Final Report

07.08.2018

COOMET.PR-S2

Supplementary Comparison
Angle of rotation of plane of polarization
(COOMET project 438/RU/08)

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Contents

1 Introduction

Participants agreed to conduct a Supplementary Comparison on Angle of rotation of plane of polarization measurements.

The aim of this comparison was to ensure the correctness and comparability of angle of rotation of plane of polarization measurements carried out by participants of the comparison, within the uncertainties claimed for their measuring facility.

2 Organization

2.1 Participants

The Technical Protocol and the Participant's details listed below has been approved by COOMET - TCPR.

Participants accepted the general instructions and the Technical Protocol written in the document and commit themselves to follow the procedures strictly.

2.2 Participants' details

Table 1. Participant's details

	NMI, address	**NMI acronym**	**Contact person**	**Contact details**
Pilot	All-Russian Research Institute for Optical and Physical Measurements (VNIIOFI), Ozernaya str. 46, 119361 Moscow, **Russia**	VNIIOFI	Prof. Gennady Vishnyakov	Phone: +7 495 781 45 76, +7 495 437 33 77 Fax: +7 495 437 34 01 E-mail: vish@vniiofi.ru
Participant 1	Physikalisch-Technische Bundesanstalt, Bundesallee 100, D-38116 Braunschweig, **Germany**	PTB	Mr. Andreas Fricke	Phone: +49 531 592 4213. E-mail: Andreas.Fricke@ptb.de

Participant 2	State Enterprise All-Ukrainian State Research and Production Centre for Standardization, Metrology, Certification and Consumers Rights Protection 4 Metrologichna st., Kyiv, **Ukraine**, 03680	Ukrmetrtestst andart	Mr. Andrey Glebov, Ms. Nataliya Parkhomenko	Phone: +38 044 526 36 98 Fax: +38 044 526 36 98 E-mail: optic@ukrcsm.kiev.ua natapar@mail.ru
Participant 3	GUM Glowny Urzad Miar 2., Elektoralna Str. 00-139 Warsaw, **Poland**	GUM	Ms. Joanna Przybylska	Phone: +48 22 581 95 58 Fax: +48 22 620 83 78 E-mail: length@gum.gov.pl

2.3 Comparison organization

The comparison covers the values of angle of rotation of plane of polarization determined by the participants for a set of three comparison standards at the specified wavelength. Full description of the comparison standards is given in Section 3.

The comparisons were conducted according to the following scheme:

The measurements of the artefacts were carried out in the following sequence:

Pilot → Participant 1 → Participant 2 → Participant 3 → Pilot

The Pilot sent the set of the comparison standards to Participant 1. Participant 1 carried out measurements of rotation of plane of polarization for each comparison standard and conveyed the set to the next Participant. As soon as the measurements had been conducted, each participant sent the results and the corresponding uncertainty budget to the Pilot via e-mail. In the end, the comparison standards were sent back to the Pilot.

3 Description of transfer standards

The comparison standards are three quartz control plates (QCP) with different value of optical rotations (see Table 2). These standards were made in accordance with the recommendations stipulated in ICUMSA Specification and Standard SPS-1(2009).

Table 2. Parameters of the transfer standards used as the comparison artifacts

Manufacturer	Serial number	Optical rotation, wavelength of He-Ne laser (632.8 nm)
Bellingham Stanley Ltd.	PZ07028	29°
Bellingham Stanley Ltd.	PZ06033	-9°
Bellingham Stanley Ltd.	PZ07035	4.5°

4 Measurement results

In the current section the results received from all the participants are presented.

For analysis of results we have taken from the uncertainty budgets of participants the value of combined standard uncertainty instead the expanded uncertainty.

Angle of rotation of plane of polarization was measured at the wavelength of He-Ne laser (632.8 nm). It was possible to recalculate the results from another working wavelength to the wavelength of Ne-Ne laser.

In the Table 4.1 measurement results provided by all participants are shown.

In the table R_i is angle of rotation of plane of polarization (optical rotation in angular degree), $u(R_i)$ - its associated combined standard uncertainty, $u^s_{\mathrm{adj}}(R_i)$- adjusted uncertainty after correction by Mandel-Paule method, R_{CRV} - the comparison reference value (CRV), Δ_i - the difference from CRV, $U(\Delta_i)$ - the expanded uncertainty of the Δ_i with k=2. More explanations are given below in section 5. Tables are divided by three different color areas corresponding three standards.

After the table a graph of Δ_i for all standards is presented (Figures 4.1-4.3).

Table 4.1. Angle of rotation of plane of polarization measurement results, angular degree

Standards	Participants	R_i	$u(R_i) \times 10^3$ (k=1)	$u^s_{adj}(R_i) \times 10^3$	R_{CRV}	$\Delta_i \times 10^4$	$U(\Delta_i) \times 10^4$
PZ07028	**VNIIOFI**	**29.75724**	**0.488**	**0.488**	**29.757**	**1.3**	**7.1**
	PTB	**29.75721**	**0.260**	**0.374**		**0.9**	**4.2**
	Ukrmetrtest -standart	**29.75440**	**1.95**	**1.95**		**-27**	**38**
	GUM	**29.755**	**2.112**	**2.112**		**-21**	**42**
PZ06033	**VNIIOFI**	**-9.05373**	**0.488**	**0.623**	**-9.053**	**-8.9**	**7.0**
	PTB	**-9.05176**	**0.260**	**0.538**		**11**	**5.3**
	Ukrmetrtest -standart	**-9.05384**	**1.14**	**1.204**		**-10**	**21**
	GUM	**-9.055**	**2.063**	**2.099**		**-22**	**40**
PZ07035	**VNIIOFI**	**4.51170**	**0.488**	**0.745**	**4.512**	**-8.1**	**8.9**
	PTB	**4.51202**	**0.260**	**0.676**		**-4.9**	**7.4**
	Ukrmetrtest -standart	**4.51226**	**0.960**	**1.113**		**-2.5**	**16**
	GUM	**4.518**	**2.073**	**2.148**		**55**	**38**

