01.12.2022

№ 770/RU/18

SUPPLEMENTARY COMPARISON OF NATIONAL STANDARD INSTRUMENTS IN THE FIELD OF MAGNETIC FLUX DENSITY AND MAGNETIC FLUX MEASUREMENTS BY SENSING COILS

KCDB BIPM ID № COOMET.EM-S26 (1715)

FINAL REPORT

Pilot laboratory:

Laboratory of metrology of magnetic
measurements and nondestructive testing (261)
UNIIM – Affiliated Branch of the D.I. Mendeleyev Institute for Metrology (VNIIM – UNIIM)
(until 21.01.2020 FSUE "UNIIM")
Russia, Yekaterinburg
620075 Russia Yekaterinburg
Krasnoarmeyskaya Str, 4
Contact person:
Tatiana Maslova
Tel: +7 (343) 217-29-24
Fax: +7 (343) 350-60-26
E-mail: sergey.serdjukov@gmail.com

1 Abstract

The COOMET.EM-S26 (1715) supplementary comparison of national standard instruments in the field of magnetic flux density and magnetic flux measurements by sensing coils has been organized within the CCEM framework to test the abilities of the metrology institutes to measure magnetic flux density and magnetic flux.

Five NMIs participated in the supplementary comparison COOMET.EM-S26 (1715). Ural Research Institute for Metrology (UNIIM) (since 2020 it has been called Affiliated Branch of D.I.Mendeleev Institute for Metrology (VNIIM-UNIIM)), Russian Federation, acted as the coordinating laboratory of the comparison.

The comparison on DC magnetic flux density and magnetic flux was carried out through the measurement of the coil constant of transfer standard coils. In general, very good agreement of the results has been observed.

2 Introduction

At a meeting at the UNIIM in Ekaterinburg on the 11th and 12th of October 2016 between the representatives of UNIIM (Russia), PTB (Germany), NIM (China), the decision was made to carry out a supplementary comparison for the measurements of magnetic flux density and magnetic flux of a DC magnetic field.

Two work items have been defined:

- First: the results of the measurements shall demonstrate the degree of equivalence between the laboratories.
- Second: the capability of the measurement equipment shall be compared.

CMC classification numbers:

10.2.1 Magnetic flux: *flux meter, flux etalon*

10.2.2 DC magnetic flux density and applied magnetic field strength: *magnetic flux density meter, magnetic field strength meter*

3 Participants

Table 1. The list of participants of comparisons

№	NMI	Address	NMI abbreviation	Contact person	E-mail, tel, fax
1	Ural Scientific Research Institute for Metrology (since 2020 Affiliated Branch of the D.I.Mendeleev Institute for Metrology	Russia, 620075, Yekaterinburg, Krasnoarmeyskaya Str, 4	VNIIM-UNIIM, in this report is indicated as UNIIM	Tatiana Maslova	E-mail: sergey.serdjukov@gmail.com Tel: +7 343 217-29-24 Fax: +7 343 350-60-26
2	Physikalisch-Technische Bundesanstalt	Germany, 38116 Braunschweig Bundesallee 100	PTB	Martin Albrecht	E-mail: martin.albrecht@ptb.de
3	Czech Metrology Institute	Czech Republic, Praha 5 15072 V Botanice 4	CMI	Michal Ulvr	E-mail: mulvr@cmi.cz
4	National Physical Laboratory	United Kingdom, Hampton Road, Teddington, Middlesex, TW11 0LW	NPL	Stuart Harmon	E-mail: stuart.harmon@npl.co.uk
5	National Institute of Metrology	China, 100013, Beijing No.18 Bei San Huan Dong Lu	NIM	Jian He	E-mail: hejian@nim.ac.cn Tel: +86 10 64525432

4 Organization of the comparisons

4.1 Organization of the comparisons

The comparison is carried out by a circulation scheme. The transfer standard was produced by UNIIM (Russia) and measured with State Primary Standard (GET 198-2017). After initial measurements at UNIIM the samples were sent to: 1. PTB, 2. CMI, 3. NPL, 4. NIM, and after this returned to UNIIM for final measurements to establish the stability. After measurements the data were sent to UNIIM for evaluation of the comparison results.

4.2 The transfer standard

Reference standard of the magnetic flux unit is a set of 4 cylindrical measuring coils of different diameters. The measuring coils are cylindrical frames made of quartz glass, on which windings with a whole number of turns of copper wire are applied. The frames with the windings are placed coaxially in protective cylinders of larger diameter. The gaps between the cylindrical frame and the protective cylinder at both ends of the coil are closed. A thermal sensor is mounted in the space between the coil and the protective cylinder. The coils are applied with tension, excluding their spontaneous movement. Each measuring coil is packed in a wooden case, ensuring its safety during transportation.

5. Description of the method of measurement

During the comparisons measurements are carried out using the measurement standards of the National Metrological Institutes (NMI).

5.1 Preparation of the transfer standard for measurements

Before taking measurements, the reference standard must be kept at ambient conditions for at least 3 hours. Recommended temperature during measurements: (23 ± 3) ^{o}C. Measurement conditions must be specified in the measurement report.

5.2 Measurements of the transfer standard

The measuring coil is placed in a region of homogeneity of the device that creates a magnetic field (electromagnet, Helmholtz coil). Ensure perpendicularity of the end surface of the coil, on which there is one screw of copper alloy, to the magnetic induction power lines. Carry out a measurement of the measuring coil constant K_{sw}, Wb/T (m^2), at least 10 times and calculate the arithmetical mean.

6. Processing of results

Processing of measurement results obtained from the comparisons carried out with the use of the Guidelines for data evaluation of COOMET supplementary comparisons (COOMET R/GM/19:2016).

