CCEM WGKC/2001-15

International Comparison
of 10 mH Inductance Standards at 1 kHz

CCEM-K3

Final Report

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June 2001

Summary

This Draft B Report describes the organisation, the equipment and the results of a CCEM comparison (CCE-88/1) of 10 mH inductance standards at a frequency of 1 kHz which took place from 1989 to 1994. Participants were ASMW/PTB in Berlin as pilot laboratory and BNM-LCIE (France), IEN (Italy), NIM (PR China), NIST (USA), NPL (UK), OFMET (Switzerland), PTB in Braunschweig (Germany), SP (Sweden) and VNIIM (Russia). At the meeting of the CCEM Working Group on Key Comparisons in July 1999 at the BIPM it was decided to include this comparison in the BIPM key comparison database as CCEM-K3.

Although the methods of measurement differed in all participating laboratories, an agreement inside the respective limits of uncertainty could be achieved by all participants. The Key Comparison Reference Value was calculated with an expanded uncertainty of $U_R = 4 \cdot 10^{-6}$ from the results of those nine institutes which realised their unit of inductance by absolute determination. Three institutes reached this reference value. Two institutes achieved results inside the limits of the expanded uncertainty of the reference value. The differences of the remaining institutes to the reference value were less than their respective expanded uncertainties.

Besides the CCEM comparison, the pilot laboratory organised additional comparisons in the frame of COOMET and a bilateral comparison resulting from a technical co-operation between PTB and the Turkish National Metrology Institute UME. The results are included in this report.

1 Participating Institutes and Counterparts

A. CCEM comparison

NIM	National Institute for Metrology, Beijing, PR CHINA Lu Wenjun
BNM-LCIE	Bureau National de Métrologie/Laboratoire Central des Industries Electriques, Fontenay-aux-Roses, FRANCE J.C. Antoine, I. Blanc
PTB-BS	Physikalisch-Technische Bundesanstalt, Braunschweig, GERMANY: R. Hanke, K. Dröge
PTB-B	Physikalisch-Technische Bundesanstalt, Berlin, GERMANY: H. Eckardt, K. Neumann, P. Räther
IEN	Istituto Elettrotecnico Nationale “Galileo Ferraris”, Torino, ITALY: F. Cabiati, G.C. Bosco
VNIIM	Vserossisky Naučny Issledovatjelsky Institut Metrologii “D.I.Mendelejew”, St. Petersburg, RUSSIA: F.E. Kurotschkin, Yu.P. Semenov
SP	Sveriges Provnings- och Forskningsinstitut, Borås, SWEDEN: G. Eklund
OFMET	Office Fédéral de Métrologie, Wabern, SWITZERLAND: M. Flüeli (since 1 January 2001: Swiss Federal Office of Metrology and Accreditation)
NPL	National Physical Laboratory, Teddington, UNITED KINGDOM: B.P. Kibble, J.H. Belliss
NIST	National Institute for Standards and Technology, Gaithersburg, Md, USA: N. Oldham, B. Waltrip

B. COOMET comparison

KSM	Komitet Standardisazii i Metrologii, Sofia, BULGARIA: N.I. Dipčikov
OMH	Orszagos Mereszegyi Hivatal, Budapest, HUNGARY as guest in COOMET M. Győző
GUM	Główny Urząd Miar, Warszawa, POLAND: K. Bielak, A. Szymczak
VNIIM	Vserossisky Naučny Issledovatjelsky Institut Metrologii “D.I.Mendelejew”, St. Petersburg, RUSSIA: Yu.P. Semenov

C. Bilateral comparison PTB – UME

UME	Ulusal Metroloji Enstitüsü, Gebze-Kocaeli, TURKEY: Y. Gülmez, E. Turhan, G. Gülmez

2 Introduction

On the occasion of its 18th session held in September, 1988, the CCE (now CCEM) decided to carry out an international comparison of 10 mH inductance standards to be measured at 1 kHz. The ASMW of the former GDR, which since 1985 had maintained the international standard of inductance of those East European countries which were members of the Council of Mutual Economic Aid (CMEA) was asked to act as the pilot laboratory. The following ten metrology institutes then showed interest in participating in the comparison: ASMW (GDR), OFMET (CH), IEN (I), BNM-LCIE (F), NIM (PR China), NIST (USA), NPL (GB), PTB (D), SP (S), VNIIM (SU). The pilot laboratory drew up the working program and the circulation scheme, constructed the thermostated transfer standards and organized the measurement cycles after they had been confirmed by the participants.

Upon German reunification in 1990, the pilot laboratory was integrated into the Electricity Division of the PTB and completed this task in Berlin (referred to as PTB-B after this date). A progress report [1] covering most of the results was presented to the 19th CCE session in 1992, the final report [2] to the 20 th session in 1995, both for the CCE loop.

In addition, the equipment of the pilot laboratory was used for the following comparisons:

- two comparisons within the framework of the Regional Metrology Organization COOMET of the East European NMIs. They took place at PTB-B in December 1992 with the Polish institute GUM and, as a guest in COOMET, the Hungarian institute OMH participating [3]; the Bulgarian, Polish and Russian institutes (KSM, GUM and VNIIM) took part in an intercomparison carried out in May 1995 [4];
- a bilateral intercomparison with UME, the Turkish NMI. It was carried out in 1997 [5].

The present report is based on Appendix F to the Multilateral Recognition Arrangement of the CIPM dated 14 October 1999 “Guidelines for CIPM key comparisons”. It describes the state of inductance measurements for the rated value of 10 mH at the frequency of 1 kHz in several member institutes of CCEM and COOMET based on the above-mentioned comparisons.

3 CCEM Intercomparison

3.1 Organization of the Comparisons

The technical protocol for the CCE (CCEM) intercomparison (measurement program, measurement conditions, circulation scheme) was planned by the pilot laboratory and adopted by the participants. A manual “Specifications and Operating Instructions“ covering the conditions of measurement and transport of the standards and describing possible faults was made available together with the standards. During transport, the thermostated travelling standards had to be accompanied by a member of the staff of the laboratory in which the measurements had just been carried out. This condition was fulfilled by all participants, except for one cycle: unfortunately, the standards were unaccompanied during the flight to the USA and back. A non-reversible change of the temperature of the battery-operated thermostat of one of the standards (L2) occurred, despite a special air cargo service provided by the airline. As the temperature of the standards which are thermostated to ± 10 mK can be determined by measurement of the copper resistance of the inductor coils, the resulting error could be corrected.
The following circulation scheme had been planned:

Cycle 1: ASMW ⇒ NIM ⇒ ASMW (October 1989)
Cycle 2: ASMW ⇒ VNIIM ⇒ SP ⇒ ASMW (April to May 1990)
Cycle 3: ASMW ⇒ BNM-LCIE ⇒ OFMET ⇒ IEN ⇒ ASMW (June to Sept. 1990)
Cycle 4: ASMW ⇒ PTB ⇒ NPL ⇒ NIST ⇒ ASMW (October to December 1990)

However, this schedule could not be adhered to, since there were a great deal of difficulties, for example:

- the measurement times planned were too short or too early for some laboratories
- the customs documents from the former GDR could not be recognized by the states of the European Union; this led to a great deal of trouble with the customs authorities in some countries;
- in the process of German reunification in 1990, the pilot laboratory was integrated into the PTB and moved from Potsdam to Berlin, resulting in the supervision task being neglected to a certain extent.

In fact, 4 cycles were carried out. Changes made followed from short-term corrections agreed with the participants, and from the fact, that intermediate measurements at the pilot laboratory had been omitted.

Cycle 1: ASMW ⇒ NIM ⇒ ASMW (October 1989)
Cycle 2: ASMW ⇒ VNIIM ⇒ ASMW (April 1990)
Cycle 3: ASMW ⇒ PTB ⇒ SP ⇒ NPL ⇒ BNM-LCIE ⇒ OFMET ⇒IEN ⇒ PTB-B (May 1990 to April 1992)
Cycle 4: PTB-B ⇒ NIST ⇒ PTB-B (July to September 1994).

3.2 Measuring Set-ups of the Pilot Laboratory and Transfer Standards

The ASMW/PTB-B measuring set-ups used to realize the unit of inductance and to compare standards with a nominal value of 10 mH consist of

- a group standard GS at the rated value of 10 mH
- a MAXWELL-WIEN bridge MWB for the realization of the unit of inductance at the rated value of 10 mH and a frequency of 1 kHz
- an inductive voltage divider bridge IVDB for 1:1 comparison of 10 mH standards at a frequency of 1 kHz
- two specially constructed thermostatically controlled 10 mH transfer standards (modified 1482-H type standards) used as travelling standards TS.

In 1995, the relative uncertainty of the measurement of a 10 mH standard (e.g. travelling standard) at PTB-B had been estimated to be less than $1 \cdot 10^{-6}$ (type A) and $5 \cdot 10^{-6}$ (type B) for $2\,\sigma$ at the beginning of the comparison and $8 \cdot 10^{-6}$ (type B) in 1992 and 1994. Because measurements with the MWB could not be carried out at this time, the values were related to the group standard [2].

Table 1: Realized time schedule of the intercomparison

Nr.	Measurement at	started	finished	1989			1990				1991				1992				1993				1994			
				Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1	ASMW	01.11.89	08.11.89			▨																				
2	NIM	09.11.89	26.11.89			▨																				
3	ASMW	27.11.89	20.04.90			▨	▨	▨																		
4	VNIIM	23.04.90	27.04.90					▨																		
5	ASMW	30.04.90	03.05.90					▨																		
6	PTB-BS	04.05.90	24.06.90					▨																		
7	SP	25.06.90	20.08.90					▨	▨																	
8	NPL	21.08.90	20.01.91						▨	▨	▨															
9	BNM-LCIE	21.01.91	23.04.91								▨	▨														
10	OFMET	24.04.91	28.06.91									▨														
11	IEN	01.07.91	31.03.92										▨	▨	▨											
12	PTB-B	01.04.92	16.04.92													▨										
13	PTB-B	23.05.94	18.07.94																					▨	▨	
14	NIST	19.07.94	28.09.94																						▨	
15	PTB-B	29.09.94	14.10.94																							▨

3.3 Measurements performed at the Pilot Laboratory

A. Group Standard GS 10 mH

At PTB, the unit of inductance is maintained at the rated value 10 mH, realized by a group of four 10 mH GR standards of the type 1482-H maintained in an air thermostat at a fixed temperature of (23.00 ± 0.05) °C. It is shown in figure 1. The GS is traceable to the units of resistance and capacitance derived from the calculable cross capacitor of PTB. Its value is determined based on absolute realisations of the unit by means of the MWB. Only in 1992 and 1994 the measurements of the travelling standards at the PTB were directly related to the group standard.

