## **BUREAU INTERNATIONAL DES POIDS ET MESURES**

## Bilateral comparison of 1 $\Omega$ and 10 k $\Omega$ standards (ongoing BIPM key comparisons BIPM.EM-K13) between the CEM (Spain) and the BIPM

June 2023

## **Final Report**

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#### 1 <u>Introduction</u>

A comparison of values assigned to 1  $\Omega$  and 10 k $\Omega$  resistance standards was carried out between the BIPM (Bureau International des Poids et Mesures) and the CEM (Centro Español de Metrología) in the period March 2022 to September 2022. Two 1  $\Omega$  and two 10 k $\Omega$  BIPM travelling standards were calibrated first at the BIPM, then at the CEM and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: March – April 2022 CEM measurements: May – June 2022 'After' measurements at the BIPM: August – September 2022

This report is organised as follows: details of the travelling standards used are listed in Section 2; the results of the BIPM measurements are given in Section 3 and the calibration report provided by the CEM is summarized in Section 4; these two last sections include the uncertainty budgets for each laboratory. Finally, the two sets of measurements are compared and analysed in Section 5. The uncertainties arising from the transfer of the standards between the two laboratories are estimated and included at this point. The final results of the comparisons are given in the form of the degrees of equivalence between the CEM and the BIPM for measurements of 1  $\Omega$  and 10 k $\Omega$  resistance standards.

This report covers the comparison of both 1  $\Omega$  standards and 10 k $\Omega$  standards. The measurements of these two different resistance values are analysed separately, but are reported together here as the two comparisons were carried out simultaneously.

#### 2 <u>Travelling standards</u>

Two 1  $\Omega$  and two 10 k $\Omega$  travelling standards provided by the BIPM were used for this comparison. The two 1  $\Omega$  standards are of CSIRO type, with working labels BIV203 (manufacturer's serial number S-64203) and BIV207 (manufacturer's serial number S-64207). The two 10 k $\Omega$  standards are TEGAM SR104 type and have the working labels B10K09 (manufacturer's serial number K203039730104) and B10K11 (serial number K205039730104). The standards were shipped by regular air freight between the laboratories.

All measurements are corrected to a reference temperature of 23.000 °C and reference pressure 1013.25 hPa using the known coefficients of the standards, given in Table 1. According to the protocol, the CEM did not apply pressure and temperature corrections to its results, but supplied the raw values and the measured temperature and pressure. The corrections were applied in the analysis made by the BIPM following the CEM results for 1  $\Omega$  and 10 k $\Omega$ .

	Relative temperature coefficients		Relative pressure coefficient		Relative power coefficient		
Standard #	$\frac{\alpha_{23}}{(10^{-6}/\text{K})}$	$\frac{\beta}{(10^{-6}/\text{K}^2)}$	$\frac{u_{\rm T}}{(10^{-6}/{\rm K})}$	$\frac{\gamma}{(10^{-9})}$ / hPa)	$u_{\rm P}/(10^{-9}/{\rm hPa})$	$P/(10^{-9}/\text{ mW})$	$\frac{u_{\rm W}}{(10^{-9}/{\rm mW})}$
BIV203	- 0.0096	+0.0016	0.001	- 0.20	0.20	- 2.0	2.0
BIV207	- 0.0094	+0.0001	0.001	- 0.25	0.20	- 2.0	1.5
B10K09	- 0.0400	- 0.0220	0.010	- 0.16	0.10	+ 1.0	3.0
B10K11	- 0.0700	- 0.0270	0.010	- 0.35	0.10	+ 2.4	2.4

 Table 1: Temperature, pressure and power coefficients of the traveling standards.

#### 3 Measurements at the BIPM

#### 3.1 Measurement of the 1 $\Omega$ standards at the BIPM:

The BIPM measurements are traceable to the quantum Hall resistance (QHR) standard via different measurement bridges and working standards for the two nominal values included in this comparison. In all cases, values are based on the revised SI value of the von Klitzing constant,  $R_{\rm K} = h/e^2 = 25$  812.807 46  $\Omega$ , using the fixed numerical values for the Planck constant *h* and the elementary charge *e*.

The 1  $\Omega$  measurements were carried out by comparison with a 100  $\Omega$  reference resistor (identifier BI100-3) whose value is calibrated against the BIPM QHR standard regularly (at least once every 6 months). The comparison was performed using a DC cryogenic current comparator operating with 50 mA current in the 1  $\Omega$  resistors.

The 1  $\Omega$  travelling standards were kept in a temperature-controlled oil bath at a temperature which is close (within a few mK) to the reference temperature of 23 °C. The oil temperature close to each standard was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT) in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The additional pressure  $P_h$  exerted by the volume of the mineral oil above the resistors (reference plane corresponding to the plane containing the resistor terminals) has been considered for every measurement.  $P_h$  is calculated using the following equation:

 $P_h = RD \times \rho \times g \times h$ With *RD* the relative density of the oil Marcol 52 type = 0.83 at 23°C;  $\rho$  the density of the pure water = 1000 kg m<sup>-3</sup> at 4 °C; g the local gravity = 9.807 m s<sup>-2</sup> and h the height of the oil above the reference plane in m. The height of the oil above the reference plane is recorded in the software of the measurement bridge and the additional pressure is calculated automatically at every measurement.

The 'dc' resistance value (or ratio) measured with the BIPM CCC-bridge results from a current signal passing through the resistors having polarity reversals with a waiting time between polarity inversions, see Figure 1. The polarity reversal frequency is of the order of 3 mHz (340 s cycle period) and the measurements are sampled only during 100 s before the change of polarity.



Figure 1: Schematic representation of the reference current signal with polarity reversals used in the BIPM CCC-bridge. The reversal cycle comprises a waiting time of about 36 s at zero current (green dotted line). The red dotted line corresponds to the sampling time period.

The travelling standards were measured 11 times during the period labelled 'before' (March 2022 – April 2022) and only 7 times during the period labelled 'after' (August 2022 – September 2022) due to the impossibility of obtaining a sufficient quantity of liquid helium because of the current worldwide shortage of this cryogen.

The individual BIPM measurement data are plotted in Figures 3 and 4 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 2 and the uncertainty budget in Table 3. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	]	Relative difference from nominal 1 $\Omega$ value ( $\mu\Omega/\Omega$ )						
Standard #	DEEODE	Std dev.	ΛΕΤΕΡ	Std dev.	INTERPOLATED	Std dev.		
Stanuaru #	DEFUKE	$u_{1\mathrm{B}}$	AFTER	$u_{1\mathrm{A}}$		$u_1$		
BIV203	+ 0.562	0.006	+ 0.593	0.015	+ 0.575	0.008		
BIV207	- 0.418	0.005	- 0.444	0.010	- 0.429 ON 04-06-2022	0.006		

Table 2: Summary of BIPM calibrations of the 1  $\Omega$  standards.

