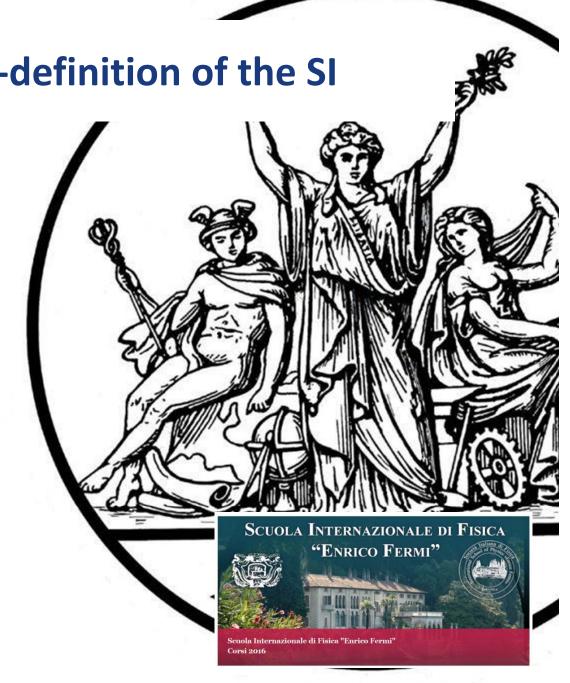
On the proposed re-definition of the SI

Martin Milton
Director, BIPM

Metrology Summer School, Varenna

Thursday 30th June 2016

Bureau
International des
Poids et
Mesures



# The International System of Units (SI)

#### **Prefixes**

#### Table 5. SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10 <sup>1</sup>	deca	da	10-1	deci	d
$10^{2}$	hecto	h	$10^{-2}$	centi	c
$10^{3}$	kilo	k	$10^{-3}$	milli	m
$10^{6}$	mega	M	$10^{-6}$	micro	μ
10 <sup>9</sup>	giga	G	$10^{-9}$	nano	n
$10^{12}$	tera	T	$10^{-12}$	pico	p
$10^{15}$	peta	P	$10^{-15}$	femto	f
$10^{18}$	exa	E	$10^{-18}$	atto	a
$10^{21}$	zetta	Z	$10^{-21}$	zepto	z
$10^{24}$	yotta	Y	$10^{-24}$	yocto	У



### **Base units**

Table 1. SI base units

Base quantity	SI base unit		
Name	Symbol	Name	Symbol
length	<i>l, x, r</i> , etc.	metre	m
mass	m	kilogram	kg
time, duration	1	second	S
electric current	<i>I, i</i>	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	$I_{\rm v}$	candela	cd

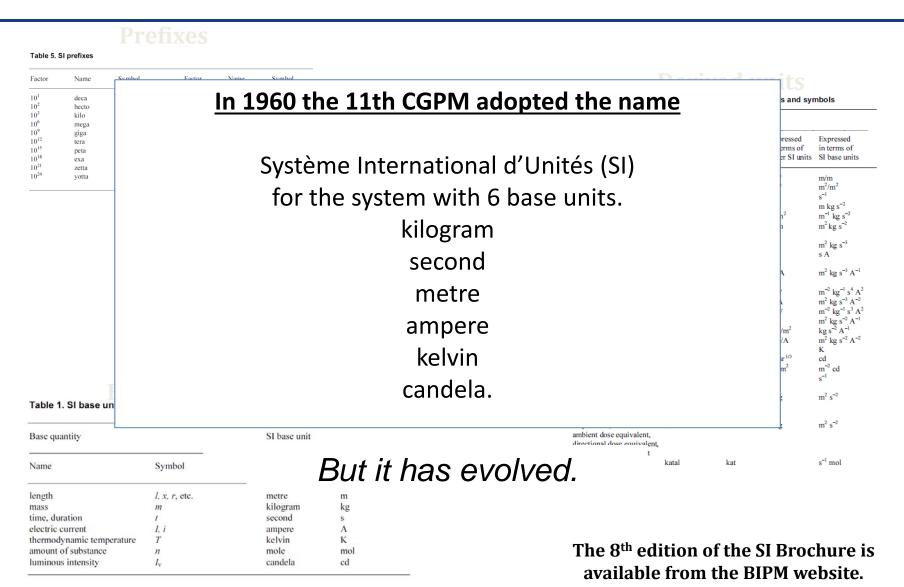
#### **Derived units**

Table 3. Coherent derived units in the SI with special names and symbols

	SI coherent derived unit (a)			
Derived quantity	Name	Symbol	Expressed in terms of other SI units	Expressed in terms of SI base units
plane angle	radian (b)	rad	1 <sup>(b)</sup>	m/m
solid angle	steradian (b)	sr (c)	1 (b)	$m^2/m^2$
frequency	hertz (d)	Hz		s <sup>-1</sup>
force	newton	N		m kg s <sup>-2</sup>
pressure, stress	pascal	Pa	N/m <sup>2</sup>	m <sup>-1</sup> kg s <sup>-2</sup>
energy, work, amount of heat	joule	J	N m	m² kg s <sup>-2</sup>
power, radiant flux	watt	W	J/s	m <sup>2</sup> kg s <sup>-3</sup>
electric charge, amount of electricity	coulomb	С		s A
electric potential difference, electromotive force	volt	V	W/A	m <sup>2</sup> kg s <sup>-3</sup> A <sup>-1</sup>
capacitance	farad	F	C/V	$m^{-2} kg^{-1} s^4 A^2$
electric resistance	ohm	Ω	V/A	m2 kg s-3 A-2
electric conductance	siemens	S	A/V	m <sup>-2</sup> kg <sup>-1</sup> s <sup>3</sup> A <sup>2</sup>
magnetic flux	weber	Wb	V s	m2 kg s-2 A-1
magnetic flux density	tesla	T	Wb/m <sup>2</sup>	kg s <sup>-2</sup> A <sup>-1</sup>
inductance	henry	H	Wb/A	m2 kg s-2 A-2
Celsius temperature	degree Celsius (e)	°C		K
luminous flux	lumen	lm	cd sr (c)	cd
illuminance	lux	lx	lm/m <sup>2</sup>	m <sup>-2</sup> cd
activity referred to a radionuclide (f)	becquerel (d)	Bq		s <sup>-1</sup>
absorbed dose, specific energy (imparted), kerma	gray	Gy	J/kg	m <sup>2</sup> s <sup>-2</sup>
dose equivalent, ambient dose equivalent, directional dose equivalent, personal dose equivalent	sievert (g)	Sv	J/kg	m <sup>2</sup> s <sup>-2</sup>
catalytic activity	katal	kat		s <sup>-1</sup> mol

