

**Bilateral Comparison of 1.018 V and 10 V Standards
between the SMD (Belgium) and the BIPM,
November to December 2024
(part of the ongoing BIPM key comparison BIPM.EM-K11.a and b)**

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Introduction

As part of the ongoing BIPM key comparison BIPM.EM-K11.a and b, a comparison of the 1.018 V and 10 V voltage reference standards of the BIPM and the *Service de Métrologie* (SMD), Brussel, Belgium, was carried out from November to December 2024. Two BIPM Zener diode-based travelling standards (Fluke 732A), BIPM_1 (Z1) and BIPM_H (ZH), were transported to SMD and back to BIPM by road. Since the laboratories are close to each other, Fluke 732A standards that have a long metrological history were selected to serve as transfer standards. In order to keep the Zeners powered during their transportation and during long measurement series, a dedicated auxiliary battery supply was designed and connected in parallel to the internal battery.

At SMD, the reference standard for DC voltage is a Programmable Josephson Voltage Standard (PJVS). The output electromotive force (EMF) of each travelling standard was measured by direct comparison with the primary standard.

At the BIPM, the output EMF of each travelling standard was calibrated before and after the measurements at SMD against the PJVS developed at the BIPM around a PTB programmable SNS (Superconductor/Normal Metal/Superconductor) array.

Results of all measurements were corrected by the BIPM for the dependence of the output voltages of the Zener standards on ambient atmospheric pressure.

Outline of the measuring method

SMD 1.018 V and 10 V measurements

At SMD, the reference standard for DC voltage is a 10 V Programmable Josephson Voltage Standard (PJVS) [1], fabricated by *Supracon AG**. It is controlled by a computer using the AC *SupraVOLT* control software [1, 2], and is used to generate programmable DC quantum voltages. The output electromotive force of each travelling standard was measured by direct comparison with the primary standard.

Each output terminal of the travelling standards was connected in series opposition to the PJVS using a low thermal EMF switch equipped with three different channels, which allows polarity reversals. The EMF differences between the standard and the PJVS array were measured using a digital nanovoltmeter *Keithley 2182A* operated on its 10 mV range and were automatically collected by the computer.

Each day, 8 data points were taken consecutively. The simple mean value was considered as the result of the day. Each individual data point represents the mean of 40 measurements (20 in positive and 20 in negative polarity). The same 40 measurements were taken at ± 0.1 mA to ensure the voltage step flatness to better than 5 m Ω . The nanovoltmeter input was shorted before each polarity reversal and restored right after.

Frequency and power of the microwave irradiating the PJVS array were set to maximize the quantum operating margins and were kept fixed throughout the whole measurement. The PJVS array was programmed to generate the closest quantum voltage level to the output voltage of the standard under measurement. Since the voltage resolution of the PJVS array is approximately equal to 144 μ V, the magnitude of the voltage recorded by the nanovoltmeter was always lower than 73 μ V.

The standards were disconnected from the mains power about two and a half hours before and remained disconnected throughout the entire measurement session, in order for the standard to let the internal temperature stabilize. The “GUARD” and “CHASSIS” binding posts were jointly connected to the common ground point of the setup which was the Earth potential. In order to quantify the effect of earthing or not the measurement setup, a series of measurements was performed after each “official” day measurement without setting the common point to the Earth potential. The results are reported in Annex 1. The internal

* Certain commercial equipment, instruments, or materials are identified in this paper to facilitate understanding. Such identification does not imply recommendation or endorsement by BIPM and SMD, nor does it imply that the materials or equipment that are identified are necessarily the best available for the purpose.

thermistor resistance was measured with a *Fluke 8508A* digital multimeter, operated on its 20 k Ω range, with a current of 10 μ A and recorded prior to the start of the measurements. The pressure was recorded at the same time using a portable gauge. The participant full report is available in Annex 2.

BIPM Measurements for 1.018 V and 10 V

The output voltage of the Zener standard to be measured was connected in series opposition to the BIPM Programmable Josephson Voltage Standard - PTB 10 V SNS array (S/N: 2013-02/4a) [3], through a low thermal EMF multiplexer [4, 5]. The binding post terminals “GUARD” and “CHASSIS” of the Zener standard were connected together and connected to a single point which is the grounding reference point of the measurement setup.

The measurements started at least two hours after the mains plug at the rear of the Zeners had been disconnected in order for the Zener internal temperature to stabilize. In this comparison, the BIPM detector was a digital nanovoltmeter *Keithley 2182A* operated on its 10 mV range. A computer was used to monitor, record the measurements, acquire the data, correct for pressure dependence, and calculate results.

The BIPM array biasing frequency was adjusted in such a way that the voltage difference between the primary and the secondary voltage standards was always below 1 μ V for both nominal voltages. In such a case, the nanovoltmeter gain error doesn't affect the measurement result and the corresponding uncertainty can also be neglected.

One individual measurement point was acquired according to the following:

- 1- The Zener and the BIPM array are set in their positive polarity, connected in series opposition and the detector data reading sequence starts;
- 2- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 1;
- 3- The polarity of the detector is reversed again to match the conditions of step 1 and the reading sequence restarts;
- 4- The Zener and the BIPM array are set in their negative polarity, connected in series opposition and the detector data reading sequence starts;
- 5- The polarity of the detector is reversed and a reading sequence is carried out. The number of measurements is twice the number acquired in step 4;
- 6- The polarity of the detector is reversed again to match the conditions of step 4 and the reading sequence restarts.

The reversal of the array polarity (by reversing the bias current) is always accompanied by a reversal of the Zener voltage standard using the multiplexer. The reversal of the detector polarity is done to cancel out any internal detector thermal EMF with a constant drift rate. Each data acquisition step consists of 50 preliminary measurements followed by 100 measurements. Each of these should not differ from the mean of the preliminary measurements by more than four times their standard deviation. If so, the software warns the operator with a beep. If many beeps occur, it means that the Zener output value has changed. The decision on restarting the “Data Acquisition” step in progress is based on considering the graphical representation of the measurements on the computer screen. The procedure to acquire one individual measurement point is repeated five times in a row and the mean value corresponds to one result on the graph (cf. Fig. 1, 2, 3, and 4).

Results at 10 V

Figure 1 shows the measured values obtained for the two standards by the two laboratories at 10 V. Figure 2 presents the voltage evolution of the arithmetic mean of the two standards which is used to compute the final result at 10 V. A linear least squares fit is applied to all of the individual BIPM results, and to the mean value of both transfer standards. The comparison result is the voltage difference between the BIPM fitted value at the mean date of the SMD measurements (06/12/2024) and the mean value of the SMD measurements, and the related uncertainties.