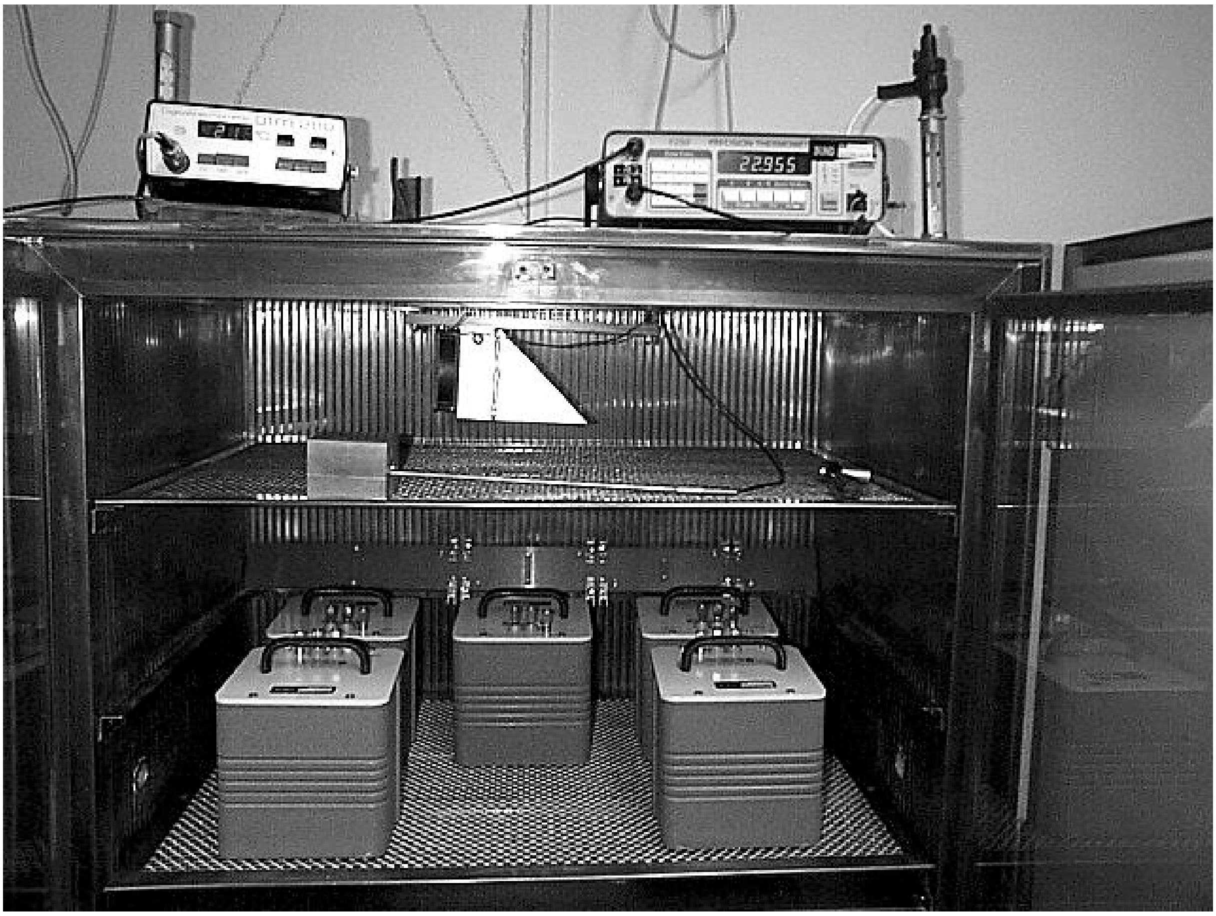

Figure 1: Pictorial view of the GS inside the air thermostat

B. Maxwell-Wien Bridge (MWB) with Wagner Balance

The bridge has been specially constructed to minimise all errors due to stray impedances and to achieve the best resolution for the standards to be measured. It consists of a small copper box directly coupled to a variable precision capacitor of the type GR 1422-CE in arm 3 and containing the bridge resistors R_2 and R_4 and the Wagner arm resistors R_A and R_B. In order to reach the high resolution of 1 nH, the bridge is designed to cover an inductance range of about 10 mH ± 100 nH. The measurement of only a single defined standard L_N of the GR 1482-H type can, therefore, be carried out. The bridge is kept and operated inside a second air thermostat at the temperature of (23.00 ± 0.05) °C (figure 2).

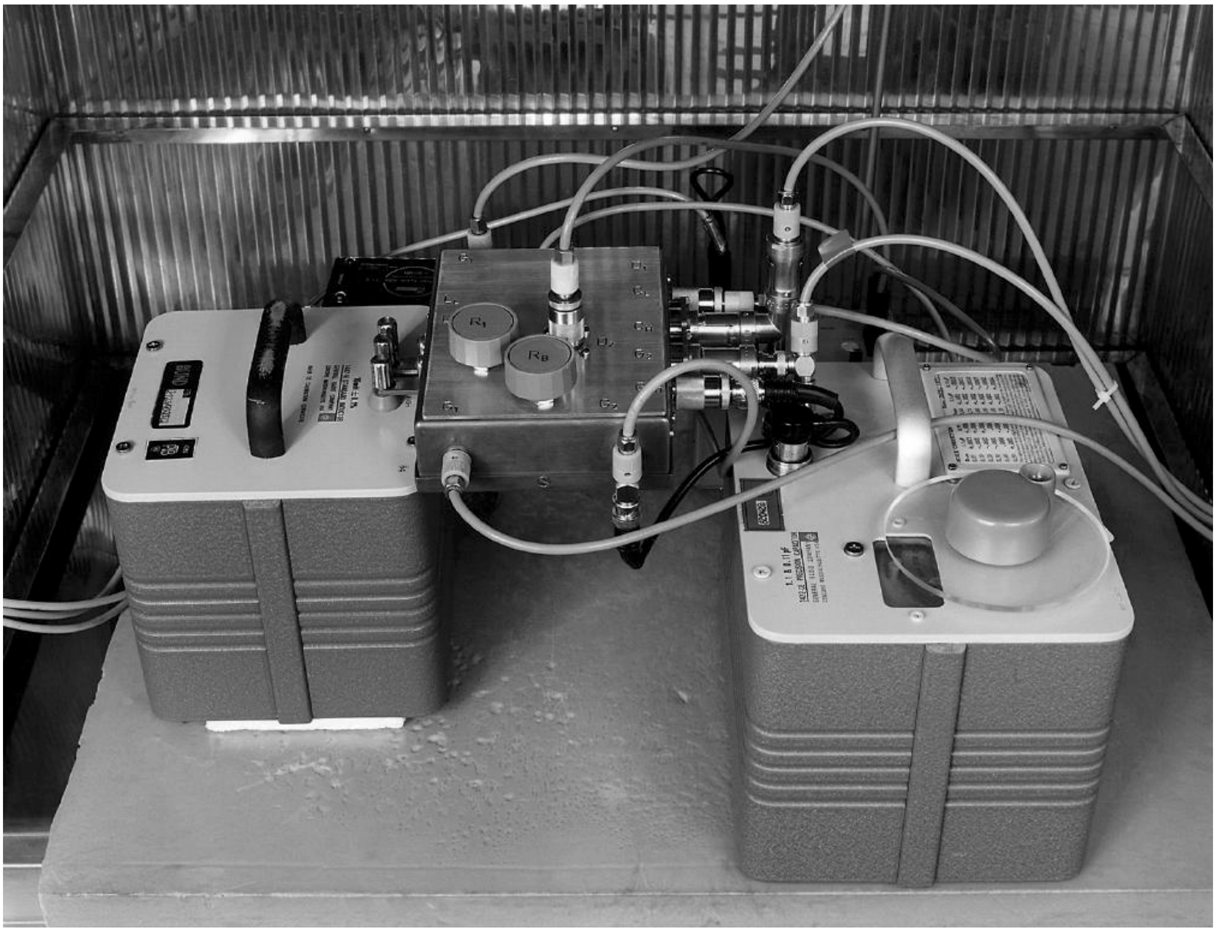

Figure 2: Pictorial view of the MWB inside the second air thermostat

To avoid residual stray effects on the measurement result acting despite the described construction, a zero substitution method [6] is applied. The remaining self-inductance of the bridge is eliminated by means this method. A complete measurement comprises two steps:

In a first step, the special inductance standard L_N is measured according to figure 3.

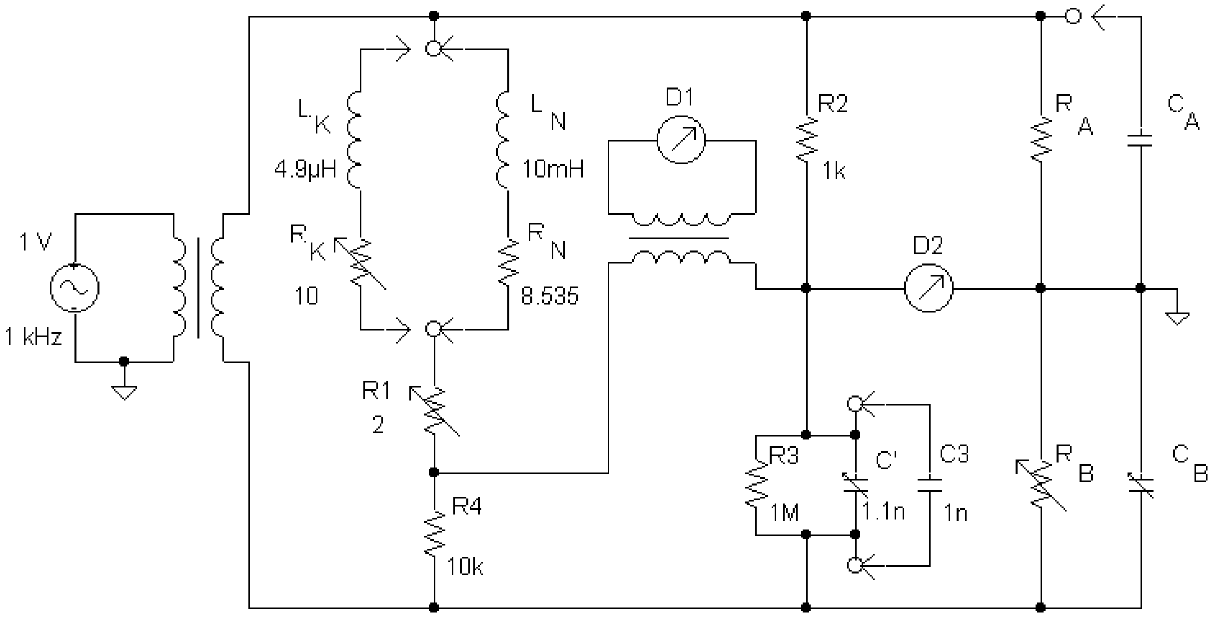

Figure 3: Maxwell-Wien bridge MWB (main measurement)

After adjustment of the real and imaginary components of the bridge, the inductance of the standard is calculated according to equ. (1)

$$L_N = R_2 R_4 (C' + C_3) - L_{\mathrm{bridge}} \qquad (1)$$

In step two, instead of the standard L_N a small inductor with the known inductance L_K in series with an adjustable resistor R_K is connected to the bridge as shown in figure 4.

