THEORY OF THE LAMB SHIFT IN MUONIC HYDROGEN

Savely Karshenboim

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&
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MAX-PLANCK-INSTITUTE
OF QUANTUM OPTICS
GARCHING

Outline

- Level structure
- QED
  - Unperturbed energy levels
  - Specific QED
  - Re-scaled QED
  - Proton-line QED
  - Hadronic vacuum polarization
- Proton structure
  - Leading term
  - External field
  - Two-photon exchange
  - Recoil proton-size
  - Proton polarizability
- Comparison of theory and experiment
  - Proton radius

Outline

- Level structure
  - QED
  - Specific QED
  - Re-scaled QED
- Proton structure
  - Leading term
  - Ex
  - Pr
- Comp

PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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(Received 28 August 1995)

Michael I. Eides Howard Grotch Valery A. Shelyuto

Theory of Light Hydrogenic Bound States

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journal homepage: www.elsevier.c

Lamb shift in muonic hydrogen—I. Veri of theoretical predictions

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Annals of Physics 327 (2012) 733–763
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Annals of Physics
journal homepage: www.elsevier.com/locate/aop

Lamb shift in light muonic atoms — Revisited

E. Borie *
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D-76344 Eggenstein-Leopoldshafen, Germany

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Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen

Aldo Antognini [a,*], Franz Kottmann [a], François Biraben [b], Paul Indelicato [b], François Nez [b], Randolf Pohl [c]
[a] *Institute for Particle Physics, ETH Zurich, 8093 Zurich, Switzerland*
[b] *Laboratoire Kastler Brossel, École Normale Supérieure, CNRS and Université P. et M. Curie, 75252 Paris, CEDEX 05, France*
[c] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*

Unperturbed energy levels: effective Dirac equation

- Effective-Dirac-equation approach

$$E = m_r(f_D - 1) - \frac{m_r^2}{2(M+m)}(f_D - 1)^2$$

- Breit-Hamiltonian approach

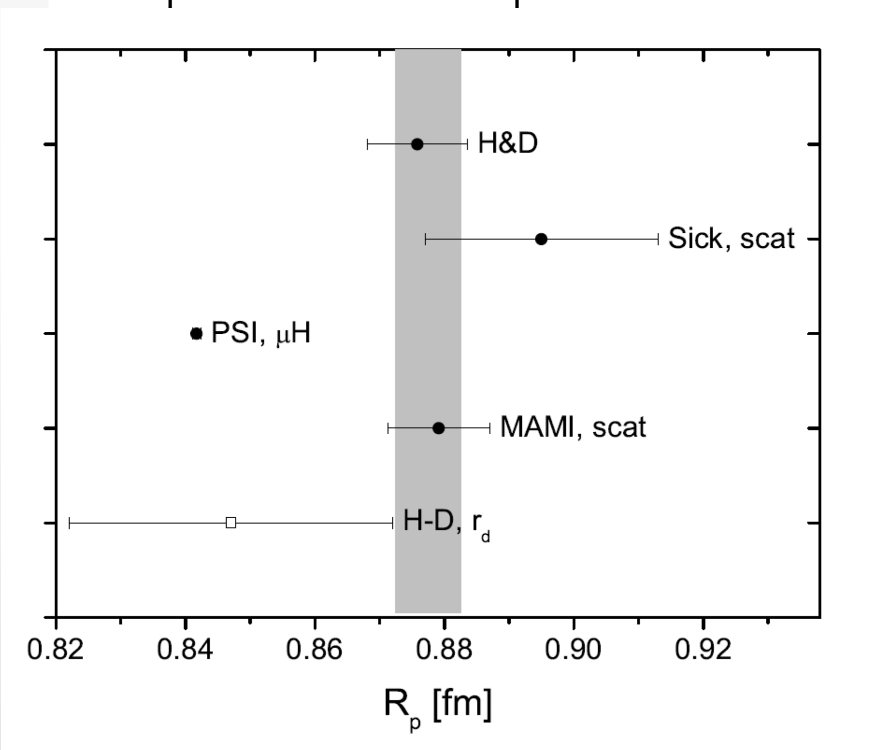
$$\Delta E_{\mathrm{BG}}(nl) = \frac{(Z\alpha)^4 m_r^3}{2n^3M^2}\left(\frac{1}{j+1/2} - \frac{2}{3}\right)(1-\delta_{l0})$$

Unperturbed energy levels: effective Dirac equation

#	Designation	Order	Ref.	ΔE [meV]
0.1	Rel	$(Z\alpha)^{4+}m$		0
0.2	Rel-Rec*	$(Z\alpha)^4 m^2/M$		0
0.3	BG*	$(Z\alpha)^4(m/M)^2 m$	[31]	**0.057 47**
0.4	BP*†	$(Z\alpha)^4(m/M)^2 m$	Table XII	$-$**0.108 35**

UNPERTURBED ENERGY LEVELS: EFFECTIVE DIRAC EQUATION

#	Designation	Order	Ref.	ΔE [meV]
				0
				0
		m	[31]	**0.057 47**
		m	Table XII	**−0.108 35**



Unperturbed energy levels: effective Dirac equation

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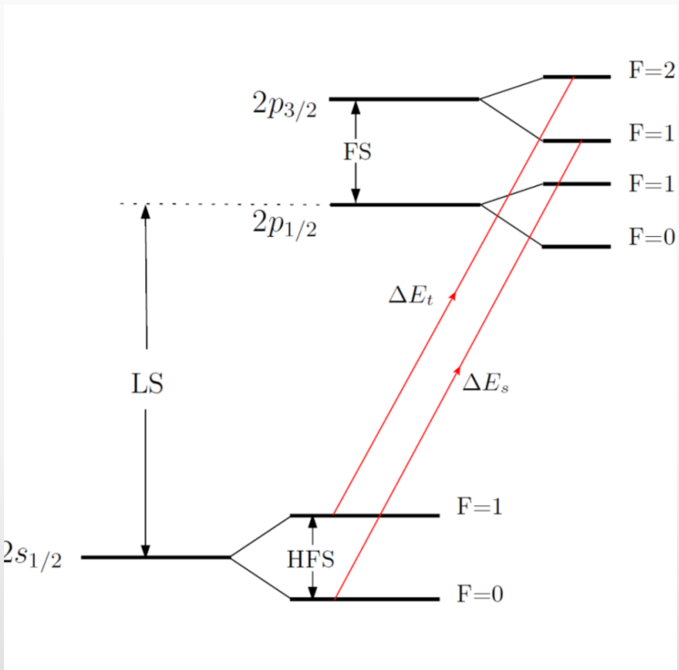
0.4	BP*†	$(Z\alpha)^4(m/M)^2 m$	Table XII	**−0.108 35**

Levels structure

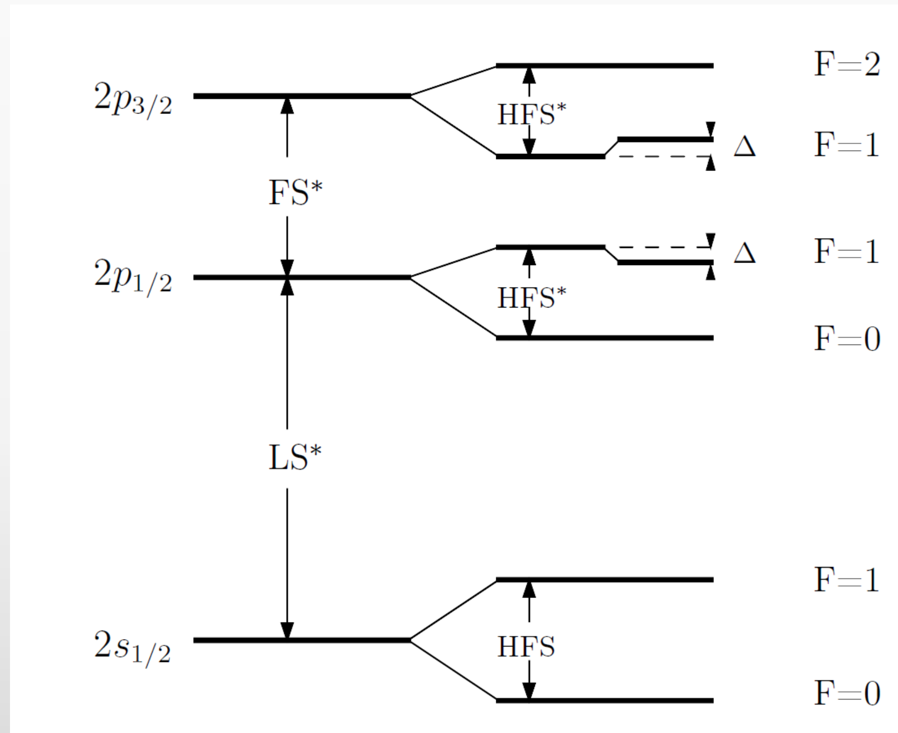
- Experiment



Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen

Aldo Antognini,[1,2*] François Nez,[3] Karsten Schuhmann,[2,4] Fernando D. Amaro,[5] François Biraben,[3] João M. R. Cardoso,[5] Daniel S. Covita,[5,6] Andreas Dax,[7] Satish Dhawar Marc Diepold,[1] Luis M. P. Fernandes,[5] Adolf Giesen,[4,8] Andrea L. Gouvea,[5] Thomas Graf,[8] Theodor W. Hänsch,[1,9] Paul Indelicato,[3] Lucile Julien,[3] Cheng-Yang Kao,[10] Paul Knowles, Franz Kottmann,[2] Eric-Olivier Le Bigot,[3] Yi-Wei Liu,[10] José A. M. Lopes,[5] Livia Ludhova,[11] Cristina M. B. Monteiro,[5] Françoise Mulhauser,[11] Tobias Nebel,[1] Paul Rabinowitz,[12] Joaquim M. F. dos Santos,[5] Lukas A. Schaller,[11] Catherine Schwob,[3] David Taqqu,[13] João F. C. A. Veloso,[6] Jan Vogelsang,[1] Randolf Pohl[1]

