BUREAU INTERNATIONAL
DES POIDS ET MESURES

Bilateral comparison of 1 Ω and 10 kΩ standards (ongoing BIPM key comparisons BIPM.EM-K13) between INRIM (Italy) and the BIPM

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Final Report

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1   Introduction

A comparison of values assigned to 1 Ω and 10 kΩ resistance standards was carried out between the BIPM (Bureau International des Poids et Mesures) and INRIM (Istituto Nazionale de Ricerca Metrologica) in the period October 2022 to February 2023. Two 1 Ω and two 10 kΩ BIPM travelling standards were calibrated first at the BIPM, then at INRIM and again at the BIPM after their return. The measurement periods are referred to as:

   'Before' measurements at the BIPM: October 2022
   INRIM measurements: November – December 2022
   'After' measurements at the BIPM: January – February 2023

This report is organised as follows: details of the travelling standards used are listed in Section 2; the results of the BIPM measurements are given in Section 3 and the calibration report provided by INRIM is summarized in Section 4; these two last sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in Section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given in the form of the degrees of equivalence between INRIM and the BIPM for measurements of 1 Ω and 10 kΩ resistance standards.

This report covers the comparison of both 1 Ω standards and 10 kΩ standards. The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

2   Travelling standards

Two 1 Ω and two 10 kΩ travelling standards provided by the BIPM were used for this comparison. The two 1 Ω standards are of CSIRO type, with working labels BIV200 (manufacturer's serial number S-64200) and BIV203 (manufacturer's serial number S-64203). The two 10 kΩ standards are TEGAM SR104 type and have the working labels B10K09 (manufacturer's serial number K203039730104) and B10K12 (serial number K201089830104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 °C and reference pressure 1013.25 hPa using the known coefficients of the standards, given in Table 1. According to the protocol, INRIM did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM following INRIM results for 1 Ω and 10 kΩ.

Standard #	Relative temperature coefficients			Relative pressure coefficient		Relative power coefficient	
	$\alpha_{23}/$ $(10^{-6}/\text{K})$	$\beta/$ $(10^{-6}/\text{K}^2)$	$u_\text{T}/$ $(10^{-6}/\text{K})$	$\gamma/$ $(10^{-9}/\text{hPa})$	$u_\text{P}/$ $(10^{-9}/\text{hPa})$	$P/$ $(10^{-9}/\text{mW})$	$u_\text{W}/$ $(10^{-9}/\text{mW})$
BIV200	− 0.0074	− 0.0004	0.001	− 0.13	0.20	− 2.0	2.0
BIV203	− 0.0096	− 0.0016	0.001	− 0.20	0.20	− 2.0	2.0
B10K09	− 0.0400	− 0.0220	0.010	− 0.16	0.10	+ 1.0	3.0
B10K12	+ 0.0100	− 0.0230	0.010	− 0.23	0.10	+ 1.0	3.0

Table 1: Temperature, pressure and power coefficients of the traveling standards.

3 Measurements at the BIPM

3.1 Measurement of the 1 Ω standards at the BIPM:

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values included in this comparison. In all cases, values are based on the SI value of the von Klitzing constant, $R_K = h/e^2 = 25\ 812.807\ 46\ \Omega$, using the fixed numerical values for the Planck constant h and the elementary charge e.

The 1 Ω measurements were carried out by comparison with a 100 Ω reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1 Ω resistors.

The 1 Ω travelling standards were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of 23 °C. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT) in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The additional pressure P_h exerted by the volume of the mineral oil above the resistors (reference plane corresponding to the plane containing the resistor terminals) has been considered for every measurement. P_h is calculated using the following equation:

$$P_h = RD \times \rho \times g \times h$$

With RD the relative density of the oil Marcol 52 type = 0.83 at 23 °C; ρ the density of the pure water = 1000 kg m^{-3} at 4 °C; g the local gravity = 9.807 m s^{-2} and h the height of the oil above the reference plane. The height of the oil above the reference plane is recorded in the software of the measurement bridge and the additional pressure is calculated automatically at every measurement.

The 'dc' resistance value (or ratio) measured with the BIPM CCC-bridge results from a current signal passing through the resistors having polarity reversals with a waiting time between polarity inversions, see Figure 1. The polarity reversal frequency is of the order of 3 mHz (340 s cycle period) and the measurements are sampled only during 100 s before the change of polarity.
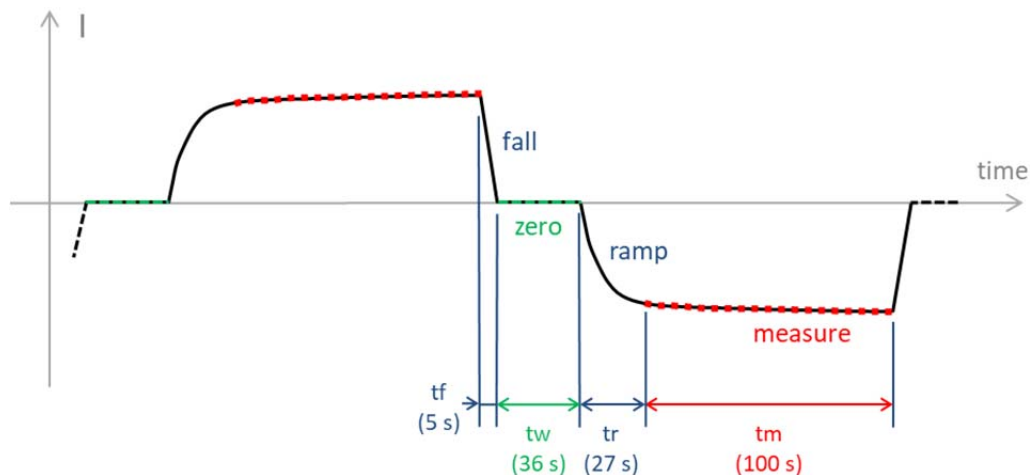

