

# **Physikalisch-Technische Bundesanstalt**

## **Fertigungsmesstechnik**

**PTB-Bericht F-43**

**Angle calibration on presision polygons**  
Final Report of EUROMET Project # 371

by

**Reinhard Probst and Reiner Wittekopf**



## Contents:

	Page
1. Introduction .....	3
2. Participating laboratories and time schedule .....	4
3. Precision polygons as transfer standards .....	5
4. Description of the task .....	7
5. Measurement methods, conditions and equipment .....	8
6. Results and evaluations .....	11
6.1 Evaluation methods .....	11
6.2 Results for the 7-sided polygon .....	15
6.3 Results for the 24-sided polygon .....	16
6.3.1 Measurements on 12 faces .....	16
6.3.2 Measurements on 24 faces .....	17
6.4 Differences between polygon positions .....	18
6.5 Interferometric measurements .....	18
7. Investigation of autocollimators .....	20
8. Conclusions .....	21
References .....	22

## Tables and Figures

Annex A: Measurement instructions

Annex B: Contour and profile diagrams for the polygon faces





## 1. Introduction

Precision polygons are basic standards for angle measurement, which are used and calibrated by the national standards laboratories in particular. The uncertainty of measurement attainable when measuring the angles between faces of a polygon largely depends on the geometry (flatness and squareness) of their reflecting faces, as well as on alignment errors and errors caused by the autocollimator sensing the faces. A CCDM comparison of two 12-sided polygons, carried out by eleven laboratories between 1980 and 1986, produced unsatisfactory results, as the measurement differences turned out to be much greater than the uncertainties estimated by the participants [1].

In 1995, OFMET and PTB proposed to carry out an interlaboratory comparison as EUROMET project "Angle calibration on precision polygons". It was decided that PTB should act as the pilot laboratory and that two polygons should be circulated, one with 24 faces and the other with seven faces. Participation in this comparison was announced by nine European national institutes, five of which took part, together with PTB, in the first circulation between April 1996 and May 1997. A second circulation was organized between October 1997 and September 1999, with seven other laboratories, three of which had joined as additional participants.

It was the main objective of the project to provide greater clearness about the expression of uncertainty in angle calibrations on polygons, in accordance with the *Guide* [2]. Interferometric measurements of the polygon faces carried out at PTB were expected to provide information about the influence of the flatness quality on the uncertainty, on the basis of an analysis of the correlation with the laboratories' results.

This final report is a revised and extended version of the draft A1 report which was distributed to the participants in June 2000. It covers the evaluation of the measurement results made available by twelve laboratories including PTB, together with summaries and extracts from the laboratories' reports in accordance with the measurement instructions for this project.

This comparison has been declared as an EUROMET key comparison and entered into the key comparison database Appendix B of the BIPM under the identifier EUROMET.L-K3.PREV.

## 2. Participating laboratories and time schedule

The following table lists the participating national standards laboratories with their responsible metrologists and the periods during which measurements took place:

### Pilot laboratory:

PTB	Physikalisch-Technische Bundesanstalt Bundesallee 100 D-38116 Braunschweig, Germany	R. Probst R. Wittekopf	January – March 1996 May – August 1997 January – March 1999
-----	---	---------------------------	--

### Laboratories of the first circulation:

METAS (formerly OFMET)	Swiss Federal Office of Metrology and Accreditation Lindenweg 50 CH-3003 Bern-Wabern, Switzerland	R. Thalmann	April – May 1996
NMi-VSL	Van Swinden Laboratory Netherlands Measurements Institute Schoemakerstraat 97 NL-2628 VK Delft, Netherlands	H. Haitjema G. Kotte	May – June 1996
VTT	VTT Manufacturing Technology PL 1702 FIN-02044 VTT, Finland	H. Lehto	July – October 1996
IMGC	Istituto di Metrologia “G. Colonnetti” Strada delle Cacce, 73 I-10135 Torino, Italy	A. Sacconi	October 1996 – February 1997
NPL	National Physical Laboratory Queens Road Teddington, Middlesex TW11 OLW United Kingdom	G.N. Peggs D.R. Flack	March – May 1997

### Laboratories of the second circulation:

SMU	Slowensky Metrologický Ústav Slovak Institute of Metrology Karloveska 63 SK-84255 Bratislava, Slovakia	J. Mokros	October – November 1997
-----	---	-----------	----------------------------

CEM	Centro Español de Metrología c/del Alfar, 2 E-28760 Tres Cantos Madrid, Spain	E. Prieto	December 1997 - February 1998
LNE	Laboratoire National d'Essais Bureau National de Métrologie 1, Rue Gaston Boissier F-75724 Paris Cedex 15, France	M. Priel G.P. Vailleau	March – May 1998
IGM *	Inspection Générale de la Métrologie, Metrology Service Emile Jacqmainlaan 154 B-1000 Brussels, Belgium	H. Pirée	May – August 1998, April – May 1999
GUM	Główny Urząd Miar Central Office of Measures 2, Elektoralna Street PL-00-950 Warsaw, Poland	Z. Ramotowski G. Rudnicka	August – October 1998
UME	Ulusal Metroloji Enstitüsü National Metrology Institute P.O. Box 21 TR-41470 Gebze-Kocaeli, Turkey	T. Yandayan	October 1998 – January 1999
CMI	Ceský Metrologický Institut OI Liberec Slunečná 23 CZ-460 01 Liberec Czech Republic	V. Stezka	July - September 1999

\*) IGM finally decided not to transmit its results, because unexpected problems with the measurement set-up prevented them from achieving the proposed uncertainty of measurement.

### 3. Precision polygons as transfer standards

The following polygons were used as transfer standards for the comparison: a 7-sided polygon (provided by NMI) and a 24-sided polygon (provided by PTB); their main features are listed in the following table:

	7-sided polygon (P7)	24-sided polygon (P24)
Manufacturer, identification	Rank Taylor Hobson Ltd. SP LE 5997	Möller / Wedel PTB 5,22-23-539-1
Pitch angle	51° 25' 42,857..."	15°
Size of reflecting faces	15 mm Ø	20 mm x 25 mm (width x height)
Diameter	60 mm	150 mm
Material	glass	glass ceramics "Zerodur"
Mass (incl. case)	495 g	2126 g
Angle deviations from nominal (max.-min.)	2,0"	2,0"
Pyramidity of faces <sup>1)</sup> (max.-min.)	8,4"	8,2"
Flatness deviations <sup>1)</sup> mean RMS	3,3 nm	6,0 nm

1) For details see in Section 6.5, Tables 11.1 and 11.2, as well as in Annex B.

P7 was made to comply with the NPL specification for the accuracy of a glass precision polygon [5]. Both polygons have central mounting holes and are accommodated in metal cases. An adjustable mounting device, which was circulated together with the polygons, could be used to facilitate the alignment of the polygons in the measurement set-up (see Annex A). The housing of P7 had circular apertures which limited the diameter of the reflecting faces to 15 mm, whereas the case of P24 had an open perimeter exposing the faces to their full rectangular size of 20 mm x 25 mm. The rather high flatness deviations of these faces are due to roll-off close to the edges, which is significantly reduced when the faces are masked to about 19 mm diameter (see Section 6.5 and Annex B). The mean RMS flatness deviation in the table above refers to this diameter.

The faces of both polygons are numbered and given on the cases, starting with No.1. The polygons can also be mounted in the inverted position (numbers on the bottom side), the counting direction with respect to the rotation of the table then being reversed.



The stability of the polygon standards was checked by three repeated calibrations performed by the pilot laboratory in the course of the intercomparison. These calibrations were found to be identical within the uncertainty of measurement, so the stability of the standards has been verified.

#### **4. Description of the task**

The aims of the comparison were:

- to determine the mutual compatibility of angle calibrations on precision polygons among laboratories realizing angular units independently and applying different methods of measurement;
- to improve the expression of uncertainty in angle measurements of this kind according to the "Guide" [ 2 ];
- to investigate the potential existence of systematic influences and to determine the limit of accuracy attainable with polygon / autocollimator systems;
- to exchange information about measurement techniques and procedures.

The results of this EUROMET comparison must also be considered with regard to the "CCL Key Comparison" of angle standards, which is being organized by CSIR, South Africa.

The laboratories applying for participation in this comparison first gave a short description of the envisaged extent of the measurements, the instrumentation available and the methods they intend to use. Recommendations were made by the pilot laboratory as regards the measurement instructions; these were distributed in advance to all participants (see Annex A). Several alternative measurements were suggested for the comparison, taking into account the different capabilities of the laboratories:

- measurement of one or of both polygons
- measurement on all faces of P24 or limitation to twelve faces
- measurement of the polygons in the normal position and in the inverted position
- measurement by standard methods 1 and 2 or other methods (see Annex A and Section 5)
- use of the participant's own mounting device instead of the device made available.

The laboratories were instructed to report the results of the measurements together with the combined standard uncertainty, and to give a description of the adjustment conditions, measuring instruments and measurement procedures as well as the evaluation of the uncertainties. This requirement was met by all the laboratories, and some laboratories also supplied more detailed supplementary reports.

## **5. Measurement methods, conditions and equipment**

Table A gives a survey of the methods and conditions of measurement and states the combined standard uncertainties in accordance with the laboratories' reports. It is indicated which of polygons P7 and P24 was measured exclusively in the normal (n) or in the inverted (i) position; where no indication is given, the measurements were performed in both positions. The indication P24/2 means that P24 was measured on twelve faces only.

Method 1 refers to the use of a single autocollimator, either by the cross calibration method or by the method of direct comparison with an angle measuring or indexing table. In method 2, two autocollimators and a rotary table for angular positioning are used.

Table B gives a survey of the measuring instruments of the laboratories. NMI did not use the measuring system RON 905 for the measurement but only for positioning of the polygons (method 2). NPL used two different Moore indexing tables, one with 1440 serrations for P24 and the other with 2160 serrations for P7, together with two different autocollimator types. Two additional autocollimators of the same type were set up with their measuring axis oriented vertically to control the levelling of the polygons.

In addition to tables A and B, the summary below gives the calibration procedures followed by the laboratories.

The cross calibration method is often used in such a way that changes of the relative positions (so-called settings) of the polygon in relation to the angle measuring or indexing table can be made with the aid of an additional indexing table. This additional table can be used either as an "intermediate" (between polygon and measuring table) or as a "subsidiary" table (beneath the measuring table supporting the polygon).

NPL reported that the polygon was levelled again at the start of each set of measurements after a setting had been made, to minimize the pyramid error. The other laboratories did not report anything in this respect; it is assumed that the pyramid errors stated in the reports were valid for all polygon settings.

### **PTB:**

P24, P7:

- (1) Cross calibration vs. numerically controlled rotary table with measuring system RON 905, polygon settings with intermediate Moore 1440 (P24) or manually on mounting device (P7)
- (2) Cross calibration vs. angle comparator WMT 220, polygon setting with numerically controlled calibration facility for the WMT 220.



**OFMET:**

P24, P7 n:

Cross calibration vs. numerically controlled rotary table / RON 905, polygon settings with intermediate Moore 1440 (P24) or manually on mounting device (P7n)

P7 i:

Direct comparison vs. rotary table / RON 905

**NMI:**

P24, P7:

NMI version of method 2: Comparison of polygon vs. two autocollimators

(a) directed at two neighbouring faces and (b) directed at two faces closest to 180°

In both cases measurements are performed with the two autocollimators while the polygon is rotated step by step over 360° on the rotary table SIP MU-214B, repeated in three rounds.

**VTT:**

P24 n:

Cross calibration vs. rotary table / RON 905, setting procedure not described.

P7 n:

Direct comparison vs. rotary table / RON 905 whose errors were corrected with results of cross calibration P24n, three times in one position.

**IMGC:**

P24 n, P24/2 i:

Cross calibration vs. Moore 1440, repeated in reversed order of rotation, polygon settings with subsidiary indexing table.

P7:

Direct comparison of each pair of adjacent polygon faces vs. the same angular interval of a Moore 1440, repeated at 0°, 90°, 180° and 270° reading of the indexing table. Measurement set-up with subsidiary rotary table (SIP) and auxiliary Moore table.

Circular aperture diaphragms 26 mm in diameter for P24 and 16 mm in diameter for P7 were used in front of the autocollimators. The faces of P24 thus were slightly cut off at the corners.

**NPL:**

P24/2:

Cross calibration vs. Moore 1440, repeated for two revolutions at each setting (manual settings on mounting device). At each setting the polygon was levelled.

P7:

Direct comparison vs. Moore 2160 with step-by-step index settings either  $51^{\circ} 30'$  or  $51^{\circ} 20'$ , difference readings with autocollimator MO(1)1873, repeated for two readings at each setting, repeated in  $0^{\circ}$ ,  $120^{\circ}$  and  $240^{\circ}$  start reading of the indexing table. At each  $120^{\circ}$  setting the polygon was levelled.

**SMÚ:**

P24, P7

Cross calibration vs. ring laser using goniometer GS1L in dynamic mode of operation (continuous rotation), five revolutions in both directions of rotation, repeated 15 times. Additional measurement of pyramidity, adjustment on mounting device of the goniometer table.

**CEM:**

P24/2, P7

1. Cross calibration vs. measuring table ("angular generator") and use of autocollimator DA 80
2. Two-autocollimator method using DA 80 and DA 400 at all n settings ( $n = 12$ ,  $n = 7$ ), readings taken from the autocollimators after stepwise positioning of the table.

**LNE:**

P24 n

Cross calibration vs. angle measuring table in 24 positions of the polygon, measured ten times in each position, each time clockwise (CW) and counterclockwise (CCW), autocollimator used in  $\pm 10''$  range.

P 24/2 i

Direct comparison with the angle measuring table after applying the table corrections obtained by previous method P24 n; four measurements with one unchanged setting of the polygon on the table, each measurement CW and CCW.

P7

Measured using correction values derived from the previous measurement of P24. Ten repeated measurements (CW and CCW) with one polygon setting.

**GUM:**

P24, P7

Two-autocollimator method using two FET 100 and table TL 78 for positioning,  $n/2$  settings of the instruments relative to the  $n = 24$  and  $n = 7$  faces, pyramidity adjustment using a DA 80 autocollimator and GUM's own polygon mounting device. An aperture diaphragm 15 mm in diameter was used with the autocollimators, thus limiting the faces of P24.

**UME:**

P24/2

Cross calibration vs. Moore 1440 using ELCOMAT HR (12 x 12 measurements), polygon settings by use of another Moore 1440 as a subsidiary indexing table.

P7

Method proposed by H. Haitjema, described in the UME report.

**CMI:**

P24/2

Comparison with a 36-sided CMI polygon with the aid of two autocollimators, the two polygons mounted with a mandrel and distance piece on the rotary table Zeiss OKT 315

P7

Two-autocollimator method with rotary table Zeiss OKT 315

## **6. Results and evaluations**

### **6.1 Evaluation methods**

#### *Preliminary remarks on the question of uncertainty evaluation*

In the measurement instructions (see Annex A) the participants were asked to report their results and uncertainties separately for the two positions (normal and inverted) of the polygons. The intention was to obtain from the evaluation of all the differences of results between the two positions an uncertainty estimation in correlation with the flatness deviations of the polygon faces, regardless of the type of autocollimator used in the single measurement. The reason for this attempt must be seen in the fact that it is practically impossible to make an a priori estimation of any "bias error" of angle measurement occurring with a certain combination of non-ideally flat polygon faces and an unknown optical transfer function of the autocollimator.

In other words: There is no independent reference value available for comparison with the single result. Under this aspect, the results of the two polygon positions should have been delivered without any uncertainty estimation due to the flatness error.

It turned out however during the intercomparison that several participants took into account the flatness deviations also as a source of uncertainty in each of the two positions, regarding positioning or excentricity errors in combination with the autocollimator. This question was discussed during the EUROMET/Length contact persons meeting in Prague 1999. It was finally recommended that the uncertainty evaluations should be reviewed, including also the influences of the polygon flatness deviations. These reviews have then been reported to the pilot laboratory, and the corresponding corrections were made in the Draft A1 report of June 2000.

In their uncertainty budgets the participants reported as the main contributions:

Type A: - repeatability of autocollimators and angle measuring tables

Type B: - autocollimator calibration error

- angle setting or measuring error of indexing or rotary table
- pyramidal error of the polygon faces
- excentricity error between polygon and autocollimator
- flatness deviation of the polygon faces
- resolution of autocollimator and measuring table

The reported *cumulative angle deviations*  $\Delta\beta_i$  relate to the datum face termed 1. Any bias error in the measurement of this face thus enters as a constant into all values of  $\Delta\beta_i$ , which can lead to systematic differences between the laboratories' results for all the other faces (see Sections 6.2 and 6.3). To eliminate this dependence on the error of a single face, it is therefore preferred to relate the values of  $\Delta\beta_i$  to the average of all faces. We refer to these values as the so-called *reduced angle deviations*  $\Delta\beta_i^r$ :

$$(1) \quad \Delta\beta_i^r = \Delta\beta_i - \frac{1}{n} \sum_{k=1}^n \Delta\beta_k \quad i = 1 \dots n : \text{Face No.}$$

From this definition it follows that  $\sum_{i=1}^n \Delta\beta_i^r = 0$ . The deviation from the nominal angle between any two faces is equal to the difference between their reduced angle deviations. It is an advantage of the reduced angle deviation that this value relates to a single polygon face,



contrary to the pitch or cumulative angle deviation which always relates to a pair of faces. The reduced angle deviation can thus be attributed to the quality and error influence of the specific face.

After the laboratories' results  $\Delta\beta_i$ , in the first step of the evaluation, the results  $\Delta\beta_i^r$  are given as absolute values. In the following, the differences of the values  $\Delta\beta_i^r$  from the reference values are given, which are calculated as the weighted mean for each face, as explained below.

The evaluation described was made on the basis of the results obtained with the polygons both in the normal and in the inverted position. In addition to this, the mean results for the normal and inverted positions have been evaluated.

#### *Uncertainties and reference values*

In the following we denote the  $m$  different results  $\Delta\beta_i^r$  of the laboratories for each polygon face No.  $i$  by an index  $j$  in a short and general form as quantities  $x_j$  ( $j = 1 \dots m$ ). From the values  $x_j$  and their associated standard uncertainties  $u_j$  the weighted mean as the reference value  $x_{ref}$  of each polygon face and its uncertainty  $u_{ref}$  are determined as:

$$(2) \quad x_{ref} = \frac{\sum_{j=1}^m u_j^{-2} \cdot x_j}{\sum_{j=1}^m u_j^{-2}}, \quad u_{ref} = \left( \sum_{j=1}^m u_j^{-2} \right)^{-1/2}$$

The uncertainties  $u_j$  which were used in the calculations (2) are the so-called “enlarged” uncertainties  $u_{j \text{ enlarged}}$  derived from the reported uncertainties of the laboratories by quadratic addition of the standard deviations  $s_j$  resulting from the evaluations of their measured differences between the normal and inverted polygon positions (Section 6.4, Tables 10.1 and 10.2):

$$(3) \quad u_{j \text{ enlarged}} = \left( u_{j \text{ reported}}^2 + s_j^2 \right)^{1/2}$$

In Table C the values of  $u_{j \text{ reported}}$ ,  $s_j$  and  $u_{j \text{ enlarged}}$  are listed for all participants and both polygons.<sup>1</sup>

The internal consistency between the results and the reference value is tested by application of the  $E_n$ -criterion [6], after calculation of the “normalized error” or  $E_n$ -number:

$$(4) \quad E_n = \frac{1}{k} \cdot \frac{x_j - x_{ref}}{\sqrt{u_j^2 - u_{ref}^2}}$$

The minus sign in the denominator of (4) reflects that  $x_j$  and  $x_{ref}$  are correlated according to (2) and that their covariance is equal to  $-u_{ref}^2$  [7]. The  $E_n$ -criterion for  $k = 2$  demands that

$$(5) \quad |E_n| \leq 1$$

Additionally, the so-called Birge ratio  $R_B$  is calculated to check the external consistency of the results with the reference value, according to [8], [9], [10]:

$$(6) \quad R_B = \frac{s_{ext}}{s_{int}}$$

with

$$(7) \quad s_{ext} = \sqrt{\frac{\sum_j [(x_j - x_{ref})/u_j]^2}{(m-1) \cdot \sum_j u_j^{-2}}}, \quad s_{int} = \left( \sum_j u_j^{-2} \right)^{-1/2}$$

As can be seen from (2),  $s_{int}$  is equal to  $u_{ref}$ , whereas  $s_{ext}$  expresses a weighted standard deviation of the results  $x_j$  with respect to  $x_{ref}$ . In the case of external consistency,  $R_B$  approaches 1 ( $s_{ext} = s_{int}$ ), for a large number  $m$  of results. For small numbers  $m$ ,  $R_B$  is related to the  $\chi^2$ -criterion according to

$$(8) \quad \left| \chi_{min}^2 - \nu \right| \leq k\sqrt{2\nu} \quad ; \quad \chi_{min}^2 = \nu \cdot R_B^2$$

with  $\nu = m - 1$  designating the number of degrees of freedom. It follows from (8) with  $k = 2$  that the  $m$  results are consistent with the reference on the condition

$$(9) \quad R_B^2 \leq 1 + \sqrt{8/(m-1)}$$

---

<sup>1</sup> in the case of VTT no value of  $s_j$  is available and  $u_{j \text{ enlarged}}$  set equal to  $u_{j \text{ reported}}$



## 6.2 Results for the 7-sided polygon

The reported results and uncertainties are given in Table 1.1 as cumulative angle deviations, related to face No. 1, measured in the normal position of this polygon. In Figure 1.1 these results are presented in a graphical form. Offset errors can be clearly seen for the results of NPL and CMI, apparently related with the datum face.

Table 1.2 and Figure 1.2 show the results as reduced angle deviations according to (1), proving better overall coincidence than in Figure 1.1. The last column of Table 1.2 gives the weighted mean as reference value for each face and its uncertainty according to (2), and in the last row the enlarged uncertainties according to (3) entering into the weighted mean values.

The differences between the reduced angle deviations and their weighted mean for each polygon face are shown in Table 1.3 and in Figure 1.3. The last column and row of Table 1.3 give the standard deviations  $s_i$  and  $s_j$  as quantities characterizing the coincidences of the results for each face and for each laboratory. It can be seen that the  $s_j$  values have a much larger spread than the  $s_i$  values which are nearly identical for all faces of the polygon.

Finally, in Table 1.4, the  $E_n$  and  $R_b$  values are listed, calculated with the values from Tables 1.2 and 1.3 using formulas (4) and (9). The  $R_b$  criterion is generally fulfilled, whereas the  $E_n$  values exceed the limits for several results of SMU and OFMET due to their rather small uncertainties, as can be seen from Tables 1.1 and 1.2.

The corresponding evaluations for the measurements on the 7-sided polygon in the inverted position as well as the mean of normal and inverted position, are given in Tables 2.1 to 2.4 and 3.1 to 3.4, together with the diagrams of Figures 2.3 and 3.3 for the values of Tables 2.3 and 3.3. The coincidences and consistencies of the results are on the whole slightly better than the results for the normal position, as can be concluded from a comparison of Tables 1.4, 2.4 and 3.4. Two  $E_n$  values in Table 2.4 and one value in Table 3.4 for the SMU results still slightly exceed the compatibility criterion.

### 6.3 Results for the 24-sided polygon

In Tables 4.1, 5.1 and 6.1 the reported results and uncertainties of the measurements on this polygon are listed as cumulative angular deviations, related to face No. 1: Thirteen results from measurements in the normal position and twelve results in the inverted position as well as the mean of the two positions. The representative diagram in Figure 4.1 shows the distribution of the results from Table 4.1, restricted to twelve faces. A large offset due to a common deviation in the datum face is apparent in the results of NMI and CEM. This offset can be largely reduced by evaluation of the reduced angular deviations, as shown in the following two Subsections for the cases of measurement on twelve and 24 faces.

#### 6.3.1 Measurements on 12 faces

Table 4.2.1 and Figure 4.2.1 give the reduced angle deviations, as calculated from the thirteen results of the common twelve odd-numbered faces in Table 4.1. The last column of Table 4.2.1 contains the weighted means as reference values for each face, the last row the enlarged uncertainties of the results and the uncertainty of the reference values.

The differences between the reduced angular deviations and the reference values are shown in Table 4.2.2, together with the standard deviations  $s_i$  and  $s_j$  characterizing the variations of the results for each face and for each laboratory. It is evident also from the diagram of Figure 4.2.2 that the results largely deviate from normal distributions and are practically divided into two groups of a very different magnitude of  $s_j$ . Most remarkable are the rather large, coincident differences of the three results NMI, CEM 1 and CEM 2. The differences of the CMI results are of a first-harmonic nature and seem to indicate an axial alignment error, whereas those of VTT and GUM are of a more irregular nature.

The resulting  $E_n$  and  $R_b$  values are given in Table 4.2.3. The exceeding  $E_n$  values are concentrated on the deviating results of NMI and CEM, in one case also of LNE due to the rather small uncertainty connected with these results. The exceeding  $R_b$  values are coincident with the corresponding  $E_n$  values of NMI and CEM, thus confirming a dominating correlation with the polygon faces 1, 5, 13 and 17.

The following results in Tables 5.1 for the inverted position and 6.1 for the mean of the normal and inverted position are evaluated and shown in Tables 5.2.1 to 5.2.3 and 6.2.1 to 6.2.3, together with Figures 5.2.2 and 6.2.2, in the same way as described above.

The number of results is now reduced to twelve because VTT did not measure the inverted position. These evaluations lead to similar conclusions as given above for Tables 4.2.1 to 4.2.3. The  $E_n$  and  $R_B$  values in Tables 5.2.3 and 6.2.3 show values and distributions only slightly different from those in Table 4.2.3. From this it can be concluded that the differences of the results in the two polygon positions are by far smaller than the differences between the results which are mostly of a systematic nature and must therefore be influenced by the measuring instruments and measuring procedures of the laboratories.

### 6.3.2 Measurements on 24 faces

As can be seen in Tables 4.1 and 5.1, the 24 faces of the polygon in the normal position were measured by eight laboratories and in the inverted position by five laboratories. From these results the reduced angular deviations were calculated and listed in Tables 7.1 (normal position), 8.1 (inverted position) and 9.1 (mean of normal and inverted position). In Tables and Figures 7.2 / 8.2 / 9.2 the differences from the reference values (weighted mean) are given and in Tables 7.3 / 8.3 / 9.3 the  $E_n$  and  $R_B$  values, in the same way as described in Section 6.3.1 for twelve faces.

It can be concluded from these evaluations that the results of LNE, OFMET, PTB and SMU are coincident with the references well below 0,1" for nearly all faces, whereas the other four results in part have much larger deviations. The  $s_i$  values in Table 7.2, expressing the standard deviations of the results for each face, range between 0,06" (faces 6 and 14) and 0,30" (face 1). Compared with this, the  $s_j$  values of the laboratories lie between 0,01" (LNE) and 0,34" (NMI). This difference in the range of the  $s_i$  and  $s_j$  values indicates that the polygon faces seem to have less influence on the results than the measuring instruments of the laboratories.

The  $E_n$  values in Tables 7.3, 8.3 and 9.3 exceed the compatibility criterion for the NMI results for faces 1, 5 and 13, in accordance with the former evaluations given in Tables 4.2.3, 5.2.3 and 6.2.3. Three other results of IMGC, SMU and LNE for faces 7 and 12 are also outside the limit. The external consistency of the results with the reference values is, however, confirmed by the  $R_B$  values for all faces except No. 1, inverted position, where this value exceeds the limit due to the large  $E_n$  of NMI (Tables 8.3 and 9.3).



#### 6.4 Differences between polygon positions

The differences between the reduced angular deviations resulting from the two positions of the polygon are summarized in Table 10.1 for the 7-sided polygon and in Tables 10.2 and 10.3 for the 24-sided polygon (12 or 24 faces measured), as far as reported by the laboratories. In the last row of these tables the standard deviations  $s_j$  for each laboratory and in the last two columns the mean values and standard deviations  $s_i$  for each face are given. In nearly all cases these mean values are smaller than the range of the standard deviations  $s_i$ , indicating that the measured differences are rather to be attributed to the laboratories than to the polygon faces.

From the comparison of the  $s_j$  values of the two polygons for each laboratory it follows that in most cases the values from P7 (Table 10.1) are larger than those from P24/2 or P24 (Tables 10.2 and 10.3). This seems to indicate that adjustments and measurements on P7 were less reproducible in the two positions due to the smaller faces and the unusual division number.

In the case of P24/2 and P24, it is remarkable that SMU and LNE obtained very small differences ( $s_j = 0,01''$ ), despite of their quite different measuring methods and instruments.

The most remarkable results are those of IMG C and NPL for P24/2 (Table 10.2), which are strongly correlated with a coefficient of 0,97 as shown in the diagram of Figure 10. This may be explained by the fact that the two laboratories used autocollimators of a similar type. It is also evident from Table C that NPL and IMG C obtained rather large  $s_j$  values in relation to their reported uncertainties for both polygons.

In the case of the CMI results, the systematic distribution of the signs of the differences in Tables 10.1 and 10.2 seems to indicate axial alignment errors between the polygon positions.

#### 6.5 Interferometric measurements

In the pilot laboratory, measurements on the two polygons were made by the method described in [3], [4] using a phase shifting interferometer (PI). One aim of this method is to obtain the flatness deviations for the faces, shown in Annex B in the form of contour line and profile diagrams. For the 24-sided polygon, the interferograms were evaluated for the full rectangular size of the faces as well as for a circular aperture limited to 18,6 mm in diameter.

In the case of circular apertures, the Zernike polynomial evaluation of the phase map data allows also the pyramidal and reduced angle deviations of the polygons from the tilt angles of the first-order fitted planes to be obtained.

The resulting RMS and P-V values of the polygon faces are given in Tables 11.1 and 11.2, together with the pyramidal deviations and the differences between the reduced angular deviations measured with the autocollimator Elcomat HR and the PI, in the case of P24 for both face sizes. With the whole face viewed the flatness deviations of P24, expressed by the P-V values, are rather large due to roll-off close to the edges. These are greatly reduced when the circular aperture is used. For these cases, Tables 11.1 and 11.2 give the RMS and P-V values as differences from their mean values, showing that the flatness quality of P7 is about twice better than that for P24.

The differences between the HR and PI results of the reduced angular deviations, given in the last two columns of Table 11.2, relate to HR measurements without and with aperture limitation to about 18 mm and thus reveal the influence of aperture errors of the autocollimator. The comparison between these results shows that the aperture influence is rather undefined and does not generally reduce the differences to the PI results but can even increase them. The HR-PI differences are always within  $\pm 0,1$  arc sec for the 7-sided polygon (Table 11.1), whereas these differences are significantly greater for several faces of the 24-sided polygon (Table 11.2). It is not, however, possible to find a correlation between these differences and the flatness or pyramidal deviations of the faces given in Table 11.2.

The interferometric results for flatness and pyramidity of the polygons raise the question as to whether there are correlations between these quantities and the differences of the results of this intercomparison. In the case of the 7-sided polygon as well as for the 24-sided polygon, when only twelve faces had been measured, a correlation could not be proved, which is perhaps also due to the poor statistics for the small number of faces. In the case of P24 however, with all faces measured, a correlation was found which is shown in the diagram of Figure 11. This diagram gives the standard deviations  $s_i^*$  of seven results according to Table 7.2, Section 6.3.2, with the results of NMI excluded, as a function of the RMS\* values from Table 11.2. The correlation coefficient was found to be  $r = 0,70$  with an error probability  $p < 0,1\%$ . (When the same analysis is made with the NMI values included, the correlation is reduced to  $r = 0,38$ .) The linear regression of the correlation is described by:

$$s_i^* = 0,0323'' + (0,0146'' / \text{nm}) \cdot \text{RMS}^*$$

The constant offset may be attributed to measurement errors which are not influenced by the polygon faces. The RMS\* values of the faces, expressing their flatness deviation from the common mean, enter with the factor  $0,0146'' / \text{nm} = 70,8 \text{ nrad} / \text{nm}$ . The inverse of this value is 14,1 mm and can be interpreted as the effective width of the face linked with this angular dependence on the flatness deviation.

## 7. Investigation of autocollimators

As is described in Section 6.3 and can be seen in Figures 4.1, 4.2.2, 5.2.2 and 6.2.2, the results of NMI and the two results of CEM for the twelve faces of P24, are of a remarkable coincidence and largely deviate from the weighted mean obtained from a total of 13 results. This fact was all the more astonishing as NMI and CEM had applied different methods of measurement, NMI using a shortened "method 2" on 24 faces and CEM the complete "methods 1 and 2" on twelve faces. The only similarity found was due to the types of autocollimators used: NMI applied two instruments of type RTH-DA 400 and CEM either employed instrument RTH-DA 80 or combined DA 80 and DA 400. It was finally decided with the pilot laboratory to carry out additional comparisons with these two autocollimator types on the same polygon at PTB. The instruments used for this purpose were the DA 400 from NMI and another DA 80 from PTB.

The results of these comparisons are shown in Figure 12. The polygon was measured using the two DA-autocollimators on 24 faces of full size and compared with the results of NMI and of PTB using the autocollimator Elcomat HR. All results are given as differences from the weighted mean of the reduced angle deviations obtained in this intercomparison. The coincidence of the results of NMI and PTB, obtained with DA 80 and DA 400, and their deviation from the Elcomat HR result can be clearly seen, which also explains the coincidence and deviation of the NMI and CEM results.

