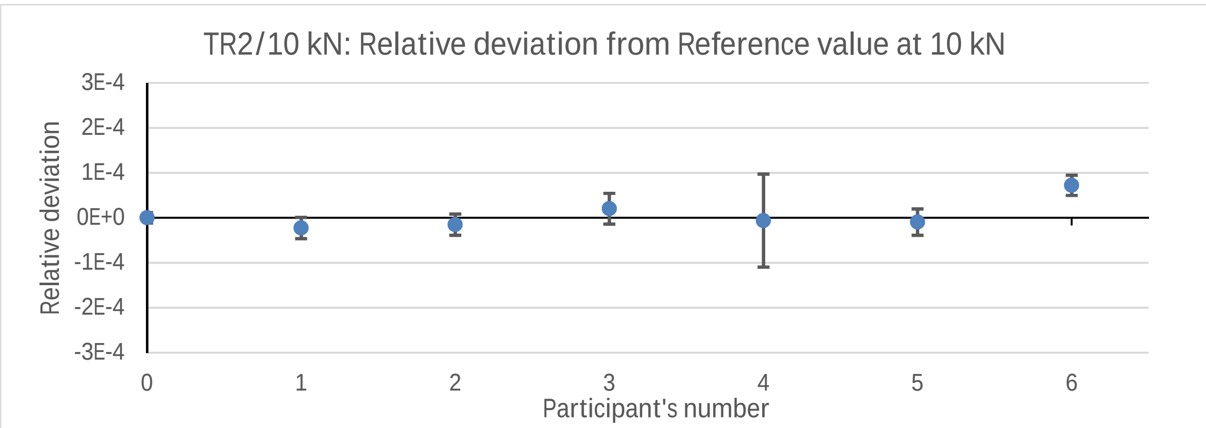


Figure 2.7 TR2/10 kN, load 10 kN, deviations from reference value with expanded uncertainties, k = 2

Transducer TR1/5 kN

Reference values for transducer TR1/5 kN, values in 10^{-5}	
Load 5 kN	
Reference value	Uncertainty $W_{d,ref}$ (k = 2)
-3.77	1.30

Table 2.7 Reference values for transducer TR1/5 kN with corresponding relative uncertainty

Laboratory	Transducer TR1/5 kN, values in 10^{-5}	
	Rel. deviation to reference value	Expanded uncertainty
	5 kN	$W_{dL,ref}$ (k = 2)
0	3.77	3.14
1	3.53	2.22
2	-	-
3	-	-
4	-1.57	20.07
5	2.28	2.31
6	-5.72	1.77

Table 2.8 The deviations to reference value of transducer TR1/5 kN for each laboratory with corresponding expanded relative uncertainties

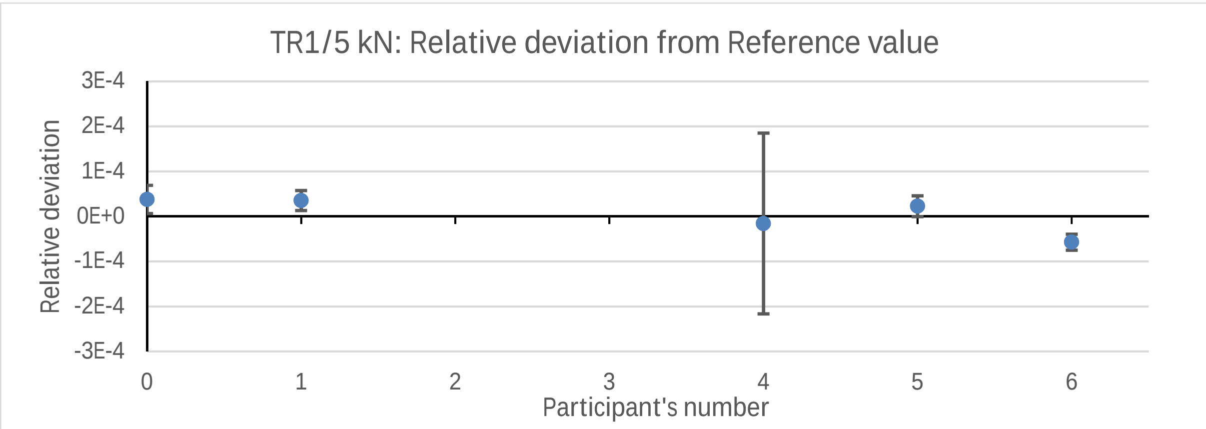


Figure 2.8 TR1/5 kN, load 5 kN, deviations from reference value with expanded uncertainties, k = 2

Transducer TR2/5 kN

Reference values for transducer TR2/5 kN, values in 10^{-5}	
Load 5 kN	
Reference value	Uncertainty $W_{\mathrm{d,ref}}$ (k = 2)
-0.83	1.07

Table 2.9 Reference values for transducer TR2/5 kN with corresponding relative uncertainty

Laboratory	Transducer TR2/5 kN, values in 10^{-5}	
	Rel. deviation to reference value	Expanded uncertainty
	5 kN	$W_{\mathrm{dL,ref}}$ (k = 2)
0	0.83	1.05
1	-1.09	2.45
2	-	-
3	-	-
4	0.59	20.10
5	-0.20	3.24
6	-1.02	2.00

Table 2.10 The deviations to reference value of transducer TR2/5 kN for each laboratory with corresponding expanded relative uncertainties

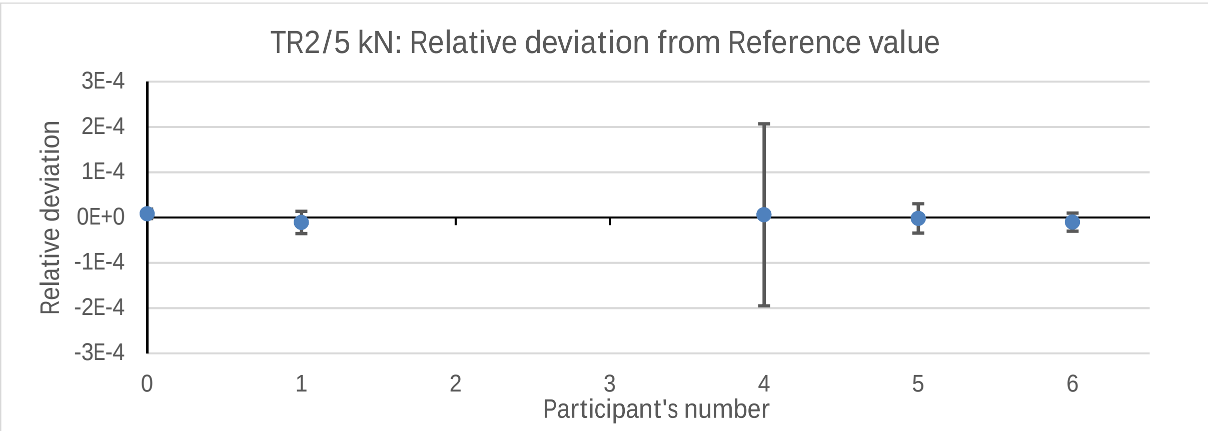


Figure 2.9 TR2/5 kN, load 5 kN, deviations from reference value with expanded uncertainties, k = 2