In processing the experimental data the following characteristics were calculated:

<u>Weighted mean</u>

- supplementary comparisons reference value X_{ref} by the formula:

$$X_{ref} = \frac{\sum_{i=1}^{N} \frac{X_i}{u^2(X_i)}}{\sum_{i=1}^{N} \frac{1}{u^2(X_i)}}, \tag{1}$$

where X_i – results of the supplementary comparisons of the i-th participant of the comparisons, Wb/T (m^2);

$u(X_i)$ – declared standard uncertainty of the i-th participant of the comparisons, Wb/T (m^2);

N – the number of the participants of comparisons;

- standard uncertainty of the reference value of the supplementary comparisons $u(X_{ref})$, Wb/T (m[2]), by the formula:

$$u(X_{ref}) = \sqrt{\frac{1}{\sum_{i=1}^{N} \frac{1}{u^2(X_i)}}}\,, \tag{2}$$

Median

- supplementary comparisons reference value X_{ref} by the formula:

$$med(x) = \begin{Bmatrix} \frac{1}{2}\left(x'_{m/2} + x'_{\frac{m}{2}+1}\right), & m\ even \\ x'_{(m+1)/2}, & m\ \ odd \end{Bmatrix}, \tag{3}$$

$$u^2(med(x)) = \frac{\pi}{2m}\hat{\sigma}^2, \tag{4}$$

$$\hat{\sigma} = 1.483 med(|d_i|), \tag{5}$$

where $d_i = x_i - med(x)$.

- value of $\tilde{E}_n$ criterion by the formula

$$\tilde{E}_n = \frac{|x_i - x_{ref}|}{2\sqrt{u^2(x) + u^2(x_{ref}) - 2cov(x, x_{ref})}}, \tag{6}$$

where $cov(x, x_{ref}) = 0$.

The declared standard uncertainties are judged as confirmed if equation

$$\tilde{E}_n < 1, \tag{7}$$

is satisfied for *i*-th participant of the comparisons.

It is suggested to use the median as a robust estimation of the reference value for coils #1 and #4 for the following reasons

- $\chi^2_{obs} > \chi^2_{0.05,m-1}$ in this case the data is inconsistent;

- uncertainty values differ slightly;

- there is one extreme value according to the criterion.

The results of processing of results obtained are shown in Tables 2 to 5.

The results of the participants of the comparison are presented in Appendices A to E.

7 Evaluation of the results

7.1 Analysis of the results of the comparisons shows that the data of national metrological institutes participating in comparisons can be recognized as consistent, which is an objective confirmation of the declared uncertainties.

7.2 The inconsistency of the data obtained on coil №4 is associated with a significant temperature dependence of the coil constant and requires additional studies.

Table 2. The results of processing the experimental data obtained on the coil №1

№	Country	NMI	Measurement result, Wb/T (m^2)	Standard uncertainty, Wb/T (m^2)	Relative extended uncertainty,%	$\tilde{E}_n$	Conclusion
1	Russia	UNIIM	0.26550	0.00027	0.20	0.51	< 1
2	Germany	PTB	0.26517	0.00020	0.15	0	< 1
3	Czech Republic	CMI	0.26539	0.00013	0.10	0.49	< 1
4	United Kingdom	NPL	0.26502	0.00012	0.09	0.34	< 1
5	China	NIM	0.26471	0.00019	0.14	0.88	< 1
Median			**0.26517**	**0.00018**	**Reference value**		
Weighted mean			0.26504	0.00006			
Checking the compatibility of results							
χ^2_{obs}			$\chi^2_{0.05,m-1}$		m	$\chi^2_{obs} > \chi^2_{0.05,m-1}$	
12.04			9.49		5	inconsistent	

Table 3. The results of processing the experimental data obtained on the coil №2

№	Country	NMI	Measurement result, Wb/T (m^2)	Standard uncertainty, Wb/T (m^2)	Relative extended uncertainty,%	$\tilde{E}_n$	Conclusion
1	Russia	UNIIM	0.52258	0.00052	0.20	0.75	< 1
2	Germany	PTB	0.52412	0.00039	0.15	0.89	< 1
3	Czech Republic	CMI	0.52307	0.00026	0.10	0.54	< 1
4	United Kingdom	NPL	0.52371	0.00024	0.09	0.60	< 1
5	China	NIM	0.52317	0.00026	0.10	0.36	< 1
Median			0.52317	0.00045			
Weighted mean			**0.52338**	**0.00013**	**Reference value**		
Checking the compatibility of results							
χ^2_{obs}			$\chi^2_{0.05,m-1}$		m	$\chi^2_{obs} > \chi^2_{0.05,m-1}$	
9.89			9.49		5	inconsistent	

Table 4. The results of processing the experimental data obtained on the coil №3

№	Country	NMI	Measurement result, Wb/T (m^2)	Standard uncertainty, Wb/T (m^2)	Relative extended uncertainty,%	$\tilde{E}_n$	Conclusion
1	Russia	UNIIM	1.2043	0.0012	0.20	0.53	< 1
2	Germany	PTB	1.2041	0.0009	0.15	0.58	< 1
3	Czech Republic	CMI	1.2030	0.0006	0.10	0.01	< 1
4	United Kingdom	NPL	1.2036	0.0013	0.21	0.24	< 1
5	China	NIM	1.2020	0.0006	0.10	0.70	< 1
Median			1.20360	0.00025			
Weighted mean			**1.20298**	**0.00035**	**Reference value**		
Checking the compatibility of results							
χ^2_{obs}			$\chi^2_{0.05,m-1}$		m	$\chi^2_{obs} < \chi^2_{0.05,m-1}$	
5.64			9.49		5	consistent	

Table 5. The results of processing the experimental data obtained on the coil №4

№	Country	NMI	Measurement result, Wb/T (m^2)	Standard uncertainty, Wb/T (m^2)	Relative extended uncertainty,%	$\tilde{E}_n$	Conclusion
1	Russia	UNIIM	19.030	0.019	0.20	0.59	< 1
2	Germany	PTB	19.013	0.016	0.17	0.30	< 1
3	United Kingdom	NPL	18.943	0.038	0.40	0.64	< 1
4	China	NIM	18.983	0.009	0.10	0.34	< 1
Median			**18.998**	**0.020**	**Reference value**		
Weighted mean			18.994	0.007			
Checking the compatibility of results							
χ^2_{obs}			$\chi^2_{0.05,m-1}$		m	$\chi^2_{obs} > \chi^2_{0.05,m-1}$	
8.29			7.81		4	inconsistent	