Additional measurements were made at PTB with the same instruments to test the influence of the aperture using an 18 mm aperture diaphragm between autocollimators and polygon, the results of which are given in Figure 13. Compared with Figure 12, a strong influence of the aperture on all three autocollimators is revealed: The coincidence between DA 80 and DA 400 has vanished, the deviations of the Elcomat HR increase to the same magnitude and the total distribution exhibits a rather random nature.

It is, however, not possible to find a correlation between the pronounced DA 80 and DA 400 deviations in Figure 12, ranging between  $+ 0,8''$  and  $- 0,6''$ , and the flatness deviations of the polygon faces given in Table 11.2.



## 8. Conclusions

The results of this intercomparison on two polygons of rather unequal size and flatness quality of the faces differed in several regards.

In the case of the 7-sided polygon the differences of the reduced angle deviations from the weighted means as reference values are more or less randomly distributed for all faces and results, with a standard deviation  $s_i$  ranging between 0,11" and 0,16" for the normal position (Table 1.3 and Figure 1.3). A similar result is valid for the inverted position or the mean of the two positions (Tables / Figures 2.3 and 3.3). The internal and external consistency of the results with the reference values is almost fully confirmed by the  $E_n$  and  $R_b$  values (Tables 1.4, 2.4 and 3.4).

In the case of the 24-sided polygon, the results are of a more complex, non-random distribution. About half the results are in a surprisingly close agreement with the reference values ( $< 0,1''$ ), whereas the other results deviate systematically by up to  $\pm 0,5''$  and even more (Figures 4.2.2 to 6.2.2 and 7.2 to 9.2).

Some of these deviations were certainly caused by the critical roll-off at the edges of the rectangular faces, as is obvious from the GUM results obtained with an aperture limited to 15 mm in diameter.

The most astonishing deviations were found with the three nearly coincident results of NMI and CEM. These deviations were proved to be caused by systematic influences from a certain type of autocollimator. Although it was proved that these deviations must also be seen in connection with the flatness defects of the polygon faces, it was not possible to specify this correlation.

It can generally be concluded that for both polygons it was in most cases not possible to find a correlation between the flatness quality of the faces and the differences between the results, also as regards the differences measured for the two polygon positions. An exception from this was found only when all the faces of the 24-sided polygon had been measured but the extraordinary results of the NMI excluded, in a comparison between the  $s_i$  values and the RMS flatness deviations of the faces limited to about 18 mm in diameter (Figure 11). The correlation found here can furnish a useful general estimation of the uncertainty in angle measurement due to the influence of non-ideal flatness.

This comparison has, however, clearly shown that it is not possible - as is often called for - to specify an absolute systematic influence of a certain autocollimator on the angles measured between individual polygon faces of non-ideal flatness, and thus to derive an absolute, quantifiable contribution to the uncertainty of angle calibration which can be attributed to this influence.

**References:**

- [1] K. Toyoda: Report of International Comparison of Angle Standards, NRLM Japan, 1990, 12.1
- [2] Guide to the Expression of Uncertainty in Measurement, BIPM et al, ed ISO, 1993
- [3] R. Probst: Measurement of Angle and Flatness Deviations of Polygon Prism Faces using a Phase-Shifting Interferometer, VDI-Reports No. 1118, 1994, 173 - 178
- [4] R. Probst, H. Kunzmann: Messung von Winkel- und Formabweichungen an Spiegelpolygonflächen mit einem Phaseninterferometer, PTB-Mitt. 103 (1993), 43 - 50
- [5] National Physical Laboratory, Metrology Center, Specification of Accuracy for a Glass Precision Polygon, MOY / SCMI / 83, Issue 5, October 1969
- [6] W. Wöger: Remarks on the  $E_n$ -Criterion Used in Measurement Comparisons, PTB-Mitt. 109 (1999), 24 - 27
- [7] L. Nielsen: Evaluation of measurement intercomparisons by the method of least squares, DFM-99-R39, 3208 LN, 1999
- [8] K. Weise, W. Wöger: Meßunsicherheit und Meßdatenauswertung, Wiley-VCH, 1999, ISBN 3-527-29610-7
- [9] B.N. Taylor et al.: Determination of  $e/h$ , ... Reviews of Modern Physics 41 (1969) 375 – 496
- [10] R.T. Birge: The calculation of errors by the method of least squares, Physical Review 40 (1932) 207 – 261

Laboratory	Method of Meas.	Polygons measured	Mounting Device	Adjustments		Comb. Stand. Uncertainty
				Eccentric.	Pyramidal.	
PTB	1	P7 P24	delivered	10 $\mu\text{m}$	$\pm 6''$ $\pm 6''$	0,11" 0,03"
OFMET	1	P7 P24	delivered	10 $\mu\text{m}$	$\pm 30''$ $\pm 4''$	0,05" / 0,06" 0,04"
NMI	2	P7 P24	delivered	5 $\mu\text{m}$	$\pm 8''$ $\pm 7''$	0,10" 0,22"
VTT	1	P7 n P24 n	delivered	60 $\mu\text{m}$	5" 10"	0,40" 0,45"
NPL	1	P7 P24/2	delivered	51 $\mu\text{m}$ 13 $\mu\text{m}$ / 50 $\mu\text{m}$	$\pm 8''$ $\pm 7''/\pm 6''$	0,24" 0,09"
IMGC	1	P7 P24 n P24/2 i	own delivered delivered	$\leq 20 \mu\text{m}$	50"/200" 16" 18"	0,10" 0,03" 0,03"
SMU	1 (dynamic)	P7 P24	own	20 $\mu\text{m}$	5"	0,05" 0,04"
LNE	1	P7 P24 n P24/2 i	delivered	$\leq 100 \mu\text{m}$	$\pm 4''$ $\pm 6''$	0,10" 0,02"
CEM1	1	P7 P24/2	delivered	$\pm(10...40) \mu\text{m}$ $\pm(35...70) \mu\text{m}$	$\pm 5''$ $\pm 5''$	0,20" 0,15"
CEM2	2	P7 P24/2	delivered	$\pm(10...40) \mu\text{m}$ $\pm(35...70) \mu\text{m}$	$\pm 5''$ $\pm 5''$	0,25" 0,25" / 0,30"
GUM	2	P7 P24	own	120 $\mu\text{m}$	$\pm 4''$	0,21" 0,25"
UME	1	P7 P24/2	delivered	$\pm 30 \mu\text{m}$	$\pm 6''$ $\pm 8''$	0,12" 0,06"
CMI	2 other	P7 P24/2	own	$\pm(5...11) \mu\text{m}$ $\pm(8...11) \mu\text{m}$	$\pm 7''$ $\pm 8''$	0,30" 0,40"

Table A : Methods and conditions of measurement of the laboratories

Abbreviations :

P7 : 7-sided polygon  
P24 : 24-sided polygon  
P24/2 : 24-sided polygon, measured on twelve faces  
n,i : measured in normal or inverted position

Method 1 : using one autocollimator  
Method 2 : using two autocollimators



Laboratory	Rotary tables Indexing tables	Angle measuring systems, Resolution	Autocollimators Type	Range / Resolution
PTB	angle measuring table, Heidenhain Moore 1440	RON 905 0,035"	Möller-Wedel ELCOMAT 2000spec., 2-axis	$\pm 1000'' / 0,01''$
PTB	angle comparator Heidenhain WMT 220	interferential phase grating 0,0012"	Möller-Wedel ELCOMAT HR, 2-axis	$\pm 150'' / 0,005''$
OFMET	angle measuring table, Heidenhain Moore 1440	RON 905 0,035"	Möller-Wedel ELCOMAT HR, 2-axis	$\pm 150'' / 0,005''$
NMI	SIP MU - 214B	RON 905 positioning only	Rank Taylor Hobson 2x DA400, 2-axis	$\pm 400'' / 0,1''$
VTT	angle measuring table VTT	RON 905 0,035"	Rank Taylor Hobson TA80, modified, 1-axis	$\pm 15'' / 0,05''$
NPL	Moore 1440 - 28 (1440 serrations) Moore 1440 - 38 (2160 serrations)	15' 10'	2x H&W TA 9057, 1-axis H&W Microptic photoelectric autocollimator	$100'' / 0,01''$ $600'' / 0,05''$
IMGC	Moore 1440 subsidiary tables : Taboni, Moore 1440 and SIP MU - 214B	15'	P7: H&W TA 3-4, 1-axis P24: H&W TA 5-1, 1-axis	$10' / 0,1''$ $100'' / 0,01''$
SMU	laser goniometer GS1L Arsenal, Kiev	ring laser system 0,01"	built-in, photoelectric, $f = 1000$ mm	pulse generation on optical axis
LNE	angle measuring table BNM-LNE	2 x RON 905 0,035"	Möller-Wedel ELCOMAT HR, 2-axis	$\pm 150'' / 0,005''$
CEM	angle measuring table Tekniker	RON 905 0,035"	RTH DA 80 RTH DA 400	$\pm 80'' / 0,1''$ $\pm 400'' / 0,1''$
GUM	rotary table Microcontrôle TL 78	only used for positioning 1/1000°	2 x Leitz FET 100 (+RTH DA 80)	$40'' / 0,025''$
UME	Moore 1440	15'	Möller-Wedel ELCOMAT HR, 2-axis	$\pm 150'' / 0,005''$
CMI	rotary table Zeiss OKT 315	36-sided polygon as reference standard	ELCOMAT 2000 H&W TA 53	$\pm 1000'' / 0,05''$ $10' / 0,2''$

Table B : Measuring instruments of the laboratories

Lab.	7-sided polygon				24-sided polygon			
	$u_j$ reported	$s_j$	$u_j$ enlarged		$u_j$ reported	$s_j$	$u_j$ enlarged	
PTB	0,11	0,03	0,11		0,03	0,04	0,05	
OFMET	0,05 / 0,06	0,05	0,07 / 0,08		0,04	0,03	0,05	
NMI	0,10	0,09	0,13		0,22	0,09	0,24	
VTT	0,40	-	0,40		0,45	-	0,45	
NPL	0,24	0,28	0,37		0,09	0,10	0,13	
IMGC	0,10	0,13	0,16		0,03	0,13	0,13	
SMU	0,05	0,03	0,06		0,04	0,01	0,04	
LNE	0,10	0,11	0,15		0,02	0,01	0,02	
CEM 1	0,20	0,08	0,21		0,15	0,04	0,16	
CEM 2	0,25	0,10	0,27		0,25 / 0,30	0,10	0,27 / 0,32	
GUM	0,21	0,13	0,25		0,25	0,05	0,26	
UME	0,12	0,05	0,13		0,06	0,03	0,07	
CMI	0,30	0,08	0,31		0,40	0,16	0,43	

values in arcsec

Table C : Combined standard uncertainties  $u_j$  as reported by the laboratories,  
 standard deviations  $s_j$  of the measured differences between normal and  
 inverted polygon positions according to Tables 10.1 and 10.2 and uncertainties  $u_j$  enlarged by inclusion of  $s_j$

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13
Face No. i	values in arcsec												
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	-1,18	-1,23	-1,18	-0,77	-1,60	-1,23	-1,43	-1,39	-1,27	-1,30	-1,56	-1,34	-1,00
3	-0,37	-0,51	-0,25	-0,29	-0,90	-0,33	-0,26	-0,33	-0,35	-0,13	-0,28	-0,27	0,10
4	0,31	0,16	0,25	0,44	-0,40	0,28	0,44	0,06	0,23	0,16	0,30	0,37	0,50
5	-0,37	-0,35	-0,33	-0,42	-1,10	-0,35	-0,49	-0,42	-0,39	-0,38	-0,40	-0,46	-0,10
6	0,19	0,21	0,09	0,46	-0,10	0,08	0,18	0,10	0,10	-0,33	0,04	0,03	0,50
7	-0,07	-0,05	-0,16	-0,20	-0,50	-0,07	0,16	0,11	-0,12	-0,14	0,09	0,03	0,30
$U_{j \text{ reported}}$	0,11	0,05	0,10	0,40	0,24	0,10	0,05	0,10	0,20	0,25	0,21	0,12	0,30

Table 1.1: Cumulative angle deviations of the 7-sided polygon related to face No. 1, **normal** position.

Last row : Combined standard uncertainties as reported by the laboratories.

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	ref. : w. m. i
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i	values in arcsec													
1	0,21	0,25	0,23	0,11	0,66	0,23	0,20	0,27	0,26	0,30	0,26	0,23	-0,04	0,23
2	-0,96	-0,98	-0,95	-0,66	-0,94	-1,00	-1,23	-1,12	-1,01	-1,00	-1,30	-1,11	-1,04	-1,08
3	-0,16	-0,26	-0,02	-0,18	-0,24	-0,10	-0,06	-0,06	-0,09	0,17	-0,02	-0,04	0,06	-0,11
4	0,52	0,41	0,48	0,55	0,26	0,51	0,64	0,33	0,49	0,46	0,56	0,60	0,46	0,52
5	-0,15	-0,10	-0,10	-0,31	-0,44	-0,12	-0,29	-0,15	-0,13	-0,08	-0,14	-0,22	-0,14	-0,19
6	0,40	0,46	0,32	0,57	0,56	0,31	0,38	0,37	0,36	-0,03	0,30	0,26	0,46	0,38
7	0,15	0,20	0,07	-0,09	0,16	0,16	0,36	0,38	0,14	0,16	0,35	0,27	0,26	0,25
$U_{j \text{ enlarged}}$	0,11	0,07	0,13	0,40	0,37	0,16	0,06	0,15	0,21	0,27	0,25	0,13	0,31	$U_{\text{ref}}$ 0,03

Table 1.2: Reduced angle deviations of the 7-sided polygon, **normal** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{\text{ref}}$ .



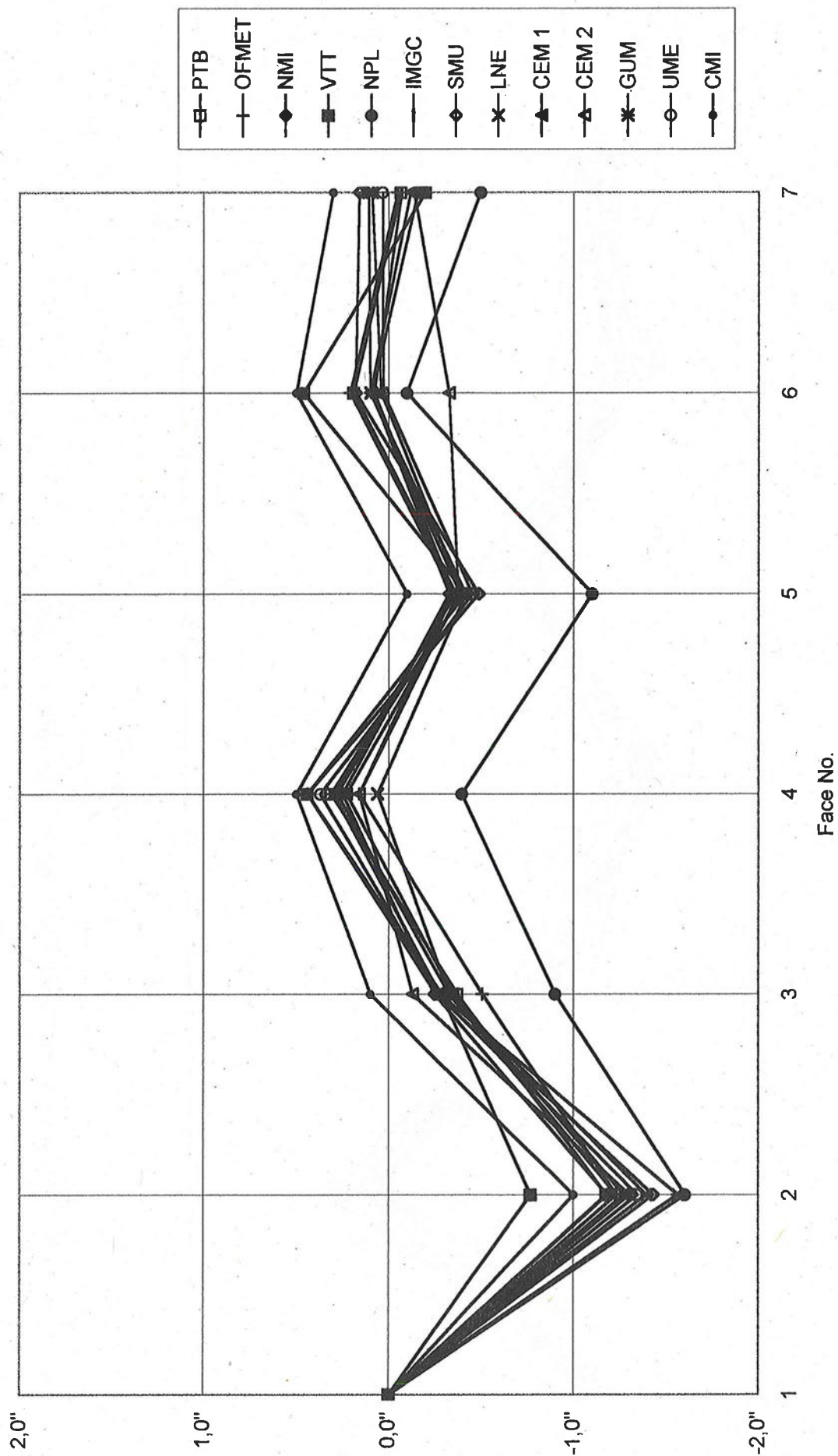


Figure 1.1 : Cumulative angle deviations of the 7-sided polygon related to face no. 1, **normal** position (Table 1.1).

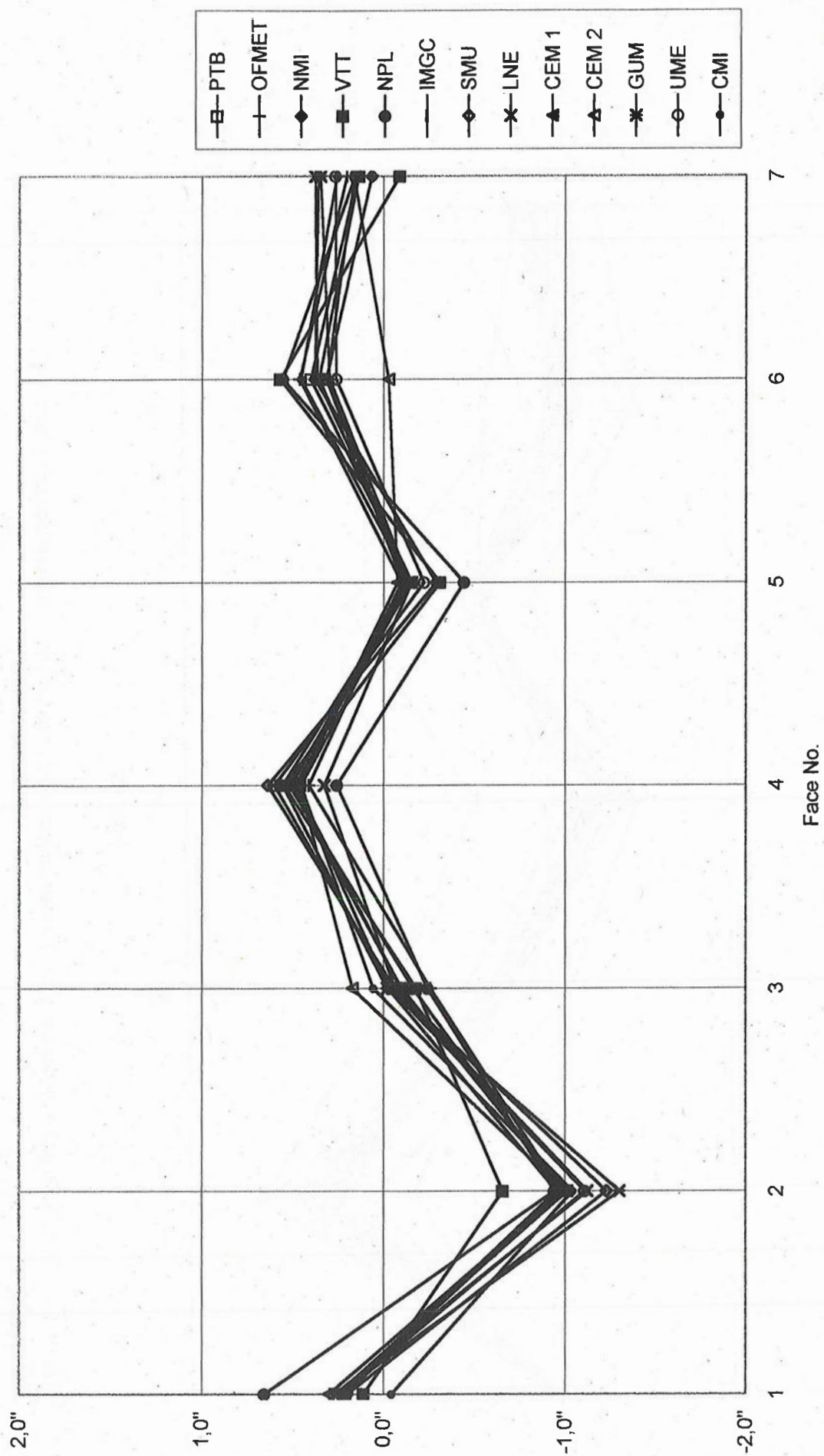


Figure 1.2 : Reduced angle deviations of the 7-sided polygon, normal position (Table 1.2).

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i	values in arcsec													
1	-0,02	0,02	0,00	-0,12	0,43	0,00	-0,03	0,04	0,03	0,07	0,03	0,01	-0,27	0,15
2	0,12	0,11	0,13	0,42	0,14	0,08	-0,15	-0,04	0,07	0,09	-0,22	-0,03	0,04	0,16
3	-0,05	-0,14	0,09	-0,07	-0,13	0,01	0,05	0,05	0,02	0,29	0,09	0,08	0,17	0,12
4	0,00	-0,11	-0,05	0,03	-0,26	-0,01	0,12	-0,19	-0,03	-0,06	0,04	0,08	-0,06	0,11
5	0,03	0,09	0,08	-0,12	-0,26	0,07	-0,10	0,03	0,05	0,11	0,05	-0,03	0,04	0,11
6	0,02	0,08	-0,06	0,19	0,18	-0,07	0,00	-0,01	-0,02	-0,41	-0,08	-0,12	0,08	0,15
7	-0,11	-0,05	-0,19	-0,34	-0,09	-0,09	0,11	0,13	-0,11	-0,09	0,10	0,02	0,01	0,13
$s_j$	0,07	0,10	0,11	0,25	0,26	0,06	0,10	0,10	0,06	0,22	0,11	0,07	0,14	

Table 1.3 : 7-sided polygon, **normal** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No. i

Last row: Standard deviations  $s_j$  of the faces for each result No. j

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$R_B$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i														
1	-0,08	0,20	-0,01	-0,15	0,58	0,01	-0,29	0,13	0,07	0,14	0,06	0,02	-0,44	0,79
2	0,57	0,87	0,51	0,53	0,19	0,27	-1,50	-0,14	0,17	0,16	-0,44	-0,10	0,07	1,22
3	-0,24	-1,19	0,35	-0,08	-0,18	0,04	0,53	0,17	0,05	0,53	0,18	0,30	0,27	0,88
4	0,00	-0,90	-0,19	0,04	-0,36	-0,03	1,20	-0,67	-0,08	-0,11	0,07	0,33	-0,11	0,90
5	0,15	0,74	0,33	-0,15	-0,35	0,22	-1,06	0,12	0,13	0,20	0,09	-0,14	0,07	0,74
6	0,10	0,68	-0,26	0,24	0,24	-0,22	0,00	-0,05	-0,06	-0,76	-0,17	-0,47	0,12	0,95
7	-0,50	-0,40	-0,74	-0,43	-0,13	-0,29	1,11	0,43	-0,28	-0,17	0,20	0,06	0,01	1,17

Table 1.4 :  $E_n$  and  $R_B$  values, 7-sided polygon, **normal** position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,35$



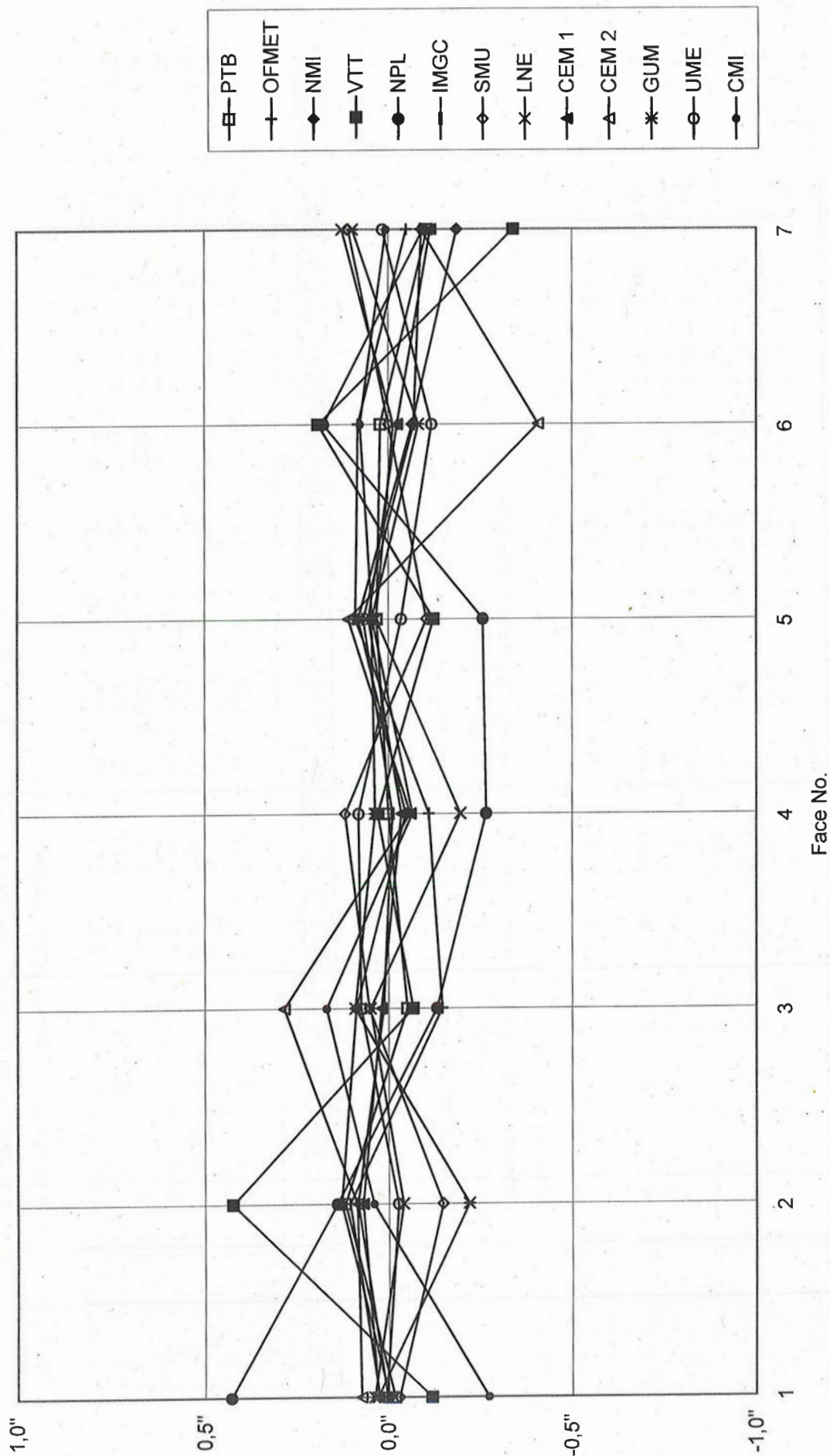


Figure 1.3 : 7-sided polygon, **normal** position. Differences of the reduced angle deviations from the weighted mean of the faces. (Table 1.3)



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12
Face No. i	values in arcsec											
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	-1,24	-1,19	-1,34	-1,50	-0,94	-1,37	-1,45	-1,34	-1,27	-1,31	-1,30	-1,00
3	-0,37	-0,44	-0,24	-0,50	-0,30	-0,20	-0,44	-0,30	0,09	-0,28	-0,35	0,00
4	0,24	0,13	0,31	0,20	0,42	0,42	0,27	0,13	0,24	0,39	0,33	0,60
5	-0,38	-0,35	-0,33	-0,30	-0,24	-0,47	-0,46	-0,36	-0,28	-0,54	-0,40	0,00
6	0,12	0,14	0,03	0,30	0,41	0,22	-0,04	0,18	-0,05	0,11	0,09	0,60
7	-0,08	-0,03	-0,04	0,00	-0,03	0,14	0,05	-0,23	-0,07	0,00	0,01	0,30
$U_j$ reported	0,11	0,06	0,10	0,24	0,10	0,05	0,10	0,20	0,25	0,21	0,12	0,30

Table 2.1 : Cumulative angle deviations of the 7-sided polygon related to face No. 1, **inverted** position.

Last row : Combined standard uncertainties as reported by the laboratories.

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12
Face No. i	values in arcsec											
1	0,25	0,25	0,23	0,26	0,10	0,18	0,30	0,27	0,19	0,23	0,23	-0,07
2	-1,00	-0,94	-1,11	-1,24	-0,84	-1,19	-1,15	-1,07	-1,08	-1,08	-1,07	-1,07
3	-0,12	-0,19	-0,01	-0,24	-0,20	-0,02	-0,14	-0,03	0,28	-0,05	-0,12	-0,07
4	0,49	0,38	0,54	0,46	0,52	0,60	0,57	0,40	0,43	0,62	0,56	0,53
5	-0,14	-0,10	-0,10	-0,04	-0,14	-0,29	-0,16	-0,09	-0,09	-0,31	-0,16	-0,07
6	0,36	0,39	0,26	0,56	0,51	0,40	0,26	0,45	0,14	0,34	0,32	0,53
7	0,16	0,22	0,19	0,26	0,07	0,32	0,35	0,04	0,12	0,23	0,24	0,23
$U_j$ enlarged	0,12	0,08	0,13	0,37	0,16	0,06	0,15	0,21	0,27	0,25	0,13	0,31

ref. :  
w. m. i

0,21  
-1,08  
-0,08  
0,52  
-0,19  
0,37  
0,24  
 $U_{ref}$   
0,04

Table 2.2 : Reduced angle deviations of the 7-sided polygon, **inverted** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{ref}$ .