Figure 1: Schematic representation of the current signal shape with polarity reversals used in the BIPM CCC-bridge. The reversal cycle comprises a waiting time of about 36 s at zero current (green dotted line). The red dotted line corresponds to the sampling time period.

The travelling standards were measured 11 times during the period labelled 'before' (October 2022) and 10 times during the period labelled 'after' (January 2023 – February 2023).

The individual BIPM measurement data are plotted in Figures 3 and 4 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 1 Ω value (μΩ/Ω)					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED	Std dev. u_1
BIV200	− 0.717	0.011	− 0.763	0.010	− 0.733 ON 24-11-2022	0.007
BIV203	+ 0.596	0.007	+ 0.521	0.008	+ 0.572 ON 24-11-2022	0.005

Table 2: Summary of BIPM calibrations of the 1 Ω standards.

Source of uncertainty	Relative standard uncertainty (nΩ/Ω)
Imperfect realisation of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the (1 Ω / BI100-3) ratio	8
Temperature correction for the 1 Ω standard	2
Pressure correction for the 1 Ω standard	3
Combined standard uncertainty u_2	**16**

Table 3: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

3.2 Measurement of the 10 kΩ standards at the BIPM:

The 10 kΩ measurements were carried out by comparison with a set of two 10 kΩ reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 kΩ travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % (± 10 %).

The travelling standards were measured 13 times during the period labelled 'before' (October 2022) and 13 times during the period labelled 'after' (January 2023 – February 2023).

The individual BIPM measurement data are plotted in Figures 5 and 6 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	Relative difference from nominal 10 kΩ value (μΩ/Ω)					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED	Std dev. u_1
B10K09	+ 0.027	0.002	+ 0.097	0.001	+ 0.055 ON 24-11-2022	0.001
B10K12	+ 1.155	0.002	+ 1.093	0.005	+ 1.130 ON 24-11-2022	0.003

Table 4: Summary of BIPM calibrations of the 10 kΩ standards.

Source of uncertainty	Relative standard uncertainty (nΩ/Ω)
Imperfect realization of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 Ω / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of 10 kΩ reference	8
Measurement of the voltage applied to the bridge	5
Measurements of the bridge unbalance voltage	5
Leakage resistances	1
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined standard uncertainty u_2	**15**

Table 5: BIPM uncertainty budget for the calibration of the 10 kΩ travelling standards.

4   Measurements at INRIM

4.1   Preparation:

INRIM received the standard resistors on the 4th of November 2022 and a visual inspection was done to assess their physical conditions. The standards were carefully packed, but when the container arrived at INRIM one of the sides had been deformed during the travel. The condition of the container was promptly reported to the BIPM. The shock markers placed inside the box were not triggered. The 1 Ω resistors were kept in a stirred temperature-controlled oil bath (Guildline 9732 VT s/n 58.805) at a temperature of (23.000 ± 0.001) °C and the 10 kΩ travelling standards were housed in a temperature-controlled air bath (Kambic TK-105 US s/n 15115002) set at the temperature of (23.000 ± 0.005) °C. They were allowed to stabilise prior to the measurements.

4.2   Traceability of DC resistance standard at INRIM:

The group of INRIM working standards consists of the resistors in Figure 2 [1]. The arrows indicated in the diagram show the comparisons and the direction of the traceability in the measurements for the calibration of all INRIM standards and for the periodic checks.

The origin of traceability is the QHE which is used for the calibration of a 12960 Ω /10 Ω Hamon network. Subsequently, by means of ratios 1:10 directly with a Measurements International MI 6010Q (DCC) bridge, or in ratio 1:100 with Hamon and DCC networks, all the elements of the scale are calibrated.

The first step of the precision measurements was carried out during the period November 14th to November 20th by means of a calibration of the QHE / 10 Ω transfer Hamon network with an associated uncertainty equal to $U = 7{\times}10^{-8}$ Ω/Ω at 2σ confidence level. The transfer of traceability from the 10 Ω Hamon network to the first element of the decade scale must take place within a short time, usually a few hours. The first group of resistors to receive traceability consists of the 10 Ω – 1 Ω – 100 Ω standards, which are calibrated and compared with each other both in a 1:10 ratio and with the 10 Ω Hamon network. Due to the impossibility of exchanging the compared resistors when they are in a ratio of 1:10, to reduce the bridge interchange error, the Hamon network is used by comparing the resistors following the diagram in Figure 2, which allows reducing this type of systematic error.

