

# **Comparison of the INMETRO and NIST Josephson Voltage Standards**

**(part of the ongoing regional key comparison SIM.EM.BIPM-K10.b1)**

Yi-hua Tang  
Quantum Electrical Metrology Division  
National Institute of Standards and Technology  
100 Bureau Drive  
Gaithersburg, MD 20899-8171, USA

Regis Pinheiro Landim, Edson Afonso, and Vitor Ferreira  
Electrical Metrology Division  
Instituto Nacional de Metrologia, Normalização e Qualidade Industrial  
Av. Nossa Senhora das Gracas, 50  
Duque de Caxias, RJ 25250-020, Brazil



# Comparison of the INMETRO and NIST Josephson Voltage Standards

(part of the ongoing regional key comparison SIM.EM.BIPM-K10.b1)

Yi-hua Tang  
Quantum Electrical Metrology Division  
National Institute of Standards and Technology  
100 Bureau Drive  
Gaithersburg, MD 20899-8171, USA

Regis Pinheiro Landim, Edson Afonso, and Vitor Ferreira  
Electrical Metrology Division  
Instituto Nacional de Metrologia, Normalização e Qualidade Industrial  
Av. Nossa Senhora das Gracas, 50  
Duque de Caxias, RJ 25250-020, Brazil

**Abstract:** A direct comparison of the 10 V Josephson Voltage Standards (JVS) of the National Institute of Standards and Technology (NIST), USA, and the Instituto Nacional de Metrologia, Normalização e Qualidade Industrial (INMETRO), Brazil, was made in June 2009. The process consisted of two comparisons, first using the NIST CJVS's measuring system (hardware and software) to measure the 10 V reference voltage provided by the INMETRO JVS and then using the INMETRO JVS's measuring system to measure the 10 V reference voltage provided by the NIST CJVS. The results of the two comparisons were in agreement to within 1.1 nV and their mean indicated that the difference between the two JVSs at 10 V was 0.54 nV with a combined standard uncertainty of 1.48 nV or a relative standard uncertainty of 1.48 parts in  $10^{10}$ .

## 1. INTRODUCTION

In the framework of the Comité International des Poids et Mesures (CIPM) Mutual Recognition Arrangement (MRA) Key Comparisons, a bilateral Regional Metrology Organization (RMO) Comparison between the Instituto Nacional de Metrologia, Normalização e Qualidade Industrial (INMETRO) JVS and the National Institute of Standards and Technology (NIST) compact JVS (CJVS) was performed at the 10 V level to check the coherence of the realization of the volt through the Josephson Voltage Standard (JVS) in June 2009. The NIST CJVS was shipped to INMETRO and the comparison was carried out in the Electrical Metrology Division of INMETRO. This report

describes the technical details of the experiments which were carried out to achieve the final result of the comparison.

The purpose of this comparison was to measure the equivalence of the INMETRO JVS to the NIST CJVS by directly comparing the two array systems. Based on the results of the INMETRO-NIST comparison (SIM.EM.BIPM-K10.b1), a link between INMETRO and the Bureau International des Poids et Mesures (BIPM) can be established via the comparison (BIPM.EM-K10.b.) between NIST and BIPM that was carried out in March 2009 [1].

## **2. COMPARISON EQUIPMENT**

### **2.1 The NIST CJVS**

The Compact Josephson Voltage Standard (CJVS), constructed at NIST, uses a fixed microwave frequency of either 76.76 GHz or 76.84 GHz and integrates the microwave frequency assembly with the cryoprobe. The unique design of the frequency assembly eliminates the need of a frequency counter, thereby reducing the weight of the system. This makes the system compact and transportable. Fig.1 shows the microwave assembly. A local 10 MHz quartz oscillator is phase-locked to a 10 MHz frequency reference from a Cesium clock provided by INMETRO. A quadrupler generates a 40 MHz frequency from the 10 MHz signal. Inside the cryoprobe, the 40 MHz signal is supplied as a reference to a Dielectric Resonance Oscillator (DRO) with an internal phase-lock loop (PLL) circuit. The DRO operates at 7.68 GHz. Its tenth harmonic, 76.8 GHz, is mixed with the 76.76 or 76.84 GHz GUNN Oscillator, creating a 40 MHz intermediate frequency (IF). This 40 MHz IF output is appropriately amplified and mixed with the original 40 MHz quartz signal to provide a dc error signal. The dc error signal is provided to the GUNN tuner to generate a phase-locked stable microwave frequency at 76.76 GHz or 76.84 GHz for the Josephson array operation. The uncertainty of the fixed 76.76 GHz or 76.84 GHz frequency is determined by the 10 MHz frequency reference and is in the range of a few parts in  $10^{12}$  or better. Commercial bias electronics and software developed at NIST control the measurement process.