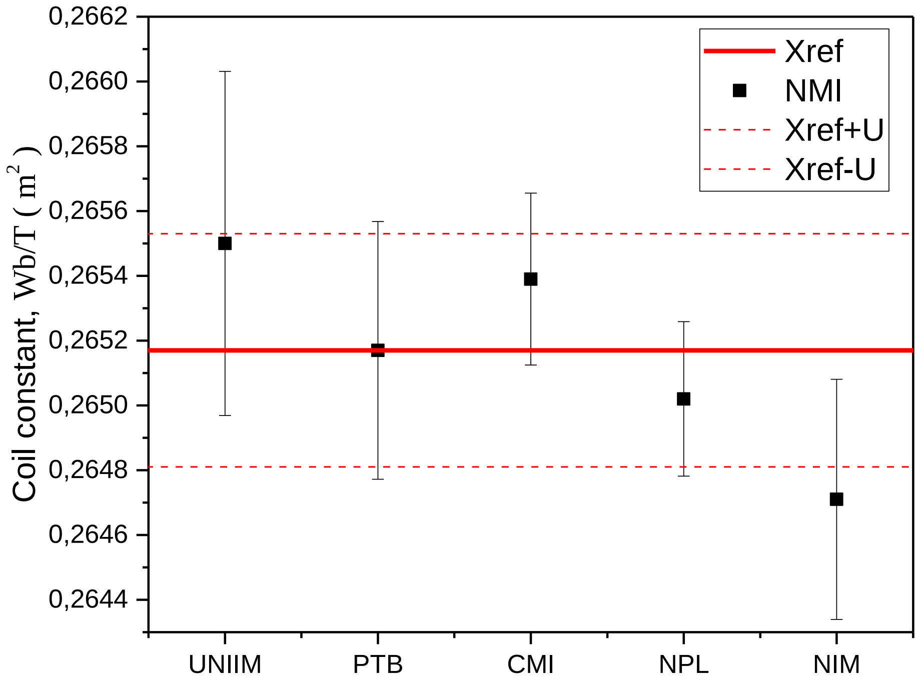


Figure 1 – The results of the comparison of the measurement of constant coil № 1

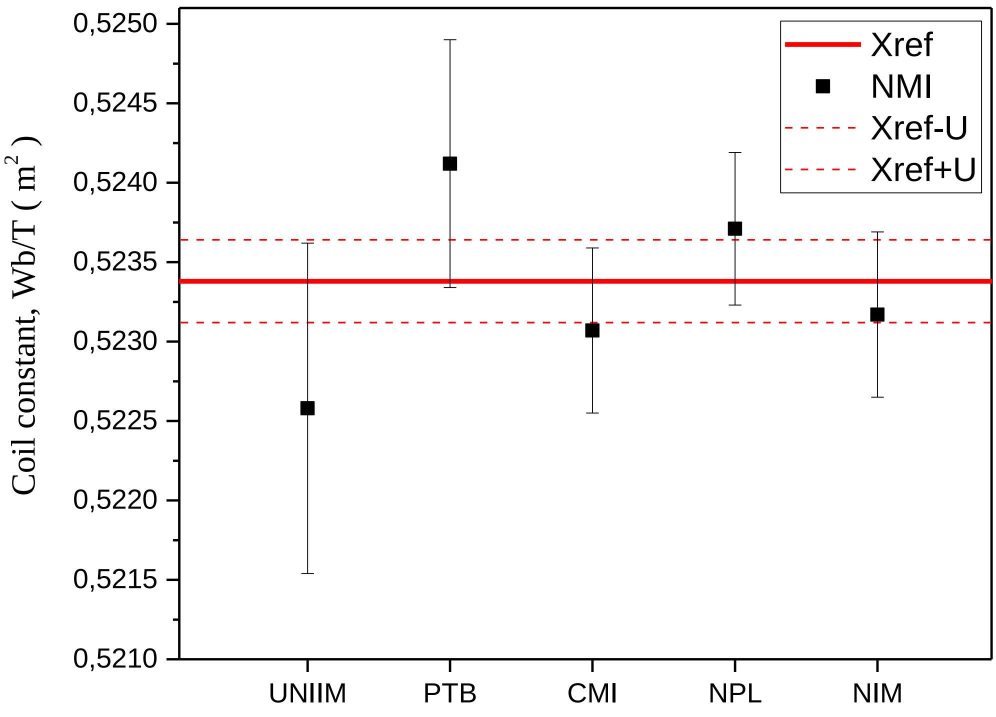


Figure 2 – The results of the comparison of the measurement of constant coil № 2

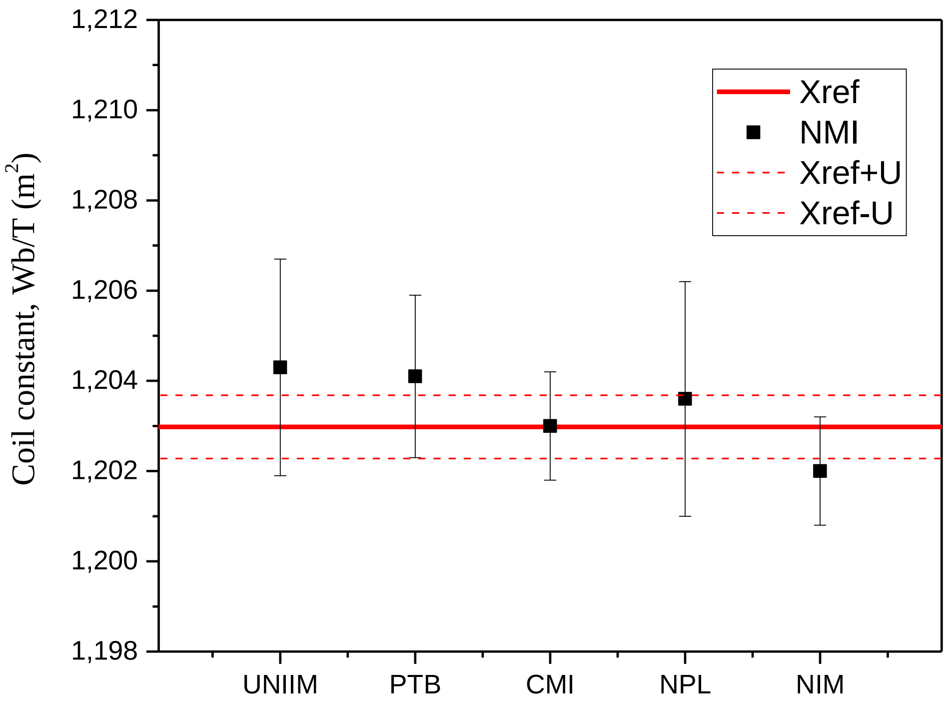


Figure 3 – The results of the comparison of the measurement of constant coil № 3

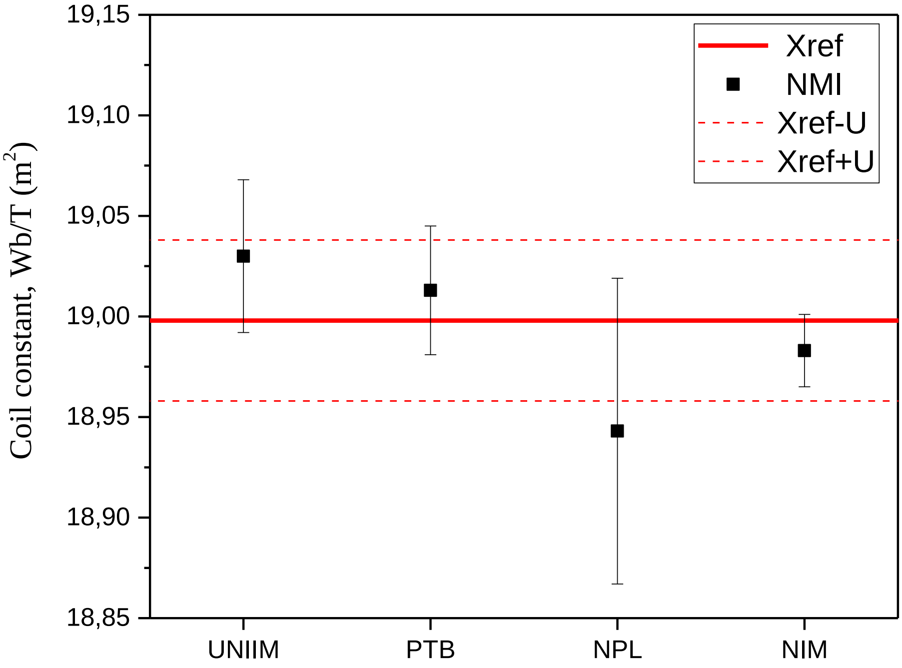


Figure 4 – The results of the comparison of the measurement of constant coil № 4

8 Conclusions

1. Fulfillment of equation (7) confirms agreement between the measurement results and estimated uncertainties of the coil constants declared by NMIs. The corresponding rows of measurement capabilities (CMCs) can be judged as confirmed.

2. Results of the comparisons (reported results of measuring of the coil constant, accompanied by expanded uncertainty) confirms equivalence of standards of units of the magnetic flux density and magnetic flux belonging to VNIIM-UNIIM (Russia), PTB (Germany), CMI (Czech Republic), NPL (United Kingdom), NIM (China).

Appendix A

CMI report of the COOMET 770/RU/18
Supplementary comparison of national standard instruments in the field of magnetic flux density and magnetic flux measurements by sensing coils

Ambient conditions

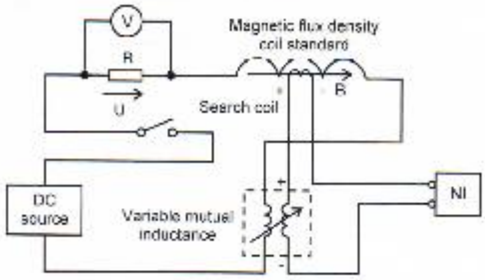
- temperature 23.8 ± 0.5 °C
- humidity 35 ± 3 %

Measurement procedure

Method used for calibration of four search coils is based on the compensation of the magnetic flux by variable mutual inductance. The search coil is placed in the center of a magnetic flux density coil standard so that the greatest magnetic flux passes through. The magnetic flux density standard is connected in series with the primary winding of the variable mutual inductance. The search coil is connected in series with the null indicator NI (a webermeter with high input resistance and high sensitivity) and, conversely, with the secondary winding of the variable mutual inductance. The value of the change in magnetic flux (caused by current shutdown) is then compensated by the variable mutual inductance. When a value is set on the variable mutual inductance such that NI shows zero, the value of search coil constant K_S is calculated from

$$K_S = \frac{K_E}{K_B},$$