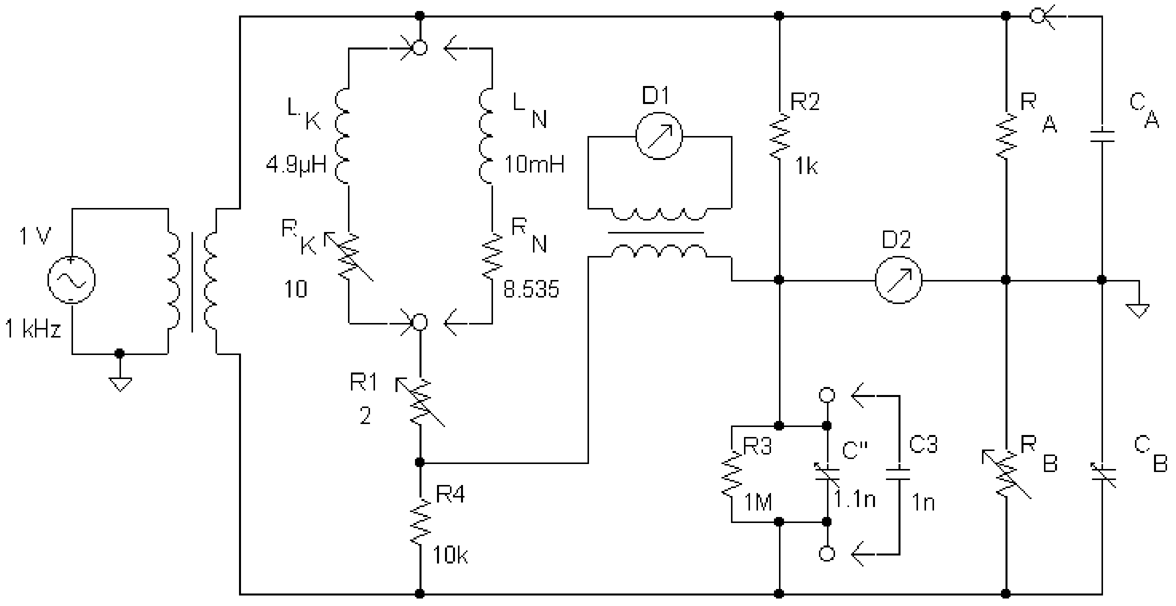

Figure 4: Maxwell-Wien bridge MWB (“short-circuit measurement”)

After adjustment of the real and imaginary components of the bridge, the zero inductance of the bridge L_{bridge} is calculated according to equ. (2)

$$L_{\text{bridge}} = R_2\, R_4\, C'' - L_K \tag{2}$$

The unknown inductance of L_N is calculated from (3), resulting from equ.’s (1) and (2).

$$L_N = R_2\, R_4\, (C' - C'' + C_3) + L_K \tag{3}$$

where

L_N	inductance standard 1482-H (General Radio)
R_2, R_4	precision resistors (Vishay) 1 kΩ and 10 kΩ
C', C''	fine adjustment capacitance values, realized and measured with a variable precision capacitor 1422-CE, resolution: 0.1 fF
C_3	highly stable, thermostatically controlled capacitor 1000 pF
L_K	short-circuit inductor with adjustable real component R_K
R_1	resistor network for the fine adjustment of the real component
L_{bridge}	self-inductance of Maxwell-Wien bridge
R_A, R_B, C_A, C_B	components of Wagner arm

Characteristics of the bridge: - resolution for the unknown standard L_N is: 1 nH
- relative 1σ uncertainty for L_N is: $u = 2.4 \cdot 10^{-6}$

C. Inductive voltage divider bridge 1:1 IVDB

An IVD bridge was constructed using a seven-decade inductive voltage divider, exclusively for the comparison of 10 mH inductance standards. The bridge is used in particular to compare the four standards of the group standard with the standard determined by means of the MWB, to measure the values of the travelling standards TS or for a direct comparison of 10 mH inductance standards. The circuit of the bridge is shown in fig. 5, its pictorial view in fig. 6.

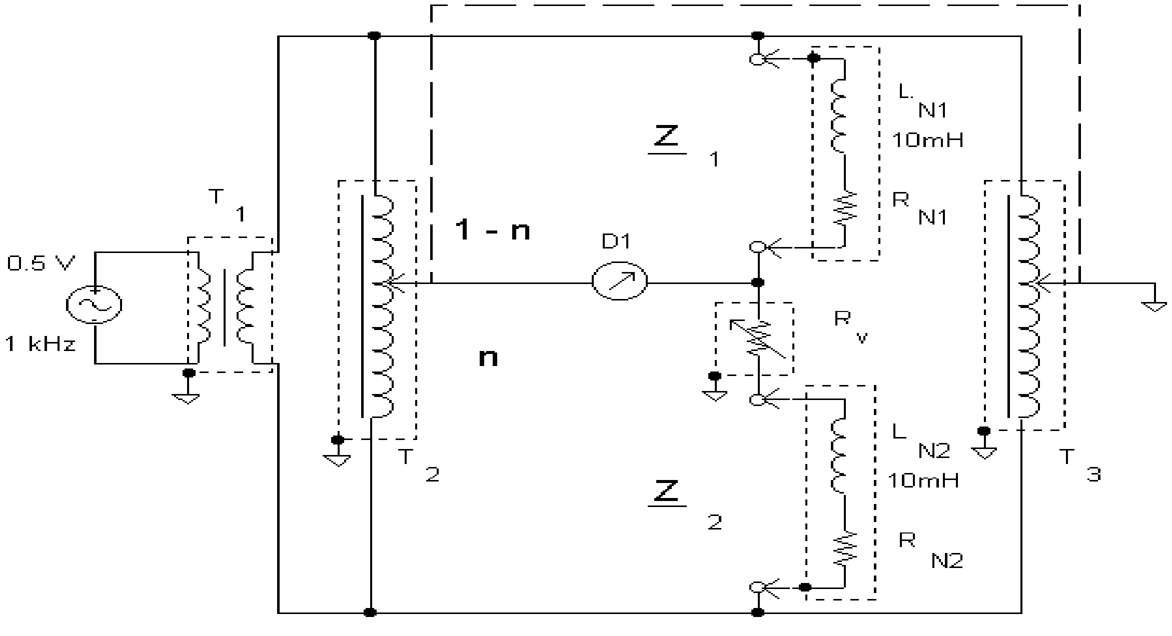

Figure 5: 1:1 Inductive Voltage Divider Bridge IVDB

where

$\underline{Z}_1, \underline{Z}_2$	impedances represented by the 10 mH inductance standards to be compared
R_v	resistor network for the fine adjustment of the real component of the bridge
T_2	main inductive voltage divider for the adjustment of the imaginary component of the bridge
T_3	Wagner divider
T_1	isolating transformer
D1	detector

Balancing condition:

$$\frac{1-n}{n} = \frac{\underline{Z}_1}{\underline{Z}_2} \approx \frac{L_{N1}}{L_{N2}} \tag{4}$$

The substitution method is used to achieve high accuracy in the comparisons. Extremely short connections to the inductance standards ensure smallest errors of measurement.

Characteristics of the bridge:

- resolution for the comparison of two standards: 1 nH
- relative uncertainty from the IVD for the 1:1 comparison in substitution: $u = 0.5 \cdot 10^{-6}$ (1σ).

Figure 6: Pictorial view of the inductive voltage divider bridge

D. Travelling Standards TS

In order to make highly precise comparisons, two thermostatically controlled transfer standards were constructed consisting of two commercial standards of the GR 1482-H type in a temperature-regulated battery- or line-operated enclosure (figure 7). In this way, the standards were kept at the same temperature during transport as well as during the measurements in the different laboratories. The temperature of the thermostat is about 30 °C with a stability of 0.01 $\mathrm{K \cdot a^{-1}}$. The dependence of the inductance on changes of the ambient temperature (punch-through) is less than $0.3 \cdot 10^{-6}\ \mathrm{K}^{-1}$. The temperature of the standards can easily be determined by measuring the dc resistances of the inductor coils. A change of the resistance value by +1 mΩ corresponds to a change in the inductance value of +10 nH (or 1 ppm). The inductances of the transfer standards have to be measured in a two-terminal configuration (one of the terminals is internally linked to the housing of the transfer standards).

The sudden increase in inductance of the standard L2 (see last column in Table 3, marked by "!") was caused by a change in the temperature of its thermostat. In all probability this happened during transport back from Washington to Berlin. However, on the assumption that a temperature-effected change by 1 mΩ of the copper resistance of the inductor coil is due to the above mentioned temperature-dependent change of the inductance value in the order of 10 nH, the increase in the inductance value of the standard could be satisfactorily explained and corrected for.