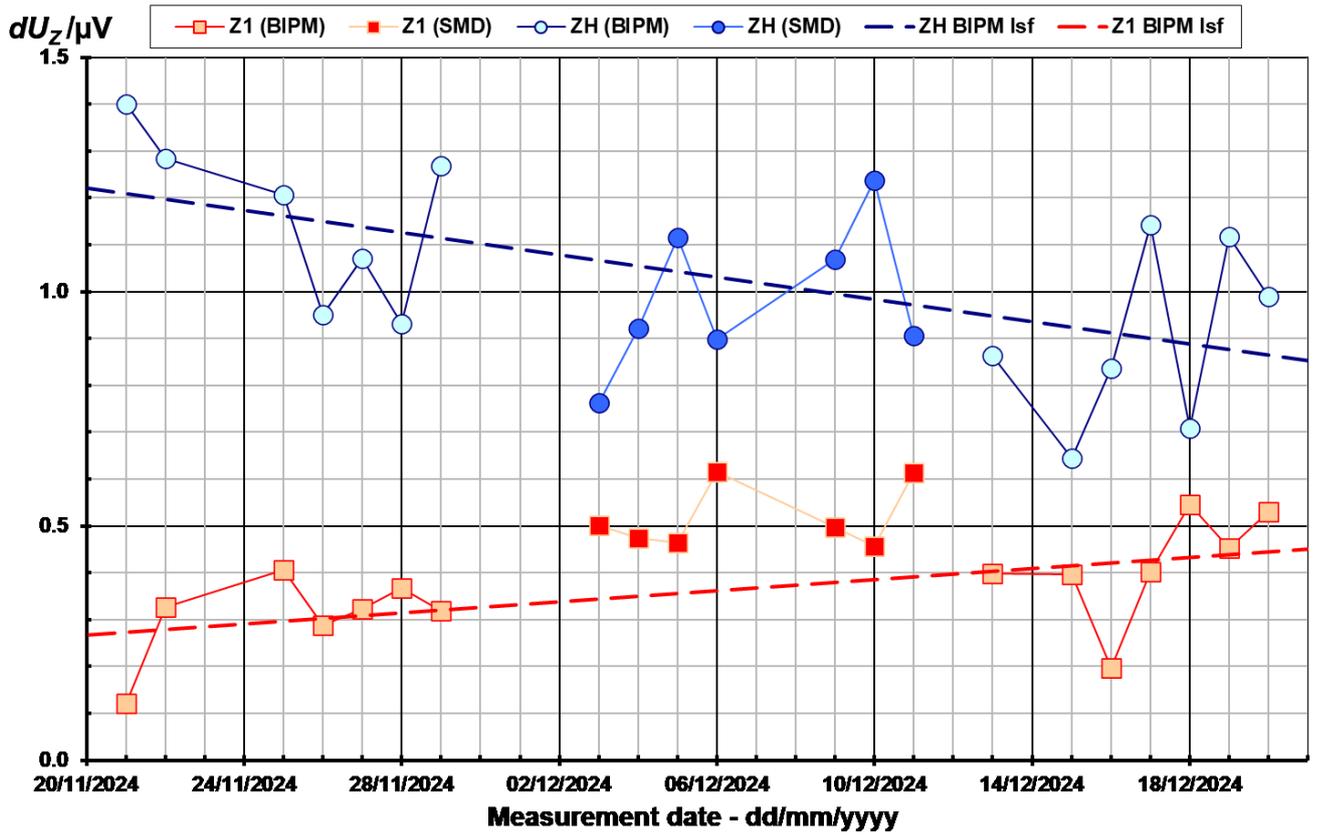


Figure 1: Voltage of Z1 (squares) and ZH (disks) at 10 V measured at both institutes (light markers for BIPM and dark markers for SMD), referred to an arbitrary offset, as a function of the measurement date with a linear least-squares fit (lsf) to the BIPM measurements.

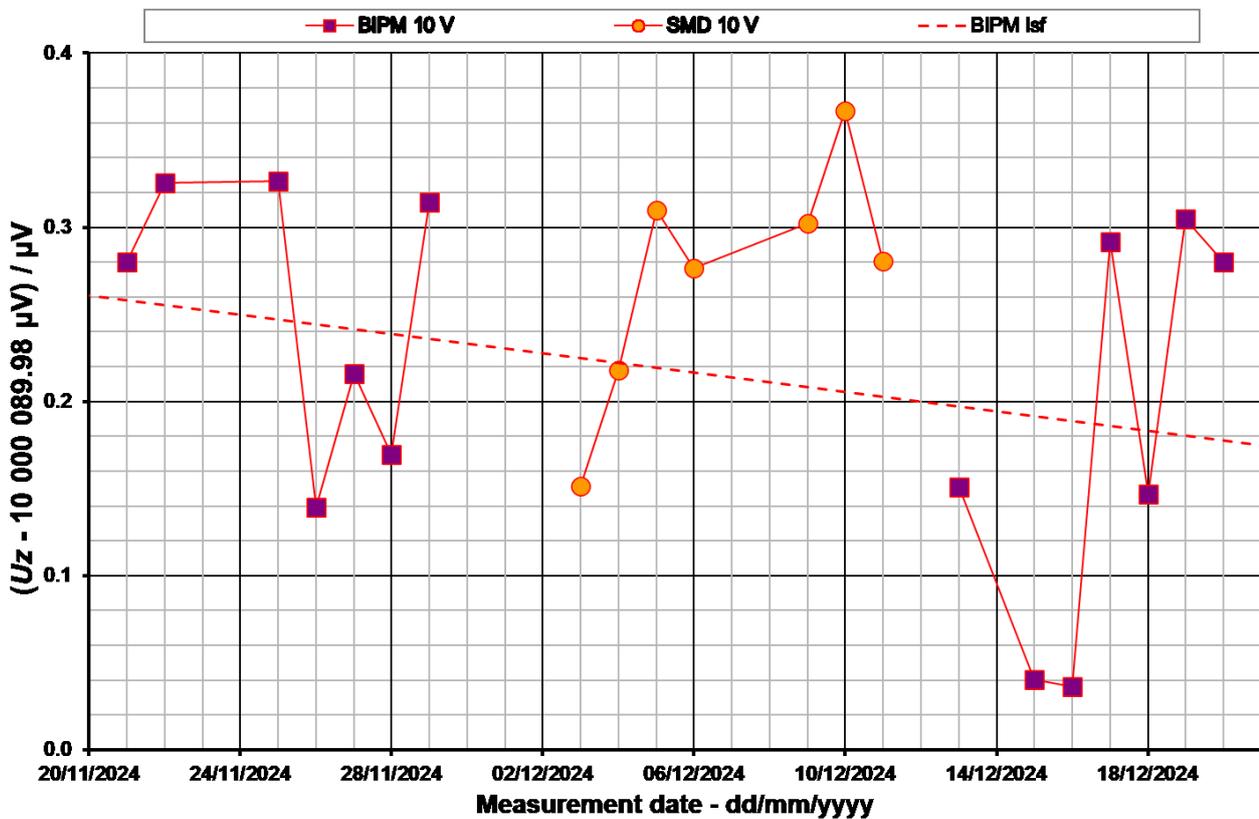


Figure 2: Voltage evolution of the arithmetic mean of the two standards at 10 V. SMD measurements are represented by disks and BIPM measurements by squares. A least-squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 10 V

BIPM uncertainty budget at 10 V

Table 1 summarizes the uncertainties related to the calibration of a Zener against the PJVS at the BIPM at the level of 10 V.

Experience has shown that flicker or $1/f$ noise ultimately limits the stability characteristics of Zener standards and it is not appropriate to use the standard deviation divided by the square root of the number of observations to characterize the dispersion of measured values. For the present standards, the relative value of the voltage noise floor due to flicker noise is about 1 part in 10^8 [6]. The Type A standard uncertainty in Table 1 therefore has a lower limit of 100 nV. However, if the standard deviation of the measurements at the mean date of the participant is larger than the flicker noise floor, it is this standard deviation which is considered to be the Type A standard uncertainty.

| PJVS & detector uncertainty components | Uncertainty (nV) |
|--|---|
| Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A) | 2 |
| Detector gain (Type B) | negligible |
| Leakage resistance (Type B) | 4 |
| Frequency (Type B) | 0.1 |
| Zener noise (Type A) | Not lower than the $1/f$ noise estimated as 100 nV, included in the comparison uncertainty budget (Table 3) and Annex 3 |
| Zener pressure correction | Included in the comparison uncertainty budget (Table 3) |

Table 1: Estimated standard uncertainties arising from the PJVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 10 V.

SMD uncertainty budget at 10 V

Tables 2a and 2b list the uncertainties related to the calibration of the Zeners at SMD for Z1 and ZH, respectively.

Note that the uncertainty of the pressure correction (in italics) is given as an indication only and does not contribute to the final uncertainty budget used for this comparison as it is applied by the BIPM and included in the comparison uncertainty budget (Table 3).

