Fundamental constants meeting - Eltville - February 2015

A determination of the molar gas constant *R* (and the Boltzmann constant *k*) by acoustic thermometry in helium

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

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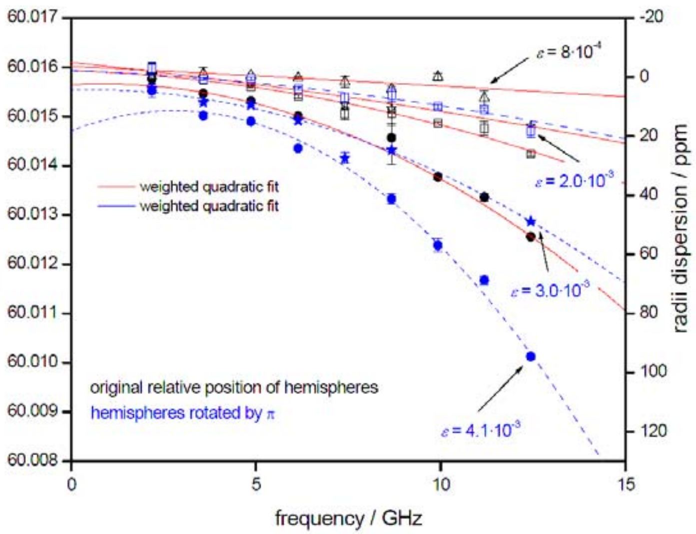
LNE Le progrès, une passion à partager

experiment design (2007)

design feature 1: use a **sphere** and induce the geometrical defect which is needed to lift the degeneracy of microwave modes by a slight misalignment of the comprising hemispheres

sphere vs ellipsoid

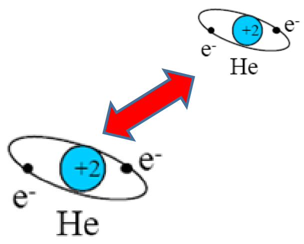
pro: the results of tests with small steel cavities made by 2007 suggested that this method might work satisfactorily by using a larger copper cavity.



con: **no geometrical perturbation models** for acoustics and microwaves (not in 2007, neither in 2015). For ellipsoids these models became available in 2007 and 2009.

design feature 2: use **helium**

He vs Ar

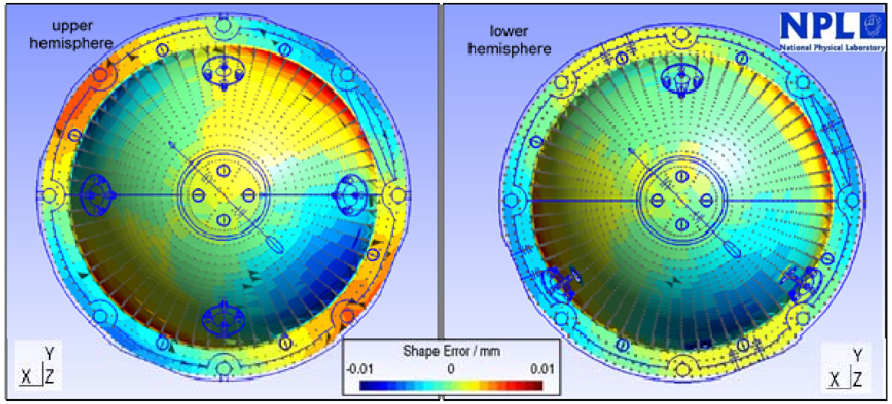


pros:
- less problems from **isotopes**
- **helium is calculable** from theory (thermodynamic, transport and electrical properties)
- in a copper cavity, some acoustic perturbations have a smaller effect (**shell recoil**)