Source of uncertainty	Relative standard uncertainty (nΩ/Ω)
Imperfect realisation of $R_{\rm K}$	2
Calibration of the BIPM 100 $\Omega$ reference (BI100-3) against $R_{\rm K}$	3
Interpolation / extrapolation of the value of BI100-3	13
Measurement of the $(1\Omega / BI100-3)$ ratio	8
Temperature correction for the 1 $\Omega$ standard	2
Pressure correction for the 1 $\Omega$ standard	3
Combined standard uncertainty <i>u</i> <sub>2</sub>	16

Table 3: BIPM uncertainty budget for the calibration of the 1  $\Omega$  travelling standards.

### **3.2** <u>Measurement of the 10 kΩ standards at the BIPM:</u>

The 10 k $\Omega$  measurements were carried out by comparison with a set of two 10 k $\Omega$  reference resistors (identifiers B10K1 and B10K2) which are calibrated regularly (at least once every 6 months) against the BIPM QHR standard. The comparison was performed using a Warshawsky bridge operating with a 0.1 mA DC current (i.e. at a measurement voltage of 1 V).

The 10 k $\Omega$  travelling standards were kept in a temperature-controlled air bath at a temperature which is close to the reference temperature of 23 °C (within 0.05 °C). The temperature of the standards was determined by means of a calibrated SPRT, in conjunction with thermocouples placed in the thermal well of each resistor. The air pressure in the laboratory was recorded using a calibrated manometer at the time of each measurement. The relative humidity in the air bath was not monitored, but the laboratory air conditioning system controls the relative humidity to 50 % ( $\pm$  10 %).

The travelling standards were measured 11 times during the period labelled 'before' (March 2022 – April 2022) and 10 times during the period labelled 'after' (August 2022 – September 2022).

The individual BIPM measurement data are plotted in Figures 5 and 6 of Section 5 (after application of the temperature and pressure corrections). The mean results are summarized in Table 4 and the uncertainty budget in Table 5. The dispersion of each group of measurements is estimated by the standard deviation.

BIPM	R	Relative difference from nominal 10 k $\Omega$ value ( $\mu\Omega/\Omega$ )						
Standard #	BEFORE	Std dev.	AFTER	Std dev.	INTERPOLATED	Std dev.		
Stanuaru #	DEFORE	$u_{1B}$ AFTER		$u_{1\mathrm{A}}$	$u_{1A}$			
B10K09	+ 0.008	0.002	+ 0.029	0.004	+ 0.016 ON 02-06-2022	0.002		
B10K11	+ 1.286	0.002	+ 1.345	0.004	+ 1.307 ON 02-06-2022	0.002		

<b>Table 4: Summarv</b>	of BIPM	calibrations	of the	10 kΩ	standards.
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Source of uncertainty	Relative standard uncertainty $(n\Omega/\Omega)$
Imperfect realization of $R_{\rm K}$	2
Calibration of the BIPM 100 $\Omega$ reference (BI100-3) against $R_{\rm K}$	3
Link 100 Ω / 10 000 Ω	5
Link 10 000 $\Omega$ / (mean reference B10K1-B10K2)	7
Extrapolation of mean value of 10 k $\Omega$ reference	8
Measurement of the voltage applied to the bridge	5
Measurements of the bridge unbalance voltage	5
Leakage resistances	1
Temperature correction for travelling standard	3
Pressure correction for travelling standard	2
Combined standard uncertainty <i>u</i> <sub>2</sub>	15

Table 5: BIPM uncertaint	y budget for the	calibration of the	10 k $\Omega$ travelling standard	ds.
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## 4 <u>Measurements at the CEM</u>

## 4.1 <u>Preparation:</u>

The CEM received the standard resistors on 5<sup>th</sup> of May 2022 and a visual inspection was done to assess their physical conditions. The 1  $\Omega$  travelling standards were kept in a stirred temperature-controlled oil bath (Kambiç OB-100/2 LTUS) at a temperature of (23.000 ± 0.003) °C and the 10 k $\Omega$  travelling standards were kept in a temperature-controlled air bath (Kambiç TK-190-US) at a temperature of (23.000 ± 0.005) °C. They were allowed to stabilise prior to the measurements.

### 4.2 <u>Traceability of DC resistance standard at CEM:</u>

The CEM resistance national standard is based on the comparison of the QHR (*i*=2) with the 10 k $\Omega$  reference group (a set of three standards, model ESI SR104) using a MI 6010C bridge. The same bridge model is used for all reported measurements. The error in ratio 10:1,  $\epsilon_{10:1}$ , has been measured with a Hamon transfer standard. The linearity of the bridge for QHR (*i*=2) to 10 k $\Omega$  ratio has also been measured using a homemade resistive transfer standard (RTS) [1]. There is a 25 years historical follow-up of the reference resistance group calibrations with the QHE. The traceability chain for the calibration of the comparison travelling standards is represented in Figure 2 below.



# Figure 2: Schematic representation of the traceability chain for the calibration of the comparison standards at the CEM.

[1] F. Raso and others, "Comparison of Two High Accuracy Room Temperature Methods to calibrate a 10 k $\Omega$  Standard with the Quantized Hall Resistance," 2018 Conference on Precision Electromagnetic Measurements (CPEM 2018), 2018, pp. 1-2.

To follow the usual traceability chain for the calibration of the comparison standards at the CEM, and for a better understanding of the ratios used in the formulas used in §4.3 and §4.4, we choose to describe the method of calibration for the 10 k $\Omega$  before the one used for the 1  $\Omega$  travelling standards.

#### 4.3 <u>Method of calibration for the 10 kΩ travelling standards:</u>

The measurements of the two 10 k $\Omega$  travelling standards have been made using a substitution method against the 10 k $\Omega$  standard reference group. A 1 k $\Omega$  tare resistor was used. Both the standards under test and the CEM standards were compared with it, obtaining the ratios:

$$r_{\rm pj} = \frac{R_{\rm pj}^{\rm MC}}{R'}$$
$$r_{\rm xi} = \frac{R_{\rm xi}^{\rm MC}}{R'}$$

Where  $R_{pj}^{MC}$  is the value of each of the *j* CEM reference standards in MC (measurement conditions during the calibration),  $R_{xi}^{MC}$  is the value of each of the *i* travelling standards in MC and *R*' is the 1 k $\Omega$  tare resistor. RC stands for reference conditions (23.000°C and 1013.25 hPa).