Other details of the CJVS are as follows:

- Precision measurement leads resistance:  $5.8 \Omega$
- Leakage resistance between the precision measurement leads:  $8.3 \times 10^{10} \Omega$

- Josephson junction array: Hypres<sup>1</sup> 10 V SN 2546E3
- Null detector: Agilent 34420A SN576912; range 1 mV
- 10 V DVM: Agilent 34420A SN612213; range 10 V
- Bias source : VMetrix JVS1000
- Software: NISTVolt for Windows (ver. NV-2-17-09)

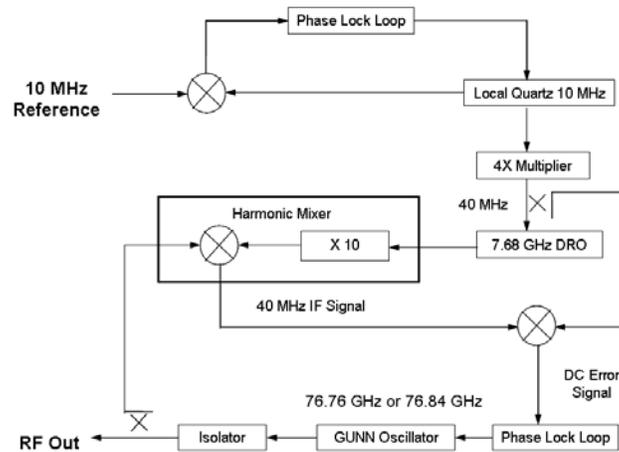


Fig. 1 Block diagram of a 76.76/76.84 GHz source for Josephson array operation.

## 2.2 The INMETRO JVS

The INMETRO Josephson Voltage Standard (JVS) is routinely used to calibrate Zener diode-based standards and it is designed to run in a fully automatic manner once the operator has adjusted the array parameters (microwave frequency and power level). INMETRO's working standards and some customer standards are directly measured against the primary standard, thereby significantly reducing the traceability chain. The INMETRO JVS system uses a 10 V SIS array (Hypres) mounted onto the lower WR-12 flange of a tube waveguide inside a magnetic shield at the bottom of an HPD CP-525 cryoprobe (which has a liquid helium level sensor and 6 RFI filters that use several stages of discrete components to intercept frequencies below 100 MHz and to block higher frequencies). This array is biased by a programmable current source. The RF source is a Millitech Gunn diode with a central frequency at 75 GHz and a  $\pm 1$  GHz mechanical tuning (hence, the Gunn can generate frequencies from 74 GHz to 76 GHz). The working

<sup>1</sup> Certain commercial equipment, instruments, or materials are identified in this report in order to facilitate understanding. Such identification does not imply recommendation or endorsement by NIST neither by INMETRO, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

frequency is locked by an EIP-578B frequency counter. This is a new system: only the 10 V array and the cryoprobe have been retained from the previous INMETRO JVS DOS-based system (which was used in the BIPM.EM-K10.b [2]). It was assembled in 2007 and has been operational since then. Step biasing, array monitoring and the connection of the Zener under test are operated automatically by software. The bias source is disconnected during the measurements and the array is floating with respect to ground, which means that the measurement ground reference point can be chosen arbitrarily. The voltage from the detector (HP 3458) and the frequency from the EIP counter are monitored and stored in an electronic file. The GPIB interface for reading the measurement instruments is optically isolated from the computer. The 10 MHz reference signal for the EIP counter distributed by a GPS receiver is also electrically isolated. An analogue oscilloscope is used to visualize the array I-V characteristics and adjust the RF power level at the beginning of the operation (during the measurements, the array is disconnected from this instrument). The laboratory temperature was regulated to better than  $\pm 0.8$  °C over a week of measurements. This minimizes the thermal voltages and ensures good voltage stability during the measurements. The room temperature and humidity are measured (and recorded during the measurements) by the software through external sensors. The system is powered by an exclusive UPS through an isolator transformer.

Other details of the JVS are as follows:

- Frequency reference (10 MPPS): Symmetricom XLi Time and Frequency System GPS receiver SN PR0743Q98293. This equipment was not used during the comparison. A Symmetricom Cesium frequency standard 5071A, option 1 (high stability) SN US45382393 was used.
- Frequency counter: EIP 578B SN 20211544 with frequency locked to the external 10 MHz reference and stability better than  $\pm 1.5$  Hz during the comparison. The INMETRO JVS array was irradiated at a frequency around 76 GHz.
- Precision measurement leads resistance:  $1.8 \Omega$ .
- Leakage resistance between the precision measurement leads:  $8.3 \times 10^{10} \Omega$ .
- Josephson junction array: Hypres SIS 10 V SN KL164B-5.
- Null detector: Agilent 34420A SN MY42002972; range 1 mV
- 10 V DVM: Agilent 3458A SN 2823A15824; range 10 V
- Bias source : VMetrix JVS1002 SN 41

- Software: NISTVolt for Windows (ver. NV-2-17-09)
- Measurement reversing switch: Dataproof scanner 160 A SN 683. This device was not used during the comparison; the output of the NIST array was connected to the INMETRO device through a NIST low thermal EMF switch.

### **3. COMPARISON SETUP**

Comparisons between the two JVS were carried out in an automatic manner following the NIST-NRC protocol [3]. For this particular scheme, both JVS were biased with a single source (either the NIST CJVS or INMETRO JVS bias source). Step jumps in either array during the data acquisition were permissible as long as two measurement conditions are met:

- a. The voltage difference between the two arrays is less than 1 mV (or 7 steps) so that the DVM range of the Agilent 34420A is 1 mV at all times. Keeping the DVM on a fixed range can avoid the impact produced by a change in the DVM gain error and linearity for different ranges.
- b. The array voltage of the JVS which provides the biasing to both arrays is within 5 mV from the target voltage 9.997 V or any other preferred voltage.