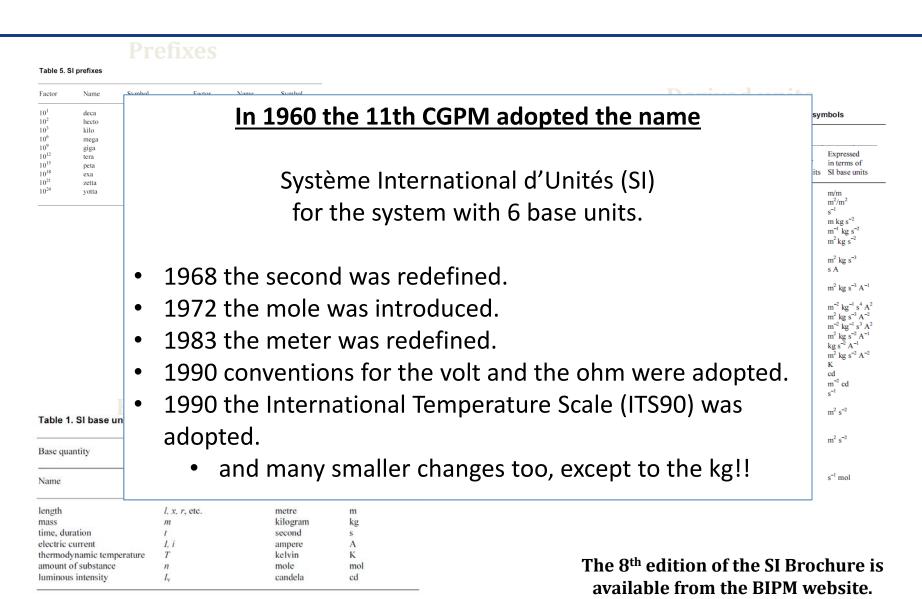
The 8<sup>th</sup> edition of the SI Brochure is available from the BIPM website.

# The International System of Units (SI)



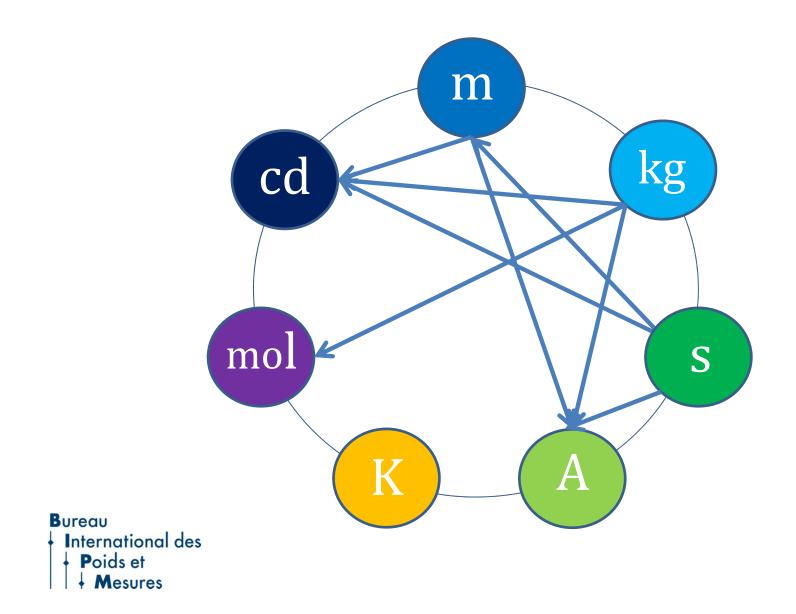
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# The International System of Units (SI)