The tare resistor is very stable in temperature; its value can be considered constant during measurements corresponding to the above equations. To eliminate its dependence the following ratio is computed:

$$r = \frac{r_{\rm xi}}{r_{\rm pj}}$$

With the substitution method the contribution of the ratio of the bridge is cancelled. The equation for the 10 k $\Omega$  measurements is the following:

$$R_{\rm xi}^{\rm MC} = r R_{\rm pj}^{\rm MC} = r R_{\rm pj}^{\rm RC} (1 + \alpha_{\rm pj} (t_{\rm pj} - 23) + \beta_{\rm pj} (t_{\rm pj} - 23)^2)$$

 $R_{xj}$  values haven't been corrected in temperature, but their known coefficients (indicated CEM0000016797 on their boxes) have been considered for the estimation of the uncertainties. These contributions have been found to be negligible. For uncertainties, the equation is:

$$R_{xi}^{RC} = r R_{pj}^{RC} \frac{(1 + \alpha_{pj}(t_{pj} - 23) + \beta_{pj}(t_{pj} - 23)^2)}{(1 + \alpha_{xj}(t_{xj} - 23) + \beta_{xj}(t_{xj} - 23)^2)}$$

#### 4.4 <u>Method of calibration for the 1 Ω travelling standards:</u>

The measurement of the Hamon transfer standard in series  $(10 \text{ k}\Omega)$  is similar as the measurements of the 10 k $\Omega$  travelling standards. The Hamon resistor in parallel (100  $\Omega$ ) is compared with an intermediate 10  $\Omega$  standard resistor in 10:1 ratio. This 10  $\Omega$  standard is then compared to the 1  $\Omega$  travelling standards in 10:1 ratio. The equations for the 1  $\Omega$  measurements are the following:

$$R_{\rm xi}^{\rm MC} = H_{\rm s} \frac{R_{\rm xi}^{\rm MC}/R_{\rm t10}}{H_{\rm p}/R_{\rm t10}} \frac{1}{N} = r_{\rm H} R_{\rm pj}^{\rm MC} \frac{R_{\rm xi}^{\rm MC}/R_{\rm t10}}{H_{\rm p}/R_{\rm t10}} \frac{1}{N}$$

With  $r_{\rm H} = \frac{H_{\rm s}}{R_{\rm pj}^{\rm MC}}$  and  $N = \frac{H_{\rm s}}{H_{\rm p}}$ , where N is approximately 100 within the Hamon specifications.

$$R_{\rm xi}^{\rm MC} = r_{\rm H} R_{\rm pj}^{\rm RC} \frac{R_{\rm xi}^{\rm MC} / R_{\rm t10}}{H_{\rm p} / R_{\rm t10}} \frac{1}{N} (1 + \alpha_{\rm pj} (t_{\rm pj} - 23) + \beta_{\rm pj} (t_{\rm pj} - 23)^2)$$
$$R_{\rm xi}^{\rm MC} = r_{\rm H} \frac{r_{\rm xi}}{r_{\rm h}} \frac{1}{N} R_{\rm pj}^{\rm RC} (1 + \alpha_{\rm pj} (t_{\rm pj} - 23) + \beta_{\rm pj} (t_{\rm pj} - 23)^2)$$
$$With r_{\rm h} = \frac{H_{\rm p}}{R_{\rm t10}} \text{ and } r_{\rm xi} = \frac{R_{\rm xi}^{\rm MC}}{R_{\rm t10}}$$

As the temperature and pressure coefficients are unknown, the resultant uncertainties of temperature appears as (— for void) in Tables 10, 11, 16, 17 and 18.

$$R_{\rm xi}^{\rm RC} = r_{\rm H} \frac{r_{\rm xi}}{r_{\rm h}} \frac{1}{N} R_{\rm pj}^{\rm RC} \frac{(1 + \alpha_{\rm pj}(t_{\rm pj} - 23) + \beta_{\rm pj}(t_{\rm pj} - 23)^2)}{(1 + \alpha_{\rm xi}(t_{\rm xi} - 23) + \beta_{\rm xi}(t_{\rm xi} - 23)^2) + \gamma_{\rm xi}(p_{\rm xi} - 101325))}$$

2

<u>Notes:</u> a  $R_{xj}$  value is obtained by comparison with each of the three CEM reference standards. The three values are consistent with each other within  $8 \times 10^{-9}$ . The results were averaged. The highest uncertainty is taken as the uncertainty of the group of the three 10 k $\Omega$  CEM reference standards.

#### 4.5 <u>Standards and instruments used by CEM:</u>

Name	Value	Manufacturer	Model No.	Serial No.	Traceability
$R_{p1}$	10 kΩ	ESI	SR104	224109	CEM, Spain
$R_{p2}$	10 kΩ	ESI	SR104	415103	CEM, Spain
R <sub>p3</sub>	10 kΩ	ESI	SR104	J-204079130104	CEM, Spain
$T_{1k\Omega}$	1 kΩ	GUIDLINE	7334/1k	69537	CEM, Spain
$R_{10\Omega}$	10 Ω	TINSLEY	5685A	260025	CEM, Spain
H <sub>1k/step</sub>	$1 \text{ k}\Omega$ /step	GUIDLINE	9350	61117	CEM, Spain

The information related to the standards used at CEM in the comparison is given in the Table 6.

 Table 6: Measurement standards used at CEM.

The CEM  $R_{p1}$ ,  $R_{p3}$ , 1 k $\Omega$  tare resistor Guildline 7334/1k and both 10 k $\Omega$  travelling standards are placed in a Kambiç TK-190-US air bath. A second air bath (Selecta) is used for  $R_{p2}$ .

The Guildline 9350 Hamon transfer standard (with  $\alpha_{\rm H}$  its temperature coefficient) and the Tinsley 5685A 10  $\Omega$  are used for the 1  $\Omega$  travelling standard measurements (Table 6). As it's unknown, we consider the manufacturer specifications.

The values of the standards in reference conditions (reference temperature of 23 °C and reference pressure of 1013.25 hPa) with their relative uncertainty at the mean date of CEM measurements are the following:

$$\begin{split} R_{\rm p1}^{\rm RC} &= 10\ 000.014\ 10\ \Omega, \pm 2.9\times 10^{-8}R,\ k = 1\\ R_{\rm p2}^{\rm RC} &= 10\ 000.014\ 47\ \Omega, \pm 2.3\times 10^{-8}R,\ k = 1\\ R_{\rm p3}^{\rm RC} &= 10\ 000.014\ 64\ \Omega, \pm 2.9\times 10^{-8}R,\ k = 1 \end{split}$$

Drifts are included in these uncertainties in the form:

$$u(R_{\rm pj}^{\rm RC}) = \sqrt{\left(\frac{U_{\rm j}}{k_{\rm j}}\right)^2 + u_{\rm j}(drift)^2}$$

With  $U_j$  = expanded uncertainty of each of the *j* CEM reference standards and  $k_j$  its coverage factor.