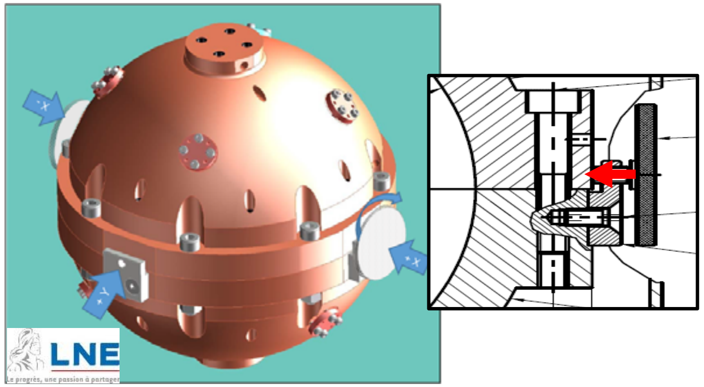
cons:
- **contamination** from common impurities has a ten-fold **effect** with respect to argon;
- in a copper cavity, some acoustic perturbations have a larger effect (**thermal accomodation**)

design features and preparation of the resonator

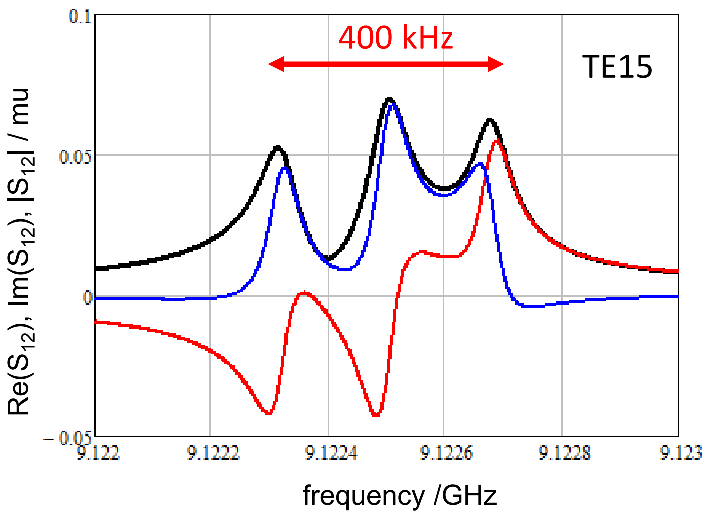
cavity: a copper sphere with an internal radius of 90 mm; the two comprising hemispheres have geometrical defects within a few microns.



the hemispheres can be micrometrically misaligned by a simple mechanical system.



we prepare the resonator for measurements by inducing (by trial and error) the minimum perturbation needed to achieve useful fitting precision (and accuracy, as tested in synthetic fits)

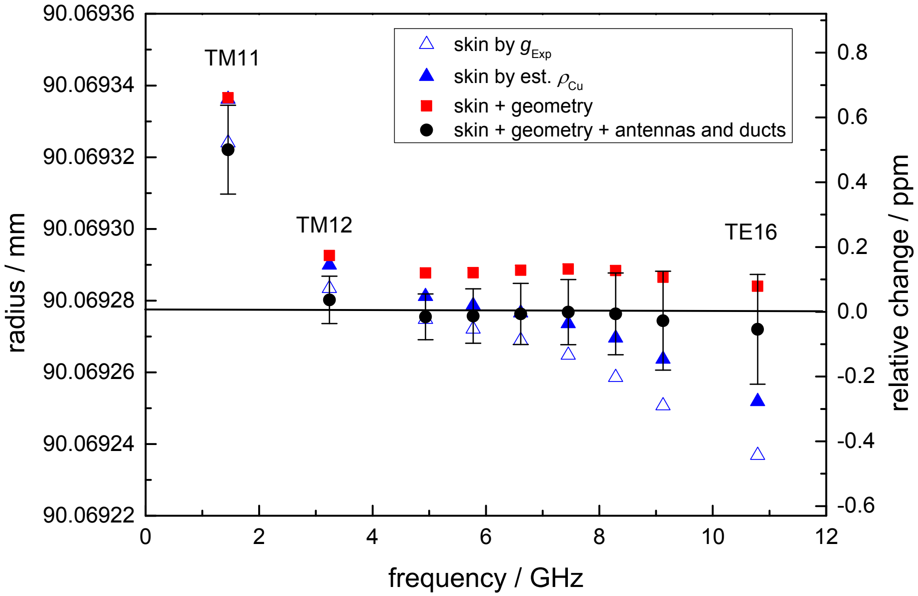


example of fitting results - TE15 triplet			
	singlet 1	singlet 2	singlet 3
frequencies / MHz	9122.3364	9122.5251	9122.6960
halfwidths / MHz	0.0353	0.0351	0.0365
u_r (fit) / ppm	0.034	0.022	0.026