Figure 4.1. Difference from CRV (Δ_i) with expanded uncertainties $U(\Delta_i)$ for quartz control plate PZ07028

Figure 4.2. Difference from CRV (Δ_i) with expanded uncertainties $U(\Delta_i)$ for quartz control plate PZ06033

Figure 4.3. Difference from CRV (Δ_i) with expanded uncertainties $U(\Delta_i)$ for quartz control plate PZ07035

5 Analysis

5.1 Calculation of the comparison reference value and its uncertainty

Preparation of this report was carried out according to Section 4 of "Guidelines for CCPR Key Comparison Report Preparation", CCPR Working Group on Key Comparisons, CCPR-G2 Rev.3, July 1, 2013.

We assume the total number of participants submitting a result is I. Each laboratory i reports a measured value, R_i, and its associated combined standard uncertainty $u(R_i)$. In the Tables 5.1 combined standard uncertainty are shown.

The comparison reference value (CRV) R_{CRV} for each reference standard is calculated using weighted mean with cut-off uncertainty. The cut-off value $u_{\text{cut-off}}$ is calculated by

$$u_{\text{cut-off}} = \text{average}\{u(R_i)\} \text{ for } u(R_i) \leq \text{median}\{ u(R_i)\};\ i=1,\ldots,I. \quad (1)$$

In the Tables 5.1 calculation cut-off results are shown.

The reported uncertainty $u(R_i)\}$ of each NMI i is adjusted by the cut-off,

$$u_{\text{adj}}(R_i) = u(R_i) \text{ for } u(R_i) > u_{\text{cut-off}}$$

$$u_{\text{adj}}(R_i) = u_{\text{cut-off}} \text{ for } u(R_i) \leq u_{\text{cut-off}},\ i=1,\ldots,I. \quad (2)$$

In the Tables 5.1 adjusted by the cut-off uncertainty $u_{\text{adj}}\ (R_i)\times 10^3$ are shown.

Table 5.1 Adjusted by the cut-off uncertainty $u_{\text{adj}}\ (R_i)$

Standards	Participants	$u(R_i)$ $\times 10^3$ $(k=1)$	$u_{\text{cut-off}}$ $\times 10^3$	$u_{\text{adj}}\ (R_i)$ $\times 10^3$	Original χ^2_{obs}	$s\times 10^4$	After s correction χ^2_{obs}
PZ07028	**VNIIOFI**	**0.488**	**0.374**	**0.488**	**3.08**	**0**	**3.08**
	PTB	**0.260**		**0.374**			
	Ukrmetrteststandart	**1.95**		**1.95**			
	GUM	**2.112**		**2.112**			
PZ06033	**VNIIOFI**	**0.488**	**0.374**	**0.488**	**17.10**	**3.866**	**7.815**
	PTB	**0.260**		**0.374**			
	Ukrmetrteststandart	**1.14**		**1.14**			
	GUM	**2.063**		**2.063**			
PZ07035	**VNIIOFI**	**0.488**	**0.374**	**0.488**	**8.20**	**5.629**	**7.815**
	PTB	**0.260**		**0.374**			
	Ukrmetrteststandart	**0.960**		**0.960**			
	GUM	**2.073**		**2.073**			

The normalised weight, w_i , for the result R_i is given by:

$$w_i = u_{\text{adj}}^{-2}(R_i) / \sum_{i=1}^{I} u_{\text{adj}}^{-2}(R_i) \ . \tag{3}$$

Then the weighted mean R_{CRV} is calculated:

$$R_{\text{CRV}} = \sum_{i=1}^{I} w_i \cdot R_i \ . \tag{4}$$

The uncertainty of the weighted mean is calculated by:

$$u(R_{\text{CRV}}) = \sqrt{\sum_{i=1}^{I} u^2(R_i) / u_{\text{adj}}^4(R_i)} \Bigg/ \sum_{i=1}^{I} u_{\text{adj}}^{-2}(R_i) \, . \tag{5}$$

The Chi-square value χ^2_{obs} is calculated for consistency check. i=1 represents the pilot lab:

$$\chi^2_{\text{obs}} = \sum_{i=1}^{I} \frac{(R_i - R_{\text{CRV}})^2}{u_{\text{adj}}^2(R_i)} \, . \tag{6}$$

If $\chi^2_{\text{obs}} \le \chi^2_{0,05}(\nu)$, consistency is satisfied. The value $\chi^2_{0,05}(\nu)$ is determined from Table 5.2 for $\nu = I - 1$. For different standards we have one case: I = 4 and $\chi^2_{0,05}(3)$ = 7.815.

Table 5.2. $\chi^2_{0,05}(\nu)$ values.

ν	$\chi^2_{0.05}(\nu)$
1	3.841
2	5.991
3	7.815
4	9.488
5	11.07
6	12.592
7	14.067
8	15.507
9	16.919
10	18.307
11	19.675
12	21.026
13	22.362
14	23.685
15	24.996
16	26.296
17	27.587
18	28.869

The original results of Chi-square test are shown in Table 5.1. Unfortunately for two standards $\chi^2_{\text{obs}} \geq \chi^2_{0,05}(\nu)$. These values are highlighted in red in Table 5.1.