The temperature coefficients of the standards are given in the Table 7. The pressure coefficients are negligible and not reported in this table.

	Relative temperature coefficients				
Standard #	$\alpha_{23} / (10^{-6}/\mathrm{K})$	eta / (10 <sup>-6</sup> /K <sup>2</sup> )			
<b>R</b> <sub>p1</sub>	- 0.05	- 0.027			
$R_{p2}$	+0.02	- 0.025			
<b>R</b> <sub>p3</sub>	- 0.08	- 0.024			

Table 7: Temperature coefficients of reference standards used at CEM.

The values of the three reference standards have been corrected to the corresponding measurement conditions (MC) in each measurement.

As mentioned above, the direct current comparator used in all measurements is a MI bridge, type MI 6010C, S/N 1100286.

Temperatures of the standards under measurement are taken using a thermometer ASL F250 with scanner, the measurement uncertainty of which is  $\underline{u}_{ter}$  ( $\underline{k}_{ter}$ = 1) = ± 0.010 °C.  $\delta_{ter}$  is the contribution to the uncertainty due to the resolution of the thermometer. In our case, the thermometer's resolution is  $\delta_{ter}$  = ± 0.000 5 °C (Tables 10, 11 and 12). Temperature corrections (~mK) of CEM reference standards are already applied by CEM to the measurements reported to the BIPM.

Pressure on the standards under measurement was taken using a barometer Ruska model 6200, the uncertainty of which is  $u_{bar}(k_{bar}=1) = \pm 2$  Pa. A pressure correction (-7 Pa) of CEM standards is already applied by CEM to the measurements reported to the BIPM.

The temperature and humidity in the lab have been registered with a Testo logger, model Testo 175-H2.

#### 4.6 **Operating conditions:**

Upon receipt the 1  $\Omega$  standards were placed in a stirred temperature controlled oil bath (Kambiç OB-100/2 LTUS) at a nominal temperature of 23 °C. The oil temperature was monitored using a Standard Platinum Resistance Thermometer. The measured oil temperature did not vary by more than ±3 mK during the entire period of the resistance measurements.

A test current of 50 mA and current reversal time of 6 seconds (filter 0.3 s) were used for all measurements.

The pressure reported to the BIPM does not include the pressure correction due to the oil level for the immersion of the 1  $\Omega$  standards. The heads of the standards (reference plane corresponding to the plane containing the resistor terminals) were 190 mm below the surface of the bath during the measurement period. The oil used has a mean specific gravity of 0.934 at 23°C.

An additional pressure  $P_h$  exerted by the volume of the oil above the resistors was added to the atmospheric pressure reported by the CEM. It is calculated using the following equation:  $P_h = RD \times \rho \times g \times H$ 

With *RD* the relative density (equal to the specific gravity) of the oil = 0.934;  $\rho$  the density of the pure water = 1000 kg m<sup>-3</sup> at 4°C; *g* the local gravity = 9.813 m s<sup>-2</sup> and *H* the height of the oil above the reference plane = 0.19 m. The computed additional pressure due to the head of oil during the measurements is 17.4 hPa.

The 10 k $\Omega$  travelling standards were placed in a temperature-controlled air bath (Kambic TK-190) at a nominal temperature of 23 °C. The air bath temperature varied by less than ± 5 mK during the entire period of the resistance measurements. The temperature, pressure and humidity were reported for each measurement. The measurement current in the 10 k $\Omega$  standards was 100  $\mu$ A and current reversal times of 12 seconds (filter 3 s) were used for all measurements.

#### 4.7 <u>CEM results at 1 $\Omega$ </u>:

The 1  $\Omega$  travelling standards were measured 10 times for both BIV203 and BIV207 in the period May 2022 – June 2022. The measurements were made at 50 mA. Power correction is not needed as applied current at CEM and BIPM are comparable.

Table 8 gives the mean values at the mean date of 4<sup>th</sup> of June 2022 for BIV203 and BIV207 before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements. The pressure of the mineral oil exerted on the resistors has been considered for every measurement and the mean pressure from Table 8 is corrected for this effect.

Standard #	Relative difference from nominal 1 Ω value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean pressure at the reference plane / hPa
BIV203	+0.662	0.012	22.965	950.97
BIV207	-0.377	0.010	22.976	950.97

#### Table 8: Summary of CEM 1 $\Omega$ calibrations.

#### 4.7.1 Corrections for temperature and pressure differences:

The value R(23) of the resistance corrected to  $T_0 = 23$  °C is:  $R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$ where R(T) is the resistance of the standard at temperature T.

The value R(1013.25) of the resistance corrected to  $P_0 = 1013.25$  hPa is:  $R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$ where R(P) is the resistance of the standard at pressure P.

The CEM results are corrected to the reference temperature and the reference pressure using the coefficients  $\alpha_{23}$ ,  $\beta$  and  $\gamma$  shown in Table 1. Applied corrections are reported in Table 9.

Reference temperature = $23.000 \text{ °C}$ Reference pressure = $1013.25 \text{ hPa}$				
	Relative corrections $(\mu\Omega/\Omega)$			
Standard #	For temperature	For pressure		
BIV203	- 0.000	- 0.012		
BIV207	- 0.000	- 0.016		

#### Table 9: Corrections applied to the CEM 1 $\Omega$ results.

The standard uncertainties of the temperature and pressure measurements at the CEM are 0.01 °C and 2 Pa respectively. Taking into account the differences from the reference temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties  $u_{\text{Temp}}$  and  $u_{\text{Press}}$  associated with the temperature and pressure corrections applied by the BIPM are estimated to be  $u_{\text{Temp}} < 0.001 \times 10^{-6}$  and  $u_{\text{Press}} = 0.013 \times 10^{-6}$  leading to a combined relative standard uncertainty  $u_3 = 0.013 \times 10^{-6}$ .  $u_3$  is reported in Table 13.

A correction for a possible dependence on the current reversal cycle has not been evaluated.