2.8 Tables of the deviation and the degree of equivalence

Lab	$d_{L,\mathrm{ref}} = d_{\mathrm{L}} - d_{\mathrm{ref}}$ in 10^{-5}											
	TR1/10 kN				TR2/10 kN				TR1/5 kN		TR2/5 kN	
	5 kN		10 kN		5 kN		10 kN		5 kN		5 kN	
	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$	$d_{L,\mathrm{ref}}$	$W_{\mathrm{dL,ref}}$
0	0.48	1.77	0.33	1.18	-1.13	1.14	-0,95	1.16	3.77	3.14	0.83	1.05
1	-3.49	2.68	-3.09	2.53	-2.89	2.47	-3.18	2.37	3.53	2.22	-1.09	2.45
2	-0.97	2.40	-1.54	2.39	-2.40	2.37	-2.52	2.36	-	-	-	-
3	0.15	3.47	2.12	3.70	0.77	3.83	1.16	3.42	-	-	-	-
4	-0.65	25.69	2.01	19.48	0.67	20.19	-1.50	10.33	-1.57	20.07	0.59	20.10
5	-1.20	2.74	-0.49	2.76	-2.11	3.18	-1.93	2.93	2.28	2.31	-0.20	3.24
6	2.21	1.89	1.71	1.79	9.00	2.33	8.53	2.25	-5.72	1.77	-1.02	2.00

Table 2.11 The relative deviations of all participants to reference values of each transducers and with used loads including the associated expanded relative uncertainties

	TR1/10 kN at 5 kN: $d_{\mathrm{Pairs}} = d_{\mathrm{L},n} - d_{\mathrm{L},n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		3.97	1.44	0.33	1.13	1.68	-1.74
1	-3.97		-2.52	-3.64	-2.84	-2.29	-5.70
2	-1.44	2.52		-1.11	-0.32	0.23	-3.18
3	-0.33	3.64	1.11		0.80	1.35	-2.07
4	-1.13	2.84	0.32	-0.80		0.55	-2.86
5	-1.68	2.29	-0.23	-1.35	-0.55		-3.41
6	1.74	5.70	3.18	2.07	2.86	3.41	

Table 2.12 Degree of equivalence for transducer TR1/10 kN, with load 5 kN

	TR1/10 kN at 5 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		3.21	2.98	3.89	25.75	3.26	2.59
1	3.21		3.83	4.58	25.86	4.05	3.53
2	2.98	3.83		4.42	25.84	3.87	3.33
3	3.89	4.58	4.42		25.96	4.61	4.16
4	25.75	25.86	25.84	25.96		25.87	25.79
5	3.26	4.05	3.87	4.61	25.87		3.58
6	2.59	3.53	3.33	4.16	25.79	3.58	

Table 2.13 Expanded relative uncertainties for values in table 2.12, transducer TR1/10 kN, load 5 kN

	TR1/10 kN at 10 kN: $d_{\text{Pairs}} = d_{\text{L},n} - d_{\text{L},n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		3.42	1.87	-1.79	-1.68	0.82	-1.38
1	-3.42		-1.55	-5.21	-5.10	-2.60	-4.80
2	-1.87	1.55		-3.66	-3.55	-1.05	-3.25
3	1.79	5.21	3.66		0.11	2.61	0.40
4	1.68	5.10	3.55	-0.11		2.50	0.29
5	-0.82	2.60	1.05	-2.61	-2.50		-2.20
6	1.38	4.80	3.25	-0.40	-0.29	2.20	

Table 2.14 Degree of equivalence for transducer TR1/10 kN, with load 10 kN

	TR1/10 kN at 10 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		3.08	2.97	4.10	19.56	3.28	2.51
1	3.08		3.72	4.67	19.69	3.97	3.36
2	2.97	3.72		4.60	19.67	3.88	3.26
3	4.10	4.67	4.60		19.88	4.80	4.32
4	19.56	19.69	19.67	19.88		19.72	19.61
5	3.28	3.97	3.88	4.80	19.72		3.54
6	2.51	3.36	3.26	4.32	19.61	3.54	

Table 2.15 Expanded relative uncertainties for values in table 2.14, transducer TR1/10 kN, load10 kN

	TR2/10 kN at 5 kN: $d_{\text{Pairs}} = d_{\text{L},n} - d_{\text{L},n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		1.76	1.27	-1.90	-1.79	0.99	-10.13
1	-1.76		-0.49	-3.66	-3.55	-0.78	-11.89
2	-1.27	0.49		-3.17	-3.06	-0.29	-11.40
3	1.90	3.66	3.17		0.10	2.88	-8.23
4	1.79	3.55	3.06	-0.10		2.78	-8.34
5	-0.99	0.78	0.29	-2.88	-2.78		-11.11
6	10.13	11.89	11.40	8.23	8.34	11.11	

Table 2.16 Degree of equivalence for transducer TR2/10 kN, with load 5 kN

	TR2/10 kN at 5 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		3.05	2.97	4.23	20.26	3.65	2.93
1	3.05		3.69	4.76	20.38	4.26	3.66
2	2.97	3.69		4.71	20.37	4.20	3.59
3	4.23	4.76	4.71		20.59	5.17	4.69
4	20.26	20.38	20.37	20.59		20.48	20.37
5	3.65	4.26	4.20	5.17	20.48		4.18
6	2.93	3.66	3.59	4.69	20.37	4.18	

Table 2.17 Expanded relative uncertainties for values in table 2.16, transducer TR2/10 kN, load 5 kN

	TR2/10 kN at 10 kN: $d_{\mathrm{Pairs}} = d_{\mathrm{L},n} - d_{\mathrm{L},n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		2.23	1.57	-2.11	0.55	0.98	-9.48
1	-2.23		-0.67	-4.34	-1.68	-1.25	-11.71
2	-1.57	0.67		-3.68	-1.02	-0.58	-11.05
3	2.11	4.34	3.68		2.66	3.09	-7.37
4	-0.55	1.68	1.02	-2.66		0.43	-10.03
5	-0.98	1.25	0.58	-3.09	-0.43		-10.46
6	9.48	11.71	11.05	7.37	10.03	10.46	

Table 2.18 Degree of equivalence for transducer TR2/10 kN, with load 10 kN

	TR2/10 kN at 10 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		2.96	2.95	3.85	10.48	3.43	2.87
1	2.96		3.60	4.37	10.69	4.00	3.54
2	2.95	3.60		4.36	10.68	4.00	3.53
3	3.85	4.37	4.36		10.96	4.70	4.31
4	10.48	10.69	10.68	10.96		10.82	10.66
5	3.43	4.00	4.00	4.70	10.82		3.93
6	2.87	3.54	3.53	4.31	10.66	3.93	