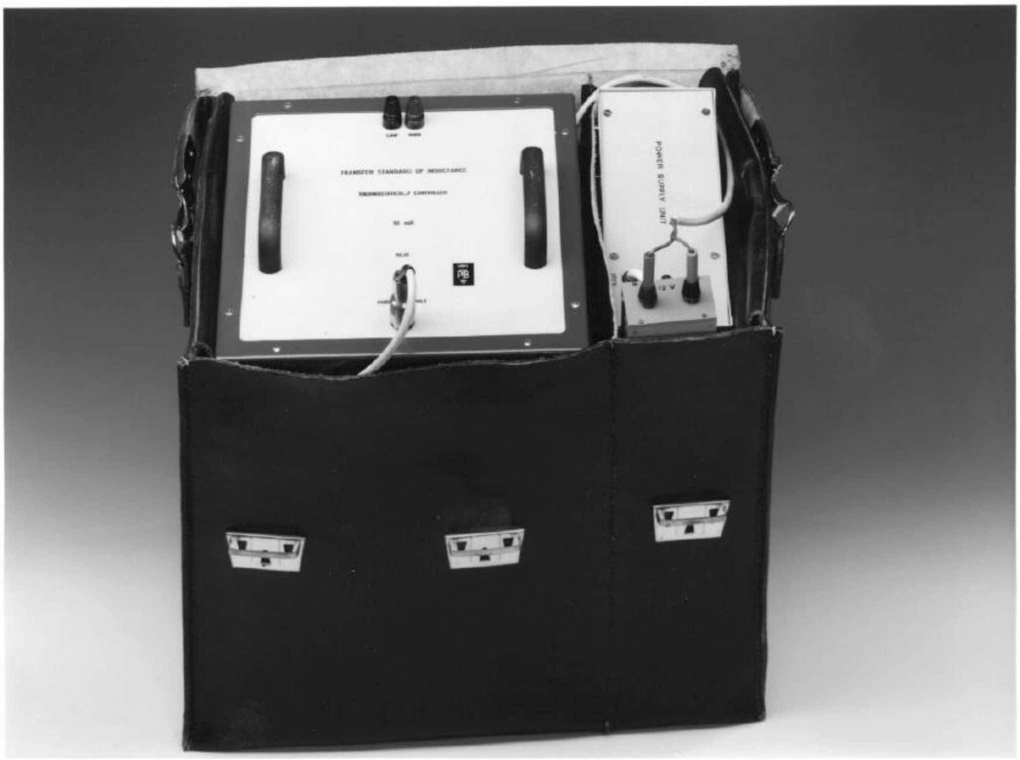

Figure 7: View of one travelling standard TS

E. Long-time measurements of the Travelling Standards at PTB

Basic requirement for a successful comparison is the stable or predictable behaviour of the travelling standards during the time of the measurement loop. To check this behaviour, both standards were observed by the pilot laboratory in the period from 1984 to1999, i.e. for about fifteen years. The values m_j of the standards defined by

$$L_j = 10\ \mathrm{mH}\ (1 + m_j \cdot 10^{-6}) \qquad (5)$$

and determined in measurements in the period between 1989 and 1999 are shown in the upper part of Table 2. They have been temperature-corrected for the same resistances of the inductor coils in the way described above. The values obtained during the time of the comparison from 1989 to 1994 have already been reported in [2] and are additionally shown as values of the pilot laboratory in Table 3 and plotted (as P) together with all results in figure 10.

Table 2: Results of the long-time measurements

Date	1989 11-07	1990 05-03	1992 04-16	1994 05-22	1994 10-14	1997 04-23	1998 10-06	1999 09-27
Number of months k	0	6	29.3	54.5	59.5	89.5	107	118.7
1. Temperature-corrected values (related to the same resistances R_L, measured on 1989-11-07: $R_{L1} = 8.674\ \Omega$, $R_{L2} = 8.856\ \Omega$), up to 1994 already reported in [2] 1σ uncertainty from Appendix A								
m No.01 $_{(8.674\Omega)}$	481	481	478	479	475	461	461	460
m No.02 $_{(8.856\Omega)}$	417	414	409	407	407	388	384	382
u (1σ)	2.7	2.7	3.7	3.7	3.7	2.7	2.7	2.7
2. Values resulting from the ASMW/PTB measurements (diagram see fig. 9) $m_{\mathrm{res\ PTB}} = 1/2\ (m\ \mathrm{No.01}_{(8.674\Omega)} + m\ \mathrm{No\ 02}_{(8.856\ \Omega)})$								
$m_{\mathrm{res\ PTB}}$	449	447.5	443.5	443	441	424.5	422.5	421
u (1σ)	2.7	2.7	3.7	3.7	3.7	2.7	2.7	2.7
3. Drift correction of the mean value of the ASMW/PTB measurements $m_{\mathrm{res,\ drift\ corrected}} = m_{\mathrm{res\ PTB}} - r \cdot \mathrm{k}$ (r from fig. 9)								
$m_{\mathrm{res\ PTB}}$	449	447.5	443.5	443	441	424.5	422.5	421
$r \cdot \mathrm{k}$	0	– 1.45	– 7.09	– 13.2	– 14.4	– 21.7	– 25.9	– 28.7
$m_{\mathrm{res\ PTB,drift\ corr.}}$	449	449	450.6	456.2	455.4	446.2	448.4	449.7
u	2.7	2.7	3.7	3.7	3.7	2.7	2.7	2.7

A diagram of the values $m_{\mathrm{res\ PTB}}$ is given in figure 8. The plots may be approximated by a straight line which indicates a linear drift in time with a drift rate r calculated to be equal to

$$r = -\,0.242/\mathrm{month}. \qquad (6)$$

Using this value, a drift correction of the measurements in relation to the initial measurement can be carried out for all the measurements throughout the comparison, for the ASMW/PTB results presented here as well as, later on, for the results of all participants.

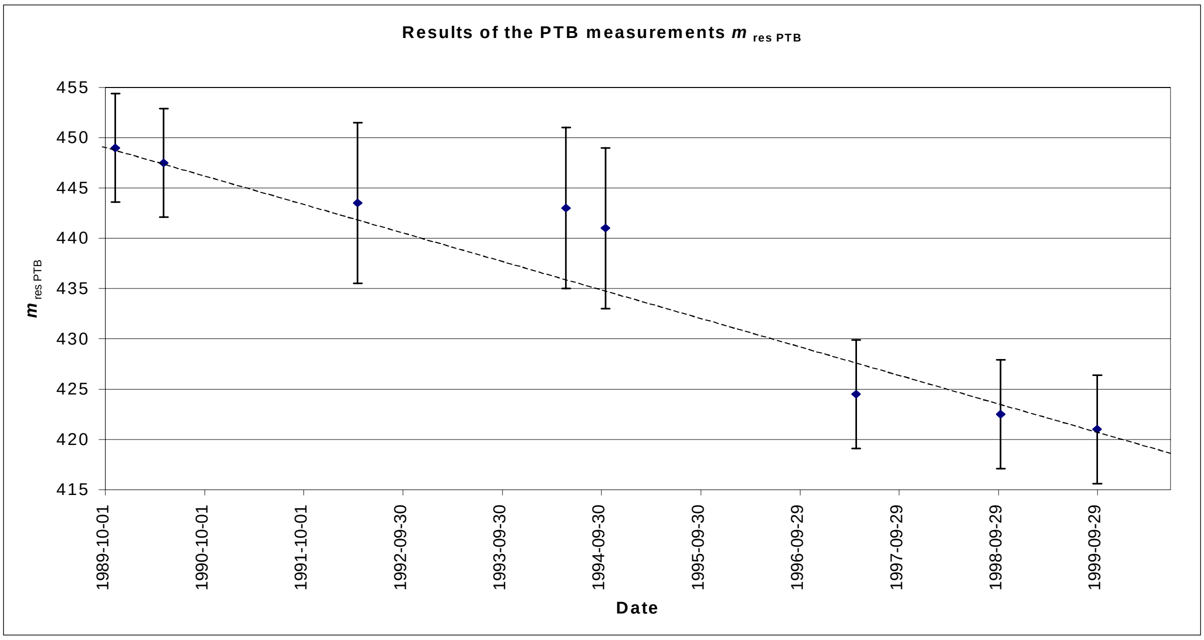

Figure 8: Values resulting from the ASMW/PTB measurements

The drift correction is carried out in the last part of Table 2; the results have been plotted in fig.9.

It should be noted that in 1991 the measurement equipment of the ASMW was transferred from Potsdam to Berlin-Friedrichshagen. The measurements of the PTB carried out in 1992 and 1994 were referred to the Group Standard, no reliable absolute measurements with the MWB could be carried out. This is the reason for the increase in the uncertainty from $u = 2.7$ to $u = 3.7$ stated for these years.

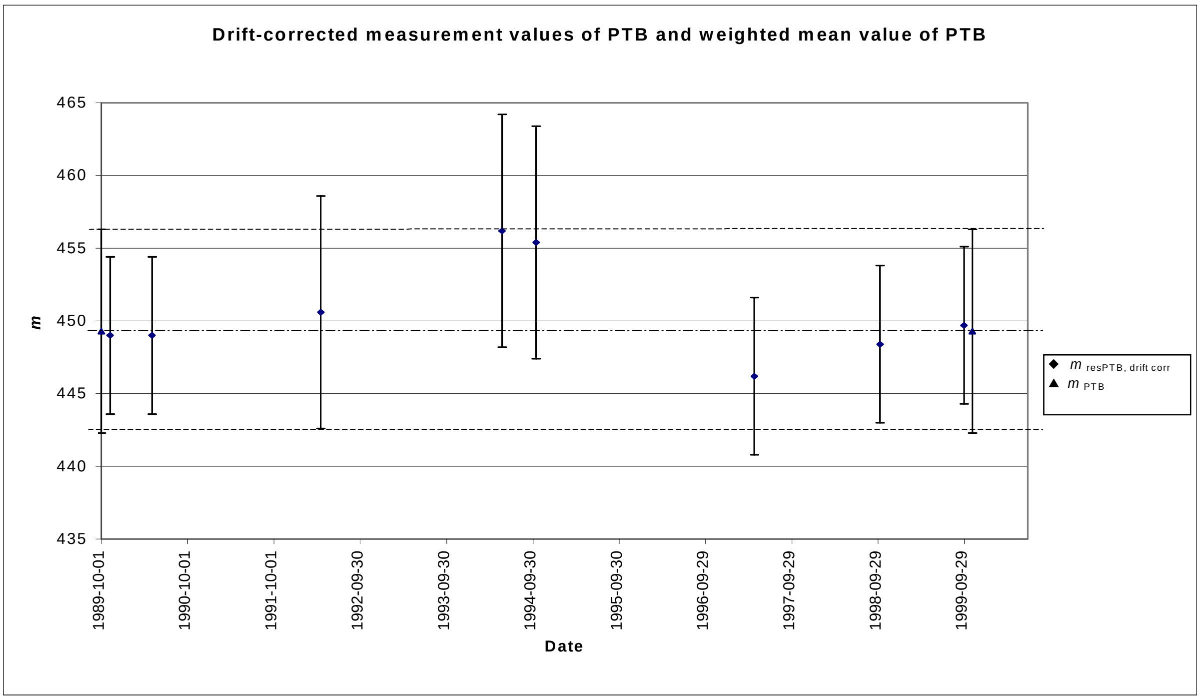

Figure 9: Drift-corrected values of the ASMW/PTB measurements and mean value of PTB

F. Final result of the ASMW/PTB measurements

From the last two lines of Table 2 the final result of the ASMW/PTB measurements for the comparison can be derived by calculating the weighted mean m_{PTB} from the $m_{\mathrm{j\ resPTB,driftcorr.}}$ and from their affiliated u_{j} according to

$$u_{PTB}^2 = \frac{1}{\sum \frac{1}{u_j^2}} = \frac{1}{0.874} = 1.144 \quad \text{and} \quad m_{PTB} = u_{PTB}^2 \sum \frac{m_i}{u_j^2} = 1.114 \times 392.72 = 449.27 \quad (7)$$

Weighted mean: $m_{\mathrm{PTB}} = 449.3$ (8)

shown additionally by the middle line in figure 9.

