BUREAU INTERNATIONAL DES POIDS ET MESURES

Bilateral comparison of 1 Ω and 10 k Ω standards (ongoing BIPM key comparisons BIPM.EM-K13) between the NPLI (India) and the BIPM

September 2022

Final report

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1 <u>Introduction</u>

A comparison of values assigned to 1 Ω and 10 k Ω resistance standards was carried out between the BIPM (Bureau International des Poids et Mesures) and the NPLI (National Physical Laboratory of India) in the period August 2021 to April 2022. Two 1 Ω and two 10 k Ω BIPM travelling standards were calibrated first at the BIPM, then at the NPLI and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: August – September 2021 NPLI measurements: December 2021 'After' measurements at the BIPM: March – April 2022

This report is organised as follows: details of the travelling standards used are listed in Section 2; the results of the BIPM measurements are given in Section 3 and the calibration report provided by the NPLI is summarized in Section 4; these two last sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in Section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given in the form of the degrees of equivalence between the NPLI and the BIPM for measurements of 1 Ω and 10 k Ω resistance standards.

This report covers the comparison of both 1 Ω standards and 10 k Ω standards (BIPM.EM-K13). The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

2 <u>Travelling standards</u>

Two 1 Ω and two 10 k Ω travelling standards provided by the BIPM were used for this comparison. The two 1 Ω standards are of CSIRO type, with working labels BIV200 (manufacturer's serial number S-64200) and BIV207 (manufacturer's serial number S-64207). The two 10 k Ω standards are TEGAM SR104 type and have the working labels B10K11 (manufacturer's serial number K205039730104) and B10K12 (serial number K201089830104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 °C and reference pressure 1013.25 hPa using the known coefficients of the standards, given in Table 1. According to the protocol, the NPLI did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM.

	Relative temperature coefficients		Relative press	Relative pressure coefficient		Relative power coefficient	
Standard #	α ₂₃ / (10 ⁻⁶ /K)	eta / (10 ⁻⁶ /K ²)	<i>u</i> _T /(10 ⁻⁶ /K)	$\frac{\gamma}{(10^{-9})}$ / hPa)	$u_{\rm P}/(10^{-9}/{\rm hPa})$	$P/(10^{-9}/\text{ mW})$	$\frac{u_{\rm W}}{(10^{-9}/{\rm mW})}$
BIV200	- 0.0074	+0.0004	0.001	- 0.100	0.200	- 2.0	1.5
BIV207	- 0.0094	+ 0.0001	0.001	- 0.250	0.200	- 2.0	1.5
B10K11	- 0.0700	- 0.0270	0.010	- 0.350	0.100	+ 2.4	2.4
B10K12	+ 0.0100	- 0.0230	0.010	- 0.226	0.100	+ 1.0	2.4

 Table 1: Temperature, pressure and power coefficients of the traveling standards.

3 <u>Measurements at the BIPM</u>

3.1 <u>Measurement of the 1 Ω standards at the BIPM:</u>

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values included in this comparison. In all cases, values are based on the revised SI value of the von Klitzing constant, $R_{\rm K} = h/e^2 = 25$ 812.807 46 Ω , using the fixed numerical values for the Planck constant *h* and the elementary charge *e*.

The 1 Ω measurements were carried out by comparison with a 100 Ω reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1 Ω resistors.

The 1 Ω travelling standards were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of 23 °C. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT) in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The additional pressure P_h exerted by the volume of the mineral oil above the resistors (reference plane corresponding to the plane containing the resistor terminals) has been considered for every measurement. P_h is calculated using the following equation:

$P_h = RD \times \rho \times g \times h$

With *RD* the relative density of the oil Marcol 52 type = 0.83; ρ the density of the pure water = 1000 kg m⁻³ at 4 °C; *g* the local gravity = 9.807 m s⁻² and *h* the height of the oil above the reference plane in m. The height of the oil above the reference plane is recorded in the software of the measurement bridge and the additional pressure is calculated automatically at every measurement.

The 'dc' resistance value (or ratio) measured with the BIPM CCC-bridge results from a current signal passing through the resistors having polarity reversals with a waiting time between polarity inversions, cf. Figure 1. The polarity reversal frequency is of the order of 3 mHz (340 s cycle period) and the measurements are sampled only during 100 s before the change of polarity.