At the BIPM, the built-in thermistor is used to characterize the internal temperature of the Zeners. The thermistor sensitivity of the Fluke 732A instrument model is not sufficient to determine a temperature coefficient. As a consequence, no temperature correction was applied.

| Quantity | Estimate | Type | Dist. | Standard uncertainty | Sensitivity | Uncertainty contribution | Degree of Freedom |
|---|---|----------|--------------|------------------------|---------------------|--------------------------|-------------------|
| Difference voltage measured with the nanovoltmeter | 55.41 μV | A | Norm. | 25.76 nV | 1 | 25.76 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.22 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Rect. | 1×10^{-5} | 55.41 μV | 0.55 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| <i>Pressure coefficient of the Zener</i> | <i>1021.5 hPa</i> | <i>B</i> | <i>Rect.</i> | <i>6.3 hPa</i> | <i>-1.26 nV/hPa</i> | <i>7.93 nV</i> | <i>14</i> |
| [7] | Combined uncertainty $u(U_Z) = 27 \text{ nV}$ | | | | | | |
| | Relative combined uncertainty $u(U_Z) / U_Z = 2.7 \text{ nV/V}$ | | | | | | |
| | Effective degrees of freedom $\nu_{eff} = 8$ | | | | | | |
| | Coverage factor $k_{0.9545} = 2.37$ | | | | | | |
| | Expanded uncertainty (95.45%) $U(U_Z) = k_{0.9545} \times u(U_Z) = 64 \text{ nV}$ | | | | | | |

Table 2a: Estimated standard uncertainties of U_Z for a Zener calibration with the SMD equipment at the level of 10 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard uncertainty | Sensitivity | Uncertainty contribution | Degree of freedom |
|---|--|------|-------|------------------------|---------------------|--------------------------|-------------------|
| Difference voltage measured with the nanovoltmeter | -62.08 μV | A | Norm. | 59.02 nV | 1 | 59.02 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.22 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Rect. | 1×10^{-5} | 62.08 μV | 0.62 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | -0.15 nV/hPa | 0.94 nV | 14 |
| [7] | Combined uncertainty $u(U_Z) = 60 \text{ nV}$ | | | | | | |
| | Relative combined uncertainty $u(U_Z) / U_Z = 6.0 \text{ nV/V}$ | | | | | | |
| | Effective degrees of freedom $\nu_{eff} = 6$ | | | | | | |
| | Coverage factor $k_{0.9545} = 2.52$ | | | | | | |
| | Expanded uncertainty (95.45%) $U(U_Z) = k_{0.9545} \times u(U_Z) = 151 \text{ nV}$ | | | | | | |

Table 2b: Estimated standard uncertainties of U_Z for a Zener calibration with the SMD equipment at the level of 10 V for Zener ZH.

Uncertainty contributions for the comparison SMD/BIPM at 10 V

Table 3 lists the results and the uncertainty contributions for the comparison SMD/BIPM at 10 V.

| | | Results/ μV | | Uncertainty/ μV | |
|----|---|------------------------|--------|----------------------------|-------|
| | | Z1 | ZH | Z1 | ZH |
| 1 | SMD ($U_{\text{SMD}} - 10 \text{ V}$) | 76.52 | 103.99 | | |
| 2 | Type A uncertainty | | | 0.026 | 0.059 |
| 3 | correlated (Type B) uncertainty | | | 0.009 | |
| 4 | BIPM ($U_{\text{BIPM}} - 10 \text{ V}$) | 76.37 | 104.02 | | |
| 5 | Type A uncertainty | | | 0.100 | 0.100 |
| 6 | correlated (Type B) uncertainty | | | <0.005 | |
| 7 | pressure correction uncertainty | | | 0.003 | 0.002 |
| 8 | ($U_{\text{SMD}} - U_{\text{BIPM}}$) | 0.15 | -0.03 | | |
| 9 | Total uncorrelated uncertainty | | | 0.103 | 0.116 |
| 10 | Total correlated uncertainty | | | 0.009 | |
| 11 | < $U_{\text{SMD}} - U_{\text{BIPM}}$ > | 0.06 | | | |
| 12 | <i>a priori</i> uncertainty | | | 0.078 | |
| 13 | <i>a posteriori</i> uncertainty | | | 0.090 | |
| 14 | comparison total standard uncertainty/μV | | | 0.09 | |

Table 3: Results and uncertainties of SMD (Belgium)/BIPM bilateral comparison of 10 V standards using two Zener travelling standards: reference date 6 December 2024. Standard uncertainties are used throughout.

In Table 3, the following elements are listed:

- (1) the value attributed by SMD to each Zener, U_{SMD} , computed as the arithmetic mean of all data from SMD and corrected for pressure differences between both laboratories by the BIPM.
- (2) SMD combined Type A uncertainty (cf. Tables 2a and 2b).
- (3) the uncertainty component arising from the realization and maintenance of the volt at SMD: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 2a and 2b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of the SMD measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 100 nV.

(7) the uncertainty due to the effects of the pressure coefficients [8] and to the differences of the mean pressures in the participating laboratories is calculated as follows:

The uncertainty of the pressure correction $u_{P,i}$ of Zener i is determined for the difference ΔP_i between the mean values of the pressures measured at both institutes which is then multiplied by the uncertainty $u(c_{P,i})$ of the relative pressure coefficients of each Zener standard:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where $U = 10 \text{ V}$, $u(c_{P,Z1}) = 0.038 \times 10^{-9} / \text{hPa}$, $u(c_{P,ZH}) = 0.025 \times 10^{-9} / \text{hPa}$, $\Delta P_{Z1} = 9.0 \text{ hPa}$ and $\Delta P_{ZH} = 9.1 \text{ hPa}$.

The uncertainty of the pressure measurement is negligible.

(8) the difference ($U_{\text{SMD}} - U_{\text{BIPM}}$) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the arithmetic mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

(12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;

(13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and the larger of (12) and (13).

To estimate the uncertainty related to the stability of the standards during transportation, we have calculated the “*a priori*” uncertainty of the mean of the results obtained for the two standards (also called statistical internal consistency). It consists of the quadratic combination of the uncorrelated uncertainties of each result. We compared this component to the “*a posteriori*” uncertainty (also called statistical external consistency) which consists of the experimental standard deviation of the mean of the results from the two travelling standards*.

If the “*a posteriori*” uncertainty is significantly larger than the “*a priori*” uncertainty, we assume that a standard has changed in an unusual way, probably during their transportation. This is not the case at the 10 V output. We use the larger of these two estimates in calculating the final uncertainty.

The comparison result is presented as the difference between the value assigned to a 10 V standard by SMD, at SMD, U_{SMD} , and that assigned by the BIPM, at the BIPM, U_{BIPM} , on the reference date of the 6th of December 2024:

$$U_{\text{SMD}} - U_{\text{BIPM}} = 0.06 \mu\text{V}; \quad u_c = 0.09 \mu\text{V}$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at SMD, at the BIPM, and the uncertainty related to the comparison.

* With only two travelling standards, the uncertainty of the standard deviation of the mean is comparable to the value of the standard deviation of the mean itself.

Results at 1.018 V

Figure 3 shows the measured values obtained for the two standards by the two laboratories at 1.018 V and Figure 4 presents the voltage evolution of the arithmetic mean of the two standards which is used to compute the final result at 1.018 V.

A linear least squares fit is applied to the results of the BIPM, before and after the measurements at SMD, to obtain the results for both standards and their uncertainties at the mean date of the SMD measurements (06/12/2024).