Table 2.19 Expanded relative uncertainties for values in table 2.18, transducer TR2/10 kN, load10 kN

	TR1/5 kN: $d_{\mathrm{Pairs}} = d_{\mathrm{L},n} - d_{\mathrm{L},n+1}$ in 10^{-5}				
Pairs	0	1	4	5	6
0		0.24	5.34	1.49	9.49
1	-0.24		5.10	1.25	9.25
4	-5.34	-5.10		-3.85	4.14
5	-1.49	-1.25	3.85		8.00
6	-9.49	-9.25	-4.14	-8.00	

Table 2.20 Degree of equivalence for transducer TR1/5 kN, with load 5 kN

	TR1/5 kN: W_{pairs} in 10^{-5}				
Pairs	0	1	4	5	6
0		4.26	20.40	4.31	4.05
1	4.26		20.28	3.70	3.39
4	20.40	20.28		20.29	20.24
5	4.31	3.70	20.29		3.45
6	4.05	3.39	20.24	3.45	

Table 2.21 Expanded relative uncertainties for values in table 2.20, transducerTR1/5 kN, load 5 kN

	TR2/5 kN: $d_{\text{Pairs}} = d_{\text{L},n} - d_{\text{L},n+1}$ in 10^{-5}				
Pairs	0	1	4	5	6
0		1.92	0.23	1.02	1.85
1	-1.92		-1.68	-0.89	-0.07
4	-0.23	1.68		0.79	1.61
5	-1.02	0.89	-0.79		0.82
6	-1.85	0.07	-1.61	-0.82	

Table 2.22 Degree of equivalence for transducer TR2/5 kN, with load 5 kN

	TR2/5 kN: W_{pairs} in 10^{-5}				
Pairs	0	1	4	5	6
0		3.06	20.18	3.73	2.72
1	3.06		20.30	4.34	3.50
4	20.18	20.30		20.41	20.25
5	3.73	4.34	20.41		4.10
6	2.72	3.50	20.25	4.10	

Table 2.23 Expanded relative uncertainties for values in table 2.22, transducerTR2/5 kN, load 5 kN

Chapter 3
Reference values for 5 kN and 10 kN forces

3.1 The deviation between pilot and laboratory

For each laboratory and each transducer, a single value will be calculated as deviation. This value is based on results of differences between the pilot and the laboratories. The model of deviation is:

$$D = X_{\mathrm{L}} - X_{\mathrm{P}} \tag{3.1}$$

with

$$X_{\mathrm{P}} = X_{\mathrm{PA}} + \Delta_{\mathrm{corr}} \tag{3.2}$$

The correction Δ_{corr} is used for the pilot measurements. It is assumed that the drift is linear.

3.1.1 The used deflection value by pilot for loop *n*

The used deflection for loop *n* is calculated from A and B measurements by pilot with the correction of the drift. The assumption for the drift is a linear drift between A and B measurements, which are made as close as possible before and after the measurement of the participating laboratory. The needed time for transport in one direction has not been equal for all participants, variation from four days up to 2 weeks. The correction of the drift is made as function of the time.

$$\Delta_{\mathrm{corr}} = \frac{X_{\mathrm{PB}} - X_{\mathrm{PA}}}{t_{\mathrm{total}}} \cdot t_1 \tag{3.3}$$

where:

$t_{\mathrm{total}} = t_1 + t_2$

t_1 = time between pilot A measurement and calibration by laboratory

t_2 = time between calibration by laboratory and pilot B measurement.

The value X_{PA} has been used as reference deflection for loop *n*.

Used reference values for each laboratory with correction of the non-symmetry in time						
Transducer	TR1/10 kN	TR1/10 kN	TR2/10 kN	TR2/10 kN	TR1/5 kN	TR2/5 kN
Load	5 kN	10 kN	5 kN	10 kN	5 kN	5 kN
Laboratory	mV/V	mV/V	mV/V	mV/V	mV/V	mV/V
1	1.0028670	2.0060427	1.0099508	2.0199301	1.8820172	2.0277785
2	1.0028783	2.0060699	1.0099567	2.0199405	-	-
3	1.0028920	2.0060970	1.0099545	2.0199356	-	-
4	1.0028935	2.0060983	1.0099528	2.0199186	1.8820019	2.0277800
5	1.0029048	2.0061159	1.0099541	2.0199260	1.8820046	2.0277806
6	1.0028769	2.0060674	1.0099955	2.0200053	1.8819374	2.0277900

Table 3.1 Used reference values for each loop after the correction of non-symmetric timing of measurement by participating laboratories

The correction Δ_{corr} is relatively small; the maximum for correction has been $+5.8 \cdot 10^{-6}$ and the minimum $-4.3 \cdot 10^{-6}$. By using the mean value $\overline{X}_{\mathrm{P}}$ as value X_{P}, the difference is not greater than $5 \cdot 10^{-6}$. In practice, the average value of A and B measurements could have been used.

3.2 Principle to use the pilot FSM as link in the comparison

The same principle given in Chapter 2 is used to handle the results of the comparison. The comparison has been made in star form; the transducers came back to the pilot after every measurement by the participating laboratory. One complete measurement, pilot – participating laboratory – pilot is called a loop.

Pilot's measurements have been made always on the same deadweight machine for the whole key-comparison. This pilot's machine is a link between all participants. The pilot's link machine is marked as PLM in this paper.

The pilot laboratory is also a participating laboratory (Lab 0). Pilot laboratory, as a participant, makes also a comparison between PLM and its own reference calibration machine (PM).

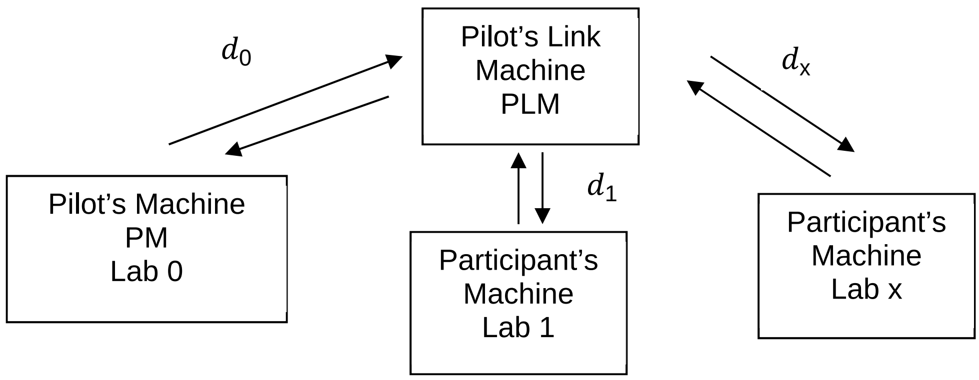
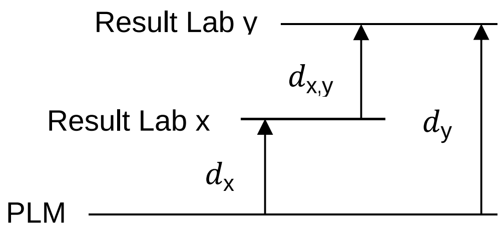


Figure 3.1 Principle of the star-type comparison with the link machine.