### 4.7.2 Uncertainty budget provided by the CEM:

Table 10 shows the uncertainty budget provided by the CEM associated to the CEM reference standards (10 k $\Omega$  references  $R_{p1}$ ,  $R_{p2}$ ,  $R_{p3}$  and the Hamon in series configuration  $R_{H}$ ). This table is common to both 1  $\Omega$  travelling standards.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
t <sub>p1</sub>	А, В	Mean t <sub>p1</sub>	23.019	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.014	$(\alpha_{p1}+2\beta_{p1}(t_{p1}-23))$	-5.10E-08	7.39E-10
t <sub>p2</sub>	А, В	Mean t <sub>p2</sub>	23.078	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.031	$(\alpha_{p2}+2\beta_{p2}(t_{p2}-23))$	1.61E-08	4.95E-10
t <sub>p3</sub>	А, В	Mean t <sub>p3</sub>	23.012	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.013	$(\alpha_{p3}+2\beta_{p3}(t_{p3}-23))$	-8.06E-08	1.01E-09
α <sub>p1</sub>	В	Indicated by manufacturer	-5.00E-08	Indicated by manufacturer	1.00E-08	( <i>t</i> <sub>p1</sub> -23)	0.019	1.85E-10
a <sub>p2</sub>	В		2.00E-08		1.00E-08	( <i>t</i> <sub>p2</sub> -23)	0.078	7.78E-10
$\alpha_{p3}$	В		-8.00E-08		1.00E-08	( <i>t</i> <sub>p3</sub> -23)	0.012	1.24E-10
$m{eta}_{p1}$	В		-2.70E-08	" "	1.00E-09	( <i>t</i> <sub>p1</sub> -23) <sup>2</sup>	0.000	3.43E-13
$\beta_{p2}$	В		-2.50E-10		1.00E-09	$(t_{p2}-23)^2$	0.006	6.06E-12
$\beta_{p3}$	В		-2.40E-08		1.00E-09	$(t_{p3}-23)^2$	0.000	1.54E-13
$R_{p1}^{RC}$	В	$R_{p1}^{RC}$	10 000.014 10	U/k and drift	2.86E-08	1	1	2.86E-08
$R_{p2}^{RC}$	В	$R_{p2}^{RC}$	10 000.014 47		2.26E-08	1	1	2.26E-08
$R_{p3}^{RC}$	В	$R_{p3}^{RC}$	10 000.014 64		2.88E-08	1	1	2.88E-08
R <sub>p1</sub> <sup>MC</sup>	В	$R_{p1}^{MC}$	10 000.014 08	2.9E-08				
$R_{p2}^{MC}$	В	$R_{p2}^{MC}$	10 000.014 48	2.3E-08				
R <sub>p3</sub> <sup>MC</sup>	В	$R_{p3}^{MC}$	10 000.014 63	2.9E-08				
R <sub>p</sub> <sup>MC</sup>			10 000.014 39				<i>и</i> (R <sub>р</sub> <sup>МС</sup> )	2.9E-08
r <sub>H</sub>		Mean r <sub>H</sub>	0.999 992 150	Bridge calibration and specs	2.89E-10	1	1	2.89E-10
t <sub>H</sub>		Mean (t <sub>x</sub> )	22.976	$\sqrt{u_A(t_x)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.013	$(\alpha_x+2\beta_x(t_x-23))$	0	—
α <sub>H</sub>		Indicated by manufacturer	0.00E-00	Indicated by manufacturer	2.89E-07	(t <sub>x</sub> -23)	-0.024	6.99E-09
R <sub>H</sub>		Mean $(R_x^{RC})$	9 999.935 50	u <sub>A</sub> (R <sub>H</sub> )	6.13E-09	$u_A(H_s) = \sqrt{\frac{\sum_{l=1}^n (H_{sl} - \overline{H_s})}{n(n-1)}}$	1	6.13E-09

Table 10: CEM uncertainty budget of the 10 k $\Omega$  references for the calibration of the 1  $\Omega$  travelling standard.

Table 11 shows the uncertainty budget provided by the CEM associated to the transfer from the
CEM's reference standard to the 1 $\Omega$ travelling standard BIV203.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
R <sub>x2</sub>	A	Mean $R_{x2}$	1.000 000 662	$u_A(R_{\rm x}) = \sqrt{\frac{\sum_{l=1}^n (R_{\rm xl} - \overline{R_{\rm x}})}{n(n-1)}}$	2.0E-9	1	1	2.02E-09
Ν	В	100	100	5 x 10 <sup>-8</sup> /sqrt(3)(relative)	2.9E-08	N <sup>2</sup>	1.00E-04	2.89E-12
r <sub>h</sub>	В	0	0	<i>u</i> (ε <sub>10:1</sub> )	2.5E-08	1	1	2.51E-08
r <sub>xi</sub>	В	0	0	<i>u</i> (ε <sub>10:1</sub> )	2.5E-08	1	1	2.51E-08
a <sub>x2</sub>	В	0	0.00E+00	0.1 x 10 <sup>-6</sup> K <sup>-1</sup>	1.00E-07	(t <sub>x2</sub> -23)	-0,035	3.46E-09
β <sub>x2</sub>	В	0	0.00E+00	0.01 x 10 <sup>-6</sup> K <sup>-2</sup>	1.00E-08	$(t_{x2}-23)^2$	1.20E-03	1.20E-11
t <sub>x2</sub>	A, B	22.965	22.965	$\sqrt{u_A(\bar{t}_i)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_{x2}+2\beta_{x2}(t_{x2}-23))$	_	_
Peltier	В	0	0	1.00E-08	1.00E-08	1	1	1.00E-08

Table 11: CEM uncertainty budget for the transfer from CEM's 10 k $\Omega$  references to the 1  $\Omega$  travelling standard BIV203.

Table 12 shows the uncertainty budget provided by the CEM associated to the transfer from the CEM's reference standard to the 1  $\Omega$  travelling standard BIV207.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
R <sub>x1</sub>	A	Mean R <sub>x1</sub>	0.999 999 623	$u_A(R_{\rm x}) = \sqrt{\frac{\sum_{l=1}^n (R_{\rm xl} - \overline{R_{\rm x}})}{n(n-1)}}$	1.7E-9	1	1	1.70E-09
N	В	100	100	5 x 10 <sup>-8</sup> /sqrt(3)(relative)	2.9E-08	N <sup>2</sup>	1.00E-04	2.89E-12
<i>r</i> <sub>h</sub>	В	0	0	<i>u</i> (ε <sub>10:1</sub> )	2.5E-08	1	1	2.51E-08
r <sub>xi</sub>	В	0	0	<i>u</i> (ε <sub>10:1</sub> )	2.5E-08	1	1	2.51E-08
α <sub>x1</sub>	В	0	0.00E+00	0.1 x 10 <sup>-6</sup> K <sup>-1</sup>	1.00E-07	(t <sub>x1</sub> -23)	-0,025	2.46E-09
β <sub>x1</sub>	В	0	0.00E+00	0.01 x 10 <sup>-6</sup> K <sup>-2</sup>	1.00E-08	$(t_{x1}-23)^2$	6.07E-04	6.07E-12
t <sub>x1</sub>	A, B	22.975	22.975	$\sqrt{u_A(\bar{t}_i)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\delta_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_{x1}+2\beta_{x1}(t_{x1}-23))$	_	_
Peltier	В	0	0	1.00E-08	1.00E-08	1	1	1.00E-08

# Table 12: CEM uncertainty budget for the transfer from CEM's 10 k $\Omega$ references to the 1 $\Omega$ travelling standard BIV207.

