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Bilateral comparison of 1 Ω and 10 kΩ standards (ongoing BIPM key comparisons BIPM.EM-K13.a and 13.b) between the INMETRO (Brazil) and the BIPM

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Final report

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1 Introduction

A comparison of values assigned to 1 Ω and 10 kΩ resistance standards was carried out between the BIPM (Bureau International des Poids et Mesures) and the INMETRO (Instituto Nacional de Metrologia, Qualidade e Tecnologia, Brazil) in the period November 2020 to September 2021. Two 1 Ω and two 10 kΩ BIPM travelling standards were calibrated first at the BIPM, then at the INMETRO and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: November – December 2020
INMETRO measurements: April – June 2021
'After' measurements at the BIPM: August – September 2021

This report is organised as follows: details of the travelling standards used are listed in Section 2; the results of the BIPM measurements are given in Section 3 and the calibration report provided by the INMETRO is summarized in Section 4; these two sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in Section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given in the form of the degrees of equivalence between the INMETRO and the BIPM for measurements of 1 Ω and 10 kΩ resistance standards.

This report covers the comparison of both 1 Ω standards (BIPM.EM-K13.a) and 10 kΩ standards (BIPM.EM-K13.b). The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

2 Travelling standards

Two 1 Ω and two 10 kΩ travelling standards provided by the BIPM were used for this comparison. The two 1 Ω standards are of CSIRO type, with working labels BIV200 (manufacturer's serial number S-64200) and BIV207 (manufacturer's serial number S-64207). The two 10 kΩ standards are TEGAM S104 type and have the working labels B10K11 (manufacturer's serial number K205039730104) and B10K12 (serial number K201089830104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 °C and reference pressure 1013.25 hPa using the known coefficients of the standards, given in Table 1. According to the protocol, the INMETRO did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM.

	Relative temperature coefficients			**Relative pressure coefficient**		**Relative power coefficient**	
Standard #	α_{23} / (10^{-6}/K)	β / (10^{-6}/K²)	u_{Temp} / (10^{-6}/ K)	γ / (10^{-9} / hPa)	u_{Press} / (10^{-9} / hPa)	P / (10^{-9} / mW)	u_{Power} / (10^{-9} / mW)
BIV200	− 0.0074	+ 0.0004	0.001	− 0.100	0.200	− 2.0	1.5
BIV207	− 0.0094	+ 0.0001	0.001	− 0.250	0.200	− 2.0	1.5
B10K11	− 0.0700	− 0.0270	0.010	− 0.350	0.100	+ 2.4	2.4
B10K12	+ 0.0100	− 0.0230	0.010	− 0.226	0.100	+ 1.0	2.4

Table 1: Temperature, pressure and power coefficients of the traveling standards.

3 Measurements at the BIPM

3.1 Measurement of the 1 Ω standards at the BIPM:

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values included in this comparison. In all cases, values are based on the revised SI value of the von Klitzing constant, $R_K = h/e^2 = 25\ 812.807\ 46\ \Omega$, using the fixed numerical values for the Planck constant h and the elementary charge e.

The 1 Ω measurements were carried out by comparison with a 100 Ω reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1 Ω resistors.

The 1 Ω travelling standards were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of 23 °C. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT) in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The additional pressure P_h exerted by the volume of the mineral oil above the resistors (reference plane corresponding to the plane containing the resistor terminals) has been considered for every measurement. P_h is calculated using the following equation:

$$P_h = RD \times \rho \times g \times h$$

With RD the relative density of the oil Marcol 52 type = 0.83; ρ the density of the pure water = 1000 kg m^{-3} at 4 °C; g the local gravity = 9.807 m s^{-2} and h the height of the oil above the reference plane in m. The height of the oil above the reference plane is recorded in the software of the measurement bridge and the additional pressure is calculated automatically at every measurement.

The ‘dc’ resistance value (or ratio) measured with the BIPM CCC-bridge results from a current signal passing through the resistors having polarity reversals with a waiting time between polarity inversions, cf. Figure 1. The polarity reversal frequency is of the order of 3 mHz (340 s cycle period) and the measurements are sampled only during 100 s before the change of polarity.

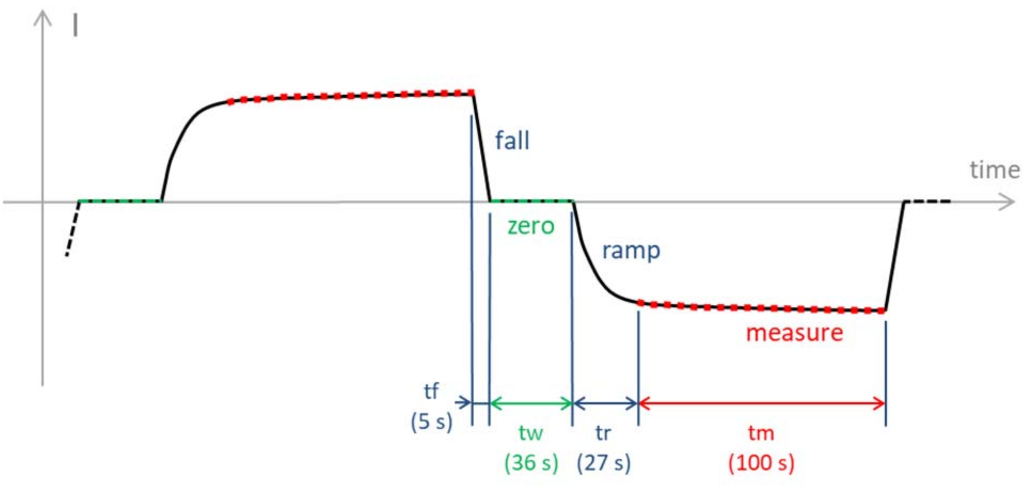

Figure 1: Schematic representation of the reference current signal with polarity reversals used in the BIPM CCC-bridge. The reversal cycle comprises a waiting time of about 36 s at zero current (green dotted line). The red dotted line corresponds to the sampling time period.