This graph can be applied even if the PLM and the PM are the same machine.



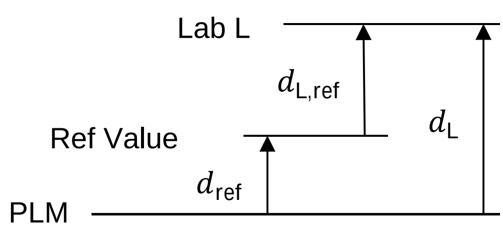


Figure 3.2 Using the pilot as a link, the deviation between two laboratories is equal to $d_{\mathrm{x,y}} = d_{\mathrm{x}} - d_{\mathrm{y}}$. To compute the deviation between two laboratories, it is not necessary to use a traceable PLM but a machine stable enough between the 2 loops.

Figure 3.3 The same approach is made for the deviation from the reference value and a laboratory. Using the pilot as a link, the deviation between a laboratory and the reference value is equal to $d_{\mathrm{L,ref}} = d_{\mathrm{L}} - d_{\mathrm{ref}}$. To compute the deviation between a laboratory and the reference value, it is not necessary to use a traceable PLM but a machine stable enough during all the comparisons.

For each loop, i.e. for each laboratory, the relative deviation d_{L} from the PLM is computed according to:

$$d_{\mathrm{L}} = \frac{X_{\mathrm{L}} - X_{\mathrm{P}}}{X_{\mathrm{P}}} \tag{3.4}$$

3.2.1 Relative standard uncertainty due to the stability of the pilot link machine w_{PLM}

For the two reasons given above, the uncertainty of the PLM is computed taking into account only components of stability. Following components are given in relative value with $k = 1$

- § stability of masse: $1.0 \cdot 10^{-6}$
- § stability of gravity: $0.2 \cdot 10^{-6}$
- § stability of the air buoyancy: $3.5 \cdot 10^{-6}$

The combined relative standard uncertainty due to the stability of pilot FSM w_{PLM} can be estimated equal to:

$$w_{\mathrm{PLM}} = 0.35 \cdot 10^{-5} \,. \tag{3.5}$$

3.2.2 Relative standard uncertainty of deflection obtained at pilot laboratory w_{P}

The measured value $\boldsymbol{X}_{\mathrm{P}}$ for each loop is based on A and B measurements according to equation (3.2). The pilot had a variation of relative reproducibility with rotation $\boldsymbol{w}_{\mathrm{RP}} = 0.2 \cdot 10^{-5} \ldots 1.0 \cdot 10^{-5}$, based on the information from pilots A and B measurements. The standard uncertainty $\boldsymbol{w}_{\mathrm{RP}}$ contains the stability of the FSM as well the instability of the force transducers. By omitting the effect of the transducer, the value of the pure relative stability of the FSM has value of 0.6×10^{-5} . This value can be assumed as maximum value of the stability of FSM by pilot. The uncertainty of the deflection by pilot ($\boldsymbol{w}_{\mathrm{PA}}$ or $\boldsymbol{w}_{\mathrm{PB}}$) includes the pilot reproducibility uncertainty ($\boldsymbol{w}_{\mathrm{RP}}$) and the stability of PLM ($\boldsymbol{w}_{\mathrm{PLM}}$):

$$w'_{\mathrm{P}} = \sqrt{w_{\mathrm{PR}}^2 + w_{\mathrm{PLM}}^2} = 0.6 \cdot 10^{-5} \tag{3.6}$$

However to get the consistency for all measurements, which are influenced by the uncertainty of the pilot's FSM we have to use a value for standard uncertainty:

$$w_{\mathrm{P}} = 0.75 \cdot 10^{-5} \,. \tag{3.7}$$

3.3 Calculation of the reference values for 5 kN and 10 kN

To get the connection between all laboratories it is necessary to create only one reference value for measured forces, 5 kN and 10 kN. This means that every laboratory should have only one value for 5 kN and 10 kN force.

3.3.1 Calibration result as relative deviation to pilot and relative uncertainty of the participant laboratory, X_{L} and w_{L}

A laboratory result is defined by the mean deflection obtained from each calibrated force transducer. The deflection is noted X_{L} for the participant laboratory number L. The relative deviation between laboratory and pilot values has been calculated using equation (3.8).

$$d_{\mathrm{L}} = \frac{X_{\mathrm{L}} - X_{\mathrm{P}}}{X_{\mathrm{P}}} \tag{3.8}$$

The relative uncertainty on this deflection is announced by the participant laboratory. This uncertainty is noted w_{L}.

3.3.2 The weighted mean relative deviation of the laboratory for each measured force (5 kN and 10 kN) and associated relative uncertainties

The mean value has been calculated as the weighted mean of all of the measured relative deviations of 5 kN and correspondingly of 10 kN values. In every case the deviation with PLM is measured with more than one transducer for one participating laboratory. The deviation value $\bar{d}_\mathrm{L}$ used for each participating laboratory is the weighted mean of the deviation obtained for this laboratory.

$$\bar{d}_\mathrm{L} = \sum_{L=1}^{n} p_L \cdot d_L \Big/ \sum_{L=1}^{n} p_L \tag{3.9}$$

with n = 2 or 4, depending of the number of measured transducers and with the weighing factor

$$p_L = \frac{1}{w_\mathrm{dL}^2} \tag{3.10}$$

The uncertainty of this deviation is calculated according to following equation:

$$w_{\bar{\mathrm{d}}\mathrm{L}} = \sqrt{1 \Big/ \sum_{L=1}^{n} \frac{1}{w_\mathrm{dL}^2}} \tag{3.11}$$

where n = 2 or 4, depending of the numbers of measured transducers.

3.3.3 The relative standard uncertainty w_dL of the weighted mean deviation between pilot and laboratory by using the pilot as a link

The standard uncertainty w_dL of the relative deviation between laboratory and pilot is calculated using the equation (3.12).

Relative uncertainty of the deviation is taking in to account:

- the relative uncertainty of the calibration laboratory,
- the relative uncertainty of the pilot
- the relative uncertainty of the drift.

$$w_\mathrm{dL} = \sqrt{w_\mathrm{L}^2 + w_\mathrm{P}^2 + w_{\Delta,\mathrm{corr}}^2} \tag{3.12}$$

Whereas the relative standard uncertainty of the pilot machine has been used the value $w_\mathrm{P} = 0.75 \cdot 10^{-5}$ from equation (3.7). The drift has been assumed as a linear function between A and B measurement by pilot. The time between these measurements is t_total and the participating laboratory has measured it in time $t_\mathrm{total}/2$ with deviation ± n days. By comparing the calculated value X_P based on the drift against the value $\overline{X}_\mathrm{P}$, which is the mean value of measurements A and B, the difference between these two values is not greater than $5 \cdot 10^{-6}$. However, the calculation is based on the method used for drift.