3.4 Transfer Uncertainty of the Comparison

The distribution of the temperature- and drift-corrected values $m_{\mathrm{j\ resPTB,drift\ corr}}$ in figure 9 shows a normal behaviour: the mean value of PTB lies inside the calculated uncertainty bars for k = 2 for all results. This means that the distribution is fully described by the uncertainty contributions taken into account in the estimation of uncertainty given in Appendix A.

On the other hand, some influences on the results have not been taken into account in the uncertainty calculation. During the time of measurement the standards were transported to the different institutes, returned to ASMW/PTB and were measured in different laboratory environments in Potsdam and Berlin. It is obvious that there is an additional influence on the results to be described by the transfer uncertainty u_{t}.

This influence of the transfer uncertainty is included in the results shown in figure 9. Moreover, the scatter may be regarded as representative also of all the measurements carried out by the different laboratories participating in the comparison, thus describing the transfer uncertainty of the comparison as well.

From this point of view two interpretations of figure 9 may be given:

1. No additional influence of the transfer conditions can be recognized, the uncertainty distribution is described by the common uncertainty calculation, the transfer uncertainty is negligible.
2. There is a transfer uncertainty, in every case overrated by the limits of the dashed lines shown in figure 9.

Although the first conclusion would simplify the uncertainty calculations, the second one will be preferred for this report for safety reasons. As the dispersion of the values $m_{\mathrm{j\ resPTB,drift\ corr}}$ about their mean can be described by the experimental standard deviation $s(m_{\mathrm{j\ resPTB,drift\ corr}})$ [14], the relation

$$s(m_{\mathrm{j\ resPTB,drift\ corr}}) = u_{\mathrm{t}} = 3.5 \quad (9a)$$

calculated from the values in Table 2 will in the following be used for the comparison.

Two dashed lines referred to above are shown in figure 9. They cover all measurement results and represent the expanded transfer uncertainty

$$U_{\mathrm{t}} = 2\, u_{\mathrm{t}} = 7. \quad (9b)$$

3.5 Measurements performed by the Participants

Six different types of measuring devices were used by the participating laboratories:

- modified Maxwell-Wien bridge	PTB, VNIIM [7], BNM-LCIE, NIST
- resonance devices (series and parallel)	NIM [13], NPL [8]
- parallel resonance circuit (1H), 10:1 step-down (IVD bridge)	SP [9]
- commercial bridge and calibrated standard	OFMET
- impedance comparison related to resistance and frequency	IEN [10]
- new digital impedance bridge	NIST [11]
- transformer (IVD) bridge	NPL [12].

The results of the participants' measurements had to be expressed in the form:

$$L = 10\ \text{mH}\ (1 + m\cdot 10^{-6}) \qquad \text{similar to eq.(5).}$$

Table 3 shows the measurement results of the participating institutes for m. The original results measured by the laboratories are given in part 1 of the table. In order to ensure comparability, the values in part 2 of Table 3 have been related to the same coil resistances, assuming that changes in these values are caused by the thermal behaviour of the thermostats of the standards. These corrections furnish consistent inductance values, even in the case of the above-mentioned greater temperature change of the thermostat of L2 during transport from the USA to Germany. The uncertainties reported by the participants at that time are shown in the lower part of the table. All temperature-corrected measurement results are shown in the diagram of figure 10 with reference to the linear drift of the standards, measured by the pilot laboratory and represented by the two straight lines. The codes of the participating laboratories are given in the headline of Table 3.

Unfortunately, the measurement uncertainties reported by the laboratories in 1992 do not give a clear and uniform picture of the uncertainties of the various measuring devices. It must be borne in mind that the "Guide to the Expression of Uncertainty in Measurement" GUM [14] was not yet available eight years ago and that the uncertainty calculations were rather made following the tradition of the respective laboratories.

The original uncertainty values given in part 3 of Table 3 are taken from the reports of the participants up to 1995 and presented in figure 10 by the corresponding error bars. They had been estimated as

- types A and B uncertainties according to INC-1 by four institutes,
- total uncertainty based on listed components by two institutes
- unspecified total uncertainty by four institutes.

The confidence level was stated by only 6 laboratories.

Considering the progress achieved in uncertainty calculations since 1992 and reflected by the GUM [14], the statement of uncertainties as shown in part 3 of Table 3 and figure 10 cannot be sufficient today.

Consequently, for the preparation of the Draft A of this report, the participants were requested in 1999 to explain and/or to specify their uncertainties reported up to 1995, paying due regard to the rules of the GUM.

The uncertainty budgets recalculated by the participating institutes are reported in Appendix A. The new calculated uncertainties from these budgets are shown in part 4 of Table 3.

3.6 Calculation of the Reference Value and Deviation of the Participants

Some influences on the measurement results had to be corrected for in order to calculate a reference value for the comparison.

1. As two travelling standards were used, the two results of the institutes were combined to a single representative value by calculation of their mean value as had been done when the m-value of PTB in Table 2 was calculated. NPL and IEN measured the standards over a longer period with very stable results. For these institutes a single mean value was calculated from all their results.

2. The comparison covered a period of about 60 months, from the beginning in November 1989 to the last measurements at PTB in October 1994. During this time, a normal drift of the inductance values of the standards could not be avoided, independent of the drift caused by the temperature of the thermostats corrected in Table 3. From the long-time measurements described in section 3.3.E, a nearly linear behaviour was found with a drift coefficient of $r = -0.242$/month (eq.6) for the mean value of both standards. This coefficient was used to correct the institutes' mean values m_{mi} to obtain their final value m_{i}, carried out in Table 4.

3. Following [15] for the determination of the Key Comparison Reference Value (KCRV) m_{R}, the weighted mean of the final values m_{i} of all the participants except OFMET was calculated. The result of OFMET had not been considered for the calculation of the reference value as their measurement was traced back to NPL.

4. As weight for the calculation of the KCRV the combined uncertainty from u_{i}, the institutes' uncertainty from part 4 of Table 3, and the transfer uncertainty u_{t} from eq. (9a) were chosen, assuming that the reference value not only depends on the uncertainties calculated by the participants, but also on the conditions which led to the introduction of the transfer uncertainty.

5. The calculation of the KCRV m_{R} is carried out in Table 5 which finally shows the results of the different institutes in relation to the calculated reference value m_{R} (= 452). A diagram of the institutes' results combined with the expanded uncertainties of their deviations from the reference value is shown in figure 11. These uncertainties are calculated according to the equation

$$U_{\mathrm{D}} = 2(u_{\mathrm{i}}^2 + u_{\mathrm{t}}^2 - u_{\mathrm{R}}^2)^{1/2} \qquad (10)$$

   which takes the correlation between m_{R} and m_{i} into account.

As a result of the calculations described above, the Key Comparison Reference Value was found to be equal to

$$m_{\mathrm{R}} = 452$$

with an expanded standard uncertainty of

$$U_{\mathrm{R}} = 4$$

calculated from the institutes' uncertainties u_{i} and the transfer uncertainty u_{t}.

Table 3: Comprehensive representation of all measurement results

1. Measured values of m and R_L for the inductors No. 01 and No. 02 (R_L - dc resistance of the inductor coils)																	
Institute	ASMW	NIM	VNIIM	ASMW	PTB-BS	SP	NPL	NPL	BNM-LCIE	OFMET	IEN	IEN	IEN	PTB-B	PTB-B	NIST	PTB-B
Code	P	1	2	P	3	4	5	5	6	7	8	8	8	P	P	9	P
Date	1989 11-07	1989 11-22	1990 04-27	1990 05-03	1990 05-	1990 07-	1990 08-21	1991 01-17	1991 04-10	1991 06-12	1991 10-25	1991 11-08	1992 03-30	1992 04-16	1994 05-22	1994 08-09	1994 10-14
m No.01	481	492	482	481	488	468	474	473	478	466	478	477	479	475	477	475	475
R_L in Ω	8.674	8.672	8.675	8.674	8.672	8.674	8.672	8.671	8.672	8.672	8.672	8.672	8.672	8.671	8.672	8.674	8.672
m No.02	417	425	419	414	421	394	407	404	406	392	409	411	410	407	406	404	426 !
R_L in Ω	8.856	8.855	8.855	8.856	8.854	8.855	8.854	8.851	8.853	8.854	8.854	8.854	8.854	8.854	8.855	8.858	8.875 !
2. Temperature corrected values (related to the same resistances R_L, measured 1989-11-07: $R_{LNo.\,01}$= 8.674 Ω, $R_{LNo.\,02}$ = 8.856 Ω)																	
m No.01	481	494	481	481	490	468	476	476	480	468	480	479	481	478	479	475	475
m No.02	417	426	420	414	423	395	409	409	409	394	411	413	412	409	407	402	407 !
3. Total uncertainties reported by the participants up to 1994 [1,2]																	
Total uncertainty	5	5	5	5	15	20	20	20	24	35	7	7	7	8	8	22	8
4. Updated uncertainties from Appendix A, according to GUM [14]																	
u_i	2.7	3.5	2.4	2.7	14.5	19	10.16	10.16	7	37.5	6.8	6.8	7.0	3.7	3.7	19	3.7
$U_i = 2u_i$	5	7	5	5	29	38	20	20	14	75	14	14	14	7	7	38	7

