BUREAU INTERNATIONAL DES POIDS ET MESURES

Bilateral Comparison of 1 Ω standards (ongoing BIPM key comparison BIPM.EM-K13.a) between the NSAI-NML (Ireland) and the BIPM

December 2011

R. Goebel[*], O. Power[**],
N. Fletcher[*] and M. Stock[*].

[*] Bureau International des Poids et Mesures, Sèvres, France.
[**] NSAI-National Metrology Laboratory, Dublin, Ireland.

1 Introduction

A comparison of values assigned to 1 Ω resistance standards was carried out between the BIPM and the National Standards Authority of Ireland - National Metrology Laboratory (NML) in the period September 2010 to December 2010.

Two 1 Ω BIPM travelling standards of CSIRO type were calibrated first at the BIPM, then at the NML and again at the BIPM after their return. The measurement periods are referred to as:

'Before' measurements at the BIPM: September 2010
NML measurements: October 2010
'After' measurements at the BIPM: November-December 2010

The BIPM calibrations are corrected to the reference temperature 23.000 °C and the reference pressure 1013.25 hPa.
According to the protocol, the NML did not apply pressure and temperature corrections to its results. The corrections were made by the BIPM, using the temperature and pressure coefficients of the standards together with the temperature and pressure measurements provided by the NML.
The calibration reports provided by the NML are summarized by the BIPM in section 3 of the present report.

There is no clear evidence of a single linear drift of each standard over the whole period of the comparison (three measurement periods, 'Before', 'NML' and 'After': see Figures 1 and 2). During each period, the mean resistance of each standard is assumed to be constant, with superimposition of a random noise.

For each period, the calibration value assigned to each standard is the mean value of the measurements performed during this period, with an associated standard uncertainty.

The difference between the NML and the BIPM calibrations of a given standard R_i can be written as:

$$\Delta_i = R_{\mathrm{NML},i} - R_{\mathrm{BIPM},i}$$

If two standards are used, the mean of the differences is:

$$\Delta_{\mathrm{NML-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}(R_{\mathrm{NML},i} - R_{\mathrm{BIPM},i}) \qquad (1)$$

This expression can also be written as:

$$\Delta_{\mathrm{NML-BIPM}} = \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{NML},i} - \frac{1}{2}\sum_{i=1}^{2}R_{\mathrm{BIPM},i} \qquad (2)$$

which is the difference of the means.

The reference standards of the two participants are closely correlated, as the NML takes its traceability from the BIPM. The effect of this correlation is reduced by the size of the time lapse since the last comparison of NML's standards with those of the BIPM, in November 2008.

2 Measurements at the BIPM

2.1 BIPM calibrations

The BIPM measurements were carried out by comparison with a 100 Ω reference resistor whose value is known with respect to the BIPM quantized Hall resistance (QHR) standard. The comparison was performed using a DC cryogenic current comparator operating with a 50 mA current in the 1 Ω resistors.
The BIPM 100 Ω reference resistor was calibrated against the QHR in September 2010.
The 1 Ω resistors were kept in a temperature controlled oil bath at a temperature which is close (within a few mK) to the reference temperature. The temperature of the standards was determined by means of a calibrated Standard Platinum Resistance Thermometer (SPRT), in conjunction with thermocouples.

The BIPM measurements are summarized in Table 1 and the uncertainty budget in Table 2.

BIPM	Relative difference from nominal 1 Ω value / 10^{-6}			
Standard #	BEFORE	St. d. mean u_{1B}	AFTER	St. d. mean u_{1A}
BIV200	– 0.812	0.001	– 0.793	0.001
BIV203	+ 0.452	0.001	+ 0.471	0.001

Mean value of 'Before' and 'After'			
Standard #	mean / 10^{-6}	Exp. Std. dev. u_1 / 10^{-9}	Systematic u_2 / 10^{-9}
BIV200	– 0.802	1	16
BIV203	+ 0.461	1	16

Table 1: Summary of the BIPM calibrations. The dispersion is estimated by the standard deviations, and 'systematic' refers to the sources of uncertainty that do not contribute to the variability of the results.