where K_E (Wb/A) is the value of the variable mutual inductance determined by direct comparison with the national standard of magnetic flux and K_B (T/A) is the value of the constant of the magnetic flux density standard determined by the nuclear magnetic resonance (NMR) method with flowing water (nutation method).



Schematic diagram of the method for calibration of search coils with variable mutual inductance.

CMI equipment

We used a webermeter F191 (Etalon, St. Peterburg) with fotoelectrical amplifier and webermeter LDJ 702P as null indicators during the measurement. Standard resistor Tinsley 0.1 Ω, type 1682 with a digital multimeter 3458A were used for current measurement in primary windings of the coils (for checking only). A multi-layer Helmholtz type solenoid, no.

page 1

052 with the constant of (1.94428 ± 0.00097) mT/A was used for search coil no. 1, 2 and 3. A single-layer Helmholtz type solenoid no. 1201 with the constant of (105.684 ± 0.052) µT/A was used for search coil no. 4. Variable mutual inductance Tinsley, type 4229B was used during the measurement of all four search coils.

Measurement results

Search coil No.	Measured search coil constant K_S (m^2)	Expanded uncertainty (%)
1	0.26539	0.1
2	0.52307	0.1
3	1.2030	0.1
4	19.180	0.1

Uncertainty budget

Source of uncertainty	Type of uncertainty	value (%)	k-factor	sensitivity coefficient	std. uncertainty (%)
standard of magnetic flux density	B	0.025	1	1	0.025
standard of magnetic flux	B	0.025	1	1	0.025
influence of homogeneity of magnetic flux density standard	B	0.01	1	1	0.01
directional dependence measurement of the search coil	B	0.015	1	1	0.015
repeatability	A	0.03	1	1	0.03
standard uncertainty	-	-	1	-	0.05
expanded uncertainty	**-**	**-**	**2**	**-**	**0.10**

Appendix B

SUPPLAMENTARY

COMPARISON OF NATIONAL STANDARD INSTRUMENTS FOR MAGNETIC FLUX DENSITY AND MAGNETIC FLUX BY SENSING COILS

Report of AG2.51 of the PTB:

Measurement using Helmholtz-coil and dc-field

The dimensions of the coils were too large to place them in the electromagnet providing sufficient homogeneity. For this reason and because we wanted to use the same coil for dc and ac measurements the coil area is detected using a NMR-calibrated Helmholtz-coil and a calibrated current source ADCMT 6245 for the generation of the field. The Helmholtz-coil current was commutated to reach the two field directions. The flux was measured using a calibrated fluxmeter MPS EF5.