Figure 1: Schematic representation of the reference current signal with polarity reversals used in the BIPM CCC-bridge. The reversal cycle comprises a waiting time of about 36 s at zero current (green dotted line). The red dotted line corresponds to the sampling time period.

The travelling standards were measured 11 times during the period labelled 'before' (August 2021 – September 2021) and 11 times during the period labelled 'after' (March 2022 – April 2022).

The individual BIPM measurement data are plotted in Figures 2 and 3 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 1 Ω value $~(\mu\Omega/\Omega)$						
Standard #	BEFORE	Std dev.	AFTED	Std dev.	INTERPOLATED	Std dev.	
	DEFORE	$u_{1\mathrm{B}}$	AFTER	$u_{1\mathrm{A}}$		u_1	
BIV200	- 0.713	0.008	- 0.723	0.004	- 0.718 0N 22-12-2021	0.005	
BIV207	- 0.420	0.009	- 0.418	0.005	- 0.419 ON 22-12-2021	0.005	

Table 2: Summary of BIPM calibrations of the 1 $\boldsymbol{\Omega}$ standards.

Source of uncertainty	Relative standard uncertainty $(n\Omega/\Omega)$
Imperfect realisation of $R_{\rm K}$	2
Calibration of the BIPM 100 Ω reference (BI100-3) against $R_{\rm K}$	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the (1 Ω / BI100-3) ratio	8
Temperature correction for the 1 Ω standard	2
Pressure correction for the 1 Ω standard	3
Combined standard uncertainty <i>u</i> ₂	16

Table 3: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

3.2 Measurement of the 10 k Ω standards at the BIPM:

The 10 k Ω measurements were carried out by comparison with a set of two 10 k Ω reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 k Ω travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % (\pm 10 %).

The travelling standards were measured 13 times during the period labelled 'before' (August 2021 – September 2021) and 11 times during the period labelled 'after' (March 2022 – April 2022).

The individual BIPM measurement data are plotted in Figures 4 and 5 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 10 k Ω value ($\mu\Omega/\Omega)$						
Standard #	REFORE	Std dev.	ΛΕΤΕΡ	Std dev.	INTERPOLATED	Std dev.	
Stanuaru #	DEFORE	$u_{1\mathrm{B}}$	AFILK	u_{1A}		u_1	
B10K11	+ 1.309	0.002	+ 1.286	0.002	+ 1.298 ON 22-12-2021	0.001	
B10K12	+ 1.111	0.002	+ 1.057	0.002	+ 1.085 ON 22-12-2021	0.001	

Table 4: Summarv	of BIPM	calibrations	of the	10 I	kΩ standards.
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Source of uncertainty	Relative standard uncertainty $(n\Omega/\Omega)$
Imperfect realization of $R_{\rm K}$	2
Calibration of the BIPM 100 Ω reference (BI100-3) against $R_{\rm K}$	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 Ω / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of $10 \text{ k}\Omega$ reference	8
Measurement of the voltage applied to the bridge	5
Measurements of the bridge unbalance voltage	5
Leakage resistances	1
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined standard uncertainty <i>u</i> ₂	15

Table 5: BIPM uncertaint	y budget for the	calibration of the	10 k Ω travelling standards.
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4 <u>Measurements at the NPLI</u>

4.1 <u>Preparation:</u>

The NPLI received the standard resistors on 26th October 2021 and a visual inspection was done to assess their physical conditions. The 1 Ω and 10 k Ω travelling standards were kept in a temperature-controlled air bath (Guildline 5032) at a temperature of (23.00 ± 0.02) °C and were allowed to stabilise for one week prior to the measurements.

4.2 <u>Traceability of DC resistance standard at NPLI :</u>

The resistance standards of the NPLI are traceable to a Quantum Hall Resistance (QHR) standard. Dissemination of resistance values from the QHR is done using a 1 k Ω reference standard (Guildline, model 7330) with a transfer uncertainty of 0.08 $\mu\Omega/\Omega$, using a Direct Current Comparator (DCC) bridge (Measurement International, Model 6010Q) for calibration. The calibration down to 1 Ω was achieved with repetitive use of 10:1 ratio. The information related to the standards used in the comparison is given in the Table 6.