Source of uncertainty	relative standard uncertainty
Imperfect realisation of R_H	2×10^{-9}
Calibration of the BIPM 100 Ω reference (BI100-3) against R_H	3×10^{-9}
Interpolation / extrapolation of the value of BI100-3	13×10^{-9}
Measurement of the (1Ω / BI100-3) ratio	8×10^{-9}
Temperature correction for the 1 Ω standard	2×10^{-9}
Pressure correction for the 1 Ω standard	3×10^{-9}
Combined uncertainty u_2	16×10^{-9}

Table 2: BIPM uncertainty budget for the calibration of the 1 Ω travelling standards.

The value attributed to i-th standard during the period of the NML measurements is the arithmetic mean of the "Before" and "After" values.

$$R_{\mathrm{BIPM},\,i} = (R_{\mathrm{Before},\,i} + R_{\mathrm{After},\,i})/2$$

For each standard, the uncertainty u_1 associated with the dispersion is the quadratic mean of the standard deviations "Before" and "After".

$$u^2_{1,\,i} = (u^2_{1\mathrm{Before},\,i} + u^2_{1\mathrm{After},\,i})/2^2$$

u_2 is the uncertainty arising from the combined contributions associated with the BIPM measurement facility and the traceability, as described in Table 2. This component is assumed to be strongly correlated between calibrations performed in the same period.

For a single standard, the BIPM uncertainty $u_{\mathrm{BIPM},\,i}$ is obtained from: $u^2_{\mathrm{BIPM},\,i} = u^2_{1,\,i} + u^2_{2,\,i}$

The $u_{2,\,i}$ are assumed to be correlated, unlike $u_{1,\,i}$.

Using expression (2), when the mean (for two standards) of the NML-BIPM relative difference is calculated, the BIPM contribution to the uncertainty is:

$$u_{\mathrm{BIPM}}^2 = \sum_{i=1}^{2} \frac{u_{1,i}^2}{2^2} + u_2^2 \qquad (3)$$

Using the values shown in Table 1, the relative standard uncertainty u_{BIPM} is

$$u_{\mathrm{BIPM}} = 16 \times 10^{-9}.$$

2.2 Uncertainty associated with the transfer

u_{d} is the uncertainty associated with the drift (or the step changes) of the travelling standards observed after their return at the BIPM.
The final resistance value attributed by the BIPM (the mean of the 'Before' and 'After' measurements) is in the middle of the step d: $d = |(R_{\mathrm{After}} - R_{\mathrm{Before}})|$

As we have no clear knowledge about the behaviour of the standards during the period between 'Before' and 'After', it is assumed that the actual resistance could have any value in the range d, with equal probability.

Assuming a rectangular probability distribution, $u_{\mathrm{d}} = \frac{d}{2} \cdot \frac{1}{\sqrt{3}}$

Another source of uncertainty associated with the transfer can be the difference in the operating currents used by the two laboratories, influencing the resistance of the standards through their power coefficients. In the present case, the nominal operating currents are identical (50 mA) in the two laboratories.
The standards being used with the same current, the influence of their power coefficient is assumed to be similar in both laboratories. The value of the relative standard uncertainty u_{P} associated with possible power effects is therefore estimated to be negligible.
For a single standard, the transfer uncertainty $u_{\mathrm{T},\,i}$ is obtained from: $u_{\mathrm{T},\,i}^2 = u_{\mathrm{d},\,i}^2 + u_{\mathrm{P},\,i}^2$

The $u_{\mathrm{d},\,i}$ are assumed to be uncorrelated.
Following the same reasoning as in expression (3), the uncertainty u_{T} associated with the transfer (for the mean of two standards) is:

$$u_{\mathrm{T}}^2 = \sum_{i=1}^{2} \frac{u_{\mathrm{d},\,i}^2}{2^2}$$

	Transfer	
Standard #	Drift u_{d} / 10^{-9}	Power u_{P} / 10^{-9}
BIV200	– 6	0.0
BIV203	– 5	0.0
Combined	4	0.0
Total u_{T}	4×10^{-9}	

Table 3: Uncertainty associated with the drift and the power coefficient of the standards.

Using the values of Table 3, the relative standard uncertainty u_{T} is:

$$u_{\mathrm{T}} = 4 \times 10^{-9}$$

3 Measurements at the NML

3.1 Measuring method:

The resistance of each travelling standard was measured by comparison with the NML 1 Ω reference standard (comprising a group of seven 1 Ω standard resistors).
The NML reference standard is maintained with respect to R_{K-90} by means of periodic comparisons with the BIPM and by extrapolation of the behaviour over time of its reference group.