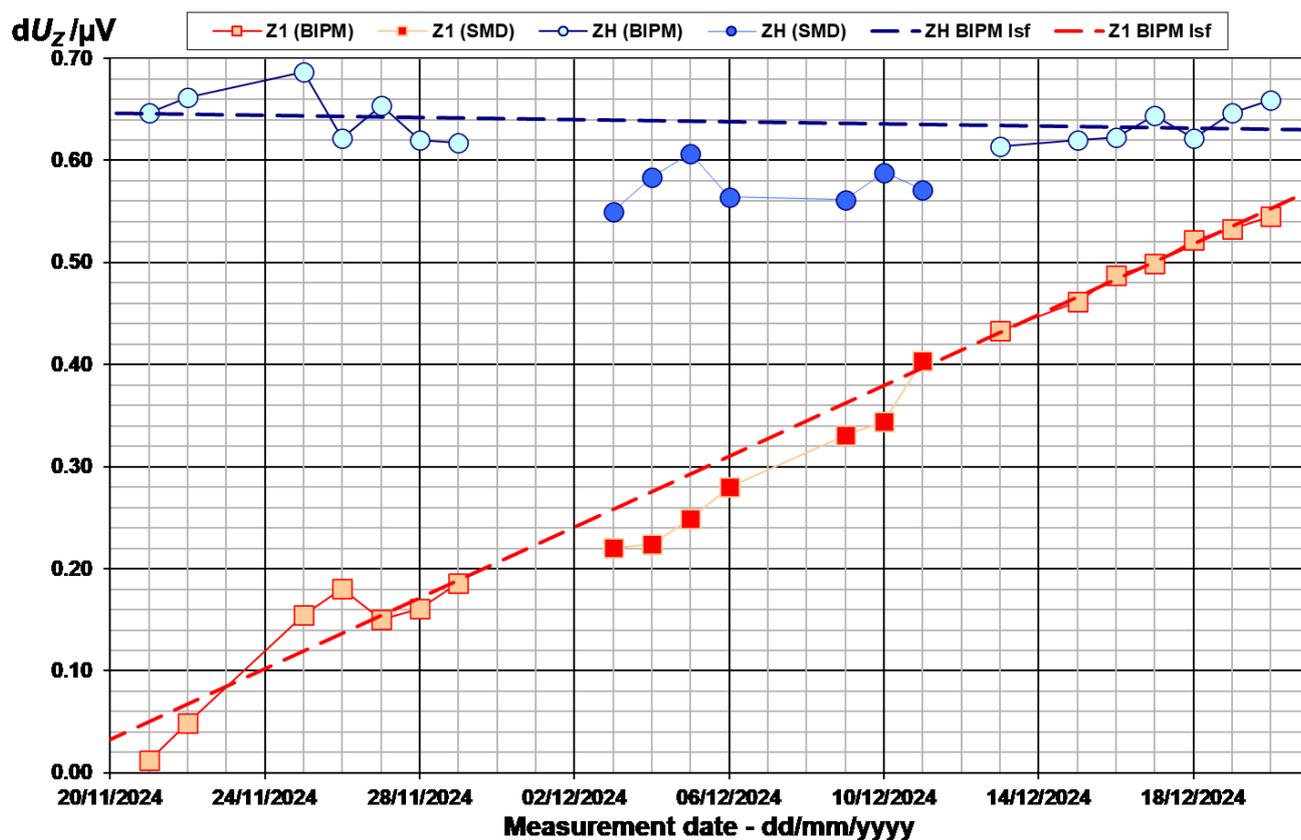


Figure 3: Voltage of Z1 (squares) and ZH (disks) at 1.018 V measured at both institutes (light markers for BIPM and dark markers for SMD), referred to an arbitrary offset, as a function of the measurement date with a linear least-squares fit (Isf) to the BIPM measurements.

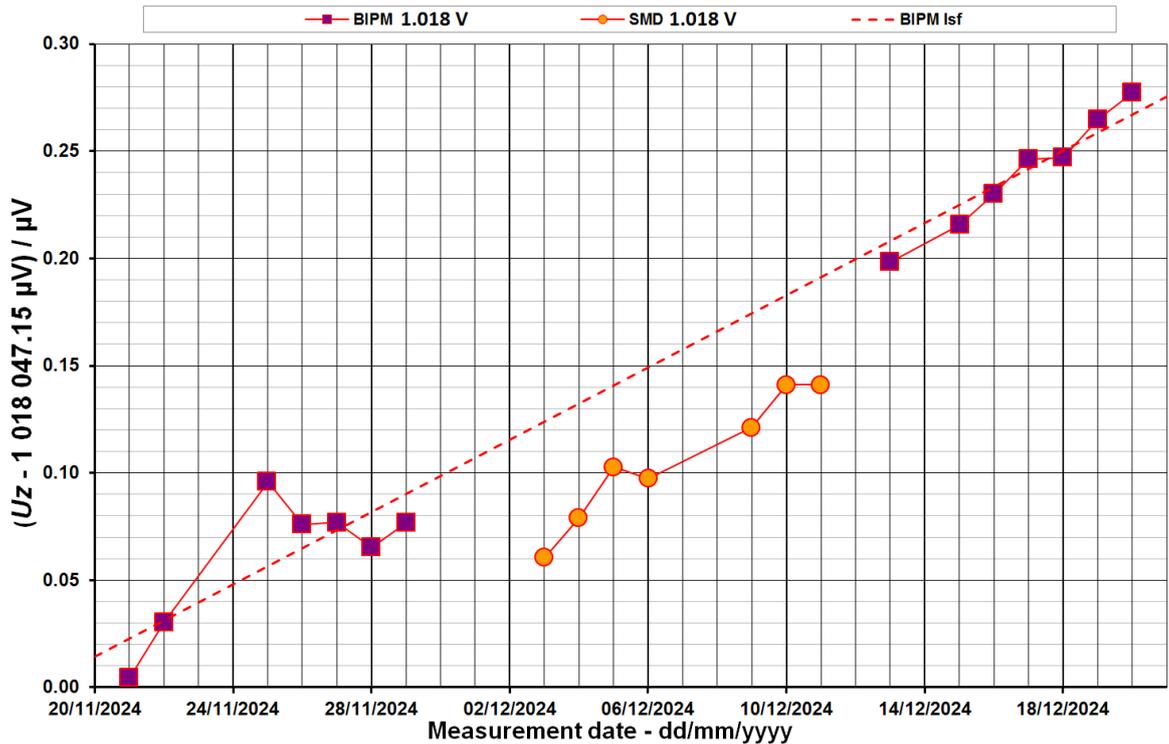


Figure 4: Voltage evolution of the arithmetic mean of the two standards at 1.018 V. SMD measurements are represented by disks and BIPM measurements by squares. A least-squares fit is applied to the BIPM measurements.

Uncertainty Budgets at 1.018 V

BIPM uncertainty budget at 1.018 V

Table 4 summarizes the uncertainties related to the calibration of a Zener against the Programmable Josephson Voltage Standard at the BIPM at the level of 1.018 V.

| PJVS & detector uncertainty components | Uncertainty (nV) |
|--|--|
| Noise of the measurement loop that includes the residual thermal EMF including the residual EMF of the reversing switch (Type A) | 2 |
| Detector gain (Type B) | negligible |
| Leakage resistance (Type B) | 0.4 |
| Frequency (Type B) | 0.01 |
| Zener noise (Type A) | Not lower than the 1/f noise estimated as 10 nV, included in the comparison uncertainty budget (Table 6) |
| Zener pressure and temperature correction | Included in the comparison uncertainty budget (Table 6) |

Table 4: Estimated standard uncertainties arising from the PJVS and the measurement setup for Zener calibrations with the BIPM equipment at the level of 1.018 V.

SMD uncertainty budget at 1.018 V

Tables 5a and 5b list the uncertainties related to the calibration of the Zeners at SMD for Z1 and ZH, respectively.

Note that the uncertainty of the pressure correction (in italics) is given as an indication only and does not contribute to the final uncertainty budget used for this comparison as it is applied by the BIPM and included in the comparison uncertainty budget (Table 6).