- Level structure



PHYSICAL REVIEW VOLUME 163, NUMBER 1 5 NOVEMBER 1967

Precise Theory of the Zeeman Spectrum for Atomic Hydrogen and Deuterium and the Lamb Shift*

Stanley J. Brodsky and Ronald G. Parsons†
Stanford Linear Accelerator Center, Stanford University, Stanford, California

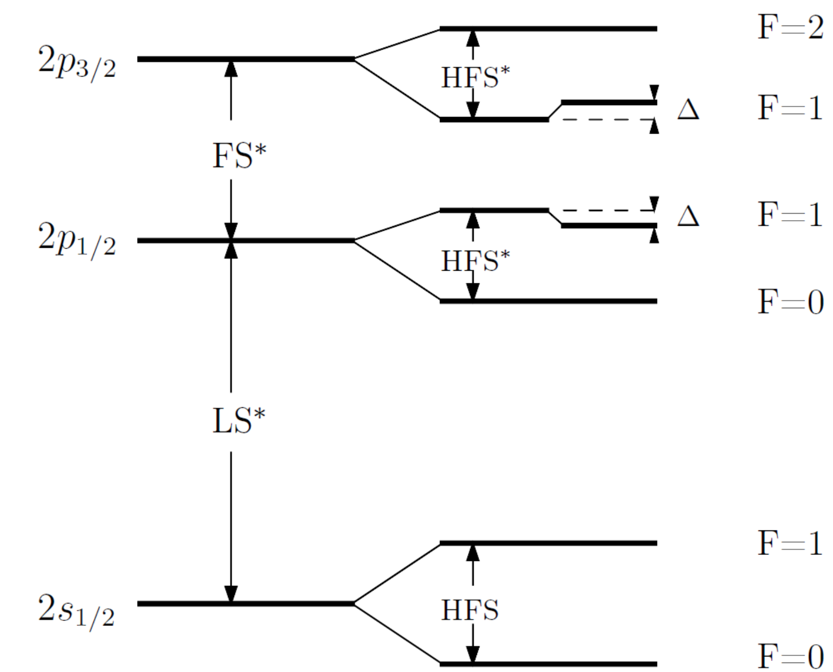
Unperturbed energy levels: Brodsky-Parsons term

- HFS mixing

$$\langle 2p_{1/2}(F=1)|H_{\rm HFS}|2p_{3/2}(F=1)\rangle \neq 0$$

$$\Delta \simeq 0.145 \text{ meV}$$

- Level structure



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PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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(Received 28 August 1995)

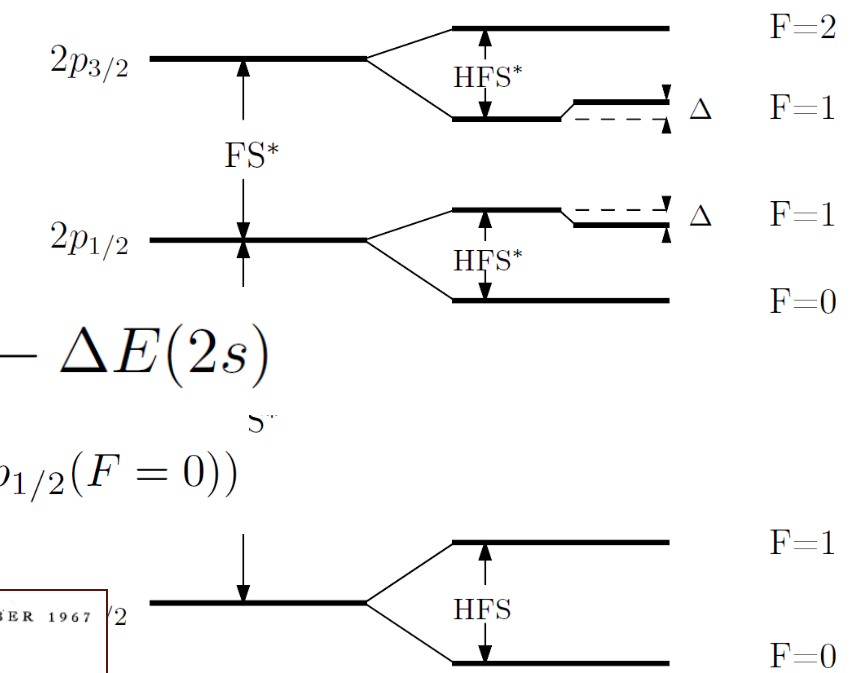
Unperturbed energy levels: Brodsky-Parsons term

- HFS mixing

$$\langle 2p_{1/2}(F=1)|H_{\mathrm{HFS}}|2p_{3/2}(F=1)\rangle \neq 0$$

- We consider

- Level structure



$$\Delta E_{\mathrm{L}}(2p_{1/2} - 2s) \;\equiv\; \Delta E(2p_{1/2}) - \Delta E(2s)$$

$$\Delta E(2p_{1/2}) \;\equiv\; \frac{3}{4}\Delta E(2p_{1/2}(F=1)) + \frac{1}{4}\Delta E(2p_{1/2}(F=0))$$

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ELEMENTARY PARTICLES AND FIELDS
Theory

Fine and Hyperfine Structure of *P*-Wave Levels in Muonic Hydrogen

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PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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(Received 28 August 1995)

Unperturbed energy levels: effective Dirac equation

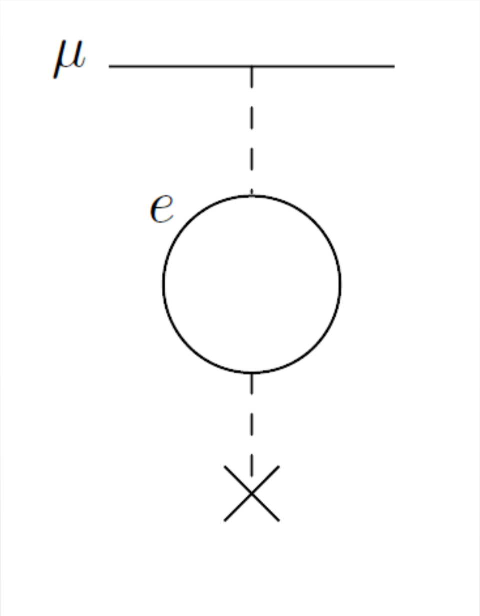
#	Designation	Order	Ref.	ΔE [meV]
0.1	Rel	$(Z\alpha)^{4+}m$		0
0.2	Rel-Rec*	$(Z\alpha)^4 m^2/M$		0
0.3	BG*	$(Z\alpha)^4(m/M)^2 m$	[31]	**0.057 47**
0.4	BP*†	$(Z\alpha)^4(m/M)^2 m$	Table XII	**−0.108 35**

Specific QED for muonic hydrogen

#	Designation	Order	Ref.	ΔE [meV]
1.1	eVP1 (NR)*	$\alpha(Z\alpha)^2 m$		**205.007 36**
1.2	eVP1 (Rel)	$\alpha(Z\alpha)^4 m$		0.020 84
1.3	eVP1 (Rel-Rec)*	$\alpha(Z\alpha)^4 \frac{m^2}{M}$	[25, 34]	−0.002 08
2	eVP2 (NR)*	$\alpha^2(Z\alpha)^2 m$	[21, 50]	**1.658 85**
3	eVP3 (NR)*	$\alpha^3(Z\alpha)^2 m$	[51, 52]	0.007 52
4	LbL*†	$\alpha^5 m$	Table III	−0.000 89(2)
5	eVP+SE	$\alpha^2(Z\alpha)^4 m$	[53]	−0.002 54
6	SE[eVP]	$\alpha^2(Z\alpha)^4 m$	[26, 54]	−0.001 52

SPECIFIC QED FOR MUONIC HYDROGEN

- The leading term



- + relativistic and recoil corrections

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Lamb shift in light muonic atoms — Revisited

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Lamb shift in muonic hydrogen—I. Verification and update of theoretical predictions

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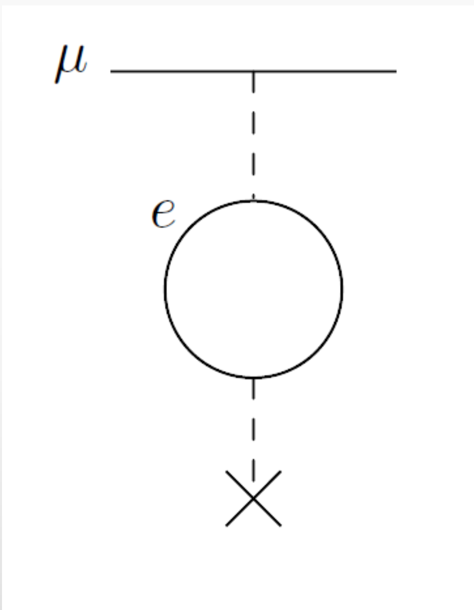
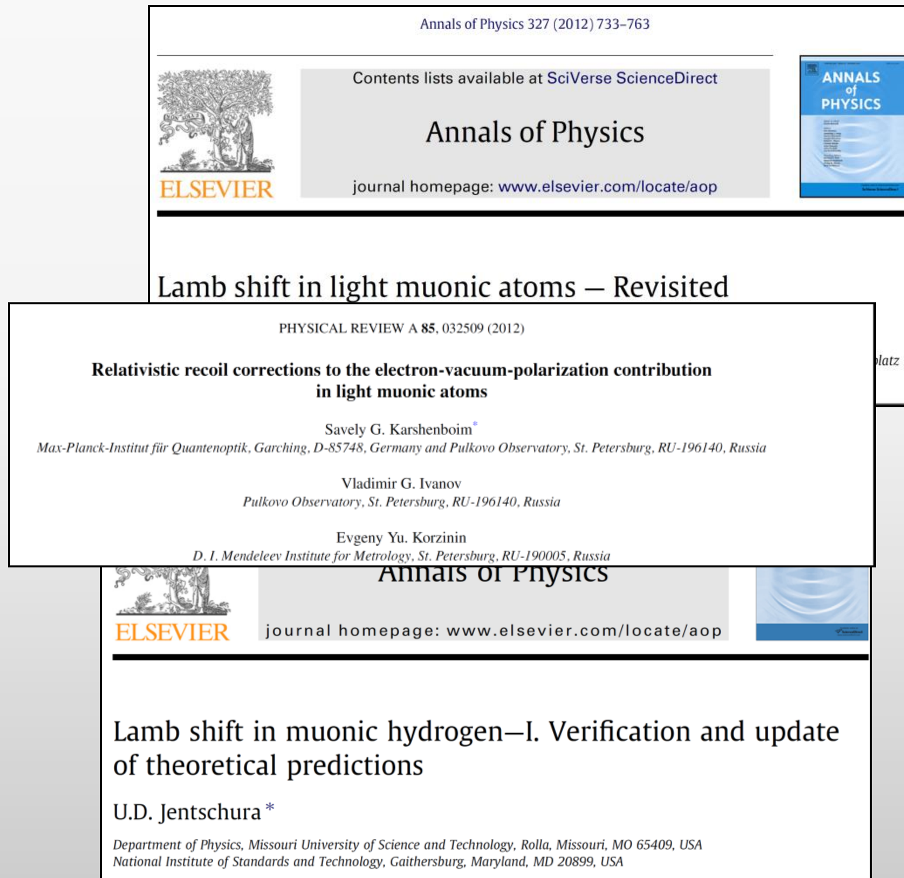
National Institute of Standards and Technology, Gaithersburg, Maryland, MD 20899, USA