The value assigned to the NML 1 Ω reference standard is the weighted mean of the values assigned to the resistances of the individual members of the group. Each assigned value is obtained from a model which incorporates the effects of temporal drift and temperature. The temporal drift parameters of the model, and their uncertainties, are estimated from a set of traceable values for the standard, obtained during previous comparisons with the BIPM. The uncertainty associated with each assigned value therefore includes the uncertainty of this historical data and the uncertainty due to the parameter fitting.

The comparison of the travelling standards (R_x) with the reference group was carried out using a substitution measuring technique. A direct current comparator resistance bridge (MIL Model 6010C) was used as a transfer standard.

3.2 Operating conditions:

At the time of the comparison, the NML oil-bath thermo-regulated at 23 °C was temporarily out of order. The travelling standards were placed in an oil-bath thermo-regulated at 20 °C and allowed to stabilise.

The actual temperature of the standards and the barometric pressure were recorded during each individual calibration.
Operating current: 50 mA dc.
Barometric pressure range: 989 hPa – 1019 hPa.

3.3 NML results:

The standards were measured 17 times in the period 4 October – 26 October 2010.
During the first part of this period, the travelling standards exhibited some instability, with changes of the order of a few parts in 10^8. Eight measurements have been used, recorded from the 14 to the 26 October, after this settling period.
The results are summarized in Table 4.

Serial No. of standard	Mean date of measurement	Resistance value (R_x – 1 Ω) / μΩ	Mean temperature / °C	Mean barometric pressure / hPa	Experimental std.dev. mean u_1 / 10^{-6}	Systematic u_2 / 10^{-6}
BIV200	19 October 2010	– 0.699	19.997	1013	0.003	0.054
BIV203	19 October 2010	+ 0.602	19.996	1013	0.004	0.054

Table 4: Summary of the NML calibrations. The relative standard uncertainty u_1 refers to the experimental standard deviation of the mean, and u_2 to the uncertainties listed in Table 7.

The NML results are corrected to the reference temperature and the reference pressure using the coefficients shown in Table 5.

	Relative temperature coefficients		Relative pressure coefficients.
Standard #	Alpha $_{23}$ / (10^{-6}/K)	Beta / (10^{-6}/K²)	/ (10^{-9}/hPa)
BIV200	– 0.0074	+ 0.0004	– 0.1
BIV203	– 0.010	– 0.0016	– 0.2

Table 5: Temperature and pressure coefficients of the travelling standards.

Standard #	Correction for temperature / 10^{-6}	Correction for pressure / 10^{-6}
BIV200	– 0.026	0.000
BIV203	– 0.016	0.000
After temperature and pressure corrections resistance value (R_x – 1 Ω) / μΩ		
BIV200	– 0.725	
BIV203	+ 0.586	

Table 6: NML calibration results after temperature and pressure correction

Owing to the small pressure coefficient and the negligible pressure correction, the associated uncertainty is assumed to be negligible.

Due to the significant temperature difference (20°C at the NML, 23°C at the BIPM), the uncertainty u_3 associated with temperature corrections is estimated to be 3 parts in 10^9.

Source of uncertainty	Relative standard uncertainty / 10^{-6}
NML 1 Ω reference group R_s	0.045
Ratio measurement R_x to R_s	0.030
Correction for temperature (applied by the BIPM)	
Combined (sum in quadrature): u_2	0.054

Table 7: Summary of the NML uncertainty budget (simplified model)

For a single standard, the NML uncertainty $u_{NML,\,i}$ is obtained from: $u_{NML,\,i}^2 = u_{1,\,i}^2 + u_{2,\,i}^2$

The $u_{2,\,i}$ are assumed to be correlated, unlike the $u_{1,\,i}$.

Using expression (2), when the mean (for two standards) of the NML-BIPM relative difference is calculated, the NML contribution to the uncertainty is:

$$u_{\mathrm{NML}}^2 = \sum_{i=1}^{2} \frac{u_{1,\,i}^2}{2^2} + u_2^2 + u_3^2 \qquad (5)$$

Using the values shown in Table 4, the relative standard uncertainty u_{NML} is

$u_{\mathrm{NML}} = 0.054 \times 10^{-6}$.

4 Comparison NML – BIPM

The differences between the values assigned by the NML at the NML, R_{NML}, and those assigned by the BIPM at the BIPM, R_{BIPM}, to each of the two travelling standards during the period of the comparison are shown in Table 8.