In this case we used the Mandel-Paule method and added the s^2 term in eq. (2) as

$$u^s_{\text{adj}}(R_i) = \sqrt{u^2_{\text{adj}}(R_i) + s^2}. \quad (7)$$

The value of s was determined by iterative process so that $\chi^2_{\text{obs}} \approx \chi^2_{0,05}(\nu)$. The values of this additional "interlaboratory variance" $s \times 10^4$ are shown in Table 5.1. Values $u^s_{\text{adj}}(R_i) \times 10^3$ are placed in the Table 4.1.

After reaching consistency, we recalculated normalised weight, weighted mean R_{CRV} and uncertainty of the weighted mean from (3) to (5).

5.2 Calculation the difference from CRV

The difference from CRV, Δ_i, for a laboratory result R_i is calculated simply as

$$\Delta_i = R_i - R_{\text{CRV}} . \quad (8)$$

The expanded uncertainty of the difference from CRV with $k=2$ is calculated using

$$U(\Delta_i) = 2\sqrt{u^2(R_i) + u^2(R_{\text{CRV}}) - 2\left[\frac{u^2(R_i)}{u^2_{\text{adj}}(R_i)} \Big/ \sum_{i=1}^{I} u^{-2}_{\text{adj}}(R_i)\right]} \quad (9)$$

for results which contributed to the weighted mean.

6 Conclusion

The COOMET.PR-S2 comparison results are presented in Table 6.1.

Table 6.1. Comparison results

Standards	**Participants**	$\Delta_i \times 10^3$	$U(\Delta_i) \times 10^3$
PZ07028	**VNIIOFI**	**0.13**	**0.71**
	PTB	**0.09**	**0.42**
	Ukrmetrtest-standart	**-2.70**	**3.80**
	GUM	**-2.10**	**4.20**
PZ06033	**VNIIOFI**	**-0.89**	**0.70**
	PTB	**1.10**	**0.53**
	Ukrmetrtest-standart	**-1.00**	**2.10**
	GUM	**-2.20**	**4.00**
PZ07035	**VNIIOFI**	**-0.81**	**0.89**
	PTB	**-0.49**	**0.74**
	Ukrmetrtest - standart	**-0.25**	**1.60**
	GUM	**5.50**	**3.80**

APPENDIX

A Measurement procedure

Each Participant measures the angle of rotation of plane of polarization of the comparison standards utilizing their own methods and apparatus.

Before measurements the comparison standards should be inspected for damage. Any flaws should be documented by taking photographs and including them with comments into the appropriate form. The Pilot is to be informed immediately by e-mail.

Angle of rotation of plane of polarization is to be measured at the wavelength of He-Ne laser (632.8 nm).

If the Participant cannot conduct measurements at the aforesaid wavelength, they are allowed to recalculate their results from their working wavelength to the wavelength of Ne-Ne laser. The rotation of 1 mm quartz α for a wavelength λ (in μm) is given by:

$$\alpha(\lambda) = -0.1963657 + 7.262667\times\lambda^{-2} + 0.1171867\times\lambda^{-4} + 0.0019554\times\lambda^{-6}.$$

If a participant conducts measurements at a wavelength λ_1 he should recalculate results to 632.8 nm using the following expression:

$$\varphi_{632.8} = \varphi_{\lambda 1}\times\alpha(0.6328)/\alpha(\lambda_1),$$

where $\varphi_{632.8}$ – optical rotation at 632.8 nm, $\varphi_{\lambda 1}$ – optical rotation at λ_1.

The measured angle of rotation of plane of polarization is to be recalculated to the temperature 20°C. The temperature dependence of the rotation is described by the following formula, in which λ is a wavelength (in μm) and $\alpha_{20^\circ C}$ the rotation at 20°C:

$$\alpha_{20^\circ C} = \alpha_{t^\circ C}/\{1.0 + (a_1 + a_2\times\lambda^{-1} + a_3\times\lambda^{-2})(t - 20)\}$$

with $a_1 = 1.280924\times10^{-4}$, $a_2 = 1.0852636\times10^{-5}$, $a_3 = -9.0311692\times10^{-7}$.

Each Participant uses their own method for temperature measurement in their facilities.

A.1 All-Russian Research Institute for Optical and Physical Measurements (VNIIOFI, Russia)

1.1. Method of measurement the angle of rotation of the polarization plane

The principle of operation of the updated polarimetry standard is based on Malus' law. The intensity of linearly polarized light beam passed through a rotating analyzer can be described by the following expression:

$$s_1(t) = I_0 \{\cos^2[\alpha(t) + \varepsilon_1] + T_\perp\}, \qquad \text{(A.1.1)}$$

where I_0 is the light intensity at the analyzer input, α is the angle between the light polarization plane and direction of the analyzer, t – time, ε_1 is the phase of the signal, and $T_\perp$ is the transmission coefficient of the analyzer for $\varphi + \varepsilon_1 = \pi/2$. For uniform circular rotation of the analyzer, $\alpha(t) = \omega_0 t$, where ω_0 is the angular frequency. Then Eq.(A.1.1) can be rewritten in the form

$$s_1(t) = \frac{I_0}{2}\{1 + 2T_\perp + \cos[2\omega_0 t + 2\varepsilon_1] + T_\perp\}.$$

If a standard quartz control plate (QCP) is placed in front of the analyzer, then the light intensity at the output is given by

$$s_2(t) = \frac{I_0}{2}\{1 + 2T_\perp + \cos[2\omega_0 t + 2\varepsilon_2] + T_\perp\},$$

where ε_2 is the phase of the second signal. The desired angle $\Delta\varphi$ of optical rotation caused by QCP is given by

$$\Delta\varphi = \varepsilon_2 - \varepsilon_1.$$

Thus, the problem of the optical rotation angle measurement reduced to measuring the phase difference between two harmonic signals. This is the major task of phase measurement in electronics.