The second group of resistors consists of 100 Ω – 1 kΩ – 10 kΩ. Traceability is transferred to the 10 kΩ resistor via a 1 kΩ step Hamon network, while the 1 kΩ resistor is calibrated against both 100 Ω and 10 kΩ, again in a 1:10 ratio.

The precision measurements on the K13 traveling standards are carried out in a 1:1 ratio with 1 Ω and 10 kΩ standards of INRIM working standard scale.

[1]: G. Boella and G. Marullo Reedtz, "A Room Temperature Setup to compare the Quantized Hall Resistance with 1 Ω Standards, IEEE Trans Instrum Meas, 41. 1, pp. 59-63, 1992.
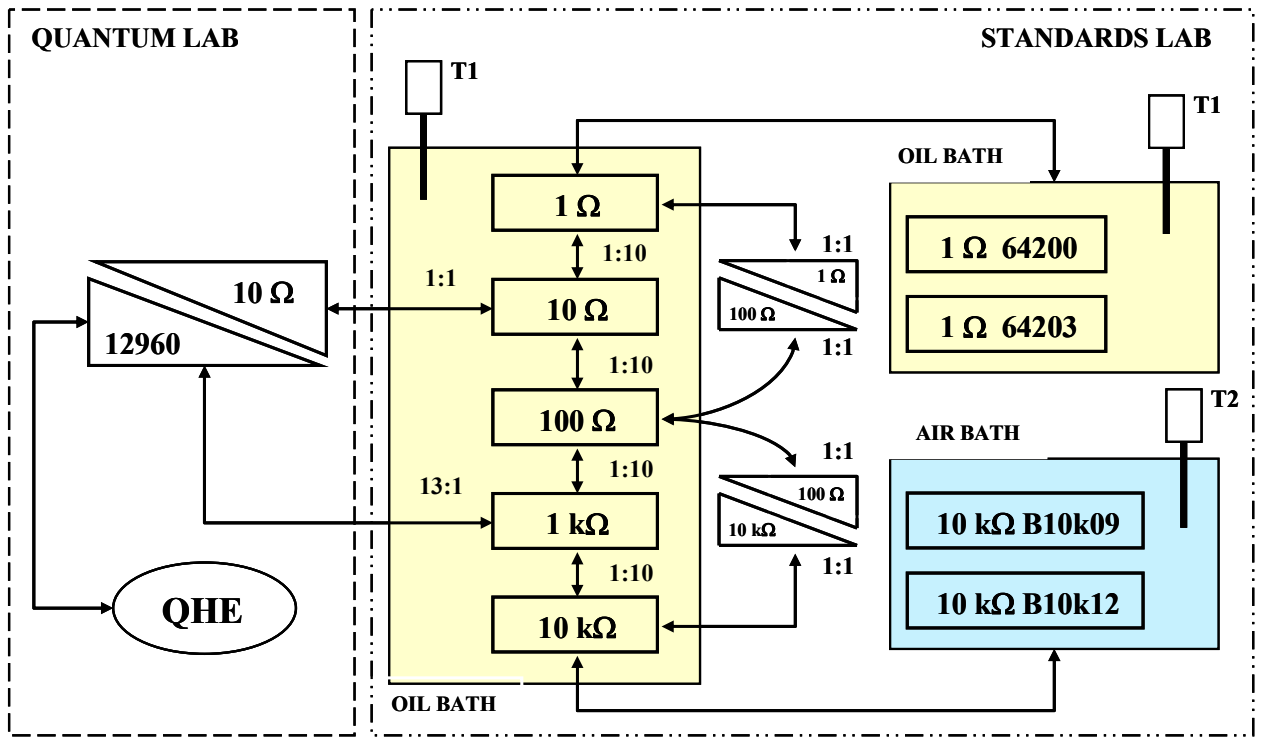
Figure 2: Complete diagram of the measurement system used for the K13 a & b key comparison. The traceability derives from the QHE realized in INRIM Quantum laboratory and transferred to the decade scale in two ways: through a specifically designed Hamon network and by 1:13 ratio direct comparison, with a MI 6010Q DCC bridge.

4.3 Operating conditions:

Upon receipt the 1 Ω standards were placed in a stirred temperature controlled oil bath (Guildline 9732 VT s/n 58.805) with a set point temperature of 23.000 °C (± 0.001 °C) and which uncertainty is $u_{temp}(k_{temp} = 1) = \pm 0.001$ °C . The oil temperature is measured with a calibrated Standard Platinum Resistance Thermometer Leeds & Northrup 25 Ω s/n 177 3055.

The pressure gauge used in INRIM official weather measurement station is based on a Vaisala BAROCAP® PTB110 sensor, with an uncertainty of $u_{pres}(k_{pres} = 1) = \pm 0.3$ hPa at 20 °C. The temperature and pressure were reported for each measurement.

A current of 50 mA and current reversal time of 8 seconds (filter 0.3 s) were used for all measurements. All measurements were also made using a 100 mA current which is the normal current used by INRIM. The values reported in this report are the ones using the 50 mA current avoiding the application of a power factor correction and its uncertainty. This has been decided after verification of the agreement between the 50 mA and the 100 mA values and the power factor coefficient (Table 1) application on both standards in the data post-treatment. The power difference is 7.5 mW (between the application of a 50 mA or 100 mA current) resulting in a correction of 15×10^{-9} on the measured values. The difference between the corrected data at 100 mA reduced to 50 mA and the data at 50 mA is comparable and within the uncertainty of the power factor coefficient.