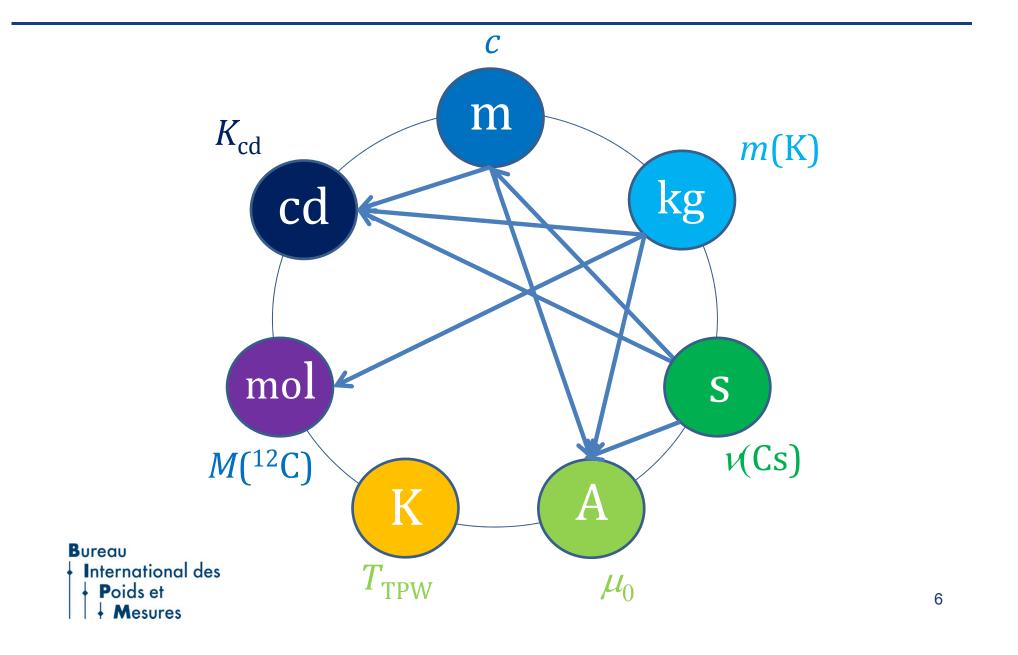


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## The base units of the SI



## The base units of the SI



### What are the references used to define the SI?

#### One is a fundamental constant.

• c is also the conversion factor between mass and energy or between length and time.

Three are just **conventions** that we should attribute a certain value to a certain material properties:

- T<sub>TPW</sub>
- ◆ M<sub>IPK</sub>

#### Three are actually **conversion factors**:

- $\mu_0$  from electrical to mechanical units
- ◆ K<sub>cd</sub> from luminous flux to luminous intensity
- ◆ M(¹²C) from mass to amount of substance.
- But we could have explained the same thing in other ways.



## A re-defintion of the SI is being proposed for 2018

### What will change?

- the ampere,
- the kilogram,
- the kelvin, and
- the mole.

## Why make the change?

- What will the consequences be?
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### How do we define the electrical units?

Why do we need the electrical units – don't the mechanical units give us everything we need?

Two laws link electrical units to mechanical units

- Coulomb's law
- Ampere's law

$$F = k_1 \frac{qq'}{r^2}$$

$$\frac{dF}{dl} = 2k_2 \frac{II'}{r}$$

Two equations that link mechanical to electrical units through a proportionality constant that depends on the choise of units system.

Using Maxwell's equations we can show that

$$\frac{k_1}{k_2} = c^2$$

We can either fix  $k_1$  or  $k_1$ .

### How do we define the electrical units? – in the SI

### The definition of the ampere gives us

Ampere's law

$$\frac{dF}{dl} = \frac{\mu_0}{2\pi} \frac{II'}{r}$$

Coulomb's law

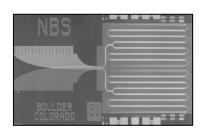
$$F = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2}$$

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular crosssection, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newton per metre of length.

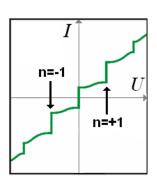
But, since 1990, macroscopic quantum effects have been the basis for the reproduction of the electrical units

# Since 1990, macroscopic quantum effects have been the basis for the reproduction of the electrical units

### **Josephson effect**

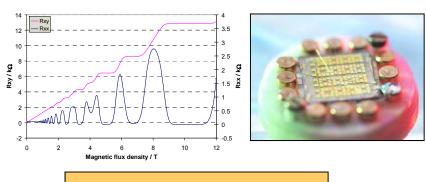






$$U_{\rm J} = n \frac{f}{K_{\rm J}}, \qquad K_{\rm J} = \frac{2e}{h}$$

### **Quantum-Hall effect**



$$R_{\mathrm{H}}(i) = \frac{R_{\mathrm{K}}}{i}, \quad R_{\mathrm{K}} = \frac{h}{e^2}$$

• Excellent reproducibility has underpinned the worldwide uniformity of electrical units since 1990.

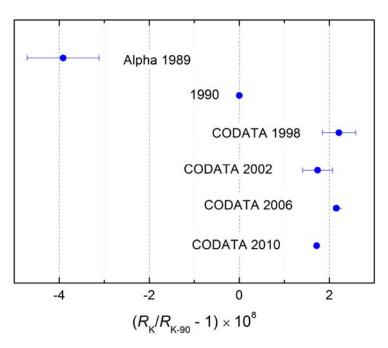
$$K_{1-90} \equiv 483 597.9 \text{ GHz/V}$$

$$R_{\text{K-90}} \equiv 25 \ 812.807 \ \Omega$$

• But: not within the SI ( $\mu_0 \equiv 4\pi \cdot 10^{-7}$  N A<sup>-2</sup>) because "conventional values"  $K_{J-90}$  and  $R_{K-90}$  were adopted in 1990.