Specific QED for muonic hydrogen

- The leading term



- + relativistic and recoil corrections



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Lamb shift in light muonic atoms — Revisited

PHYSICAL REVIEW A **85**, 032509 (2012)

Relativistic recoil corrections to the electron-vacuum-polarization contribution in light muonic atoms

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Max-Planck-Institut für Quantenoptik, Garching, D-85748, Germany and Pulkovo Observatory, St. Petersburg, RU-196140, Russia

Vladimir G. Ivanov
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Evgeny Yu. Korzinin
D. I. Mendeleev Institute for Metrology, St. Petersburg, RU-190005, Russia

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Lamb shift in muonic hydrogen—I. Verification and update of theoretical predictions

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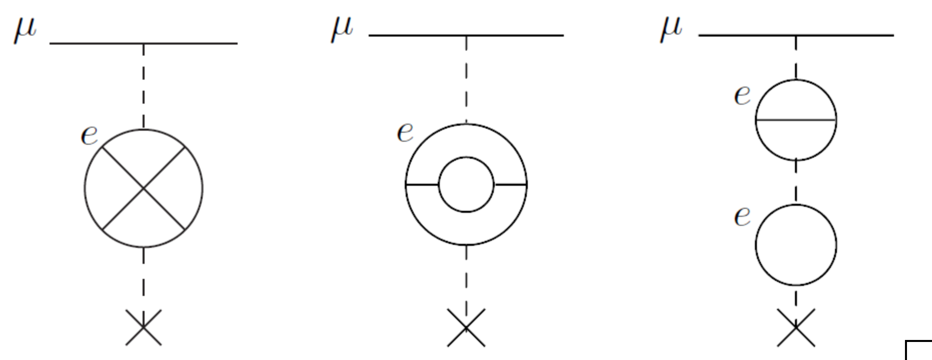
Department of Physics, Missouri University of Science and Technology, Rolla, Missouri, MO 65409, USA
National Institute of Standards and Technology, Gaithersburg, Maryland, MD 20899, USA

Specific QED for muonic hydrogen: eVP (3)



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PHYSICAL REVIEW A **81**, 060501(R) (2010)

Nonrelativistic contributions of order $\alpha^5 m_\mu c^2$ to the Lamb shift in muonic hydrogen and deuterium, and in the muonic helium ion

S. G. Karshenboim*
D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia and
Max-Planck-Institut für Quantenoptik, Garching D-85748, Germany

V. G. Ivanov
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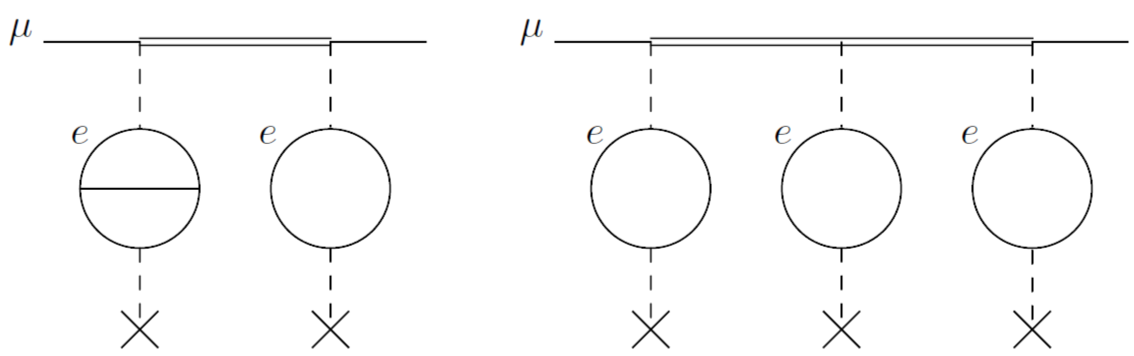
D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia

E. Yu. Korzinin and V. A. Shelyuto
D. I. Mendeleev Institute for Metrology, St. Petersburg RU-190005, Russia

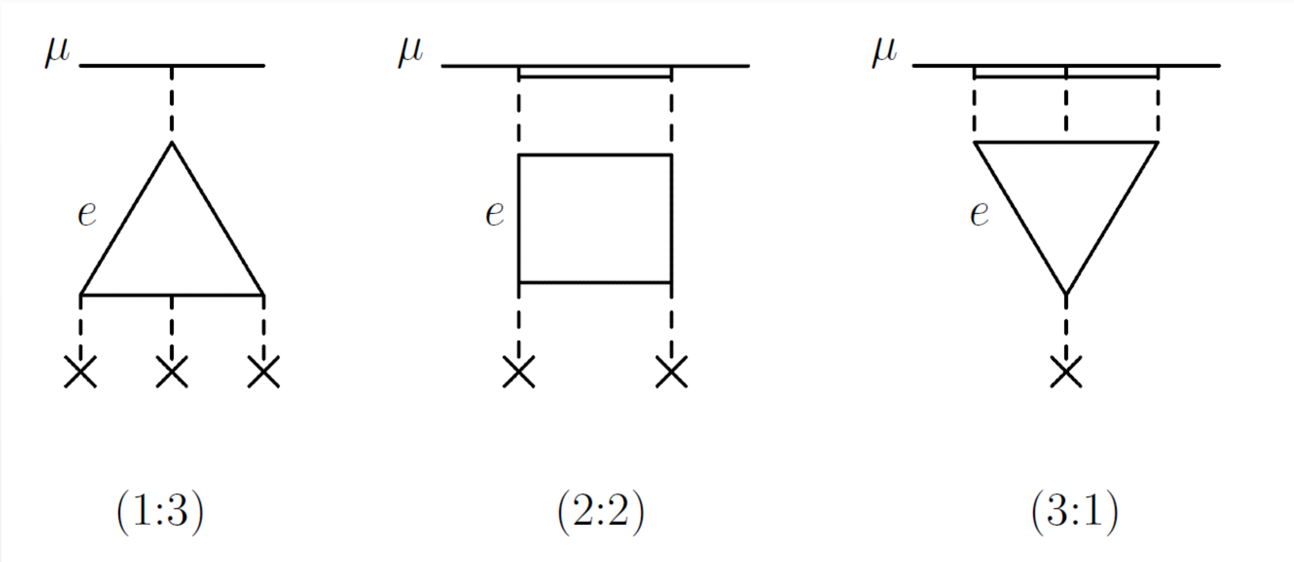
PRL **103,** 079901 (2009) PHYSICAL REVIEW LETTERS week 14 AUGU

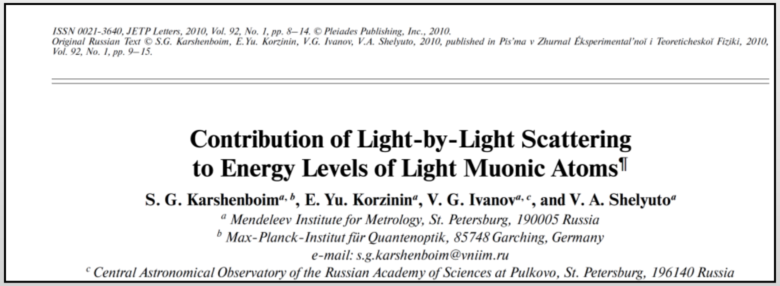
Erratum: Sixth-Order Vacuum-Polarization Contribution to the Lamb Shift of Muonic Hydrogen [Phys. Rev. Lett. 82, 3240 (1999)]

T. Kinoshita and M. Nio



Specific QED for muonic hydrogen: Light-by-Light





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Original Russian Text © S.G. Karshenboim, E.Yu. Korzinin, V.G. Ivanov, V.A. Shelyuto, 2010, published in Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2010, Vol. 92, No. 1, pp. 9–15.