1.2. Description of measuring facility

The national standard of the optical rotation angle consists of:

• a digital polarimeter;

• a set of standard quartz control plate (QCP);

• a climate controlled chamber with active thermal stabilization and a multichannel digital thermometer with remote temperature sensors;

• a barometer for measuring the atmospheric pressure in the chamber;

• a hygrometer for measuring the humidity of the air in the chamber.

The multichannel digital thermometer with remote temperature sensors is calibrated by VNIIM absolutely with an uncertainty 5 mK.

The climate controlled chamber is controlled by an air-conditioning unit. The temperature of the ambient air is 20 ±0,5°C.

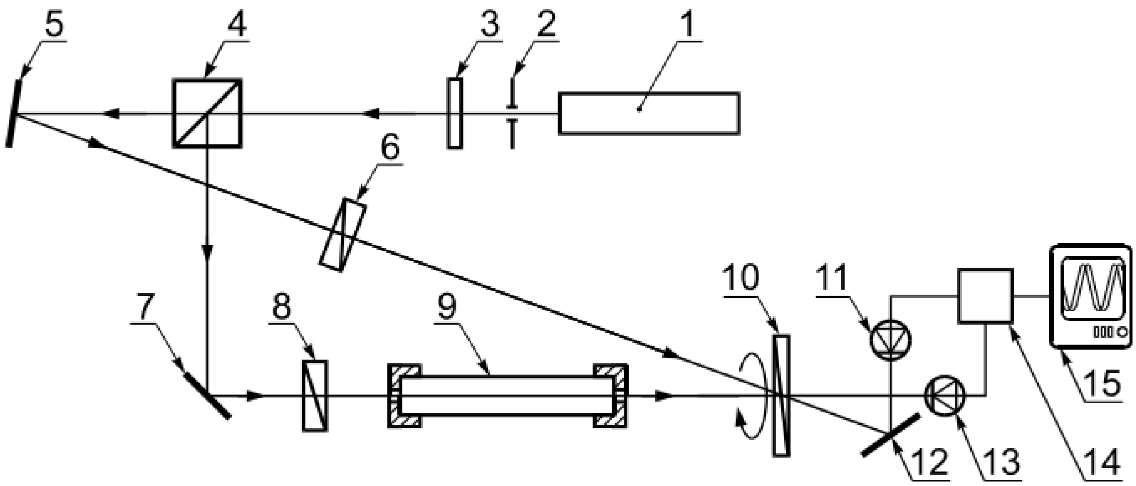


Fig.A.1.1. Optical setup of the digital polarimeter: 1 - light source (frequency-stabilized He–Ne laser); 2 - optical isolator; 3 - neutral density filter; 4 - beam splitter; 5, 7, 12 - mirrors; 6, 8 - polarizers; 9 - quartz control plate (QCP); 10 - rotating analyzer; 11, 13 - photodetectors; 14 - ADC; 15 - personal computer.

The optical setup (Fig.A.1.1) of the standard consists two photoelectric channels. A beam splitter 4 divides the light from He-Ne laser 1 into two beams. The first (object) laser beam passes through the Glan–Thomson polarizer 8, QCP 9, rotating analyzer 10 and is detected by a photodetector 13. The second (reference) beam passes through polarizer Glan–Thomson 6, rotating analyzer 10 and is detected by a photodetector 11. The rotating analyzer 10 is an ordinary thin film polarizer. The reference and object laser beams pass through the same part of the rotating analyzer 10, coincided with the axis of rotation, so that inhomogeneities of the polarizer will not affect the signals from the detectors 11 and 13.

In addition, since both beams pass through the same analyzer 10, any nonuniformity in its angular rotation velocity will have the same effect on the shape of the electrical signal from the detectors 11 and 13; therefore the phase shift between the signals remains constant. A high quality 2-channel analog-to-digital converter (ADC) 14 is used to digitize these signals, after that the signals are processed by personal computer 15.

The measurements are made in two stages. During the first stage (first exposure, as in interferometry), the QCP 9 is absent. Two signals are delivered to the computer through the ADC 14. A slight shift in the sinusoids is caused by a deviation in the direction of the polarization of the light after the polarizes 6 and 8, which remains fixed through all the measurements. We calculate the initial phase shift φ_{12} between the reference and object signals (Fig.A.1.2) as

$$\varphi_{12} = 2(\varphi_1 - \varphi_2),$$

where φ_1 and φ_2 are the initial phases of the reference and object signals.

After this, the QCP 9 is placed on the special mount of polarimeter and the second stage of measurements is made. Signals during the second exposure are shown in Fig.A.1.3. It can be seen that one signal has been shifted relative to the other. This happens because the QCP further rotates the polarization plane of the laser beam by angle $\Delta\varphi$. After the second exposure the following phase difference is measuring

$$\tilde{\varphi}_{12} = 2(\varphi_1 - \varphi_2 + \Delta\varphi) = 2\Delta\varphi + \varphi_{12}.$$

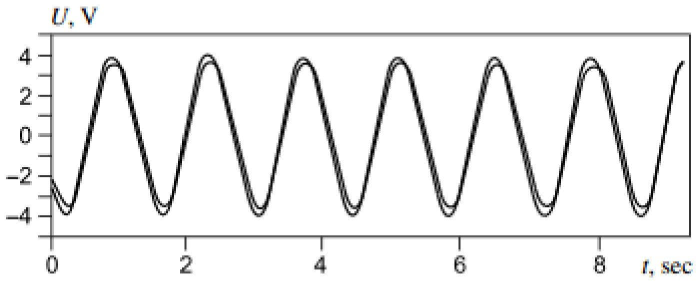


Fig.A.1.2. Plots of the reference and object signals. First exposure.