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	0,03	0,04	0,02	0,05	-0,11	-0,03	0,08	0,06	-0,02	0,02	0,02	-0,28	0,10
2	0,08	0,14	-0,03	-0,16	0,24	-0,11	-0,08	0,01	0,00	0,00	0,01	0,01	0,11
3	-0,04	-0,11	0,07	-0,16	-0,12	0,06	-0,06	0,06	0,37	0,04	-0,03	0,01	0,14
4	-0,03	-0,14	0,02	-0,06	0,00	0,08	0,04	-0,12	-0,09	0,10	0,04	0,01	0,08
5	0,05	0,08	0,09	0,14	0,04	-0,10	0,02	0,10	0,10	-0,12	0,02	0,11	0,08
6	-0,01	0,01	-0,11	0,18	0,13	0,03	-0,12	0,08	-0,23	-0,03	-0,05	0,15	0,12
7	-0,08	-0,02	-0,05	0,01	-0,18	0,08	0,10	-0,20	-0,12	-0,01	-0,01	-0,01	0,09
$s_j$	0,06	0,10	0,07	0,14	0,15	0,08	0,09	0,11	0,19	0,07	0,03	0,14	

Table 2.3 : 7-sided polygon, **inverted** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No. i

Last row: Standard deviations  $s_j$  of the faces for each result No. j

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$R_B$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i													
1	0,15	0,26	0,08	0,06	-0,37	-0,32	0,29	0,15	-0,04	0,04	0,08	-0,46	0,47
2	0,35	0,96	-0,12	-0,22	0,76	-1,15	-0,26	0,03	0,00	0,00	0,03	0,01	0,93
3	-0,16	-0,75	0,30	-0,21	-0,38	0,67	-0,20	0,14	0,68	0,08	-0,13	0,02	0,75
4	-0,15	-1,00	0,08	-0,09	-0,01	0,82	0,15	-0,28	-0,17	0,21	0,16	0,01	0,73
5	0,20	0,59	0,34	0,19	0,14	-1,09	0,07	0,24	0,18	-0,25	0,08	0,18	0,72
6	-0,04	0,10	-0,46	0,25	0,43	0,27	-0,41	0,19	-0,43	-0,06	-0,21	0,25	0,59
7	-0,35	-0,17	-0,21	0,02	-0,56	0,80	0,35	-0,48	-0,23	-0,02	-0,02	-0,02	0,68

Table 2.4 :  $E_n$  and  $R_B$  values, 7-sided polygon, **inverted** position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,36$

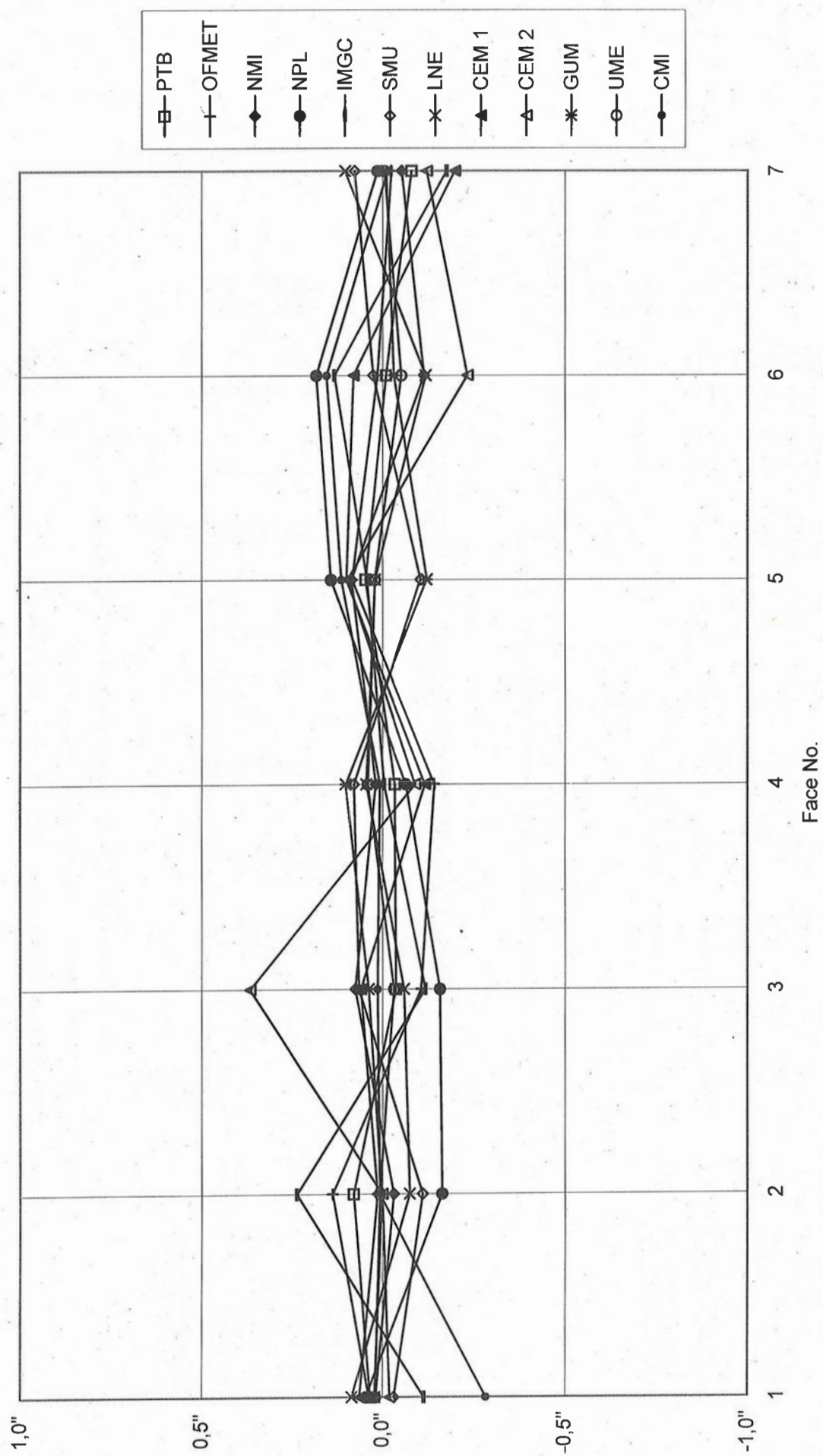


Figure 2.3 : 7-sided polygon, **inverted** position. Differences of the reduced angle deviations from the weighted mean of the faces. (Table 2.3)



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12
Face No. i	values in arcsec											
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	-1,21	-1,21	-1,26	-1,55	-1,09	-1,40	-1,42	-1,31	-1,29	-1,44	-1,32	-1,00
3	-0,37	-0,48	-0,25	-0,70	-0,32	-0,23	-0,39	-0,33	-0,02	-0,28	-0,31	0,05
4	0,28	0,15	0,28	-0,10	0,35	0,43	0,17	0,18	0,20	0,35	0,35	0,55
5	-0,37	-0,35	-0,33	-0,70	-0,30	-0,48	-0,44	-0,38	-0,33	-0,47	-0,43	-0,05
6	0,16	0,18	0,06	0,10	0,25	0,20	0,03	0,14	-0,19	0,08	0,06	0,55
7	-0,07	-0,04	-0,10	-0,25	-0,05	0,15	0,08	-0,18	-0,11	0,05	0,02	0,30
$U_j$ reported	0,11	0,08	0,10	0,24	0,10	0,05	0,10	0,20	0,25	0,21	0,12	0,30

Table 3.1 : Cumulative angle deviations of the 7-sided polygon related to face No. 1, mean of normal and inverted position.

Last row : Combined standard uncertainties as reported by the laboratories.

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	ref. : w. m. i
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	0,23	0,25	0,23	0,46	0,16	0,19	0,28	0,27	0,25	0,25	0,23	-0,06	0,22
2	-0,98	-0,96	-1,03	-1,09	-0,92	-1,21	-1,14	-1,04	-1,04	-1,19	-1,09	-1,06	-1,09
3	-0,14	-0,22	-0,02	-0,24	-0,15	-0,04	-0,10	-0,06	0,23	-0,03	-0,08	-0,01	-0,09
4	0,50	0,40	0,51	0,36	0,51	0,62	0,45	0,45	0,45	0,59	0,58	0,49	0,52
5	-0,15	-0,10	-0,10	-0,24	-0,13	-0,29	-0,16	-0,11	-0,08	-0,22	-0,19	-0,11	-0,19
6	0,38	0,43	0,29	0,56	0,41	0,39	0,31	0,41	0,06	0,32	0,29	0,49	0,37
7	0,15	0,21	0,13	0,21	0,11	0,34	0,36	0,09	0,14	0,29	0,25	0,24	0,25
$U_j$ enlarged	0,11	0,08	0,14	0,37	0,16	0,06	0,15	0,22	0,27	0,25	0,13	0,31	$U_{ref}$ 0,04

Table 3.2 : Reduced angle deviations of the 7-sided polygon, mean of normal and inverted position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{ref}$ .



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i													
1	0,01	0,03	0,01	0,24	-0,05	-0,03	0,06	0,05	0,03	0,03	0,01	-0,28	0,11
2	0,10	0,13	0,05	-0,01	0,17	-0,12	-0,05	0,05	0,05	-0,10	0,00	0,03	0,09
3	-0,05	-0,13	0,08	-0,15	-0,06	0,05	-0,01	0,03	0,32	0,06	0,02	0,09	0,12
4	-0,02	-0,13	-0,02	-0,17	-0,01	0,10	-0,08	-0,08	-0,08	0,07	0,06	-0,03	0,08
5	0,04	0,09	0,09	-0,05	0,06	-0,10	0,03	0,08	0,11	-0,04	0,00	0,08	0,07
6	0,01	0,05	-0,09	0,18	0,04	0,02	-0,06	0,03	-0,32	-0,05	-0,08	0,12	0,13
7	-0,10	-0,04	-0,12	-0,04	-0,14	0,09	0,11	-0,16	-0,11	0,04	0,00	-0,01	0,09
$s_j$	0,06	0,10	0,08	0,16	0,10	0,09	0,07	0,09	0,20	0,06	0,04	0,13	

Table 3.3 :

7-sided polygon, mean of normal and inverted position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No. i

Last row: Standard deviations  $s_j$  of the faces for each result No. j

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$R_B$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i													
1	0,04	0,20	0,03	0,32	-0,17	-0,24	0,21	0,11	0,05	0,05	0,05	-0,45	0,42
2	0,43	0,79	0,21	-0,01	0,52	-1,03	-0,17	0,11	0,09	-0,21	-0,02	0,05	0,91
3	-0,20	-0,82	0,29	-0,20	-0,18	0,45	-0,03	0,08	0,59	0,12	0,07	0,14	0,73
4	-0,08	-0,81	-0,07	-0,23	-0,03	0,79	-0,26	-0,19	-0,14	0,13	0,22	-0,05	0,75
5	0,17	0,56	0,33	-0,07	0,18	-0,85	0,10	0,19	0,20	-0,07	-0,02	0,13	0,69
6	0,04	0,32	-0,33	0,25	0,11	0,13	-0,21	0,08	-0,59	-0,11	-0,31	0,19	0,55
7	-0,40	-0,25	-0,47	-0,06	-0,42	0,75	0,37	-0,38	-0,20	0,08	0,01	-0,01	0,74

Table 3.4 :

$E_n$  and  $R_B$  values, 7-sided polygon, mean of normal and inverted position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,36$

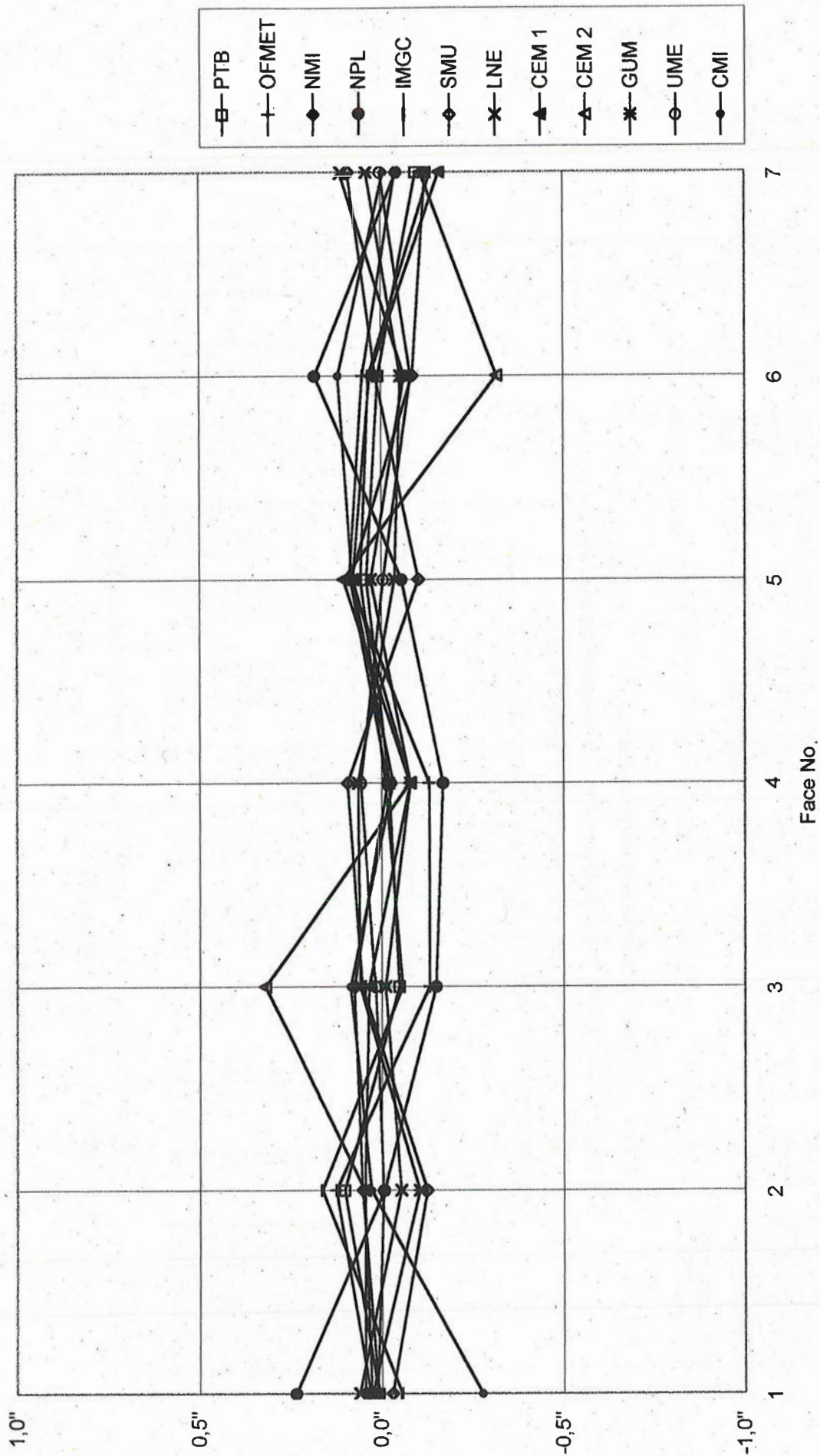


Figure 3.3 : 7-sided polygon, mean of normal and inverted position.  
Differences of the reduced angle deviations from the weighted mean of the faces. (Table 3.3)

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13
Face No. i													
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	0,52	0,55	-0,63	0,13	-	0,60	0,55	0,56	-	-	0,61	-	-
3	-0,12	-0,10	-1,26	0,11	-0,18	-0,17	-0,13	-0,13	-0,98	-1,00	-0,07	-0,09	-0,20
4	0,26	0,32	-0,83	0,00	-	0,55	0,29	0,30	-	-	0,75	-	-
5	0,34	0,38	-1,01	0,40	0,30	0,33	0,36	0,34	-0,85	-0,88	0,47	0,39	0,50
6	0,13	0,17	-0,69	0,31	-	0,05	0,17	0,11	-	-	0,13	-	-
7	-0,10	-0,07	-0,60	-0,26	-0,26	-0,32	-0,04	-0,17	-0,40	-0,53	-0,48	-0,12	0,20
8	-0,34	-0,33	-1,19	0,44	-	-0,54	-0,34	-0,37	-	-	-0,39	-	-
9	0,12	0,18	-0,41	-0,26	0,08	0,12	0,17	0,11	-0,23	-0,35	-0,06	0,15	0,30
10	-0,05	-0,02	-0,97	0,10	-	0,05	-0,07	-0,05	-	-	0,24	-	-
11	0,39	0,40	-0,88	0,84	0,25	0,28	0,38	0,38	-0,64	-0,60	0,47	0,37	0,40
12	1,41	1,43	0,70	1,03	-	1,72	1,41	1,44	-	-	1,51	-	-
13	0,22	0,22	-0,04	0,34	0,09	0,06	0,23	0,19	0,23	0,16	-0,13	0,20	-0,10
14	0,42	0,44	-0,58	0,41	-	0,44	0,44	0,43	-	-	0,44	-	-
15	1,09	1,14	0,31	0,73	1,04	1,15	1,13	1,10	0,50	0,39	1,03	1,17	0,60
16	0,47	0,49	0,05	0,31	-	0,73	0,48	0,48	-	-	0,59	-	-
17	0,23	0,26	-1,07	0,33	0,12	0,19	0,27	0,22	-0,99	-0,99	0,35	0,26	-0,40
18	0,40	0,42	-0,54	0,59	-	0,53	0,35	0,40	-	-	0,56	-	-
19	0,38	0,42	-0,61	0,61	0,30	0,40	0,40	0,38	-0,52	-0,56	0,67	0,43	-0,30
20	0,76	0,80	-0,37	0,42	-	0,87	0,78	0,76	-	-	0,85	-	-
21	0,53	-0,53	-0,26	0,86	0,45	0,47	0,46	0,48	-0,04	-0,15	0,41	0,50	0,00
22	0,08	0,07	-0,41	-0,04	-	0,17	0,05	0,03	-	-	0,02	-	-
23	-0,57	-0,57	-1,50	-0,31	-0,75	-0,79	-0,56	-0,63	-1,27	-1,25	-0,72	-0,61	-1,00
24	-0,48	-0,51	-1,33	-0,02	-	-0,70	-0,53	-0,53	-	-	-0,61	-	-
U <sub>j reported</sub>	0,03	0,04	0,22	0,45	0,09	0,03	0,04	0,02	0,15	0,25	0,25	0,06	0,40

Table 4.1 : Cumulative angle deviations of the 24-sided polygon related to face No. 1, **normal** position.

Last row : Combined standard uncertainties as reported by the laboratories.



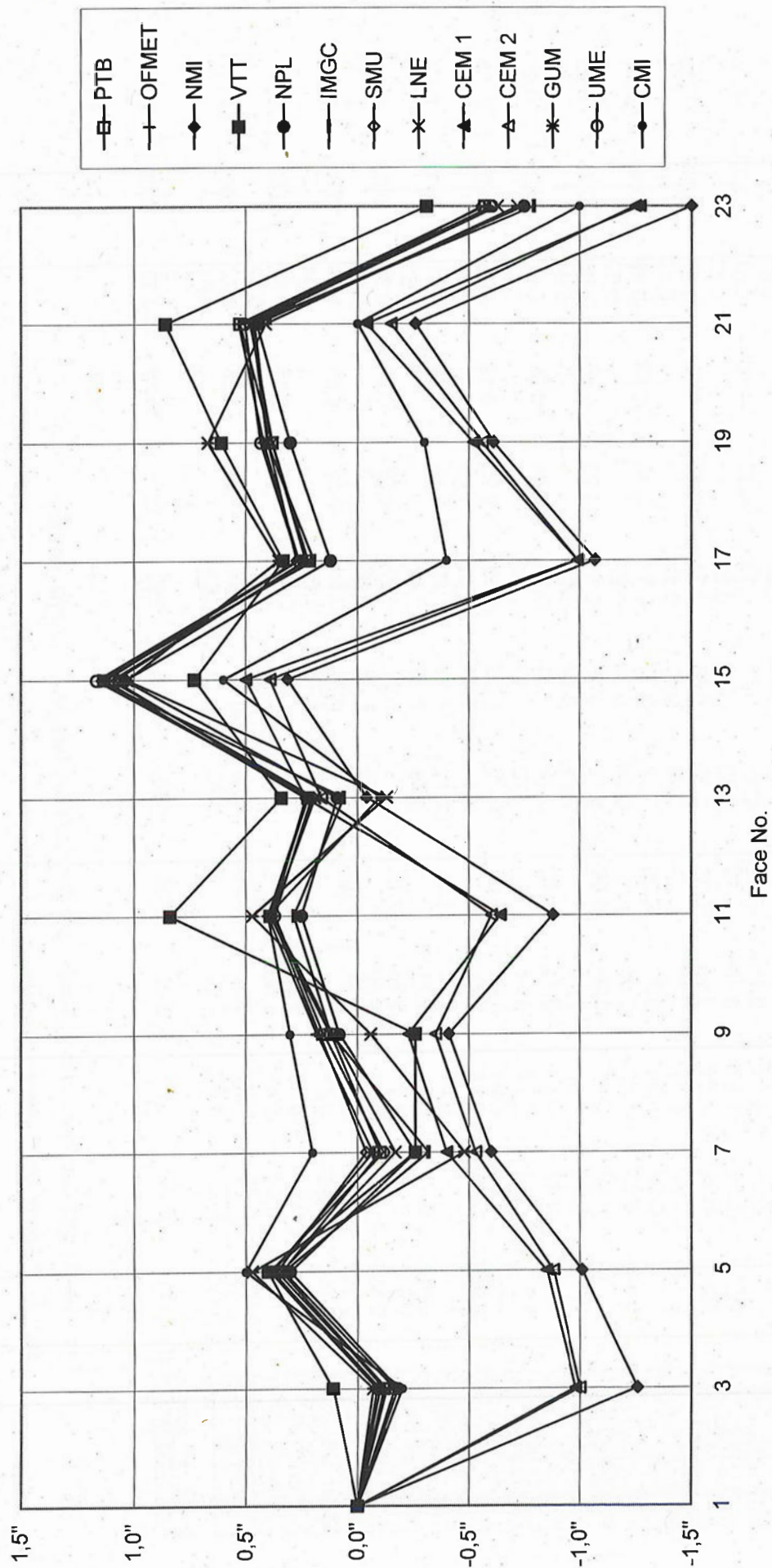


Figure 4.1: Cumulative angle deviations of the 24-sided polygon related to face No. 1, measured on faces No. 1, 3, 5, ..., 23 in normal position (Table 4.1).



Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	ref. : w. m. i
Result No. i	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i														
1	-0,21	-0,23	0,61	-0,28	-0,12	-0,14	-0,22	-0,19	0,43	0,48	-0,16	-0,22	0,00	-0,19
3	-0,33	-0,33	-0,65	-0,17	-0,30	-0,31	-0,35	-0,32	-0,55	-0,52	-0,23	-0,31	-0,20	-0,33
5	0,13	0,15	-0,40	0,12	0,18	0,18	0,14	0,15	-0,42	-0,40	0,31	0,17	0,50	0,14
7	-0,31	-0,30	0,01	-0,54	-0,38	-0,46	-0,26	-0,36	0,03	-0,05	-0,64	-0,34	0,20	-0,33
9	-0,09	-0,05	0,20	-0,54	-0,04	-0,03	-0,05	-0,08	0,20	0,13	-0,22	-0,07	0,30	-0,07
11	0,18	0,17	-0,27	0,56	0,13	0,14	0,16	0,19	-0,21	-0,12	0,31	0,15	0,40	0,17
13	0,01	-0,01	0,57	0,06	-0,03	-0,08	0,01	0,00	0,66	0,64	-0,29	-0,02	-0,10	0,01
15	0,88	0,91	0,92	0,45	0,92	1,01	0,91	0,91	0,93	0,87	0,87	0,94	0,60	0,91
17	0,02	0,03	-0,46	0,05	0,00	0,05	0,05	0,03	-0,56	-0,51	0,19	0,04	-0,40	0,02
19	0,17	0,19	0,00	0,33	0,18	0,26	0,18	0,19	-0,09	-0,08	0,51	0,21	-0,30	0,18
21	0,32	0,30	0,35	0,58	0,33	0,33	0,24	0,29	0,39	0,33	0,25	0,28	0,00	0,29
23	-0,78	-0,80	-0,89	-0,59	-0,87	-0,94	-0,78	-0,82	-0,84	-0,77	-0,88	-0,83	-1,00	-0,81
U <sub>j</sub> enlarged	0,05	0,05	0,24	0,45	0,13	0,13	0,04	0,02	0,16	0,27	0,26	0,07	0,43	U <sub>ref</sub> 0,02

Table 4.2.1 : Reduced angle deviations of the 24-sided polygon, measured on 12 faces in **normal** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference values  $u_{ref}$ .

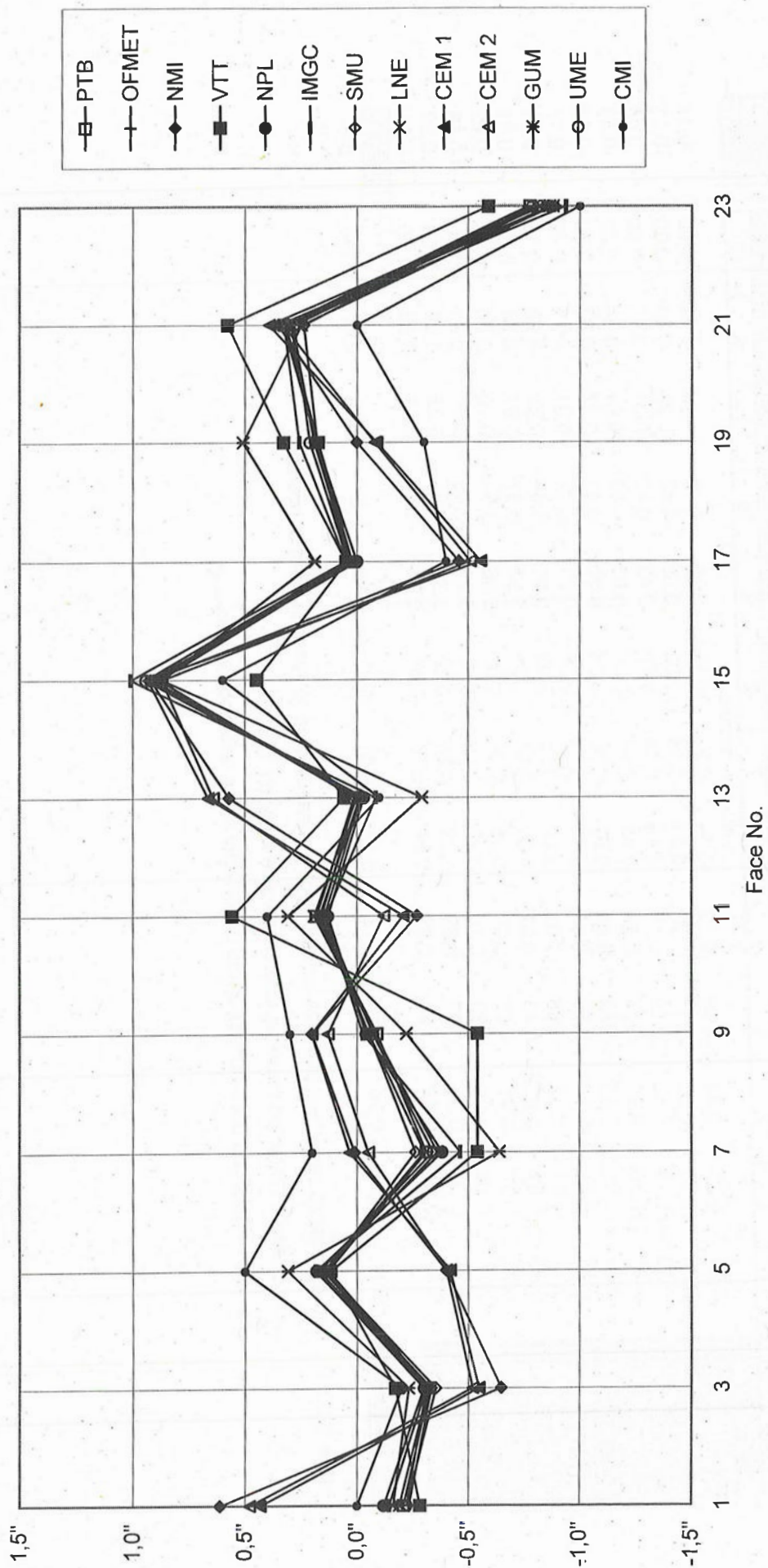


Figure 4.2.1 : Reduced angle deviations of the 24-sided polygon, measured on faces No.1,3,5,...,23 in **normal** position (Table 4.2.1).

Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. i	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i	values in arcsec													
1	-0,02	-0,04	0,80	-0,09	0,07	0,04	-0,03	0,00	0,62	0,67	0,03	-0,03	0,19	0,31
3	0,00	0,00	-0,32	0,16	0,03	0,02	-0,02	0,01	-0,22	-0,19	0,10	0,02	0,13	0,14
5	-0,01	0,01	-0,54	-0,02	0,04	0,05	0,00	0,01	-0,56	-0,54	0,17	0,03	0,36	0,29
7	0,02	0,03	0,34	-0,21	-0,05	-0,14	0,07	-0,03	0,36	0,28	-0,31	-0,01	0,53	0,24
9	-0,02	0,01	0,27	-0,48	0,03	0,04	0,01	-0,01	0,27	0,20	-0,15	0,00	0,37	0,21
11	0,01	0,00	-0,44	0,39	-0,04	-0,03	-0,01	0,02	-0,38	-0,29	0,14	-0,03	0,23	0,23
13	0,00	-0,02	0,56	0,05	-0,04	-0,09	0,00	-0,01	0,65	0,63	-0,30	-0,03	-0,11	0,31
15	-0,03	0,00	0,01	-0,46	0,01	0,10	0,00	0,00	0,02	-0,04	-0,04	0,03	-0,31	0,15
17	0,00	0,01	-0,48	0,03	-0,02	0,03	0,03	0,01	-0,58	-0,53	0,17	0,02	-0,42	0,26
19	-0,01	0,00	-0,18	0,14	0,00	0,07	-0,01	0,01	-0,27	-0,26	0,32	0,03	-0,48	0,21
21	0,03	0,01	0,06	0,29	0,04	0,04	-0,05	0,00	0,10	0,04	-0,04	-0,01	-0,29	0,12
23	0,03	0,01	-0,08	0,22	-0,06	-0,13	0,03	-0,01	-0,03	0,04	-0,07	-0,02	-0,19	0,10
$s_j$	0,02	0,02	0,43	0,27	0,04	0,08	0,03	0,01	0,42	0,40	0,19	0,02	0,34	

Table 4.2.2 : 24-sided polygon, measured on 12 faces in **normal** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No. i

Last row: Standard deviations  $s_j$  of the faces for each result No. j



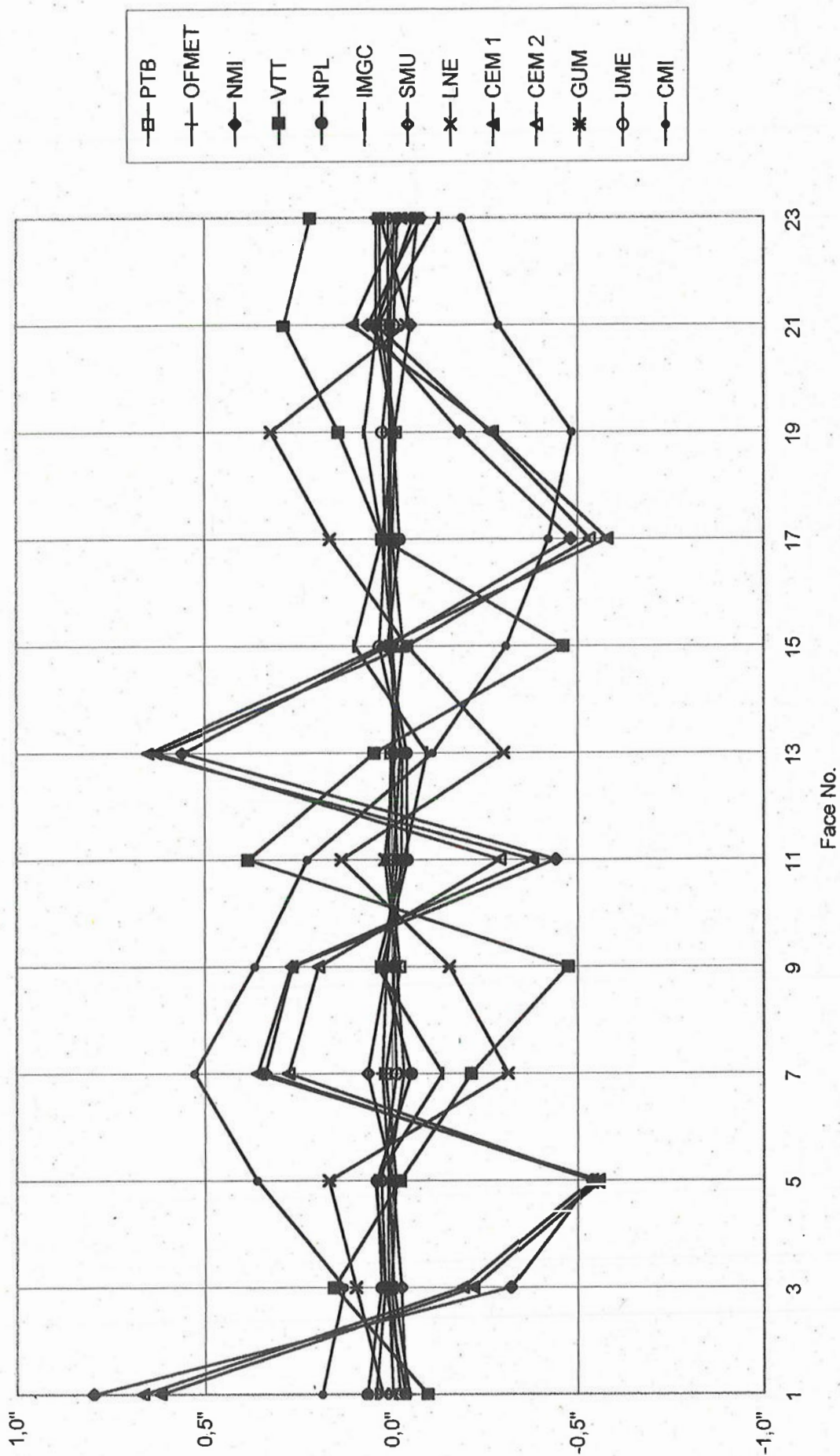


Figure 4.2.2 : 24-sided polygon, measured on 12 faces in **normal** position.  
Differences of the reduced angle deviations from the weighted mean of the faces.