Contribution of Light-by-Light Scattering to Energy Levels of Light Muonic Atoms¶

S. G. Karshenboim[a, b], E. Yu. Korzinin[a], V. G. Ivanov[a, c], and V. A. Shelyuto[a]

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[b] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
e-mail: s.g.karshenboim@vniim.ru
[c] *Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo, St. Petersburg, 196140 Russia*

Specific QED for muonic hydrogen

#	Designation	Order	Ref.	ΔE [meV]
1.1	eVP1 (NR)*	$\alpha(Z\alpha)^2 m$		**205.007 36**
1.2	eVP1 (Rel)	$\alpha(Z\alpha)^4 m$		0.020 84
1.3	eVP1 (Rel-Rec)*	$\alpha(Z\alpha)^4 \frac{m^2}{M}$	[25, 34]	−0.002 08
2	eVP2 (NR)*	$\alpha^2(Z\alpha)^2 m$	[21, 50]	**1.658 85**
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4	LbL*†	$\alpha^5 m$	Table III	−0.000 89(2)
5	eVP+SE	$\alpha^2(Z\alpha)^4 m$	[53]	−0.002 54
6	SE[eVP]	$\alpha^2(Z\alpha)^4 m$	[26, 54]	−0.001 52

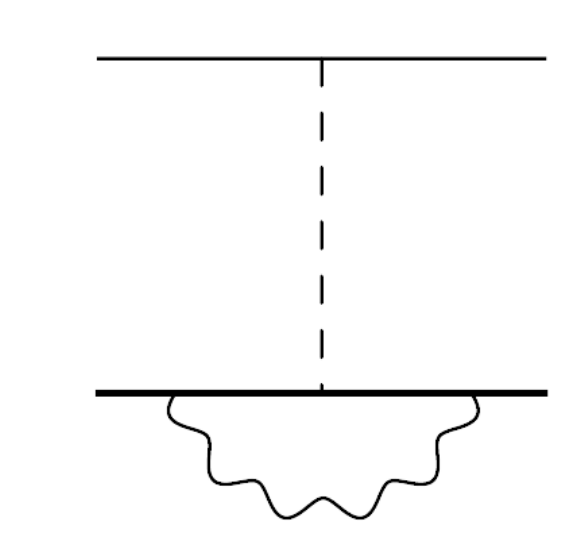
Re-scaled hydrogenic theory

#	Designation	Order	ΔE [meV]
7.1	QED (Rad)*	$\alpha(Z\alpha)^4 m$	$\mathbf{-0.663\,45}$
7.2	QED (Rad)	$\alpha(Z\alpha)^5 m$	$-0.004\,43$
7.3.	QED (Rad-Rec)	$\alpha(Z\alpha)^5 \frac{m^2}{M}$	$0.000\,19$
8	QED (Rec)*	$(Z\alpha)^5 m^2/M$	$\mathbf{-0.044\,97}$

Proton-line QED

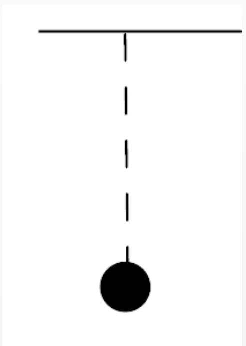
- Infrared divergence in the radius
- Finite corrections to the energy levels

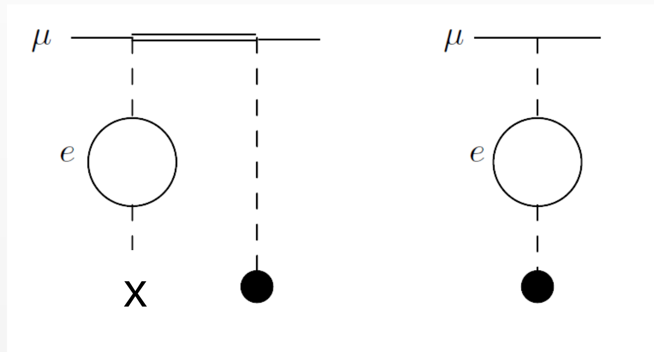
$$\Delta R_p^2 = \frac{\alpha}{\pi}\frac{1}{m_p^2}\ln\left(\frac{m_p^2}{\lambda^2}\right)$$

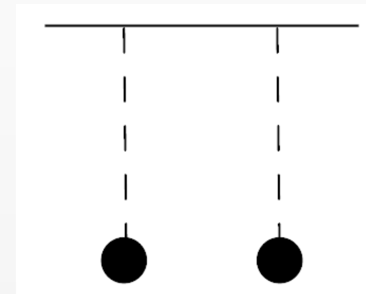


$$\Delta E_{\rm pQED}(nl) = \frac{4(Z^2\alpha)(Z\alpha)^4}{\pi n^3}\frac{m_r^3}{M^2} \times\left\{\left[\frac{1}{3}\ln\frac{M}{(Z\alpha)^2 m_r} + \frac{11}{72}\right]\delta_{l0} - \frac{1}{3}\ln k_0(nl)\right\},$$

External-field proton-size contributions



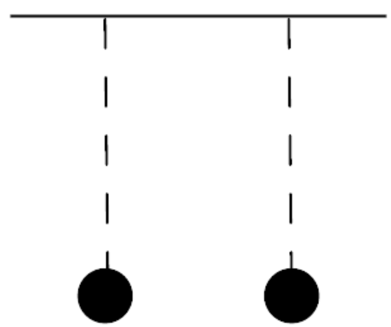




#	Designation	Order	Ref.	ΔE [meV]	
				Value	Estimation
10	PS (NR)	$(Z\alpha)^4 m$		$\mathbf{-5.1974}\, r_p^2$	$-\mathit{3.7}$
11	PS (Rel)	$(Z\alpha)^6 m$	[65–67]	$-0.0016\, r_p^2 - 0.000\,04 (r_p^2)^2$	-0.0011
12	PS (eVP)	$\alpha(Z\alpha)^4 m$	Eq. (15)	$-0.0282\, r_p^2$	-0.020
13	PS (SE)	$\alpha(Z\alpha)^4 m$	[6]	$0.0006\, r_p^2$	0.0005
14.1	PS (Fri) term	$(Z\alpha)^5 m$	[69]	$-0.0251(35) + 0.062\,44\, r_p^2$	0.019

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: PROBLEM

- Friar term



$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5 m_r^4}{\pi} I_{\rm Fr} \,,$$
$$I_{\rm Fr} = \frac{\pi}{48}\int d^3r\, d^3r' \rho_E(\mathbf{r})\rho_E(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^3$$
$$= \frac{\pi}{48}\langle r^3\rangle_2 \,,$$

$$I_{\rm Fr} = \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\, q^2\right]$$

Volume 80B, number 3 PHYSICS LETTERS

FINITE-SIZE CORRECTIONS TO THE ENERGY LEVELS OF LIGHT MUONIC ATOMS ☆

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PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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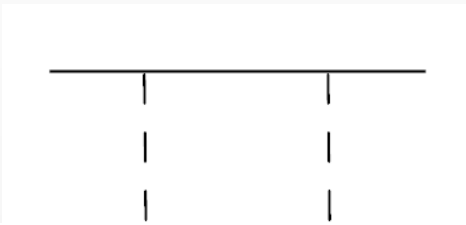
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany

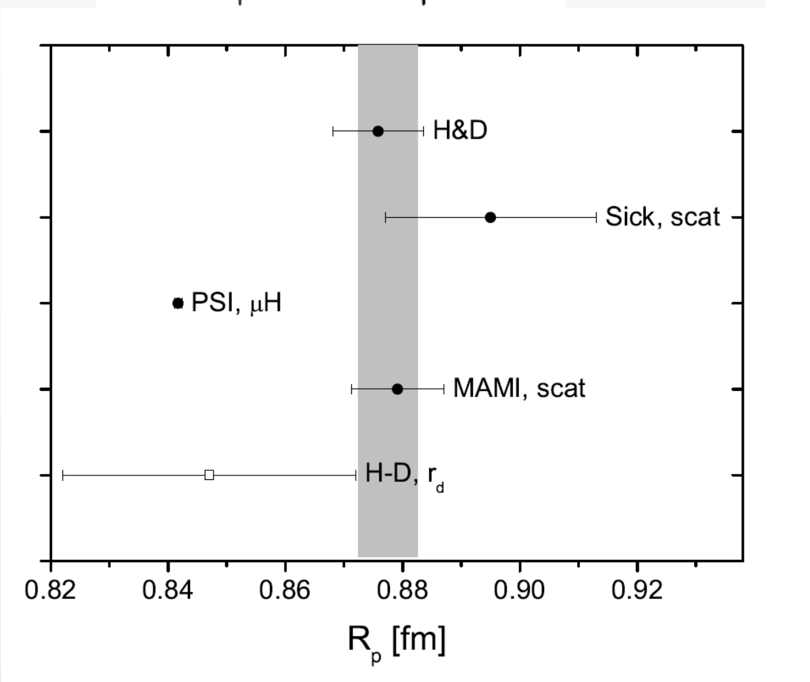
(Received 28 August 1995)

$$G'_E(0) = -\frac{1}{6}R_p^2$$

EXTERNAL-FIELD PROTON-SIZE CONTRIBUTION: PROBLEM

- Friar term





$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5\, m_r^4}{\pi}\, I_{\rm Fr}\ ,$$

$$I_{\rm Fr} = \frac{\pi}{48}\int d^3r\, d^3r'\rho_E(\mathbf{r})\rho_E(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^3$$

$$= \frac{\pi}{48}\langle r^3\rangle_2\ ,$$

$${}_{\rm Fr} = \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\, q^2\right]$$

$$G_E'(0) = -\frac{1}{6}R_p^2$$

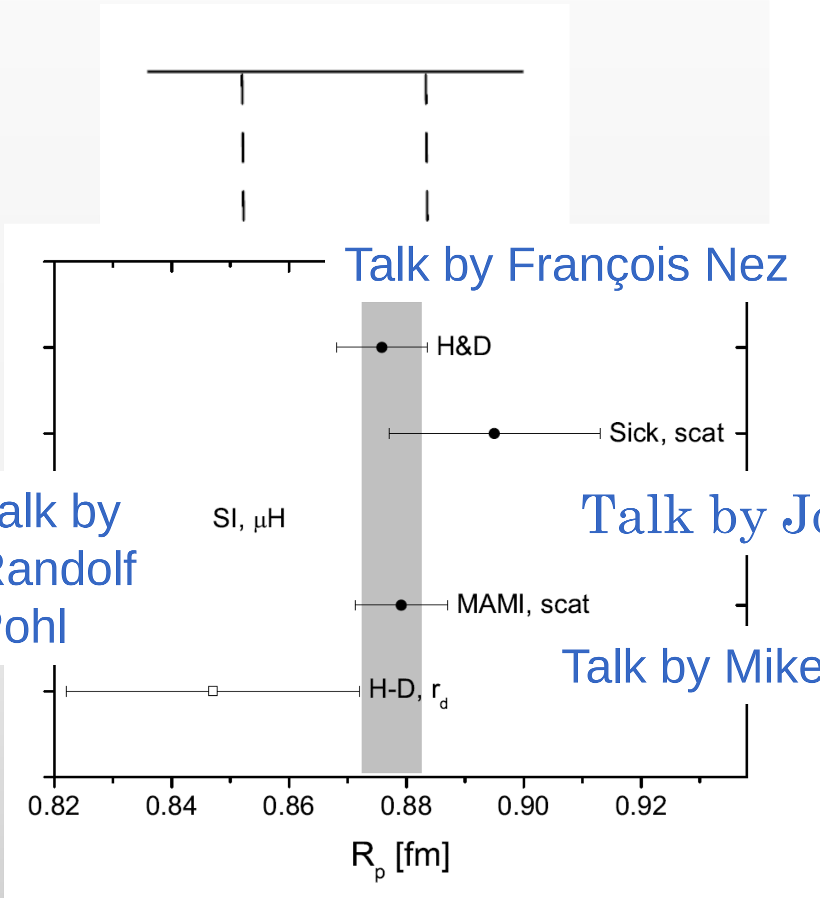
External-field proton-size contribution: problem