Once both arrays were biased within 1 mV of the nominal target voltage 9.997 V, the data acquisition system was run following the + - + - measurement sequence where + represents both arrays biased positively, and - represents both arrays biased negatively. There was no mechanical switch in the measurement loop to eliminate the thermal voltages from the switch as shown in Fig. 2. The polarity change of the arrays was accomplished electronically by the bias source.

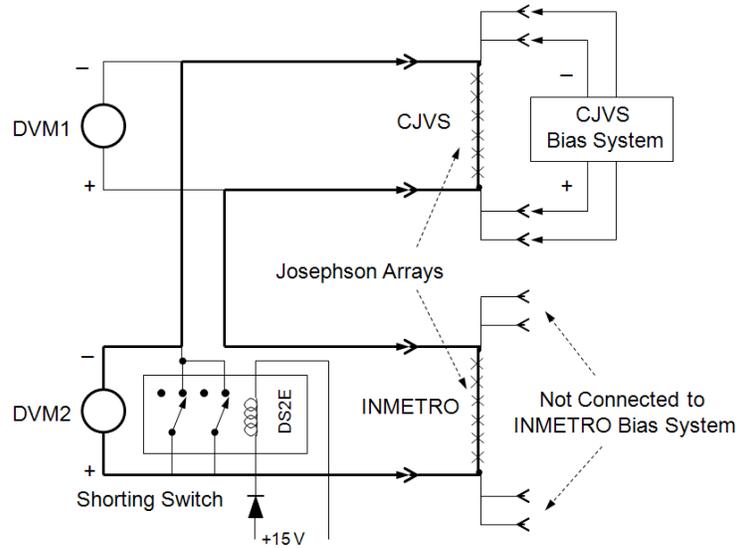


Fig. 2 The setup using the NIST CJVS to bias both the NIST CJVS and INMETRO JVS arrays. A shorting switch (a latching relay DS2E) connected to the DVM2 and controlled by the NISTVolt software makes the automatic process possible.

Simultaneous biasing of both arrays was achieved by the insertion of a shorting switch in parallel with the nanovoltmeter. Fig. 2 shows the setup using the NIST CJVS to measure the INMETRO JVS. With the nanovoltmeter shorted, the NIST CJVS bias source JBS1000 applied a bias voltage to both the NIST CJVS and the INMETRO JVS arrays at the same nominal voltage. Once both arrays reached the nominal voltage with the difference between the two arrays within 1 mV, the NIST CJVS bias source was gradually disconnected from the two arrays via an optoisolator. The gradually increasing impedance of the optoisolator greatly reduced the occurrences of step jumps in both arrays when the bias was disconnected from the arrays. Opening the shorting switch usually resulted in a one or two step jump between the two arrays. An automatic switch controlled by the NISTVolt software was implemented in this comparison to improve the efficiency of the process. Note that the shorting switch produced no thermal EMF measurement contribution since it was an open circuit and not in the potential measurement loop.

#### 4. MEASUREMENT PROTOCOL

In a direct Josephson comparison, the parameter of interest is the amount by which the difference voltage deviates from its theoretical value. In the case of using the NIST CJVS bias source to bias both arrays, comparisons are made by connecting the arrays in series opposition and measuring the difference voltage  $V_d$  with a sensitive digital voltmeter (DVM) such that

$$V_d = V_{a1} - V_{a2} = \frac{N_1 \cdot f_1 - N_2 \cdot f_2}{K_{J-90}} \quad (1)$$

where  $V_{a1}$  is the voltage from the NIST CJVS array and  $V_{a2}$  is the voltage from the INMETRO JVS array,  $N_1$  and  $N_2$  are the step numbers of the NIST CJVS and INMETRO JVS arrays,  $f_1$  and  $f_2$  are the microwave frequencies used for the NIST CJVS and INMETRO JVS arrays, respectively, and  $K_{J-90}$  is the Josephson constant adopted internationally since January 1, 1990. The purpose of a direct JVS comparison is to measure the real difference,  $\delta$ , of two JVS voltages from the theoretical value  $V_d$ .

$$V_d - \frac{V_m}{1 + E_g} = V_o + m.t + V_n + \delta \quad (2)$$

where  $V_o + mt$  represents an offset voltage with a fixed and a linearly drifting component. The offset voltage is assumed to include both the voltmeter offset and thermal emfs in the measurement loop.  $V_n$  is the random time dependent noise in the meter readings and any other unaccounted for effects such as DVM nonlinearity and nonlinear thermal emf drift.  $E_g$  is the gain error of the voltmeter.  $\delta$  is the amount by which the measured voltage between the two standards differs from its theoretical value. Contributions to  $\delta$  are

1. A discrepancy between  $f_1$  or  $f_2$  used in the equation and the actual frequencies applied to the Josephson arrays.
2. Leakage current  $I_L$  that results in a voltage drop across the resistance of the measurement loop.
3. Uncorrected thermal offset and drift.
4. Any additional unknown effects.