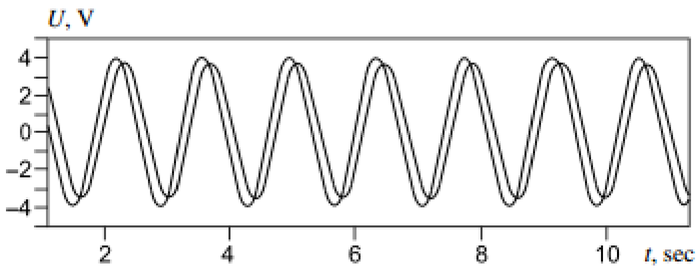


Fig.A.1.3. Plots of the reference and object signals. Second exposure.

Half of the difference between the phases of the first and second exposures yields the desired angle of optical rotation:

$$\Delta\varphi = \left(\tilde{\varphi}_{12} - \varphi_{12}\right)/2 \qquad \text{(A.1.2)}$$

For measuring the phase difference (A.1.2) we have used an algorithm based on a one-dimensional fast Fourier transform analogous to that used in interferometry and profilometry for automatic fringe counting.

1.3. Measurement results

Number of comparison standard	Optical rotation reduced to 20°C and 632.8 nm	Uncertainty of measurement
1 PZ07028	29.75724	0.488×10^{-3}
2 PZ06033	-9.05373	0.488×10^{-3}
3 PZ07035	4.51170	0.488×10^{-3}

1.4. Uncertainty budget

In the following table are listed the relevant contributions of uncertainty.

Table A.1.1

Uncertainty sources	**Standard Uncertainty Type A**	**Standard Uncertainty Type B**
Random errors of optical rotation angle measurement	0.00040°	
Quarts plate surface imperfection		0.00013º
Inclination of the quartz plate with respect to the polarimeter optical axis resulted from quartz plate casing imperfection		0.00003º
Inclination of the quartz plate with respect to the polarimeter optical axis resulted from unaccurate alignment of the mount of quartz plate casing		0.00003º
Optical rotation dispersion and laser wavelength instability		0.000006º
Temperature dependence of specific rotation and error of temperature measurement		0.00003º
Error of phase difference measurement of signals received from two photodetector		0.00024º
Combined uncertainty (Quadratic summation) Type B		0.00030º
Total combined standard uncertainty	0.000488º	
Expanded uncertainty (k=2)	0.001º	

A.2 PTB

2.1 Parameters to be determined

The measurement value is the angle of optical rotation, measured in degrees of arc at the wavelength of 632.8 nm (He-Ne Laser) and at the temperature of 20.00 ˚C. The optical rotation at other wavelengths is calculated with the "Keitel formula".

2.2 Measurement facilities and technical requirements

For the pre-check of purity, axis error, parallelism and flatness special optical facilities are used.

For the measurement of the optical rotation an automatic photoelectric polarimeter is used.

The optical setup consists of a light source, a He-Ne Laser with a wavelength of 632.8 nm, a $\lambda/4$-waveplate, a polarizer, coupled with a motor driven divided circle, two thermalized probe holders, a Faraday-modulator, a second polarizer as analyzer and a photomultiplier as detector. All components are mounted on a warp resistant stable breadboard.

The two polarizers are highest quality Glan-Tompson-Polarizers with an extinction of approx. 10^{-6}.

The divided circle, mounted together with the polarizer, is a precision type ERO 725 from Heidenhain with 36000 pitches and four measurement heads. The signals of the heads are interpolated by 1024. The average of the counts of the four heads is served with a resolution of 0.035 arcsec. The absolute accuracy is approx. 0.5 arcsec. The divided circle is mounted to a motor driven air bearing.

One of the thermalized probe holders contains the reference quartz with a length of 50 mm to monitor the possible variation of the wavelength and the temperature. The second holder contains the quartz plate to be measured. The holders are thermalized exactly to 20.00 ˚C with an external water circulation.

Both holders are mounted to a motor driven linear stage for positioning several measurement positions, so as zero position without quartz plate, position for the reference quartz and the position to measure the quartz plate. The holder contains also calibrated precision thermal sensors to measure the temperature of the reference quartz and the quartz plate under test very exactly. So the measured angle of rotation at ambient conditions can be calculated to 20.000 ˚C.

For a better resolution, the plane of polarization of the measurement beam is modulated with a Faraday-Modulator. The frequency is 130 Hz and the amplitude of modulation is approx. 0.7 ˚. So the point of extinction can be determined very exactly and more sensitive with a synchronic demodulator.

The photoelectric sensor is a photomultiplier (PMT) with a sufficient sensitivity in the point of the crossed position of the polarizer and analyzer.

The measurement room is controlled by an air-conditioning unit. The temperature of the ambient air is 20 ±0,5°C.

The probe holders are thermalized with an external thermostated water circulation. The constancy of temperature inside the holder nearby of the quartz plate and the reference quartz is 0.01 K over 12 hours.

The temperature measurement system TEMP 12 is calibrated by PTB absolutely with an uncertainty 5 mK.

2.3 Data to be recorded and evaluation of the data

The following data are recorded during a measurement and stored in a measurement file and in the measurement protocol:

- Angular position Zero point without QCP;
- Angular position of the 50 mm reference quartz;
- Angular position of the QCP in 4 azimuthal positions;
- Temperature of the 50 mm reference quartz;
- Temperature of the QCP;
- Room temperature.

2.4 Required reference standards

The wavelength of the measurement beam and the measurement temperature is observed through measuring the rotation angle of the 50 mm reference quartz. The actual measured angle of the reference quartz will be compared with the exactly stated and well known rotation angle and the recorded histories during each calibration.