Laboratory	PTB	OFMET	NMI	VTT	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	R <sub>B</sub>
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	13	
Face No. i														
1	-0,22	-0,45	1,68	-0,11	0,26	0,16	-0,46	-0,05	2,01	1,24	0,05	-0,27	0,22	1,73
3	0,03	-0,03	-0,67	0,17	0,11	0,07	-0,31	0,34	-0,71	-0,36	0,19	0,15	0,15	0,67
5	-0,11	0,09	-1,13	-0,02	0,16	0,18	-0,01	0,41	-1,80	-1,00	0,33	0,22	0,42	1,41
7	0,19	0,26	0,71	-0,24	-0,20	-0,51	0,87	-1,04	1,17	0,52	-0,62	-0,09	0,61	1,22
9	-0,23	0,15	0,56	-0,53	0,11	0,16	0,20	-0,40	0,87	0,37	-0,30	-0,02	0,43	0,81
11	0,10	-0,05	-0,93	0,43	-0,16	-0,12	-0,19	0,62	-1,23	-0,54	0,27	-0,19	0,26	1,05
13	0,02	-0,22	1,18	0,05	-0,15	-0,36	-0,03	-0,29	2,12	1,17	-0,59	-0,21	-0,13	1,62
15	-0,32	-0,02	0,02	-0,51	0,04	0,38	-0,02	0,06	0,08	-0,07	-0,08	0,26	-0,36	0,49
17	-0,01	0,06	-1,01	0,03	-0,08	0,10	0,34	0,30	-1,88	-0,99	0,33	0,13	-0,49	1,42
19	-0,12	0,03	-0,39	0,16	-0,02	0,28	-0,09	0,21	-0,88	-0,49	0,64	0,20	-0,56	0,83
21	0,32	0,10	0,13	0,32	0,16	0,14	-0,67	0,10	0,34	0,08	-0,08	-0,05	-0,33	0,54
23	0,34	0,09	-0,16	0,24	-0,23	-0,48	0,38	-0,26	-0,08	0,08	-0,14	-0,12	-0,22	0,49

Table 4.2.3 : E<sub>n</sub> and R<sub>B</sub> values, 24-sided polygon, measured on 12 faces in **normal** position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,35$

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12
Face No. i												
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	0,56	0,57	-0,72	-	-	0,56	-	-	-	0,61	-	-
3	-0,13	-0,12	-1,43	-0,10	-0,10	-0,12	-0,13	-0,99	-1,14	0,02	-0,10	-0,10
4	0,33	0,35	-0,94	-	-	0,30	-	-	-	0,75	-	-
5	0,34	0,36	-1,04	0,38	0,38	0,36	0,33	-0,75	-0,88	0,48	0,37	0,60
6	0,13	0,13	-0,80	-	-	0,18	-	-	-	0,13	-	-
7	-0,03	-0,16	-0,84	0,04	0,07	-0,04	-0,16	-0,27	-0,43	-0,55	-0,12	0,50
8	-0,35	-0,37	-1,24	-	-	-0,36	-	-	-	-0,36	-	-
9	0,20	0,12	-0,61	0,15	0,15	0,17	0,10	-0,12	-0,13	-0,15	0,12	0,60
10	0,00	-0,02	-1,15	-	-	-0,06	-	-	-	0,23	-	-
11	0,40	0,37	-0,80	0,49	0,50	0,38	0,35	-0,52	-0,60	0,45	0,37	0,90
12	1,57	1,51	0,70	-	-	1,42	-	-	-	1,52	-	-
13	0,26	0,17	-0,13	0,29	0,32	0,24	0,18	0,31	0,26	-0,14	0,22	0,30
14	0,48	0,43	-0,65	-	-	0,44	-	-	-	0,42	-	-
15	1,13	1,11	0,29	1,08	1,14	1,14	1,09	0,60	0,57	0,98	1,12	1,00
16	0,55	0,52	-0,01	-	-	0,48	-	-	-	0,55	-	-
17	0,20	0,22	-1,13	0,30	0,34	0,26	0,22	-0,90	-0,92	0,33	0,21	0,00
18	0,44	0,47	-0,60	0,39	0,49	0,34	0,37	-0,43	-0,56	0,53	0,37	0,20
19	0,37	0,40	-0,61	-	-	0,38	0,37	-	-	0,66	0,37	-
20	0,81	0,79	-0,39	-	-	0,77	-	-	-	0,77	-	-
21	0,50	0,52	-0,34	0,57	0,55	0,46	0,48	0,02	-0,04	0,44	0,49	0,40
22	0,10	0,05	-0,40	-	-	0,04	-	-	-	0,01	-	-
23	-0,59	-0,63	-1,58	-0,45	-0,45	-0,57	-0,62	-1,19	-1,31	-0,76	-0,57	-0,70
24	-0,54	-0,54	-1,21	-	-	-0,52	-	-	-	-0,58	-	-
U <sub>j</sub> reported	0,03	0,04	0,22	0,09	0,03	0,04	0,02	0,15	0,30	0,25	0,06	0,40

Table 5 1 : Cumulative angle deviations of the 24-sided polygon related to face No. 1, **inverted** position.

Last row : Combined standard uncertainties as reported by the laboratories.

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	ref. : w. m. i
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	-0,22	-0,20	0,69	-0,26	-0,28	-0,22	-0,18	0,35	0,43	-0,15	-0,21	-0,31	-0,19
3	-0,35	-0,32	-0,75	-0,36	-0,38	-0,34	-0,31	-0,64	-0,71	-0,13	-0,31	-0,41	-0,33
5	0,12	0,16	-0,36	0,12	0,10	0,14	0,15	-0,40	-0,45	0,33	0,17	0,29	0,13
7	-0,25	-0,36	-0,16	-0,22	-0,21	-0,26	-0,34	0,08	0,00	-0,70	-0,32	0,19	-0,31
9	-0,02	-0,08	0,08	-0,11	-0,13	-0,05	-0,08	0,23	0,30	-0,30	-0,09	0,29	-0,07
11	0,18	0,17	-0,12	0,23	0,22	0,16	0,17	-0,17	-0,17	0,30	0,16	0,59	0,16
13	0,04	-0,03	0,56	0,03	0,04	0,02	0,00	0,66	0,69	-0,29	0,01	-0,01	0,01
15	0,91	0,91	0,98	0,82	0,86	0,92	0,91	0,95	1,00	0,83	0,91	0,69	0,91
17	-0,02	0,02	-0,45	0,04	0,05	0,04	0,04	-0,55	-0,49	0,18	0,00	-0,31	0,02
19	0,15	0,20	0,08	0,13	0,21	0,16	0,19	-0,08	-0,13	0,51	0,17	-0,11	0,17
21	0,28	0,32	0,35	0,31	0,26	0,24	0,30	0,37	0,39	0,29	0,28	0,09	0,29
23	-0,81	-0,83	-0,90	-0,71	-0,73	-0,79	-0,80	-0,84	-0,88	-0,91	-0,77	-1,01	-0,80
$U_{j \text{ enlarged}}$	0,05	0,05	0,24	0,13	0,13	0,04	0,02	0,16	0,32	0,26	0,07	0,43	$U_{\text{ref}}$ 0,02

Table 5.2.1 : Reduced angle deviations of the 24-sided polygon, measured on 12 faces in **inverted** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference values  $u_{\text{ref}}$ .



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	-0,03	-0,01	0,87	-0,07	-0,09	-0,03	0,00	0,54	0,62	0,04	-0,02	-0,12	0,33
3	-0,02	0,01	-0,42	-0,03	-0,05	-0,01	0,02	-0,31	-0,38	0,20	0,02	-0,08	0,18
5	-0,02	0,03	-0,49	-0,02	-0,04	0,00	0,01	-0,53	-0,58	0,20	0,03	0,16	0,27
7	0,06	-0,04	0,16	0,09	0,10	0,05	-0,03	0,40	0,31	-0,38	-0,01	0,50	0,23
9	0,05	-0,01	0,14	-0,04	-0,06	0,02	-0,02	0,30	0,37	-0,23	-0,02	0,36	0,19
11	0,02	0,01	-0,28	0,06	0,06	-0,01	0,00	-0,33	-0,33	0,14	0,00	0,43	0,21
13	0,03	-0,04	0,54	0,01	0,02	0,00	-0,02	0,65	0,68	-0,30	0,00	-0,02	0,31
15	0,00	0,01	0,07	-0,09	-0,05	0,01	0,00	0,05	0,09	-0,07	0,01	-0,22	0,08
17	-0,04	0,01	-0,46	0,02	0,04	0,02	0,02	-0,56	-0,51	0,17	-0,02	-0,33	0,25
19	-0,03	0,03	-0,10	-0,05	0,03	-0,02	0,01	-0,25	-0,30	0,34	-0,01	-0,28	0,17
21	-0,01	0,04	0,06	0,02	-0,02	-0,05	0,01	0,09	0,10	0,01	0,00	-0,20	0,08
23	-0,01	-0,02	-0,09	0,09	0,07	0,01	0,00	-0,03	-0,08	-0,10	0,03	-0,21	0,08
$s_j$	0,03	0,03	0,41	0,06	0,06	0,03	0,02	0,41	0,43	0,22	0,02	0,29	

Table 5.2.2 : 24-sided polygon, measured on 12 faces in **inverted** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No. i

Last row: Standard deviations  $s_j$  of the faces for each result No. j



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	R <sub>B</sub>
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i													
1	-0,35	-0,09	1,84	-0,29	-0,36	-0,45	0,11	1,75	0,97	0,08	-0,14	-0,14	1,62
3	-0,22	0,13	-0,87	-0,12	-0,21	-0,16	0,52	-1,00	-0,59	0,40	0,15	-0,09	0,92
5	-0,17	0,29	-1,03	-0,06	-0,14	0,04	0,36	-1,72	-0,91	0,39	0,24	0,18	1,32
7	0,65	-0,45	0,33	0,35	0,38	0,67	-1,06	1,28	0,49	-0,76	-0,09	0,59	1,24
9	0,50	-0,08	0,30	-0,17	-0,24	0,22	-0,52	0,98	0,58	-0,45	-0,16	0,42	0,87
11	0,16	0,10	-0,59	0,25	0,22	-0,07	0,07	-1,07	-0,52	0,27	-0,02	0,50	0,86
13	0,27	-0,41	1,14	0,06	0,09	0,06	-0,61	2,11	1,06	-0,59	-0,02	-0,03	1,59
15	0,02	0,06	0,14	-0,35	-0,19	0,14	-0,06	0,15	0,15	-0,15	0,04	-0,25	0,33
17	-0,41	0,06	-0,97	0,08	0,14	0,27	0,60	-1,83	-0,79	0,33	-0,13	-0,38	1,36
19	-0,27	0,29	-0,21	-0,18	0,12	-0,21	0,38	-0,81	-0,47	0,67	-0,07	-0,33	0,77
21	-0,09	0,36	0,12	0,08	-0,09	-0,66	0,26	0,28	0,16	0,01	-0,02	-0,23	0,48
23	-0,09	-0,25	-0,19	0,35	0,28	0,14	-0,06	-0,11	-0,12	-0,20	0,22	-0,24	0,40

Table 5.2.3 : E<sub>n</sub> and R<sub>B</sub> values, 24-sided polygon, measured on 12 faces in **inverted** position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,36$

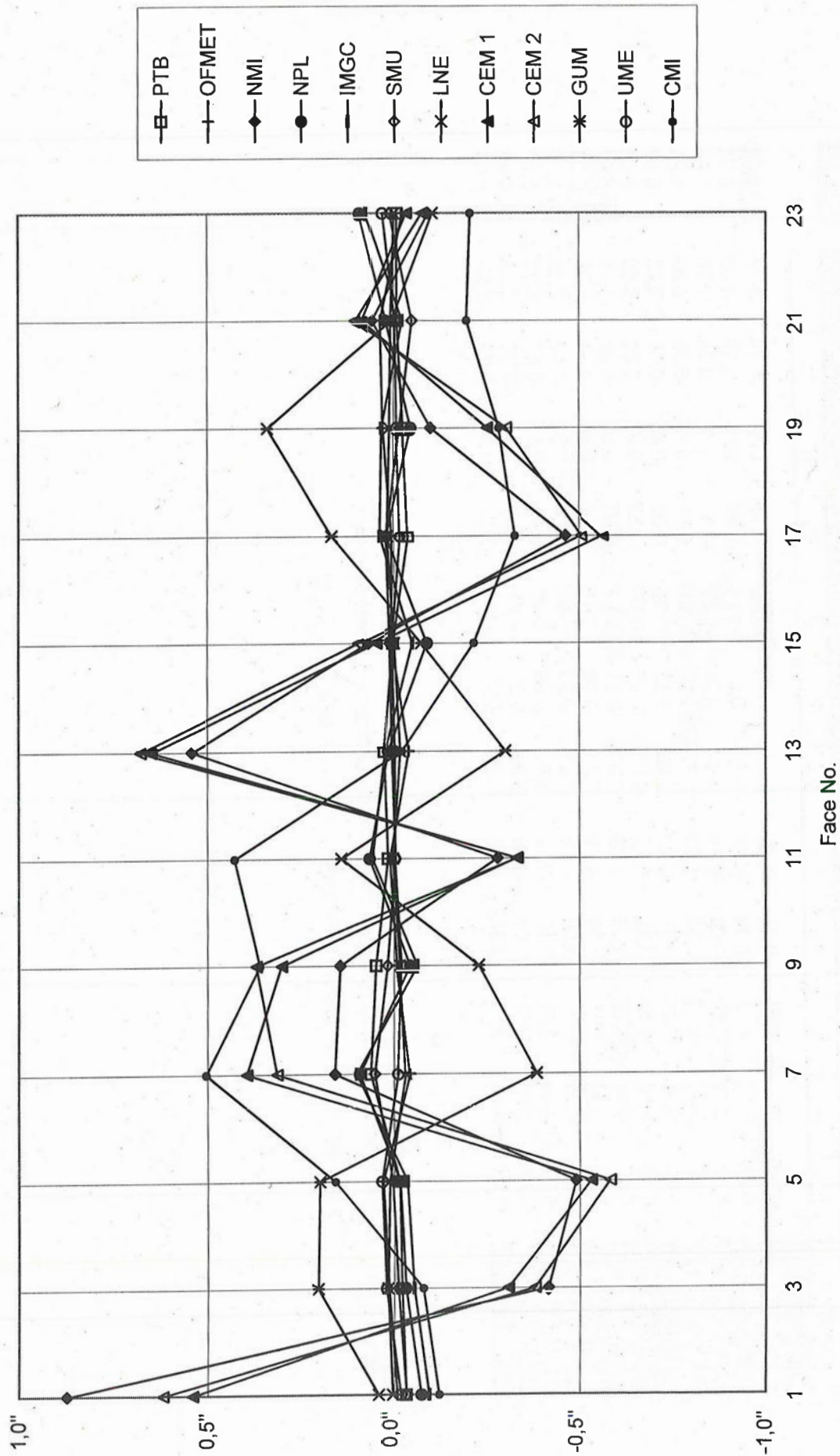


Figure 5.2.2 : 24-sided polygon, measured on 12 faces in **inverted** position.  
 Differences of the reduced angle deviations from the weighted mean of the faces.

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12
Face No. i	values in arcsec											
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
2	0,54	0,56	-0,68	-	-	0,56	-	-	-	0,61	-	-
3	-0,12	-0,11	-1,35	-0,14	-0,13	-0,13	-0,13	-0,99	-1,07	-0,03	-0,10	-0,15
4	0,30	0,34	-0,89	-	-	0,30	-	-	-	0,75	-	-
5	0,34	0,37	-1,03	0,34	0,36	0,36	0,34	-0,80	-0,88	0,48	0,38	0,55
6	0,13	0,15	-0,75	-	-	0,18	-	-	-	0,13	-	-
7	-0,07	-0,12	-0,72	-0,11	-0,13	-0,04	-0,17	-0,34	-0,48	-0,52	-0,12	0,35
8	-0,34	-0,35	-1,22	-	-	-0,35	-	-	-	-0,38	-	-
9	0,16	0,15	-0,51	0,12	0,13	0,17	0,11	-0,18	-0,24	-0,11	0,13	0,45
10	-0,02	-0,02	-1,06	-	-	-0,07	-	-	-	0,24	-	-
11	0,40	0,39	-0,84	0,37	0,39	0,38	0,37	-0,58	-0,60	0,46	0,37	0,65
12	1,49	1,47	0,70	-	-	1,42	-	-	-	1,52	-	-
13	0,24	0,20	-0,09	0,19	0,19	0,24	0,19	0,27	0,21	-0,14	0,21	0,10
14	0,45	0,44	-0,62	-	-	0,44	-	-	-	0,43	-	-
15	1,11	1,13	0,30	1,06	1,15	1,14	1,10	0,55	0,48	1,01	1,14	0,80
16	0,51	0,51	0,02	-	-	0,48	-	-	-	0,57	-	-
17	0,21	0,24	-1,10	0,21	0,27	0,27	0,22	-0,95	-0,96	0,34	0,23	-0,20
18	0,42	0,45	-0,57	-	-	0,35	-	-	-	0,55	-	-
19	0,38	0,41	-0,61	0,35	0,45	0,39	0,38	-0,48	-0,56	0,67	0,40	-0,05
20	0,78	0,80	-0,38	-	-	0,78	-	-	-	0,81	-	-
21	0,51	0,53	-0,30	0,51	0,51	0,46	0,48	-0,01	-0,10	0,43	0,50	0,20
22	0,09	0,06	-0,41	-	-	0,05	-	-	-	0,02	-	-
23	-0,58	-0,60	-1,54	-0,60	-0,62	-0,57	-0,63	-1,23	-1,28	-0,74	-0,59	-0,85
24	-0,51	-0,53	-1,27	-	-	-0,53	-	-	-	-0,60	-	-
U <sub>j reported</sub>	0,03	0,04	0,22	0,09	0,03	0,04	0,02	0,15	0,28	0,25	0,06	0,40

Table 6.1 : Cumulative angle deviations of the 24-sided polygon related to face No. 1, mean of normal and inverted position.  
Last row : Combined standard uncertainties as reported by the laboratories .



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	ref. : w. m. i
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	-0,21	-0,21	0,65	-0,19	-0,21	-0,22	-0,19	0,39	0,46	-0,15	-0,21	-0,15	-0,19
3	-0,34	-0,32	-0,70	-0,33	-0,35	-0,35	-0,32	-0,59	-0,61	-0,18	-0,31	-0,30	-0,33
5	0,12	0,16	-0,38	0,15	0,14	0,14	0,15	-0,41	-0,42	0,32	0,17	0,40	0,14
7	-0,28	-0,33	-0,07	-0,30	-0,34	-0,26	-0,35	0,06	-0,02	-0,67	-0,33	0,20	-0,32
9	-0,05	-0,06	0,14	-0,08	-0,08	-0,05	-0,08	0,22	0,22	-0,26	-0,08	0,30	-0,07
11	0,18	0,17	-0,19	0,18	0,18	0,16	0,18	-0,19	-0,14	0,31	0,15	0,50	0,17
13	0,03	-0,02	0,56	0,00	-0,02	0,01	0,00	0,66	0,67	-0,29	0,00	-0,05	0,01
15	0,89	0,91	0,95	0,87	0,93	0,91	0,91	0,94	0,94	0,85	0,93	0,65	0,91
17	0,00	0,03	-0,45	0,02	0,05	0,04	0,03	-0,55	-0,50	0,19	0,02	-0,35	0,02
19	0,16	0,20	0,04	0,15	0,23	0,17	0,19	-0,08	-0,10	0,51	0,19	-0,20	0,18
21	0,30	0,31	0,35	0,32	0,30	0,24	0,29	0,38	0,36	0,27	0,28	0,05	0,29
23	-0,79	-0,81	-0,89	-0,79	-0,83	-0,79	-0,81	-0,84	-0,82	-0,89	-0,80	-1,00	-0,81
U <sub>j enlarged</sub>	0,05	0,05	0,24	0,13	0,13	0,04	0,02	0,16	0,32	0,26	0,07	0,43	U <sub>ref</sub> 0,02

Table 6.2.1 : Reduced angle deviations of the 24-sided polygon, measured on 12 faces, **mean of normal and inverted** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference values  $u_{ref}$ .

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i	values in arcsec												
1	-0,03	-0,03	0,84	0,00	-0,03	-0,03	0,00	0,58	0,64	0,03	-0,03	0,03	0,32
3	-0,01	0,00	-0,37	0,00	-0,02	-0,02	0,01	-0,26	-0,28	0,15	0,02	0,03	0,15
5	-0,01	0,02	-0,51	0,01	0,00	0,00	0,01	-0,54	-0,56	0,18	0,03	0,26	0,28
7	0,04	-0,01	0,25	0,02	-0,02	0,06	-0,03	0,38	0,30	-0,35	-0,01	0,52	0,23
9	0,01	0,00	0,21	-0,01	-0,01	0,02	-0,01	0,29	0,28	-0,19	-0,01	0,36	0,17
11	0,01	0,00	-0,36	0,01	0,01	-0,01	0,01	-0,35	-0,31	0,14	-0,01	0,33	0,20
13	0,01	-0,03	0,55	-0,01	-0,04	0,00	-0,01	0,65	0,65	-0,30	-0,02	-0,07	0,31
15	-0,01	0,00	0,04	-0,04	0,02	0,00	0,00	0,03	0,03	-0,06	0,02	-0,26	0,08
17	-0,02	0,01	-0,47	0,00	0,03	0,02	0,01	-0,57	-0,52	0,17	0,00	-0,37	0,26
19	-0,02	0,02	-0,14	-0,03	0,05	-0,01	0,01	-0,26	-0,28	0,33	0,01	-0,38	0,19
21	0,01	0,02	0,06	0,03	0,01	-0,05	0,01	0,10	0,07	-0,02	0,00	-0,24	0,09
23	0,01	-0,01	-0,09	0,02	-0,03	0,02	0,00	-0,03	-0,02	-0,09	0,01	-0,20	0,06
$s_j$	0,02	0,02	0,41	0,02	0,03	0,03	0,01	0,41	0,41	0,21	0,02	0,31	

Table 6.2.2 : 24-sided polygon, measured on 12 faces, **mean of normal and inverted** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviation  $s_i$  of all results for each face No. i

Last row: Standard deviations  $s_j$  from all faces for each result No. j

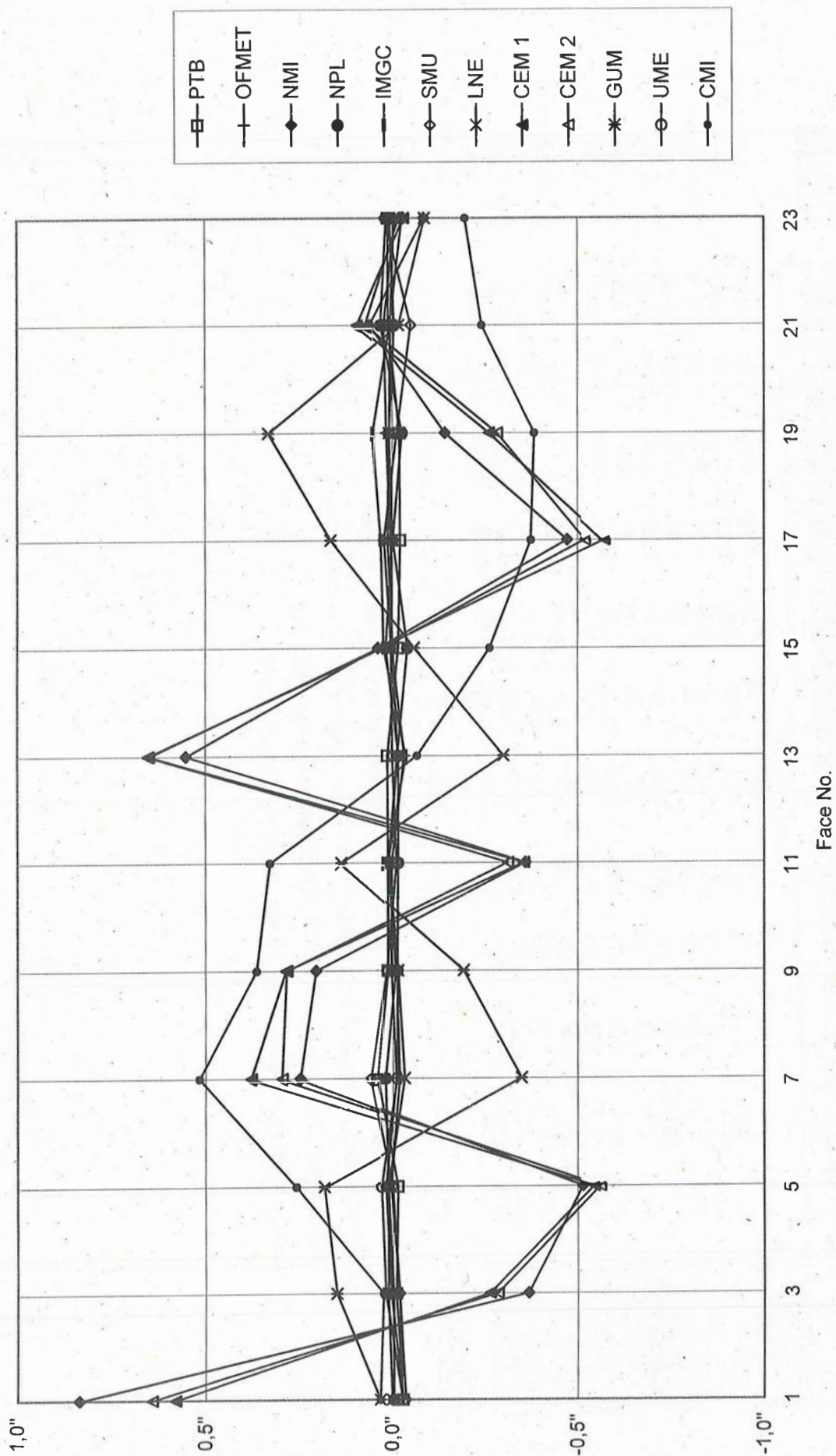


Figure 6.2.2 : 24-sided polygon, measured on 12 faces, **mean of normal and inverted position**.  
Differences of the reduced angle deviations from the weighted mean of the faces.



Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	$R_B$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12	
Face No. i													
1	-0,28	-0,27	1,76	-0,01	-0,10	-0,45	0,04	1,88	1,01	0,07	-0,20	0,04	1,63
3	-0,09	0,05	-0,77	0,00	-0,07	-0,23	0,43	-0,85	-0,45	0,30	0,15	0,03	0,77
5	-0,14	0,19	-1,08	0,05	0,02	0,01	0,38	-1,76	-0,88	0,36	0,23	0,30	1,34
7	0,42	-0,09	0,52	0,08	-0,07	0,77	-1,05	1,23	0,46	-0,69	-0,09	0,60	1,15
9	0,13	0,03	0,43	-0,03	-0,04	0,21	-0,47	0,93	0,44	-0,38	-0,09	0,42	0,76
11	0,13	0,03	-0,76	0,04	0,05	-0,13	0,35	-1,15	-0,49	0,27	-0,11	0,38	0,90
13	0,15	-0,31	1,16	-0,05	-0,13	0,02	-0,44	2,11	1,02	-0,59	-0,12	-0,08	1,57
15	-0,15	0,02	0,08	-0,15	0,09	0,06	-0,01	0,11	0,04	-0,11	0,15	-0,31	0,26
17	-0,21	0,05	-0,99	0,00	0,12	0,30	0,44	-1,86	-0,81	0,33	0,00	-0,43	1,36
19	-0,20	0,16	-0,30	-0,10	0,20	-0,15	0,29	-0,85	-0,44	0,65	0,07	-0,45	0,77
21	0,11	0,23	0,13	0,12	0,03	-0,66	0,19	0,31	0,11	-0,03	-0,04	-0,28	0,47
23	0,13	-0,08	-0,18	0,06	-0,10	0,26	-0,15	-0,10	-0,03	-0,17	0,05	-0,23	0,28

Table 6.2.3:  $E_n$  and  $R_B$  values, 24-sided polygon, measured on 12 faces, mean of normal and inverted position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,36$

Laboratory	PTB	OFMET	NMI	VTI	IMGC	SMU	LNE	GUM	ref.: w. m. i
Result No. j	1	2	3	4	5	6	7	8	
Face No. i	values in arcsec								
1	-0,25	-0,28	0,59	-0,29	-0,26	-0,26	-0,24	-0,28	-0,25
2	0,27	0,27	-0,04	-0,16	0,35	0,29	0,32	0,33	0,30
3	-0,37	-0,38	-0,67	-0,18	-0,42	-0,39	-0,37	-0,35	-0,38
4	0,01	0,04	-0,24	-0,29	0,29	0,03	0,06	0,47	0,05
5	0,08	0,10	-0,42	0,11	0,07	0,10	0,10	0,19	0,09
6	-0,12	-0,11	-0,10	0,02	-0,21	-0,09	-0,13	-0,15	-0,12
7	-0,36	-0,35	-0,01	-0,55	-0,58	-0,30	-0,41	-0,76	-0,38
8	-0,59	-0,61	-0,60	0,15	-0,80	-0,60	-0,61	-0,67	-0,61
9	-0,13	-0,10	0,18	-0,55	-0,14	-0,09	-0,13	-0,34	-0,12
10	-0,30	-0,30	-0,38	-0,19	-0,21	-0,33	-0,29	-0,04	-0,30
11	0,14	0,12	-0,29	0,55	0,03	0,12	0,14	0,19	0,13
12	1,15	1,15	1,29	0,74	1,46	1,15	1,20	1,23	1,18
13	-0,03	-0,06	0,55	0,05	-0,20	-0,03	-0,05	-0,41	-0,05
14	0,17	0,16	0,01	0,12	0,18	0,18	0,19	0,16	0,18
15	0,83	0,86	0,90	0,44	0,89	0,87	0,86	0,75	0,86
16	0,22	0,21	0,64	0,02	0,47	0,22	0,24	0,31	0,23
17	-0,02	-0,02	-0,48	0,04	-0,07	0,01	-0,02	0,07	-0,02
18	0,15	0,14	0,05	0,30	0,27	0,09	0,16	0,28	0,15
19	0,13	0,14	-0,02	0,32	0,14	0,14	0,14	0,39	0,14
20	0,50	0,52	0,22	0,13	0,61	0,52	0,52	0,57	0,52
21	0,27	0,25	0,33	0,57	0,21	0,20	0,24	0,13	0,24
22	-0,18	-0,21	0,18	-0,33	-0,09	-0,21	-0,21	-0,26	-0,20
23	-0,82	-0,85	-0,91	-0,60	-1,05	-0,82	-0,87	-1,00	-0,86
24	-0,73	-0,79	-0,74	-0,31	-0,96	-0,79	-0,77	-0,89	-0,78
U <sub>j</sub> enlarged	0,05	0,05	0,24	0,45	0,13	0,04	0,02	0,26	0,02
U <sub>ref</sub>									

Table 7.1 : Reduced angle deviations of the 24-sided polygon, measured on 24 faces in **normal** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{ref}$ .