The pressure reported to the BIPM does not include the pressure correction due to the oil level for the immersion of the 1 Ω standards. The heads of the standards (reference plane corresponding to the plane containing the resistor terminals) were 32 mm below the surface of the bath during the measurement period.

The oil used is also Marcol 52 type with RD the relative density of the oil = 0.83 at 23 °C; ρ the density of the pure water = 1000 kg m^{-3} at 4 °C; g the local gravity of the building = 9.805 m s^{-2} and h the height of the oil above the reference plane in m = 0.032 m.

An additional pressure P_h exerted by the volume of the oil above the resistors was added to the atmospheric pressure reported by INRIM. It is calculated using the following equation:

$$P_h = RD \times \rho \times g \times H$$

The computed additional pressure due to the head of oil during the measurements is 2.6 hPa.

The 10 kΩ travelling standards were placed in a temperature-controlled air bath (Kambic TK-105 US s/n 15115002) set at the temperature of 23.000°C (± 0.005 °C) and the uncertainty of which is $u_{\text{temp}}(k_{\text{temp}} = 1) = \pm 0.002$ °C .

The temperature and pressure were reported for each measurement. The measurement current in the 10 kΩ standards was 100 μA.

The measurements were carried out every day, starting at 5:00, i.e. when low level electrical and thermal noise and drift usually occur. Furthermore in the period between 18th and 23rd November no measurements were made on the K13 standards due to the calibration of INRIM working standards.

4.4   INRIM results at 1 Ω:

The 1 Ω travelling standards were measured 30 times for both BIV200 and BIV203 in the period November 2022 – December 2022. As described above, the measurements were made at 50 mA and at 100mA. Power correction is not needed if we consider only the 50 mA series of measurements as the applied currents at INRIM and BIPM are comparable (see § 4.3 above).

Table 6 gives the mean values at the mean date of 24th of November 2022 for BIV200 and BIV203 before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements. The pressure of the mineral oil exerted on the resistors has been considered for every measurement and the mean pressure from Table 6 is corrected for this effect.

Standard #	Relative difference from nominal 1 Ω value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean pressure at the reference plane / hPa
BIV200	− 0.780	0.026	22.999	986.80
BIV203	+ 0.480	0.026	22.999	986.80

Table 6: Summary of INRIM 1 Ω calibrations.

4.4.1   Corrections for temperature and pressure differences:

The value $R(23)$ of the resistance corrected to $T_0 = 23$ °C is:
$$R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$
where $R(T)$ is the resistance of the standard at temperature T.

The value $R(1013.25)$ of the resistance corrected to $P_0 = 1013.25$ hPa is:
$$R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$

where $R(P)$ is the resistance of the standard at pressure P.

INRIM results are corrected to the reference temperature and the reference pressure using the coefficients α_{23}, β and γ shown in Table 1. Applied corrections are reported in Table 7.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa		
	Relative corrections ($\mu\Omega/\Omega$)	
Standard #	For temperature	For pressure
BIV200	− 0.000	− 0.003
BIV203	− 0.000	− 0.005

Table 7: Corrections applied to INRIM 1 Ω results.

The standard uncertainties of the temperature and pressure measurements at INRIM are 0.001 °C and 0.3 hPa respectively. Taking into account the differences from the reference temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure corrections applied by the BIPM are estimated to be $u_{\text{Temp}} < 0.001 \times 10^{-6}$ and $u_{\text{Press}} = 0.005 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.005 \times 10^{-6}$. u_3 is reported in Table 9.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

4.4.2   Uncertainty budget provided by INRIM:

Table 8 shows the uncertainty budget provided by INRIM associated with the transfer from INRIM reference standard to the 1 Ω travelling standards BIV200 and BIV203.

	Source of uncertainties	*Type*	$u_r (\times 10^{-9})$	ν
Transfer $H_{ss} \rightarrow H_{sp}$	Link from R_K to Hamon 36×360 Ω (H_{SS} = 12960 Ω)	B	35.2	∞
	H_{SS}/H_{sp} (10 Ω) ratio correction	B	17.1	∞
	H_{sp} temperature coefficient	B	0.03	∞
	temperature difference H_{SS} and H_{sp}	B	0.58	∞
	H_{sp} drift (1 day)	B	0.69	∞
	$u_c(R_{HSP})$		**39.1**	
Transfer $H_{sp} \rightarrow R_x$ (1 Ω)	Ratio noise Hsp – 1 Ω	A	31.4	50
	Bridge ratio error correction	B	46.2	∞
	Effect due to H_{sp} temperature coefficient	B	5.8	∞
	H_{sp} drift (1 day)	B	0.3	∞
	Emf	B	9.99	∞
	Temperature instability on R_x	B	7.1	∞
	Resistors Johnson noise	B	0.74	∞
	$u_c (R_x)$ **type B only**		**62.1**	

Table 8: INRIM uncertainty budget associated to the transfer from INRIM reference standard to the 1 Ω travelling standards BIV200 and BIV203.