At the BIPM, the built-in thermistor is used to characterize the internal temperature of the Zeners. The thermistor sensitivity of the Fluke 732A instrument model is not sufficient to determine a temperature coefficient. As a consequence, no temperature correction was applied.

| Quantity | Estimate | Type | Dist. | Standard uncertainty | Sensitivity | Uncertainty contribution | Degree of freedom |
|---|---|----------|--------------|------------------------|---------------------|--------------------------|-------------------|
| Difference voltage measured with the nanovoltmeter | 49.60 μV | A | Norm. | 25.86 nV | 1 | 25.86 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.5 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Rect. | 1×10^{-5} | 49.60 μV | 0.50 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| <i>Pressure coefficient of the Zener</i> | <i>1021.5 hPa</i> | <i>B</i> | <i>Rect.</i> | <i>6.3 hPa</i> | <i>-0.09 nV/hPa</i> | <i>0.57 nV</i> | <i>14</i> |
| [7] | Combined uncertainty $u(U_Z) = 27 \text{ nV}$ | | | | | | |
| | Relative combined uncertainty $u(U_Z) / U_Z = 27 \text{ nV/V}$ | | | | | | |
| | Effective degrees of freedom $\nu_{eff} = 8$ | | | | | | |
| | Coverage factor $k_{0.9545} = 2.37$ | | | | | | |
| | Expanded uncertainty (95.45%) $U(U_Z) = k_{0.9545} \times u(U_Z) = 64 \text{ nV}$ | | | | | | |

Table 5a: Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard uncertainty | Sensitivity | Uncertainty contribution | Degree of freedom |
|---|---|------|-------|------------------------|--------------|--------------------------|-------------------|
| Difference voltage measured with the nanovoltmeter | 4.84 μ V | A | Norm. | 7.31 nV | 1 | 7.31 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.5 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Rect. | 1×10^{-5} | 4.84 μ V | 0.05 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | -0.15 nV/hPa | 0.94 nV | 14 |
| [7] | Combined uncertainty $u(U_z) = 12$ nV | | | | | | |
| | Relative combined uncertainty $u(U_z) / U_z = 12$ nV/V | | | | | | |
| | Effective degrees of freedom $\nu_{eff} = 38$ | | | | | | |
| | Coverage factor $k_{0.9545} = 2.07$ | | | | | | |
| | Expanded uncertainty (95.45%) $U(U_z) = k_{0.9545} \times u(U_z) = 24$ nV | | | | | | |

Table 5b: Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener ZH.

Uncertainty contributions for the comparison SMD/BIPM at 1.018 V

Table 6 lists the results and the uncertainty contributions for the comparison SMD/BIPM at 1.018 V.

| | | Results/ μV | | Uncertainty/ μV | |
|----|---|------------------------|-------|----------------------------|-------|
| | | Z1 | ZH | Z1 | ZH |
| 1 | SMD ($U_{\text{SMD}} - 1.018 \text{ V}$) | 69.94 | 24.88 | | |
| 2 | Type A uncertainty | | | 0.026 | 0.007 |
| 3 | correlated (Type B) uncertainty | | | 0.009 | |
| 4 | BIPM ($U_{\text{BIPM}} - 1.018 \text{ V}$) | 69.97 | 24.94 | | |
| 5 | Type A uncertainty | | | 0.010 | 0.010 |
| 6 | correlated (Type B) uncertainty | | | <0.005 | |
| 7 | pressure correction uncertainty | | | 0.001 | 0.000 |
| 8 | ($U_{\text{SMD}} - U_{\text{BIPM}}$) | -0.03 | -0.06 | | |
| 9 | Total uncorrelated uncertainty | | | 0.028 | 0.012 |
| 10 | Total correlated uncertainty | | | 0.009 | |
| 11 | $\langle U_{\text{SMD}} - U_{\text{BIPM}} \rangle$ | -0.047 | | | |
| 12 | <i>a priori</i> uncertainty | | | 0.015 | |
| 13 | <i>a posteriori</i> uncertainty | | | 0.015 | |
| 14 | comparison total standard uncertainty/μV | | | 0.017 | |

Table 6: Results and uncertainties of SMD (Belgium)/BIPM bilateral comparison of 1.018 V standards using two Zener travelling standards: reference date 6 December 2024. Standard uncertainties are used throughout.

In Table 6, the following elements are listed:

- (1) the value attributed by SMD to each Zener U_{SMD} , computed as the arithmetic mean of all data from SMD and corrected for pressure differences between both laboratories by the BIPM.
- (2) the SMD Type A uncertainty (cf. Tables 5a and 5b).
- (3) the uncertainty component arising from the realization and maintenance of the volt at SMD: it is the quadratic combination of the Type B components of the participant uncertainty budget listed in Tables 5a and 5b. This uncertainty is completely correlated between the different Zeners used for the comparison.

(4-6) the corresponding quantities for the BIPM referenced to the mean date of SMD measurements. In this case, the Type A uncertainty is limited by the flicker noise level of 10 nV.

(7) the uncertainty due to the effects of the pressure coefficients [7] and to the differences of the mean pressures in the participating laboratories is calculated as follows:

The uncertainty of the pressure correction $u_{P,i}$ of Zener i is determined for the difference ΔP_i between the mean values of the pressures measured at both institutes which is then multiplied by the uncertainty $u(c_{P,i})$ of the relative pressure coefficients of each Zener standard:

$$u_{P,i} = U \times u(c_{P,i}) \times \Delta P_i$$

where $U = 1.018 \text{ V}$, $u(c_{P,Z1}) = 0.083 \times 10^{-9} / \text{hPa}$, $u(c_{P,ZH}) = 0.035 \times 10^{-9} / \text{hPa}$, $\Delta P_{Z1} = 9.0 \text{ hPa}$ and $\Delta P_{ZH} = 8.9 \text{ hPa}$.

The uncertainty of the pressure measurement is negligible.

(8) the difference ($U_{\text{SMD}} - U_{\text{BIPM}}$) for each Zener, and (9) the uncorrelated part of the uncertainty, calculated as the quadratic sum of lines 2, 5 and 7.

(10) the correlated part of the uncertainty, calculated as the quadratic sum of lines 3 and 6, for each travelling standard.

(11) the result of the comparison is the arithmetic mean of the differences of the calibration results for the different standards.

(12 and 13) the uncertainty related to the transfer, estimated by comparing the following uncertainties:

(12) the *a priori* uncertainty, determined as the standard uncertainty of the mean, obtained by propagating the uncorrelated uncertainties for both Zeners;

(13) the *a posteriori* uncertainty, which is the standard deviation of the mean of the two results.

(14) the total uncertainty of the comparison, which is the root sum square of the correlated part of the uncertainty (10) and the larger of (12) and (13).

In this case the *a priori* uncertainty is comparable to the *a posteriori* uncertainty. We conclude that at 1.018 V both Zeners behaved consistently within the uncertainty of the comparison.

The result of the comparison is presented as the difference between the value assigned to a 1.018 V standard by SMD, at SMD, U_{SMD} , and that assigned by the BIPM, at the BIPM, U_{BIPM} , on the reference date of the 6th of December 2024:

$$U_{\text{SMD}} - U_{\text{BIPM}} = -0.047 \mu\text{V}; \quad u_c = 0.017 \mu\text{V}$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at the BIPM and at SMD and the uncertainty related to the comparison.

Conclusion

The final result of the comparison is presented as the difference between the values assigned to DC voltage standards by SMD, at the level of 1.018 V and 10 V, at SMD, U_{SMD} , and those assigned by the BIPM, at the BIPM, U_{BIPM} , at the reference date of the 6th of December 2024.

$$U_{\text{SMD}} - U_{\text{BIPM}} = -0.047 \mu\text{V}; \quad u_c = 0.017 \mu\text{V}, \text{ at } 1.018 \text{ V}$$

$$U_{\text{SMD}} - U_{\text{BIPM}} = 0.06 \mu\text{V}; \quad u_c = 0.09 \mu\text{V}, \text{ at } 10 \text{ V}$$

where u_c is the combined standard uncertainty associated with the measured difference, including the uncertainty of the realization of the volt at the BIPM and at SMD, and the uncertainty related to the comparison.

These are very good results for both nominal voltages. The comparison results show that the voltage standards maintained by SMD and the BIPM are equivalent. However, the result at 1.018 V is just outside the confidence interval for $k = 2$. Since the SMD results for both Zeners at 1.018 V show a similar deviation from the BIPM linear fit (applied to the measurement before and after the comparison), we might expect a small systematic error in the SMD measurement setup. This assumption is confirmed by the systematic error brought to the 1.018 V measurements when the standards chassis and guard binding posts were not connected to the Earth potential (see Annex 1).