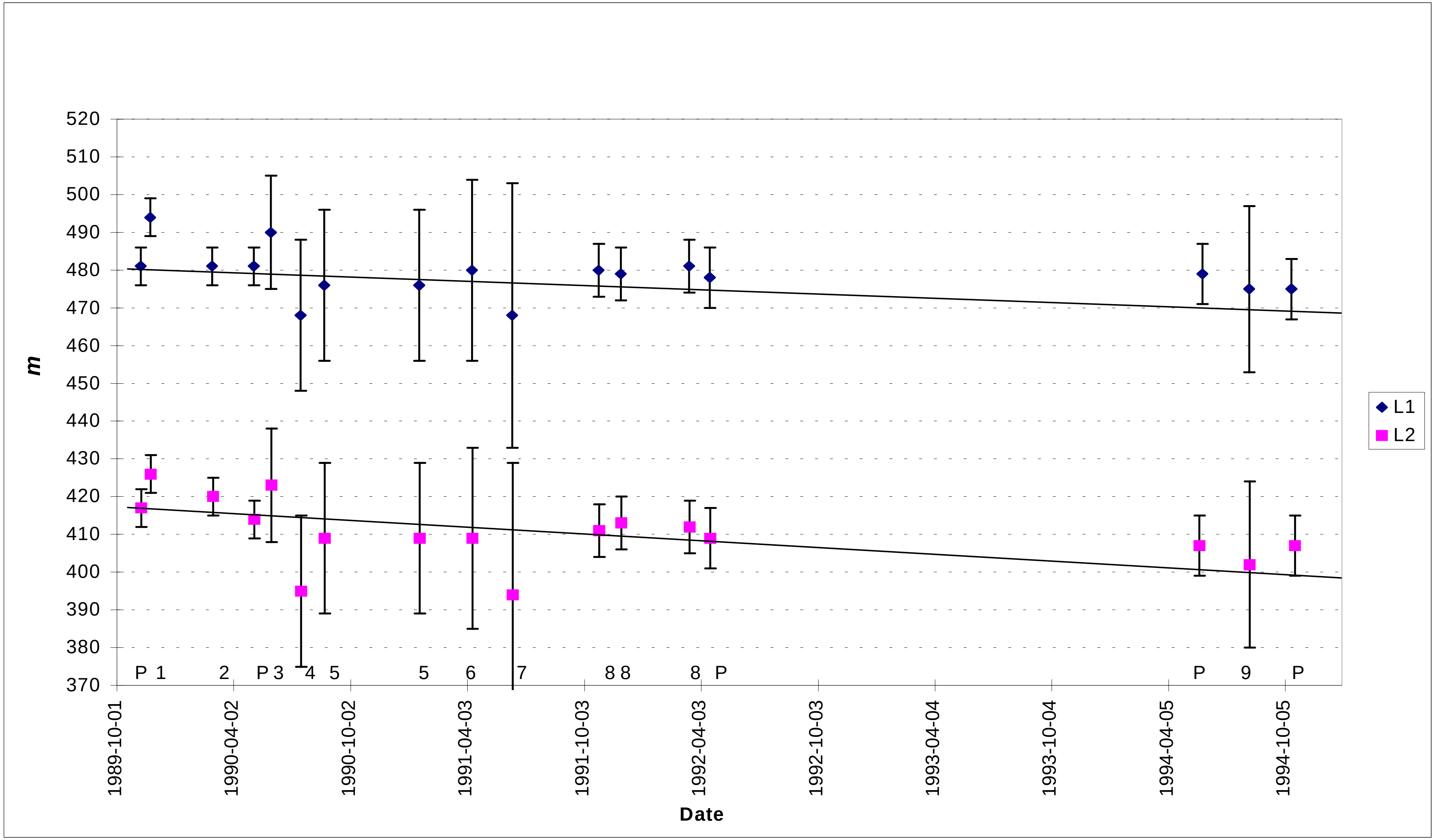

Figure 10: Diagram of all temperature-corrected measurement results (uncertainties according to [1,2])

Table 4: Calculation of the institutes' values m_i and their deviation D from the reference value m_R

Institute	ASMW	NIM	VNIIM	PTB-BS	SP	NPL	NPL	BNM-LCIE	OFMET	IEN	IEN	IEN	NIST
Code	P	1	2	3	4	5	5	6	7	8	8	8	9
Date	1989 11-07	1989 11-22	1990 04-27	1990 05-	1990 07-	1990 08-21	1991 01-17	1991 04-10	1991 06-12	1991 10-25	1991 11-08	1992 03-30	1994 08-09
1. Temperature-corrected values of the participants from Table 3													
m No. 01	481	494	481	490	468	476	476	480	468	480	479	481	475
m No. 02	417	426	420	423	395	409	409	409	394	411	413	412	402
Mean value m_{mi} of the institute	449	460	450.5	456.5	431.5	442.5		444.5	431		446		438,5
2. Drift-corrected mean values of the institutes [$m_{mi,driftcorr.} = m_{mi} + k \cdot 0.242$]													
Number of months k	0	0.5	5.5	6	8	12		17	19		26		57
Correction	0	0.1	1.4	1.6	2.0	2.9		4.1	4.6		6.3		13.8
Drift corrected value $m_{mi,drift\ corr.}$	449.3 *	460.1	451.9	458.1	433.5	445.4		448.6	435.6		452.3		452.3
Value resulting for the institute m_i	449	460	452	458	434	445		449	436		452		452
3. Deviation D of m_i to the reference value $m_R = m_{mean} = 452.2$ from eq (12) and expanded uncertainty U_D of D from eq. (10)													
$D = m_{mi,driftcorr} - m_R$	– 2.9	+ 7.9	– 0.3	+ 5.9	– 18.7	– 6.8		– 3.6	– 16.6		0.1		0.1
U_D (from Table 5)	8	9	7	29	38	21		15	75		15		38

* from section 3.3.E, eq (8)

Table 5: Calculation of the reference value m_R as weighted mean m_{mean}

$$m_{mean} = u^2_{mean} \sum \frac{m_i}{u^2_{i+t}} \qquad\qquad u^2_{mean} = \frac{1}{\sum \frac{1}{u^2_{i+t}}} \qquad (11)$$

m_i *is* taken from Table 4 (the values of $m_{mi,\text{drift corr}}$) and u_{i+t} is calculated from u_i in Table 3 and u_t (eq.9a)

	ASMW/ PTB-B	NIM	VNIIM	PTB-BS	SP	NPL	BNM/ LCIE	OFMET*	IEN	NIST	
m_i	449.3	460.1	451.9	458.1	433.5	445.4	448.6	435.6	452.3	452.3	
u_i	2.7	3.5	2.4	14.5	19	10.15	7	37.5	7	19	
u_t	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	
$u_{i+t}^2 = u_i^2 + u_t^2$	19.54	24.50	18.01	222.50	373.25	115.27	61.25	1418.50	61.25	373.25	
$\frac{m_i}{u^2_{i+t}}$	22.99	18.78	25.09	2.06	1.16	3.86	7.32	(0.31)*	7.38	1.21	$\sum \frac{m_i}{u^2_{i+t}} = 89.85$
$\frac{1}{u^2_{i+t}}$	0.0512	0.0408	0.0555	0.0045	0.0027	0.0087	0.0163	(0.0007)*	0.0163	0.0027	$\sum \frac{1}{u^2_{i+t}} = 0.1987$
$U_D = 2(u_{i+t}^2 - u_R^2)^{1/2}$	7.6	8.8	7.2	29.5	38.4	21.0	15.0	75.2	15.0	38.4	From eq. (10)

$$u_R^2 = u^2_{mean} = \frac{1}{\sum \frac{1}{u^2_{i+t}}} = \frac{1}{0.1987} = 5.033 \qquad m_R = m_{mean} = u^2_{mean} \sum \frac{m_i}{u^2_{i+t}} = 5.033 \times 89.85 = 452.21 \qquad u_R = 2.24$$

$$\underline{\underline{m_R = 452}} \qquad \underline{\underline{U_R = 4}} \qquad (12)$$

* The values of OFMET were not included into the calculation of the mean value

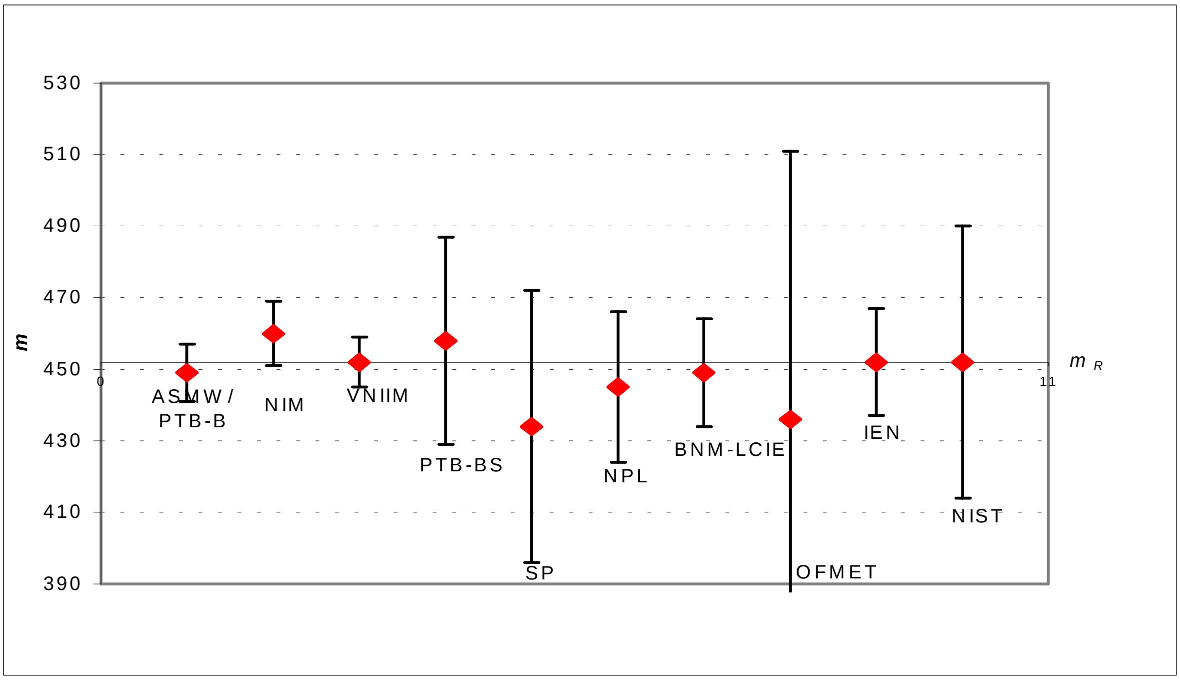

Figure 11: Reference value $m_R = 452$ and measurement values of the participants

3.7 Conclusions

Considering the above results, the following conclusions can be drawn:

1. Since the stability of the standards (see fig. 9) could be maintained throughout the period of the comparison, the long circulation time in cycle 3 (approximately 2 years) did not affect the results.

2. Although the methods of measurement differed in all participating laboratories, an agreement inside the respective limits of uncertainty could be achieved by all participants (fig. 11).

3. A Key Comparison Reference Value of $m_R = 452$ with an expanded uncertainty of $U_R = 4$ was found. Three institutes reached this reference value. Two institutes achieved results inside the limits of the expanded uncertainty of the reference value.