Solving Eq. 2 for  $\delta$  gives

$$\delta = V_d - \frac{V_m}{1 + E_g} - V_o - m.t - V_n \quad (3)$$

The unknowns in this equation are  $V_o$ ,  $m$  and  $\delta$ . They can be estimated by taking sets of measurements with two or more polarity reversals. Rather than using a reversing switch, the polarity of each array is reversed by changing the array bias to reverse the signs of  $N_1$  and  $N_2$  but not the magnitude. The data set is an array  $V_d(i)$ ,  $V_m(i)$ ,  $t(i)$ , and  $P(i)$  for  $i = 1$  to  $N$  where  $V_d(i)$  is the  $i^{\text{th}}$  theoretical difference in array voltages,  $V_m(i)$  is the  $i^{\text{th}}$  meter reading,  $t(i)$  is the time of the  $i^{\text{th}}$  reading,  $P(i)$  is the polarity of  $V_{a1}$  and  $V_{a2}$  for the  $i^{\text{th}}$  reading, and  $N$  is the total number of readings. Eq. 3 is a model for the data set. Best estimates for  $V_o$  and

$m$  and  $\delta$  are computed using a 3 parameter least square fit that minimizes the RSS sum of the residuals  $R(i)$  in Eq. 3 where:

$$R(i) = V_d(i) - \frac{V_m(i)}{1 + E_g} - V_o - m.t(i) - \delta \quad (4)$$

## 5. MEASUREMENT RESULTS AND UNCERTAINTY

### 5.1 Results using the NIST CJVS bias source JVS1000

The measurements from comparing the NIST CJVS against the INMETRO JVS were made on June 25 by automatically operating the JBS1000. The NISTVOLT software was used for the data acquisition and calculation of the difference between the two JVS systems. The difference between the two arrays was always controlled within 7 steps so that the DVM was always on the 1 mV range. The frequency of the CJVS was 76.76 GHz and the frequency of the INMETRO JVS was 75.991 GHz. Fig. 3 shows the results from the NIST CJVS measuring the INMETRO JVS. A total of 58 points were taken with 29 points using the DVM in the normal polarity mode for the measurements and 29 points using the DVM in the reversed polarity mode. No obvious offset related to the DVM polarity mode was observed. Each point was calculated from 4 sets of DVM measurements with array polarity + - + -. Each set consisted of 15 DVM readings. The average time to finish a comparison was 6 min. No leakage corrections for the cryoprobes were applied to the measurements. The leakage errors were taken into account as a Type B uncertainty. Fig. 3 shows a typical comparison with 4 data sets.

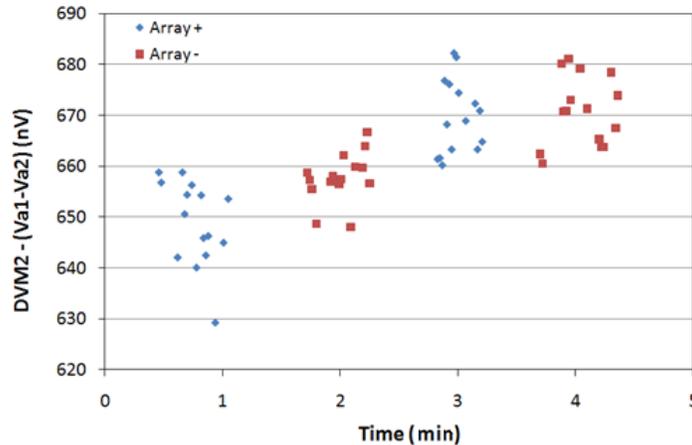


Fig. 3 Raw data of a typical measurement point (voltage difference between the NIST CJVS and the INMETRO JVS). The drifting of the data points was due to the DVM and thermal voltage in the measurement loop.

During the measurements with the DVM in the normal polarity mode, it was necessary to make a microwave power adjustment several times to optimize the step quality of the INMETRO JVS. One outlier was removed from the measurements due to an apparent step collapse. Fig. 4 shows the measurements made by the NIST CJVS against the INMETRO JVS.

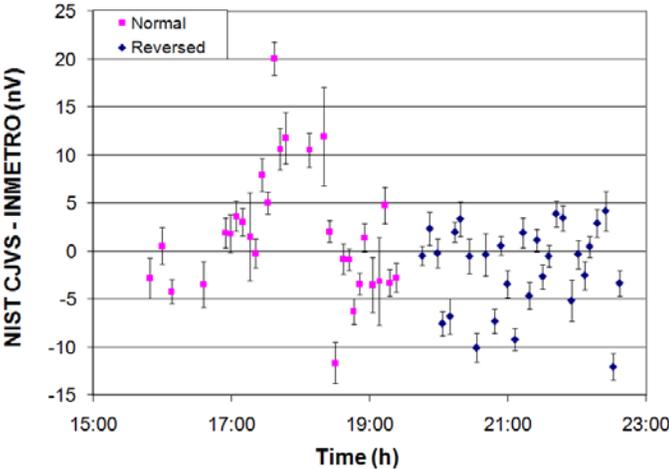


Fig. 4. Differences between the measured value and the theoretical value of the NIST CJVS - INMETRO JVS at the nominal voltage of 9.997 V using the CJVS as the bias source where “Normal” and “Reversed” refer to the DVM polarity mode for the measurements. The error bar shows the Type A uncertainty of each measurement.