The results of the dc measurements were calculated using our evaluated software for flux measurements and winding area. The uncertainty was calculated by the same software in conformance with the GUM with a coverage factor k=2. Temperature 23,5°C.

coil number	Winding area in m^2	uncertainty in m^2	relative uncertainty in %, k=2, level of confidence 95%
1	0,26517	0,00040	0,15
2	0,52412	0,00079	0,15
3	1,2041	0,00180	0,15
4	19,013	0,03200	0,17

Method of the calculation of the result and uncertainty

$$A = \frac{\Delta B_S}{\Delta \varphi}$$

Model

$$A = \frac{\Delta B_S}{\Delta \varphi} \cdot \cos\phi \cdot k_{Feld} \cdot k_{Drift} \cdot (1 + \alpha \cdot \Delta T)$$

A	Winding area of the DUT
ΔB_S	difference of the flux density of the source
$\Delta \varphi$	flux difference measured using a fluxmeter
ϕ	angle of the coil in the field
k_{Feld}	correction factor for the differences in the field
k_{Drift}	factor for the drift of the fluxmeter in both directions
α	temperature coefficient of the DUT
ΔT	error of the measured temperature

Analysis of the uncertainty

Uncertainties of the measured values, sensitivity coefficients contribution to the standard uncertainty

X_i	$u(x_i)$	c_i	$u_i(y)$
x_i	$u(x_i)$	$\frac{\delta C_X}{\delta x_i}$	$c_i \cdot u(x_i)$
ΔB_S	$u(\Delta B_S)$	$\frac{A}{\Delta B_S}$	$u(\Delta B_S)\frac{A}{\Delta B_S}$
$\Delta\varphi$	$u(\Delta\varphi)$	$-\frac{A}{\Delta\varphi}$	$-u(\Delta\phi)\frac{A}{\Delta\varphi}$
ϕ	$u(\phi)$	1)	$2{,}5 \cdot 10^{-4} \cdot A$
k_{Feld}	$u(k_{Feld})$	$\frac{A}{k_{Feld}}$	$u(k_{Feld})\frac{A}{k_{Feld}}$
k_{Drift}	$u(k_{Drift})$	$\frac{A}{k_{Drift}}$	$u(k_{Drift})\frac{A}{k_{Drift}}$
α	$u(\alpha)$	$\frac{A \cdot \Delta T}{1 + \alpha \cdot \Delta T}$	$u(\alpha)\frac{A \cdot \Delta T}{1+\alpha \cdot \Delta T}$
ΔT	$u(\Delta T)$	$\frac{A \cdot \alpha}{1 + \alpha \cdot \Delta T}$	$u(\Delta T)\frac{A \cdot \alpha}{1+\alpha \cdot \Delta T}$

1) Assumption: < 2°; according to GUM < $2{,}5 \cdot 10^{-4}$

Alternative method using Helmholtz-coil and ac-field

For the second method to measure the winding area an ac measuring equipment was used. This method is not evaluated until now. We try to evaluate it during the comparison.

A coil with a measured coil constant and frequency response is used to create an ac field which is detected using the coils under test. The extrapolation of the amplitude over the frequency to dc is calculated. From this value the winding area is calculated.

coil number	Winding area In m^2	uncertainty in m^2	relative uncertainty in %, k=2, level of confidence 95%
1	0,26482	0,00027	0,2
2	0,52244	0,00052	0,2
3	1,19993	0,00120	0,2
4	18,426	0,03700	0,4

Supplementary comparison of national standard instruments in the field of magnetic flux density and magnetic flux measurements by sensing coils

1 General Information

Laboratory name:	UNIIM – Affiliated Branch of the D.I. Mendeleyev Institute for Metrology 620075, Russia, Yekaterinburg, Krasnoarmeyskaya Str, 4
Authors:	T.I. Maslova, E.A. Volegova, S.V. Serdyukov, A.S.Volegov
Report date:	

Laboratory ambient temperature:

Dates of measurement:

Comparison transfer standards:

Description:	Set of four cylindrical sensing coils
Serial numbers:	№ 1, № 2, № 3, № 4
Coil constant range:	0,26 m^2 to 19 m^2

2 Introduction

This report details are as part of a supplementary comparison in the area of the measurement of magnetic flux density and magnetic flux by sensing coil transfer standard. The aim of this comparison is to test the principle techniques and methods implemented by the NMIs of the countries participating. Additionally, to determine the degree of equivalence of the measurement standards between the NMIs.

Ambient conditions

- temperature
- humidity

3 Experimental Details

DC voltage integration method was used to determine the coil constant for the standard search coils. A magnetic flux density was produced in an electromagnet and measured using an NMR magnetometer, and an NMR resonance frequency was measured using an external frequency meter with quantum frequency measure. The search coil was connected to a calibrated integrator to determine the magnetic flux density when the flux is changed.

The search coil is fixed in the uniformity zone of the electromagnet. The uniformity zones are 12 cm in the range 0.1–1.5 T and 7 cm in the range 1.5–2 T.

A magnetic flux density 0.9 T was produced in an electromagnet, the integrator was zeroed, and then a magnetic flux density 1.0 T was produced in an electromagnet.

The coil constant was determined from the change in magnetic flux measured by the integrator and the change in magnetic flux density measured by the NMR magnetometer and the external frequency meter with quantum frequency measure.