The travelling standards were measured 14 times during the period labelled ‘before’ (November 2020 – December 2020) and 10 times during the period labelled ‘after’ (August 2021 – September 2021).

The individual BIPM measurement data are plotted in Figures 2 and 3 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	**Relative difference from nominal 1 Ω value (µΩ/Ω)**					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED	Std dev. u_1
BIV200	− 0.734	0.010	− 0.734	0.009	− 0.734 ON 23-05-2021	0.007
BIV207	− 0.403	0.009	− 0.419	0.009	− 0.413 ON 26-05-2021	0.006

Table 2: Summary of BIPM calibrations of the 1 Ω standards.

Source of uncertainty	**relative standard uncertainty (nΩ/Ω)**
Imperfect realisation of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the (1Ω / BI100-3) ratio	8
Temperature correction for the 1 Ω standard	2
Pressure correction for the 1 Ω standard	3
Combined standard uncertainty u_2	**16**

Table 3: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

3.2 Measurement of the 10 kΩ standards at the BIPM:

The 10 kΩ measurements were carried out by comparison with a set of two 10 kΩ reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 kΩ travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % (± 10 %).

The travelling standards were measured 12 times during the period labelled ‘before’ (November 2020 – December 2020) and 11 times during the period labelled ‘after’ (August 2021 – September 2021).

The individual BIPM measurement data are plotted in Figures 4 and 5 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	**Relative difference from nominal 10 kΩ value (μΩ/Ω)**					
Standard #	BEFORE	Std dev. u_{1B}	AFTER	Std dev. u_{1A}	INTERPOLATED	Std dev. u_1
B10K11	+ 1.261	0.001	+ 1.309	0.003	+ 1.292 ON 30-05-2021	0.001
B10K12	+ 1.081	0.002	+ 1.111	0.002	+ 1.096 ON 19-04-2021	0.001

Table 4: Summary of BIPM calibrations of the 10 kΩ standards.

Source of uncertainty	**relative standard uncertainty (nΩ/Ω)**
Imperfect realization of R_K	2
Calibration of the BIPM 100 Ω reference (BI100-3) against R_K	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 Ω / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of 10 kΩ reference	8
Measurement of the voltage applied to the bridge	5
Measurements of the bridge unbalance voltage	5
Leakage resistances	1
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined standard uncertainty u_2	**15**

Table 5: BIPM uncertainty budget for the calibration of the 10 kΩ travelling standards.

4 Measurements at the INMETRO

4.1 Preparation:

The standard resistors were received on 26 March 2021 and were placed in the laboratory for stabilization prior to measurements. A visual check was made to ensure that the standards were in working condition.

4.2 Method of calibration at INMETRO:

The measurements of the travelling standards were carried out using a cryogenic current comparator (CCC) resistance bridge manufactured by the NPL [1], whose type B uncertainty components are shown in Table 6. All travelling standards were allowed to stabilize for a couple of weeks prior to the measurements.

The resistance of each 10 kΩ travelling standard was measured by comparison with one INMETRO 100 Ω reference standard, manufactured by IET Labs, Inc., model SR-102/DC, coded as “7D”. “7D” and the 10 kΩ travelling standards were maintained in separate temperature controlled air baths at 23 °C. For the 10 kΩ resistors, the nominal current was set to 30 µA, while the current of “7D” was 3 mA.

“7D” was calibrated against INMETRO quantum Hall resistance (QHR) using the aforementioned CCC bridge. Table 7 summarizes the uncertainty of such measurement. The QHR device is operated on the $i = 2$ plateau, obtained at temperatures around 370 mK and magnetic flux density of approximately 10.5 T at a nominal DC current level of 23.250 µA, while the current of “7D” was 3 mA.

As for the measurement of the 1 Ω travelling standards, two different schemes were applied. In the first one, each 1 Ω travelling standard was measured by direct comparison with “7D”. The 1 Ω travelling standards were maintained in a temperature controlled oil bath at 23 °C. The nominal current was set to 10 mA, while the current of “7D” was 0.1 mA. In such setup, currents higher than these prevent the CCC bridge from reaching balance.

In the second scheme, the calibration of the 1 Ω travelling standards against the 100 Ω reference standard was performed in two stages, using an intermediate 10 Ω resistor, coded as “PT17”. “PT17” was maintained in the same oil bath as the 1 Ω travelling standards. In this case, a nominal current of 30 mA (i.e. the maximum current that the CCC can provide) could be applied to the 1 Ω travelling standards. “PT17” was calibrated against one INMETRO 100 Ω reference standard, coded as “7E”. Table 8 states the uncertainty of such measurement. Both “PT17” and “7E” are model 5685A resistors manufactured by Tinsley Instrumentation Ltd.

Type B uncertainty components of the CCC				
Source of uncertainty	**Probability distribution**	**Estimate (nΩ/Ω)**	***k***	**Standard uncertainty (nΩ/Ω)**
CCC ratio error	Normal	0.20	1.00	0.20
Leakage resistances	Rectangular	0.20	1.73	0.12
Calibration of balance	Rectangular	0.40	1.73	0.23
SQUID servo loop gain	Rectangular	0.60	1.73	0.35
Null detector servo gain	Rectangular	0.60	1.73	0.35
CCC relative combined standard uncertainty, $u_C\,(CCC)_B$ [nΩ/Ω]				**0.60**

Table 6: INMETRO uncertainty budget of the CCC resistance bridge.