### 5.2 Results using the INMETRO JVS bias source JVS1002

The measurements were repeated using the INMETRO JVS bias source JVS1002 to bias both arrays on June 26th. The INMETRO JVS also used the software NISTVolt to automatically take the measurements. Identical DVM and measurement conditions were used. A total of 40 points were taken with 20 points using the DVM in the normal polarity mode and 20 points using the DVM in the reversed polarity mode. No obvious offset related to the DVM polarity mode was observed. The average time to finish a point was 8.5 min. Fig. 5 shows the results from the INMETRO JVS measuring the NIST CJVS.

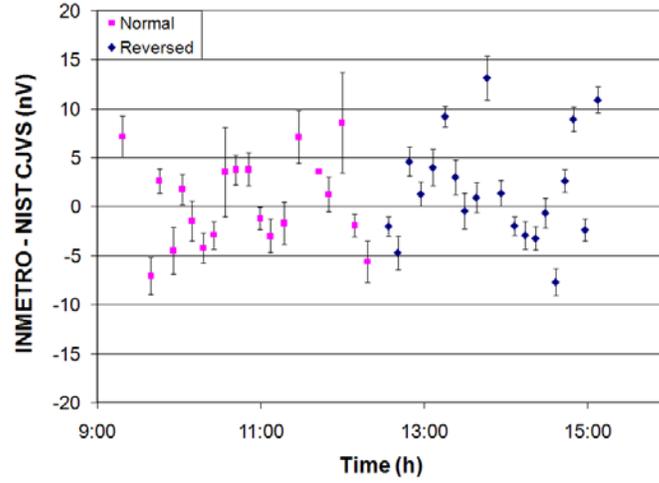


Fig. 5 Differences between the measured value and the theoretical value of INMETRO JVS – NIST CJVS at the nominal voltage of 9.997 V using the INMETRO bias source. The error bar shows the Type A uncertainty of each measurement only.

During the data acquisition for the comparisons, the lab temperature was controlled at  $22.0\text{ }^{\circ}\text{C} \pm 0.2\text{ }^{\circ}\text{C}$  and the relative humidity was controlled at  $53\% \pm 4\%$ .

### 5.3 Uncertainty

#### 5.3.1 Type A uncertainty

For the measurements made by the NIST CJVS, the difference  $D_{NIST}$  is the mean of the two data sets with normal DVM polarity and reversed DVM polarity modes

$$D_{NIST} = \frac{\frac{\sum_i D_i^{NIST+}}{N_{NIST+}} + \frac{\sum_i D_i^{NIST-}}{N_{NIST-}}}{2} \quad (5)$$

where  $D_i^{NIST+}$  is the  $i$ th measurement and  $N_{NIST+}$  is the number of measurements taken by the NIST CJVS with DVM normal polarity mode,  $D_i^{NIST-}$  is the  $i$ th measurement and  $N_{NIST-}$  is the number of measurements taken by NIST CJVS with DVM reversed polarity mode. In the comparison, an equal number of DVM normal polarity and reversed polarity mode measurements were taken. Type A uncertainty of the NIST CJVS measurements is the pooled standard deviation of the mean of all the measurements

$$u_A^{NIST} = \sqrt{\left(\frac{u_A^{NIST+}}{2}\right)^2 + \left(\frac{u_A^{NIST-}}{2}\right)^2} \quad (6)$$

where  $u_A^{NIST+}$  and  $u_A^{NIST-}$  is the standard deviation of the mean for data sets with normal and reversed DVM polarity modes, respectively. The same calculation applies to the measurements made by the INMETRO JVS against the NIST CJVS. Table 1 summarizes the differences between the two JVS systems along with the associated Type A uncertainties.

Table 1. Differences between the two JVSs and associated Type A uncertainties

	Made by NIST	Made by INMETRO
INMETRO - NIST (nV)	-0.03	1.11
Number of measurements (each polarity)	29	20
Standard deviation (nV)	4.00	3.54
Type A uncertainty (nV)	0.74	0.79

### 5.3.2 Type B uncertainty

The sources of Type B uncertainty are: the frequency stability of the Gunn diodes, the cryoprobe leakage currents, and the detector gain and linearity. Most of the effects of detector gain and frequency stability are already contained in the Type A uncertainty. Since both arrays had their polarities reversed during the measurements, the effect of the residual thermal EMFs (i.e., non-linear drift) and electromagnetic interferences are also contained in the Type A uncertainty of the measurements. Uncertainty components related to the RF power rectification and sloped Shapiro voltage steps are considered negligible since no such physical effects were observed. Table 2 lists the Type B components from various sources for each JVS system when the NIST CJVS made the measurements against the INMETRO JVS. Some sources of Type B uncertainty are indicated in Table 2, with the correlated NIST and INMETRO components listed. The combined Type B uncertainty is the root sum square (RSS) of all the components, according to the expression below.

$$u_B^{NIST} = \sqrt{u_B^2(\text{correlated}) + \left(u_B^{NIST}\right)^2(\text{uncorrelated})} \quad (7)$$

Table 2. Type B uncertainty components of each JVS system when the NIST CJVS measured the INMETRO JVS.