Calculation of coil constant (Ks) is obtained by using following equation:

$$K_S = \frac{\Phi}{B_1 - B_2}$$

Measurement results

Search coil №	Measured search coil constant K_s, Wb/T	Expanded uncertainty, %
1	0.26550	0.2
2	0.52258	0.2
3	1.2043	0.2
4	19.030	0.2

Uncertainty budget

Source of uncertainty	Type of uncertainty	Value (%)		sensitivity coefficient	std. uncertainty (%)
Standard of magnetic flux density	B	0.001	1	2	0.002
Standard of magnetic flux	B	0.1	1	1	0.1
Influence of homogeneity of magnetic flux density standard	B	0.001	1	1	0.001
Repeatability	A	0.01	1	1	0.01
Standard uncertainty	-	-	1	-	0.10
Expanded uncertainty	-	-	2	-	0.20

Appendix D

Supplementary comparison of national standard instruments in the field of magnetic flux density and magnetic flux measurements by sensing coils

1. General Information

Laboratory name:	National Physical Laboratory (NPL) Hampton Road Teddington TW11 0LW United Kingdom
Authors:	Daniel Brunt and Stuart Harmon
Report date:	20th November 2019
Laboratory ambient temperature:	20 °C ± 1 °C
Measurements carried out by:	Daniel Brunt
Dates of measurement:	15 – 28 August 2019

Comparison transfer standards:

Description:	Set of four cylindrical sensing coils
Serial numbers:	№ 1, № 2, № 3 and № 4
Coil constant range:	0.26 m^2 to 19 m^2

2. Introduction

This report details NPL's results as part of a supplementary comparison in the area of the measurement of magnetic flux density and magnetic flux by sensing coil transfer standard. The aim of this comparsion is to test the principle techniques and methods implemented by the NMIs of the countries participating. Additionally, to determine the degree of equivalence of the measurement standards between the NMIs.

The effective area of the four search coil transfer standards was determined either by the mutual inductance bridge method or the extraction method. The measurement details, results and uncertainty budgets are detailed below.

3. Experimental Details

The mutual inductance bridge method was used to determine the effective area for the standard search coils № 1, № 2 and № 3, while the extraction method was used for search coil № 4.

In all cases, prior to measurements the standard search coils were allowed to stabilise for a period of at least three hours in the calibration laboratory. All measurements were carried out at an ambient temperature of 20 °C ± 1 °C.

Traceability to the SI for the measurements detailed in this report are obtained from standards realised and maintained by NPL, and include:

- Frequency
- Inductance
- Resistance via the quantized Hall effect
- Voltage via the Josephson effect

The measurement of the effective area of search coils at NPL is covered by United Kingdom Accreditation Service (UKAS) accreditation in accordance with ISO/IEC 17025:2017 'General requirements for the competence of testing and calibration laboratories'. The Best Measurement Uncertainty for the determination of effective area over the range 12 Hz to 60 Hz is ± 0.090% (k=2). NPL's entry in the BIPM Calibration and Measurement Capabilities (CMC's) database for effective area determination is equivalent to the accredited scope of measurement.

3.1 Mutual inductance bridge method

Figure 1 shows a simplified measurement circuit of the mutual inductance bridge used for the effective area determination. To limit coupling between separate components of the measurement setup, each distinct component is distributed across the calibration laboratory.

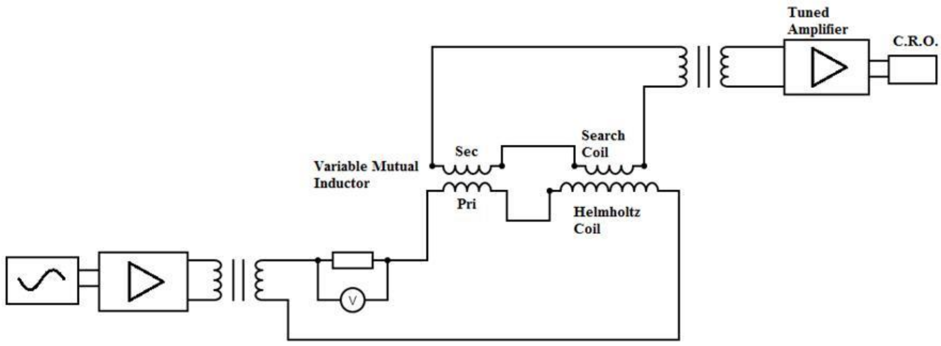


Figure 1: Simplified measurement circuit for the mutual inductance bridge method.

The search coil was positioned in a region of uniform magnetic field at the centre of a calibrated Helmholtz coil. The bridge was energised with a low frequency (20 Hz) supply and a current of 0.15 A, producing a magnetic field strength of 472.9 A/m at the centre of the Helmholtz coil. The mutual inductance between the search coil and the Helmholtz coil was balanced by the variable mutual inductor and the effective area of the search coil was calculated according to:

$$NA = \frac{M}{\mu_0\left(\frac{H}{I}\right)}$$

Where H/I is the magnetic field strength to current ratio of the Helmholtz coil (A/m/A)
M is the corrected average reading of the mutual inductor (H)
NA is the effective area of the search coil (m^2)
μ_0 is the permeability of free space ($1.256\,637\,062\,12\times10^{-6}$ N/A^2)

3.2 Extraction Method

A magnetic flux density of 0.055 T (43286 A/m) was produced in an electromagnet and measured using an NMR magnetometer. The search coil was connected to a calibrated integrator to determine the magnetic flux density when the search coil was removed from the field.

With the search coil located at an appropriate distance away from the electromagnet, the integrator was zeroed, and the search coil then inserted into a region of uniform magnetic field at the centre of the electromagnet. The integrator was zeroed again, and the search coil withdrawn to an appropriate distance away from the electromagnet.

The effective area was determined from the change in magnetic flux measured by the integrator.

4. Results

The effective area for each search coil determined by either the mutual inductance bridge method or the extraction method along with their associated measurement uncertainties are given in Table 1.

Standard coil №	Method	Effective area (m^2)	Expanded uncertainty (±%)
1	Mutual bridge	0.26502	0.09
2	Mutual bridge	0.52371	0.09
3	Mutual bridge	1.2036	0.21
4	Extraction	18.943	0.40

Table 1: Effective area and measurement uncertainties.

The reported expanded uncertainties are based on a standard uncertainty multiplied by a coverage factor $k = 2$, providing a coverage probability of approximately 95%.

5. Uncertainty Budgets

The uncertainty budgets for standard search coils № 1, № 2, № 3 and № 4 are shown in Tables 2, 3, 4 and 5 respectively.