4. All the deviations $D = m_i - m_R$ achieved by the participants lie inside their uncertainties U_D.

4 COOMET Comparisons

4.1 Introduction

Beginning in the eighties, international measurement standards for physical units were developed, maintained and used for international traceability by those East European countries which were members of the Council of Mutual Economic Aid (CMEA). The ASMW of the former GDR developed the CMEA standard of inductance at 10 mH and had maintained it since 1985. In 1973, 1980, 1986 and 1989, international comparisons of inductance had been carried out by the ASMW.

After German reunification, this work was continued by the laboratory now integrated into the PTB. Two comparisons organised within the framework of the Regional Metrology Organisation COOMET of the East European NMIs were carried out at the PTB-B: in December 1992 with the Polish institute GUM and, as a guest in COOMET, with the Hungarian institute OMH participating [3]; the Bulgarian, Polish and Russian institutes (KSM, GUM and VNIIM) took part in a comparison carried out in May 1995 [4].

4.2 Measuring Set-ups of the Participants and Results

Since 1980, all participants had traced their 10 mH standards back to the CMEA standard at ASMW. Except VNIIM, which could determine its unit by means of a modified Maxwell-Wien bridge [7], the participants' own measurements were carried out using commercial bridges with relative uncertainties in the order of 10^{-5} to 10^{-4} .

Deviating from the CCEM comparison, the participants did not measure the PTB travelling standards at the site of their laboratories. Instead of this, their standards were compared against the travelling standards at the PTB. Then the values of the travelling standards could be calculated from the values of the participants' standards measured in their institutes before and after the comparisons at PTB in Berlin.

It is evident that the uncertainties achieved by this procedure cannot be compared with those reported from the CCEM comparison. But, nevertheless, the differences between the institutes and the ASMW/PTB did not exceed 10^{-5} in most cases, which may be explained by the permanent comparisons over a long period of time and the fact that the institutes' own measurements could be carried out using the substitution method.
Considering that the COOMET comparisons had another goal than the comparison within the scope of CCEM – maintaining the units on a uniform level to ensure reliable measurements in the region but not to achieve uniform realisations of the units – the results fulfilled this purpose very well.

The equipment used in the comparisons and the results given in relation to the reference value of the CCEM comparison are shown in Table 6 with combined measurement uncertainties for k = 2 (2 σ). The difference to the reference value was calculated from the relation

$$m_{\mathrm{PTB}} - m_{\mathrm{R}} = -3 \qquad (13)$$

derived from Table 4.

Table 6: Equipment of the participants and comparison results

Institute	Unit traced back to	Unit maintained by	Measuring device	$m_{\text{inst}} - m_{\text{R}}$ 1992	$m_{\text{inst}} - m_{\text{R}}$ 1995
GUM	ASMW in 1989 $m_{\text{inst}} = m_{\text{ASMW}}$	4 standards 10 mH Genrad type	Genrad bridge type 1660-A (Owen bridge)	+ 1 $U = 26$	– 6 $U = 26$
KSM	ASMW in 1989 $m_{\text{inst}} = m_{\text{ASMW}}$	4 standards 10 mH Genrad type	Genrad bridge type 1632-A	./.	– 12 $U = 60$
OMH	ASMW in 1989 $m_{\text{inst}} = m_{\text{ASMW}}$	4 standards 10 mH Sullivan and Genrad type	Genrad bridge type 1660-A (Owen bridge)	+ 3 $U = 20$	./.
VNIIM	Resistance and Capacitance unit of VNIIM	4 standards 10 mH VNIIM type ЭИ - 1	Modified Maxwell-Wien bridge	./.	– 5 $U = 5$

The results are illustrated in figure 12 in relation to the Key Comparison Reference Value $m_{\text{R}} = 452$ of the CCEM K3 comparison.

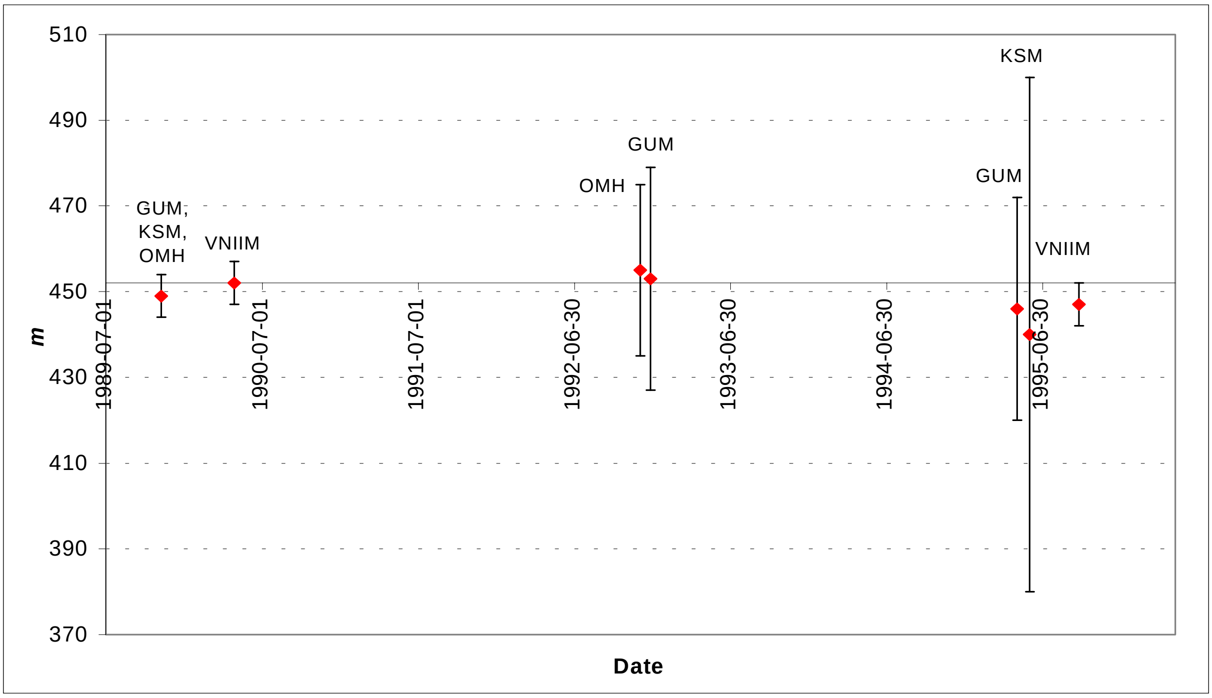

Figure 12: Key Comparison Reference Value $m_R = 452$ and measurement values of the participants of the COOMET Comparisons

5 Comparison with UME

5.1 Introduction

In co-operation between PTB and UME in 1996 and 1997, a new Maxwell-Wien bridge with Wagner arm was designed and constructed for the realisation of the unit of inductance in Turkey. After this work had been concluded in October 1997, a comparison between PTB and UME was carried out at UME at 10 mH and 1 kHz. The PTB's travelling standard L2 was used in the measurements [5].

5.2 Measuring Device at UME

For the determination of the value of 10 mH a Maxwell-Wien bridge with Wagner arm was designed whose construction is nearly identical with that of the PTB (figures 3 and 4). The differences to the bridge of the PTB are in particular:

- C_4 is a GR-1422-CB type capacitor whose value is fixed at 1000 pF
- C_3 is a variable air capacitor built at UME to make fine adjustments. Its value can be varied in the range from 0.25 pF to 10 pF with a resolution of 0.1 fF.
- R_k is an adjustable resistor series-connected to the L_x inductor in order to balance the real parameters of the bridge. R_k is formed by a special combination of four resistors and one potentiometer, which allows the sensitivity of the resistive balancing to be increased to 1 ppm. It is capable of providing 0.7 ppm sensitivity by 6 μΩ resolution.
- R_3 is a 1 MΩ resistor with a temperature coefficient of ± 15 ppm/K.

A special UME-made inductor of 8.92 μH with a variable resistance of 0.1 mΩ is used as L_k. The device ensures high reproducibility. According to PTB' s uncertainty budget in annex A the relative standard deviation for 10 measurements is smaller than $1 \cdot 10^{-6}$; the relative type B uncertainty is estimated to be smaller than $5 \cdot 10^{-6}$, both for 1 σ.

5.3 Comparison and Results

In October 1997, 10 measurements of the PTB's transfer standard No L2 at the frequency of 1 kHz were carried out using the Maxwell-Wien bridge of UME in order to verify its uncertainty. The results are shown in Table 7 together with the values obtained at PTB. The estimated relative 2 σ uncertainty of UME's measurement values was found to be $10 \cdot 10^{-6}$, with the type A and type B components combined.

Table 7: Comparison results of UME and PTB-B

PTB-B	UME	PTB-B
22.10.1997	29.10.1997	07.11.1997
(10.00398 ± 0.00005) mH (8.869 ± 0.001) Ω	(10.00393 ± 0.00010) mH (8.870 ± 0.001) Ω	(10.00398 ± 0.00005) mH (8.869 ± 0.001) Ω

By correction of the results for the same resistance values, a difference of 6 in m between PTB-B and UME was found and from eq. (13) the difference

$$D = m_{UME} - m_R = -9 \qquad U_D = 12 \tag{14}$$

can be stated, with an expanded combined uncertainty calculated according to eq. (10).

6 Presentation of the Results in the Key Comparison Data Base

To present the results of key comparisons and supplementary comparisons in the Key Comparison Data Base KCDB of the BIPM, the use of an EXCEL template has been suggested [16].

The template should consist of 3 pages containing the results of the individual measurements of the participating laboratories, statements about the equivalence of their results in relation to a key comparison reference value KCRV and a graph showing their degrees of equivalence.

The requested template is shown in Appendix B. The results of those ten institutes are presented, which had taken part in the initial comparison between 1989 and 1994. In addition, the result of the bilateral comparison between PTB and UME of 1998 is shown.

The KCRV was calculated as the weighted mean of the measurement results of those nine institutes which realized the unit of inductance by absolute determination, see 3.6.