4.3 Model equation for determination of the 100 Ω reference standard value from the QHR:

$$R_X = r \times R_S \times Z_S \times Z_X$$

Where:

- R_X is the resistance value (at T_0 and P_0) to be determined.
- r is the R_{master}-to-R_{slave} ratio. For the 100 Ω case, QHR, $i = 2$ is the reference standard, which is connected to the slave branch of the CCC resistance bridge, and r is around 0.01.
- R_S is the stated value of the standard resistor at the reference temperature T_0 and the pressure reference P_0; in the particular case of the QHR, R_S is independent of environmental variables.
- Z_S is the correction applied to the stated value of R_S, which is referenced to T_0 and P_0, to T_S and P_S. In general, it is defined as follows:

  $$Z_S = 1 + \alpha_{S\_T0}(T_S - T_0) + \beta_{S\_T0}(T_S - T_0)^2 + \gamma_{S\_P0}(P_S - P_0)$$

  T_S and P_S are the values of temperature and pressure of R_S, respectively, recorded during the measurements. α_{S_T0} and β_{S_T0} are the temperature coefficients of R_S, and γ_{S_P0} is the pressure coefficient of R_S. It should be noted that $Z_S = 1$ in the specific case where R_S is the QHR, since its value requires no correction whatsoever.
- Z_X is the correction to T_0 and P_0 applied to the value of R_X. Z_X is defined as follows:

  $$Z_X = 1 - \alpha_{X\_T0}(T_X - T_0) - \beta_{X\_T0}(T_X - T_0)^2 - \gamma_{X\_P0}(P_x - P_0)$$

  T_X and P_X are the values of temperature and pressure of R_X, respectively, recorded during the measurements. α_{X_T0} and β_{X_T0} are the temperature coefficients of R_X, and γ_{X_P0} is the pressure coefficient of R_X.

Uncertainties associated with the value of the 100 Ω ("7D") measurement		
Source of uncertainty	**Type**	**Relative standard uncertainty (nΩ/Ω)**
Imperfect realization of the quantum Hall resistance	B	2.6
CCC resistance bridge	B	0.8
Temperature of the 100 Ω resistor	B	0.3
Standard deviation of the mean	A	1.4
Combined relative standard uncertainty (*k* = 1)		**3.1**

Table 7: INMETRO uncertainty budget for the calibration of the 100 Ω reference resistance standard.

Notes on Table 7:

a- No relative pressure coefficient (γ) of the 100 Ω reference resistor could be deduced due to the little pressure variation over the period of measurements.

b- The CCC resistance bridge uncertainty is given by $\{[(R_S^2 \times Z_S^2 \times Z_X^2 \times u(r)_B^2]^{1/2}\}/R_X$, where $u(r)_B$ is type B uncertainty of the ratio, $u(r)_B = u_C(CCC)_B \times r$.

Uncertainties associated with the value of the 10 Ω (“PT17”) measurement		
Source of uncertainty	**Type**	**Relative standard uncertainty (nΩ/Ω)**
Link QHR / 100 Ω	B	4.6
CCC resistance bridge (Table 6)	B	0.6
Temperature correction	B	0.1
Standard deviation of the mean	A	4.4
Combined relative standard uncertainty (k = 1)		**6.4**

Table 8: INMETRO uncertainty budget for the calibration of the 10 Ω reference resistance standard.

Notes on Table 8:

a- No relative pressure coefficient (γ) of the 10 Ω reference resistor could be deduced due to the little pressure variation over the period of measurements.

b- The calibrations of the 100 Ω and 10 Ω resistors occurred with less than 30-day difference; therefore, no drift component applies.

c- The calculation of the 10 Ω reference standard value is based on the same model equation as for the 100 Ω reference standard, with the difference that $r \approx 0.1$.

Reference:
[1] Williams, J. M., Janssen, T. J. B. M., Rietveld, G. & Houtzager, E. (2010). An automated cryogenic current comparator resistance ratio bridge for routine resistance measurements. *Metrologia, 47*(3), 167–174. https://doi.org/10.1088/0026-1394/47/3/007

4.4 Operating conditions:

The temperature of the resistors was well controlled by the calibration baths. The resistors temperatures were measured with individual, calibrated temperature probes, PT-100 platinum resistance thermometer type. The measured oil temperature did not vary by more than ±12 mK about the reference value during the entire period of the resistance measurements (the top temperature was 23.012 °C and the bottom temperature was 22.992 °C). The measured air bath temperature did not vary by more than ±66 mK about the reference value (the top temperature was 23.053 °C and the bottom temperature was 22.934 °C).

The relative humidity inside the air baths was not monitored, since its influence is negligible at the measured resistance range; still, the air humidity in the laboratory was recorded at the beginning of each measurement set. For this purpose, a Sato Keiryoki datalogger model SK-L200THIIα was used. The relative humidity reached a minimum of 52.4 %rh and a maximum of 59.9 %rh (±7.0 %rh) over the entire period of measurements.

The atmospheric pressure in the laboratory was recorded at the time of each measurement set, using a calibrated barometer whose expanded uncertainty is U = 0.0053 kPa and coverage factor is k = 3.18 yielding the standard uncertainty u_{Barom} = 0.002 kPa. For the 1 Ω, u_{Barom} = 0.002 kPa is combined with the uncertainty of the oil column above the resistor, which is u_{Oil} = 0.012 kPa. u_{Press} = 0.012 kPa in this case. For the 10 kΩ, $u_{\mathrm{Barom}} = u_{\mathrm{Press}}$ = 0.002 kPa since there is no oil column above the resistor. Over the entire period of measurements, the atmospheric pressure ranged from 100.105 kPa to 102.221 kPa. The height h of the oil column above the 1 Ω travelling standards kept in the oil bath was measured with an uncalibrated scale; h = 0.115 m (from the connector plane), and the uncertainty estimate is 1 mm with rectangular distribution.