- Friar term

$$E(2s_{1/2}) = -\frac{2(Z\alpha)^5 m_r^4}{\pi} I_{\rm Fr}\,,$$

$$I_{\rm Fr} = \frac{\pi}{48}\int d^3r\, d^3r' \rho_E(\mathbf{r})\rho_E(\mathbf{r}')|\mathbf{r}-\mathbf{r}'|^3$$

$$= \frac{\pi}{48}\langle r^3\rangle_2\,,$$



Talk by François Nez

$$I_{\rm Fr} = \int^{\infty} \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$

Talk by Randolf Pohl

Talk by John Arrington

Talk by Mike Distler

$$G'_E(0) = -\frac{1}{6}R_p^2$$

External-field proton-size contribution: method

$$I = \int_0^\infty dq \ldots \equiv I_< + I_> \equiv \int_0^{q_0} dq \ldots + \int_{q_0}^\infty dq \ldots$$

27 January 1997

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PHYSICS LET

Physics Letters A 225 (1997) 97–106

Nuclear structure-dependent radiative corrections to the hydrogen hyperfine splitting

Savely G. Karshenboim [1]

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PHYSICAL REVIEW D **90,** 053012 (2014)

Self-consistent value of the electric radius of the proton from the Lamb shift in muonic hydrogen

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The recoil correction to the proton-finite-size contribution to the Lamb shift in muonic hydrogen

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Poster by Evgeny Korzinin

External-field proton-size contribution: results

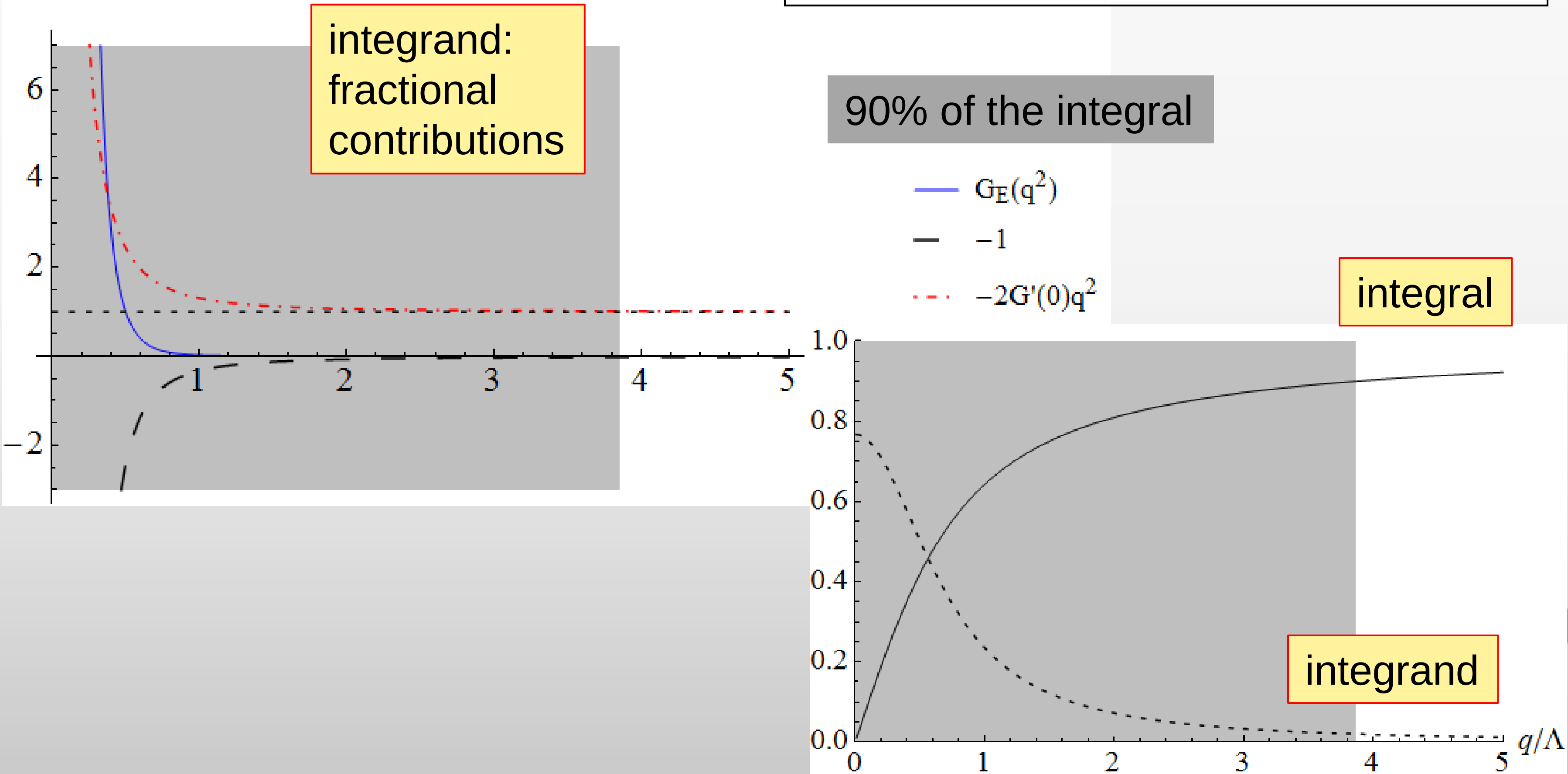
#	Designation	Order	Ref.	ΔE [meV]	
				Value	Estimation
10	PS (NR)	$(Z\alpha)^4 m$		$-\mathbf{5.1974}\, r_p^2$	$-\boldsymbol{3.7}$
11	PS (Rel)	$(Z\alpha)^6 m$	[65–67]	$-0.0016\, r_p^2 - 0.000\,04 (r_p^2)^2$	-0.0011
12	PS (eVP)	$\alpha(Z\alpha)^4 m$	Eq. (15)	$-0.0282\, r_p^2$	-0.020
13	PS (SE)	$\alpha(Z\alpha)^4 m$	[6]	$0.0006\, r_p^2$	0.0005
14.1	PS (Fri) term	$(Z\alpha)^5 m$	[69]	$-0.0251(35) + 0.062\,44\, r_p^2$	0.019

The Lamb shift in muonic hydrogen: Consistency problem

- The integrand includes

  Three terms:

$$I_3^{\mathrm{E}} \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G'_E(0)\,q^2\right]$$

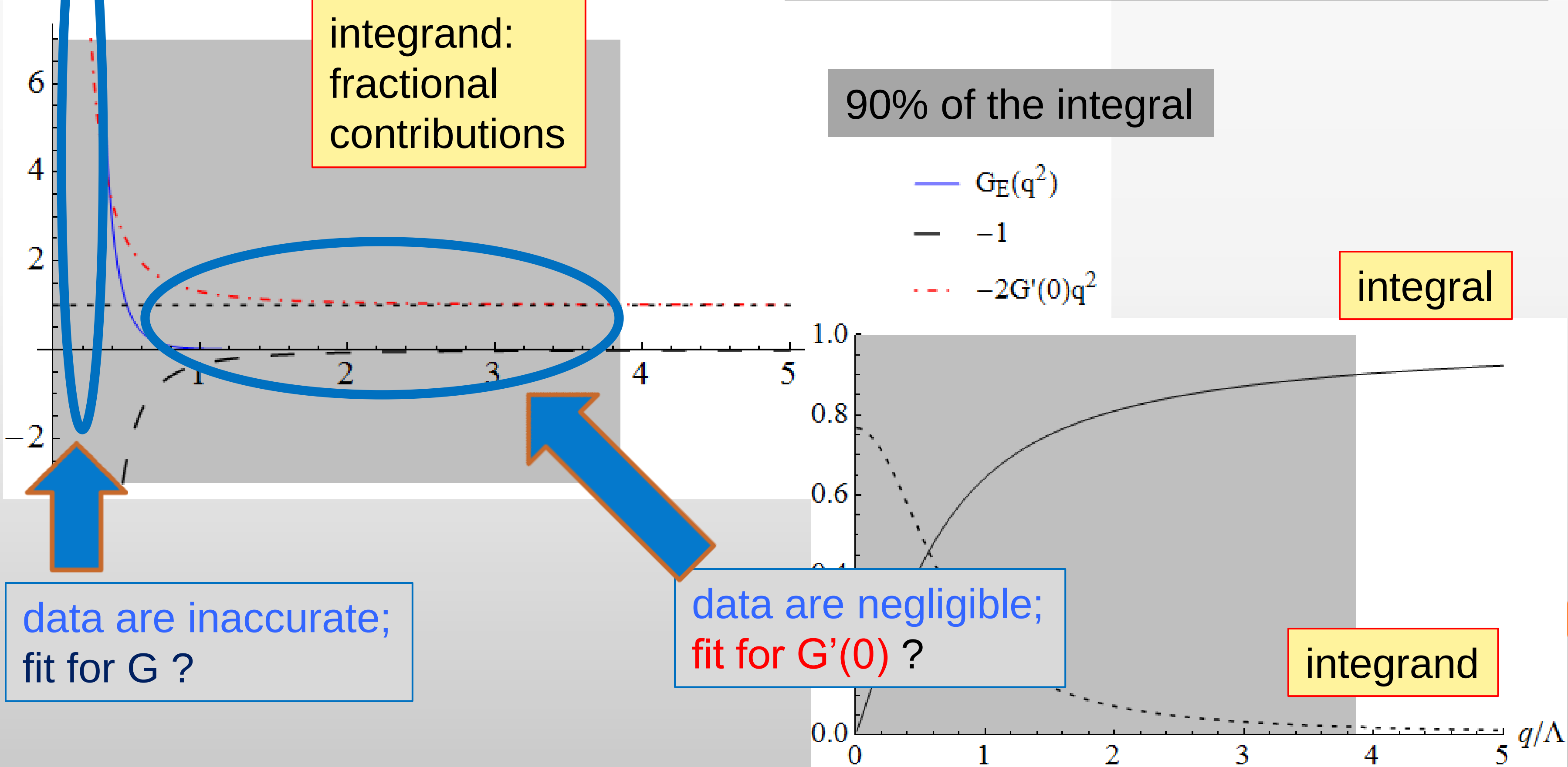


The Lamb shift in muonic hydrogen: Consistency problem

- The integrand includes

  Three terms:

$$I_3^{\mathrm{E}} \equiv \int_0^\infty \frac{dq}{q^4}\left[\left(G_E(q^2)\right)^2 - 1 - 2G_E'(0)\,q^2\right]$$