From the uncertainty components reported in the above Tables 10, 11 and 12, the type B uncertainty for a given  $1\Omega$  travelling standard is calculated using:

$$u(B) = \sqrt{\left(uR_{\rm p}^{\rm MC}\right)^2 + (uN)^2 + (ur_{\rm h})^2 + (ur_{\rm xi})^2 + (u\alpha_{\rm xi})^2 + (u\beta_{\rm xi})^2 + (uPeltier)^2}.$$

#### 4.7.3 Uncertainties associated with the measurement of 1 $\Omega$ resistors:

Table 13 shows the corrected measurements of the 1  $\Omega$  standards at CEM at the mean date of 4<sup>th</sup> of June 2022 for BIV203 and BIV207 as well as the associated uncertainty components.

CEM results	Relative difference from	stand	Relative lard uncertain	nties
after corrections	nominal value (μΩ/Ω)	Repeatability $u_1 (\mu \Omega / \Omega)$	Systematic $u_2 (\mu \Omega / \Omega)$	Corrections $u_3 (\mu \Omega / \Omega)$
BIV203	+ 0.650	0.012	0.048	0.013
BIV207	- 0.393	0.010	0.048	0.013

#### Table 13: Summary of the CEM results at 1 $\Omega$ , after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 13 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.  $u_2$  is calculated using the bolded type B contributions from Table 10 combined with components from Table 11 for BIV203 and from Table 12 for BIV207.

#### 4.8 <u>CEM results at 10 k $\Omega$ </u>:

The 10 k $\Omega$  travelling standards were measured 11 times for both B10K09 and B10K11 in the period May 2022 – June 2022. The measurements were made at 100  $\mu$ A. Power correction is not needed as applied currents at CEM and BIPM are comparable. Table 14 gives the mean values at the mean date of 2<sup>nd</sup> of June 2022 for B10K09 and B10K11, before application of temperature and pressure corrections. The repeatability is estimated by the standard deviation of the series of measurements.

Standard #	Relative difference from nominal 10 kΩ value (μΩ/Ω)	Std dev. (μΩ/Ω)	Mean temperature / °C	Mean atmospheric pressure / hPa
B10K09	+0.059	0.011	23.010	934.89
B10K11	+ 1.343	0.012	23.008	934.89

Table 14: Summary	of	CEM	10	kΩ	calibrations.
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#### 4.8.1 Corrections for temperature and pressure differences:

The value R(23) of the resistance corrected to  $T_0 = 23$  °C is:  $R(23) = R(T) \times [1 - \alpha_{23}(T - T_0) - \beta(T - T_0)^2]$ where R(T) is the resistance of the standard at temperature T.

The value R(1013.25) of the resistance corrected to  $P_0 = 1013.25$  hPa is:  $R(1013.25) = R(P) \times [1 - \gamma(P - P_0)]$ where R(P) is the resistance of the standard at pressure P.

The CEM results are corrected to the reference temperature and the reference pressure using the coefficients  $\alpha_{23}$ ,  $\beta$  and  $\gamma$  shown in Table 1. Applied corrections are reported in Table 15.

Reference temperature = 23.000 °C Reference pressure = 1013.25 hPa							
	Relative corrections ( $\mu\Omega/\Omega$ )						
Standard #	For temperature	For pressure					
B10K09	- 0.000	- 0.013					
B10K11	+ 0.001	- 0.027					

#### Table 15: Corrections applied to the CEM 10 k $\Omega$ results.

The standard uncertainties of the temperature and pressure measurements at the CEM are 0.01 °C and 2 Pa respectively. Taking into account the differences from the reference temperature, the reference pressure and the uncertainties associated with the coefficients, the relative standard uncertainties  $u_{\text{Temp}}$  and  $u_{\text{Press}}$  associated with the temperature and pressure difference corrections applied by the BIPM are estimated to be  $u_{\text{Temp}} < 0.001 \times 10^{-6}$  and  $P_{\text{ress}} = 0.008 \times 10^{-6}$  leading to a combined relative standard uncertainty  $u_3 = 0.008 \times 10^{-6}$ .  $u_3$  is reported in Table 19.

#### 4.8.2 Uncertainty budget provided by the CEM:

Table 16 shows the uncertainty budget provided by the CEM associated to the CEM reference standards (10 k $\Omega$  references  $R_{p1}$ ,  $R_{p2}$ ,  $R_{p3}$ ). This table is common to both 10 k $\Omega$  travelling standards.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
t <sub>p1</sub>	А, В	Mean t <sub>p1</sub>	23.042	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\partial_{ter}}{\sqrt{3}}\right)^2}$	0.016	$(\alpha_{p1}+2\beta_{p1}(t_{p1}-23))$	-5.23E-08	8.48E-10
t <sub>p2</sub>	А, В	Mean t <sub>p2</sub>	23.087	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\partial_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_{p2}+2\beta_{p2}(t_{p2}-23))$	1.56E-08	1.63E-10
t <sub>p3</sub>	А, В	Mean <i>t</i> <sub>p3</sub>	23.015	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\partial_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_{p3}+2\beta_{p3}(t_{p3}-23))$	-8.07E-08	8.08E-10
α <sub>p1</sub>	В	Indicated by manufacturer	-5.00E-08	Indicated by manufacturer	1.00E-08	( <i>t</i> <sub>p1</sub> -23)	0.042	4.17E-10
α <sub>p2</sub>	В		2.00E-08		1.00E-08	( <i>t</i> <sub>p2</sub> -23)	0.087	8.71E-10
$\alpha_{p3}$	В		-8.00E-08		1.00E-08	( <i>t</i> <sub>p3</sub> -23)	0.015	1.54E-10
β <sub>p1</sub>	В		-2.70E-08		1.00E-09	( <i>t</i> <sub>p1</sub> -23) <sup>2</sup>	0.002	1.74E-12
$\beta_{p2}$	В		-2.50E-10		1.00E-09	$(t_{p2}-23)^2$	0.008	7.58E-12
$\beta_{p3}$	В		-2.40E-08		1.00E-09	$(t_{p3}-23)^2$	0.000	2.38E-13
R <sub>p1</sub> <sup>RC</sup>	В	$R_{p1}^{RC}$	10 000.014 10	U/k and drift	2.86E-08	1	1	2.86E-08
$R_{p2}^{RC}$	В	$R_{p2}^{RC}$	10 000.014 47		2.26E-08	1	1	2.26E-08
$R_{p3}^{RC}$	В	$R_{p3}^{RC}$	10 000.014 64		2.88E-08	1	1	2.88E-08
R <sub>p1</sub> <sup>MC</sup>	В	R <sub>p1</sub> <sup>MC</sup>	10 000.014 08	2.9E-08				
R <sub>p2</sub> <sup>MC</sup>	В	R <sub>p2</sub> <sup>MC</sup>	10 000.014 48	2.3E-08				
R <sub>p3</sub> <sup>MC</sup>	В	R <sub>p3</sub> <sup>MC</sup>	10 000.014 63	2.9E-08				
R <sub>p</sub> <sup>MC</sup>			10 000.014 39	2.9E-08				

Table 16: Uncertainty budget associated to the CEM 10 k $\Omega$  reference standards.