To secure the good condition of the whole facility, the very well known QCP of PTB “IP 110” will be measured to validate the whole measurement facility in comparison with the history of “IP 110”.

2.5 Description of the measurement procedure

It is determined the difference of rotation angles of the zero point and the rotation angle with QCP, corrected in dependence from the wavelength and temperature (Zuckerindustrie 123 (1998) Nr. 5, 329-339).

2.6 Determination of correlations

The measured quantities are regarded as uncorrelated.

2.7 Measurement results

Number of comparison standard	Optical rotation reduced to 20°C and 632.8 nm	Uncertainty of measurement
1 PZ07028	29.75721	0.260×10^{-3}
2 PZ06033	-9.05176	0.260×10^{-3}
3 PZ07035	4.51202	0.260×10^{-3}

2.8 Uncertainty contributions of the input quantities and of the sensitivity coefficients

In the following table the relevant contributions of uncertainty are listed.

Amount	Standard measurement uncertainty	Distribution	Coefficient of sensitivity	Uncertainty contribution
Angle measurement	0.0001^o	Rectangle	1	0.0001^o
Alignment of extinction	0.0002^o	Rectangle	1	0.0002^o
Temperature	0.01K	Rectangle	0.01^o/K	0.0001^o
Wavelength	0.0008 nm	Normal distribution	0.1^o/nm	0.00008^o

2.9 Standard uncertainty and expanded uncertainty for *k*=2

For the calibration certificate, the expanded measurement uncertainty, which is obtained from the standard measurement uncertainty by multiplication by the coverage factor $k = 2$, is calculated in accordance with GUM. The total combined uncertainty of the angle of rotation amounts to 0.000260° and the expanded uncertainty (k=2) amounts to 0.000520°.

A.3 Ukrmetrteststandart

3.1 Description of measuring facility:

Type: Polarimeter, Standard Quartz Plates.

The method applied to measure the angle of rotation of plane of polarization: Comparison with Standard Quartz Plate.

The uncertainty claimed for the equipment: 3.2×10^{-3} (k = 2).

Description of how the comparison standard was adjusted: The beam was met the faces of the plate in their centre during the measurement.

3.2 Measurement results

Number of comparison standard	Angle of rotation of plane of polarization reduced to 20°C and 632,8 nm	Uncertainty of measurement (k = 2)
PZ06033	-9.05384	2.28E-03
PZ07028	29.75440	3.90E-03
PZ07035	4.51226	1.9E-03

3.3 Uncertainty budget

Number of comparison standard	PZ06033	
Standard uncertainty Contribution due to :	Standard Uncertainty Type A	Standard Uncertainty Type B
random errors for the angle of rotation of plane of polarization	2.49E-04	
uncertainty of Standard Quartz Plate		8.66E-04
uncertainty due to axis error		5.77E-04
uncertainty for the temperature of the quartz plate		4.00E-04
Combined uncertainty (Quadratic summation)	2.49E-04	1.12E-03
Total combined standard uncertainty	1.14E-03	
Expanded uncertainty (k=2)	2.28E-03	
Level of confidence	95%	

Number of comparison standard	PZ07035	
Standard uncertainty Contribution due to :	Standard Uncertainty Type A	Standard Uncertainty Type B
random errors for the angle of rotation of plane of polarization	1,87E-04	
uncertainty of Standard Quartz Plate		8,66E-04
uncertainty due to axis error		2,89E-04
uncertainty for the temperature of the quartz plate		2,31E-04
Combined uncertainty (Quadratic summation)	1,87E-04	9,42E-04
Total combined standard uncertainty	9.60E-04	
Expanded uncertainty (k=2)	1.92E-03	
Level of confidence	95%	

Number of comparison standard	PZ07028	
Standard uncertainty Contribution due to :	Standard Uncertainty Type A	Standard Uncertainty Type B
random errors for the angle of rotation of plane of polarization	2,38E-04	
uncertainty of Standard Quartz Plate		8,66E-04
uncertainty due to axis error		8,66E-04
uncertainty for the temperature of the quartz plate		1,50E-03
Combined uncertainty (Quadratic summation)	2,38E-04	1,94E-03
Total combined standard uncertainty	1,95E-03	
Expanded uncertainty (k=2)	3,90E-03	
Level of confidence	95%	

A.4 GUM

4.1 Description of measuring facility:

a. Type: Automatic Polarimeter POL S-2.

b. The method applied to measure optical rotation: Angle of rotation of plane of polarizatio n was measured at the wavelength λ=546.1 nm and the results were recalculated to the wa velength of He-Ne laser and to the temperature 20°C. Calibration results of the quartz con trol plates have been referred to the national standard of the unit of the optical rotation whi ch consists of the set of 5 quartz control plates, calibrated in PTB, Germany.

c. The uncertainty claimed for the equipment: 0.002°

d. Description of how the comparison standard was adjusted: Zero adjustment is determined by measuring the empty cell of polarimeter.