Since the two laboratories are located within a short distance, it was decided to implement Fluke 732A type rather than Fluke 732B type as travelling standards. The older model is known to have a greater voltage stability and from the BIPM measurements only we could evaluate a dispersion of the measurements to be 30 % less than for the 732B type, based on the last bilateral comparison with SMD (2021). However (and unfortunately), it is impossible to envisage the transportation of such old standards over longer distances.

Annex 1: Effect of grounding the SMD measurement setup to the earth potential

The following graphs (Figures A1-1 and A1-2) present the SMD measurements when the standards chassis and guard binding posts were not connected to the Earth potential. Each measurement point is performed applying the comparison protocol except that the chassis and guard of the Zener standards are not connected to the Earth potential.

An average systematic error of 45 nV can be observed at the 1.018 V level while this effect is not visible at the 10 V level.

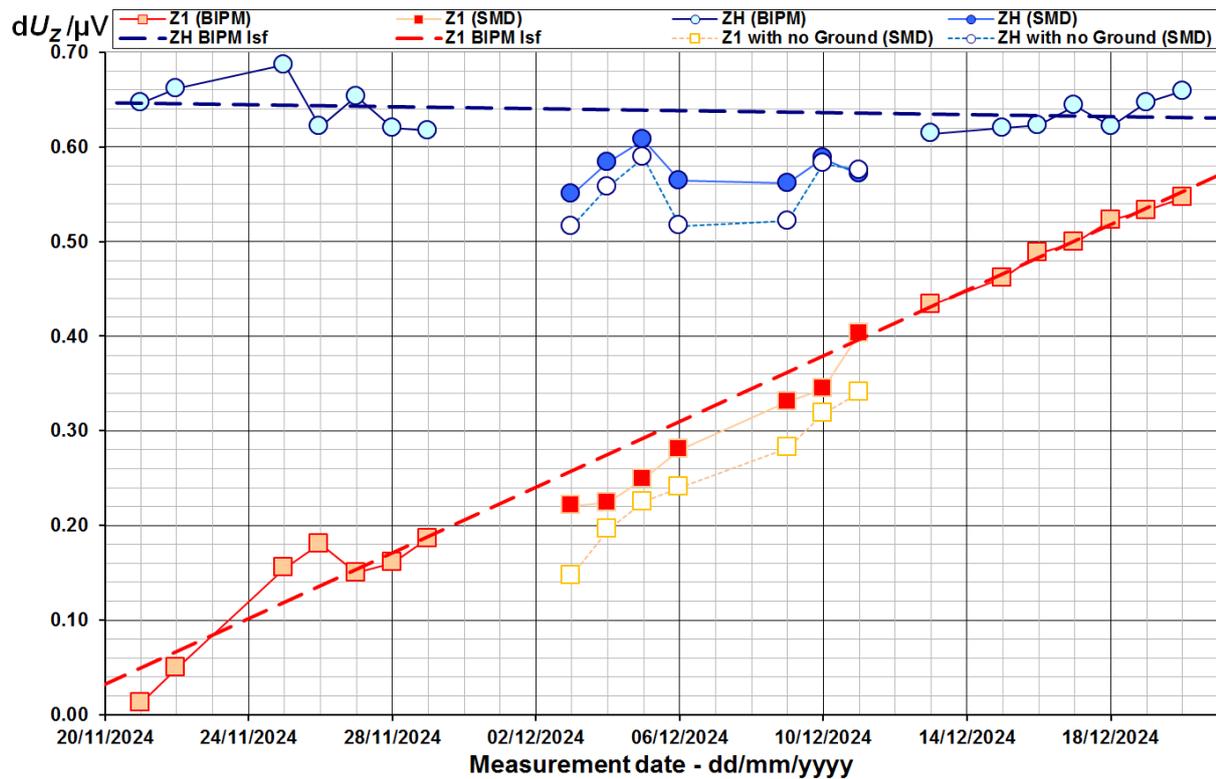


Figure A1-1: This graph is a copy of Figure 3 in addition to which SMD measurements at 1.018 V with the standards chassis and guard binding posts not connected to the Earth have been inserted. They appear as empty squares and empty circles on the graph.

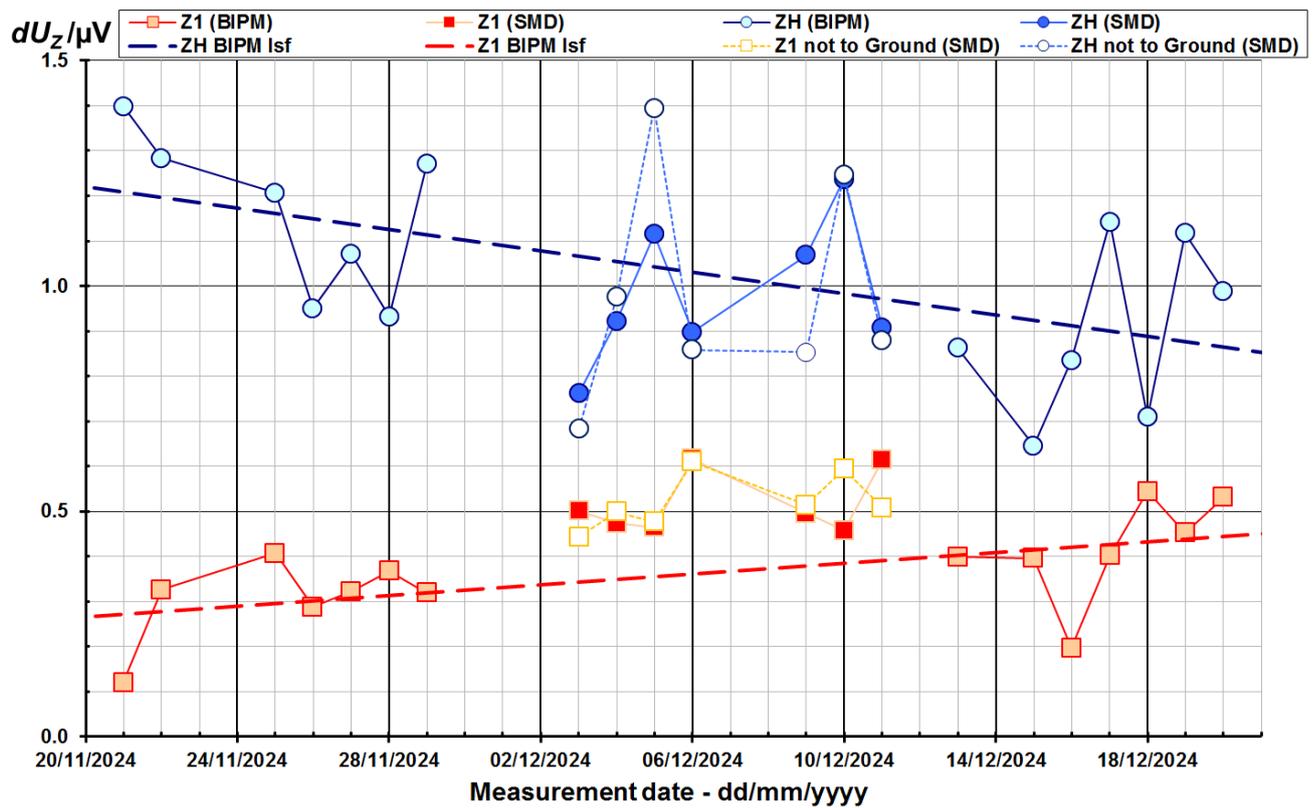


Figure A1-2: This graph is a copy of Figure 1 in addition to which SMD measurements at 10 V with the standards chassis and guard binding posts not connected to the Earth have been inserted. They appear as empty squares and empty circles on the graph.

Annex 2: Participant full report

OUTLINE OF THE MEASURING METHOD

SMD 1.018 V and 10 V measurements

At SMD, the reference standard for DC voltage is a 10 V Programmable Josephson Voltage Standard (PJVS) fabricated by Supracon AG. It is controlled by a computer using the AC SupraVOLT control software [1], and is used to generate programmable DC quantum voltages. The output electromotive force (EMF) of each travelling standard was measured by direct comparison with the primary standard.