Strategy of the evaluation

- Split the integral

$$I = \int_0^\infty dq ... \equiv I_< + I_> \equiv \int_0^{q_0} dq ... + \int_{q_0}^\infty dq ...$$

- Low momentum

$$\left(G_E(q^2)\right)^2 \quad \simeq \quad 1 - \frac{R_E^2}{3} q^2 + C^{\rm dip}(1 \pm 1)\, q^4$$

- High momentum

$$I_{3>}^{\rm E} = \int_{q_0}^\infty \frac{dq}{q^4} \left(G_E(q^2)\right)^2 - \frac{1}{3q_0^3} + \frac{1}{3}\frac{R_E^2}{q_0}$$

$$\delta I_{3>}^{\rm E} = \frac{1}{3q_0^3}\frac{2\delta G_E(q_0^2)}{G_E(q_0^2)}\left(G_{\rm dip}(q_0^2)\right)^2$$

PHYSICAL REVIEW C **83**, 015203 (2011)

Realistic transverse images of the proton charge and magnetization densities

Siddharth Venkat,[1,2] John Arrington,[3] Gerald A. Miller,[2,*] and Xiaohui Zhan[3]
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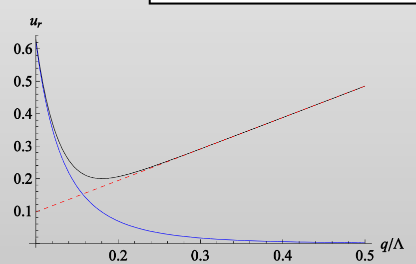
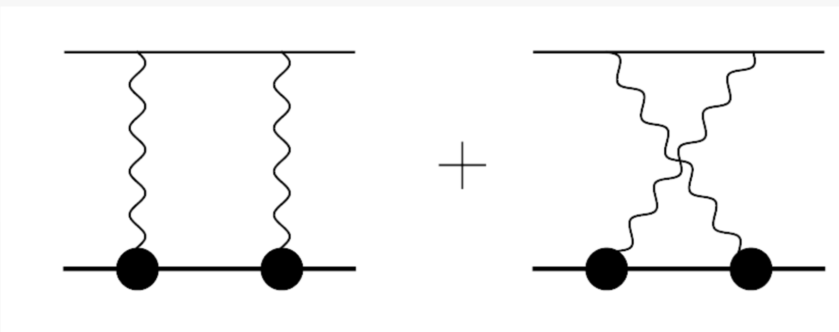
[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

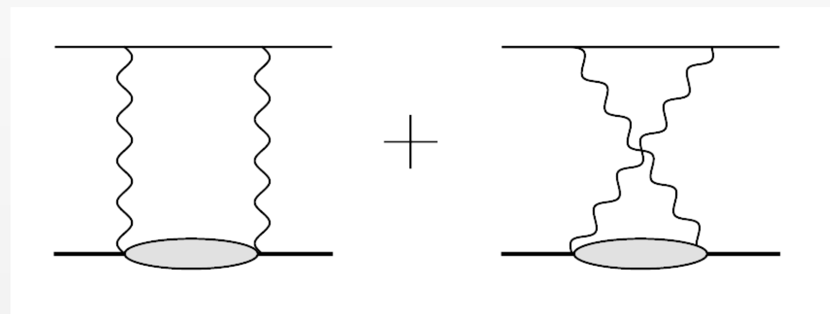
- Minimization of the uncertainty



Two-photon exchange

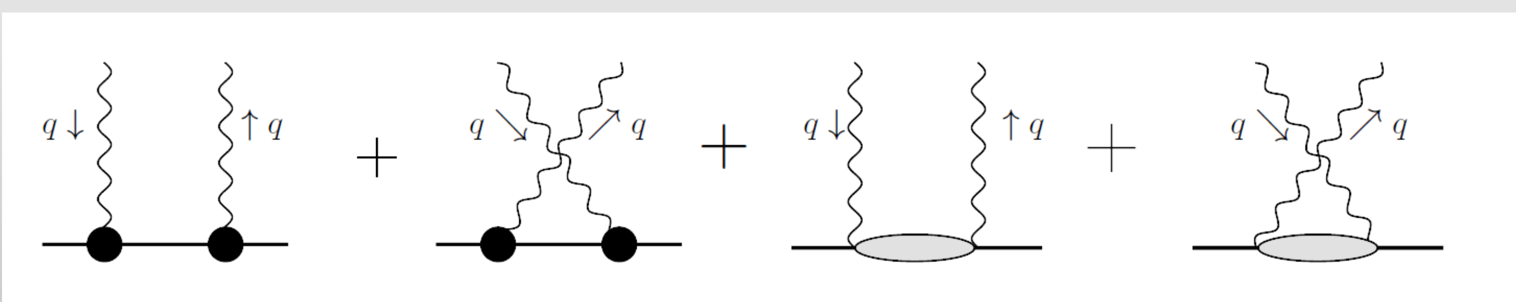
- Elastic
- Inelastic





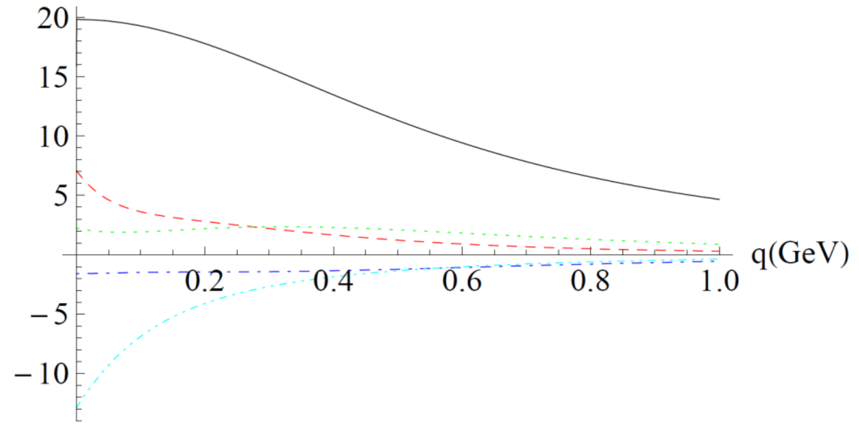
- Virtual Compton amplitude

Talk by Mike Birse



Elastic TPE: Recoil & finite size

$$
\begin{aligned}
I_{\rm rec} &= I_\kappa + I_{\rm EF} + I_{\rm M1} + I_{\rm M2}\ , \\
I_\kappa &= \kappa \int_0^\infty \frac{dq}{q^4} \left\{ (2+\kappa) f_{M1} + f_{M2} \right\}\ , \\
I_{\rm EF} &= \int_0^\infty \frac{dq}{q^4} f_{EF}(m, M; q^2) \left[\left(G_E(q^2)\right)^2 - 1 \right]\ , \\
I_{\rm M1} &= \int_0^\infty \frac{dq}{q^4} f_{M1} \left[\left(G_M(q^2)\right)^2 - (1+\kappa)^2 \right]\ , \\
I_{\rm M2} &= \int_0^\infty \frac{dq}{q^4} f_{M2} \left[G_M(q^2) G_E(q^2) - (1+\kappa) \right]\ ,(21)
\end{aligned}
$$



PHYSICAL REVIEW A VOLUME 53, NUMBER 4

Theory of the Lamb shift in muonic hydrogen

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(Received 28 August 1995)

Proton Polarizability and Lamb Shift in the Muonic Hydrogen Atom

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PHYSICAL REVIEW A **84**, 020102(R) (2011)

Higher-order proton structure corrections to the Lamb shift in muonic hydrogen

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Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory

M.C. Birse[a] and J.A. McGovern
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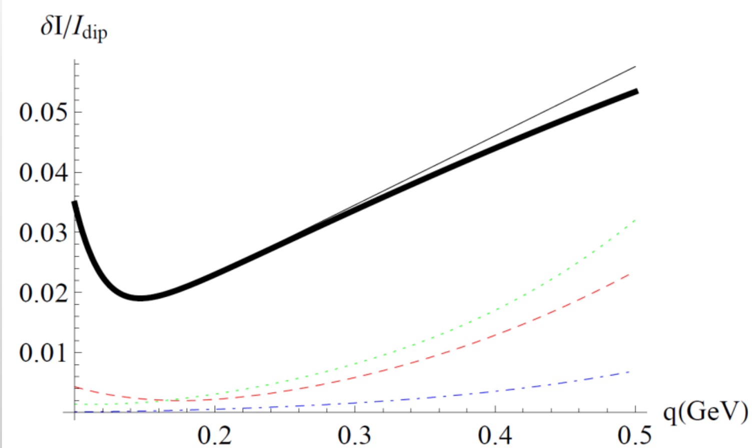