4.4.3 Uncertainties associated with the measurement of 1 Ω resistors:

Table 9 shows the corrected measurements of the 1 Ω standards at INRIM at the mean date of 24th of November 2022 for BIV200 and BIV203 as well as the associated uncertainty components.

INRIM results after corrections	Relative difference from nominal value (µΩ/Ω)	Relative standard uncertainties		
		Repeatability u_1 (µΩ/Ω)	Systematic u_2 (µΩ/Ω)	Corrections u_3 (µΩ/Ω)
BIV200	− 0.783	0.026	0.062	0.005
BIV203	+ 0.475	0.026	0.062	0.005

Table 9: Summary of INRIM results at 1 Ω, after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 9 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.

4.5   INRIM results at 10 kΩ:

The 10 kΩ travelling standards were measured 30 times for both B10K09 and B10K12 in the period November 2022 – December 2022. The measurements were made at 100 µA with a current reversal time of 12 seconds (filter 0.3 s) for all measurements. Power correction is not needed as applied currents at INRIM and BIPM are comparable. Table 10 gives the mean values at the mean date of 24th of November 2022 for B10K09 and B10K12, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 10 kΩ value (µΩ/Ω)	Std dev. (µΩ/Ω)	Mean temperature / °C	Mean atmospheric pressure / hPa
B10K09	+ 0.012	0.021	22.978	988.35
B10K12	+ 1.095	0.017	22.978	988.35

Table 10: Summary of INRIM 10 kΩ calibrations.

4.5.1   Corrections for temperature and pressure differences:

The value $R(23)$ of the resistance corrected to $T_0 = 23$ °C is:
$$R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$
where $R(T)$ is the resistance of the standard at temperature T.

The value $R(1013.25)$ of the resistance corrected to $P_0 = 1013.25$ hPa is:
$$R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$
where $R(P)$ is the resistance of the standard at pressure P.

INRIM results are corrected to the reference temperature and the reference pressure using the coefficients α_{23}, β and γ shown in Table 1. Applied corrections are reported in Table 11.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa		
	Relative corrections (µΩ/Ω)	
Standard #	For temperature	For pressure
B10K09	− 0.001	− 0.004
B10K12	+ 0.000	− 0.006

Table 11: Corrections applied to INRIM 10 kΩ results.

The standard uncertainties of the temperature and pressure measurements at INRIM are 0.002 °C and 0.3 hPa respectively. Taking into account the differences from the reference temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp} and u_{Press} associated with the temperature and pressure difference corrections applied by the BIPM are estimated to be $u_{Temp} < 0.001 \times 10^{-6}$ and $u_{Press} = 0.002 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.003 \times 10^{-6}$. u_3 is reported in Table 13.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

4.5.2 Uncertainty budget provided by INRIM:

Table 12 shows the uncertainty budget provided by INRIM associated with the transfer from INRIM reference standard to the 10 kΩ travelling standards B10K09 and B10K12.

Source of uncertainties	*Type*	$u_r\,(\times 10^{-9})$	ν
Link from R_K to standard 100 Ω ESI	B	61.7	50
Ratio noise 100 Ω ESI/Hamon $LN_{100\,\Omega}$	A	1.9	50
Ratio noise Hamon $LN_{10\,k\Omega}$/10kΩ	A	5.9	50
Bridge ratio error correction	B	8.0	∞
Hamon ratio transfer error	B	57.7	∞
Temperature coefficient	B	0.23	∞
Hamon drift	B	negl.	∞
Emf	B	0.02	∞
Temperature instability on R_x	B	3.12	∞
Resistors Johnson noise	B	0.74	∞
$u_c\,(R_x)$ type B only		**84.9**	

Table 12: Uncertainty budget associated to the transfer from INRIM reference standard to the 10 kΩ travelling standards B10K09 and B10K12.

4.5.3 Uncertainties associated with the measurement of 10 kΩ resistors:

Table 13 shows the corrected measurements of the 10 kΩ standards at INRIM as well as the uncertainty components associated with these measurements.

INRIM results after corrections	Relative difference from nominal value (μΩ/Ω)	Relative standard uncertainties		
		Repeatability u_1 (μΩ/Ω)	Systematic u_2 (μΩ/Ω)	Corrections u_3 (μΩ/Ω)
B10K09	+ 0.007	0.021	0.085	0.003
B10K12	+ 1.089	0.017	0.085	0.003

Table 13: Summary of INRIM results at 10 kΩ, after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 13 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.

5   Comparison INRIM – BIPM

The individual measurement results for each of the four standards are shown in Figures 3 to 6. The plots also show the mean value of INRIM measurements with the uncertainty bar corresponding to the expanded uncertainty ($k = 2$) of the comparison U_c provided below, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of Figures 3 to 6 indicates that this is an appropriate model as both 1 Ω standards and 10 kΩ standards fit this model well. We treat the 1 Ω and 10 kΩ results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of INRIM measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of INRIM measurements.