4.2 Measurement results

Number of comparison standard	Optical rotation reduced to 20°C and 632.8 nm	Expanded uncertainty of measurement (k = 2)
1 PZ07028	29.755	0.004
2 PZ06033	-9.055	0.004
3 PZ07035	4.518	0.004

4.3 Uncertainty budget

PZ07028

Standard uncertainty Contribution due to:	Standard Uncertainty Type A	Standard Uncertainty Type B
1) the experimental standard deviation of the mean $u\,(\alpha_{\text{śr}})$	0.000018170	
2) correction connected with the deviation of the measurement temperature of 20 °C $u\,(\delta\,\alpha_t)$		0.000094090
3) correction connected with the polarimeter, taken from the calibration certificate $u\,(\delta\,\alpha_p)$		0.002

Combined uncertainty (Quadratic summation)	0.000018170	0.002094090
Total combined standard uncertainty	0.002112260	
Expanded uncertainty (*k*=2)	0.004	
Level of confidence	95%	

PZ06033

Standard uncertainty Contribution due to:	Standard Uncertainty Type A	Standard Uncertainty Type B
1) the experimental standard deviation of the mean $u(\alpha_{śr})$ 2) correction connected with the deviation of the measurement temperature of 20 °C $u(\delta\alpha_t)$ 3) correction connected with the polarimeter, taken from the calibration certificate $u(\delta\alpha_p)$	0.000034087	 0.000028626 0.002
Combined uncertainty (Quadratic summation)	0.000034087	0.002028626
Total combined standard uncertainty	0.002062713	
Expanded uncertainty (*k*=2)	0.004	
Level of confidence	95%	