The uncertainty $w_{\Delta,\mathrm{corr}}$ of the drift is based as variation of the drift during four days. By this way the mean value of the drifts is $2.0 \cdot 10^{-6}$ and standard deviation is $1.7 \cdot 10^{-6}$. Only for transducer TR2/5 kN the drift is closely one decade lower. The result of the change is to see on figures 1.3 and 1.4 but the original phenomenon of change is not known.

The relative uncertainty of the drift a_{drift} is based on the variation width during four days and assumed to have rectangular distribution. By this way the uncertainty is connected individually to the drift of the transducer.

$$a_{\mathrm{drift}} = \frac{\Delta X_t}{t_{\mathrm{total}}} \cdot 4 \tag{3.13}$$

The relative uncertainty of the drift is:

$$w_{\Delta,\mathrm{corr}} = \frac{a_{\mathrm{drift}}}{2 \cdot \sqrt{3} \cdot X_{\mathrm{P}}} \tag{3.14}$$

3.4 Reference value and associated relative uncertainties

The reference value has been calculated from all of the mean relative deviations $\bar{d}_{\mathrm{L}}$ of 5 kN and correspondingly of 10 kN values as the weighted mean value based on weighted mean values of each laboratory for 5 kN and 10 kN. The pilot is as laboratory 0 included with a deviation from the PL equal to 0 because the pilot calibration machine is the PLM. The weighting factor is the uncertainty $w_{\bar{\mathrm{d}}\mathrm{L}}$ of the mean deviation $\bar{d}_{\mathrm{L}}$.

The reference deviation d_{ref} is calculated using equation (3.14) and (3.15).

For 5 kN: $$d_{\mathrm{ref,\,5\,kN}} = \sum_{L=1}^{7} p_L \cdot \bar{d}_L \Bigg/ \sum_{L=1}^{7} p_L \tag{3.14}$$

For 10 kN: $$d_{\mathrm{ref,\,10\,kN}} = \sum_{L=1}^{7} p_L \cdot \bar{d}_L \Bigg/ \sum_{L=1}^{7} p_L \tag{3.15}$$

where the weighing factor is

$$p_L = \frac{1}{w_{\bar{d}\mathrm{L}}^2} \tag{3.16}$$

The uncertainty of the reference deviation is calculated considering that values are not correlated:

For 5 kN: $$w_{\mathrm{d,ref,5\,kN}} = \sqrt{1 \Bigg/ \sum_{L=1}^{7} \frac{1}{w_{\bar{d}\mathrm{L}}^2}} \tag{3.17}$$

For 10 kN: $$w_{\mathrm{d,ref,10\,kN}} = \sqrt{1 \Bigg/ \sum_{L=1}^{7} \frac{1}{w_{\bar{d}\mathrm{L}}^2}} \tag{3.18}$$

The results and assigned uncertainties, concerning deviation with the reference value, are given for all participants, including the pilot laboratory, in Figures 3.4 and 3.5. The data are given in tables 3.4 to 3.10.

Reference values as weighted mean, values in 10^{-5}			
Load 5 kN		Load 10 kN	
Ref. value	Uncertainty $W_{\text{d,ref,5 kN}}$ (k = 2)	Ref. value	Uncertainty $W_{\text{d,ref,10 kN}}$ (k = 2)
-1.01	0.26	0.30	0.33

Table 3.2 The reference values for forces of 5 kN and 10 kN with expanded uncertainties

3.5 The degree of equivalence

The degree of equivalence of each participating laboratory is expressed by (according to M. G. Cox, Metrologia 2002,39):

- its deviation from the comparison reference value, equation (3.19)
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (3.20).

$$d_{\text{L,ref}} = \bar{d}_{\text{L}} - d_{\text{ref}} \tag{3.19}$$

The uncertainty of the relative deviation between laboratory and reference value is calculated using the equation (3.22) with the standard uncertainty of the deviation and standard uncertainty of the reference value:

- the uncertainty between the calibration laboratory and the pilot, caused by the deviation and including the uncertainty of laboratory,
- the uncertainty of the reference value.

$$W_{\text{dL,ref}} = 2 \cdot \sqrt{w_{\text{dL}}^2 - w_{\text{ref}}^2} \tag{3.20}$$

The values are given in table 3.5.

The degree of equivalence between pairs of laboratories is expressed by:

- its difference of their deviations from each other, equation (3.21),
- and by the uncertainty of this deviation at the 95 % level of confidence, equation (3.22).

$$d_{\text{Pairs}} = \bar{d}_{\text{L},n} - \bar{d}_{\text{L},n+1} \tag{3.21}$$

$$W_{\text{Pairs}} = 2 \cdot \sqrt{w_{d\text{L},n}^2 + w_{d\text{L},n+1}^2} \tag{3.22}$$

The values of pairs and their uncertainties are given in tables 3.6 to 3.9.

3.6 Consistency check

According to the proposal of M. G. Cox “The evaluation of key comparison data” the consistency should be checked with χ^2-test. The results are following:

Force	5 kN	10 kN
χ_{obs}^2	13.73	50.81
$\chi^2(n)$	12.59	12.59

Table 3.3 Result of the consistency check

According this result the comparison nearly has the required consistency for 5 kN. For 10 kN the uncertainty for PLM should be increased up to value $2.42 \cdot 10^{-5}$ to have the consistency.

3.7 Tables and diagrams of laboratory mean values

Laboratory	Values in 10^{-5}, k = 2			
	F = 5 kN		F = 10 kN	
	$\bar{d}_{\mathrm{L}}$	$W_{\bar{\mathrm{d}}L}$	$\bar{d}_{\mathrm{L}}$	$W_{\bar{\mathrm{d}}L}$
0	0.00	0.84	0.00	1.06
1	-2.29	1.34	-2.79	1.85
2	-1.66	1.82	-1.72	1.81
3	0.66	2.66	1.96	2.60
4	-1.22	10.60	-0.06	9.16
5	-1.28	1.51	-0.90	2.12
6	-1.82	1.13	4.66	1.55

Table 3.4 Mean relative deviation between participants laboratories and pilot laboratory