Laboratory	PTB	OFMET	NMI	VTT	IMGC	SMU	LNE	GUM	$s_i$	$s_i^*$
Result No. j	1	2	3	4	5	6	7	8		
Face No. i	values in arcsec									
1	-0,01	-0,03	0,84	-0,05	-0,01	-0,01	0,00	-0,03	0,30	0,02
2	-0,03	-0,03	-0,34	-0,46	0,05	-0,01	0,02	0,03	0,19	0,18
3	0,01	0,00	-0,29	0,19	-0,05	-0,01	0,01	0,03	0,13	0,08
4	-0,04	0,00	-0,29	-0,34	0,24	-0,02	0,01	0,42	0,25	0,24
5	-0,01	0,01	-0,52	0,01	-0,02	0,01	0,00	0,10	0,19	0,04
6	0,00	0,02	0,02	0,14	-0,09	0,03	-0,01	-0,02	0,06	0,07
7	0,03	0,04	0,37	-0,17	-0,20	0,08	-0,03	-0,37	0,22	0,16
8	0,02	0,00	0,01	0,75	-0,19	0,01	0,00	-0,06	0,29	0,31
9	-0,01	0,03	0,30	-0,43	-0,02	0,03	-0,01	-0,21	0,21	0,17
10	0,00	0,00	-0,08	0,10	0,08	-0,03	0,01	0,26	0,11	0,10
11	0,01	-0,01	-0,42	0,42	-0,10	-0,01	0,01	0,06	0,23	0,17
12	-0,03	-0,03	0,10	-0,45	0,28	-0,03	0,01	0,05	0,20	0,21
13	0,01	-0,01	0,60	0,09	-0,15	0,02	0,00	-0,36	0,27	0,15
14	-0,01	-0,02	-0,17	-0,06	0,00	0,00	0,01	-0,02	0,06	0,02
15	-0,02	0,01	0,04	-0,42	0,04	0,01	0,00	-0,10	0,15	0,16
16	-0,02	-0,02	0,40	-0,22	0,24	-0,02	0,00	0,08	0,19	0,14
17	-0,01	0,00	-0,46	0,05	-0,05	0,03	0,00	0,09	0,17	0,05
18	0,00	0,00	-0,10	0,15	0,13	-0,06	0,01	0,14	0,09	0,08
19	-0,01	0,01	-0,16	0,18	0,01	0,00	0,00	0,26	0,13	0,11
20	-0,01	0,01	-0,30	-0,39	0,10	0,00	0,00	0,06	0,18	0,16
21	0,04	0,02	0,09	0,33	-0,03	-0,04	0,00	-0,10	0,13	0,14
22	0,03	0,00	0,38	-0,13	0,11	-0,01	-0,01	-0,05	0,15	0,07
23	0,03	0,01	-0,05	0,25	-0,19	0,04	-0,01	-0,14	0,13	0,14
24	0,04	-0,01	0,03	0,46	-0,18	-0,02	0,00	-0,11	0,19	0,21
$s_j$	0,02	0,02	0,34	0,32	0,14	0,03	0,01	0,18		

Table 7.2: 24-sided polygon, measured on 24 faces in **normal** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last two columns : Standard deviations  $s_i$  of the results for each face No. i

Standard deviations  $s_i^*$  of seven results, excluding NMI ( see Section 6.5, Figure 11 )

Last row : Standard deviations  $s_j$  of the faces for each result No. j



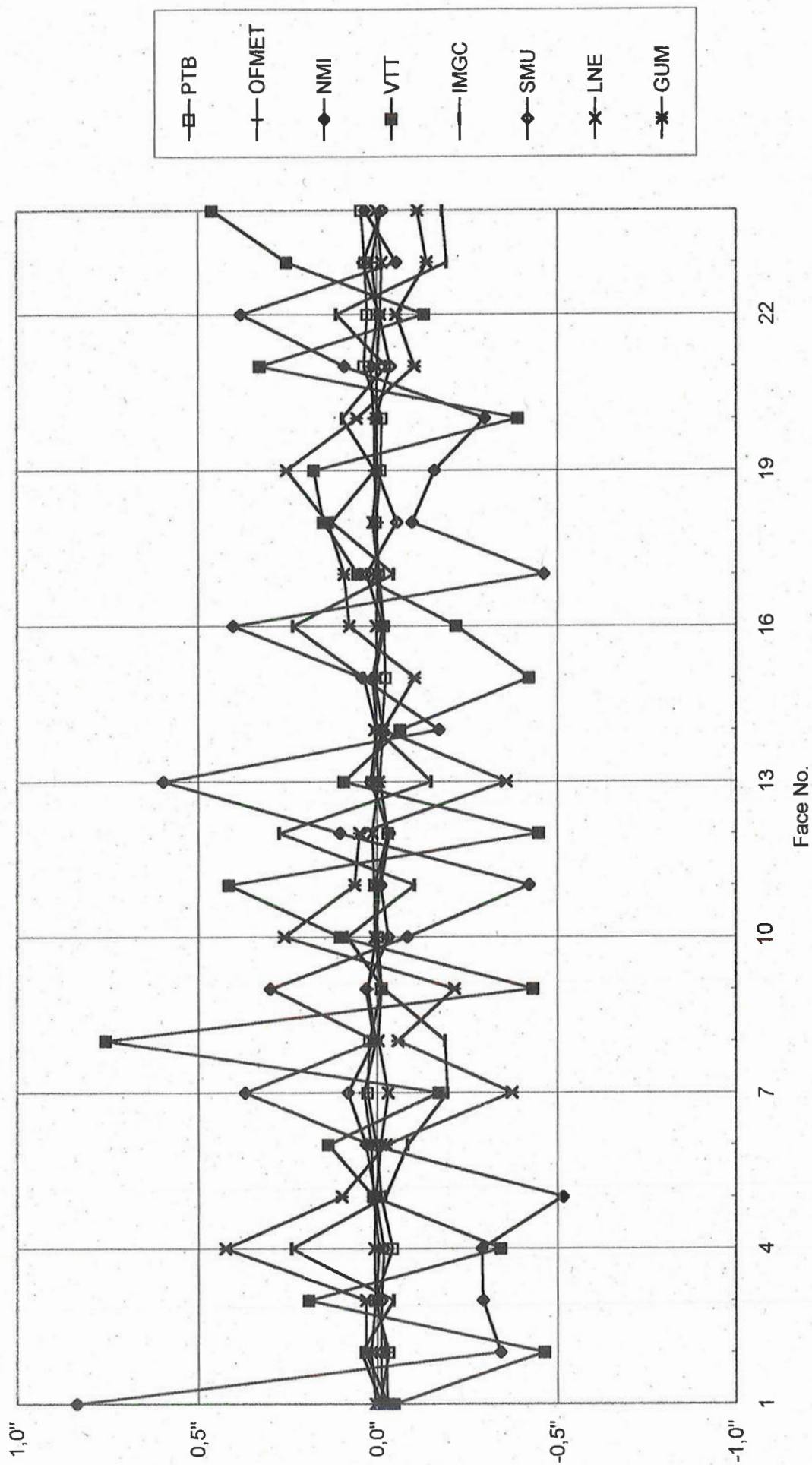


Figure 7.2 : 24-sided polygon, measured on 24 faces in **normal** position.  
Differences of the reduced angle deviations from the weighted mean of the faces.

Laboratory	PTB	OFMET	NMI	VTT	IMGC	SMU	LNE	GUM	$R_B$
Result No. j	1	2	3	4	5	6	7	8	
Face No. i									
1	-0,07	-0,31	1,76	-0,05	-0,04	-0,18	0,15	-0,06	1,35
2	-0,35	-0,28	-0,72	-0,52	0,17	-0,14	0,59	0,06	0,81
3	0,07	0,02	-0,62	0,21	-0,18	-0,17	0,18	0,06	0,54
4	-0,45	-0,05	-0,61	-0,38	0,92	-0,26	0,29	0,82	1,14
5	-0,12	0,11	-1,09	0,01	-0,09	0,07	0,11	0,19	0,85
6	0,02	0,17	0,04	0,15	-0,32	0,43	-0,38	-0,05	0,46
7	0,29	0,39	0,78	-0,19	-0,74	1,09	-1,07	-0,72	1,39
8	0,22	0,04	0,02	0,84	-0,72	0,12	-0,13	-0,11	0,86
9	-0,13	0,28	0,63	-0,48	-0,07	0,42	-0,39	-0,41	0,79
10	-0,02	0,03	-0,18	0,12	0,32	-0,43	0,19	0,50	0,57
11	0,07	-0,06	-0,89	0,46	-0,39	-0,13	0,26	0,12	0,83
12	-0,32	-0,31	0,22	-0,50	1,05	-0,45	0,48	0,10	1,02
13	0,15	-0,08	1,26	0,10	-0,57	0,23	-0,17	-0,69	1,19
14	-0,14	-0,17	-0,36	-0,07	0,00	0,00	0,26	-0,03	0,34
15	-0,25	0,07	0,09	-0,47	0,14	0,17	0,00	-0,20	0,46
16	-0,21	-0,22	0,85	-0,24	0,90	-0,20	0,08	0,15	0,99
17	-0,05	0,04	-0,97	0,06	-0,17	0,39	-0,13	0,18	0,81
18	0,03	-0,01	-0,20	0,17	0,49	-0,75	0,41	0,27	0,72
19	-0,11	0,07	-0,34	0,20	0,02	0,02	-0,03	0,49	0,48
20	-0,15	0,08	-0,63	-0,43	0,37	0,04	0,02	0,11	0,66
21	0,38	0,18	0,19	0,37	-0,10	-0,50	0,01	-0,20	0,58
22	0,29	-0,02	0,81	-0,15	0,43	-0,09	-0,31	-0,10	0,75
23	0,37	0,14	-0,11	0,28	-0,73	0,51	-0,50	-0,27	0,81
24	0,47	-0,11	0,07	0,51	-0,69	-0,20	0,08	-0,21	0,76

Table 7.3 :  $E_n$  and  $R_B$  values, 24-sided polygon, measured on 24 faces in **normal** position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,44$

Laboratory	PTB	OFMET	NMI	SMU	GUM	ref. : w. m. i.
Result No. j	1	2	3	4	5	
Face No. i	values in arcsec					
1	-0,28	-0,26	0,65	-0,26	-0,26	-0,25
2	0,28	0,31	-0,07	0,30	0,35	0,29
3	-0,41	-0,38	-0,78	-0,38	-0,24	-0,39
4	0,05	0,09	-0,29	0,04	0,49	0,06
5	0,06	0,10	-0,39	0,10	0,22	0,08
6	-0,15	-0,13	-0,15	-0,08	-0,13	-0,12
7	-0,31	-0,42	-0,19	-0,30	-0,81	-0,34
8	-0,63	-0,63	-0,59	-0,62	-0,62	-0,63
9	-0,08	-0,14	0,04	-0,09	-0,41	-0,10
10	-0,28	-0,28	-0,50	-0,32	-0,03	-0,30
11	0,12	0,11	-0,15	0,12	0,19	0,11
12	1,29	1,25	1,35	1,16	1,26	1,22
13	-0,02	-0,09	0,52	-0,02	-0,40	-0,04
14	0,20	0,17	0,00	0,18	0,16	0,18
15	0,85	0,85	0,94	0,88	0,72	0,86
16	0,27	0,26	0,64	0,22	0,29	0,25
17	-0,08	-0,04	-0,48	0,00	0,07	-0,04
18	0,16	0,21	0,05	0,08	0,27	0,14
19	0,09	0,14	0,04	0,12	0,40	0,12
20	0,53	0,53	0,26	0,51	0,51	0,52
21	0,22	0,26	0,31	0,20	0,18	0,22
22	-0,18	-0,21	0,25	-0,22	-0,25	-0,20
23	-0,87	-0,89	-0,93	-0,83	-1,02	-0,86
24	-0,82	-0,80	-0,56	-0,78	-0,84	-0,80
U <sub>j</sub> enlarged	0,05	0,05	0,24	0,04	0,26	U <sub>ref</sub> 0,03

Table 8.1 : Reduced angle deviations of the 24-sided polygon, measured on 24 faces in **inverted** position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{ref}$ .



Laboratory	PTB	OFMET	NMI	SMU	GUM	$s_i$
Result No. $i$	1	2	3	4	5	
Face No. $i$	values in arcsec					
1	-0,03	-0,01	0,91	-0,01	-0,01	0,41
2	-0,01	0,02	-0,36	0,01	0,05	0,17
3	-0,02	0,01	-0,39	0,01	0,15	0,20
4	-0,01	0,03	-0,35	-0,02	0,43	0,28
5	-0,02	0,02	-0,47	0,02	0,13	0,23
6	-0,03	-0,01	-0,03	0,04	-0,02	0,03
7	0,03	-0,08	0,15	0,04	-0,47	0,24
8	0,00	0,00	0,04	0,01	0,00	0,02
9	0,02	-0,04	0,14	0,01	-0,31	0,17
10	0,02	0,02	-0,20	-0,02	0,26	0,17
11	0,01	0,00	-0,26	0,01	0,07	0,13
12	0,06	0,02	0,13	-0,07	0,03	0,07
13	0,02	-0,05	0,56	0,02	-0,37	0,33
14	0,02	-0,01	-0,18	0,00	-0,02	0,08
15	-0,01	-0,01	0,08	0,02	-0,15	0,08
16	0,02	0,01	0,39	-0,03	0,04	0,17
17	-0,04	0,00	-0,44	0,04	0,11	0,21
18	0,02	0,07	-0,09	-0,06	0,13	0,09
19	-0,03	0,02	-0,08	0,00	0,28	0,14
20	0,01	0,01	-0,26	-0,01	-0,01	0,12
21	0,00	0,04	0,09	-0,02	-0,05	0,05
22	0,02	-0,01	0,45	-0,02	-0,05	0,21
23	-0,01	-0,03	-0,07	0,03	-0,16	0,07
24	-0,03	-0,01	0,24	0,01	-0,05	0,12
$s_j$	0,03	0,03	0,34	0,03	0,20	

Table 8.2: 24-sided polygon, measured on 24 faces in **inverted** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last column: Standard deviations  $s_i$  of the results for each face No.  $i$

Last row: Standard deviations  $s_j$  of the faces for each result No.  $j$

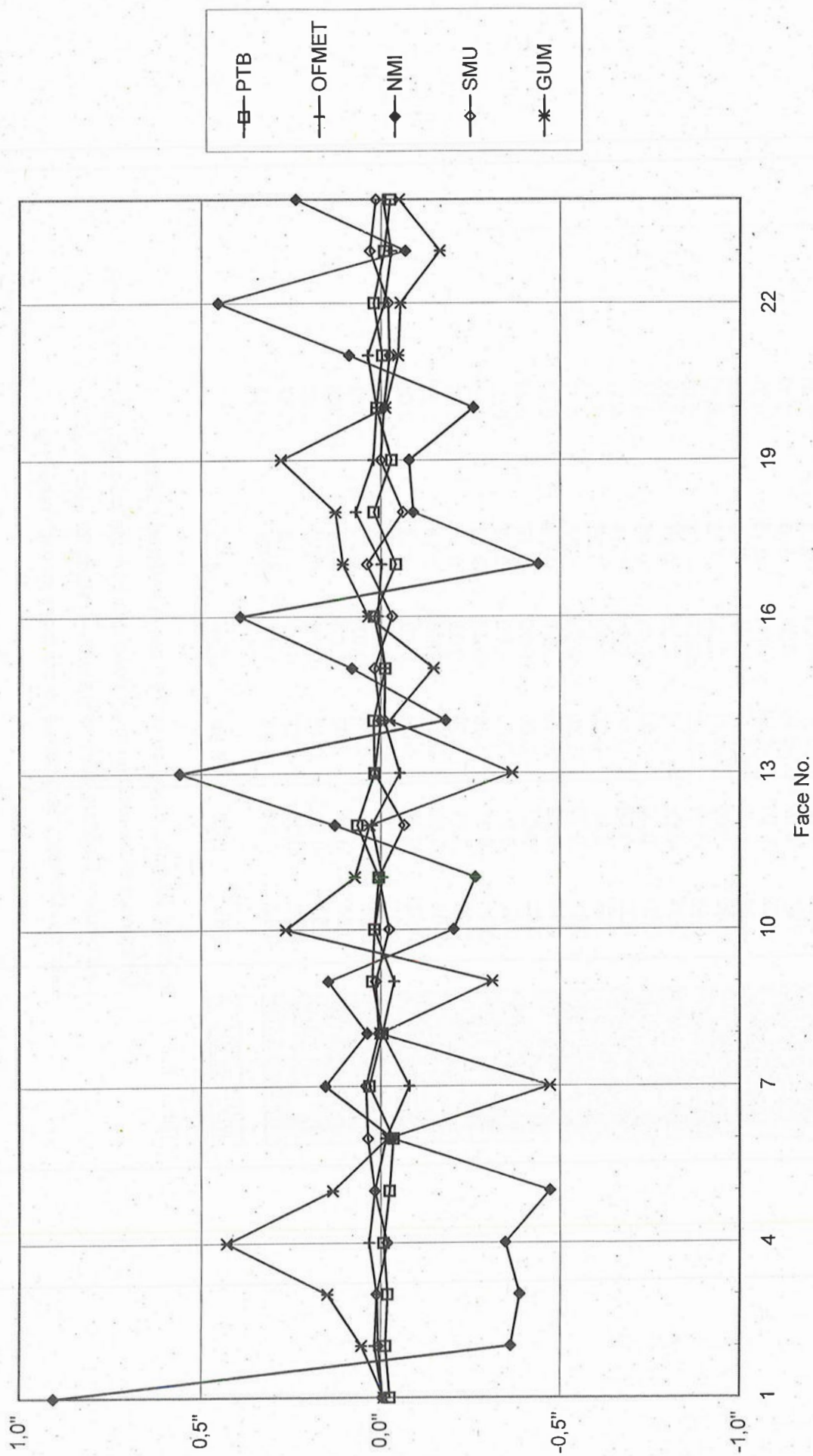


Figure 8.2 : 24-sided polygon, measured on twelve faces in **inverted** position.  
Differences of the reduced angle deviations from the weighted mean of the faces.

Laboratory	PTB	OFMET	NMI	SMU	GUM	$R_B$
Result No. j	1	2	3	4	5	
Face No. i						
1	-0,30	-0,07	1,92	-0,09	-0,02	1,92
2	-0,15	0,20	-0,76	0,11	0,10	0,80
3	-0,21	0,14	-0,82	0,19	0,29	0,90
4	-0,09	0,38	-0,73	-0,28	0,83	1,17
5	-0,28	0,19	-1,00	0,26	0,26	1,08
6	-0,41	-0,18	-0,07	0,56	-0,04	0,58
7	0,36	-0,94	0,32	0,64	-0,92	1,38
8	-0,06	-0,06	0,08	0,08	0,00	0,12
9	0,27	-0,44	0,31	0,21	-0,60	0,82
10	0,20	0,20	-0,43	-0,37	0,51	0,76
11	0,06	-0,05	-0,56	0,09	0,14	0,58
12	0,77	0,29	0,27	-1,04	0,06	1,09
13	0,20	-0,63	1,18	0,27	-0,71	1,49
14	0,23	-0,12	-0,38	-0,01	-0,05	0,44
15	-0,14	-0,14	0,17	0,28	-0,28	0,43
16	0,22	0,10	0,83	-0,50	0,07	0,93
17	-0,48	-0,01	-0,93	0,62	0,20	1,13
18	0,23	0,82	-0,19	-0,97	0,24	1,08
19	-0,34	0,25	-0,16	0,01	0,54	0,66
20	0,14	0,14	-0,54	-0,13	-0,02	0,57
21	-0,04	0,43	0,19	-0,38	-0,09	0,51
22	0,24	-0,11	0,96	-0,31	-0,10	1,01
23	-0,10	-0,34	-0,14	0,50	-0,31	0,59
24	-0,30	-0,06	0,50	0,24	-0,09	0,60

Table 8.3:  $E_n$  and  $R_B$  values, 24-sided polygon, measured on 24 faces in **inverted** position.  
The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,55$



Laboratory	PTB	OFMET	NMI	SMU	GUM	ref. : w. m. i
Result No. i	1	2	3	4	5	
Face No. i	values in arcsec					
1	-0,27	-0,27	0,62	-0,26	-0,27	-0,25
2	0,27	0,29	-0,06	0,29	0,34	0,28
3	-0,39	-0,38	-0,73	-0,39	-0,30	-0,39
4	0,03	0,07	-0,27	0,03	0,48	0,04
5	0,07	0,10	-0,41	0,10	0,20	0,09
6	-0,14	-0,12	-0,13	-0,09	-0,14	-0,11
7	-0,33	-0,38	-0,10	-0,30	-0,79	-0,34
8	-0,61	-0,62	-0,60	-0,61	-0,65	-0,61
9	-0,11	-0,12	0,11	-0,09	-0,38	-0,10
10	-0,29	-0,29	-0,44	-0,33	-0,04	-0,30
11	0,13	0,12	-0,22	0,12	0,19	0,12
12	1,22	1,20	1,32	1,15	1,24	1,19
13	-0,03	-0,07	0,53	-0,03	-0,41	-0,04
14	0,18	0,17	0,00	0,18	0,16	0,17
15	0,84	0,86	0,92	0,87	0,73	0,86
16	0,24	0,24	0,64	0,22	0,30	0,24
17	-0,05	-0,03	-0,48	0,00	0,07	-0,03
18	0,15	0,18	0,05	0,08	0,27	0,13
19	0,11	0,14	0,01	0,13	0,39	0,13
20	0,52	0,53	0,24	0,51	0,54	0,52
21	0,25	0,26	0,32	0,20	0,15	0,23
22	-0,18	-0,21	0,21	-0,22	-0,26	-0,20
23	-0,85	-0,87	-0,92	-0,83	-1,01	-0,85
24	-0,78	-0,79	-0,65	-0,79	-0,87	-0,78
U <sub>j enlarged</sub>	0,05	0,05	0,24	0,04	0,26	U <sub>ref</sub> 0,03

Table 9.1 :

Reduced angle deviations of the 24-sided polygon, measured on 24 faces, mean of normal and inverted position.

Last column: Weighted mean as reference value for each face.

Last row : Enlarged uncertainties  $u_j$  and uncertainty of the reference  $u_{ref}$ .

Laboratory	PTB	OFMET	NMI	SMU	GUM	$s_i$
Result No. j	1	2	3	4	5	
Face No. i	values in arcsec					
1	-0,01	-0,01	0,87	-0,01	-0,02	0,40
2	-0,01	0,01	-0,34	0,01	0,06	0,16
3	0,00	0,01	-0,34	0,00	0,09	0,17
4	-0,01	0,02	-0,31	-0,01	0,44	0,27
5	-0,02	0,01	-0,49	0,01	0,12	0,24
6	-0,03	-0,01	-0,02	0,02	-0,03	0,02
7	0,00	-0,05	0,24	0,03	-0,45	0,25
8	0,00	-0,01	0,02	0,00	-0,03	0,02
9	0,00	-0,01	0,21	0,01	-0,27	0,17
10	0,01	0,02	-0,14	-0,02	0,27	0,15
11	0,01	0,00	-0,34	0,00	0,07	0,16
12	0,03	0,01	0,13	-0,04	0,05	0,06
13	0,01	-0,04	0,57	0,01	-0,37	0,34
14	0,01	-0,01	-0,17	0,00	-0,02	0,08
15	-0,02	0,00	0,06	0,02	-0,12	0,07
16	0,01	0,00	0,40	-0,02	0,06	0,18
17	-0,03	0,00	-0,45	0,03	0,10	0,22
18	0,02	0,05	-0,08	-0,05	0,14	0,09
19	-0,02	0,01	-0,12	0,00	0,27	0,14
20	0,00	0,01	-0,28	0,00	0,02	0,13
21	0,02	0,03	0,09	-0,03	-0,08	0,06
22	0,02	-0,01	0,41	-0,02	-0,06	0,19
23	0,00	-0,02	-0,07	0,02	-0,16	0,07
24	0,01	-0,01	0,13	0,00	-0,08	0,08
$s_j$	0,02	0,02	0,34	0,02	0,20	

Table 9.2 : 24-sided polygon, measured on 24 faces, **mean of normal and inverted** position.

Differences of the reduced angle deviations from the weighted mean of the faces.

Last row: Standard deviations  $s_j$  of the faces for each result No. j

Last column: Standard deviations  $s_i$  of the results for each face No. i

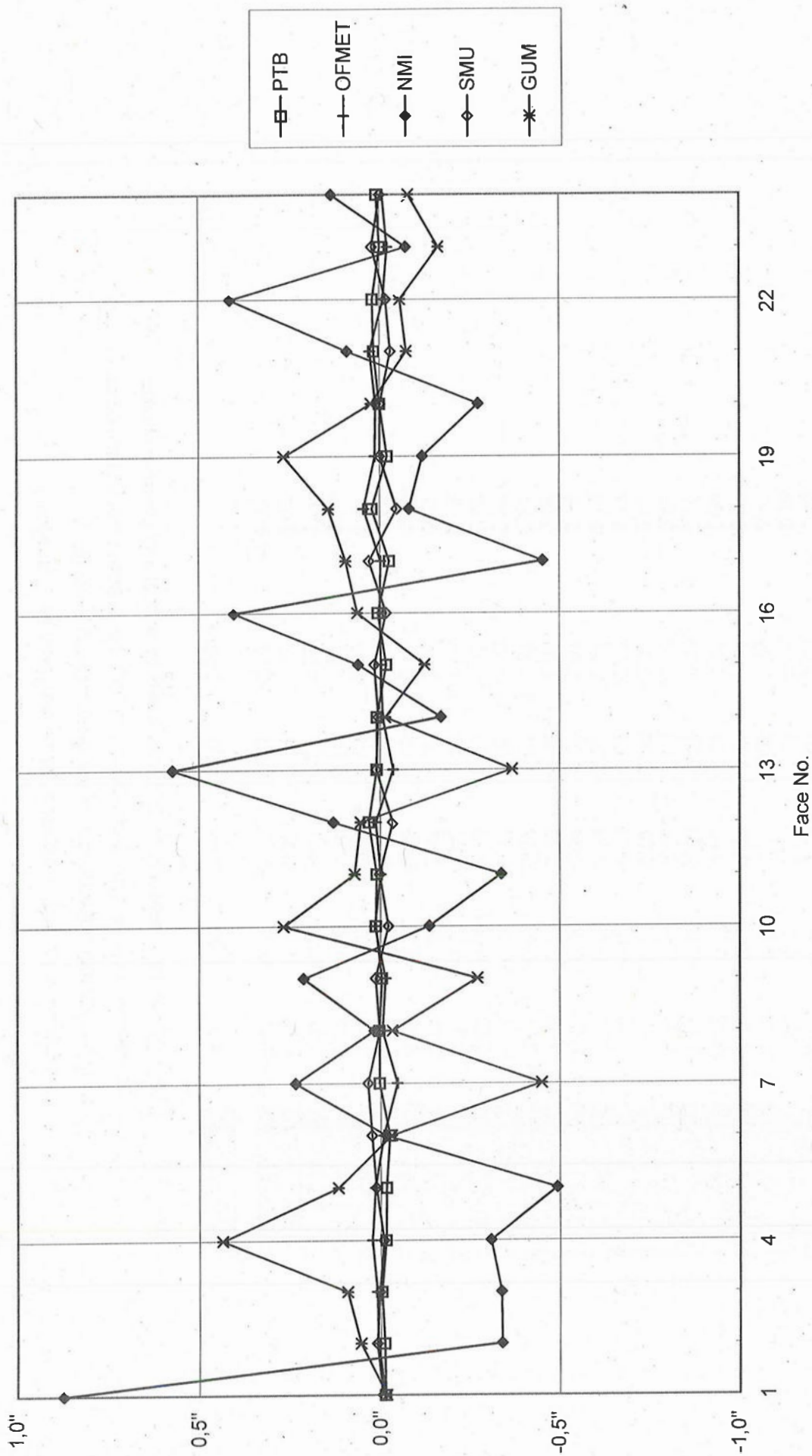


Figure 9.2 : 24-sided polygon, measured on 24 faces, mean of normal and inverted position.  
Differences of the reduced angle deviations from the weighted mean of the faces.



Laboratory	PTB	OFMET	NMI	SMU	GUM	$R_B$
Result No. j	1	2	3	4	5	
Face No. i						
1	-0,16	-0,17	1,85	-0,11	-0,03	1,85
2	-0,12	0,09	-0,72	0,17	0,11	0,74
3	-0,03	0,12	-0,71	0,04	0,18	0,74
4	-0,17	0,28	-0,65	-0,13	0,84	1,10
5	-0,19	0,17	-1,04	0,20	0,23	1,09
6	-0,30	-0,10	-0,03	0,39	-0,06	0,41
7	0,03	-0,56	0,50	0,56	-0,87	1,18
8	0,04	-0,07	0,04	0,03	-0,06	0,10
9	-0,05	-0,17	0,45	0,21	-0,53	0,72
10	0,16	0,18	-0,29	-0,35	0,52	0,68
11	0,12	-0,01	-0,71	0,03	0,14	0,73
12	0,38	0,14	0,28	-0,56	0,11	0,62
13	0,11	-0,44	1,21	0,17	-0,71	1,45
14	0,10	-0,09	-0,36	0,08	-0,03	0,38
15	-0,21	-0,03	0,13	0,24	-0,24	0,37
16	0,07	0,00	0,85	-0,28	0,12	0,88
17	-0,31	-0,03	-0,96	0,49	0,18	1,07
18	0,27	0,54	-0,17	-0,75	0,28	0,83
19	-0,23	0,16	-0,25	0,02	0,51	0,62
20	0,01	0,14	-0,58	-0,01	0,05	0,59
21	0,19	0,32	0,19	-0,48	-0,15	0,54
22	0,23	-0,12	0,87	-0,28	-0,11	0,93
23	-0,01	-0,25	-0,16	0,34	-0,32	0,49
24	0,10	-0,11	0,28	-0,02	-0,16	0,34

Table 9.3 :  $E_n$  and  $R_B$  values, 24-sided polygon, measured on 24 faces, mean of normal and inverted position.

The shaded values do not fulfil the criteria  $|E_n| \leq 1$  and  $R_B \leq 1,55$

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	mean	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12		
Face No. i	values in arcsec													
1	-0,03	0,00	0,00	0,40	0,13	0,02	-0,03	-0,02	0,11	0,03	0,00	0,03	0,05	0,12
2	0,03	-0,04	0,16	0,30	-0,16	-0,04	0,03	0,05	0,08	-0,22	-0,04	0,03	0,02	0,14
3	-0,04	-0,07	-0,01	0,00	0,10	-0,04	0,08	-0,07	-0,11	0,03	0,08	0,13	0,01	0,08
4	0,03	0,03	-0,06	-0,20	-0,01	0,04	-0,24	0,08	0,03	-0,06	0,04	-0,07	-0,03	0,10
5	-0,02	0,00	0,00	-0,40	0,02	0,00	0,01	-0,05	0,01	0,17	-0,06	-0,07	-0,03	0,13
6	0,04	0,07	0,06	0,00	-0,20	-0,02	0,11	-0,10	-0,17	-0,04	-0,06	-0,07	-0,03	0,09
7	-0,02	-0,02	-0,12	-0,10	0,09	0,04	0,03	0,09	0,04	0,12	0,03	0,03	0,02	0,07
$s_j$	0,03	0,05	0,09	0,28	0,13	0,03	0,11	0,08	0,10	0,13	0,05	0,08		

Table 10.1 : 7-sided polygon. Differences of the reduced angle deviations measured in the normal and inverted position.

Last row: Standard deviation  $s_j$  from all faces for each result No. j

Last columns: Mean and standard deviation  $s_i$  of all results for each face No. i

Laboratory	PTB	OFMET	NMI	NPL	IMGC	SMU	LNE	CEM 1	CEM 2	GUM	UME	CMI	mean	$s_i$
Result No. j	1	2	3	4	5	6	7	8	9	10	11	12		
Face No. i	values in arcsec													
1	0,01	-0,04	-0,07	0,14	0,14	0,00	-0,01	0,08	0,05	-0,02	-0,02	0,31	0,05	0,11
3	0,02	-0,02	0,10	0,06	0,07	-0,01	-0,01	0,09	0,19	-0,11	0,00	0,21	0,05	0,09
5	0,01	-0,02	-0,04	0,06	0,09	0,00	0,01	-0,02	0,05	-0,03	0,00	0,21	0,03	0,07
7	-0,06	0,05	0,17	-0,16	-0,25	0,00	-0,02	-0,05	-0,05	0,06	-0,02	0,01	-0,03	0,11
9	-0,07	0,02	0,13	0,07	0,11	0,00	0,00	-0,03	-0,17	0,08	0,02	0,01	0,01	0,08
11	0,00	-0,01	-0,15	-0,10	-0,08	0,00	0,03	-0,04	0,05	0,00	-0,01	-0,19	-0,04	0,07
13	-0,03	0,01	0,02	-0,06	-0,12	-0,01	0,01	0,00	-0,05	-0,01	-0,03	-0,09	-0,03	0,04
15	-0,03	-0,01	-0,05	0,10	0,15	-0,01	0,01	-0,02	-0,13	0,04	0,03	-0,09	0,00	0,08
17	0,04	0,00	-0,01	-0,04	-0,01	0,01	-0,01	-0,01	-0,02	0,00	0,04	-0,09	-0,01	0,03
19	0,02	-0,02	-0,07	0,05	0,05	0,02	0,01	-0,01	0,05	-0,01	0,04	-0,19	0,00	0,07
21	0,04	-0,03	0,01	0,02	0,06	0,00	-0,01	0,02	-0,06	-0,05	0,00	-0,09	-0,01	0,04
23	0,03	0,02	0,01	-0,16	-0,21	0,01	-0,02	0,00	0,11	0,03	-0,05	0,01	-0,02	0,09
$s_j$	0,04	0,03	0,09	0,10	0,13	0,01	0,01	0,04	0,10	0,05	0,03	0,16		

Table 10.2 : 24-sided polygon, measured on 12 faces.

Differences of the reduced angle deviations measured in the normal and inverted position.

Last row: Standard deviation  $s_j$  from all faces for each result No. j

Last columns: Mean and standard deviation  $s_i$  of all results for each face No. i

The framed values of NPL and IMGC are strongly correlated, as shown in Figure 10.



Laboratory	PTB	OFMET	NMI	SMU	GUM	mean	$s_i$
Result No. j	1	2	3	4	5		
Face No. i	values in arcsec						
1	0,03	-0,02	-0,06	0,00	-0,01	-0,01	0,03
2	-0,01	-0,04	0,03	-0,01	-0,01	-0,01	0,02
3	0,04	0,00	0,11	-0,01	-0,10	0,01	0,08
4	-0,04	-0,05	0,05	-0,01	-0,01	-0,01	0,04
5	0,02	0,00	-0,03	0,00	-0,02	-0,01	0,02
6	0,03	0,02	0,05	-0,01	-0,01	0,02	0,03
7	-0,05	0,07	0,18	0,00	0,06	0,05	0,08
8	0,04	0,02	-0,01	0,02	-0,04	0,01	0,03
9	-0,05	0,04	0,14	0,00	0,08	0,04	0,07
10	-0,02	-0,02	0,12	-0,01	0,00	0,01	0,06
11	0,02	0,01	-0,14	0,00	0,01	-0,02	0,07
12	-0,14	-0,10	-0,06	-0,01	-0,02	-0,07	0,05
13	-0,01	0,03	0,03	-0,01	0,00	0,01	0,02
14	-0,03	-0,01	0,01	0,00	0,01	0,00	0,02
15	-0,02	0,01	-0,04	-0,01	0,04	0,00	0,03
16	-0,05	-0,05	0,00	0,00	0,03	-0,01	0,03
17	0,06	0,02	0,00	0,01	0,01	0,02	0,02
18	-0,01	-0,07	0,00	0,01	0,02	-0,01	0,03
19	0,04	0,00	-0,06	0,02	0,00	0,00	0,04
20	-0,03	-0,01	-0,04	0,01	0,07	0,00	0,04
21	0,05	-0,01	0,02	0,00	-0,04	0,00	0,03
22	0,00	0,00	-0,07	0,01	0,00	-0,01	0,03
23	0,05	0,04	0,02	0,01	0,03	0,03	0,02
24	0,09	0,01	-0,18	-0,01	-0,04	-0,03	0,10
$s_j$	0,05	0,04	0,08	0,01	0,04		

Table 10.3 : 24-sided polygon, measured on 24 faces. Differences of the reduced angle deviations measured in the normal and inverted position.