Acknowledgements

The author should like to thank all participants in the comparison for having carried out the measurements with high quality and within the stipulated time frame and for having submitted their reports. Special thanks are due to *Hans Bachmair* who, after German unification, made it possible that the work started by ASMW was continued at PTB and who also made important contributions to the work performed and the contents of the report. *Dominique Reymann,* in particular, and *Yuri Semenov*, members of the WGKC subgroup for CCEM-K3, have substantially improved the quality of the report through their detailed comments on the Draft B report. Express thanks for this are due to them. Last but not least the author should like to thank all staff members of ASMW and PTB involved in the comparison, especially *Klaus Neumann* and *Peter Räther* for having set up the measuring facilities and kept them ready for operation over more than fifteen years, and for the contribution they made to the comparison measurements together with *Margit Geiersbach.*

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Appendix A: Uncertainty Budgets

Uncertainty budget of the pilot laboratory

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
1. Realisation of 10 mH by means of the MWB		
Repeatability (n=10)	A	0.1
Capacitance standard 1 nF of MWB	B	0.8
AC resistance of 1 kΩ resistor of MWB	B	0.8
AC resistance of 10 kΩ resistor of MWB	B	1.0
Variable capacitor C'_3	B	0.5
Inductor L_k	B	1.5
Temperature measurement of the 10 mH reference standard	B	1.0
Combined uncertainty (1σ)		2.4
2. Measurement of the travelling standards		
Repeatability (n=10)		0.1
Reference standard 10 mH		2.4
Ratio of the IVD bridge		0.5
Temperature instability of the thermostats		1.0
Drift deviation of the standards from linearity		0.5
Combined uncertainty (1σ)		2.7
Expanded uncertainty (2σ)		5.4

Carrying out the measurements in 1992 and 1994 the reference standard was referred to the Group Standard. A relative standard uncertainty contribution of 2.5×10^{-6} for the instability of the group has to be added in part 2. This leads to

$u = 3.7$ and $U = 7.4$

that must be considered for these years.

Uncertainty budgets of the participants updated in 1999

A. NIM:

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Adjustable voltage of the resonant bridge	A	0.4
Resolution of the detector	A	0.3
Instability of the working frequency	A	0.12
Statistic standard deviation	A	2.0
Ratio error of the resonant bridge	B	0.3
Inner load effect of the ratio arm	B	0.33
Residual inductance of the resistance box	B	0.15
Trace uncertainty of the standard capacitor	B	0.37
Effect of the voltage coefficient of the capacitors	B	2.53
Transfer uncertainty from 10000 pF to 1000 pF	B	1.0
Uncertainty of the IVD of the modified MWB		0.5
Ground capacitance of the high potential terminal of the inductor	B	0.6
Combined uncertainty (1σ)		3.55
Expanded uncertainty (2σ)		7

B. VNIIM

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Variability of repeated observations	A	1
Capacitance standard 1 nF (arm 3 of MWB-V)	B	1
AC resistance of 1 kΩ resistor (arm 2 of MWB-V)	B	1
AC resistance of 10 kΩ resistor (arm 4 of MWB-V)	B	0.6
Variable capacitor 1pF (arm 3 of MWB-V)	B	0.2
Variation of residual inductance in R_1 (arm 3 of MWB-V)	B	0.3
Temperature measurement of the inductance standard	B	1.5
Combined uncertainty (1σ)		2.4
Expanded uncertainty (2σ)		4.8

C. PTB-BS

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability	A	7
Capacitors of the MWB	B	11.5
AC resistors of the MWB	B	4
Bridge capacitances	B	0.6
Auxiliary inductor in the "short-circuit" balance	B	2
Auxiliary resistor in the "short-circuit" balance	B	5
Temperature measurement of the travelling standards	B	0.2
Combined uncertainty (1σ)		14.5
Expanded uncertainty (2σ)		29

D. SP

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatabality	A	1
Realization of the unit (1 H, 400 mH, 200 mH)	B	18
Bridge measurement of 100 mH (maintenance level)	B	5
Stability of maintained level (100 mH)	B	1
Bridge measurement of the travelling standards 10 mH	B	2
Temperature measurement of 100 mH standard	B	2
100 mH standard ambient humidity measurement	B	2
Temperature (resistance) measurement of the travelling standards	B	1
Combined uncertainty (1σ)		19
Expanded uncertainty (2σ)		38

E. NPL

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability	A	3.0
NPL capacitance standard to NPL transfer standard inductor	B	7.2
Transfer inductors to NPL Primary inductance standard	B	5.3
Inductance measurement bridge	B	3.0
Temperature measurement of PTB inductor	B	2.3
Combined uncertainty (1σ)		10.16
Expanded uncertainty (2σ)		20.3

F. BNM-LCIE

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability	A	0.5
Calibration of the capacitance in the MWB	B	6
Calibration of the resistance P in the MWB	B	2
Calibration of the resistance Q in the MWB	B	2
Parasitic capacitances due to connections	B	0.05
Reproducibility of mutual inductances and connections	B	0.6
Temperature measurement of the travelling standards	B	2.2
Combined uncertainty (1σ)		7
Expanded uncertainty (2σ)		14

G. OFMET

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability (n=15)	A	2.5
Calibration of the reference standard	B	35
Long term stability of the reference standard	B	10
Temperature dependence of the reference standard	B	2.5
Humidity	B	5
Ambient influences	B	5
Combined uncertainty (1σ)		37.5
Expanded uncertainty (2σ)		75

H. IEN

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability	A	1.4
Possible effect of different measurand definitions	B	5
AC-DC difference of the reference resistor	B	3
Effect of distortion of currents and voltages	B	2
Induced EMF's from residual magnetic fields	B	2
Traceability of the reference resistor	B	1
Traceability of the measuring frequency	B	0.1
Uncertainty of the voltage ratios	B	0.8
Combined uncertainty (1σ)		6.8
Expanded uncertainty (2σ)		14

I. NIST

Source of uncertainty	Type	Relative standard uncertainty in 10^{-6}
Repeatability (n=15)	A	5
Calibration of the reference standard	B	6
Binary inductive voltage divider	B	1
Ratio measurement by DVM	B	12
Frequency	B	1
Lead impedances	B	12
BIVD loading	B	5
Combined uncertainty (1σ)		19
Expanded uncertainty (2σ)		38

Appendix B: Data in accordance with the KCDB template [16]

Key comparison CCEM-K3

MEASURAND : **Inductance at f = 1 kHz**
NOMINAL VALUE : **10 mH**

m_i : result of the measurement carried out by laboratory i given as the relative difference from the nominal value

$$L_i = L_0 \times (1 + m_i \times 10^{-6}),\ \text{with } L_0 = 10\ \text{mH}.$$

u_i : combined standard uncertainty of m_i reported by laboratory i

u_t : estimated standard uncertainty of m_i due to the travelling standards
The transfer uncertainty of the travelling standards u_t is estimated from the standard deviation of one observation of the results of the long-time measurements of the pilot laboratory: $u_t = 3.5$.

u_{i+t} : total standard uncertainty of m_i

Lab i	**m_i**	**u_i**	**u_{i+t}**	**Date of measurement**
ASMW/PTB-B*	449	3	4	89-10 ... 99-10
BNM-LCIE	449	7	8	91-04
IEN	452	7	8	91-10 ... 92-03
NIM	460	4	5	89-11
NIST	452	19	19	94-08
NPL	445	10	11	90-08 ... 91-04
OFMET	436	38	38	91-06
PTB-BS*	458	15	15	90-05
SP	434	19	19	90-07
UME	443	5	6	98-10
VNIIM	452	2	4	89-11

* **ASMW/PTB-B**: ASMW up to 1990 and PTB in Berlin after 1990
PTB-BS: PTB in Braunschweig

Key comparison CCEM-K3

MEASURAND : **Inductance at f = 1 kHz**
NOMINAL VALUE : **10 mH**

The key comparison reference value L_R of this comparison is $L_R = L_0 \times (1 + m_R \times 10^{-6})$, with L_0 = 10 mH.
m_R is obtained from the weighted mean of the results of all the participants except OFMET and UME
using weights proportional to the reciprocal of the quadratic sum of the uncertainty of the laboratories and the transfer uncertainty.
The standard uncertainty u_R of m_R is the standard uncertainty of this weighted mean. It takes into account the uncertainty due to the travelling standards.

$m_R = 452$ $u_R = 2$

The degree of equivalence of each laboratory with respect to the reference value is given by a pair of terms:
$D_i = m_i - m_R$ and its expanded uncertainty (k = 2), $U_i = 2\,(u_{i+t}^2 - u_R^2)^{1/2}$.

The degree of equivalence between two laboratories i and j is given by a pair of terms:
$D_{ij} = D_i - D_j = (m_i - m_j)$ and its expanded uncertainty (k = 2), U_{ij}.
$U_{ij} = 2\,(u_{i+t}^2 + u_{j+t}^2)^{1/2}$

Lab *j* ⇨

Lab *i* ⇩			**ASMW/ PTB-B**		**BNM-LCIE**		**IEN**		**NIM**		**NIST**		**NPL**		**OFMET**		**PTB-BS**		**SP**		**UME**		**VNIIM**	
	D_i	U_i	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}	D_{ij}	U_{ij}
ASMW/PTB-B	**-3**	8			**1**	18	**-3**	18	**-11**	13	**-3**	4	**4**	23	**14**	76	**-9**	31	**16**	40	**6**	15	**-3**	12
BNM-LCIE	**-4**	15	**-1**	18			**- 4**	22	**-12**	19	**- 4**	42	**3**	27	**13**	77	**-10**	34	**15**	42	**5**	20	**-3**	18
IEN	**0**	15	**3**	18	**4**	22			**-8**	19	**0**	42	**7**	27	**17**	77	**-6**	34	**19**	42	**9**	20	**0**	18
NIM	**8**	9	**11**	13	**12**	19	**8**	19			**8**	40	**15**	24	**25**	76	**2**	31	**27**	40	**17**	16	**8**	13
NIST	**0**	38	**3**	40	**4**	42	**0**	42	**-8**	40			**7**	44	**17**	85	**-6**	49	**19**	55	**9**	41	**0**	40
NPL	**-7**	21	**- 4**	23	**-3**	27	**-7**	27	**-15**	24	**-7**	44			**10**	55	**-13**	37	**12**	44	**2**	25	**-7**	23
OFMET	**-17**	75	**-14**	76	**-13**	77	**-17**	77	**-25**	76	**-17**	85	**-10**	55			**-23**	81	**2**	85	**-8**	76	**-16**	76
PTB-BS	**6**	29	**9**	31	**10**	34	**6**	34	**-2**	32	**6**	49	**13**	37	**23**	81			**25**	49	**15**	32	**6**	31
SP	**-19**	38	**-16**	40	**-15**	42	**-19**	42	**-27**	40	**-19**	55	**-12**	44	**-2**	85	**-25**	49			**-10**	41	**-18**	40
UME	**-9**	12	**-6**	15	**-5**	20	**-9**	20	**-17**	16	**-9**	41	**-2**	25	**8**	76	**-15**	32	**10**	41			**-9**	15
VNIIM	**0**	7	**3**	12	**3**	18	**0**	18	**-8**	13	**0**	40	**7**	23	**16**	76	**-6**	31	**18**	40	**9**	15		

This value takes into account the correlation between OFMET and NPL results.

Key comparison CCEM-K3

MEASURAND : **Inductance at *f* = 1 kHz**
NOMINAL VALUE : **10 mH**