NML - BIPM	
Standard #	$(R_{\mathrm{NML}} - R_{\mathrm{BIPM}})$ / (1 Ω) / 10^{-6}
BIV200	0.077
BIV203	0.125
mean	**+ 0.101**

Table 8: Differences between the values assigned by the NML (R_{NML}) and by the BIPM (R_{BIPM}) to the two travelling standards.

The mean difference between the NML and the BIPM calibrations is:

$$(R_{\mathrm{NML}} - R_{\mathrm{BIPM}}) / (1\ \Omega) = +\,0.101 \times 10^{-6}$$

The relative combined standard uncertainty of the comparison, u_{C}, is:

$$u_{\mathrm{C}}^2 = u_{\mathrm{BIPM}}^2 + u_{\mathrm{NMLI}}^2 + u_{\mathrm{T}}^2$$

where u_{BIPM} = 0.016 x 10^{-6},
u_{NML} = 0.054 x 10^{-6},
u_{T} = 0.004 x 10^{-6}
as calculated in sections 2 and 3: u_{C} = 0.057 x 10^{-6}

The final result of the comparison is presented as the degree of equivalence D between the NML and the BIPM for values assigned to 1 Ω resistance standards, and its expanded relative uncertainty (expansion factor k = 2, corresponding to a confidence level of 95 %) , U_{C}

$$\boldsymbol{D = [(R_{\mathrm{NML}} - R_{\mathrm{BIPM}}) / 1\ \Omega] = +\,0.101 \times 10^{-6}}$$

$$\boldsymbol{U_{\mathrm{C}} = 0.11 \times 10^{-6}}$$

The difference between the NSAI-NML and the BIPM calibrations lies within the expanded uncertainty.