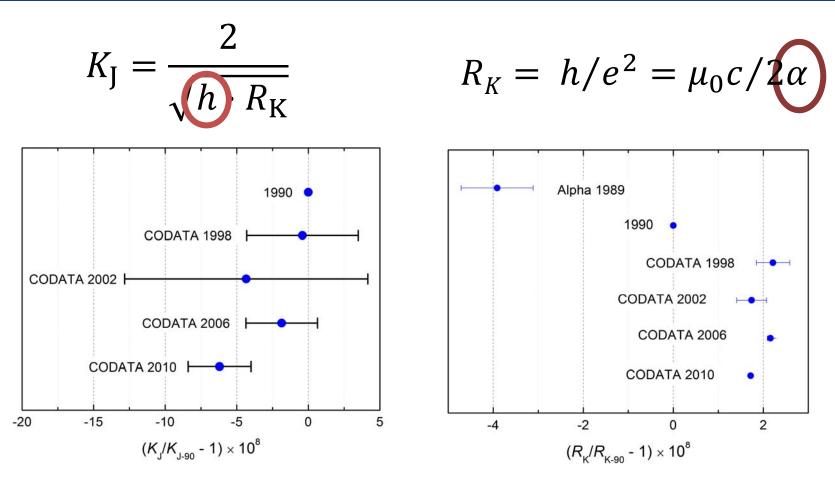
### The success of the 1990 convention

$$R_K = h/e^2 = \mu_0 c/2\alpha$$



NB Standard uncertainties (not expanded k=2)

## The success of the 1990 convention



NB Standard uncertainties (not expanded k=2)

# **But** – there is another way to link electrical units to mechanical units

- An experiment that links electrical power to mechanical power.
- The « moving coil <u>watt balance</u> »
- Now called the Kibble Balance.

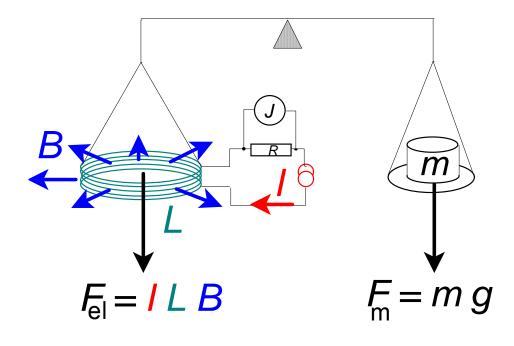


Bryan Kibble (1938 - 2016)



## The Kibble balance principal – the static phase

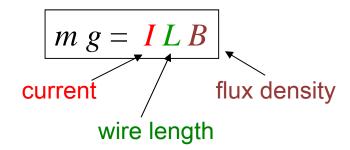
# Phase 1: static experiment (weighing mode)



### **Ampere's Law**

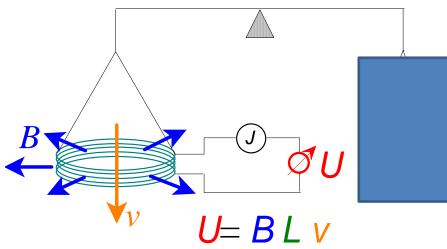
$$mg = -I\frac{d\Phi}{dz}$$

In a radial magnetic field, this can be simplified to



## The Kibble balance principal – the moving phase

Phase 2: dynamic experiment (moving mode)

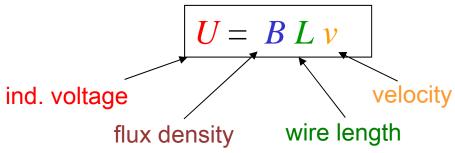


Coil is moved through the magnetic field and a voltage is induced.

### Faraday's Law

$$U = -\frac{d\Phi}{dt} = -v\frac{d\Phi}{dz}$$

In a radial magnetic field, this can be simplified to



## The Kibble Balance equations together

In the static phase

$$mg = IBL$$

In the dynamic phase

$$U = v B L$$

If the coil and the field are constant:

$$UI = m g v$$

- An experiment that sets electrical power = mechanical power
- And does not involve the magnetic field (B) and hence not  $\mu_0$ .
- Note: the Kibble Balance does not realize a direct conversion of electrical to mechanical energy.

$$Mg = -i \frac{\partial \overline{\Phi}}{\partial y} . \tag{1}$$

Suppose that the coil, in a separate measurement, moves with velocity dy/dt in the same flux  $\Phi$ . Then an e.m.f. V' is generated,

$$V' = -\frac{\partial \Phi}{\partial t} = -\frac{\partial \Phi}{\partial y} \frac{dy}{dt}$$
. (2)

Eliminating  $\partial \Phi / \partial y$  between (1) and (2), we have

$$M_{\varphi} \frac{dy}{dt} = i V' = \frac{V V'}{R}$$
 (3)

where i is known in terms of the potential drop V it produces across a resistor R. V, V and R would be measured in the maintained units of the laboratory; these are related to the

B.P. Kibble, Division of Electrical Science, National Physical Laboratory, "A measurement of the gyromagnetic ratio of the proton by the strong field method", Atomic Masses and Fundamental Constants 5, Sanders J. H. and Wapstra A. H., Eds., Plenum Press, 1976, pages 549 and 550.

# Bringing in the electrical quantum effects - a link between the kg and the Planck constant

U is measured using the Josephson effect. I is measured using  $\underline{U}_2/R$  with the Josephson and the quantum Hall effects.

$$UI = U_1 \frac{U_2}{R} = (h/2e)^2 / (h/e^2) f_1 f_2 = \frac{h}{4} f_1 f_2$$

• Assuming the exactness of the formluae for  $K_J$  and  $R_K$ 

# Bringing in the electrical quantum effects - a link between the kg and the Planck constant

*U* is measured using the Josephson effect.

I is measured using V/R with the Josephson and the quantum Hall effects.

$$UI = U_1 \frac{U_2}{R} = (h/2e)^2 / (h/e^2) f_1 f_2 = \frac{h}{4} f_1 f_2$$

• Assuming the exactness of the formluae for  $K_1$  and  $R_K$ 

$$m g v = \frac{h}{4} f_1 f_2$$

- No dependence on  $\mu_0$
- A possible basis for a defintion of the kg?
- if we can measure h with an uncertainty of some parts in 108.