The local acceleration of gravity is 9.788 m s^{-2}, and the uncertainty estimate is 0.002 m s^{-2} with rectangular distribution. The oil density is 848.5 kg m^{-3}, and the uncertainty estimate is 16.5 kg m^{-3} with rectangular distribution.

4.5 INMETRO results at 1 Ω:

The 1 Ω travelling standards were measured 40 times for BIV200 and 32 times for BIV207 in the period April 2021 to June 2021. The measurements were made at 10 mA, 30 mA, 20 seconds cycle rate and also 100 seconds cycle rate. Because no correction for a possible dependence on the current reversal cycle time is applied, measurements made at different cycles are compiled together. A power coefficient correction was applied to all INMETRO measurements to be comparable to 50 mA (BIPM measurement current). Table 9 gives the mean values at the mean date of 23rd of May 2021 for BIV200 at both currents, 25th of May 2021 for BIV207 at 10 mA and 28th of May 2021 for BIV207 at 30 mA, before application of temperature correction, atmospheric pressure and power coefficient corrections. The repeatability is estimated by the standard deviation of the series of measurements. The pressure of the mineral oil exerted on the resistors has been considered for every measurement and the mean pressure from Table 9 is corrected for this effect.

Standard #	Relative difference from nominal 1 Ω value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean pressure at the reference plane / hPa
BIV200 at 10 mA	− 0.724	0.056	23.001	1025.46
BIV200 at 30 mA	− 0.721	0.009	23.001	1023.03
BIV207 at 10 mA	− 0.397	0.034	23.000	1023.75
BIV207 at 30 mA	− 0.402	0.016	23.002	1022.26

Table 9: Summary of INMETRO 1 Ω calibrations.

4.5.1 Corrections for temperature, pressure and power differences:

The value R(23) of the resistance corrected to T_0 = 23 °C is:

$$R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$

where $R(T)$ is the resistance of the standard at temperature T.

The value R(1013.25) of the resistance corrected to P_0 = 1013.25 hPa is:

$$R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$

where $R(P)$ is the resistance of the standard at pressure P.

From BIPM experiments, the power coefficient of BIV200 and BIV207 from 10 mA to 50 mA has been measured to be – 2.0 × 10^{-9} /mW with a standard uncertainty u_{Power} = 1.5 × 10^{-9} /mW. The power difference in the 1 Ω travelling standards between the INMETRO and the BIPM is either 2.4 mW or 1.6 mW depending on the measuring current used by the INMETRO (10 mA or 30 mA, cf. section 4.2).

The INMETRO results are corrected to the reference temperature, the reference pressure and from the power difference using the coefficients α_{23}, β, γ and the power coefficient shown in Table 1. Applied corrections are reported in Table 10.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa			
	Relative corrections (µΩ/Ω)		
Standard #	For temperature	For pressure	For power difference
BIV200 at 10 mA	+ 0.000	+ 0.001	− 0.005
BIV200 at 30 mA	+ 0.000	+ 0.001	− 0.003
BIV207 at 10 mA	+ 0.000	+ 0.003	− 0.005
BIV207 at 30 mA	+ 0.000	+ 0.002	− 0.003

Table 10: Corrections applied to the INMETRO 1 Ω results.

The standard uncertainties of the temperature and pressure measurements at the INMETRO are 0.002 °C and 0.12 hPa respectively. The uncertainty of the power coefficient of the BIPM travelling standards is 1.5×10^{-9} /mW. Taking into account the differences from the reference temperature, the reference pressure, the power difference, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp}, u_{Press} and u_{Power} associated with the temperature, pressure and power difference corrections applied by the BIPM are estimated to be $u_{\text{Temp}} < 0.001 \times 10^{-6}$, $u_{\text{Press}} = 0.002 \times 10^{-6}$ and $u_{\text{Power}} = 0.004 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.005 \times 10^{-6}$. u_3 is reported in Table 13.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

4.5.2 Uncertainty budget provided by the INMETRO:

Uncertainties associated with the value of the 1 Ω measurements versus 100 Ω		
Source of uncertainty	**Type**	**Relative standard uncertainty (nΩ/Ω)**
Reference value of 100 Ω (Table 7)	B	3.1
Drift of the 100 Ω reference resistor	B	5.9
CCC resistance bridge (Table 6)	B	0.6
Temperature of the 100 Ω reference resistor	B	0.2
Combined relative standard uncertainty (*k* = 1)		**6.7**

Table 11: INMETRO uncertainty budget for the calibration of the 1 Ω travelling standard against the 100 Ω reference standard.

Note on Table 11: the calculation of the 1 Ω travelling standard value is based on the same model equation as for the 100 Ω reference standard (section 4.3).

Uncertainties associated with the value of the 1 Ω measurements versus 10 Ω		
Source of uncertainty	**Type**	**Relative standard uncertainty (nΩ/Ω)**
Reference value of 10 Ω (Table 8)	B	6.4
CCC resistance bridge (Table 6)	B	0.6
Temperature correction	B	0.8
Combined relative standard uncertainty (*k* = 1)		**6.5**

Table 12: INMETRO uncertainty budget for the calibration of the 1 Ω travelling standard against the 10 Ω reference standard.

Note on Table 12: the calculation of the 1 Ω travelling standard value is based on the same model equation as for the 10 Ω reference standard (section 4.3).

4.5.3 Uncertainties associated with the measurement of 1 Ω resistors:

Table 13 shows the corrected measurements of the 1 Ω standards at INMETRO at the mean date of 23rd of May 2021 for BIV200 and 26th of May 2021 for BIV207 as well as the associated uncertainty components. As no significant differences between measurements carried out at 10 mA and 30 mA were observed, the arithmetic mean of the overall measurements made for these two current values has been considered.