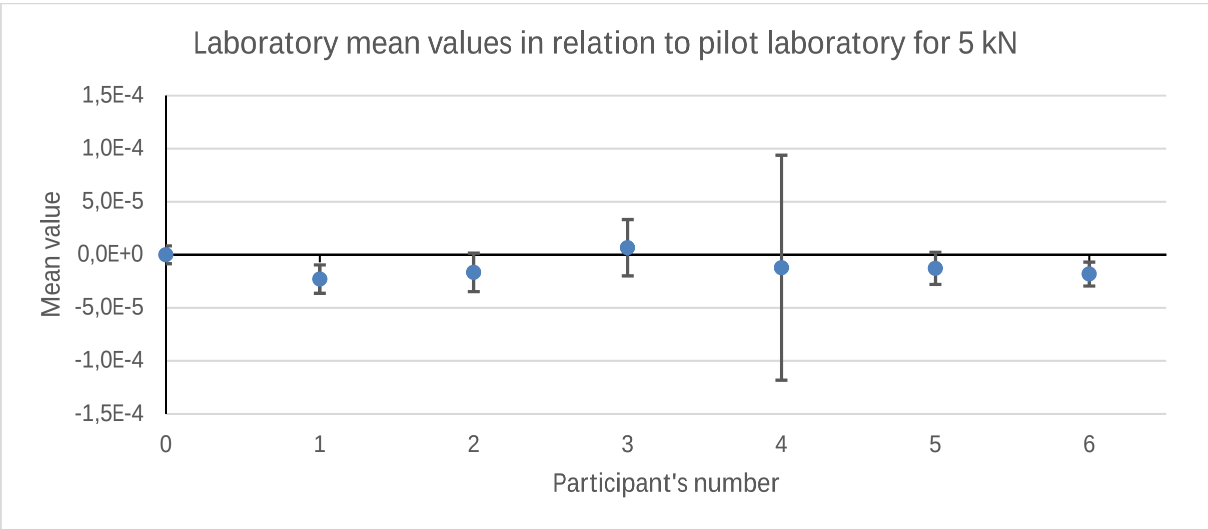


Figure 3.4 Laboratory mean values in relation to pilot laboratory for 5 kN with expanded (k = 2) uncertainties

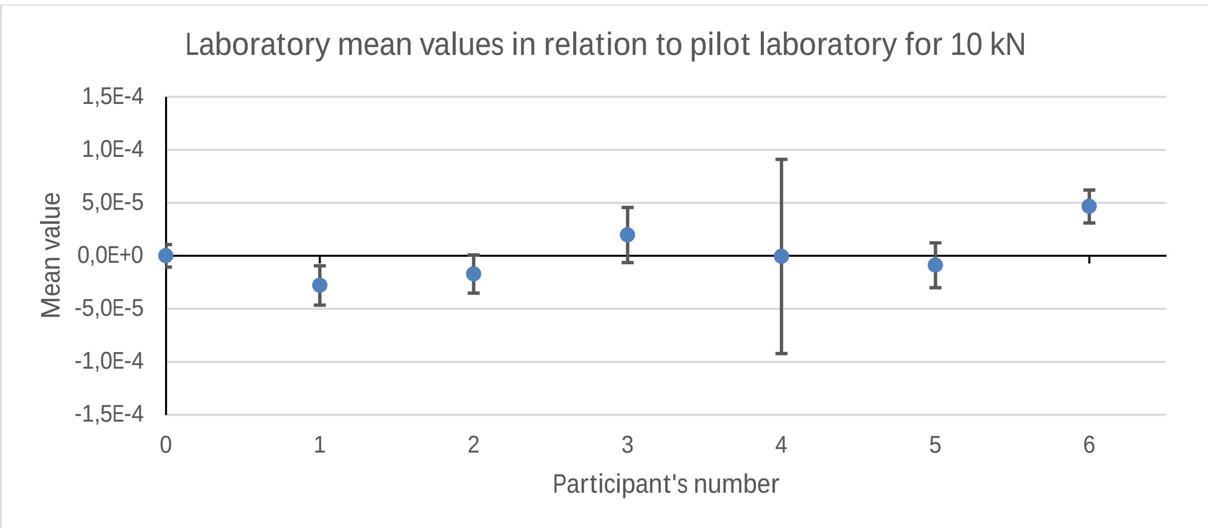


Figure 3.5 Laboratory mean values in relation to pilot laboratory for 10 kN with expanded (k = 2) uncertainties

3.8 Tables and diagrams of the degrees of the equivalence

Laboratory	Values in 10^{-5}, k = 2			
	F = 5 kN		F = 10 kN	
	$d_{L,\text{ref}}$	$W_{dL,\text{ref}}$	$d_{L,\text{ref}}$	$W_{dL,\text{ref}}$
0	1.01	0.66	-0.30	0.83
1	-1.29	1.23	-3.09	1.73
2	-0.65	1.74	-2.01	1.68
3	1.67	2.61	1.67	2.51
4	-0.21	10.59	-0.35	9.14
5	-0.27	1.42	-1.19	2.01
6	-0.81	1.00	4.36	1.41

Table 3.5 Degree of equivalence between laboratories and reference value

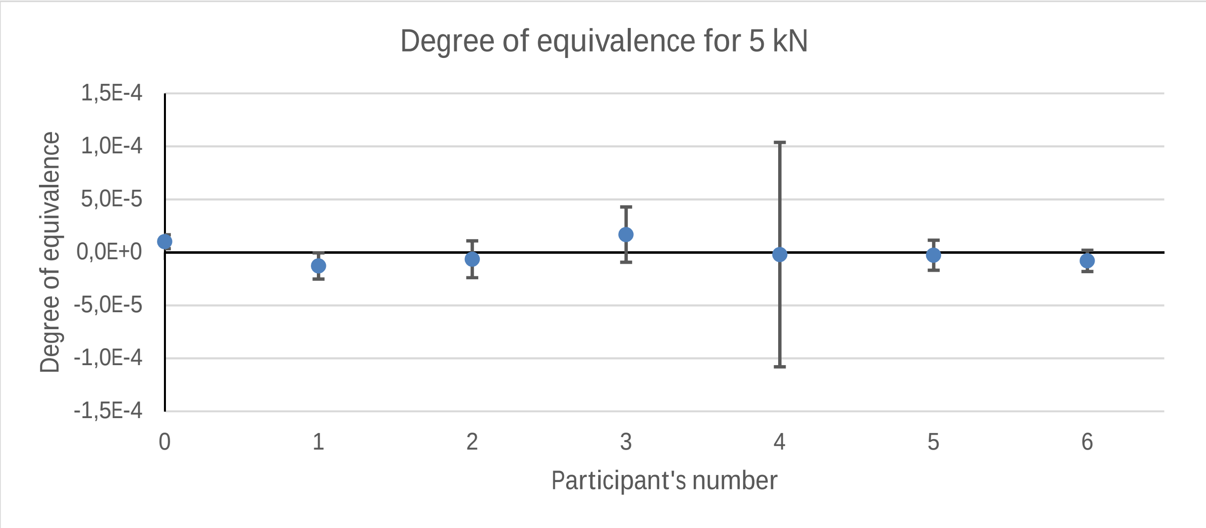


Figure 3.6 Degree of equivalence of laboratories for 5 kN with expanded (k = 2) uncertainties