Name	Manufacturer	Model No.	Serial No.	Traceability
10 Ω Standard resistor	Guildline	7330	72972	NPL, India
1 kΩ Standard resistor	Guildline	7330	72910	NPL, India
DCC bridge	Guildline	6622A-HV	72917	NPL, India

4.3 <u>Method of calibration:</u>

The calibrations of the travelling standards (R_x) are carried out by comparison to the reference standards (R_s) of NPLI using the DCC bridge. The ratio (R_x/R_s) method is used for comparison.

The 1 Ω travelling standards (R_x) measurements were carried out by comparison with a 10 Ω reference resistor (R_s). The comparison was performed using the DCC bridge operating at test current (I_x) of 50 mA.

The 10 k Ω travelling standards (R_x) measurements were carried out by comparison with a 1 k Ω reference resistor (R_s) using the DCC bridge operating at test current (I_x) of 100 μ A.

The travelling standards were measured 15 times during the period December 2021.

4.4 **Operating conditions:**

The 1 Ω and 10 k Ω travelling standards were kept in a temperature-controlled air bath (Guildline 5032) at a temperature (23.00 ± 0.02) °C. The temperature inside the air bath was monitored and recorded using a thermo-hygrometer (FLUKE 1620A). The relative humidity was 50 %RH (±10 %RH) over the entire period of measurements.

The atmospheric pressure did vary from 989 hPa to 992 hPa (± 2 hPa) during the period of measurements. It was monitored using a calibrated digital barometer (Druck PTX5072).

The reversal rate for the measurement is 20 s for 1 Ω (R_x) and 60 s for 10 k Ω (R_x). This reversal rate is set on the basis of normal ohm mode test setup instructions from the manual of the DCC bridge (Guildline 6622 A-HV).

4.5 <u>NPLI results at 1 Ω </u>:

The 1 Ω travelling standards were measured 15 times for both BIV200 and BIV207 in the period December 2021. The measurements were made at 50 mA. Power correction is not needed as applied current at NPLI and BIPM are comparable.

Table 7 gives the mean values at the mean date of 22^{nd} of December 2021 for BIV200 and BIV207 before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 1 Ω value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean pressure at the reference plane / hPa
BIV200	- 0.371	0.174	22.98	990
BIV207	-0.028	0.107	22.98	990

4.5.1 Corrections for temperature and pressure differences:

The value R(23) of the resistance corrected to $T_0 = 23$ °C is: $R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$ where R(T) is the resistance of the standard at temperature T.

The value R(1013.25) of the resistance corrected to $P_0 = 1013.25$ hPa is: $R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$ where R(P) is the resistance of the standard at pressure P.

The NPLI results are corrected to the reference temperature and the reference pressure using the coefficients α_{23} , β and γ shown in Table 1. Applied corrections are reported in Table 8.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa					
	Relative corrections ($\mu\Omega/\Omega$)				
Standard #	For temperature	For pressure			
BIV200	-0.000	- 0.002			
BIV207	- 0.000	- 0.006			

Table 8: Corrections applied to the NPLI 1 Ω results.

To take into account the differences between temperature measurement methods as the NPLI uses an air bath rather than an oil bath for its measurements and because the thermometer is not positioned in the center of the resistor as for the BIPM, we increase the uncertainty on the temperature measurement. The stability of the air bath (from the Guildline 5032 datasheet) is ± 0.015 °C over 24 hours for a stability of ± 2 °C of the ambient temperature. The temperature uniformity is ± 0.2 °C relative to the chamber center. We consider that the difference between the temperature taken in the thermometer well of the resistor and the temperature on a wall of the bath could be comparable to the temperature uniformity inside the air bath. The uncertainty on the temperature measurement has been increased to 0.2 °C instead of 0.02°C

to cover this effect.

The standard uncertainties of the temperature and pressure measurements at the NPLI are then 0.2 °C and 2 hPa respectively. Taking into account the differences from the reference

temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\text{Temp}} = 0.002 \times 10^{-6}$ and $u_{\text{Press}} = 0.005 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.005 \times 10^{-6}$. u_3 is reported in Table 10.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

Uncertainty budget provided by the NPLI: 4.5.2

Table 9 shows the uncertainty budget (type B) provided by the NPLI for the 1 Ω travelling standard measurements.