Elastic TPE: Recoil & finite size

#	Designation	ΔE [meV]	
		Value	Estimation
14.1	eTPE:Fri	$0.062\, r_p^2 - 0.025(4)$	*0.019*
14.2	eTPE:κ^*	$-0.003\,05$	*−0.003*
14.3	eTPE:EF*	$0.001\,07\, r_p^2 + 0.001\,36(4)$	*0.002*
14.4	eTPE:M1*	$0.001\,88(3)$	*0.002*
14.5	eTPE:M2*	$-0.000\,016\, r_p^2 - 0.000\,90$	*−0.0009*
14	eTPE	$0.064\, r_p^2 - 0.026(4)$	*0.019*

Proton polarizability: subtraction and dispersion relations

$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

$$\Delta E_{\rm inel}(nl) = -\frac{2\alpha^2}{mM}|\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2} \int_{\nu_{\rm th}}^\infty d\nu \left[\frac{\widetilde{\gamma}_1(\nu,q^2)F_1(\nu,q^2)}{\nu} + \frac{\widetilde{\gamma}_2(\nu,q^2)F_2(\nu,q^2)}{q^2/M}\right],$$

$$\Delta E_{\rm inel}(2p_{1/2} - 2s_{1/2}) = 13.0(0.6)\ \mu{\rm eV}$$

PHYSICAL REVIEW A **87**, 052501 (2013)

Muonic-hydrogen Lamb shift: Dispersing the nucleon-excitation uncertainty with a finite-energy sum rule

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[2]*Departamento de Fisica Teorica I, Universidad Complutense de Madrid, Madrid 28040, Spain*
[3]*Department of Physics and Center for Exploration of Energy and Matter, Indiana University, Bloomington, Indiana 47403, USA*

Proton polarizability: subtraction and dispersion relations

$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

$$\Delta E_{\rm sub}(nl) = \frac{4\pi\alpha^2}{m}|\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2}\frac{\gamma_1(\tau_\mu)}{\sqrt{\tau_\ell}}\overline{T}_1(0,q^2)$$

$$\overline{T}_1(0,0) = \beta_M$$

Proton polarizability: subtraction and dispersion relations

$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

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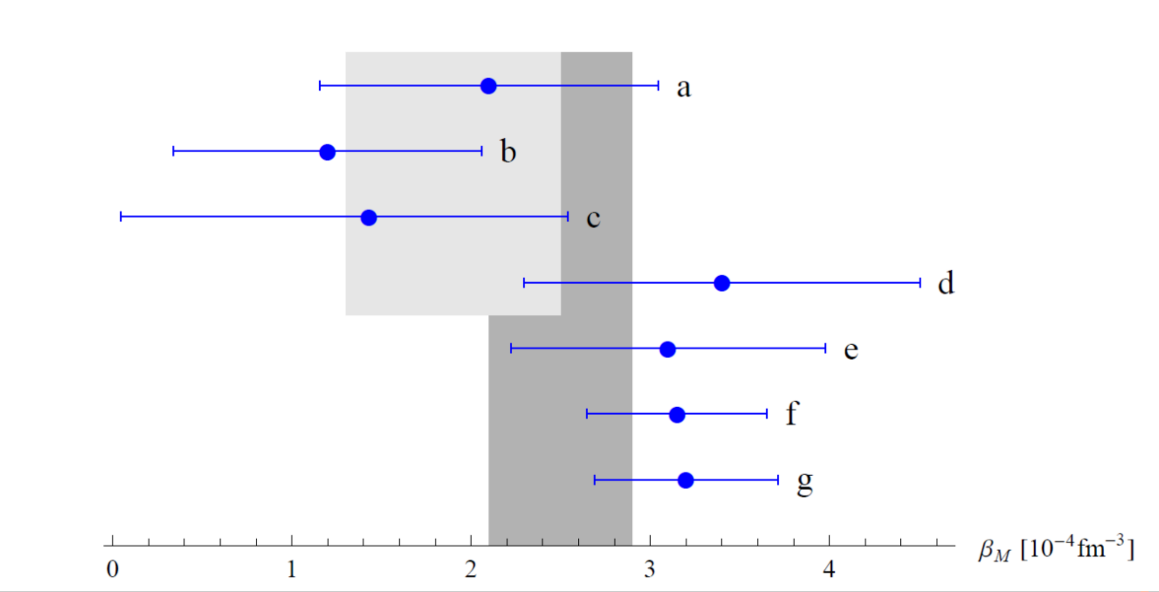
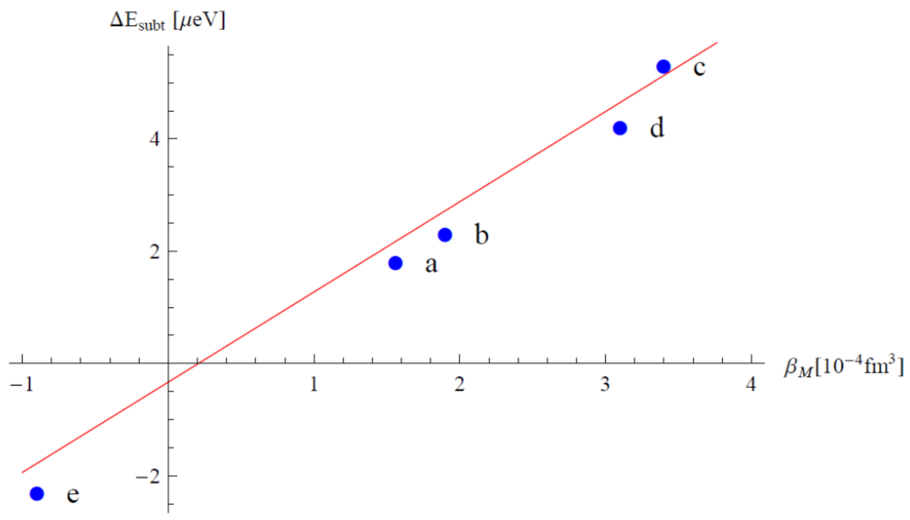
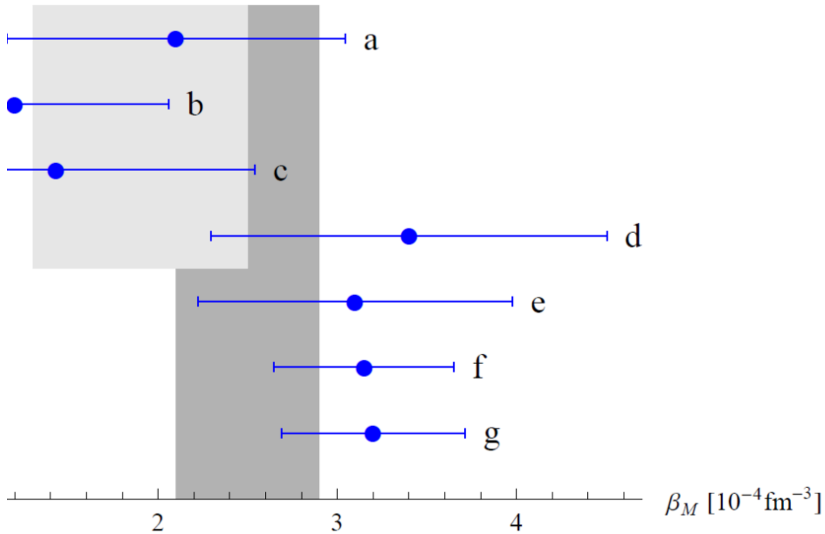


PROTON POLARIZABILITY: SUBTRACTION AND DISPERSION RELATIONS

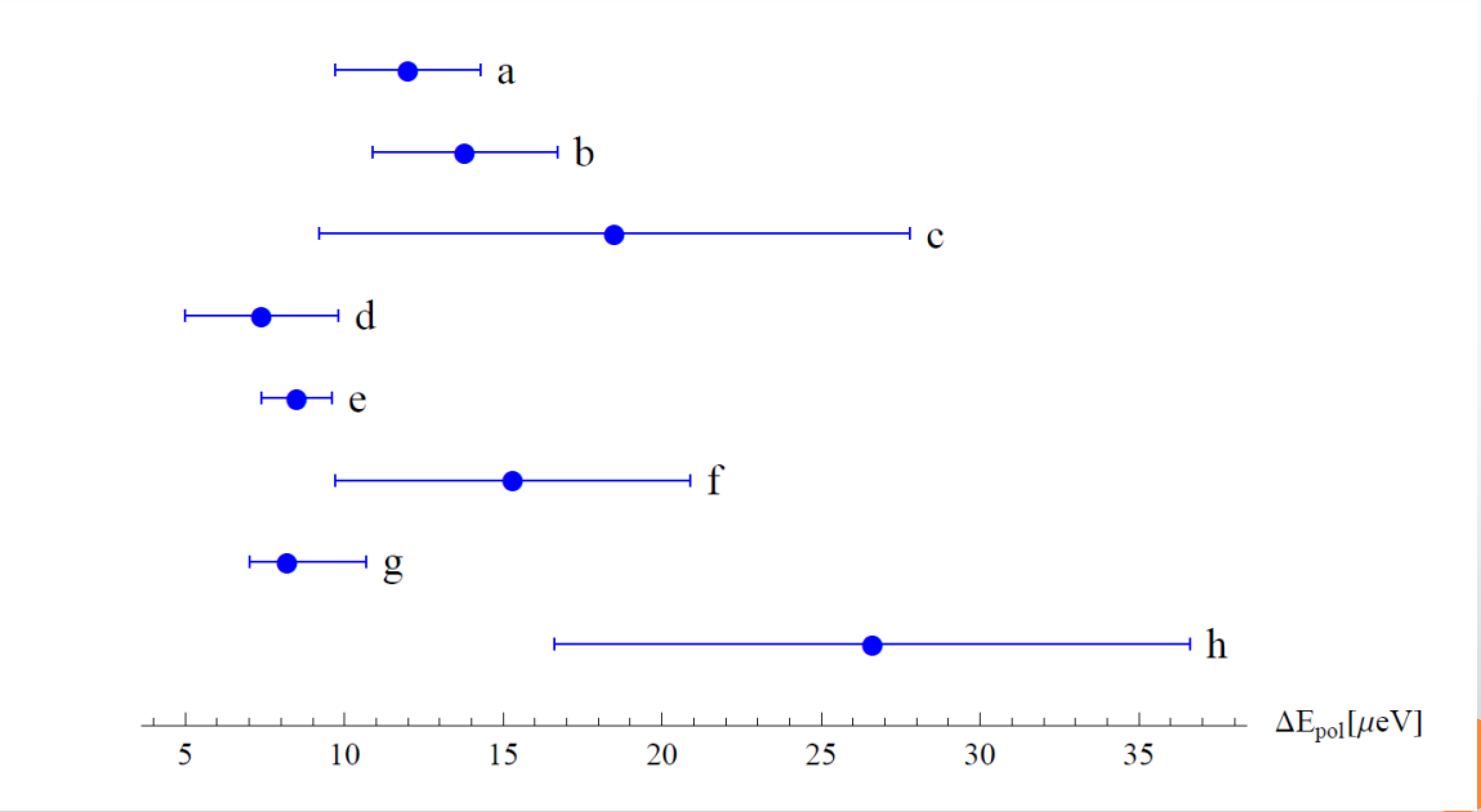
$$\Delta E_{\rm pol}(nl) = \Delta E_{\rm inel}(nl) + \Delta E_{\rm sub}(nl)$$