INMETRO After corrections	**Relative difference from nominal value (µΩ/Ω)**	**Relative standard uncertainties**		
		Repeatability u_1 (µΩ/Ω)	Systematic u_2 (µΩ/Ω)	Corrections u_3 (µΩ/Ω)
BIV200 (mean of 10 mA and 30 mA meas.)	-0.726	0.049	0.007	0.005
BIV207 (mean of 10 mA and 30 mA meas.)	-0.401	0.028	0.007	0.005

Table 13: Summary of the INMETRO results at 1 Ω, after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 13 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components. Repeatability u_1 is the standard deviation of all the measurements made at 10 mA and 30 mA for a taken resistor and not the standard deviation at 10 mA combined to the standard deviation at 30 mA.

4.6 <u>INMETRO results at 10 kΩ</u>:

The 10 kΩ travelling standards were measured 17 times for B10K11 and 7 times for B10K12 in the period April 2021 to June 2021. The measurements were made at 20 seconds cycle rate and also 100 seconds cycle rate. Because no correction for a possible dependence on the current reversal cycle is applied, measurements made at different cycles are compiled together. Table 14 gives the mean values at the mean date of 30th of May 2021 for B10K11 and of 19th of April 2021 for B10K12, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	**Relative difference from nominal 10 kΩ value (µΩ/Ω)**	**Std dev. (µΩ/Ω)**	**Mean temperature / °C**	**Mean atmospheric pressure / hPa**
B10K11	+ 1.320	0.014	23.001	1015.42
B10K12	+ 1.120	0.006	22.998	1010.93

Table 14: Summary of INMETRO 10 kΩ calibrations.

4.6.1 Corrections for temperature, pressure and power difference:

The value $R(23)$ of the resistance corrected to T_0 = 23 °C is:

$$R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$$

where $R(T)$ is the resistance of the standard at temperature T.

The value $R(1013.25)$ of the resistance corrected to P_0 = 1013.25 hPa is:

$$R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$$

where $R(P)$ is the resistance of the standard at pressure P.

From BIPM experiments, the power coefficient of B10K11 has been measured to be 2.4×10^{-9} /mW with a standard uncertainty $u_{\mathrm{Power}} = 2 \times 10^{-9}$ /mW for currents ranging from 50 µA to 100 µA. The power coefficient of B10K12 under a current of 50 µA to 100 µA has been measured to be 1×10^{-9} /mW with a standard uncertainty $u_{\mathrm{Power}} = 2 \times 10^{-9}$ /mW for the same current range. The power difference in the 10 kΩ travelling standards between the INMETRO (30 µA) and the BIPM (100 µA) is 0.09 mW. Since the difference is smaller than the dispersion, there is no correction applied to the power effect.

The INMETRO results are corrected to the reference temperature and the reference pressure using the coefficients α_{23}, β, γ and the power coefficient shown in Table 1. Applied corrections are reported in Table 15.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa			
	Relative corrections (µΩ/Ω)		
Standard #	For temperature	For pressure	For power difference
B10K11	+ 0.000	+ 0.001	+ 0.000
B10K12	+ 0.000	− 0.001	+ 0.000

Table 15: Corrections applied to the INMETRO 10 kΩ results.

The standard uncertainties of the temperature and pressure measurements at the INMETRO are 0.002 °C and 0.02 hPa respectively. The uncertainty of the power coefficient of the BIPM travelling standards is 2.4×10^{-9} /mW at maximum (B10K11). Taking into account the differences from the reference temperature, the reference pressure, the power difference, and the uncertainties associated with the coefficients, the relative standard uncertainties u_{Temp}, u_{Press} and u_{Power} associated with the temperature, pressure and power difference corrections applied by the BIPM are estimated to be $u_{\mathrm{Temp}} < 0.001 \times 10^{-6}$, $u_{\mathrm{Press}} < 0.001 \times 10^{-6}$ and $u_{\mathrm{Power}} < 0.001 \times 10^{-6}$ leading to a combined relative standard uncertainty $u_3 = 0.001 \times 10^{-6}$. u_3 is reported in Table 17.

4.6.2 Uncertainty budget provided by the INMETRO:

Uncertainties associated with the value of the 10 kΩ measurements		
Source of uncertainty	**Type**	**Relative standard uncertainty (nΩ/Ω)**
Reference value of 100 Ω (Table 7)	B	3.1
Drift of the 100 Ω reference resistor	B	5.9
CCC resistance bridge (Table 6)	B	0.6
Temperature of the 100 Ω reference resistor	B	0.2
Combined relative standard uncertainty (k = 1)		**6.7**

Table 16: Uncertainty budget for the calibration of the 10 kΩ travelling standards.

Note on Table 16: the calculation of the 10 kΩ travelling standard value is based on the same model equation for determination of the 100 Ω reference standard value from the QHR, with the difference that r^{-1} is adopted instead of r, since the reference resistor is now connected to the master branch of the CCC resistance bridge rather than to the slave branch; $r^{-1} \approx 100$.

4.6.3 Uncertainties associated with the measurement of 10 kΩ resistors:

Table 17 shows the corrected measurements of the 10 kΩ standards at INMETRO as well as the uncertainty components associated with these measurements.

INMETRO After corrections	**Relative difference from nominal value (μΩ/Ω)**	**Relative standard uncertainties**		
		Repeatability u_1 (μΩ/Ω)	Systematic u_2 (μΩ/Ω)	Corrections u_3 (μΩ/Ω)
B10K11	+1.321	0.014	0.007	0.001
B10K12	+1.119	0.006	0.007	0.001

Table 17: Summary of the INMETRO results at 10 kΩ, after corrections.

Note: The distinction between ‘systematic’ and ‘repeatability’ is made in Table 17 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components. Repeatability u_1 is the standard deviation of the overall measurement delivered by the INMETRO and not the combined standard deviation (at 20 seconds cycle rate and 100 seconds cycle rate).