## A re-definition of the SI is being proposed for 2018

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- the kilogram,
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- the mole.

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- What will the consequences be?
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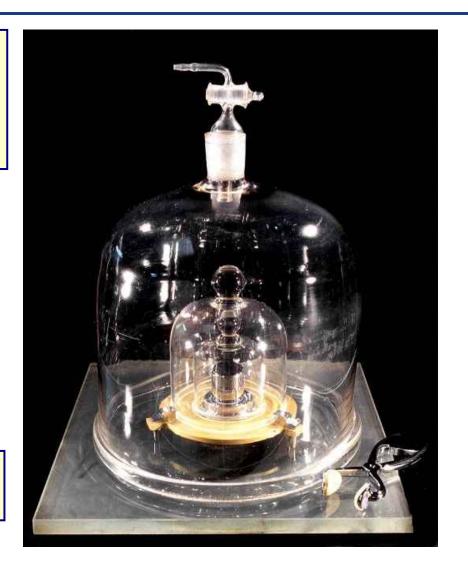
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# The definition of the kilogram in the SI

The kilogram is the unit of mass - it is equal to the mass of the international prototype of the kilogram.

- manufactured around 1880 and ratified in 1889
- represents the mass of 1 dm<sup>3</sup> of H<sub>2</sub>O at its maximum density (4 °C)
- alloy of 90% Pt and 10% Ir
- cylindrical shape,  $\emptyset = h \sim 39 \text{ mm}$
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.



### But

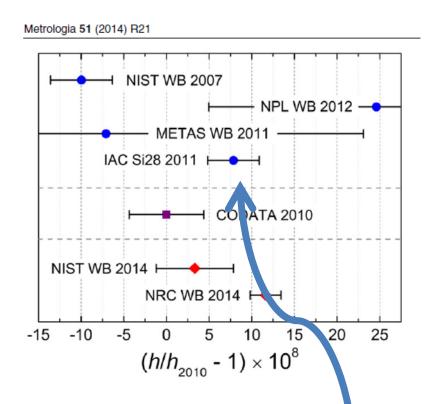
• We just discussed how we could define the kg using:

$$m g v = \frac{h}{4} f_1 f_2$$

- If the electrical units are defined through  $K_J$  and  $R_K$  then the KB gives h.
- If we can measure h with an uncertainty of some parts in  $10^8$ .
- ◆ Then the same Kibble Balance (used in reverse) can define the kilogram to some part in 10<sup>8</sup> if we fix the Planck Constant.

# Why did'nt we agree to implement this many years ago??

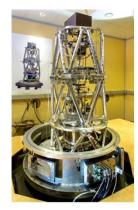
# It has not been easy to agree on the best value of the Planck constant



Many Kibble balances have been commissioned to resolve the discrepancy – and hence to realise the kg.







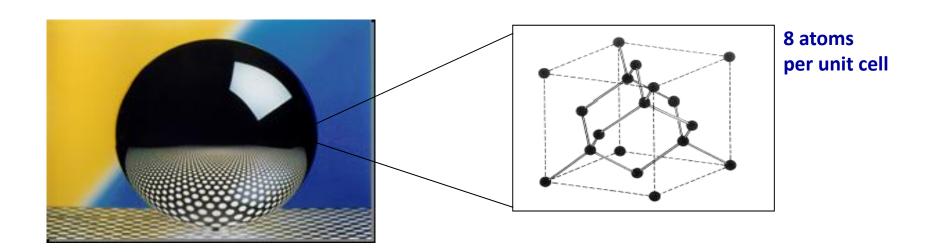






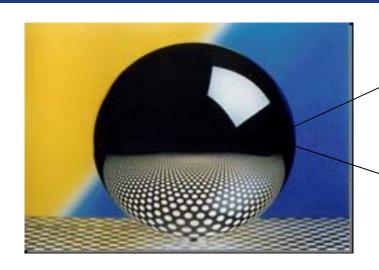
Values for *h* are available from other methods, including one that can be used to realised the kg.

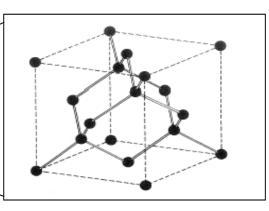
# X-ray crystal density technique (XRCD)



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## X-ray crystal density technique (XRCD)





8 atoms per unit cell

Uncertainty budget (2014)

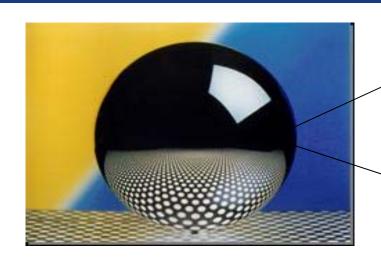


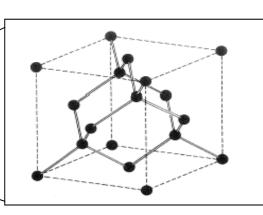
#### Only one sphere (AVO28-S5c):

Quantity	Relative uncertainty/10 <sup>-9</sup>	Contribution/%
Molar mass	5	6
Lattice parameter	5	6
Surface	10	23
Sphere volume	16	59
Sphere mass	4	4
Point defects	3	2
Total	21	100

$$M = N \overline{m}_{Si} = \frac{8V}{a_0^3} \overline{m}_{Si}$$
$$= \frac{8V}{a_0^3} \frac{\overline{M}}{N_A}$$

## X-ray crystal density technique (XRCD)





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 $M = N \overline{m}_{Si} = \frac{8V}{a_0^3} \overline{m}_{Si}$  $= \frac{8V}{a_0^3} \frac{\overline{M}}{N_A}$ 

 $N_A$  can be converted to a measurement of h because of our knowledge of the Bohr atom.

$$h.N_A = \frac{cA(e)_r M_u \alpha^2}{2R_m}$$

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- the ampere,
- the kilogram,
- the kelvin, and
- the mole.