Table 17 shows the uncertainty budget provided by the CEM associated to the transfer from the CEM's reference standard to the 10 k $\Omega$  travelling standard B10K09.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
r	A, B	Mean r	0.999 998 620	Bridge calibration and specs	2.89E-10	1	1.000 001 380	2.89E-10
t <sub>x</sub>	А, В	Mean (t <sub>x</sub> )	23.011	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\partial_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_x+2\beta_x(t_x-t_{x'}))$	-4.05E-14	4.05E-16
a <sub>x</sub>	A, B	Indicated by manufacturer	-4.00E-08	Indicated by manufacturer	1.00E-08	$(t_x-t_{x'})$	0.011	1.06E-10
β <sub>x</sub>	В		-2.20E-08		1.00E-09	$(t_x - t_x)^2$	0.000	1.12E-13
px	В	Mean (p <sub>x</sub> )	93 505	$u(\rho_x) = \sqrt{\left(\frac{U_{\text{bar}}}{k_{\text{bar}}}\right)^2 + (u_A(p_x))^2}$	36	γ×	0.00E+00	-
γx	В	Indicated by manufacturer	0.00E+00	Indicated by manufacturer	0.00E+00	(p <sub>x</sub> -p <sub>x'</sub> )	-07 820	
R <sub>x</sub> <sup>RC</sup>	А	Mean $(R_x^{RC})$	10 000.000 59	$u_A(R_x) = \sqrt{\frac{\sum_{l=1}^n (R_{xl} - \overline{R_x})}{n(n-1)}}$	2.74E-09	1	1	2.74E-09

Table 17: CEM uncertainty budget for the transfer from the CEM's 10 k $\Omega$  references to the 10 k $\Omega$  travelling standard B10K09.

Table 18 shows the uncertainty budget provided by the CEM associated to the transfer from the CEM's reference standard to the 10 k $\Omega$  travelling standard B10K11.

Quantity	Туре	Estimation	Estimation value	Uncertainty	Uncertainty value	Sensitivity coefficient	Sensitivity coefficient value	Uncertainty contribution
r	A, B	Mean r	0.999 999 906	Bridge calibration and specs	2.89E-10	1	1.000 000 09	2.89E-10
tx	А, В	Mean (t <sub>x</sub> )	23.008	$\sqrt{u_A(t_j)^2 + \left(\frac{U_{ter}}{k_{ter}}\right)^2 + \left(\frac{\partial_{ter}}{\sqrt{3}}\right)^2}$	0.010	$(\alpha_x+2\beta_x(t_x-t_x))$	-7.04E-08	7.04E-10
α <sub>x</sub>	A, B	Indicated by manufacturer	-7.00E-08	Indicated by manufacturer	1.00E-08	$(t_{x}-t_{x'})$	0.008	7.83E-11
βx	В		-2.70E-08		1.00E-09	$(t_{x}-t_{x'})^{2}$	0.000	6.13E-14
px	В	Mean (p <sub>x</sub> )	93 505	$u(\rho_x) = \sqrt{\left(\frac{U_{\text{bar}}}{k_{\text{bar}}}\right)^2 + (u_A(p_x))^2}$	36	γx	0.00E+00	_
γ×	В	Indicated by manufacturer	0.00E+00	Indicated by manufacturer	0.00E+00	(p <sub>x</sub> -p <sub>x'</sub> )	-07 820	
R <sub>x</sub> <sup>RC</sup>	A	Mean (R <sub>x</sub> <sup>RC</sup> )	10 000.013 41	$u_A(R_x) = \sqrt{\frac{\sum_{l=1}^n (R_{xl} - \overline{R_x})}{n(n-1)}}$	2.40E-09	1	1	2.40E-09

# Table 18: CEM uncertainty budget for the transfer from the CEM's 10 k $\Omega$ references to the 10 k $\Omega$ travelling standard B10K11.

From the uncertainty components reported in the above Tables 16, 17 and 18, the type B uncertainty for a given travelling standard is calculated using:

$$u(B) = \sqrt{\left(uR_{\rm p}^{\rm MC}\right)^2 + (ur)^2 + (ut_{\rm x})^2 + (u\alpha_{\rm xi})^2 + (u\beta_{\rm xi})^2}$$

#### 4.8.3 Uncertainties associated with the measurement of 10 k $\Omega$ resistors:

CEM results	Relative difference from	stand	Relative lard uncertain	nties
after corrections	nominal value (μΩ/Ω)	Repeatability $u_1 (\mu \Omega / \Omega)$	Systematic $u_2 (\mu \Omega / \Omega)$	Corrections $u_3 (\mu \Omega / \Omega)$
B10K09	+0.046	0.011	0.029	0.008
B10K11	+ 1.317	0.012	0.029	0.008

Table 19 shows the corrected measurements of the 10 k $\Omega$  standards at CEM as well as the uncertainty components associated with these measurements.

Table 19: Summary of the CEM results at 10 k $\Omega,$  after corrections.

Note: The distinction between 'systematic' and 'repeatability' is made in Table 19 because our model is that the latter can reasonably be reduced by taking an average across several transfer standards. The former cannot be reduced in this way. This does not correspond exactly to the more usual division into type A and type B components.  $u_2$  is calculated using the bolded type B contributions from Table 16 combined to components from Table 17 for B10K09 and from Table 18 for B10K11.

#### 5 <u>Comparison CEM – BIPM</u>

The individual measurement results for each of the four standards are shown in Figures 3 to 6. The plots also show the mean value of the CEM measurements with the uncertainty bar corresponding to the expanded uncertainty (k = 2) of the comparison  $U_c$  provided below, and a linear fit to the BIPM before and after measurements. We assume that the value of each standard is subject to a simple linear drift during the period of the comparison. Inspection of Figures 3 to 6 indicates that this is an appropriate model as both 1  $\Omega$  standards and 10 k $\Omega$  standards fit this model well. We treat the 1  $\Omega$  and 10 k $\Omega$  results as two separate cases.