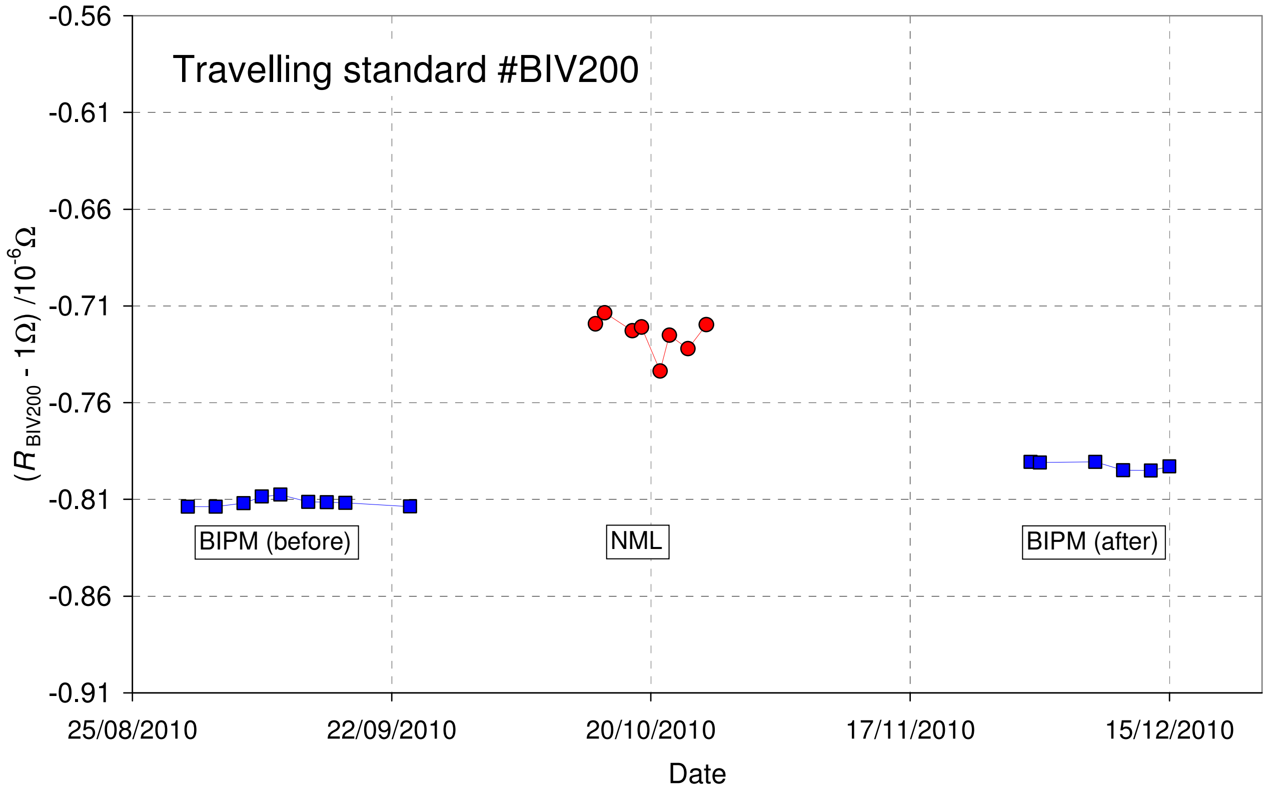

Figure 1: Calibrations at the BIPM (squares) and at the NML (circles) of the travelling standard ref. BIV200, expressed as the relative deviation from the nominal 1 Ω value.

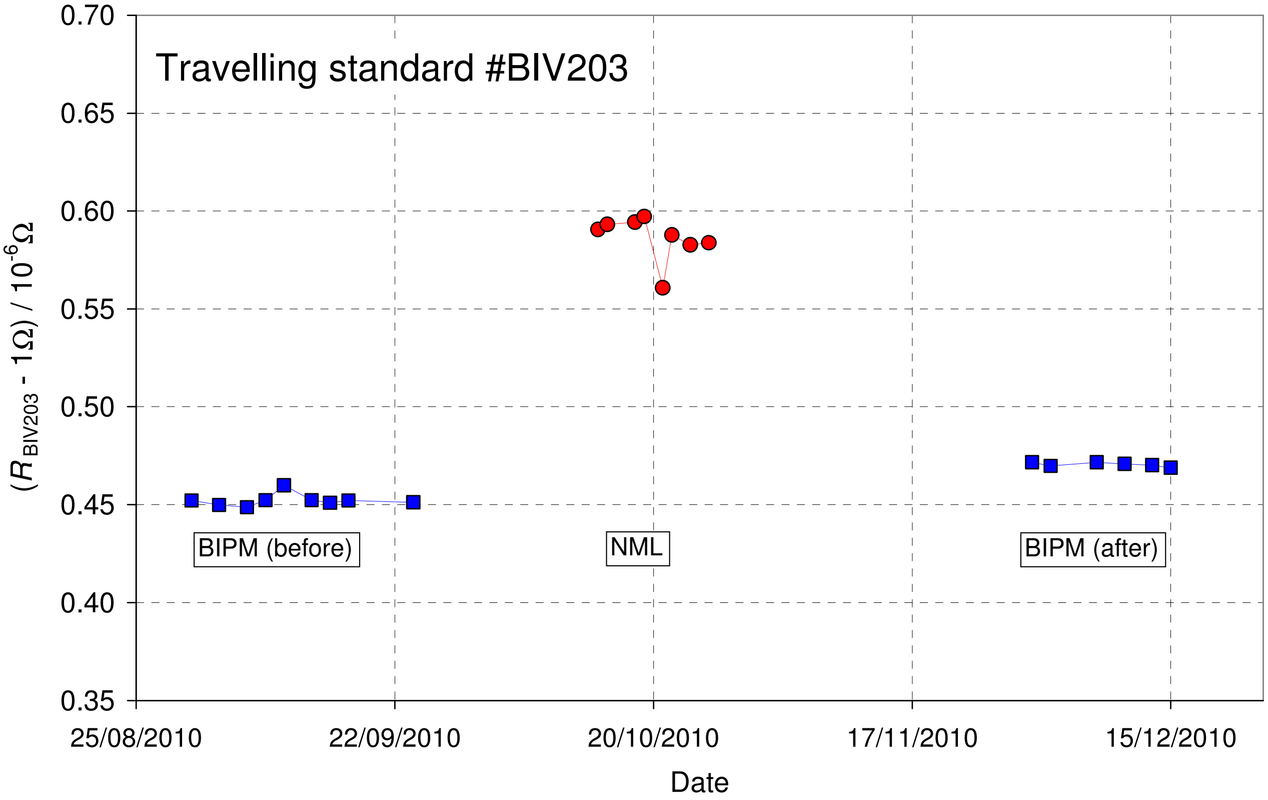

Figure 2: Calibrations at the BIPM (squares) and at the NML (circles) of the travelling standard ref. BIV203, expressed as the relative deviation from the nominal 1 Ω value.