The difference between INRIM and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\text{INRIM},i} - R_{\text{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\text{INRIM−BIPM}} = \frac{1}{2}\sum_{i=1}^{2}\left(R_{\text{INRIM},i} - R_{\text{BIPM},i}\right)$$

For each standard, the uncertainty u_1 associated with the interpolated BIPM value is calculated from the linear fit, as shown in Tables 2 and 4; u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard R_i, the BIPM uncertainty $u_{\text{BIPM},i}$ is obtained from: $u_{\text{BIPM},i}^2 = u_{1,i}^2 + u_{2,i}^2$. When the mean (for two standards) of INRIM − BIPM relative difference is calculated, the BIPM contribution to the uncertainty is,

$$u_{\text{BIPM}}^2 = \sum_{i=1}^{2}\frac{u_{1,i}^2}{2^2} + u_2^2$$

Similarly, for INRIM measurements, we expect the uncertainty components u_2 and u_3 of Tables 9 and 13 to be correlated between standards, and u_1 to be uncorrelated. We therefore calculate the total uncertainty as:

$$u_{\text{INRIM}}^2 = \sum_{i=1}^{2}\frac{u_{1,i}^2}{2^2} + u_2^2 + u_3^2$$

5.1   Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In the present case, from inspection of the BIPM 'before' and 'after' measurements in Figures 3 to 6, we can see that any such effects are insignificant compared to the overall uncertainty of the comparison. For simplicity, we do not include any extra uncertainty components.

5.2   Results at 1 Ω

The differences between the values assigned by INRIM, R_{INRIM}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of INRIM measurements are shown in Table 14.

INRIM − BIPM	
Standard #	$10^6 \times (R_{INRIM} - R_{BIPM}) / (1\ \Omega)$
BIV200	− 0.050
BIV203	− 0.097
Mean	**− 0.074**

Table 14: INRIM – BIPM differences for the two 1 Ω travelling standards.

The mean difference between INRIM and the BIPM calibrations is:

$$(R_{INRIM} - R_{BIPM}) / (1\ \Omega) = -0.074 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_C, is:

$$u_c^2 = u_{BIPM}^2 + u_{INRIM}^2$$

where,
$$u_{BIPM} = 0.017 \times 10^{-6},$$
$$u_{INRIM} = 0.065 \times 10^{-6},$$

giving:
$$u_C = 0.067 \times 10^{-6}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between INRIM and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_C:

$$D = (R_{INRIM} - R_{BIPM}) / 1\ \Omega = -0.074 \times 10^{-6}$$

$$U_C = 0.134 \times 10^{-6}$$

The difference between INRIM and the BIPM calibration results is within the expanded uncertainty.

5.3   Results at 10 kΩ

The difference between the value assigned by INRIM, R_{INRIM}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of INRIM measurements are shown in Table 15.

INRIM − BIPM	
Standard #	$10^6 \times (R_{\text{INRIM}} - R_{\text{BIPM}}) / (10 \text{ k}\Omega)$
B10K09	− 0.048
B10K12	− 0.041
Mean	**− 0.045**

Table 15: INRIM – BIPM differences for the two 10 kΩ travelling standards.

The mean difference between INRIM and the BIPM calibrations is:

$$(R_{\text{INRIM}} - R_{\text{BIPM}}) / (10 \text{ k}\Omega) = -0.045 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_C, is:

$$u_c^2 = u_{\text{BIPM}}^2 + u_{\text{INRIM}}^2$$

where,
$$u_{\text{BIPM}} = 0.015 \times 10^{-6},$$
$$u_{\text{INRIM}} = 0.086 \times 10^{-6},$$

giving:
$$u_C = 0.087 \times 10^{-6}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between INRIM and the BIPM for the value assigned to 10 kΩ resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_C:

$$D = (R_{\text{INRIM}} - R_{\text{BIPM}}) / 10 \text{ k}\Omega = -0.045 \times 10^{-6}$$