Uncertainty component	NIST (nV)	INMETRO (nV)
Frequency reference (correlated)	0.01	0.01
Frequency counter (correlated)		1.08
Leakage (correlated)	0.70	0.21
Detector gain and linearity (uncorrelated)	0.88	
Combined Type B uncertainty $u_B^{NIST}$ (nV)	1.57	

1. Frequency reference.

The frequency reference for both JVSs used in the comparison was a Cesium clock (Symmetricom 5071A). The Allan deviation of the oscillator is better than  $1 \times 10^{-12}$  for averaging times ranging from 10 s to 1000 s. The NIST CJVS uses a fixed frequency and does not have a frequency counter. The uncertainty contribution from the NIST CJVS frequency to the voltage measurement is therefore proportional to the Allan deviation during the voltage measurement which usually took around 500 s. For the uncertainty in the INMETRO frequency counter, we can use the manufacturer's specification of  $\pm 14.4$  Hz with an assumed rectangular distribution. Thus, the INMETRO frequency counter uncertainty can be calculated as 1.08 nV [4].

2. The leakage due to the cryoprobes.

The measurement of the leakage resistance between the ground and the measurement lead of each JVS cryoprobe was performed for both the NIST CJVS and INMETRO JVS, following the NIST methodology described in Fig 6. The measurements were made with the array immersed in liquid helium.

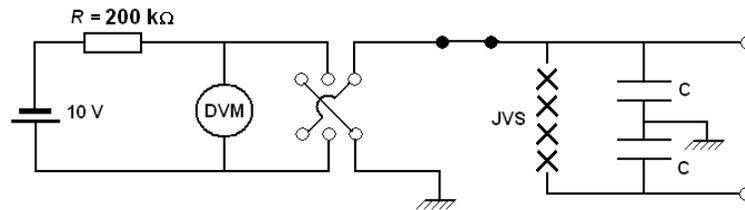


Fig. 6 Schematic of the experiment setup to measure the leakage resistance between each measurement lead and the ground.

To estimate the leakage, the battery was first disconnected from the probe. The DVM then recorded the battery voltage as a base line. Then the battery was connected to one of the precision leads and the ground. The voltage drop after the capacitor fully

charged is associated with the leakage resistance. The third step was to reverse the battery polarity to check the symmetry of the leakage resistance. Finally, the battery was disconnected from the probe to ensure there was no drift in the voltage during the measurement. The measurement generally produced a leakage current that was about 4 times greater than the direct leakage measurement between the two precision leads because the capacitor leakage paths were effectively in parallel rather than in series. Thus the leakage resistance was calculated using the formula  $R_i = 4U / i_F$  where  $U$  is the voltage of the source and  $i_F$  is the leakage current. The DVM measured the voltage time dependence across a  $200\text{ k}\Omega$  resistor ( $R$ ) that was connected from one side of the array to the ground. The other side, at a potential of  $10\text{ V}$ , was left open so that the only path to ground was through  $R_i/2$ . Fig. 7 and Fig. 8 are the records obtained from the measurements performed on the NIST CJVS and the INMETRO JVS probes.

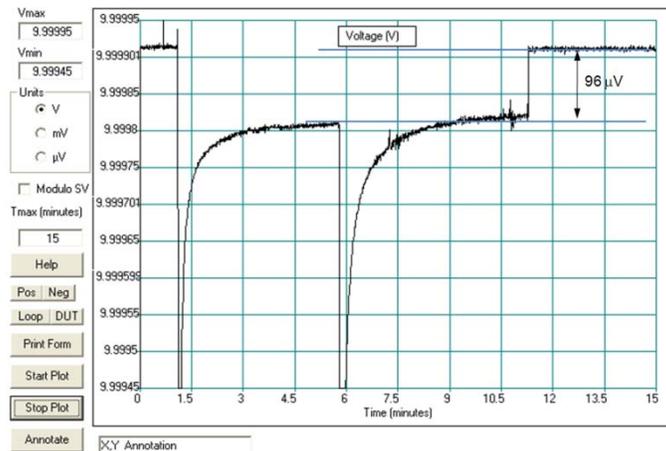


Fig. 7 Charge and discharge through the filter capacitors between a precision lead and ground on the NIST CJVS probe.

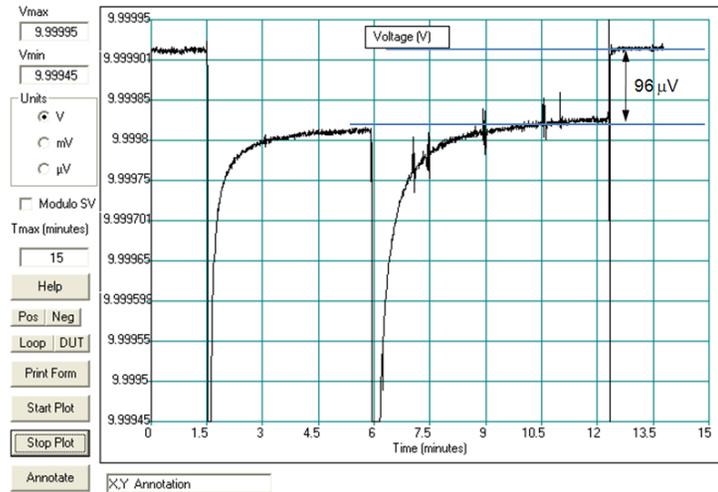


Fig. 8 Charge and discharge through the filter capacitors between a precision lead and ground on the INMETRO JVS probe.

Table 3. Uncertainty due to the leakage currents for both Josephson systems at the nominal voltage  $U$  of 10 V.