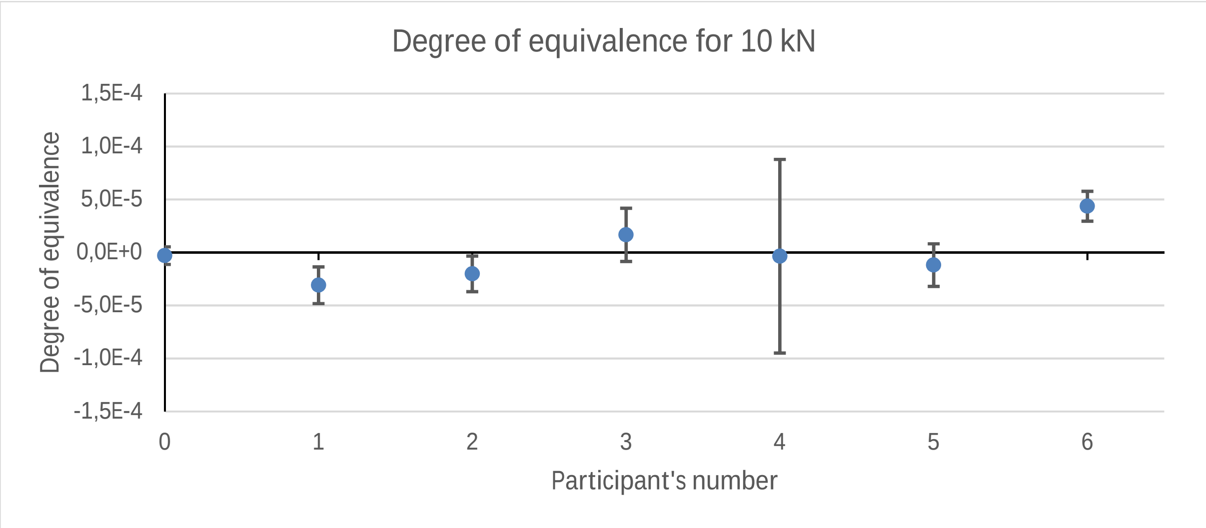


Figure 3.7 Degree of equivalence of laboratories for 10 kN with expanded (k = 2) uncertainties

	For 5 kN: $d_{\text{Pairs}} = d_{L,n} - d_{L,n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		2.29	1.66	-0.66	1.22	1.28	1.82
1	-2.29		-0.64	-2.96	-1.07	-1.01	-0.48
2	-1.66	0.64		-2.32	-0.44	-0.38	0.16
3	0.66	2.96	2.32		1.89	1.95	2.48
4	-1.22	1.07	0.44	-1.89		0.06	0.59
5	-1.28	1.01	0.38	-1.95	-0.06		0.53
6	-1.82	0.48	-0.16	-2.48	-0.59	-0.53	

Table 3.6 Degree of equivalence between two laboratories for force 5 kN

	For 5 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		1.40	1.86	2.69	10.61	1.56	1.20
1	1.40		2.13	2.88	10.66	1.88	1.59
2	1.86	2.13		3.13	10.73	2.24	2.01
3	2.69	2.88	3.13		10.90	2.97	2.79
4	10.61	10.66	10.73	10.90		10.68	10.63
5	1.56	1.88	2.24	2.97	10.68		1.74
6	1.20	1.59	2.01	2.79	10.63	1.74	

Table 3.7 Associated expanded uncertainties for values in table 3.6 for force 5 kN

	For 10 kN: $d_{\text{Pairs}} = d_{L.n} - d_{L.n+1}$ in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		2.79	1.72	-1.96	0.06	0.90	-4.66
1	-2.79		-1.08	-4.76	-2.74	-1.90	-7.45
2	-1.72	1.08		-3.68	-1.66	-0.82	-6.38
3	1.96	4.76	3.68		2.02	2.86	-2.70
4	-0.06	2.74	1.66	-2.02		0.84	-4.72
5	-0.90	1.90	0.82	-2.86	-0.84		-5.55
6	4.66	7.45	6.38	2.70	4.72	5.55	

Table 3.8 Degree of equivalence between two laboratories for force 10 kN

	For 10 kN: W_{pairs} in 10^{-5}						
Pairs	0	1	2	3	4	5	6
0		1.92	1.87	2.65	9.17	2.18	1.63
1	1.92		2.41	3.05	9.30	2.65	2.23
2	1.87	2.41		3.02	9.29	2.62	2.19
3	2.65	3.05	3.02		9.48	3.22	2.88
4	9.17	9.30	9.29	9.48		9.36	9.24
5	2.18	2.65	2.62	3.22	9.36		2.46
6	1.63	2.23	2.19	2.88	9.24	2.46	

Table 3.9 Associated expanded uncertainties for values in table 3.8 for force 10 kN

3.9 Link to the Force Key Comparison CCM.F-K1.a and CCM.F-K1.b

The results of this regional EUROMET comparison must be clearly linked to the results of the corresponding CIPM Key Comparison. To link the results of this comparison to the CIPM Key Comparison Reference Value (CIPM KCRV) the performance of the laboratories that participated in both comparisons, MIKES (laboratory number 0) and PTB (laboratory number 2), must be analyzed in both comparisons. The degrees of equivalence for MIKES and PTB in the CIPM Key Comparison and the weighted mean deviation from the CIPM KCRV with an associated uncertainty are given in table 3.10. Based on the weighted mean deviation from the CIPM KCRV it can be concluded that, at 5 kN force, the CIPM KCRV is $0.23 \cdot 10^{-5}$ lower than the weighted mean deviation of MIKES and PTB. At 10 kN force, the CIPM KCRV is $0.37 \cdot 10^{-5}$ lower than weighted mean deviation of MIKES and PTB.

Laboratory	Values in 10^{-5}, $k = 2$			
	F = 5 kN		F = 10 kN	
	$d_{L,\text{ref CIPM}}$	$W_{dL,\text{ref CIPM}}$	$d_{L,\text{ref CIPM}}$	$W_{dL,\text{ref CIPM}}$
MIKES	0.60	1.90	1.10	1.90
PTB	-0.40	2.50	-0.90	2.50
W. mean	0.23	1.51	0.37	1.51

Table 3.10 Degrees of equivalence for MIKES and PTB in CIPM Key Comparison ($d_{L,\text{ref CIPM}}$ = deviation from CIPM KCRV, $W_{dL,\text{ref CIPM}}$ = expanded uncertainty of deviation)

The degrees of equivalence for MIKES and PTB in this EUROMET Key Comparison and the weighted mean deviation from the EUROMET Reference Value (EUROMET RV) with an associated uncertainty are given in table 3.11. Based on the weighted mean deviation from the EUROMET RV it can be concluded that, at 5 kN force, the EUROMET RV is $0.80 \cdot 10^{-5}$ lower than the weighted mean deviation of MIKES and PTB. At 10 kN force, the EUROMET RV is $0.63 \cdot 10^{-5}$ greater than weighted mean deviation of MIKES and PTB.