Source	± Limits	units	Probability Distribution & Type	Standard uncertainty u(i)	Temperature Coefficient in ppm/°C	Sensitivity Coefficient (ci)	Standard uncertainty (u(i) *c(i)) in μΩ/Ω	Degree of freedom (yi)
Uncertainty of standard resistor R_s	0.7	μΩ/Ω	Normal, B	0.35		1	0.35	Infinity
Temperature deviation considered for standard resistor <i>R</i> _s	0.02	°C	Rectangular, B	0.01	0.2	1	0.002	Infinity
Instability & Linearity of DCC bridge	0.4	μΩ/Ω	Rectangular, B	0.23		1	0.23	Infinity
Drift of standard R_s	0.1	μΩ/Ω	Rectangular, B	0.06		1	0.06	infinity
Combined relative standard uncertainty (k = 1) ($\mu\Omega/\Omega$)	0.423							

uncertainty (k = 1) ($\mu\Omega/\Omega$)

Table 9: NPLI uncertainty budget for the calibration of the 1 Ω travelling standard.

Uncertainties associated with the measurement of 1 Ω resistors: 4.5.3

Table 10 shows the corrected measurements of the 1 Ω standards at NPLI at the mean date of 22nd of December 2021 for BIV200 and BIV207 as well as the associated uncertainty components.

NPLI results	Relative difference from	stand	Relative lard uncertai	nties
after corrections	nominal value (μΩ/Ω)	Repeatability $u_1 (\mu \Omega / \Omega)$	Systematic $u_2 (\mu \Omega / \Omega)$	Corrections $u_3 (\mu \Omega / \Omega)$
BIV200	-0.373	0.174	0.423	0.005
BIV207	-0.034	0.107	0.423	0.005

Table 10: Summary of the NPLI results at 1 Ω , after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 10 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.

4.6 <u>NPLI results at 10 k Ω </u>:

The 10 k Ω travelling standards were measured 15 times for both B10K11 and B10K12 in the period December 2021. The measurements were made at 100 μ A. Power correction is not needed as applied currents at NPLI and BIPM are comparable. Table 11 gives the mean values at the mean date of 22nd of December 2021 for B10K11 and B10K12, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 10 kΩ value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean atmospheric pressure / hPa
B10K11	+ 1.582	0.047	22.98	990
B10K12	+ 1.426	0.036	22.98	990

Table 1	1: S	ummary	of N	PLI 1	10 k	Ωc	alibrat	ions.

4.6.1 Corrections for temperature and pressure differences:

The value R(23) of the resistance corrected to $T_0 = 23$ °C is: $R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$ where R(T) is the resistance of the standard at temperature T.

The value R(1013.25) of the resistance corrected to $P_0 = 1013.25$ hPa is: $R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$ where R(P) is the resistance of the standard at pressure P.

The NPLI results are corrected to the reference temperature and the reference pressure using the coefficients α_{23} , β and γ shown in Table 1. Applied corrections are reported in Table 12.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa					
	Relative corrections ($\mu\Omega/\Omega$)				
Standard #	Standard # For temperature For pressure				
B10K11 - 0.001 - 0.008					
B10K12	B10K12 + 0.000 - 0.005				

Table 12: Corrections applied to the NPLI 10 k Ω results.

NPLI and the BIPM are using air baths for 10 k Ω measurements. The remaining difference is the definition point of the taken temperature. As at the NPLI the thermometer is not placed in the resistor temperature well as for the BIPM, we increase the uncertainty on the temperature measurement. The stability of the air bath (from the Guildline 5032 datasheet) is \pm 0.015 °C over 24 hours for a stability of \pm 2 °C of the ambient temperature. The temperature uniformity is \pm 0.2 °C relative to the chamber center. We consider that the difference between the temperature taken in the thermometer well of the resistor and the temperature on a wall of the bath could be comparable to the temperature uniformity inside the air bath.

The uncertainty on the temperature measurement has been increased to 0.2 $^{\circ}$ C instead of 0.02 $^{\circ}$ C to cover this effect.