Each output terminal of the travelling standards was connected in series opposition to the PJVS using a low thermal EMF polarity reversal switch with three different channels. The EMF differences between the standard and the PJVS array are measured using a digital nanovoltmeter Keithley 2182A operated on its 10 mV range and are automatically collected by the computer [1].

Each day, 8 data points are taken consecutively. The simple mean value being considered as the result of the day. Each individual data point represents the mean of 40 measurements (20 in positive and 20 in negative polarity). The same 40 measurements are taken at ± 0.1 mA to ensure a step flatness better than 5 m Ω . The nanovoltmeter input was shorted before each polarity reversal and restored right after.

Frequency and power of the microwave irradiating the PJVS array were set to maximize the quantum operating margins and were kept fixed throughout the whole measurement. The PJVS array was programmed to generate the closest quantum voltage level to the output voltage of the standard under measurement. Since the voltage resolution of the PJVS array is approximately equal to 144 μ V, the magnitude of the voltage recorded by the nanovoltmeter was always lower than 73 μ V.

The standards are disconnected from the mains about two and a half hours before and remain disconnected throughout the entire measurement session, in order for the standard to let the internal temperature stabilize. The GUARD and CHASSIS binding posts are jointly connected to the common ground point of the setup. The internal thermistor resistance is measured with a Fluke 8508A digital multimeter, operated on its 10 k Ω range, with a current of 10 μ A and recorded prior to the start of the measurements. The pressure was recorded at the same time using a portable pressure gauge.

UNCERTAINTY BUDGETS AT 10 V

SMD uncertainty budget at 10 V

Tables 1a and 1b list the uncertainties related to the calibration of the Zeners at SMD. The values correspond to the calibration of BIPM Zeners - Z1 and ZH.

Note that the uncertainty of the temperature and pressure corrections (in italics) are not given as no coefficients were provided and thus do not contribute to the final uncertainty budget used for this comparison. They will be applied by the BIPM and included in the comparison uncertainty budget.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|------------------|------|-------|--------------------------|-------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 55.41 μ V | A | Norm. | 25.76 nV | 1 | 25.76 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16 x 10 ⁻¹¹ | 10 V | 0.21 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |

| | | | | | | | |
|---|-----------------|---|-------|--------------------|---------------|-----------|----------|
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 55.41 μ V | 0.55 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Temperature coefficient of the Zener | 3763.5 Ω | B | Rect. | 5 Ω | | | 10 |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 27.3 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 2.73 nV/V | |
| Effective degrees of freedom | | | | | | 8 | |
| Coverage factor | | | | | | 2.37 | |
| Expanded uncertainty (95%) | | | | | | 64.6 nV | |

Table 1a. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 10 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|----------------------|------|-------|------------------------|---------------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | -62.08 μV | A | Norm. | 59.02 nV | 1 | 59.02 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.21 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 62.08 μV | 0.62 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Temperature coefficient of the Zener | 4665.2 Ω | B | Rect. | 5.0 Ω | | | 10 |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 59.7 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 5.97 nV/V | |
| Effective degrees of freedom | | | | | | 6 | |
| Coverage factor | | | | | | 2.52 | |
| Expanded uncertainty (95%) | | | | | | 150.2 nV | |

Table 1b. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 10 V for Zener ZH.

UNCERTAINTY BUDGETS AT 1.018 V

SMD uncertainty budget at 1.018 V

Tables 2a and 2b list the uncertainties related to the calibration of the Zeners at SMD. The values correspond to the calibration of BIPM Zeners - Z1 and ZH.

Note that the uncertainty of the temperature and pressure corrections (in italics) are not given as no coefficients were provided and thus do not contribute to the final uncertainty budget used for this comparison. They will be applied by the BIPM and included in the comparison uncertainty budget.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|------------------|------|-------|------------------------|---------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 49.60 μ V | A | Norm. | 25.85 nV | 1 | 25.85 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.6 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 1.018 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 49.60 μ V | 0.49 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| <i>Temperature coefficient of the Zener</i> | 3763.5 Ω | B | Rect. | 5.0 Ω | | | 10 |
| <i>Pressure coefficient of the Zener</i> | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 27.4 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 27.4 nV/V | |
| Effective degrees of freedom | | | | | | 8 | |
| Coverage factor | | | | | | 2.37 | |
| Expanded uncertainty (95%) | | | | | | 64.8 nV | |

Table 2a. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|------------------|------|-------|------------------------|--------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 4.84 μ V | A | Norm. | 7.31 nV | 1 | 7.31 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.6 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 1.018 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 4.84 μ V | 0.05 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Temperature coefficient of the Zener | 4665.2 Ω | B | Rect. | 5.0 Ω | | | 10 |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 11.61 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 11.61 nV/V | |
| Effective degrees of freedom | | | | | | 38 | |
| Coverage factor | | | | | | 2.07 | |
| Expended uncertainty (95%) | | | | | | 24.0 nV | |

Table 2b. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener ZH.

RESULTS

The value attributed by SMD to each Zener U_{SMD} , computed as the simple mean of all data from SMD and uncorrected for temperature and pressure.

| | Results/ μ V | |
|-----------------------------------|------------------|---------|
| | Z1 | ZH |
| SMD ($U_{\text{SMD}} - 10$ V) | 76.507 | 103.968 |
| SMD ($U_{\text{SMD}} - 1.018$ V) | 69.642 | 24.873 |

BIBLIOGRAPHY

[1] Josephson Voltage Standard SupraVOLTcontrol Manual, Supracon AG, Wildenbruchstr. 15, 07745 Jena, Germany, May 2015.

ANNEX 1

The same measurements were taken except that the GUARD and CHASSIS binding posts were connected together but not to the common ground of the system. Here are the uncertainty tables and the results.

UNCERTAINTY BUDGETS AT 10 V

SMD uncertainty budget at 10 V

Tables 3a and 3b list the uncertainties related to the calibration of the Zeners at SMD. The values correspond to the calibration of BIPM Zeners - Z1 and ZH.

Note that the uncertainty of the temperature and pressure corrections (in italics) are not given as no coefficients were provided and thus do not contribute to the final uncertainty budget used for this comparison. They will be applied by the BIPM and included in the comparison uncertainty budget.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|------------------|------|-------|------------------------|---------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 55.41 μ V | A | Norm. | 22.68 nV | 1 | 22.68 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.21 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 55.41 μ V | 0.55 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| <i>Temperature coefficient of the Zener</i> | 3763.5 Ω | B | Rect. | 5.0 Ω | | | 10 |
| <i>Pressure coefficient of the Zener</i> | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 24.4 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 2.44 nV/V | |
| Effective degrees of freedom | | | | | | 8 | |
| Coverage factor | | | | | | 2.37 | |
| Expended uncertainty (95%) | | | | | | 57.8 nV | |

Table 3a. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 10 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|----------------------|------|-------|------------------------|---------------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | -62.08 μV | A | Norm. | 95.20 nV | 1 | 95.20 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 0.14 nV/Hz | 0.20 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 10 V | 0.21 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 62.08 μV | 0.62 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Temperature coefficient of the Zener | 4665.2 Ω | B | Rect. | 5.0 Ω | | | 10 |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 95.6 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 9.56 nV/V | |
| Effective degrees of freedom | | | | | | 6 | |
| Coverage factor | | | | | | 2.52 | |
| Expended uncertainty (95%) | | | | | | 240.6 nV | |

Table 3b. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 10 V for Zener ZH.

UNCERTAINTY BUDGETS AT 1.018 V

SMD uncertainty budget at 1.018 V

Tables 4a and 4b list the uncertainties related to the calibration of the Zeners at SMD. The values correspond to the calibration of BIPM Zeners - Z1 and ZH.