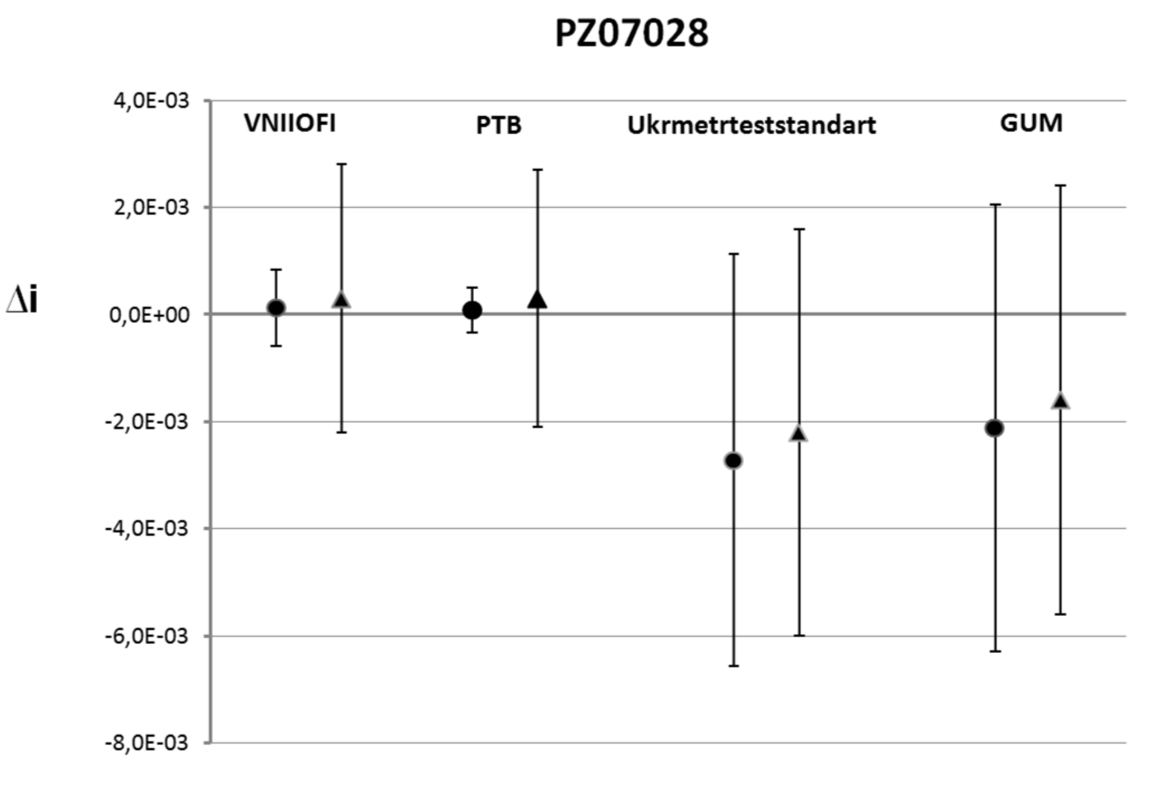
PZ07035

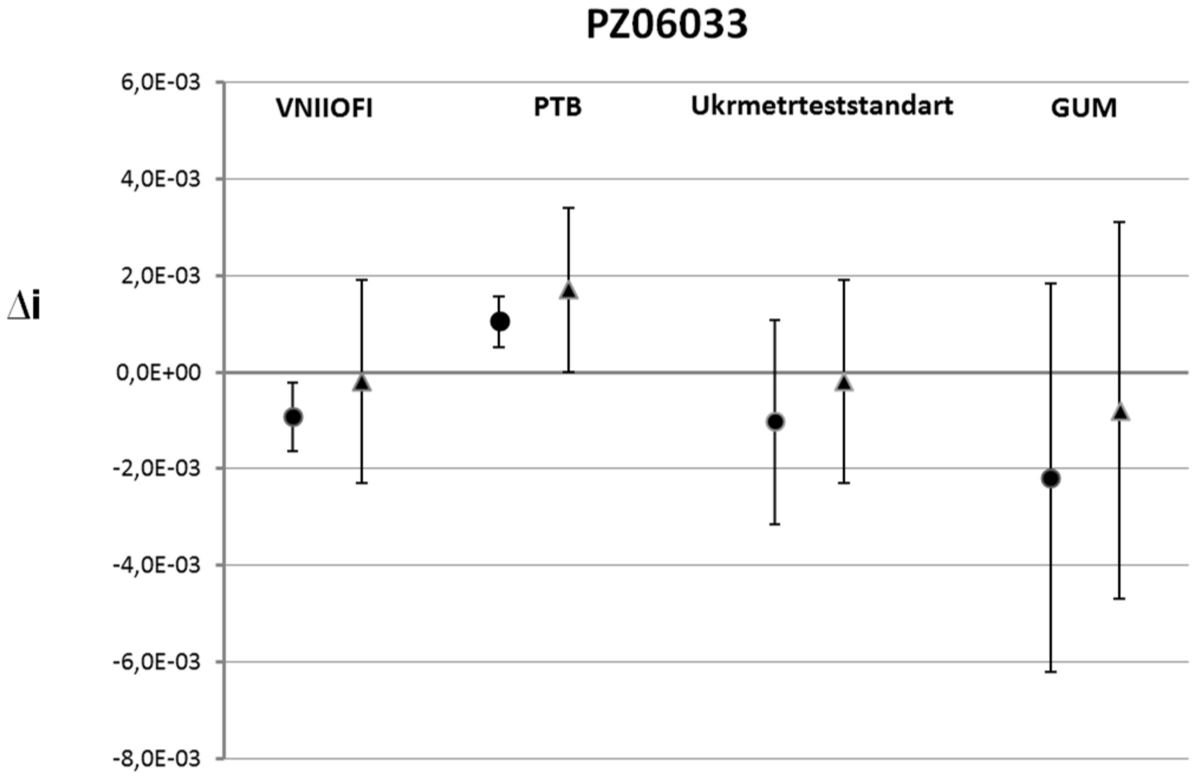
Standard uncertainty Contribution due to:	Standard Uncertainty Type A	Standard Uncertainty Type B
1) the experimental standard deviation of the mean $u(\alpha_{śr})$ 2) correction connected with the deviation of the measurement temperature of 20	0.000041937	 0.000031053

°C $u(\delta\ \alpha_t)$ 3) correction connected with the polarimeter, taken from the calibration certificate $u(\delta\ \alpha_p)$		0.002
Combined uncertainty (Quadratic summation)	0.000041937	0.002031053
Total combined standard uncertainty	0.002072990	
Expanded uncertainty (*k*=2)	0.004	
Level of confidence	95%	

B Alternative evaluation

An alternative evaluation based on the publication [1] (using model parameter m=2) was performed by PTB and resulted in slightly different values (see Figures B1).





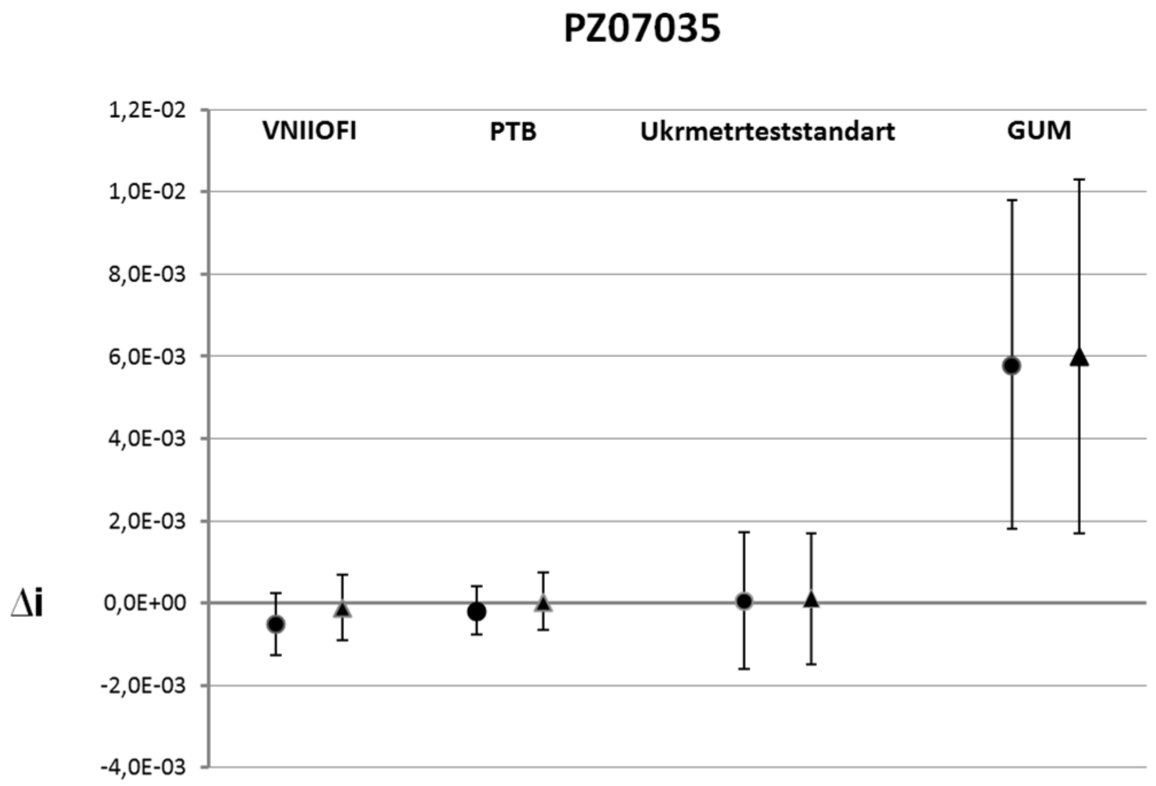


Figure B1. Difference from CRV (Δ_i) with expanded uncertainties $U(\Delta_i)$:

● - evaluation based on the "Guidelines for CCPR Key Comparison Report Preparation";

▲- evaluation based on the publication [1] carried out by PTB.

[1] Elster, C. and Toman, B. (2010). Analysis of key comparisons: estimating laboratories' biases by a fixed effects model using Bayesian model averaging. Metrologia, 47(3):113-119.