Last row: Standard deviation  $s_j$  from all faces for each result No. j

Last columns: Mean and standard deviation  $s_i$  of all results for each face No. i

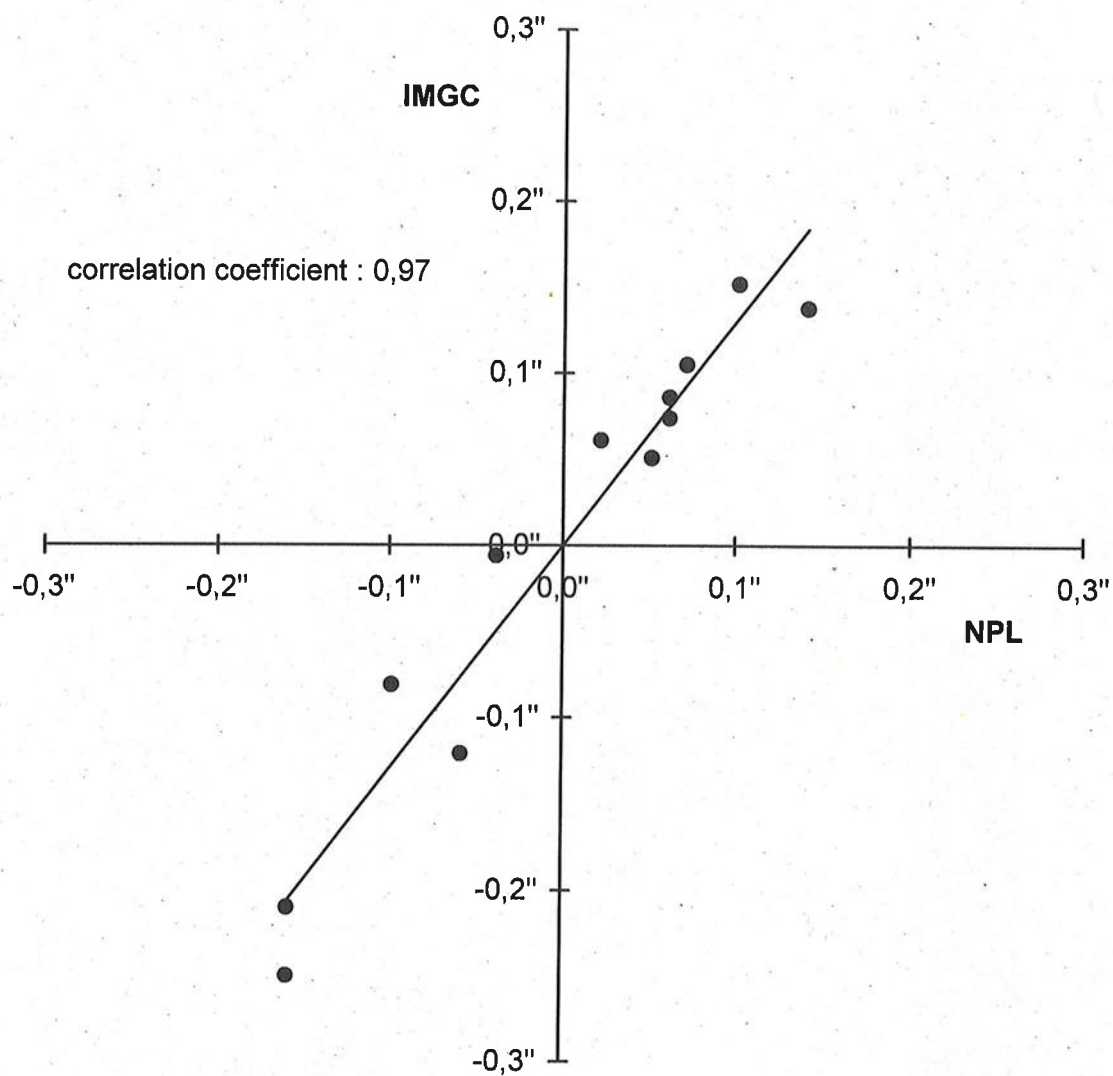


Figure 10 : Correlation between IMGC and NPL of the differences measured on twelve faces of the 24-sided polygon in the normal and the inverted position; according to Table 10.2.

Face No.	RMS nm,	P-V nm	RMS * nm	P-V * nm	pyramidal deviation arcsec	Angle difference HR - PI arcsec
1	3,6	21	2,6	12	0,4	0,06
2	6,9	42	6,7	33	0,7	0,03
3	4,6	21	3,0	17	2,6	-0,07
4	4,3	22	0,9	4	4,5	0,04
5	7,0	37	3,7	17	-2,7	-0,03
6	4,0	21	2,2	12	1,4	0,03
7	6,4	28	4,3	23	5,7	-0,05
mean face				17		
				3,3		

Table 11.1 : Interferometric results of the RMS and P-V flatness deviations of the 7-sided polygon, together with pyramidal deviations and differences of the reduced angle deviations between autocollimator and interferometer results.

\* geometrical differences from the mean face

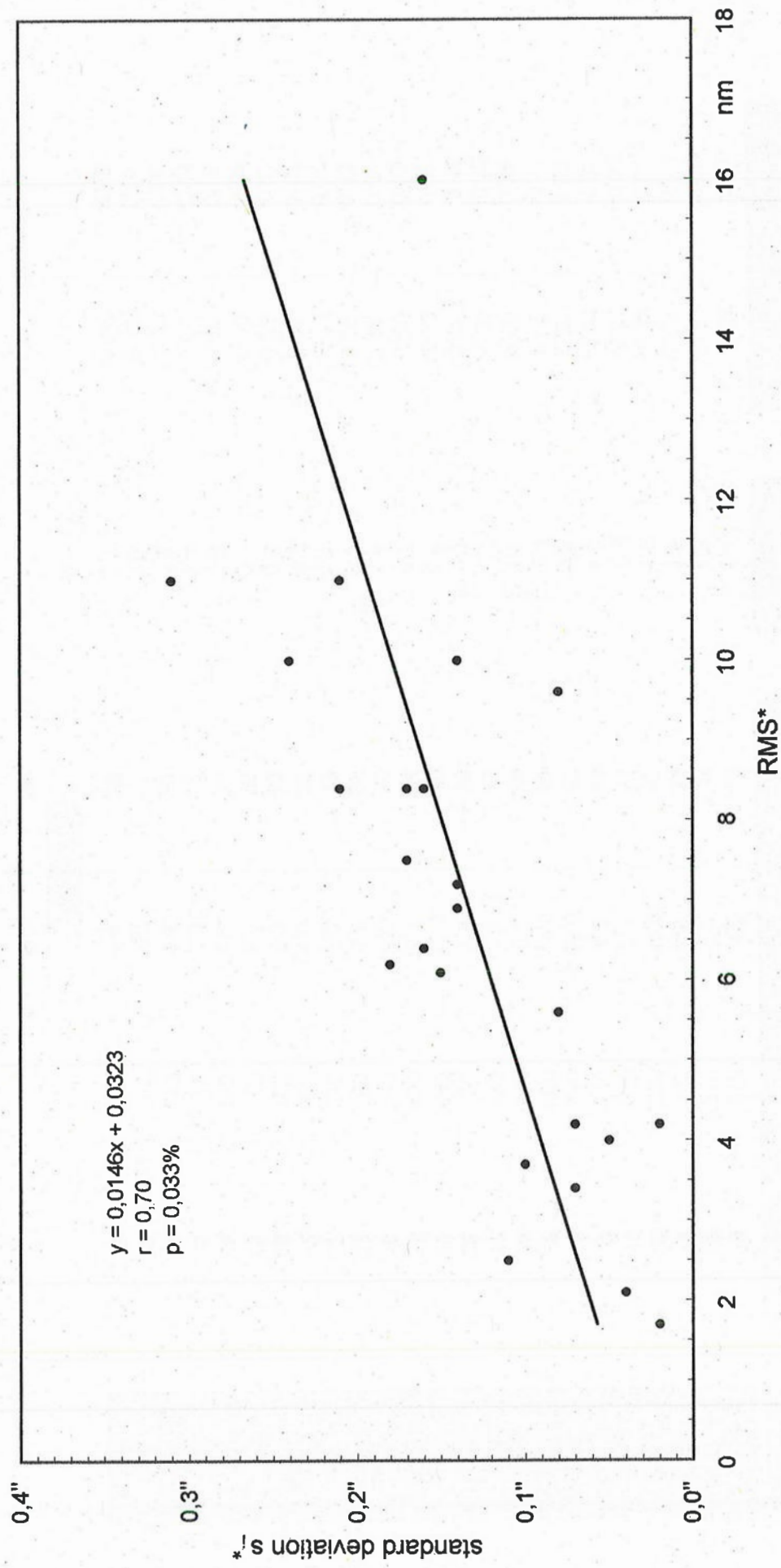


Face No.	face 20 mm x 25 mm		face 18,6 mm Ø		pyramidal deviation arcsec	Angle difference HR - PI	
	RMS nm	P-V nm	RMS* nm	P-V* nm		HR : 20 x 25 arcsec	HR : Ø 18 arcsec
1	14	110	4,2	22	5,4	-0,03	-0,07
2	30	250	6,2	31	1,3	-0,01	0,05
3	12	93	5,6	24	1,7	0,01	-0,15
4	29	230	10,0	57	1,5	-0,32	0,01
5	15	110	2,1	11	-0,4	-0,02	-0,03
6	17	130	4,2	23	4,0	-0,01	0,04
7	33	180	16,0	82	5,8	0,21	0,30
8	22	190	11,0	54	-2,4	0,13	-0,13
9	24	190	8,4	39	1,1	0,08	0,25
10	17	120	3,7	18	-1,7	-0,23	-0,10
11	23	120	7,5	36	1,3	0,01	-0,16
12	33	260	11,0	60	4,6	-0,13	0,03
13	17	130	6,1	34	0,7	0,20	0,05
14	15	150	1,7	10	1,4	0,02	0,02
15	22	180	6,5	29	2,7	0,00	0,13
16	27	250	7,2	36	1,1	-0,13	0,04
17	12	91	4,0	23	-0,2	0,00	-0,03
18	20	190	9,6	48	0,3	0,03	-0,21
19	13	100	2,5	15	0,1	-0,10	-0,03
20	21	150	8,4	36	3,9	-0,09	0,19
21	15	120	6,9	32	2,4	0,08	-0,21
22	21	180	3,4	20	0,5	-0,01	0,09
23	25	160	10,0	54	2,6	0,19	0,11
24	19	130	8,4	38	2,3	0,14	-0,16
					mean face		
					6,0		
					30		

Table 11.2 :

Interferometric results of the RMS and P-V flatness deviations of the 24-sided polygon, faces in full size and limited by circular aperture, together with pyramidal deviations and differences of the reduced angle deviations between autocollimator and interferometer results.

\* geometrical differences from the mean face



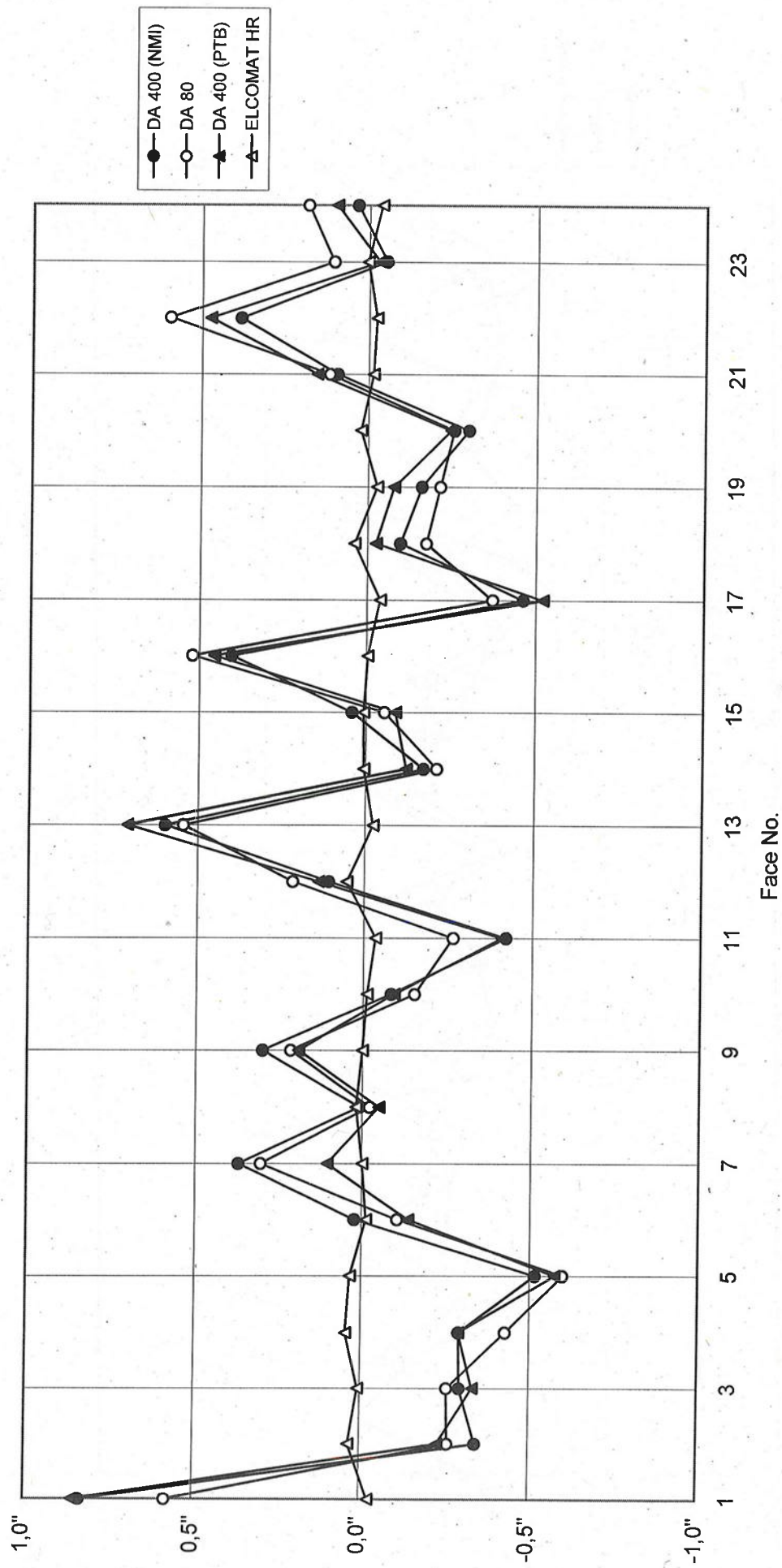


Figure 12 : 24-sided polygon, faces in full size. Differences of NMI- and PTB-results from the reference values, using autocollimators DA 80, DA 400 and ELCOMAT HR.



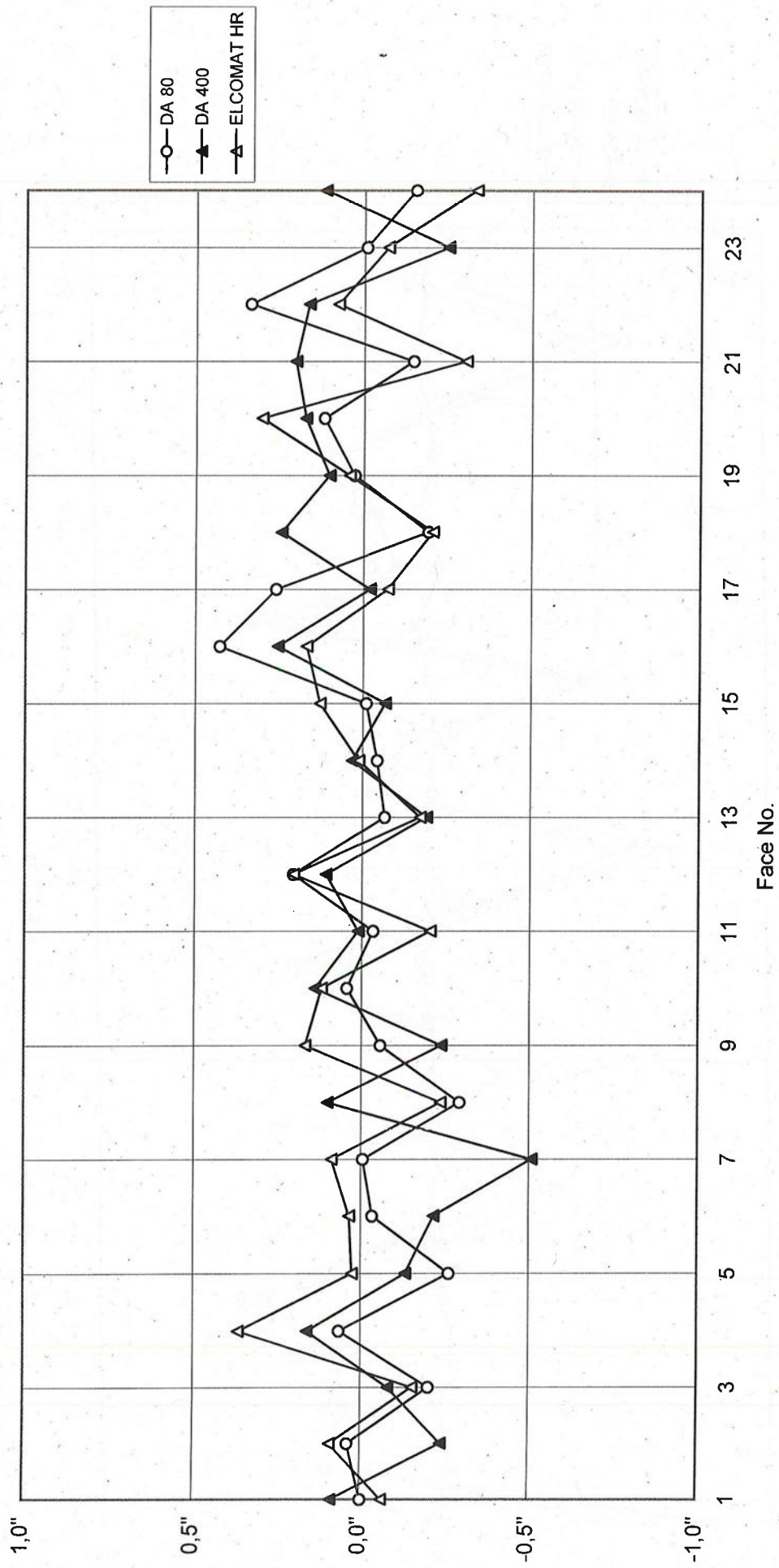


Figure 13 : 24-sided polygon, faces limited by circular aperture 18 mm in diameter. Differences of PTB-results from the reference values using autocollimators DA 80 , DA 400 and ELCOMAT HR.

**Annex A:**

**Measurement instructions**





**EUROMET Project No. 371**  
**Angle calibration on precision polygons**  
**- Instructions -**

**1 Objectives**

The project covers an international comparison of angle calibrations to be carried out on two precision polygons with 7 and 24 faces, respectively.

For the calibration of the precision polygons, the two known standard measuring methods using one or two autocollimators and a rotary table are to be applied for exact angle positioning and angle measurement [1], [2]. Besides these two methods, other optical measuring procedures may be applied which achieve a comparable accuracy, for example, the method making use of a continuously rotating ring laser or an interferential method of measurement. Other non-optical measuring methods, such as the mechanical probing of the polygon faces, are unsuitable and must therefore be excluded.

It is the project's main objective to provide information about the type A and B uncertainties of measurement for the calibration of precision polygons according to the "ISO guide to the expression of uncertainty in measurement" [3]. Above all the type B measurement uncertainty whose causes have so far been clarified only to a rather limited extent, is to be evaluated and explained by the participants, taking the special features of their respective measurement facilities into account.

The proposal that the comparison measurements should be carried out with the polygons not only in the normal position (identification No. on top), but also in the inverted position (identification No. facing downward), should be seen in this context. The differences between the measurements carried out in the two positions can give information about the contributions to the type B uncertainty resulting from errors in the optical probing of the polygon faces, and this for the individual participant and for the comparison among all participants.

## 2 Precision polygons to be calibrated

### 2.1 7-faced polygon

pitch angle	51° 25' 42,857..."
mirror size	15 mm in diameter (limited by the aperture of the case)
diameter	60 mm
material	glass
weight incl. case	495 g
manufacturer	Rank Taylor Hobson Ltd
identification number	SP LE 5997

### 2.2 24-faced polygon

pitch angle	15°
mirror size	20 mm x 25 mm (width x height)
diameter	150 mm
material	glass-ceramic (Zerodur)
weight incl. case	2126 g
manufacturer	Möller/Wedel
identification number	PTB 5.22-23-539-1

## 3 Definitions (Figure 1)

The precision polygon is a square prism with  $n$  reflecting side faces serving as measuring faces. The example of  $n = 8$  in Figure 1 shows the measuring plane to which the individual measuring faces are perpendicular in ideal circumstances. In practice, the measuring faces deviate from the square position in relation to the measuring plane by small tilts referred to as pyramid errors. In that case, the measuring plane is defined as the plane for which the sum of the squares of the pyramid errors of all measuring faces is a minimum. The normals to the faces  $N_i$  or their projections on the measuring plane determine the plane angles of the polygon:

The pitch angles  $\alpha_i$  are the angles between two adjacent normals  $N_{i-1}$  and  $N_i$  of the measuring faces with the counting index  $i$  ( $i=1,2,\dots,n$ ). The deviations

of the pitch angles from their nominal values of  $360^\circ/n$  are referred to as *pitch angle deviations*  $\Delta\alpha_i$  :

$$(1) \quad \Delta\alpha_i = \alpha_i - \frac{360^\circ}{n} \quad (i = 2, 3, \dots)$$

The cumulative pitch angles  $\beta_i$  are the angles between the normal  $N_1$  of the first face and the normal  $N_i$  of the  $i$ -th face in ascending count direction. According to the above definition, the following is valid:

$$(2) \quad \beta_i = \sum_{k=2}^i \alpha_k \quad (i = 2, 3, \dots)$$

The deviations of the cumulative pitch angles from their nominal values are referred to as *cumulative angle deviations*  $\Delta\beta_i$  :

$$(3) \quad \Delta\beta_i = \beta_i - \frac{360^\circ}{n} \cdot (i - 1) = \sum_{k=2}^i \Delta\alpha_k \quad (i = 2, 3, \dots)$$

The positive count direction of the polygon angle corresponds to the count direction of the face index  $i$  indicated on the polygon housing. Note that with the polygon in the normal and the inverted position the count directions are opposite in relation to the direction of rotation of the rotary table.

#### 4 Mounting device (Figure 2)

To facilitate the polygon adjustments and to exclude measurement errors which may occur when different polygon mounting devices are used, it is recommended to use the mounting device delivered together with the polygons. Two interchangeable mounting shafts will be made available which give a defined positional height of the reflecting faces for both polygons. The enclosed intermediate ring must be used when the 24-faced polygon is in the normal position.

If the mounting device delivered cannot be used for some reason or other, it is also possible for the participants to use their own device. This device is to be explained and represented in an outline drawing in Annex 3.



## 5 Adjustments

### 5.1 Eccentricity

The mounting device is placed on the rotary table and laterally adjusted, with the table rotating, by probing the mounting shaft with a precision indicator centrically to the axis of rotation (maximum run-out  $\pm 100 \mu\text{m}$ ). In this position, the base plate of the mounting device is clamped tightly to the measuring table. The polygon is then placed on the mounting shaft and manually fastened by slightly tightening the securing screw.

### 5.2 Adjustment of the autocollimator

The autocollimator must be adjusted as exactly as possible with its optical axis perpendicular and in true alignment to the table's axis of rotation and centrically to the center of the polygon faces. The optical path length between autocollimator and polygon should be as small as possible and shielded against air turbulences and thermal effects.

### 5.3 Pyramidality adjustment

Before further adjustments are made, the polygon faces must be examined for contamination and cleaned, if necessary (slight contaminations with a soft, dry cotton cloth, stronger contaminations with alcohol). Then the polygon is adjusted with the aid of the autocollimator and the adjusting screws of the mounting device in the plane perpendicular to the table's axis of rotation (measuring plane) in such a way that the pyramid errors of all measuring faces are at least within  $\pm 10''$ . The minimum condition according to the definition of the measuring plane in point 3 should be complied with as well as possible.

## 6 Measurements

The polygons are to be measured in both the normal (ident.No. on top) and the inverted position (ident.No. facing downward). To reduce the time and effort required for the measurements on the 24-faced polygon, only the 12 faces with odd numbers (faces Nos. 1, 3, 5, ..., 23) may be measured instead of all 24 faces.

### 6.1 Method 1

This measuring method makes use of an autocollimator and an indexing or angle measuring table. Complete application of this method means that the

polygon is measured in all positions relative to the measuring table ( $n \times n$  individual measurements) and evaluated according to the known procedure [1], [2]. If the accuracy of the angle measuring table is adequate or if the calibration values of the table are known, the measuring method can be shortened by reducing the number of the relative positions.

## 6.2 Method 2

A comparison with the fixed pitch angle formed by two autocollimators. The angle between the autocollimators is varied at least up to  $180^\circ$  in steps of the polygon pitch angle. The evaluation of the measurements is also described in [1], [2].

## 6.3 Other method

A method deviating from method 1 or 2 is to be described in Annex 4.

# 7 Measurement results and reports

According to the definitions given in point 3, the following measurement results have to be reported in the calibration tables of Annexes 1 and 2:

- the pitch angle deviations  $\Delta\alpha_i$
- the cumulative angle deviations  $\Delta\beta_i$

For each of the two polygons, for each of the two positions (normal and inverted) and, if several different methods of measurement have been applied, also for each method, a separate form (Annexes 1 and 2) is to be used to report the results of the measurements together with the combined standard uncertainty  $u_c(\beta)$  according to the *Guide* [3]. This uncertainty is to be understood as a mean value attributed to all the accumulated pitch angles  $\beta_i$  measured.

The differences between the measurements of the normal and of the inverted position of the polygon should however not enter into the evaluated uncertainty. This difference must be evaluated after the intercomparison together with the corresponding differences obtained by all the participants to estimate the uncertainty of the measurement method (probing of the polygon faces with one or two autocollimators) and to separate this from the uncertainty caused by the polygon itself (e.g. due to the flatness deviation of the faces).

In addition to the measurement report forms in Annexes 1 and 2, please

describe in Annex 3 the measurement equipment and - of applicable - your own mounting device, further in Annex 4 the measurement procedure (measurement conditions, number of repeated measurements, number of settings of the polygon relative to the rotary table, influence quantity like temperature etc.) and finally in Annex 5, the evaluation of the uncertainty (Type A and Type B evaluation with any information available for the evaluation of the uncertainty components, if possible in the form of an uncertainty budget. This description should show the origin of the main contributions to the uncertainty in your measurements).

The complete measurement report (Annexes 1 - 5) should be sent to the pilot laboratory of the PTB within two months after completion of your measurements.

## **8 Receipt and passing-on of the comparison pack**

Each participant must inform the PTB's pilot laboratory, and the preceding and following participant, of the receipt and dispatch of the polygons. Please use the enclosed report telefax to inform the pilot laboratory after receipt and inspection of the comparison pack.

The completeness and intactness of the objects in the package must be confirmed (date and signature) on the control list contained in the transport box, when the objects are received and passed on by the participant.

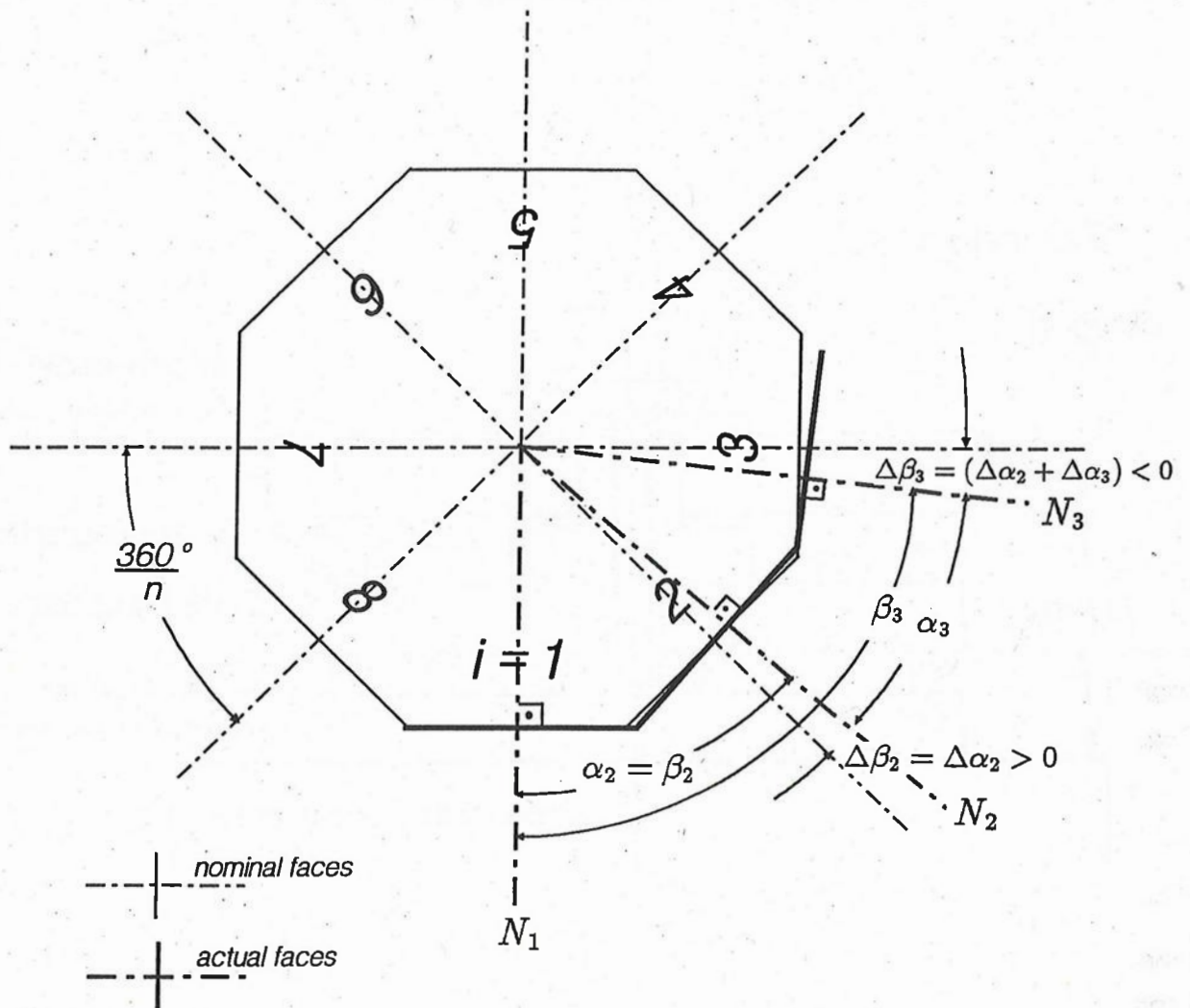
Each participant must bear the costs for the transport of the test pieces to the next participant. The method of transport shall be decided by each participant.

### **References:**

- [1] P.J. Sim, Angle Standards and their Calibration  
in: P.L.Hewitt (ed.), Modern Techniques in Metrology, World Scientific, Singapore, 1984, pp. 102-121
- [2] K.Toyoda, Report of International Comparison of Angle Standards, NRLM Japan, 1990.12.1
- [3] Guide to the Expression of Uncertainty in Measurement, International Organization of Standardization (ISO), Geneva, 1993



Figure 1 : Definitions  
( example :  $n = 8$  )



measuring face index  $i = 1 \dots n$

normals to the faces  $N_i$

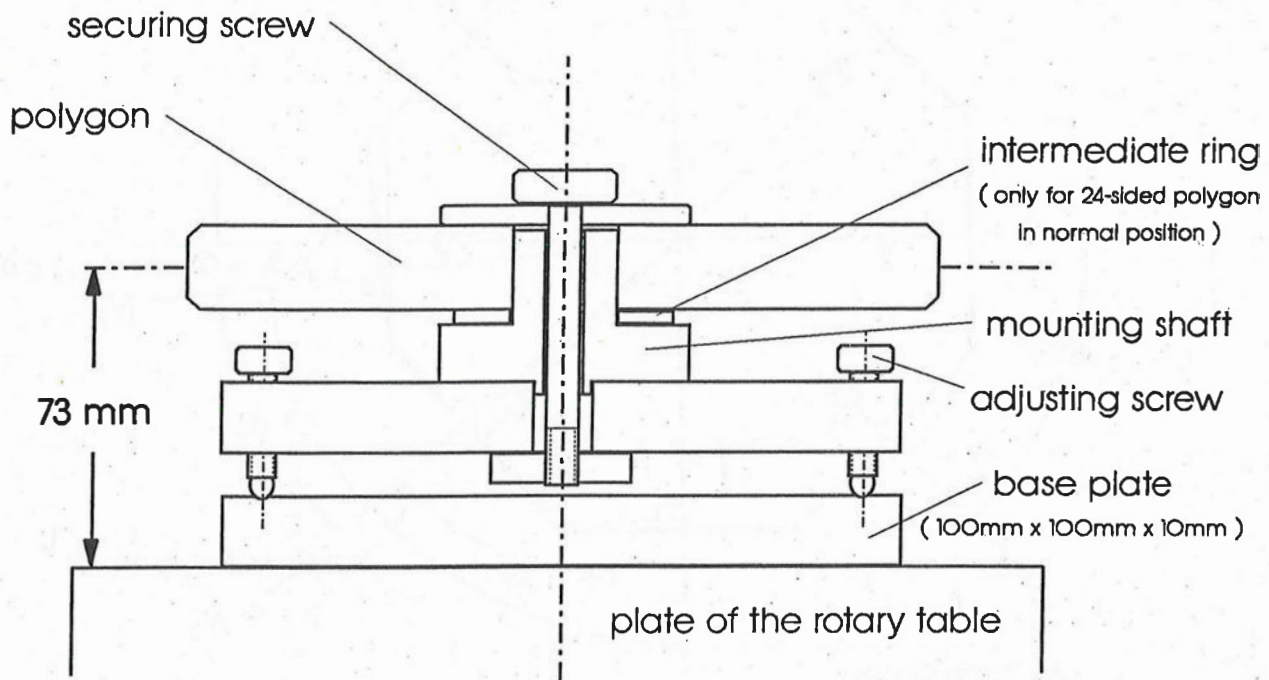
pitch angles  $\alpha_i$  ( angles between  $N_{i-1}$  and  $N_i$  )

pitch angle deviations  $\Delta\alpha_i = \alpha_i - \frac{360^\circ}{n}$  ( $i = 2, 3, \dots$ )

cumulative pitch angles  $\beta_i = \sum_{k=2}^i \alpha_k$  ( $i = 2, 3, \dots$ )

cumulative angle deviations  $\Delta\beta_i = \beta_i - \frac{360^\circ}{n} \cdot (i - 1) = \sum_{k=2}^i \Delta\alpha_k$  ( $i = 2, 3, \dots$ )

Figure 2 : Mounting device



# EUROMET Project No. 371 - Measurement Report -

Annex 1

7-sided polygon

please mark what is applicable ☒ X

Normal position  
( Ident.No. upward )

☐

Inverted position  
( Ident.No. downward )

☐

Laboratory :

Method of measurement :  
(measurement procedure  
in Annex 4)

- ☐ method 1 ( 1 autocollimator )  
☐ method 2 ( 2 autocollimators )  
☐ other method

Mounting device :

- ☐ delivered device  
☐ own device ( Annex 3 )

Adjustments :

eccentricity of the mounting shaft .....  $\mu\text{m}$   
 pyramid errors of the polygon faces ....."