5 Comparison INMETRO – BIPM

The individual measurement results for each of the four standards are shown in Figures 2 to 5. The plots also show the mean value of the INMETRO measurements with the uncertainty bar corresponding to the expanded uncertainty ($k = 2$) of the comparison U_c provided below, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of Figures 2 to 5 indicates that this is an appropriate model as both 1 Ω standards and 10 kΩ standards fit this model well. We treat the 1 Ω and 10 kΩ results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of the INMETRO measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of the INMETRO measurements.

The difference between the INMETRO and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{INMETRO},i} - R_{\mathrm{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\mathrm{INMETRO-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}\left(R_{\mathrm{INMETRO},i} - R_{\mathrm{BIPM},i}\right)$$

For each standard, the uncertainty u_1 associated with the interpolated BIPM value is calculated from the linear fit; u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard R_i, the BIPM uncertainty $u_{\mathrm{BIPM},i}$ is obtained from: $u^2_{\mathrm{BIPM},i} = u^2_{1,i} + u^2_{2,i}$. When the mean (for two standards) of the INMETRO − BIPM relative difference is calculated, the BIPM contribution to the uncertainty is,

$$u^2_{\mathrm{BIPM}} = \sum_{i=1}^{2}\frac{u^2_{1,i}}{2^2} + u^2_2$$

Similarly, for the INMETRO measurements, we expect the uncertainty components u_2 and u_3 of Tables 9 and 13 to be correlated between standards, and u_1 to be uncorrelated. We therefore calculate the total uncertainty as:

$$u^2_{\mathrm{INMETRO}} = \sum_{i=1}^{2}\frac{u^2_{1,i}}{2^2} + u^2_2 + u^2_3$$

5.1 Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In the present case, from inspection of the BIPM ‘before’ and ‘after’ measurements in Figures 2 to 5, we can see that any such effects are negligible compared to the overall uncertainty of the comparison. For simplicity, we do not include any extra uncertainty components.

5.2 Results at 1 Ω

The differences between the values assigned by the INMETRO, R_{INMETRO}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of the INMETRO measurements are shown in Table 18.

INMETRO − BIPM	
Standard #	$10^6 \times (R_{\text{INMETRO}} - R_{\text{BIPM}}) / (1\ \Omega)$
BIV200	+ 0.008
BIV207	+ 0.012
Mean	**+ 0.010**

Table 18: INMETRO – BIPM differences for the two 1 Ω travelling standards.

The mean difference between the INMETRO and the BIPM calibrations is:

$$\boldsymbol{(R_{\text{INMETRO}} - R_{\text{BIPM}}) / (1\ \Omega) = +0.010 \times 10^{-6}}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_c^2 = u_{BIPM}^2 + u_{\text{INMETRO}}^2$$

where,

$$u_{\text{BIPM}} = 0.017 \times 10^{-6},$$
$$u_{\text{INMETRO}} = 0.030 \times 10^{-6},$$

giving:

$$\boldsymbol{u_{\text{C}} = 0.034 \times 10^{-6}}$$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the INMETRO and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_{C}:

$$\boldsymbol{D = (R_{\text{INMETRO}} - R_{\text{BIPM}}) / 1\ \Omega = +0.010 \times 10^{-6}}$$

$$\boldsymbol{U_{\text{C}} = 0.068 \times 10^{-6}}$$

The difference between the INMETRO and the BIPM calibration results is within the expanded uncertainty.

5.3 Results at 10 kΩ

The difference between the value assigned by the INMETRO, R_{INMETRO}, and those assigned by the BIPM, R_{BIPM}, to each of the two travelling standards on the mean date of the INMETRO measurements are shown in Table 19.

INMETRO − BIPM	
Standard #	$10^6 \times (R_{\mathrm{INMETRO}} - R_{\mathrm{BIPM}})$ / (10 kΩ)
B10K11	+ 0.029
B10K12	+ 0.023
Mean	+ **0.026**

Table 19: INMETRO – BIPM differences for the two 10 kΩ travelling standards.

The mean difference between the INMETRO and the BIPM calibrations is:

$$(\boldsymbol{R}_{\mathbf{INMETRO}} - \boldsymbol{R}_{\mathbf{BIPM}}) \,/\, (\mathbf{10\ k\Omega}) = +\,\mathbf{0.026 \times 10^{-6}}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_c^2 = u_{BIPM}^2 + u_{\mathrm{INMETRO}}^2$$

where, $u_{\mathrm{BIPM}} = 0.015 \times 10^{-6}$,
$u_{\mathrm{INMETRO}} = 0.010 \times 10^{-6}$,

giving: $\boldsymbol{u}_{\mathbf{C}} = \mathbf{0.018 \times 10^{-6}}$

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the INMETRO and the BIPM for the value assigned to 10 kΩ resistance standards, and its expanded relative uncertainty (expansion factor $k = 2$, corresponding to a confidence level of 95 %), U_{C}:

$$\boldsymbol{D} = (\boldsymbol{R}_{\mathbf{INMETRO}} - \boldsymbol{R}_{\mathbf{BIPM}}) \,/\, \mathbf{10\ k\Omega} = +\,\mathbf{0.026 \times 10^{-6}}$$

$$\boldsymbol{U}_{\mathbf{C}} = \mathbf{0.036 \times 10^{-6}}$$

The difference between the INMETRO and the BIPM calibration results is within the expanded uncertainty.