The standard uncertainties of the temperature and pressure measurements at the NPLI are 0.2 °C and 2 hPa respectively. Taking into account the differences from the reference temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure difference corrections applied by the BIPM are estimated to be $u_{\text{Temp}} = 0.014 \times 10^{-6}$ and $u_{\text{Press}} = 0.002 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_{3\text{A}} = 0.014 \times 10^{-6}$ for B10K11. For B10K12, $u_{\text{Temp}} = 0.002 \times 10^{-6}$ and $u_{\text{Press}} = 0.002 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_{3\text{B}} = 0.003 \times 10^{-6}$ and $u_{\text{Press}} = 0.002 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_{3\text{B}} = 0.003 \times 10^{-6}$ is reported in Table 14 and has been chosen to be: $u_3^2 = u_{3A}^2 + u_{3B}^2$

4.6.2 Uncertainty budget provided by the NPLI:

Table 13 shows the uncertainty budget (type B) provided by the NPLI for the 10 k Ω travelling standard measurements.

Source	± Limits	units	Probability Distribution & Type	Standard uncertainty u(i)	Temperature Coefficient in ppm/°C	Sensitivity Coefficient (ci)	Standard uncertainty (u(i) *c(i)) in μΩ/Ω	Degree of freedom (yi)
Uncertainty of standard resistor $R_{\rm s}$	0.08	μΩ/Ω	Normal, B	0.04		1	0.04	Infinity
Temperature deviation considered for standard resistor <i>R</i> _s	0.02	°C	Rectangular, B	0.01	0.2	1	0.002	Infinity
Instability & Linearity of DCC bridge	0.04	μΩ/Ω	Rectangular, B	0.02		1	0.02	Infinity
Drift of standard $R_{\rm s}$	0.1	μΩ/Ω	Rectangular, B	0.06		1	0.06	infinity
Combined relative standard	0.075							

uncertainty (k = 1) ($\mu\Omega/\Omega$) 0.075

Table 13: Uncertainty budget for the calibration of the 10 k Ω travelling standards.

4.6.3 Uncertainties associated with the measurement of 10 k Ω resistors:

Table 14 shows the corrected measurements of the 10 k Ω standards at NPLI as well as the uncertainty components associated with these measurements.

NPLI results after corrections	Relative difference from	Relative standard uncertainties			
	nominal value (μΩ/Ω)	Repeatability $u_1 (\mu \Omega / \Omega)$	Systematic $u_2 (\mu \Omega / \Omega)$	Corrections $u_3 (\mu \Omega / \Omega)$	
B10K11	+1.573	0.047	0.075	0.014	
B10K12	+1.421	0.036	0.075	0.014	

Note: The distinction between 'systematic' and 'repeatability' is made in Table 14 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.

5 <u>Comparison NPLI – BIPM</u>

The individual measurement results for each of the four standards are shown in Figures 2 to 5. The plots also show the mean value of the NPLI measurements with the uncertainty bar corresponding to the expanded uncertainty (k = 2) of the comparison U_c provided below, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of Figures 2 to 5 indicates that this is an appropriate model as both 1 Ω standards and 10 k Ω standards fit this model well. We treat the 1 Ω and 10 k Ω results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of the NPLI measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of the NPLI measurements.

The difference between the NPLI and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\text{NPLI},i} - R_{\text{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\text{NPLI-BIPM}} = \frac{1}{2} \sum_{i=1}^{2} \left(R_{\text{NPLI},i} - R_{\text{BIPM},i} \right)$$

For each standard, the uncertainty u_1 associated with the interpolated BIPM value is calculated from the linear fit, as shown in Tables 2 and 4; u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard R_i , the BIPM uncertainty $u_{\text{BIPM},i}$ is obtained from: $u_{\text{BIPM},i}^2 = u_{1,i}^2 + u_{2,i}^2$. When the mean (for two standards) of the NPLI – BIPM relative difference is calculated, the BIPM contribution to the uncertainty is,

$$u_{\rm BIPM}^2 = \sum_{i=1}^2 \frac{u_{1,i}^2}{2^2} + u_2^2$$

Similarly, for the NPLI measurements, we expect the uncertainty components u_2 and u_3 of Tables 10 and 14 to be correlated between standards, and u_1 to be uncorrelated. We therefore calculate the total uncertainty as:

$$u_{\rm NPLI}^2 = \sum_{i=1}^2 \frac{u_{1,i}^2}{2^2} + u_2^2 + u_3^2$$

5.1 Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In the present case, from inspection of the BIPM 'before' and 'after' measurements in Figures 2 to 5, we can see that any such effects are negligible compared to the overall uncertainty of the comparison. For simplicity, we do not include any extra uncertainty components.