## Why make the change?

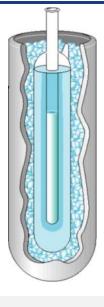
- What will the consequences be?
- + How should we present the changes?

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## The base unit of temperature - kelvin

The 1954 definition

$$T_{\text{TPW}} = 273.16 \text{ K}$$



### **Some limitations**

- Defines only one temperature,
- Based on uncontaminated(?)
   water, and a specified isotopic
   content,
- Influenced by: gradients, annealing etc.

**Note**: The ITS-90 is decoupled from the present definition of the kelvin.

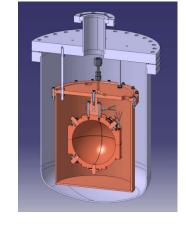
If an energy E is measured at a thermodynamic temperature T and if E is described by a function f(kT)

- At present, k is determined from  $E = f(kT_{TPW}) : T_{TPW}$  is exact.
- In the new SI, T measured from E = f(kT): k is exact.

### New measurements of the Boltzmann constant

- Acoustic Gas Thermometry (NPL, LNE-INM, INRIM, CEM, NIM...)
- Dielectric Constant Gas Thermometry (PTB, ...)
- Johnson Noise Thermometry (NIST,...)
- Doppler-Broadening Thermometry (Univ. Paris N./LNE-INM, DFM,...)





LNE-INM

- The Consultative Committee for Thermometry (CCT) recommends:
  - achieve an uncertainty of ≤ 1 ppm in k (~0.3 mK at  $T_{TPW}$ ), ideally with confirmation by different methods.

NPL

It seems that this goal is well within reach.

Michael de Podesta's talk

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## The mole – based on the Avogadro constant

### 1971

"The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogramme of carbon 12.

When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, or other particles, or specified groups of such particles".

But ..

The mole is widely thought to be defined by the Avogadro constant.

## The mole – based on the Avogadro constant

### 1971

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### 2018

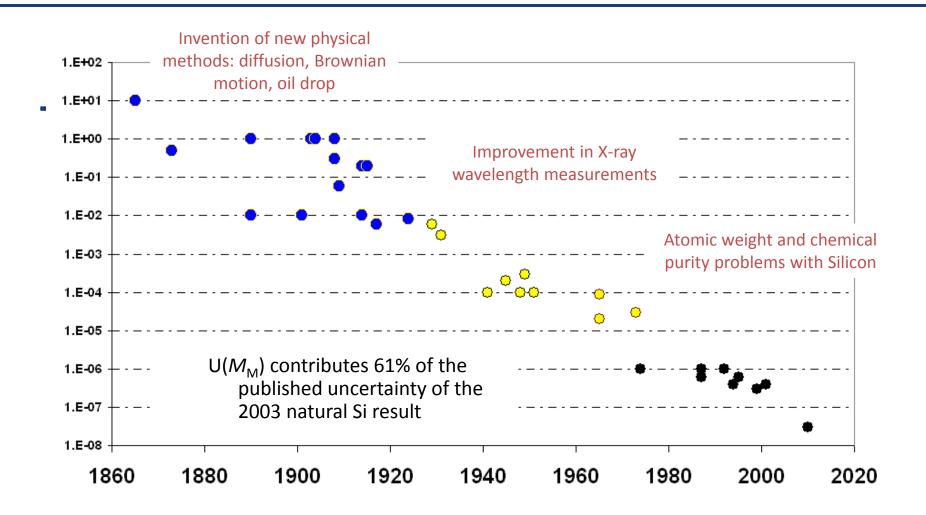
"The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles.

It is defined by taking the fixed numerical value of the Avogadro constant  $N_A$  to be 6.022 140 857 ×10<sup>23</sup> when expressed in the unit mol<sup>-1</sup>. "

## The mole – based on the Avogadro constant

- The proposed new definition of the mole would "reverse" the present definition
  - specify the number of entities in one mole
    - ◆ equal to N<sub>A</sub> exactly.
  - add some uncertainty in the mass of one mole
    - one mole of carbon-12 = 12g +/-  $u(a^2)$ .
- The molar masses and the atomic masses will have the same (relative) uncertainties.
- A single entity will be an exact amount of substance.
- The old and new definitions will be the same in practice
  - to within +/-  $u(a^2)$

## The Avogadro constant



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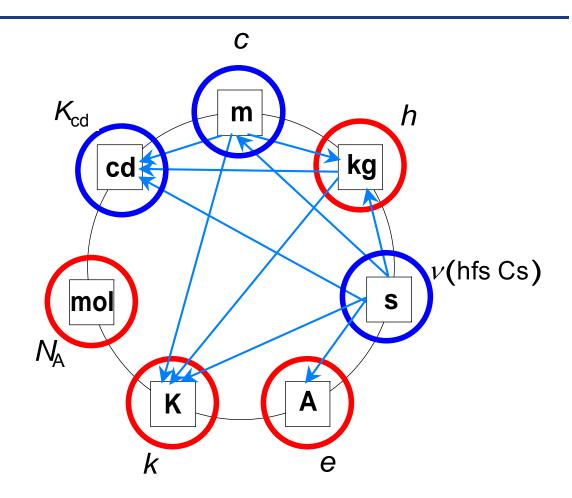
#### Proposal for an SI, with 4 new definitions