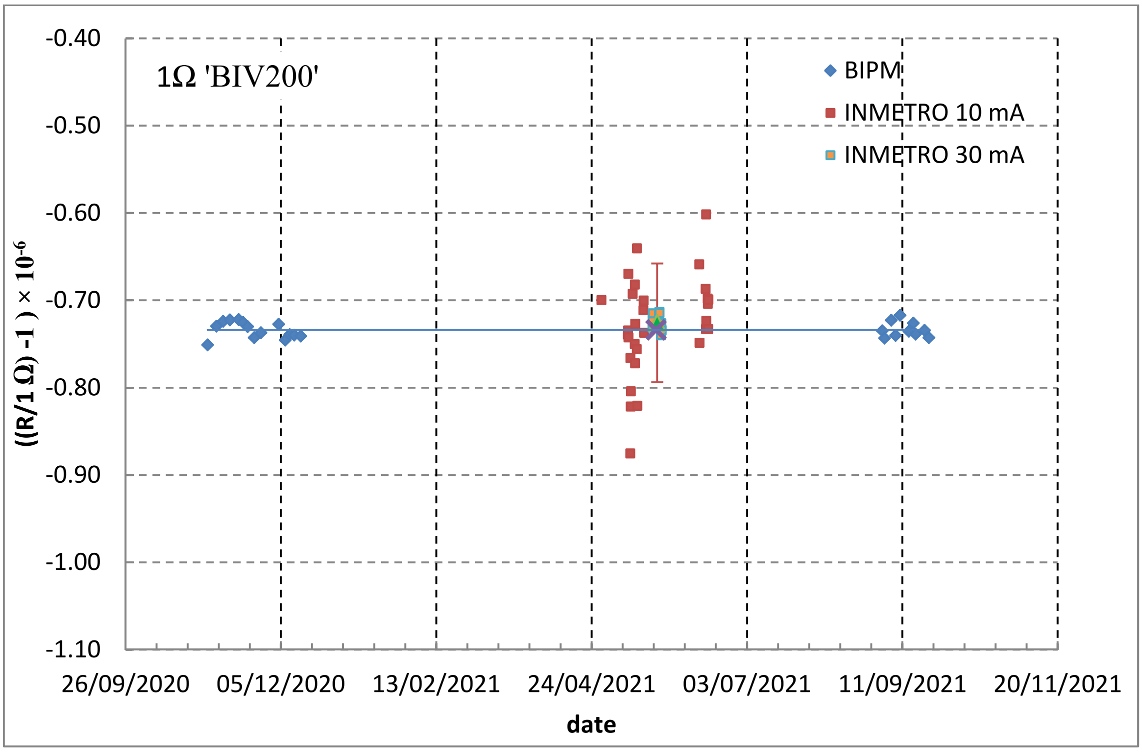

Figure 2: results for 1 Ω standard BIV200. BIPM (blue diamonds) and INMETRO (red and orange squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INMETRO and the green triangle is the mean value of INMETRO measurements. Uncertainty bar shows the expanded uncertainty of the comparison of the mean INMETRO results.

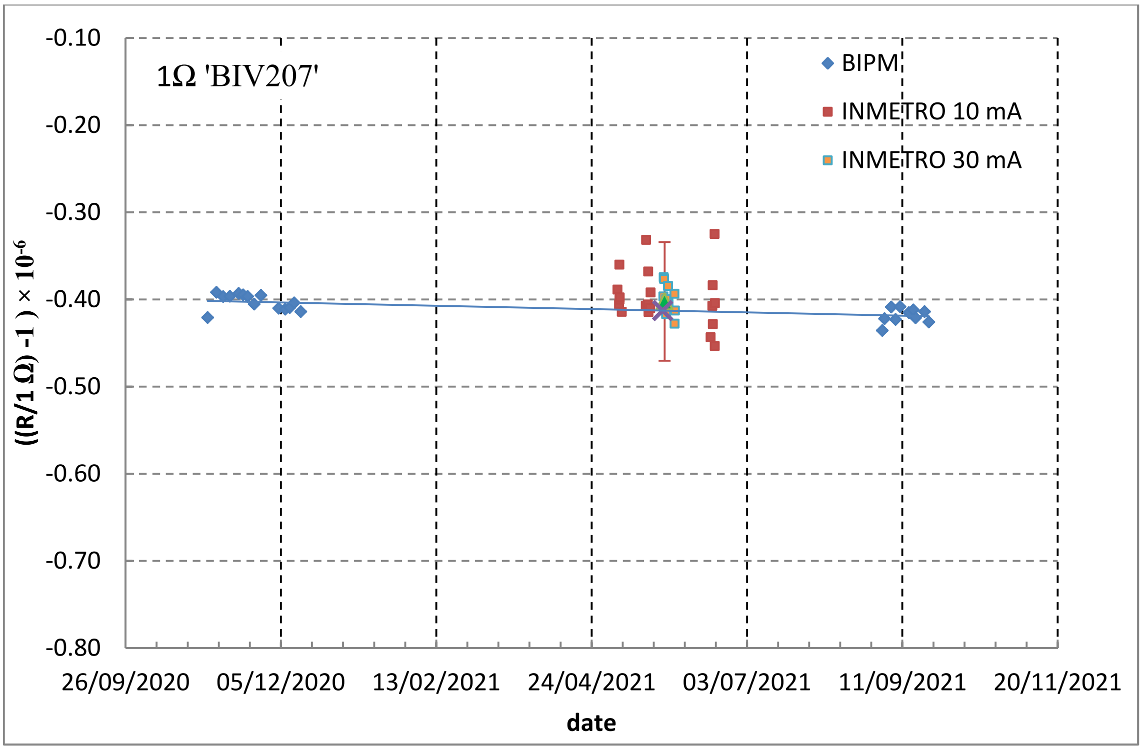

Figure 3: results for 1 Ω standard BIV207. BIPM (blue diamonds) and INMETRO (red and orange squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INMETRO and the green triangle is the mean value of INMETRO measurements. Uncertainty bar shows the expanded uncertainty of the comparison of the mean INMETRO results.