$$U_C = 0.174 \times 10^{-6}$$

The difference between INRIM and the BIPM calibration results is within the expanded uncertainty.
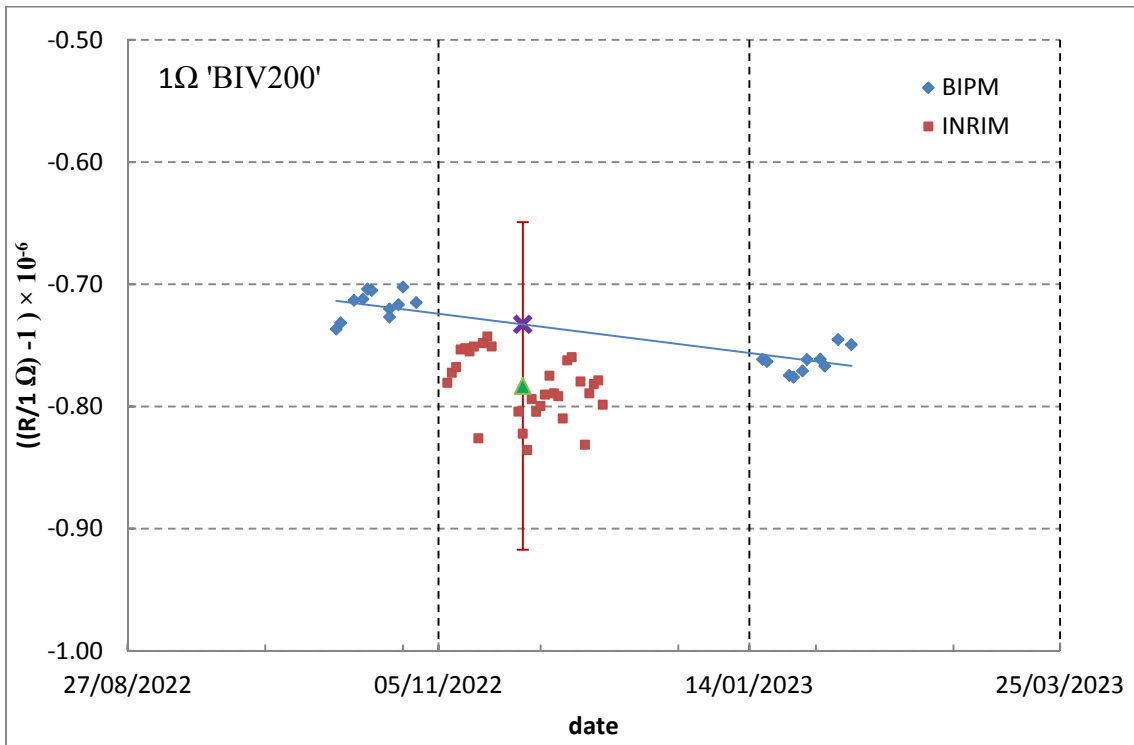
Figure 3: Results for 1 Ω standard BIV200. BIPM (blue diamonds) and INRIM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INRIM and the green triangle is the mean value of INRIM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean INRIM results.
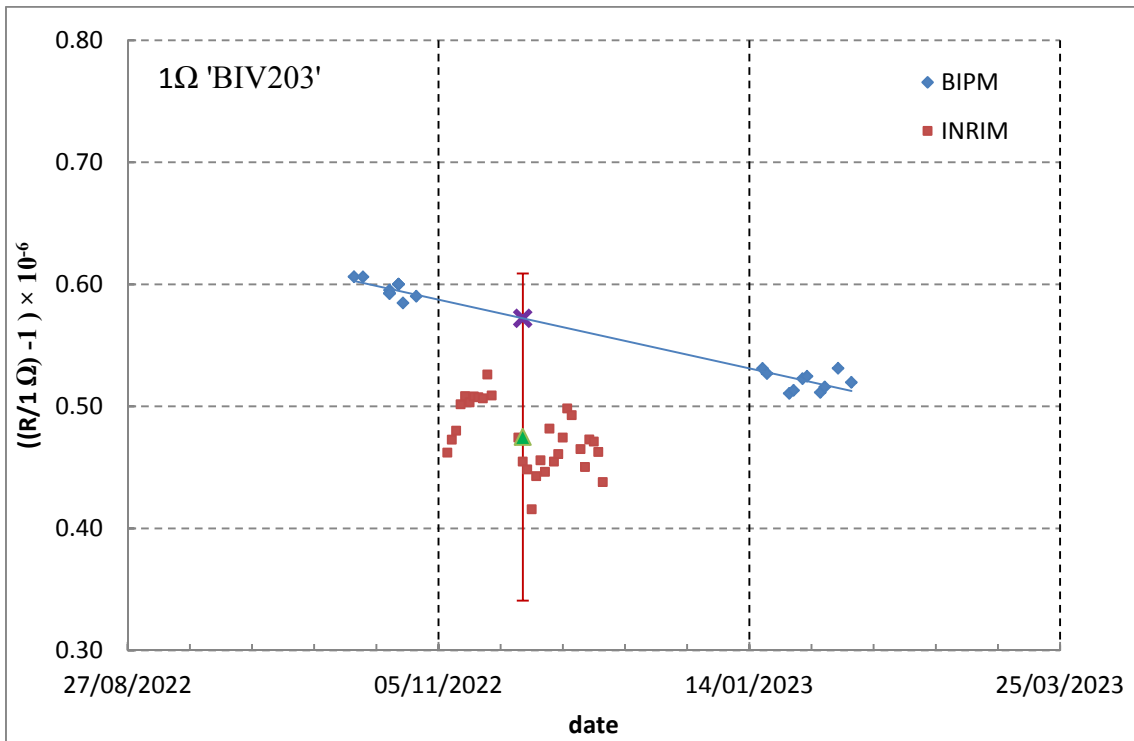