	NIST CJVS	INMETRO JVS
Leads resistance $r_B$ ( $\Omega$ )	5.8	1.8
Leakage resistance $R_i$ ( $\Omega$ )	$8.3 \times 10^{10}$	$8.3 \times 10^{10}$
Uncertainty (nV) $U(r_B / R_i)$	0.70	0.21

### 3. The DVM gain and linearity.

The uncertainty due to DVM gain and linearity errors was estimated by multiplying the measured DVM gain error by the Mean Polarized Null Voltage (MPNV) that was calculated from the comparisons. The relevant parameter for computing the uncertainty due to the DVM gain and linearity error is the mean of all of the values  $P(\text{Set}) \times \text{VDVM}(\text{Set})$  where  $P(\text{Set})$  is the polarity mode of the measurement and  $\text{VDVM}$  is one of the  $i = 1$  to 15 null voltage measurements in each of the 4 data sets acquired using the NISTVolt protocol. The average of all MPNVs from the NIST CJVS measurements was -0.13 mV and the DVM gain error calculated from a NIST programmable JVS calibration was -6.9 parts in  $10^6$ , resulting in a combined Type B uncertainty component of 0.88 nV as listed in Table 2.

Similarly, the Type B uncertainty from the INMETRO JVS measurements has components that are both correlated and uncorrelated to the NIST measurements. They are summarized in Table 4. The INMETRO JVS used the Agilent 34420A (SN MY42002972) in

the comparison to measure the difference between the two arrays. The gain error of the DVM on the 1 mV range was measured using the Measurements International MI 8000A and was -10.3 parts in  $10^6$ . The average of all of the 20 pair MPNVs from the INMETRO JVS measurements was -0.03 mV, resulting in the combined Type B uncertainty component of 0.26 nV as listed in Table 4.

$$u_B^{INMETRO} = \sqrt{u_B^2(\text{correlated}) + (u_B^{INMETRO})^2(\text{uncorrelated})} \quad (8)$$

Table 4. Type B uncertainty components of each JVS system when the INMETRO JVS measured the NIST CJVS. The combined Type B uncertainty is the root sum square (RSS) of all the components.

Uncertainty component	NIST (nV)	INMETRO (nV)
Frequency reference (correlated)	0.01	0.01
Frequency counter (correlated)		1.08
Leakage (correlated)	0.70	0.21
Detector gain and linearity (uncorrelated)		0.26
Type B uncertainty (RSS) $u_B^{INMETRO}$ (nV)	1.33	

The pooled Type A uncertainty of the final result (mean value of the two results) is then:

$$u_A^2 = \frac{(u_A^{NIST})^2 + (u_A^{INMETRO})^2}{4} \quad (9)$$

The pooled Type B uncertainty of the final result (mean value of the two results) is then:

$$u_B^2 = u^2(\text{correlated}) + \frac{(u_B^{NIST})^2(\text{uncorrelated}) + (u_B^{INMETRO})^2(\text{uncorrelated})}{4} \quad (10)$$

The final reported difference between the INMETRO JVS and the NIST CJVS shown in Table 5 is the mean difference of the measurements made by the NIST CJVS and the INMETRO JVS. The combined uncertainty associated with the difference is the RSS of the pooled Type A and pooled Type B uncertainties.

Table 5. The differences between the INMETRO JVS and the NIST CJVS at 9.997 V with associated Type A and Type B uncertainties.

INMETRO - NIST (nV)	0.54
Pooled Type A uncertainty $u_A$ (nV)	0.54
Pooled Type B uncertainty $u_B$ (nV)	1.38
Combined standard uncertainty $u_c$ (nV)	1.48

## 6. DISCUSSION AND CONCLUSION

Two comparisons between the NIST CJVS and the INMETRO JVS were carried out in June 2009 as SIM.EM.BIPM-K10.b.1, first using the NIST CJVS's measuring system (hardware and software) to measure 10 V against the INMETRO JVS and then using the INMETRO JVS's measuring system to measure 10 V against the NIST CJVS. The results of the two comparisons were in agreement to within 1.1 nV and their mean indicated that the difference between the two JVSs at 10 V was 0.54 nV with a total combined standard uncertainty of 1.48 nV or a total combined relative uncertainty of 1.48 parts in  $10^{10}$ .

A link between INMETRO and BIPM can be established via the NIST-BIPM direct JVS comparison performed in March 2009 and reported in the BIPM Key Comparison Data Base (KCDB) BIPM.EM-K10.b. The same NIST CJVS, including all its hardware and software was used in the NIST-BIPM direct JVS comparison. The difference between NIST and BIPM was -0.80 nV with a total combined standard uncertainty of 0.95 nV or a total combined relative uncertainty of 9.5 parts in  $10^{11}$ .