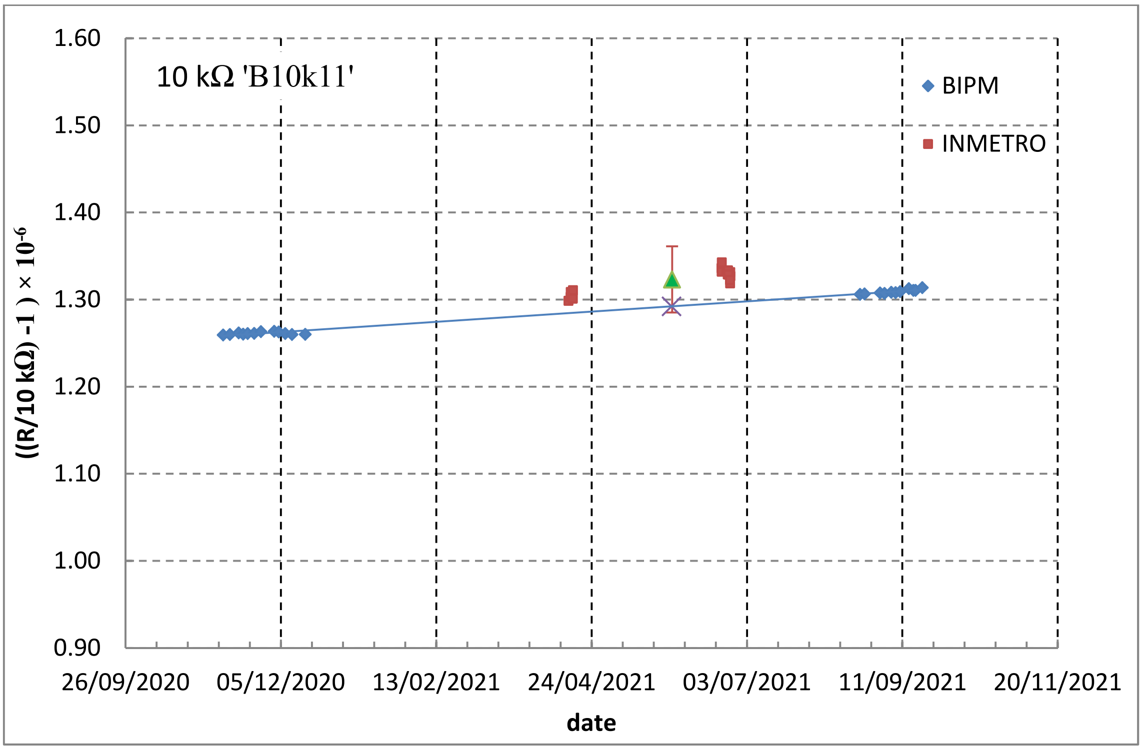

Figure 4: results for 10 kΩ standard B10K11. BIPM (blue diamonds) and INMETRO (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INMETRO and the green triangle is the mean value of INMETRO measurements. Uncertainty bar shows the expanded uncertainty of the comparison of the mean INMETRO results.

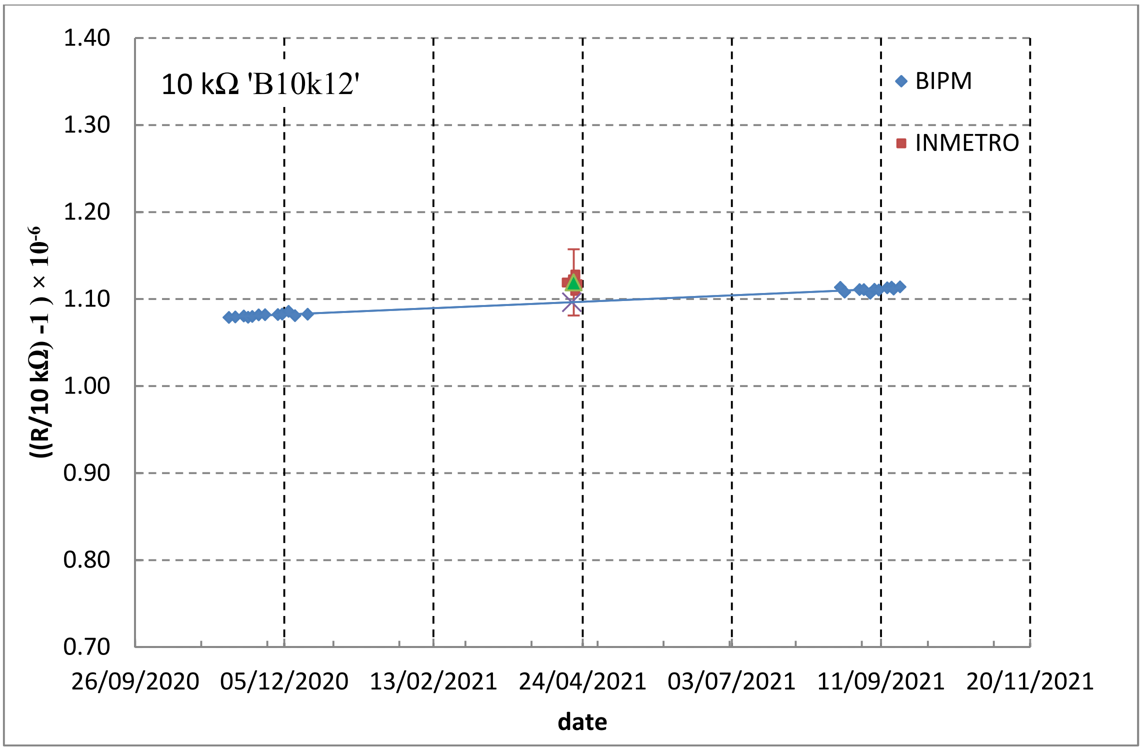

Figure 5: results for 10 kΩ standard B10K12. BIPM (blue diamonds) and INMETRO (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at INMETRO and the green triangle is the mean value of INMETRO measurements. Uncertainty bar shows the expanded uncertainty of the comparison of the mean INMETRO results.