Figure 4: Results for 1 Ω standard BIV203. BIPM (blue diamonds) and INRIM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INRIM and the green triangle is the mean value of INRIM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean INRIM results.
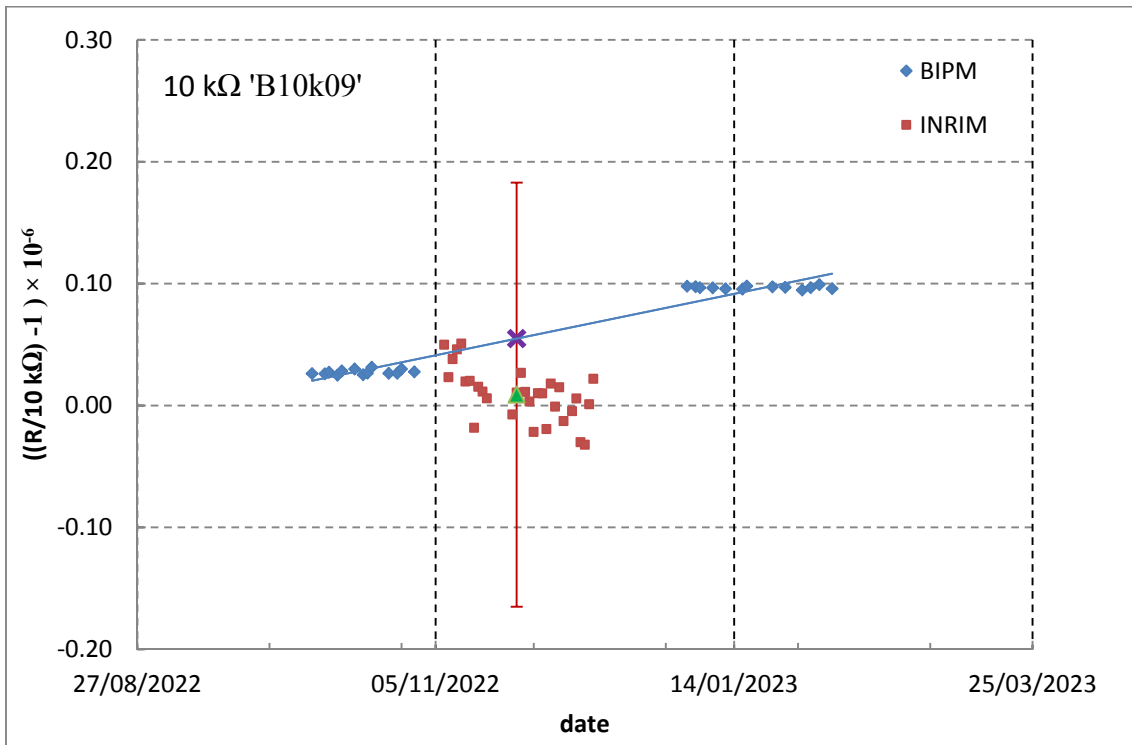
Figure 5: Results for 10 kΩ standard B10K09. BIPM (blue diamonds) and INRIM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INRIM and the green triangle is the mean value of INRIM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean INRIM results.
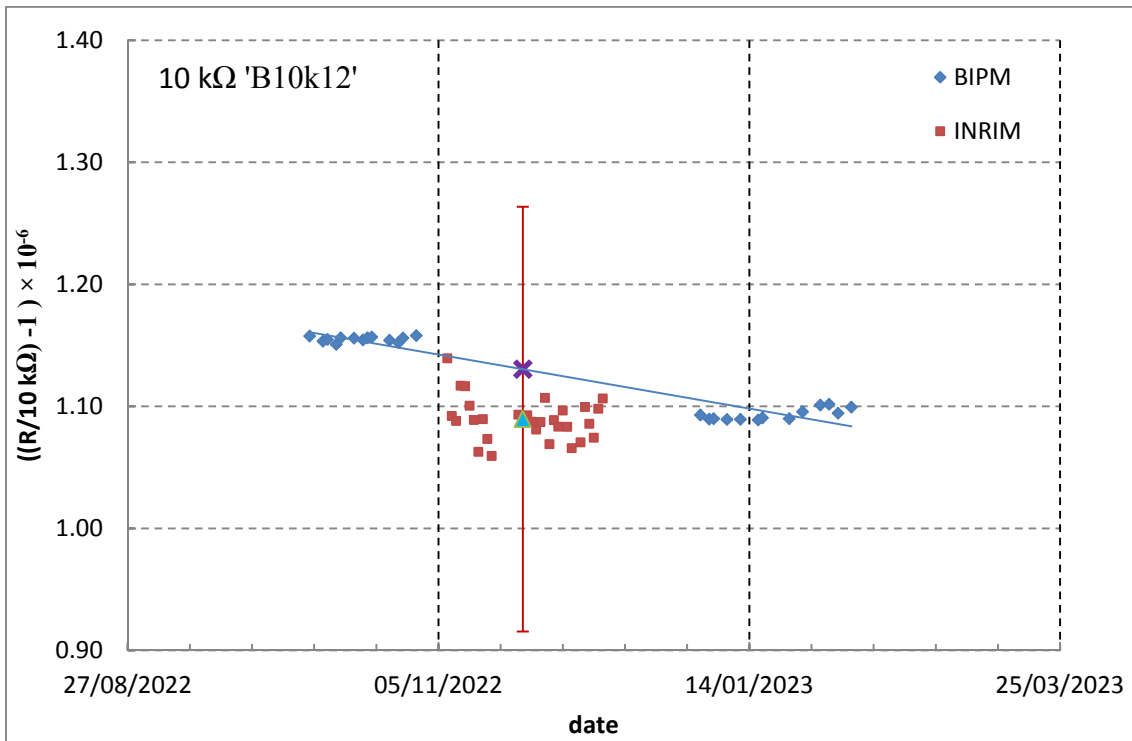

Figure 6: Results for 10 kΩ standard B10K12. BIPM (blue diamonds) and INRIM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INRIM and the green triangle is the mean value of INRIM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean INRIM results.