Within this model, the result of the comparison for a given standard is the difference between the mean of the CEM measurements and the interpolated value of the linear fit to the BIPM measurements on the mean date of the CEM measurements.

The difference between the CEM and the BIPM calibrations of a given standard  $R_i$  can be written as:

$$\Delta_i = R_{\text{CEM},i} - R_{\text{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\text{CEM-BIPM}} = \frac{1}{2} \sum_{i=1}^{2} \left( R_{\text{CEM},i} - R_{\text{BIPM},i} \right)$$

For each standard, the uncertainty  $u_1$  associated with the interpolated BIPM value is calculated from the linear fit, as shown in Tables 2 and 4;  $u_2$  is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 3 or 5. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard  $R_i$ , the BIPM uncertainty  $u_{\text{BIPM},i}$  is obtained from:  $u_{\text{BIPM},i}^2 = u_{1,i}^2 + u_{2,i}^2$ . When the mean (for two standards) of the CEM – BIPM relative difference is calculated, the BIPM contribution to the uncertainty is,

$$u_{\rm BIPM}^2 = \sum_{i=1}^2 \frac{u_{1,i}^2}{2^2} + u_2^2$$

Similarly, for the CEM measurements, we expect the uncertainty components  $u_2$  and  $u_3$  of Tables 13 and 19 to be correlated between standards, and  $u_1$  to be uncorrelated. We therefore calculate the total uncertainty as:

$$u_{\text{CEM}}^2 = \sum_{i=1}^{2} \frac{u_{1,i}^2}{2^2} + u_2^2 + u_3^2$$

#### 5.1 Uncertainty associated with the transfer

Changes in the values of the standards due to the effects of transport can add an extra uncertainty component to a comparison. In the present case, from inspection of the BIPM 'before' and 'after' measurements in Figures 3 to 6, we can see that any such effects are negligible compared to the overall uncertainty of the comparison. For simplicity, we do not include any extra uncertainty components.

#### 5.2 Results at 1 $\Omega$

The differences between the values assigned by the CEM,  $R_{\text{CEM}}$ , and those assigned by the BIPM,  $R_{\text{BIPM}}$ , to each of the two travelling standards on the mean date of the CEM measurements are shown in Table 20.

CEM – BIPM					
Standard #	$10^6  imes (R_{ ext{CEM}} - R_{ ext{BIPM}}) / (1 \ \Omega)$				
BIV203	+ 0.075				
BIV207	+ 0.036				
Mean	+ 0.056				

Table 20: CEM – BIPM differences for the two 1  $\Omega$  travelling standards.

The mean difference between the CEM and the BIPM calibrations is:

 $(R_{\text{CEM}} - R_{\text{BIPM}}) / (1 \Omega) = +0.056 \times 10^{-6}$ 

The relative combined standard uncertainty of the comparison,  $u_{\rm C}$ , is:

 $u_c^2 = u_{\rm BIPM}^2 + u_{\rm CEM}^2$ 

 $u_{\text{BIPM}} = 0.017 \times 10^{-6},$  $u_{\text{CEM}} = 0.050 \times 10^{-6},$ 

where,

giving:  $u_{\rm C} = 0.053 \times 10^{-6}$ 

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the CEM and the BIPM for values assigned to 1  $\Omega$  resistance standards, and its expanded relative uncertainty (expansion factor k = 2, corresponding to a confidence level of 95 %),  $U_{\rm C}$ :

$$D = (R_{\text{CEM}} - R_{\text{BIPM}}) / 1 \Omega = +0.056 \times 10^{-6}$$
$$U_{\text{C}} = 0.106 \times 10^{-6}$$

The difference between the CEM and the BIPM calibration results is within the expanded uncertainty.

#### 5.3 Results at 10 k $\Omega$

The difference between the value assigned by the CEM,  $R_{\text{CEM}}$ , and those assigned by the BIPM,  $R_{\text{BIPM}}$ , to each of the two travelling standards on the mean date of the CEM measurements are shown in Table 21.

CEM – BIPM	
Standard #	$10^6 \times (R_{ ext{CEM}} - R_{ ext{BIPM}}) / (10 \text{ k}\Omega)$
B10K09	+ 0.030
B10K11	+ 0.010
Mean	+ 0.020

Table 21: CEM – BIPM differences for the two 10 k $\Omega$  travelling standards.

The mean difference between the CEM and the BIPM calibrations is:

 $(R_{\rm CEM} - R_{\rm BIPM}) / (10 \text{ k}\Omega) = +0.020 \times 10^{-6}$ 

The relative combined standard uncertainty of the comparison,  $u_{\rm C}$ , is:

 $u_c^2 = u_{\rm BIPM}^2 + u_{\rm CEM}^2$ 

who

where,  

$$u_{\text{BIPM}} = 0.015 \times 10^{-6},$$
  
 $u_{\text{CEM}} = 0.031 \times 10^{-6},$   
giving:  
 $u_{\text{C}} = 0.034 \times 10^{-6}$ 

The final result of the comparison is presented as a degree of equivalence, composed of the deviation, D, between the CEM and the BIPM for the value assigned to  $10 \text{ k}\Omega$  resistance standards, and its expanded relative uncertainty (expansion factor k = 2, corresponding to a confidence level of 95 %),  $U_{\rm C}$ :

$$D = (R_{\text{CEM}} - R_{\text{BIPM}}) / 10 \text{ k}\Omega = +0.020 \times 10^{-6}$$
$$U_{\text{C}} = 0.068 \times 10^{-6}$$

The difference between the CEM and the BIPM calibration results is within the expanded uncertainty.



Figure 3: Results for  $1 \Omega$  standard BIV203. BIPM (blue diamonds) and CEM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at CEM and the green triangle is the mean value of CEM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean CEM results.



Figure 4: Results for  $1 \Omega$  standard BIV207. BIPM (blue diamonds) and CEM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at CEM and the green triangle is the mean value of CEM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean CEM results.



Figure 5: Results for 10 k $\Omega$  standard B10K09. BIPM (blue diamonds) and CEM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at CEM and the green triangle is the mean value of CEM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean CEM results.



Figure 6: Results for 10 k $\Omega$  standard B10K11. BIPM (blue diamonds) and CEM (red squares) measurements. The cross corresponds to the extrapolated BIPM measurement at the mean date of measurement at CEM and the green triangle is the mean value of CEM measurements. The uncertainty bar shows the expanded uncertainty of the comparison of the mean CEM results.