Laboratory	Values in 10^{-5}, $k = 2$			
	F = 5 kN		F = 10 kN	
	$d_{L,\text{ref EUR}}$	$W_{dL,\text{ref EUR}}$	$d_{L,\text{ref EUR}}$	$W_{dL,\text{ref EUR}}$
MIKES	1.01	0.66	-0.30	0.83
PTB	-0.65	1.74	-2.01	1.68
W. mean	0.80	0.61	-0.63	0.74

Table 3.11 Degrees of equivalence for MIKES and PTB in EUROMET Key Comparison ($d_{L,\text{ref EUR}}$ = deviation from EUROMET RV, $W_{dL,\text{ref EUR}}$ = expanded uncertainty of deviation)

In order to establish a link between these two comparisons it must be assumed that the performance of MIKES and PTB has not changed between the comparisons. Based on this assumption and the weighted mean deviations of MIKES and PTB to the reference values in both comparisons it can be concluded that, at 5 kN EUROMET RV is $0.57 \cdot 10^{-5}$ lower than the CIPM KCRV. At 10 kN, EUROMET RV is $1.00 \cdot 10^{-5}$ greater than the CIPM KCRV. To link the results of the participating laboratories to the CIPM KCRV, the degrees of equivalence (given in table 3.5) need to be corrected by these differences between the EUROMET RV and the CIPM KCRV. The corrected degrees of equivalence with associated uncertainties, calculated as the squared sum of the uncertainties given in table 3.5 and the uncertainty associated with the correction from EUROMET RV to the CIPM KCRV, are given in table 3.12 and figures 3.8 and 3.9.

Laboratory	Values in 10^{-5}, k = 2			
	F = 5 kN		F = 10 kN	
	$d_{L,\text{ref CIPM}}$	$W_{dL,\text{ref CIPM}}$	$d_{L,\text{ref CIPM}}$	$W_{dL,\text{ref CIPM}}$
0	0.44	1.76	0.70	1.88
1	-1.85	2.05	-2.09	2.42
2	-1.22	2.38	-1.01	2.38
3	1.11	3.07	2.67	3.02
4	-0.78	10.71	0.65	9.29
5	-0.84	2.16	-0.19	2.62
6	-1.37	1.92	5.36	2.20

Table 3.12 Degree of equivalence between laboratories and CIPM Key Comparison Reference Value ($d_{L,\text{ref CIPM}}$ = deviation from CIPM KCRV, $W_{dL,\text{ref CIPM}}$ = expanded uncertainty of deviation)

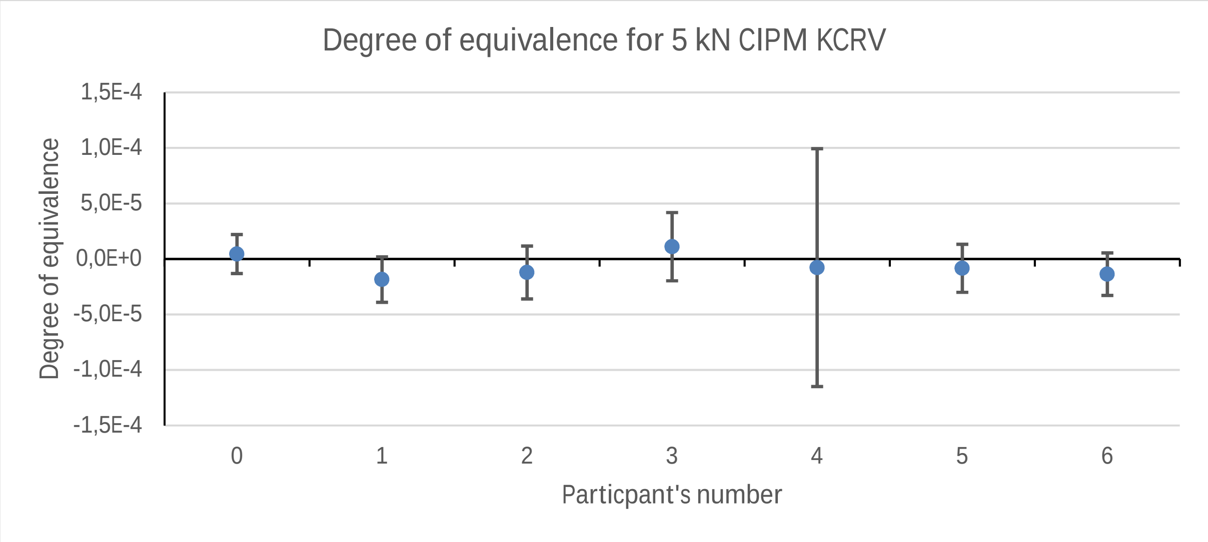


Figure 3.8 Degree of equivalence of laboratories for 5 kN with expanded (k = 2) uncertainties

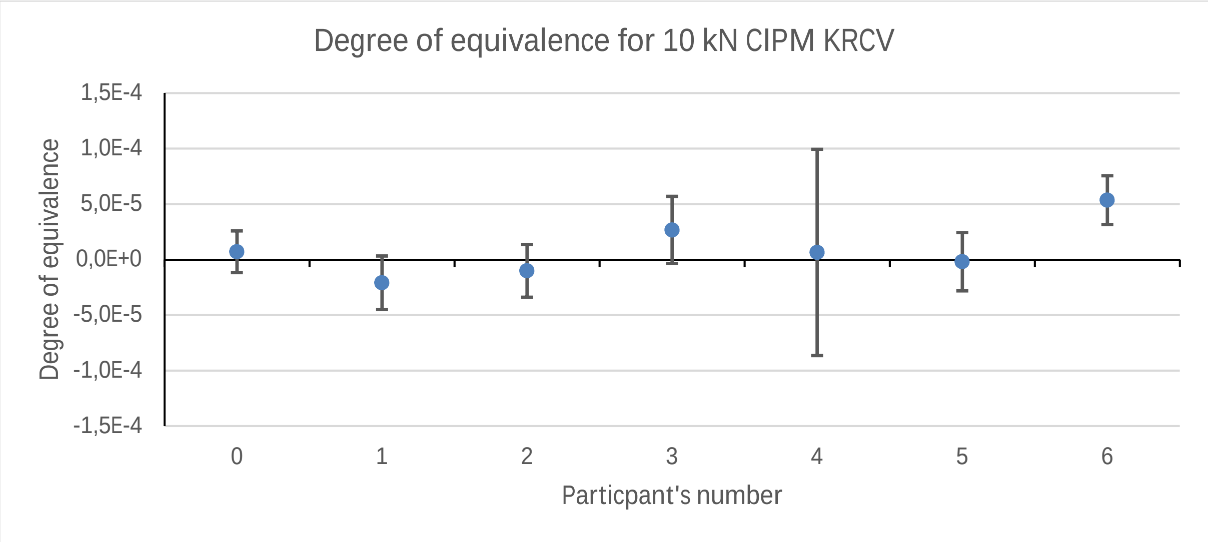


Figure 3.9 Degree of equivalence of laboratories for 10 kN with expanded (k = 2) uncertainties