UNCERTAINTY IN THE CALIBRATION OF SEARCH COIL № 1						
Source of Uncertainty	**Value ± %**	**Probability distribution**	**Divisor**	c_i	u_i ± %	V_i *or* V_{eff}
Calibration of Helmholtz coil	0.0500	normal	2	1	0.0250	inf.
Resolution of Mutual Inductor	0.0095	rectangular	1.7321	1	0.0055	inf.
Calibration of Mutual Inductor	0.0500	normal	2	1	0.0250	3751
MI frequency dependence	0.0050	rectangular	1.7321	1	0.0029	inf.
Interaction between parts of circuit	0	negligible	1	1	0	inf.
Non-uniformity of field	0.0020	rectangular	1.7321	1	0.0012	inf.
Coils not coaxial	0.0076	rectangular	1.7321	1	0.0044	inf.
Coils displaced from centre of axis	0.0189	rectangular	1.7321	1	0.0109	inf.
Measurement scatter	0.0095	rectangular	1.7321	1	0.0055	inf.
Repeatability	0.0208	normal	1	1	0.0208	15
Standard uncertainty					0.0435	284
Expanded uncertainty			k =	2	0.0870	
Expanded uncertainty quoted ± 0.09%						

Table 2: Uncertainty budget for the calibration of standard search coil № 1.

UNCERTAINTY IN THE CALIBRATION OF SEARCH COIL № 2						
Source of Uncertainty	**Value ± %**	**Probability distribution**	**Divisor**	c_i	u_i ± %	V_i or V_{eff}
Calibration of Helmholtz coil	0.0500	normal	2	1	0.0250	inf.
Resolution of Mutual Inductor	0.0048	rectangular	1.7321	1	0.0028	inf.
Calibration of Mutual Inductor	0.0650	normal	2	1	0.0325	10369
MI frequency dependence	0.0050	rectangular	1.7321	1	0.0029	inf.
Interaction between parts of circuit	0	negligible	1	1	0	inf.
Non-uniformity of field	0.0050	rectangular	1.7321	1	0.0029	inf.
Coils not coaxial	0.0076	rectangular	1.7321	1	0.0044	inf.
Coils displaced from centre of axis	0.0189	rectangular	1.7321	1	0.0109	inf.
Measurement scatter	0.0006	rectangular	1.7321	1	0.0003	inf.
Repeatability	0.0070	normal	1	1	0.0070	15
Standard uncertainty					0.0435	13388
Expanded uncertainty			k =	2	0.0870	
Expanded uncertainty quoted ± 0.09%						

Table 3: Uncertainty budget for the calibration of standard search coil № 2.

UNCERTAINTY IN THE CALIBRATION OF SEARCH COIL № 3						
Source of Uncertainty	**Value ± %**	**Probability distribution**	**Divisor**	c_i	u_i ± %	V_i or V_{eff}
Calibration of Helmholtz coil	0.0500	normal	2	1	0.0250	inf.
Resolution of Mutual Inductor	0.0021	rectangular	1.7321	1	0.0012	inf.
Calibration of Mutual Inductor	0.1000	normal	2	1	0.0500	46615
MI frequency dependence	0.0050	rectangular	1.7321	1	0.0029	inf.
Interaction between parts of circuit	0	negligible	1	1	0	inf.
Uniformity of field	0.0100	rectangular	1.7321	1	0.0058	inf.
Coils not coaxial	0.0076	rectangular	1.7321	1	0.0044	inf.
Coils displaced from centre of axis	0.0189	rectangular	1.7321	1	0.0109	inf.
Measurement scatter	0.1085	rectangular	1.7321	1	0.0626	inf.
Repeatability	0.0556	normal	1	1	0.0556	15
Standard uncertainty					0.1016	167
Expanded uncertainty			k =	2	0.2032	
Expanded uncertainty quoted ± 0.21%						

Table 4: Uncertainty budget for the calibration of standard search coil № 3.

UNCERTAINTY IN THE CALIBRATION OF SEARCH COIL № 4						
Source of Uncertainty	**Value ± %**	**Probability distribution**	**Divisor**	**c_i**	**u_i ± %**	***V_i or V_{eff}***
Integrator calibration						
Calibration of Mutual Inductor	0.0430	normal	2	1	0.0215	inf.
Resolution of DMM (int.)	0.0519	rectangular	1.7321	1	0.0300	inf.
1 Ohm resistor for current	0.0025	normal	2	1	0.0013	inf.
Calibration of DMM x2	0.0200	normal	2	2	0.0200	inf.
Resolution of DMM (resistor)	0.0010	rectangular	1.7321	1	0.0006	inf.
Repeatability	0.0548	normal	1	1	0.0548	9
Measurement						
Calibration of NMR probe	0.0100	normal	2	1	0.0050	inf.
Calibration of frequency meter	0.0010	normal	2	1	0.0005	inf.
Value of proton gyromagnetic ratio	0.0001	rectangular	1.7321	1	0.0001	inf.
Resolution of NMR probe for alignment	0.0022	rectangular	1.7321	1	0.0012	inf.
Uniformity of field	0.1608	rectangular	1.7321	1	0.0928	inf.
Coils not coaxial	0.2412	rectangular	1.7321	1	0.1393	inf.
Repeatability of NMR probe	0.00005	normal	2	1	0.00002	9
DMM resolution	0.0101	rectangular	1.7321	1	0.0058	inf.
DMM calibration	0.0200	normal	2	1	0.0100	inf.
Repeatability	0.0140	rectangular	1	1	0.0140	9
Standard uncertainty					0.1820	1095
Expanded uncertainty			k =	2	0.3640	
Expanded uncertainty quoted ± 0.40%						

Table 5: Uncertainty budget for the calibration of standard search coil № 4.

Appendix E

National Institute of Metrology (NIM), China

Transmission of International Comparison Results

The title of international comparison:

COMPARISON OF NATIONAL STANDARD INSTRUMENTS FOR MAGNETIC FLUX DENSITY AND MAGNETIC FLUX BY SENSING COILS

Serial number for international comparison:

KCDB BIPM ID No

Comparison experiment period: 2020/03/25-2020/04/10

Experiment reporter: Wenjie Gong, Jian He

Phone number of experiment reporter: +86-10-64525463

E-mail: wjgong@nim.ac.cn, hejian@nim.ac.cn

NIM address: No.18, Bei-San-Huan Dong Str., Beijing 100029, China

Phone/Fax number of Department of Metrology Services: +86-10-64213104

Phone/Fax number of Department of R&D and Planning (International Cooperation): +86-10-64218565

Transmission date: **2020/04/24**

COMPARISON OF NATIONAL STANDARD INSTRUMENTS FOR MAGNETIC FLUX DENSITY AND MAGNETIC FLUX BY SENSING COILS

Report on measurement results of coil constant in NIM

1. Description of the measuring method in NIM

a) Magnetic flux measurement: Fluxmeter TA102E-2
b) Magnetic field measurement: NMR Metrolab PT2025
c) Resistance: Multimeter Keithley 2001.
d) Temperature measurement: thermometer Fluke 54IIB

Statement: All measurement instruments in the test are traceable to national standard of China.