Calibration table :

Unit : second of arc

faces	pitch angle dev.	faces	cum. angle dev.
-	-	1-1	$\Delta\beta_1 = 0$
1-2	$\Delta\alpha_2$	1-2	$\Delta\beta_2$
2-3	$\Delta\alpha_3$	1-3	$\Delta\beta_3$
3-4	$\vdots$	1-4	$\vdots$
4-5		1-5	
5-6		1-6	
6-7		1-7	
7-1	$\Delta\alpha_1$	1-1	$\Delta\beta_1 = 0$

Combined standard uncertainty:  $u_c(\beta) = \dots\dots\dots$   
 according to the *Guide* [3]

( Evaluation of uncertainty in Annex 5 )



# EUROMET Project No. 371 - Measurement Report -

Annex 2

24-sided polygon

please mark what is applicable ☒

Normal position  
( Ident.No. upward )

☐

Inverted position  
( Ident.No. downward )

☐

Laboratory :

Method of measurement :  
(measurement procedure  
in Annex 4)

- ☐ method 1 ( 1 autocollimator )  
☐ method 2 ( 2 autocollimators )  
☐ other method

Mounting device :

- ☐ delivered device  
☐ own device ( Annex 3 )

Adjustments :

eccentricity of the mounting shaft . . . . .  $\mu\text{m}$

pyramid errors of the polygon faces . . . . . "

Calibration table :

Unit : second of arc

faces	pitch angle dev .	faces	cum. angle dev .	faces	pitch angle dev .	faces	cum. angle dev .
-	-	1 - 1	$\Delta\beta_1 = 0$	12 - 13		1 - 13	
1 - 2	$\Delta\alpha_2$	1 - 2	$\Delta\beta_2$	13 - 14		1 - 14	
2 - 3	$\Delta\alpha_3$	1 - 3	$\Delta\beta_3$	14 - 15		1 - 15	
3 - 4	$\vdots$	1 - 4	$\vdots$	15 - 16		1 - 16	
4 - 5		1 - 5		16 - 17		1 - 17	
5 - 6		1 - 6		17 - 18		1 - 18	
6 - 7		1 - 7		18 - 19		1 - 19	
7 - 8		1 - 8		19 - 20		1 - 20	
8 - 9		1 - 9		20 - 21		1 - 21	
9 - 10		1 - 10		21 - 22		1 - 22	
10 - 11		1 - 11		22 - 23	$\vdots$	1 - 23	$\vdots$
11 - 12		1 - 12		23 - 24	$\Delta\alpha_{24}$	1 - 24	$\Delta\beta_{24}$
				24 - 1	$\Delta\alpha_1$	1 - 1	$\Delta\beta_1 = 0$

Combined standard uncertainty:  $u_c(\beta) = \dots\dots\dots$   
according to the *Guide* [3]

( Evaluation of uncertainty in Annex 5 )

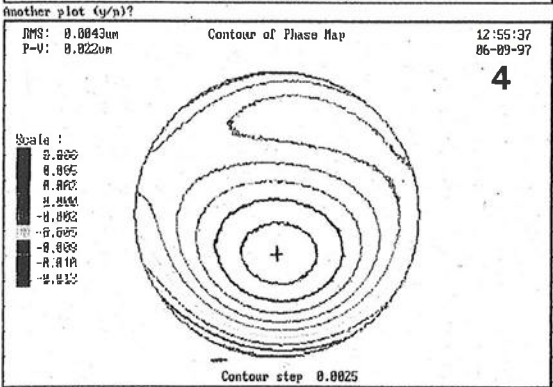
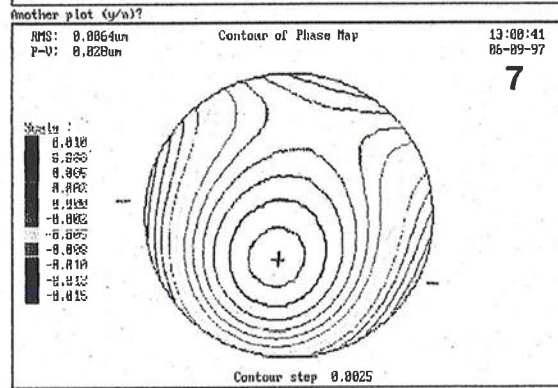
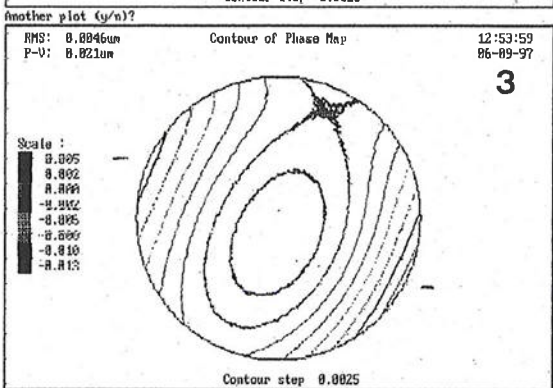
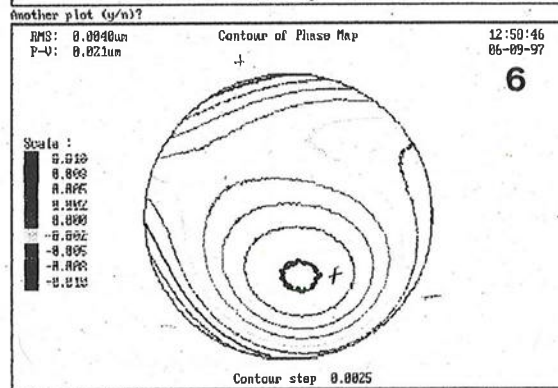
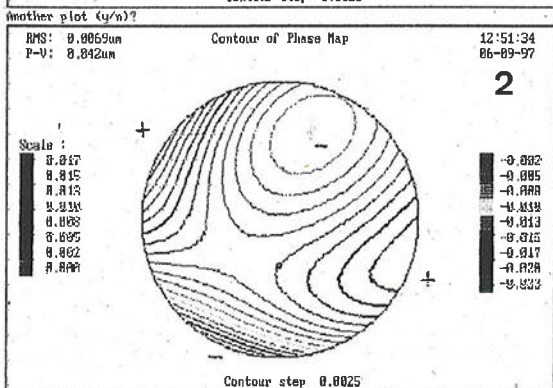
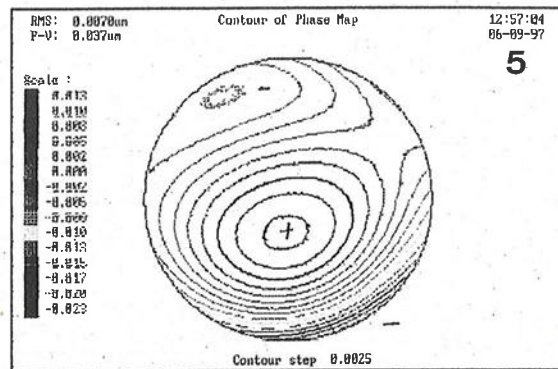
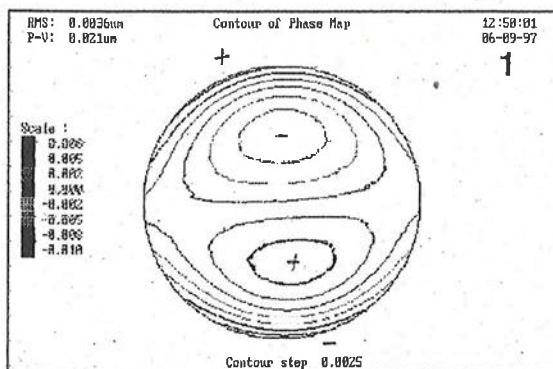
## **Annex B:**

### **Contour and profile diagrams of the polygon faces**

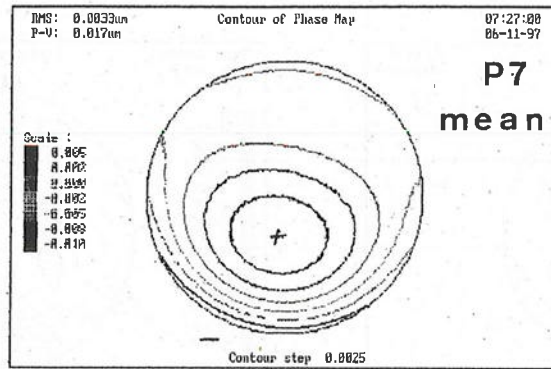
- a:** Contour diagrams of the faces of the 7-sided polygon, size 15 mm in diameter.
- b:** Contour and x-profile diagram of the mean calculated from the seven contour diagrams **a**.
- c-d:** Contour and x-profile diagrams of the differences between the contours **a** and the mean contour **b** of the 7-sided polygon.
- e-g:** Contour diagrams of the faces of the 24-sided polygon, size 20 mm x 25 mm.
- h:** Contour and x-profile diagram of the mean calculated from the 24 contour diagrams **e-g**, limited to 18,6 mm in diameter.
- i-n:** Contour and x-profile diagrams of the differences between the contours **e-g** and the mean contour **b** of the 24-sided polygon, limited to 18,6 mm in diameter.



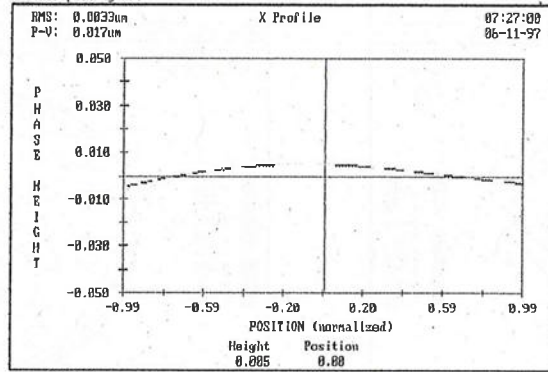




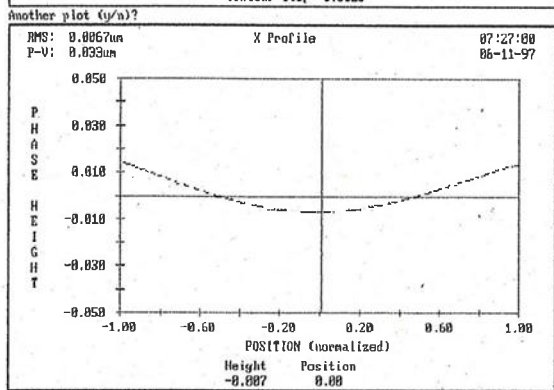
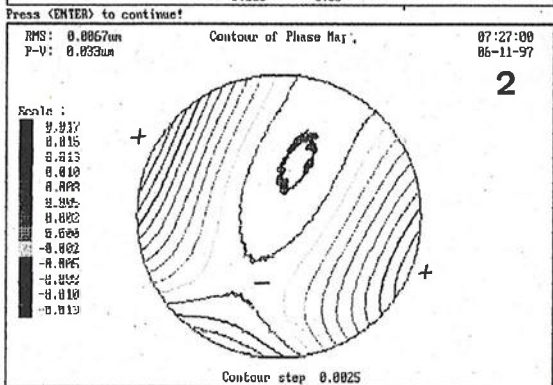
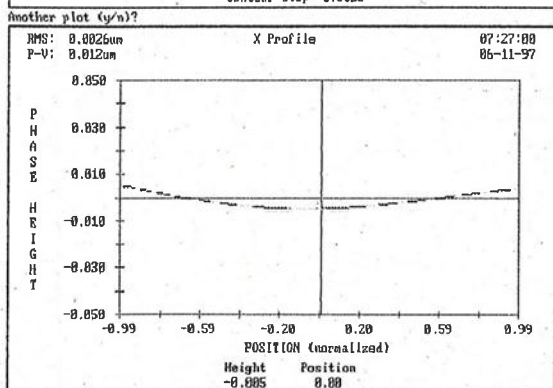
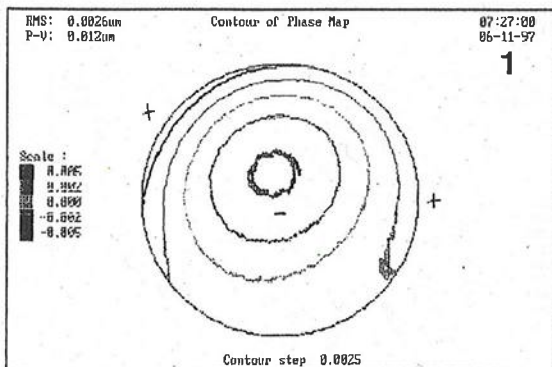
Another plot (y/n)?



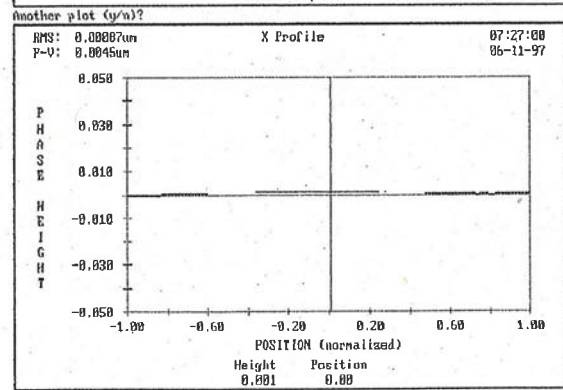
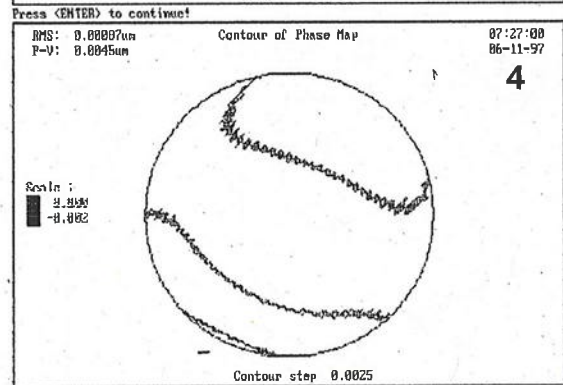
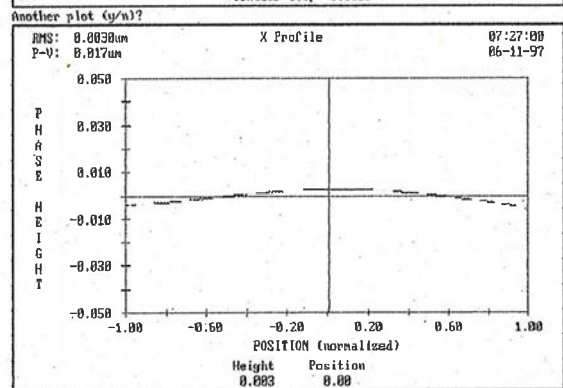
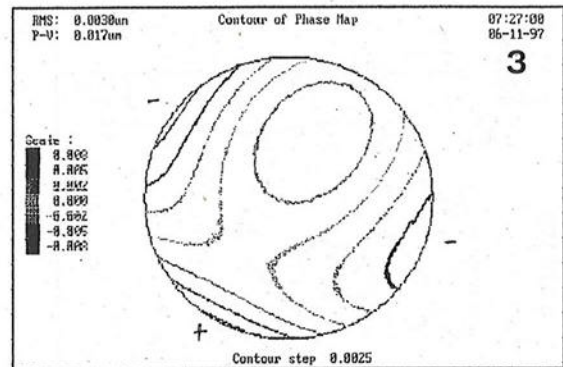
Another plot (y/n)?



Press <ENTER> to continue!

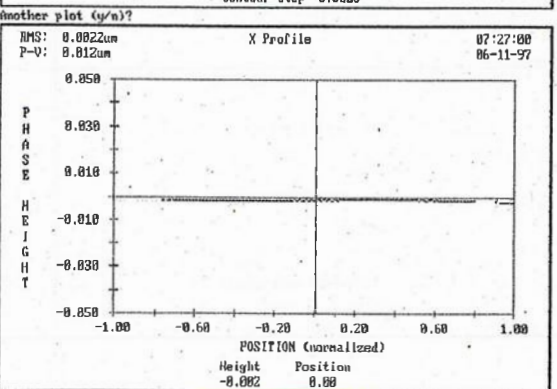
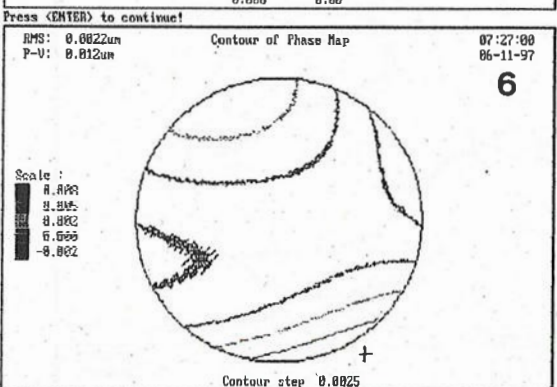
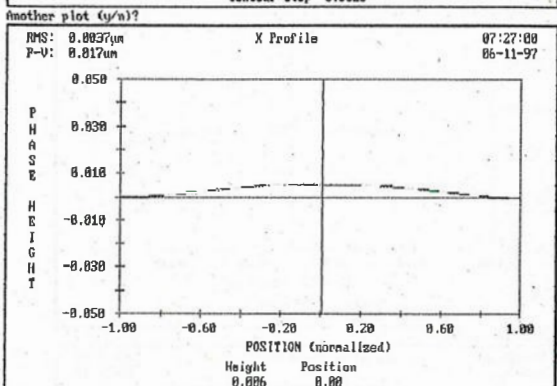
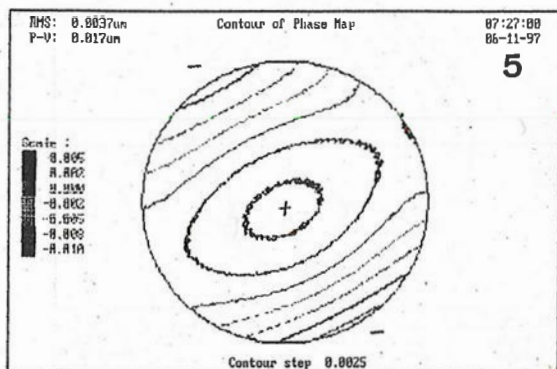


Press (ENTER) to continue!

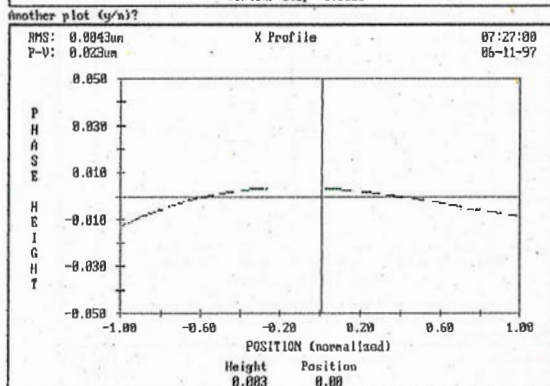
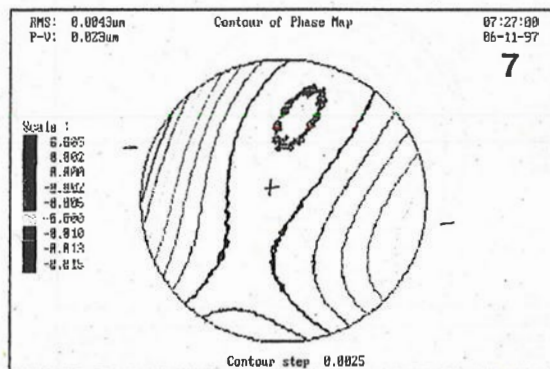


Press (ENTER) to continue!

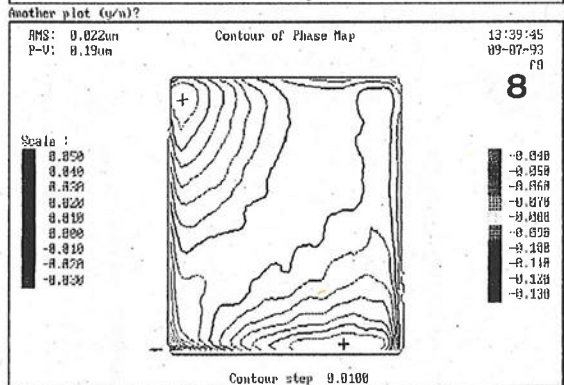
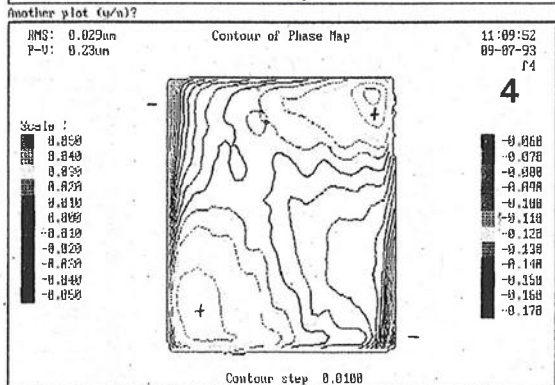
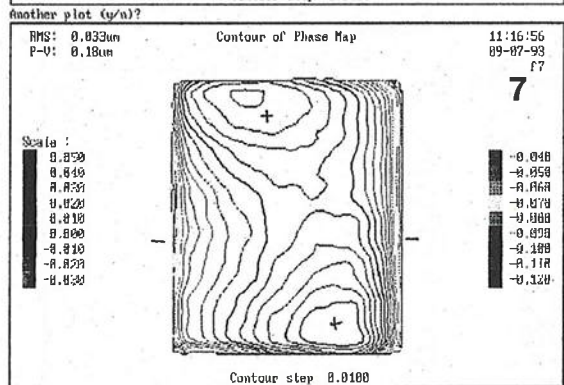
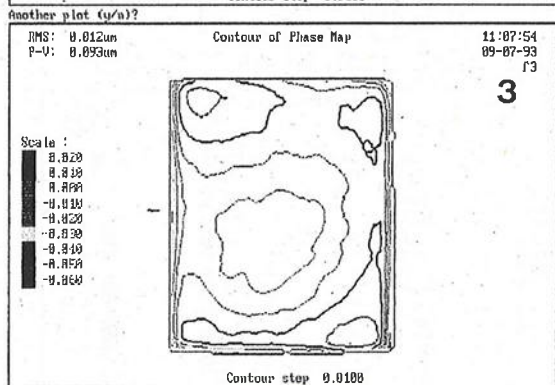
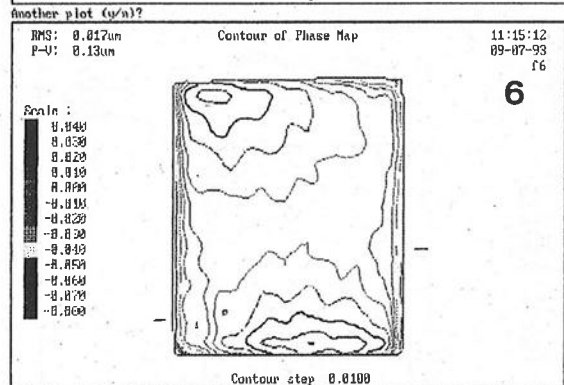
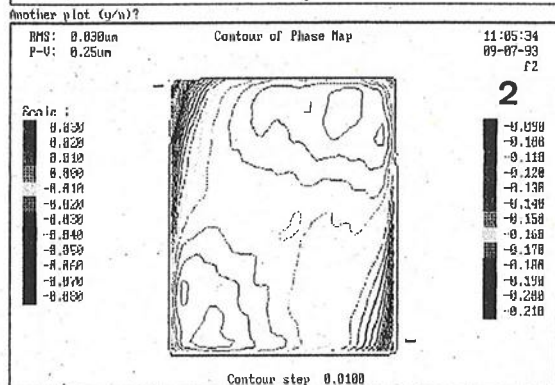
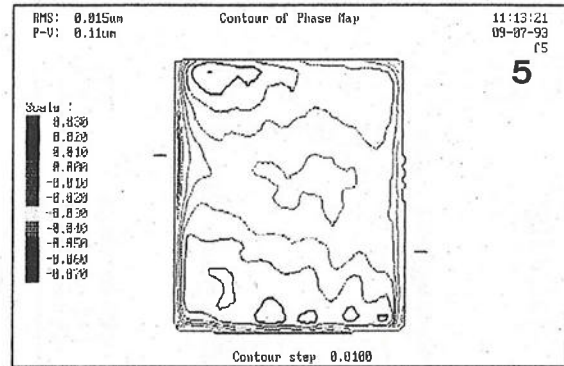
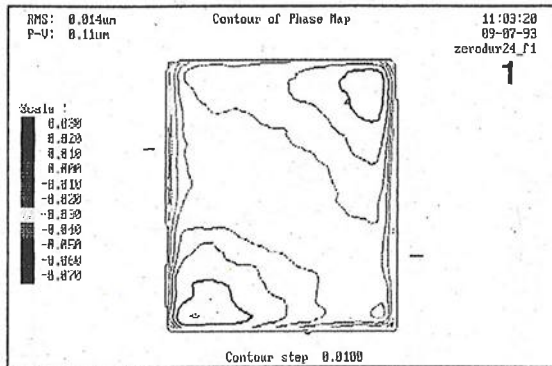




Press <ENTER> to continue!

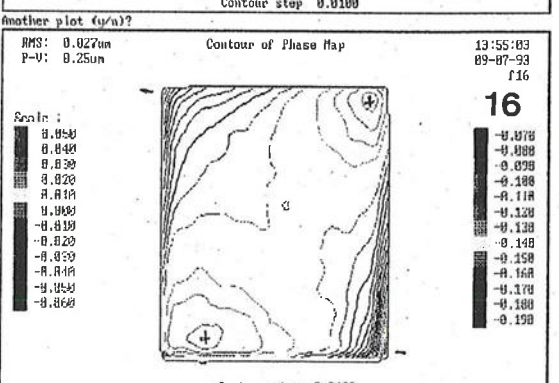
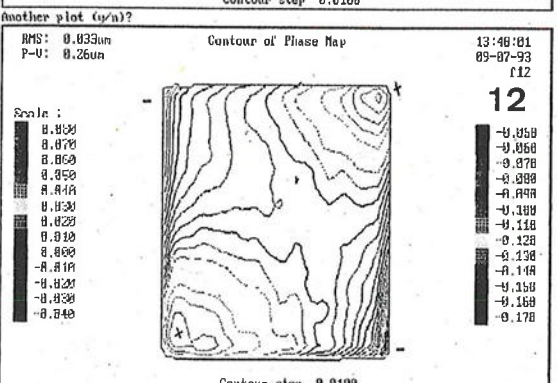
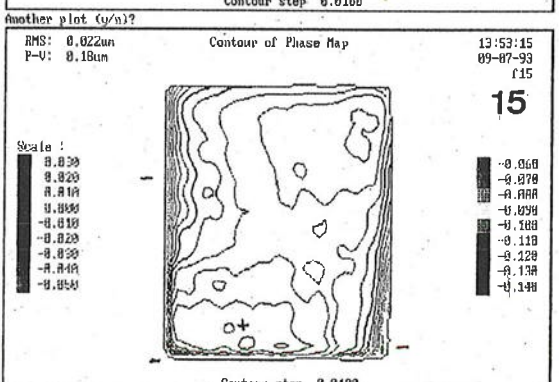
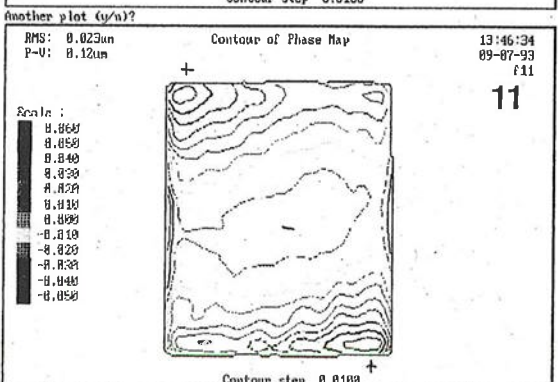
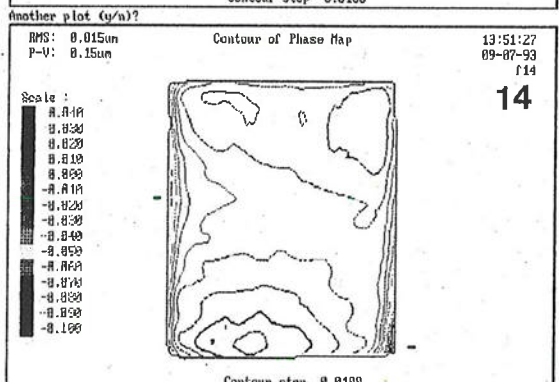
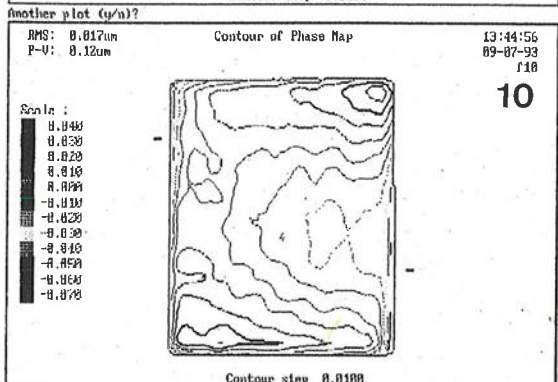
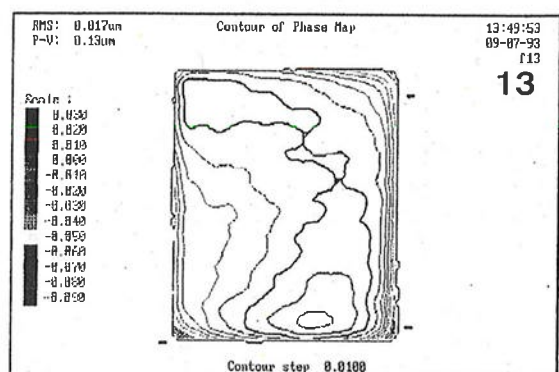
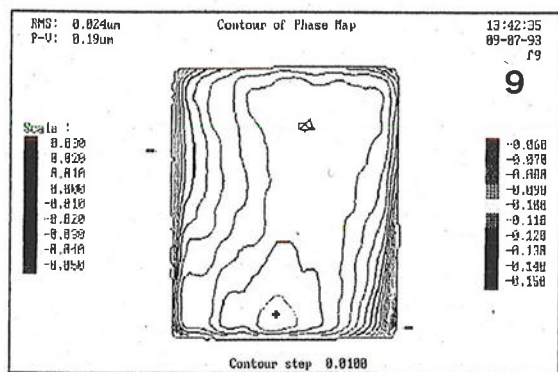


Press <ENTER> to continue!