# Definitions based on fundamental (or conventional) constants:

- metre (*c*)
- kilogram (h)
- ampere (*e*)
- candela (K<sub>cd</sub>)
- mole (N<sub>A</sub>)
- kelvin (*k*)

# Definition based on material property:

• second (133Cs)



Bureau

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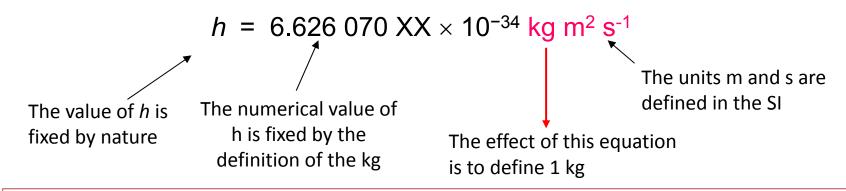
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(I. Mills et al., *Metrologia*, 2006, 43, 227-246)

# Proposed new definitions for the kilogram

The kilogram, unit of mass, is defined by taking the fixed numerical value of the Planck constant h to be 6.626 070 XX  $\times 10^{-34}$  when expressed in the unit J s which is equal to kg m<sup>2</sup> s<sup>-1</sup> where the metre and the second are defined in terms of c and  $\Delta v_{cs}$ .



#### How would this work in practice?

- The watt balance equates electrical and mechanical power
  - electrical power can be expressed in terms of h using the Josephson and quantum
     Hall effects
- The "Avogadro" Experiment determines the mass of a single <sup>28</sup>Si atom
  - $-m_{\rm u}$  can be expressed in terms of h using extremely accurate measurements of the Rydberg constant.

# Proposed new definition for the ampere

"The ampere ... is defined by taking the fixed numerical value of the elementary charge e to be 1.602 176 620 8  $\times 10^{-19}$  when expressed in the unit C, which is equal to A s, where the second is defined in terms of  $\Delta v_{Cs}$ .

#### How would this work in practice?

Since *h* is fixed by the definition of the kilogram and *e* by the definition of the ampere, then we also have an impedence and a voltage standard because:

- $\triangleright$  The quantum Hall effect defines an impedence in terms of  $h/e^2$
- > The Josephson effects defines a voltage in terms of 2e/h

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# What will the consequences be?

In order to ensure that there is no change in the kg as disseminated to users, the CCM (Consultative Committee for Mass) made the following recommendation in 2010 (and confirmed it in February 2013):

- 1. Condition on measurements of the Planck constant to be met before redefining the kilogram:
  - i. at least 3 independent results (eg watt balance and XRCD) with  $u_r < 5 \times 10^{-8}$
  - ii. at least 1 result with  $u_r \le 2 \times 10^{-8}$
  - iii. results consistent
- 2. Traceability to the IPK of BIPM working standards and of mass standards used to determine *h* needs to be re-established ("Extraordinary Calibrations")
- 3. A *mise-en-pratique* for the definition of the kilogram to be agreed.



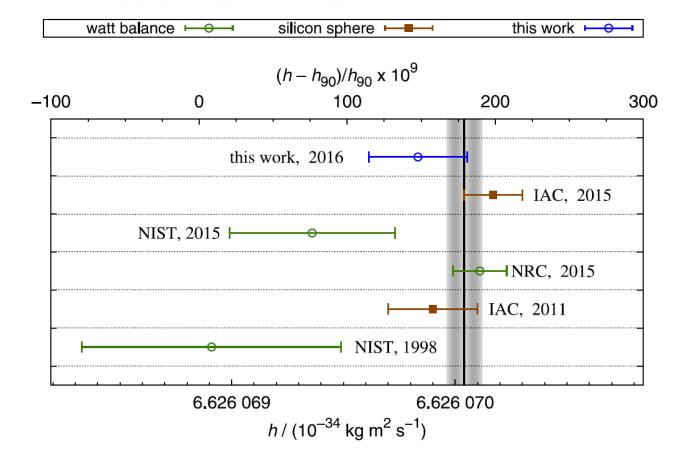


# Invited Article: A precise instrument to determine the Planck constant, and the future kilogram

D. Haddad,<sup>1,2,a)</sup> F. Seifert,<sup>1,2</sup> L. S. Chao,<sup>1</sup> S. Li,<sup>1,b)</sup> D. B. Newell,<sup>1</sup> J. R. Pratt,<sup>1</sup> C. Williams,<sup>1,2</sup> and S. Schlamminger<sup>1,c)</sup>

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#### What will the consequences be?