Description and composition of the standard:
Measurement is carried out by fluxmeter, NMR, digital multimeter, thermometer. Magnetic field is produced by permanent magnet in method 1 and electromagnet in method 2.

Measurement conditions: coil constant is measured at ambient temperature. Measured quantity value is coil constant, in unit of W/T or m^2.

Measurement method: Coil constant are measured by method 1 and method2, respectively.

Method 1: Measuring the coil constant by "pull out" method in a static magnetic field which is produced by permanent magnet.

Method 2: Measuring the coil constant by rotating coil 180° in a magnetic field which is produced by electromagnet and current source. The flux change is measured by fluxmeter.

Reference (average) value of *NS* is the average value *NS* in method 1 and method 2.

Temperature of coil is 20.3°C in the measurement.

Calculation of coil constant (*NS*) in the two method is obtained by using following equation.

$$NS = \frac{\Phi}{\mu_0 H} \cdot \frac{R_{coil} + R_{fluxmeter}}{R_{coil}}$$

2. Measurement results of coil constant

Results of coil constant by method 1 and method 2, and reference values (average values) of the two method are given in table 2.1 and table 2.2, respectively.

Table 2.1 Results of coil constant

Coil No.	Coil resistance (Ω)	Method 1			Method 2		
		NS(m^2)	U_{rel}(*k*=2)	*U* (m^2) (*k*=2)	*NS* (m^2)	U_{rel}(*k*=2)	*U* (m^2) (*k*=2)
1	100.793	0.26469	0.14%	0.00037	0.26471	0.14%	0.00037
2	141.833	0.52302	0.10%	0.00052	0.52317	0.10%	0.00052
3	1265.52	1.2021	0.10%	0.0012	1.2020	0.10%	0.0012
4	30249	18.973	0.10%	0.019	18.983	0.10%	0.019

Table2.2 Reference value of coil constant (Average value of NS in method 1 and method 2)

Coil No.	Coil resistance (Ω)	**Reference value(average)**		
		NS (m^2)	U_{rel} (*k*=2)	*U* (m^2) (*k*=2)
1	100.793	0.26470	0.10%	0.00015
2	141.833	0.52310	0.07%	0.00037
3	1265.52	1.2021	0.07%	0.0008
4	30249	18.978	0.07%	0.013

3. Uncertainty calculation of the coil constant are given in following tables 3.1-3.8 of uncertainty budgets.

3.1 Uncertainty budget of Method 1: Measuring the coil constant by pull out method in a static magnetic field which is produced by permanent magnet.

Table 3.1 Uncertainty budget Coil 1# in method 1

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.03%
Magnetic field stability (temperature fluctuation)		0.04%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.045%
Thermal expansion of coil		0.002%
Repeatability	0.01%	
Combined uncertainty	0.00002264%	

(Quadratic summation)	
Total combined standard uncertainty	0.068%
Expanded uncertainty k=2	0.14%
	0.00037 m^2
Level of confidence	95%

Table 3.2 Uncertainty budget Coil 2# in method 1

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (temperature fluctuation)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%
Repeatability	0.01%	
Combined uncertainty (Quadratic summation)	0.00002264%	
Total combined standard uncertainty	0.048%	
Expanded uncertainty k=2	0.10%	
	0.00052 m^2	
Level of confidence	95%	

Table 3.3 Uncertainty budget Coil 3# in method 1

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%

Magnetic field homogeneity		0.030%
Magnetic field stability (temperature fluctuation)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%
Repeatability	0.014%	
Combined uncertainty (Quadratic summation)	0.00002264%	
Total combined standard uncertainty	0.048%	
Expanded uncertainty k=2	0.10%	
	0.0012 m^2	
Level of confidence	95%	

Table 3.4 Uncertainty budget Coil 4# in method 1

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (temperature fluctuation)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%
Repeatability	0.01%	
Combined uncertainty	0.00002360%	

(Quadratic summation)	
Total combined standard uncertainty	0.049%
Expanded uncertainty *k*=2	0.10%
	0.019 m^2
Level of confidence	95%

3.2 Uncertainty budget of Method 2: Measuring the coil constant by pull out method in a static magnetic field which is produced by permanent magnet.

Table 3.5 Uncertainty budget Coil 1# in method 2

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (current source stability)		0.04%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.045%
Thermal expansion of coil		0.001%
Repeatability	0.01%	
Combined uncertainty (Quadratic summation)	0.00004639%	
Total combined standard uncertainty	0.068%	
Expanded uncertainty *k*=2	0.14%	
	0.00037 m^2	
Level of confidence	95%	

Table 3.6 Uncertainty budget Coil 2# in method 2

Relative standard uncertainty	A	B

Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (current source stability)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%
Repeatability	0.01%	
Combined uncertainty (Quadratic summation)	0.00002264%	
Total combined standard uncertainty	0.048%	
Expanded uncertainty k=2	0.10%	
	0.00052 m^2	
Level of confidence	95%	

Table 3.7 Uncertainty budget Coil 3# in method 2

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (current source stability)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%

Repeatability	0.01%	
Combined uncertainty (Quadratic summation)	0.00002264%	
Total combined standard uncertainty	0.048%	
Expanded uncertainty k=2	0.10%	
	0.0012 m^2	
Level of confidence	95%	

Table 3.8 Uncertainty budget Coil 4# in method 2

Relative standard uncertainty	A	B
Contribution due to		
NMR for magnetic field		0.00025%
Magnetic field homogeneity		0.030%
Magnetic field stability (current source stability)		0.005%
resistance (coil resistance measurement)		0.0025%
resistance (Fluxmeter resistance measurement)		0.0025%
Fluxmeter		0.035%
Thermal expansion of coil		0.001%
Repeatability	0.01%	
Combined uncertainty (Quadratic summation)	0.00002264%	
Total combined standard uncertainty	0.048%	
Expanded uncertainty k=2	0.10%	
	0.019 m^2	
Level of confidence	95%	

Reference value of coil constant is average value of method 1 and method 2. So the uncertainty of reference value is calculated using uncertainty of method 1 and uncertainty of method 2. And the uncertainty of reference value is given in above table 2.2.