The degree of equivalence of INMETRO with respect to BIPM is given in Table 7 by the following relations, both expressed in nV:

$$d_{INMETRO-BIPM} = d_{INMETRO-NIST} + d_{NIST-BIPM} \quad (11)$$

$$u_{INMETRO-BIPM}^2 = u_{INMETRO-NIST}^2 + u_{NIST-BIPM}^2 \quad (12)$$

Table 7. A link between INMETRO and BIPM via the NIST-BIPM direct JVS comparison listed in KCDB BIPM.EM-K10.b

	Difference (nV)	Uncertainty (nV)
INMETRO - NIST	0.54	1.48
NIST - BIPM	-0.80	0.95
INMETRO - BIPM	-0.26	1.76

The difference between INMETRO and BIPM, thus was found to be -0.26 nV with a standard uncertainty of 1.76 nV or a relative uncertainty of 1.76 parts in  $10^{10}$ . The NIST

CJVS and INMETRO JVS comparison has significantly improved upon the results of the INMETRO-BIPM direct JVS comparison that was carried out in April 2006 as a part of the on-going key comparison BIPM.EM-K10.b. This improvement is due to INMETRO's use of a new microwave assembly to make the phase lock operation work properly. The frequency variation of the new microwave assembly with the EIP counter was  $\pm 1$  Hz during the comparison.

## References

- [1] S. Solve, R. Chayramy, M. Sock and Y. Tang, *Comparison of the Josephson Voltage Standards of NIST and the BIPM (part of the ongoing BIPM key comparison BIPM.EM-K10.b)*, Technical Supplement of Metrologia, **46**, 01010, 2009  
(<http://www.iop.org/EJ/abstract/0026-1394/46/1A/01010>)
- [2] D. Reymann, S. Solve, E. Afonso, V. Ferreira and R. P Landim, *Comparison of the Josephson Voltage Standards of the INMETRO and the BIPM*, Rapport BIPM-2006/06, 2006.
- [3] Y. Tang, B. Wood and C. Hamilton, *A Two-way Josephson Voltage Standard Comparison between NIST and NRC*, IEEE Trans. Instrum. Meas. Vol.58, pp. 821-826, April 2009.
- [4] C.A. Hamilton, Y. Tang, "Evaluating the Uncertainty of Josephson Voltage Standards," Metrologia, Vol.36, pp. 53-58, 1999.

## Appendix A

During the comparison made by the NIST CJVS against the INMETRO JVS an overnight run from 8 pm on June 25 to 8 am on June 26 was carried out automatically with DVM reversed polarity. A total of 124 measurements were made as shown in Fig.A1.

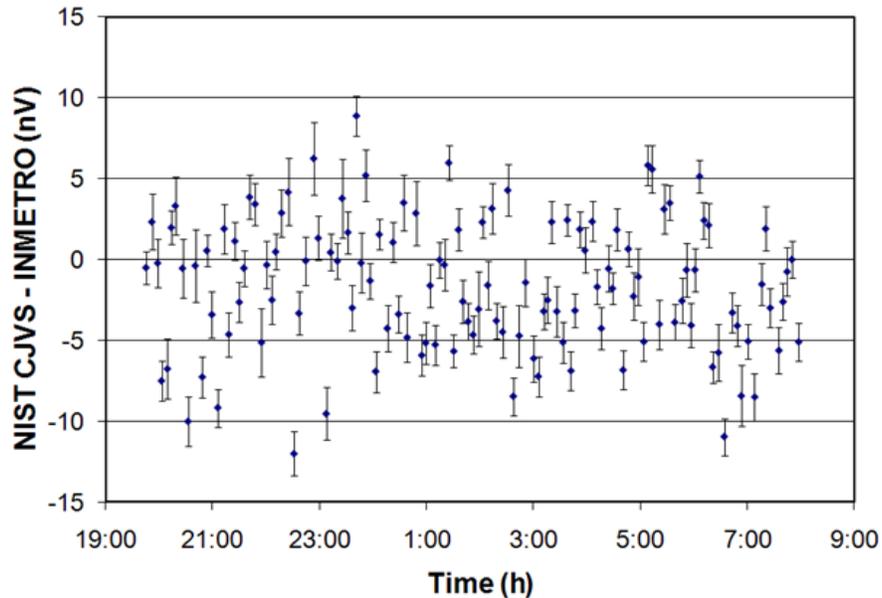


Fig.A1 Automatic measurements carried out overnight from 8 pm on June 25 to 8 am on June 26. The error bar shows the Type A uncertainty of each measurement.

In most of the JVS comparisons the conventional method to calculate the Type A uncertainty is to use the standard deviation of the mean of the measurements. The automatic data acquisition allows more measurements to be done in a short time period. The question is whether the conventional calculation for the Type A uncertainty is valid for hundreds of points. In our opinion when the standard deviation of the mean becomes much smaller than the  $1/f$  noise floor of the null detector, the Type A uncertainty should be represented by the  $1/f$  noise floor instead of the standard deviation of the mean. For the example of the NIST - INMETRO JVS comparison the  $1/f$  noise floor of the null detector, the Agilent 34420A, was estimated to be 0.75 nV [1]. The standard deviation of the mean for the 124 points was 0.37 nV. Table A1 lists the results of automatic overnight measurements using the  $1/f$  noise floor for the Type A uncertainty. The results are consistent with the previous analysis described in this report.

Table A1 Results of automatic overnight measurements

INMETRO JVS – NIST CJVS (nV)	1.61
Type A (nV) - 1/f noise floor	0.75
Type B (nV)	1.34
Combined $u_c$ (nV)	1.53

We also applied the 1/f noise floor as the Type A uncertainty for the analysis in Section 5.3, which did not make substantial changes to the combined standard uncertainty. This is because the conventional calculation for Type A uncertainty is close to the 1/f noise floor of the null detector with a limited number of measurements between 20 to 30 for each DVM polarity.