#### Mass metrology:

- mass values will not change  $m(IPK)_{new} = m(IPK)_{present} \equiv 1 \text{ kg}$
- mass uncertainties will increase  $u_r(m(IPK)_{new})$  approx 2 x 10<sup>-8</sup>  $u_r(m(IPK)_{present}) = 0$

#### **Electrical metrology:**

When the 1990 values are replaced, small step changes are inevitable

- The relative change from  $R_{K-90}$  to  $R_{K}$  will be of the order 2×10<sup>-8</sup>
- The relative change from  $K_{J-90}$  to  $K_J$  will be of the order  $1 \times 10^{-7}$
- The changes should only be visible to labs operating primary quantum standards; calibrations of even the most stable standard resistors and Zener references should be largely unaffected

#### What is the motivation for the new definitions?

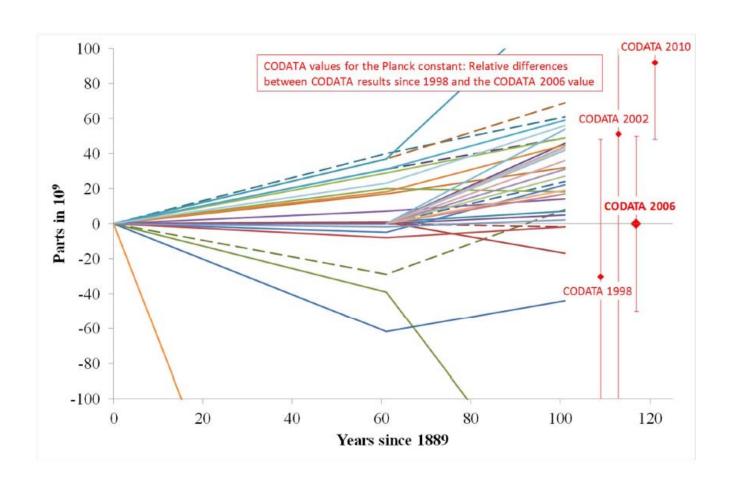
#### Various motivations have been articulated:

- 1. To solve the "kg problem".
- 2. To bring the electrical units back into the SI.
- 3. To reduce the uncertainty of certain fundamental constants.
- 4. Because it is a great ambition from the 19<sup>th</sup> century:

"If, then, we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules."

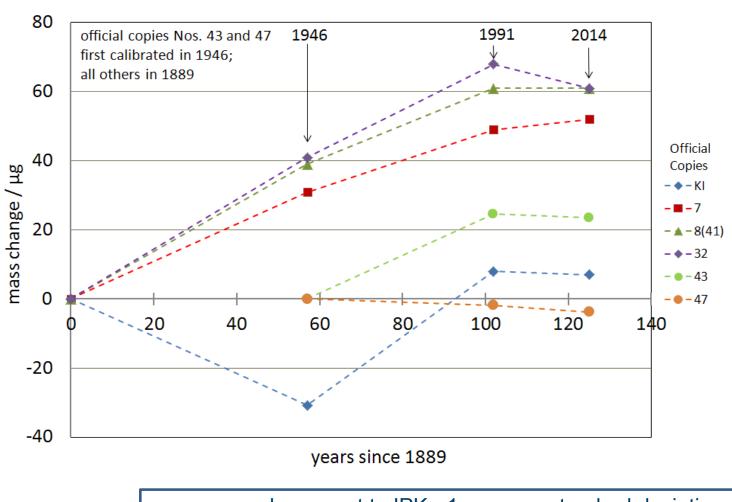
James Clerk Maxwell, 1870

# Why make the change ? – the IPK





# Why make the change ? – the IPK



Bureau
International des
Poids et
Mesures

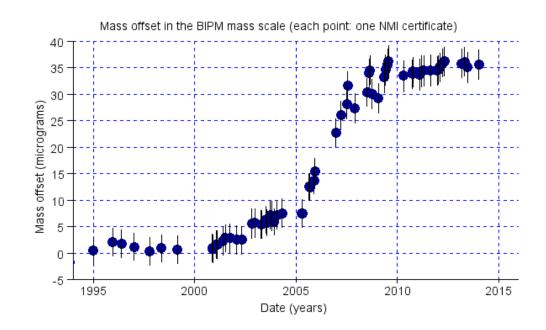
average change wrt to IPK: -1 μg

standard deviation: 3 μg

IPK and six official copies form a very consistent set of mass standards

# Why make the change ? - the BIPM "as-maintained" mass unit.

- Since the IPK is only accessible at the time of agreed *Periodic Verifications*, the BIPM must maintain a mass unit in its laboratoires using working standards – « the BIPM asmaintained mass unit »
- This was last traceable to the IPK in 1992.



 As a result of the 2014 measurements with the IPK, the BIPM as-maintained mass unit has been found to be 35 mg different from the IPK:

$$m(X)$$
maintained –  $m(X)$ IPK = + 35 mg

- All BIPM working standards have lost mass wrt to the IPK since 1992 (3rd PV), between 18 mg and 88 mg
- The relative drift within the set of working standards had been noticed by BIPM, but not the common drift (because IPK was not available)
- The undetected common drift has led to the offset of the BIPM mass unit

# How can we explain the new definitions?

- ◆ The new definitions will "facilitate universality of access to the agreed basis for worldwide measurements".
  - This has been an ambition for the "metric system" that goes back more than 200 years. The 2018 definitions will make it possible for the first time.
- The changes will underpin future requirements for increases in accuracy
  - As science and technology advances, the demands for the accuracy of measurements will continue to increase accuracy. The 2018 definitions will provide for these needs for many years to come.
- The new definitions use "the rules of nature to create the rules of measurement".
  - The use of constants in nature enable you to link from the smallest to the largest measurements quantities. It will tie measurements at the atomic (and quantum) scales to those at the macroscopic level. This introduces the appeal of a fundamental ("quantum") basis for the changes.



#### **Conclusions**

- ◆ The principle of the re-definition of four base units was approved in 2011.
- It will be based on a redefinition of the kilogram, ampere, kelvin and mole based on fixed values for:
  - The Planck constant
  - The elementary charge
  - The Boltzmann constant
  - The Avogadro constant
- A roadmap has been developed to coordinate all technical and awareness activities,
  - All activities appear to be on target for re-definition in 2018.