Guide to converting numerical equations into quantity equations for KCDB applications

When the KCDB was launched in the year 2000, the numerical equation format was chosen.

The KCDB was revised and extended in 2019 to include an interactive web platform. Several Consultative Committees requested the implementation of the quantity equation format in the revised KCDB, a format which is frequently used and required by accreditation bodies. This request was approved by CIPM members at the Consultative Committee President's meeting, which was held at the BIPM in June 2018 .

This document describes the format of quantity equations. It also gives some examples of how to transform numerical equations into quantity equations. It is based on guidance previously issued by the Consultative Committee for Length and has been adapted to fit a more general purpose.

Examples below are given for two terms to illustrate the general concept. Other examples including higher order and more complex relations follow the same general principles and are provided at the end of the document.

Numerical equation

CMCs are currently registered in the KCDB as numerical equations, such as:

$$U = \mathrm{Q}[49, 0.083L] \text{ nm, with } L \text{ in millimetres,}$$

which is a shorthand format for

$$U = \sqrt{49^2 + 0.083L^2} \text{ nm, with } L \text{ in millimetres.}$$

Quantity equation

The quantity equation for the same uncertainty, expressed in the shorthand format becomes

$$U = \mathrm{Q}[49 \text{ nm}, 83 \times 10^{-9}\, L]$$

which explicitly means

$$U = \sqrt{(49\ \mathrm{nm})^2 + (83\ \times\ 10^{-9}L)^2}\ .$$

The advantage of this representation is that the second term, in this example ‘83 × 10^{-9}’, represents a relative uncertainty. As such, the same equation may easily be applied to any unit for *L*. The first term carries a unit, in the example ‘nm’.

Converting a numerical equation into quantity equation format

The principle of the conversion from numerical to quantity equation format is given below long length, L.

Let

$$U = \sqrt{A^2 + (BL)^2}\ \mathrm{u}, \text{with } L \text{ in } \mathrm{v} \tag{1}$$

be a numerical equation, where A, B, L are dimensionless numbers and u and v are units of length.

Let

$$U = \sqrt{a^2 + (bl)^2} \tag{2}$$

be the desired corresponding quantity equation, where l is the quantity length whose value is expressed in L v.

The conversion is achieved by the two equations below:

$$a = A\,\mathrm{u} \tag{3}$$

$$b = B\,\frac{\mathrm{u}}{\mathrm{v}} \tag{4}$$

as is easily shown by substitution:

$$\sqrt{a^2 + (bl)^2} = \sqrt{(A\,\mathrm{u})^2 + \left(B\,\frac{\mathrm{u}}{\mathrm{v}}\,l\right)^2} = \sqrt{A^2 + \left(B\,\frac{l}{\mathrm{v}}\right)^2}\ \mathrm{u} = \sqrt{A^2 + (BL)^2}\ \mathrm{u}, \text{with } L \text{ in } \mathrm{v}$$

Example of conversion

Given the numerical equation

$$U = \sqrt{49^2 + (0.083L\,)^2}\ \text{nm with } L \text{ in millimetres}$$

then

$$A = 49, \quad B = 0.083, \quad \mathrm{u} = \mathrm{nm}, \quad \mathrm{v} = \mathrm{mm}$$

Therefore

$$a = A\,\mathrm{u} = 49\ \mathrm{nm}, \quad b = B\,\frac{\mathrm{u}}{\mathrm{v}} = 0.083\,\frac{\mathrm{nm}}{\mathrm{mm}} = 0.083 \times 10^{-6} = 83 \times 10^{-9}$$

resulting in the quantity equation

$$U = \sqrt{a^2 + (bl\,)^2} = \sqrt{(49\ \mathrm{nm})^2 + (83 \times 10^{-9} l\,)^2}$$

The following table gives some examples of equations presented in the former format and how the information should appear in the KCDB.

#	Initial format		Targeted format				
	Uncertainty	*U* unit	EQUATION	EQUATION comment	*U* unit	*U* min	*U* max
1	0.001*T*, *T* in K	mK	$1 \times 10^{-6}\, T$	*T* representing temperature	mK	*to complete*	*to complete*
2	(0.002 + 3E-05*p*), *p* in nm	nm	(0.002 nm + 3E-05*p*)	*p* representing pitch	nm	*to complete*	*to complete*
3	(25 + 5E-03*h*), *h* in µm	nm	(25 nm + 5E-6*h*)	*h* representing grating interval	nm	*to complete*	*to complete*
4	Q[40, 3*L*], *L* in m	µm	Q[40 µm , 3E-06*L*]	*L* representing pitch	µm	*to complete*	*to complete*
5	Q[22, 0.66*L*], *L* in mm	nm	Q[22 nm , 6.6E-7*L*]	*L* representing pitch	nm	*to complete*	*to complete*
6	Q[70, 0.06*L* , 14*P*], *L* in mm, *P* in µm	nm	Q[70 nm , 6E-8*L* , 1.4E-2*P*]	*L* representing pitch, *P* representing parallelism	nm	*to complete*	*to complete*
7	Q[2.4E-6/*L* , 2.2E-9 , 4.6E-04*α*], *L* in mm, *α* in 1/K	nm	Q[2.2E-9 nm , 2.4 nm^2/*L* , 4.6E-4*α* K nm]	*L* representing length, *α* representing thermal expansivity	nm	*to complete*	*to complete*
8	0.228/*t*	s/s	(0.228 s) /*t*	*t* representing time	s/s	*to complete*	*to complete*
9	(*Qm* + 0.05), *Qm* in g/min	%	(0.05 % + *Qm* %/ (g min^{-1}))	*Qm* representing flowrate	%	*to complete*	*to complete*
10	(0.005/*Qm* + 0.06), *Qm* in g/min	%	(0.06 % + 0.005 % (g min^{-1} /*Qm*))	*Qm* representing flowrate	%	*to complete*	*to complete*
11	0.10 / (1 - exp(-*q* /3.4)), with *q* in mL/h	%	0.1 % / [1 - exp(-*q* /(3.4 mL/h))]	*q* representing flowrate	%	*to complete*	*to complete*
12	0.0035 x RH-reading + 0.15 %rh	%rh	(0.15 %rh + 3.5E-3 × RH-reading)	%rh representing relative humidity	%rh	*to complete*	*to complete*