$$\Delta E_{\rm sub}(nl) = \frac{4\pi\alpha^2}{m}|\psi_{nl}(0)|^2 \int_0^\infty \frac{dq^2}{q^2}\frac{\gamma_1(\tau_\mu)}{\sqrt{\tau_\ell}}\overline{T}_1(0,q^2)$$



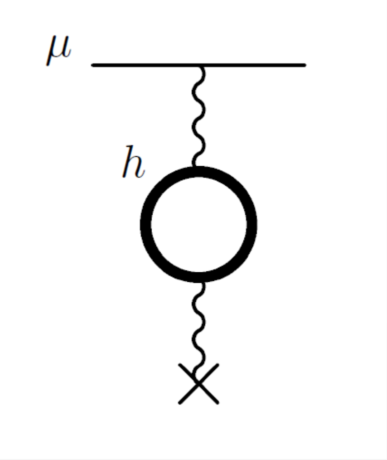


Proton polarizability: other calculations



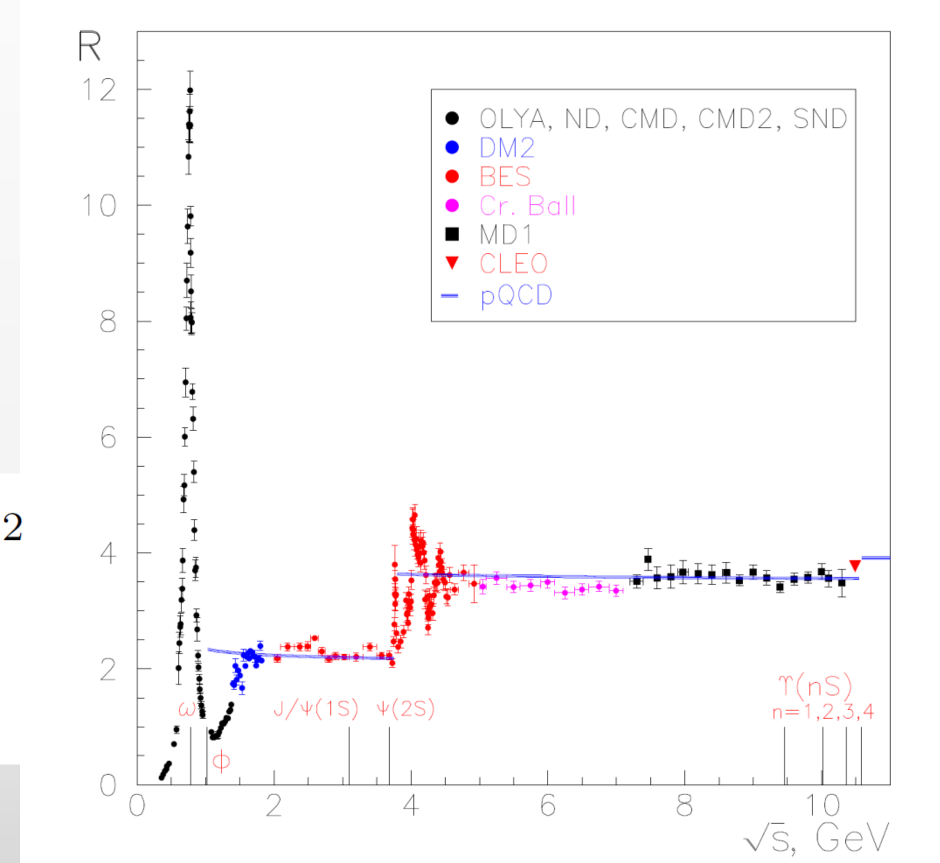
Talk by Mike Birse and Poster by Franziska Hagelstein

Hadronic vacuum polarization



$$\Delta E_{\rm hVP}(nl) = -4\pi(Z\alpha)\Pi'_h(0)|\psi_{nl}(0)|^2$$

$$\Pi_h(q^2) = \frac{\alpha}{\pi}\, q^2 \int \frac{ds\, \rho_h(s)}{q^2+s}\,.$$



$$\rho_h(s) = \frac{R(s)}{3s}$$

Talk by Ivan Logashenko

Theoretical summary

#	ΔE [meV]	Ref.
Unperturbed quantum mechanics		
0	$-\mathbf{0.05088}$	Table I
Specific QED		
1	$\mathbf{205.02612}$	Table II
2	$\mathbf{1.65885}$	Table II
3	0.00752	Table II
4	$-0.00089(2)$	Table II
5	-0.00254	Table II
6	-0.00152	Table II
Re-scaled QED		
7	$-\mathbf{0.66769}$	Table IV
8	$-\mathbf{0.04497}$	Table IV
Proton-line QED		
9	-0.01041	Eq. (12)
Proton-finite-size		
10	$-\mathbf{5.1974}\ r_p^2$	Table V
12	$-0.0282\ r_p^2$	Table V
13	$0.0006\ r_p^2$	Table V
14	$0.06354\ r_p^2 - 0.0259(35)$	Table VI
Proton polarizability		
15	$0.0088(21)$	Eq. (31)
Hadronic VP		
16	$0.0106(10)$	Eq. (35)
Total	$205.9067(42) - 5.1620\ r_p^2$	

Theory vs. experiment: proton charge radius

$$\Delta E(2p_{1/2} - 2s_{1/2}) = \left[205.9067(42) - 5.1620\, r_p^2\right] \text{ meV}$$

$$\Delta E_L \equiv \Delta E(2p_{1/2} - 2s) = 202.2622(23)\,\text{meV}$$

$$R_p = 0.840\,25(55) \text{ fm}$$

THEORY VS. EXPERIMENT: PROTON CHARGE RADIUS

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$$R_p = 0.840\,25(55) \ \text{fm}$$

Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen

Aldo Antognini,[1,2*] François Nez,[3] Karsten Schuhmann,[2,4] Fernando D. Amaro,[5] François Biraben,[3] João M. R. Cardoso,[5] Daniel S. Covita,[5,6] Andreas Dax,[7] Satish Dhawan Marc Diepold,[1] Luis M. P. Fernandes,[5] Adolf Giesen,[4,8] Andrea L. Gouvea,[5] Thomas Graf,[8] Theodor W. Hänsch,[1,9] Paul Indelicato,[3] Lucile Julien,[3] Cheng-Yang Kao,[10] Paul Knowles,[1] Franz Kottmann,[2] Eric-Olivier Le Bigot,[3] Yi-Wei Liu,[10] José A. M. Lopes,[5] Livia Ludhova,[11] Cristina M. B. Monteiro,[5] Françoise Mulhauser,[11] Tobias Nebel,[1] Paul Rabinowitz,[12] Joaquim M. F. dos Santos,[5] Lukas A. Schaller,[11] Catherine Schwob,[3] David Taqqu,[13] João F. C. A. Veloso,[6] Jan Vogelsang,[1] Randolf Pohl[1]

nd the magnetic radius, r_M = 0.87(6)
adius, r_E = 0.84087(39) femtometer,
10-CODATA value and at 7σ variance

Theory vs. experiment: proton charge radius

$$\Delta E(2p_{1/2} - 2s_{1/2}) = \left[205.9067(42) - 5.1620\, r_p^2\right] \text{ meV}$$

$$\Delta E_L \equiv \Delta E(2p_{1/2} - 2s) = 202.2622(23)\, \text{meV}$$

$$R_p = 0.840\,25(55) \text{ fm}$$

PHYSICAL REVIEW D **90,** 053012 (2014)

Self-consistent value of the electric radius of the proton from the Lamb shift in muonic hydrogen

Savely G. Karshenboim*

Max-Planck-Institut für Quantenoptik, Garching 85748, Germany and Pulkovo Observatory, St. Petersburg 196140, Russia

value of the electric radius o

be $R_E = 0.840\,22(56)$ fm.

The results are obtained in cooperation with

- Vladimir Ivanov (Pulkovo Obs)
- Evgeny Korzinin (VNIIM)
- Valery Shelyuto (VNIIM)

I am grateful for fruitful and stimulating discussions

Jose Manuel Alarcon, Igor Anikin, Aldo Antognini, Andrej Arbuzov, John Arrington, Jan Bernauer, Michael Birse, Edith Borie, Vladimir Braun, Carl Carlson, Victor Chernyak, Michael Distler, Dieter Drechsel, Simon Eidelman, Misha Eides, Ron Gilman, Misha Gorshteyn, Richard Hill, Franz Kottmann, Vadim Lensky, Ina Lorenz, Judith McGovern, Ulf Meissner, Makiko Nio, Krzysztof Pachucki, Vladimir Pascalutsa, Gil Paz, Randolf Pohl, Guy Ron, Akaki Rusetsky, Ingo Sick, Marc Vanderhaeghen