5.2 Results at 1 Ω

The differences between the values assigned by the NPLI, R_{NPLI} , and those assigned by the BIPM, R_{BIPM} , to each of the two travelling standards on the mean date of the NPLI measurements are shown in Table 15.

NPLI – BIPM			
Standard #	$10^6 imes (R_{ m NPLI} - R_{ m BIPM}) / (1 \ \Omega)$		
BIV200	+ 0.345		
BIV207	+ 0.385		
Mean	+ 0.365		

Table 15: NPLI – BIPM differences for the two 1 Ω travelling standards.

The mean difference between the NPLI and the BIPM calibrations is:

 $(R_{\text{NPLI}} - R_{\text{BIPM}}) / (1 \Omega) = +0.365 \times 10^{-6}$

The relative combined standard uncertainty of the comparison, $u_{\rm C}$, is:

 $u_c^2 = u_{\rm BIPM}^2 + u_{\rm NPLI}^2$

 $u_{\rm BIPM} = 0.017 \times 10^{-6},$

 $u_{\rm NPLI} = 0.435 \times 10^{-6}$

 $u_{\rm C} = 0.435 \times 10^{-6}$

where,

giving:

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, *D*, between the NPLI and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor *k* = 2, corresponding to a confidence level of 95 %), *U*_C:

$$D = (R_{\text{NPLI}} - R_{\text{BIPM}}) / 1 \Omega = +0.365 \times 10^{-6}$$
$$U_{\text{C}} = 0.870 \times 10^{-6}$$

The difference between the NPLI and the BIPM calibration results is within the expanded uncertainty.

5.3 Results at 10 k Ω

The difference between the value assigned by the NPLI, R_{NPLI} , and those assigned by the BIPM, R_{BIPM} , to each of the two travelling standards on the mean date of the NPLI measurements are shown in Table 16.

NPLI – BIPM				
Standard #	$10^6 imes (R_{ m NPLI} - R_{ m BIPM}) / (10 \text{ k}\Omega)$			
B10K11	+ 0.275			
B10K12	+ 0.336			
Mean	+ 0.306			

Table 16: NPLI -	- BIPM difference	es for the two	$10 \text{ k}\Omega$ trave	lling standards.
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The mean difference between the NPLI and the BIPM calibrations is:

 $(R_{\rm NPLI} - R_{\rm BIPM}) / (10 \text{ k}\Omega) = +0.306 \times 10^{-6}$

The relative combined standard uncertainty of the comparison, $u_{\rm C}$, is:

 $u_c^2 = u_{\text{BIPM}}^2 + u_{\text{NPLI}}^2$

whe

where,

$$u_{\text{BIPM}} = 0.015 \times 10^{-6},$$

 $u_{\text{NPLI}} = 0.082 \times 10^{-6},$
giving:
 $u_{\text{C}} = 0.083 \times 10^{-6}$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the NPLI and the BIPM for the value assigned to 10 k Ω resistance standards, and its expanded relative uncertainty (expansion factor k = 2, corresponding to a confidence level of 95 %), $U_{\rm C}$:

$$D = (R_{\text{NPLI}} - R_{\text{BIPM}}) / 10 \text{ k}\Omega = +0.306 \times 10^{-6}$$
$$U_{\text{C}} = 0.166 \times 10^{-6}$$

The difference between the NPLI and the BIPM calibration results is not within the expanded uncertainty. Therefore, there is no agreement between NPLI and BIPM calibration results of $10 \text{ k}\Omega$ resistance standards.



Figure 2: Results for 1 Ω standard BIV200. BIPM (blue diamonds) and NPLI (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at NPLI and the green triangle is the mean value of NPLI measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean NPLI results.



Figure 3: Results for 1 Ω standard BIV207. BIPM (blue diamonds) and NPLI (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at NPLI and the green triangle is the mean value of NPLI measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean NPLI results.



Figure 4: Results for 10 k Ω standard B10K11. BIPM (blue diamonds) and NPLI (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at NPLI and the green triangle is the mean value of NPLI measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean NPLI results.



Figure 5: Results for 10 k Ω standard B10K12. BIPM (blue diamonds) and NPLI (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at NPLI and the green triangle is the mean value of NPLI measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean NPLI results.