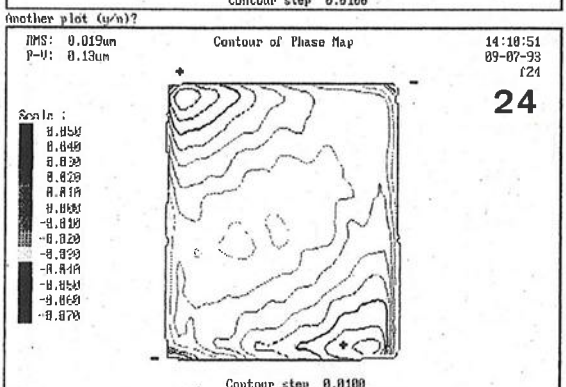
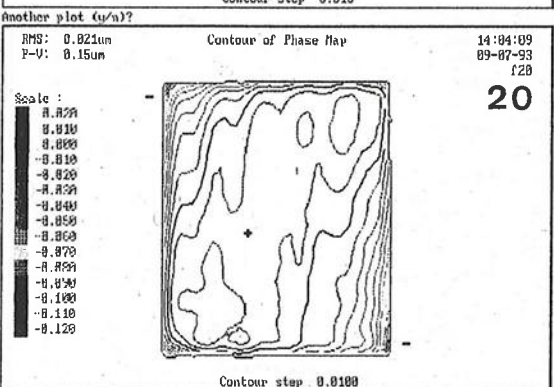
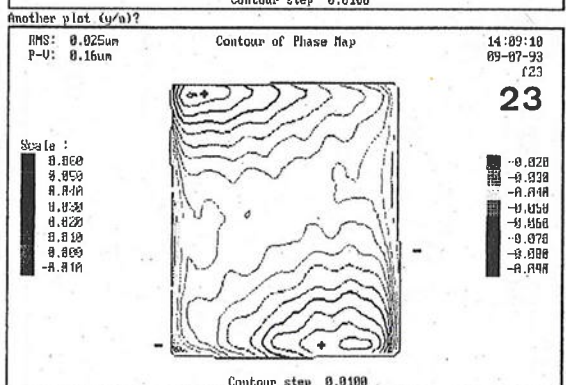
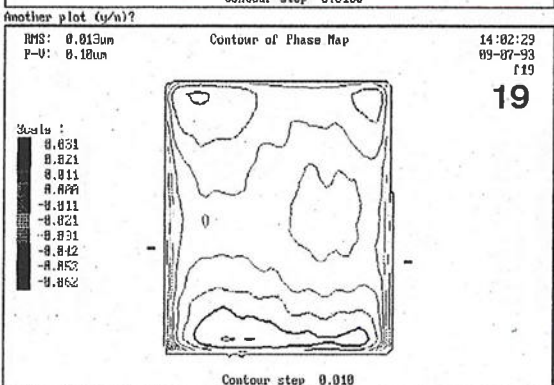
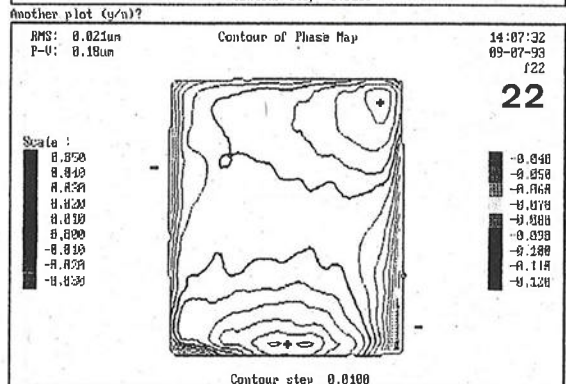
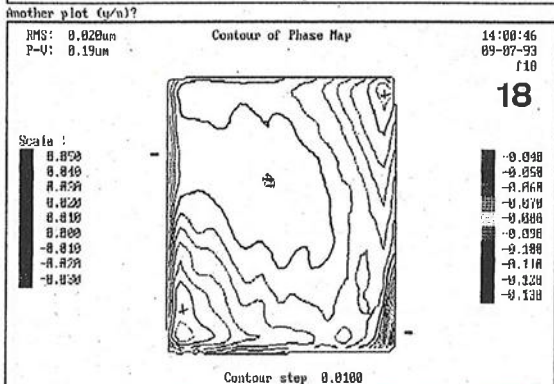
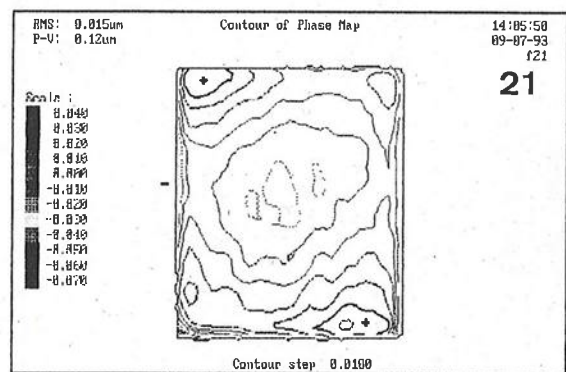
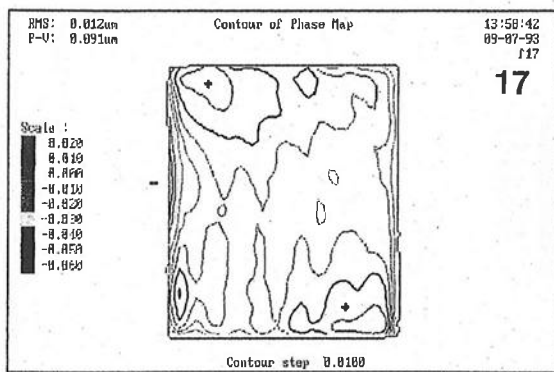


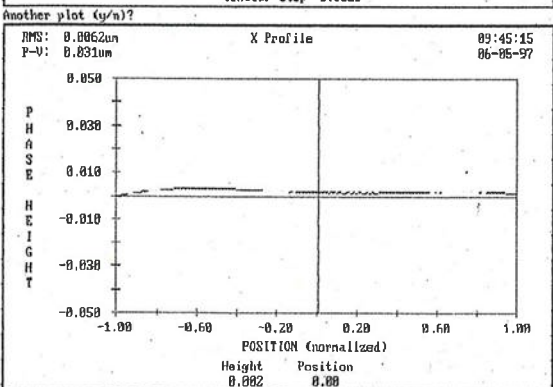
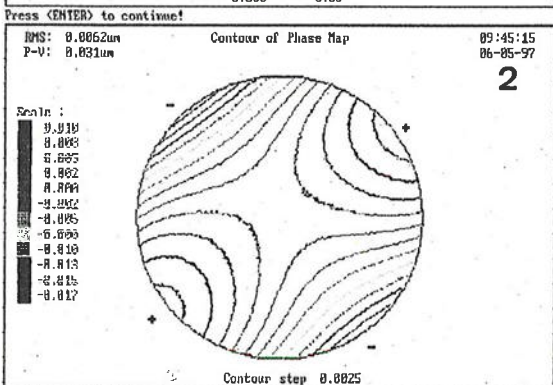
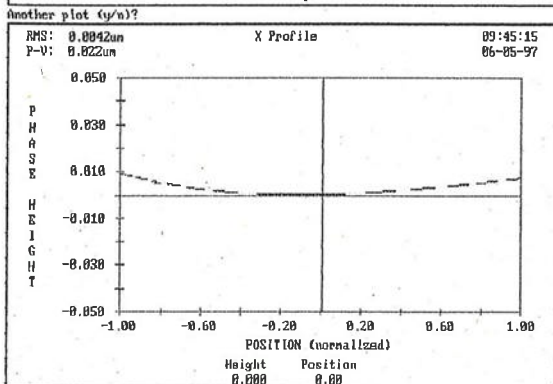
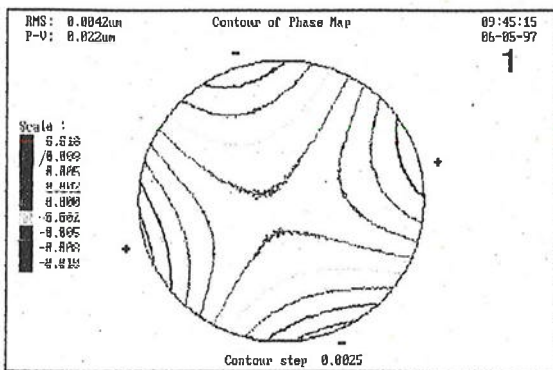
e



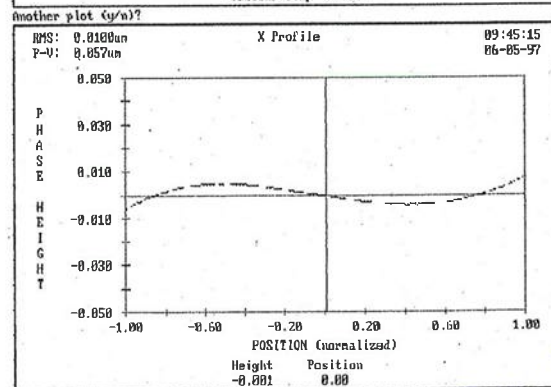
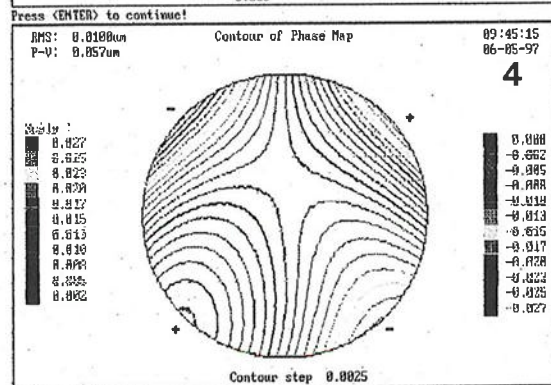
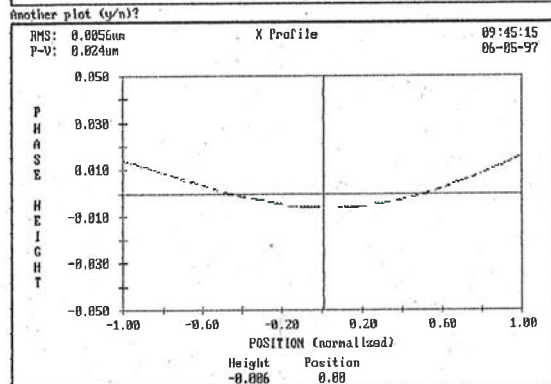
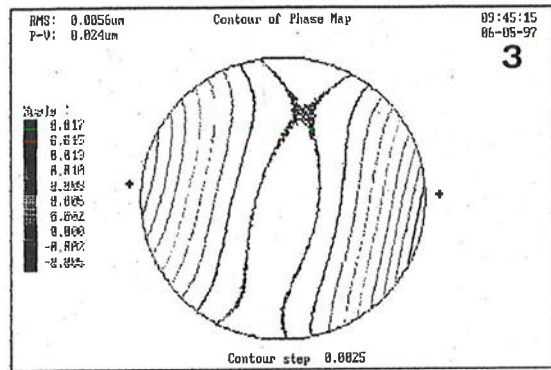








Press <ENTER> to continue!

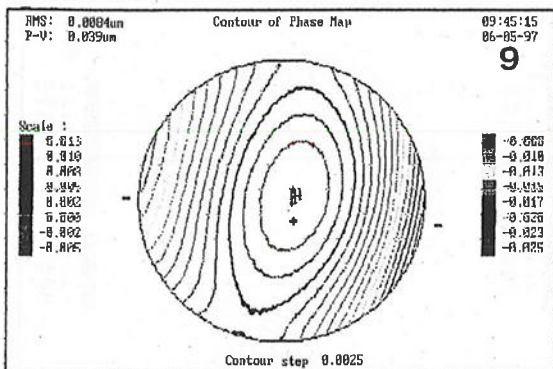


Press <ENTER> to continue!

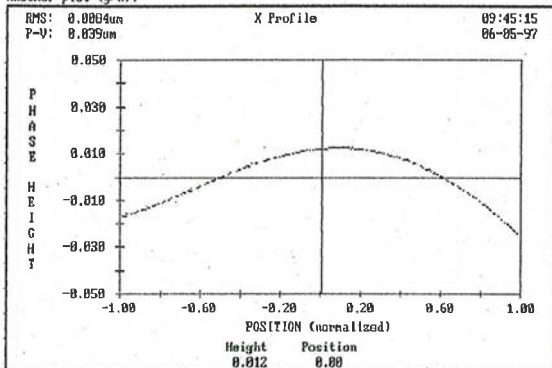




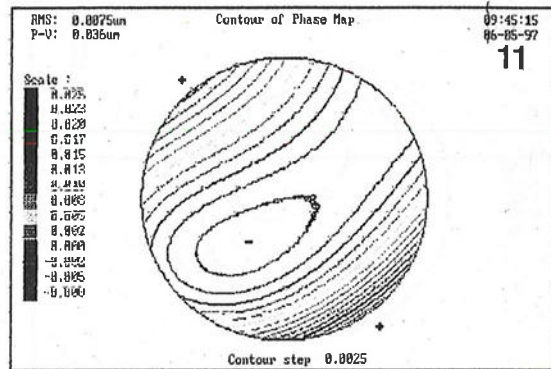




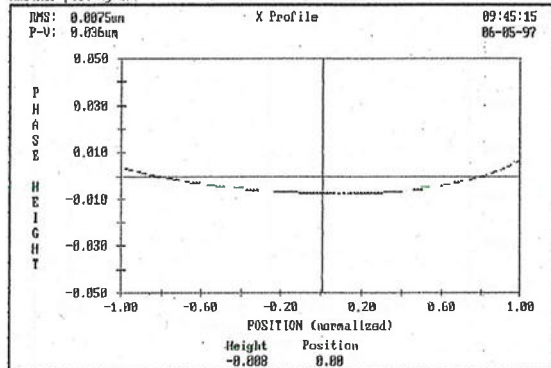
Another plot (y/n)?



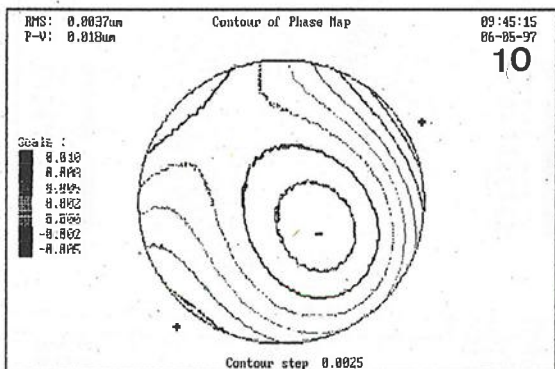
Press <ENTER> to continue!



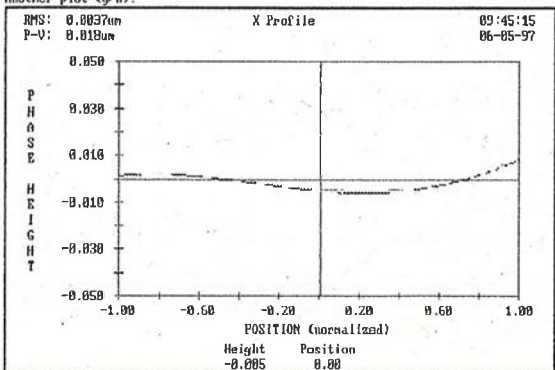
Another plot (y/n)?



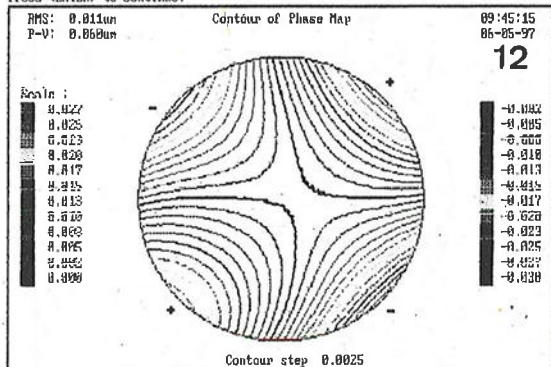
Press <ENTER> to continue!



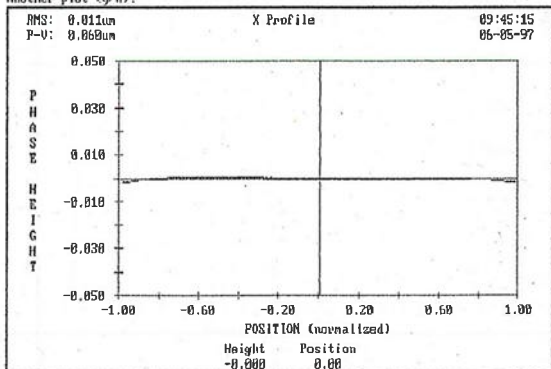
Another plot (y/n)?



Press <ENTER> to continue!

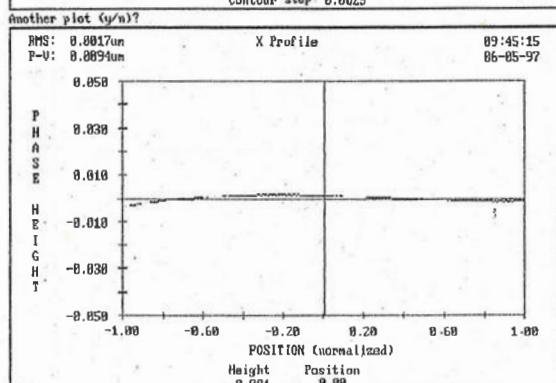
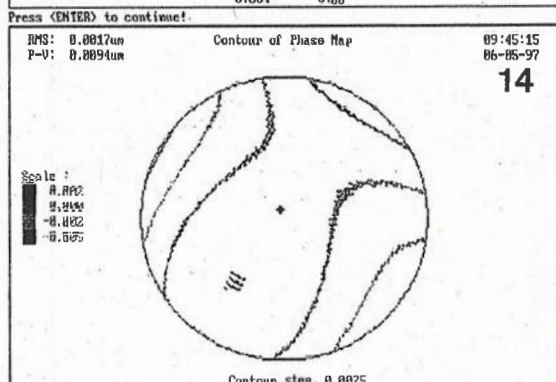
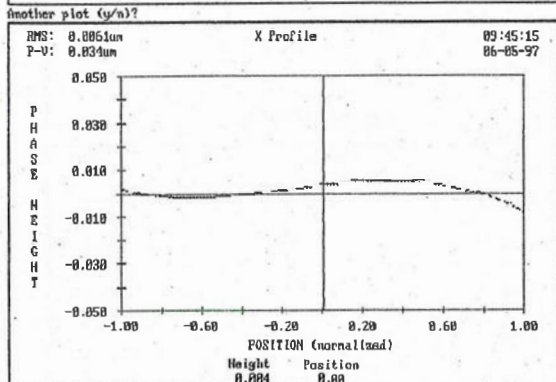
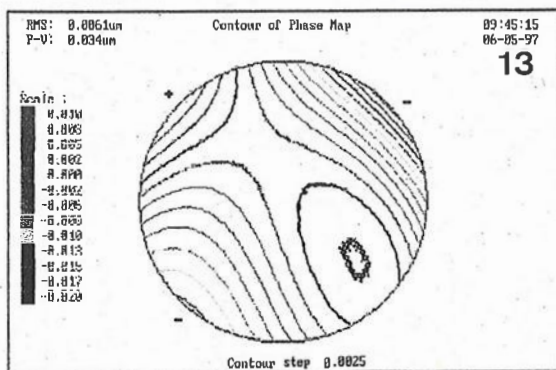


Another plot (y/n)?

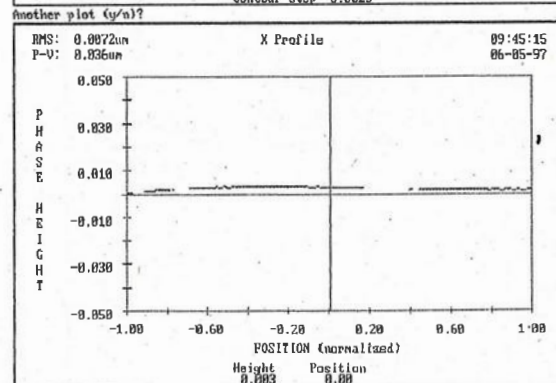
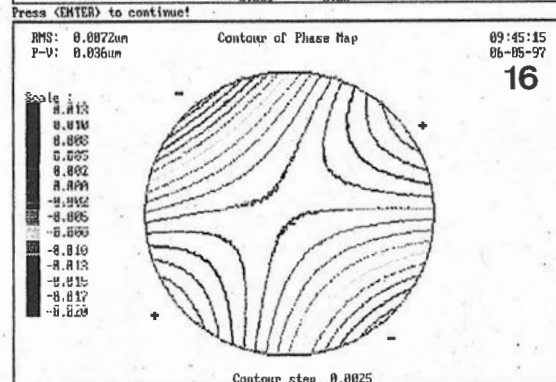
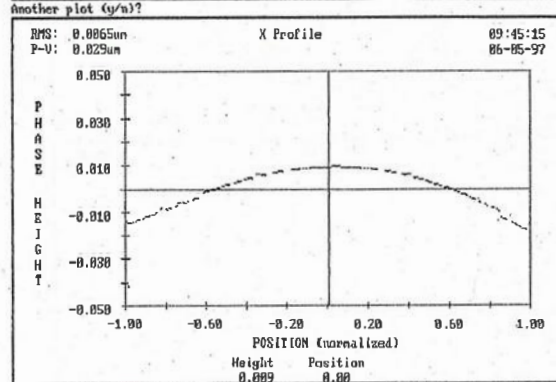
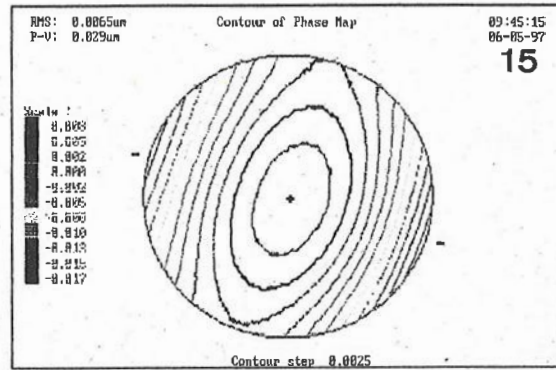


Press <ENTER> to continue!

k

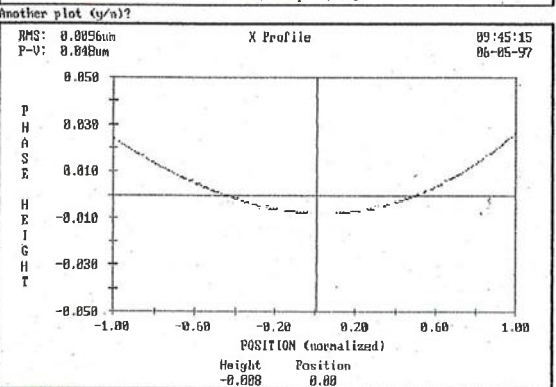
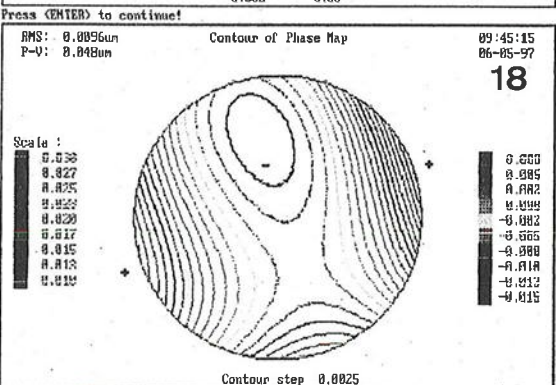
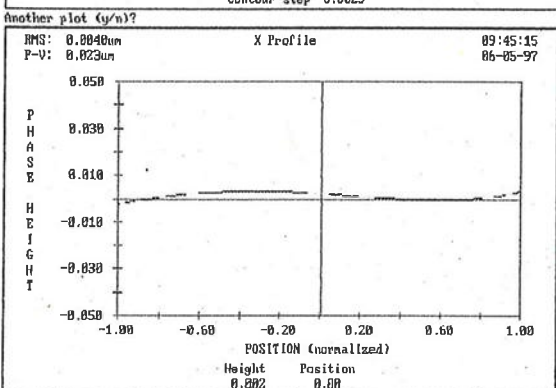
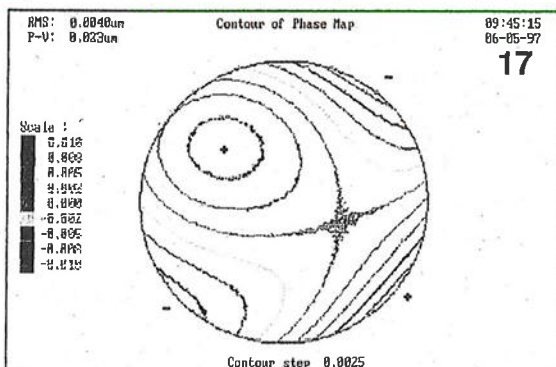


Press (ENTER) to continue!

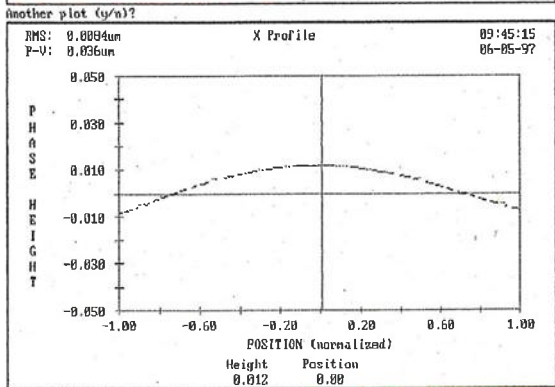
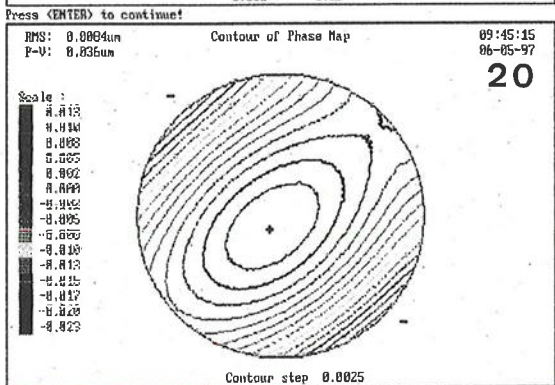
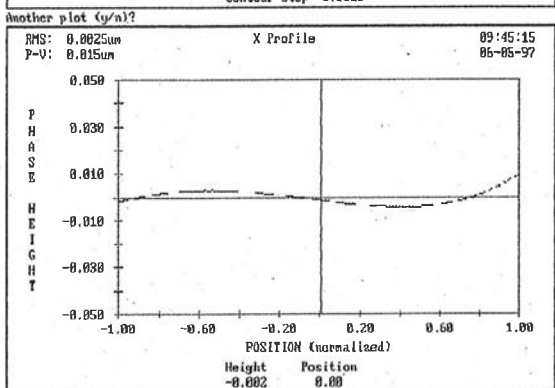
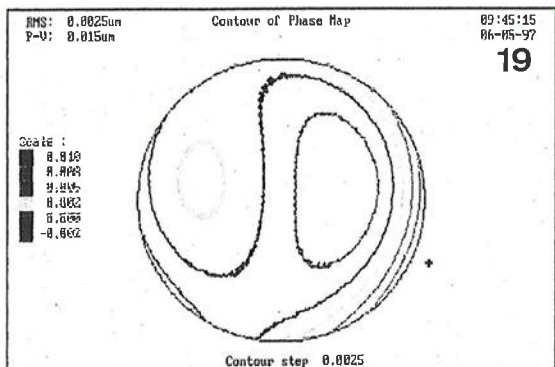


Press (ENTER) to continue!



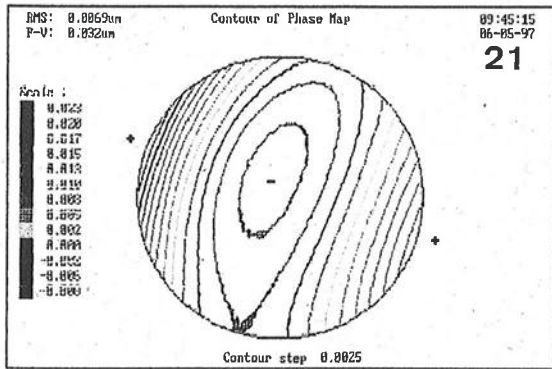


Press <ENTER> to continue!

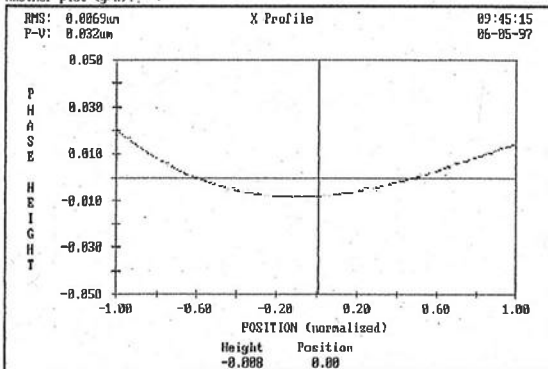


Press <ENTER> to continue!

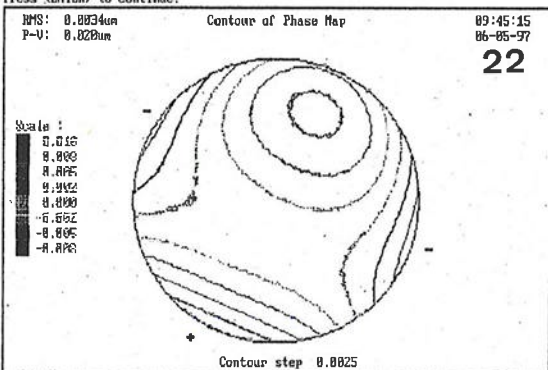




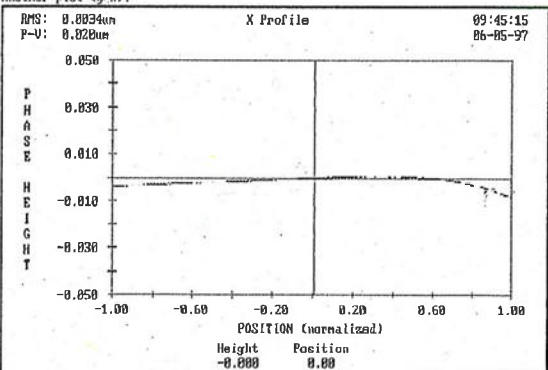
Another plot (y/n)?



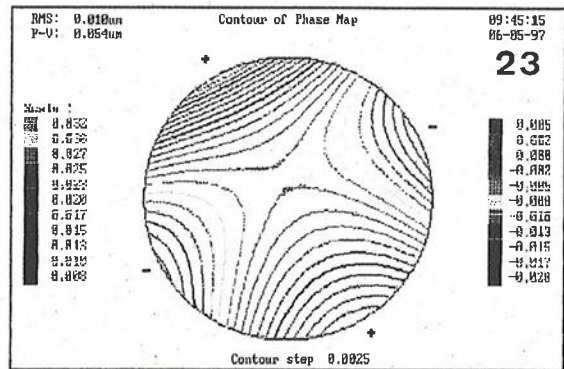
Press (ENTER) to continue!



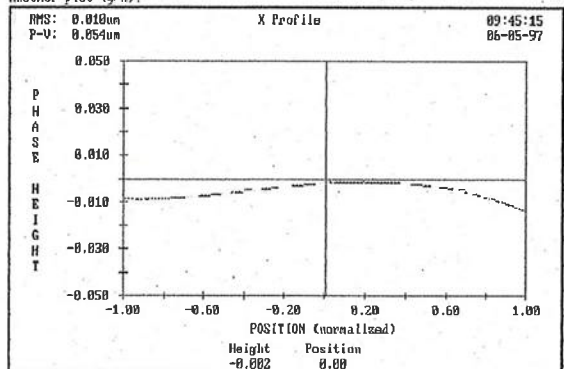
Another plot (y/n)?



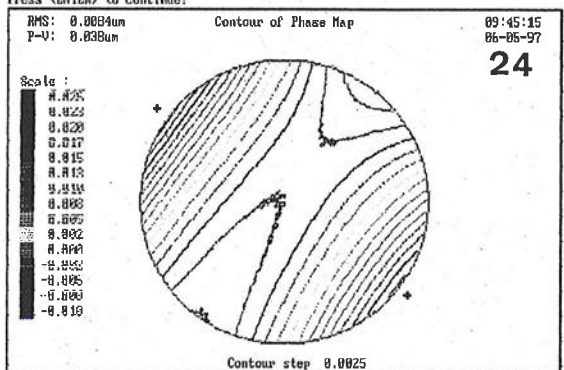
Press (ENTER) to continue!



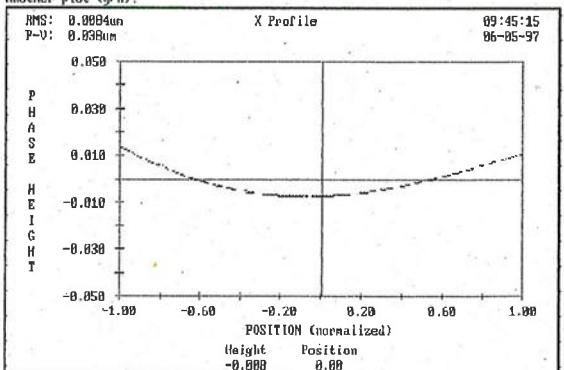
Another plot (y/n)?



Press (ENTER) to continue!



Another plot (y/n)?



Press (ENTER) to continue!