Note that the uncertainty of the temperature and pressure corrections (in italics) are not given as no coefficients were provided and thus do not contribute to the final uncertainty budget used for this comparison. They will be applied by the BIPM and included in the comparison uncertainty budget.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|------------------|------|-------|------------------------|---------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 49.56 μ V | A | Norm. | 25.74 nV | 1 | 25.74 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.6 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 1.018 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 49.60 μ V | 0.49 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| <i>Temperature coefficient of the Zener</i> | 3763.5 Ω | B | Rect. | 5.0 Ω | | | 10 |
| <i>Pressure coefficient of the Zener</i> | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 27.3 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 27.3 nV/V | |
| Effective degrees of freedom | | | | | | 8 | |
| Coverage factor | | | | | | 2.37 | |
| Expanded uncertainty (95%) | | | | | | 64.6 nV | |

Table 4a. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener Z1.

| Quantity | Estimate | Type | Dist. | Standard Uncertainty | Sensitivity | Uncertainty contribution | Degrees of freedom |
|--|--------------------|------|-------|------------------------|--------------------|--------------------------|--------------------|
| Difference voltage measurements with the nanovoltmeter | 4.81 μV | A | Norm. | 12.21 nV | 1 | 12.21 nV | 6 |
| Reference frequency | 70.099999809 GHz | B | Rect. | 1.44 Hz | 14.6 pV/Hz | 0.02 nV | ∞ |
| Voltage due to the leakage current | 0 V | B | Rect. | 2.16×10^{-11} | 1.018 V | 0.02 nV | ∞ |
| Voltage due to residual thermal EMF | 0 V | B | Rect. | 5.77 nV | 1 | 5.77 nV | ∞ |
| Voltage due to the gain error of the nanovoltmeter | 0 V | B | Norm. | 1×10^{-5} | 4.84 μV | 0.05 nV | 100 |
| Voltage due to the non-linearity of the nanovoltmeter | 0 V | B | Rect. | 6.93 nV | 1 | 6.93 nV | ∞ |
| Temperature coefficient of the Zener | 4665.2 Ω | B | Rect. | 5.0 Ω | | | 10 |
| Pressure coefficient of the Zener | 1021.5 hPa | B | Rect. | 6.3 hPa | | | 14 |
| Combined uncertainty $u(U_z)$ | | | | | | 15.2 nV | |
| Relative combined uncertainty $u(U_z)/U_z$ | | | | | | 15.2 nV/V | |
| Effective degrees of freedom | | | | | | 14 | |
| Coverage factor | | | | | | 2.20 | |
| Expended uncertainty (95%) | | | | | | 33.3 nV | |

Table 4b. Estimated standard uncertainties for a Zener calibration with the SMD equipment at the level of 1.018 V for Zener ZH.

RESULTS

The value attributed by SMD to each Zener U_{SMD} , computed as the simple mean of all data from SMD and uncorrected for temperature and pressure.

| | Results/ μV | |
|--|------------------------|---------|
| | Z1 | ZH |
| SMD ($U_{\text{SMD}} - 10 \text{ V}$) | 76.510 | 103.965 |
| SMD ($U_{\text{SMD}} - 1.018 \text{ V}$) | 69.599 | 24.849 |

Annex 3:

At the BIPM, the Type A uncertainty of the Zener standards is evaluated by long series of measurements from which an Allan Variance is calculated. The following graphs (Figures A3-1 and A3-2) present the Allan Variance calculated from a series of 4096 individual consecutive measurements performed with the same time interval for both voltages on Z1. The $1/f$ noise floor appears to be 10 nV at 1.018 V for an integration time of $t=10$ s while it is 100 nV at 10 V for an integration time of $t=2$ s

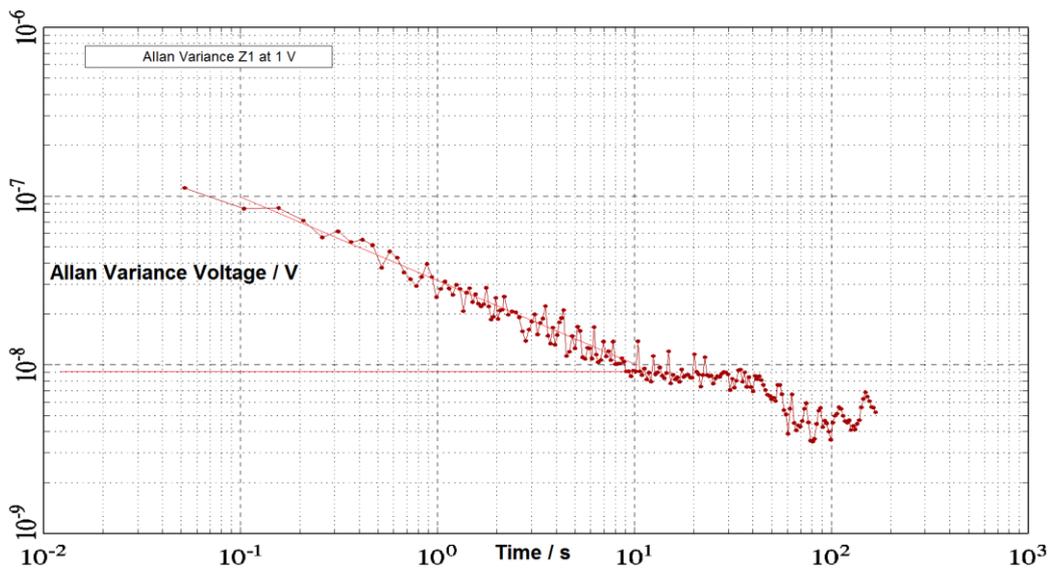


Figure A3-1: Allan Variance calculated from a series of 4096 individual consecutive measurements of the 1.018 V output of the Zener Fluke 732A “Z1”. The $1/f$ noise floor of 10 nV is reached after an integration time of 10 s.

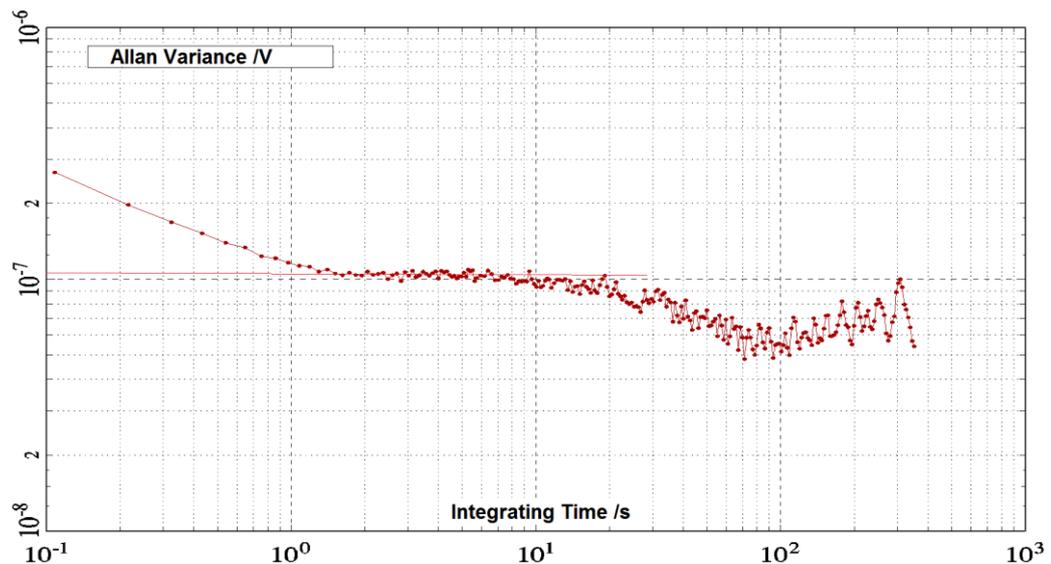


Figure A3-2: Allan Variance calculated from a series of 4096 individual consecutive measurements of the 10 V output of the Zener Fluke 732A “Z1”. The $1/f$ noise floor of 100 nV is reached after an integration time of 2 s.

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