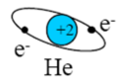
this level of precision is achieved for five TM modes
(TM11 to TM15) and 4 TE modes (TE13 to TE16)
antennas are designed to minimize coupling with the cavity

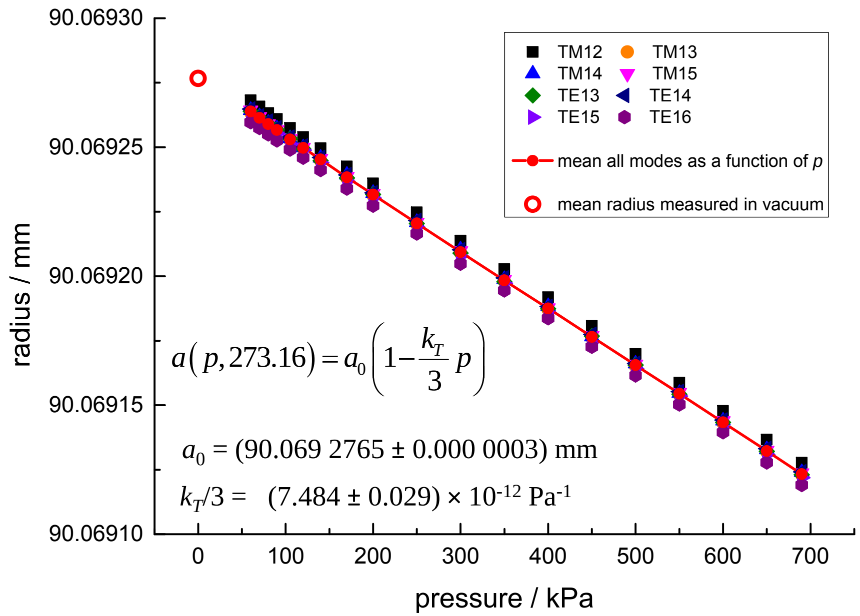
microwave determination of the mean cavity radius

perturbations and corrections:
surface resistivity, two antennas (as waveguides), three ducts, geometry (ellipsoidal approximation)

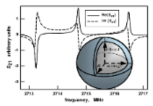


using $n_{He}(p, 273.16)$ from theory





- discrepancy of TM11 mode is due to perturbation from loop antennas (as demonstrated by substitution tests);
- approximating the shape of our assembled cavity with an ellipsoid works well: i) it reduces the dispersion of 8 modes TM12 to TE16 by a factor 5; the relative separation within the triplets gives shape factors which are consistent for the whole set of 11 mw modes and extremely stable as a function of *p* and *T*:

$\varepsilon_1 = (1.969 \pm 0.005) \times 10^{-4}$  $\varepsilon_2 = (1.018 \pm 0.002) \times 10^{-4}$

- applying (or not) the corresponding geometrical correction on both AC and MW fields changes the determined values of *R* and *k* by only 0.2 ppm

uncertainty budget for microwave determination of the squared cavity radius			
source	u / nm	u_r / ppm	comments
riproducibility upon change of antennas or underestimate of perturbation from loop probes	21	0.23	section 3.1
upper limit to the thickness of a possible dielectric layer on resonator surface	15	0.17	
estimate of surface resistivity	10	0.11	section 3.2
total (quadrature sum) a_0	23.2	0.26	
total squared radius a_0^2		**0.52**	

speed of sound in helium

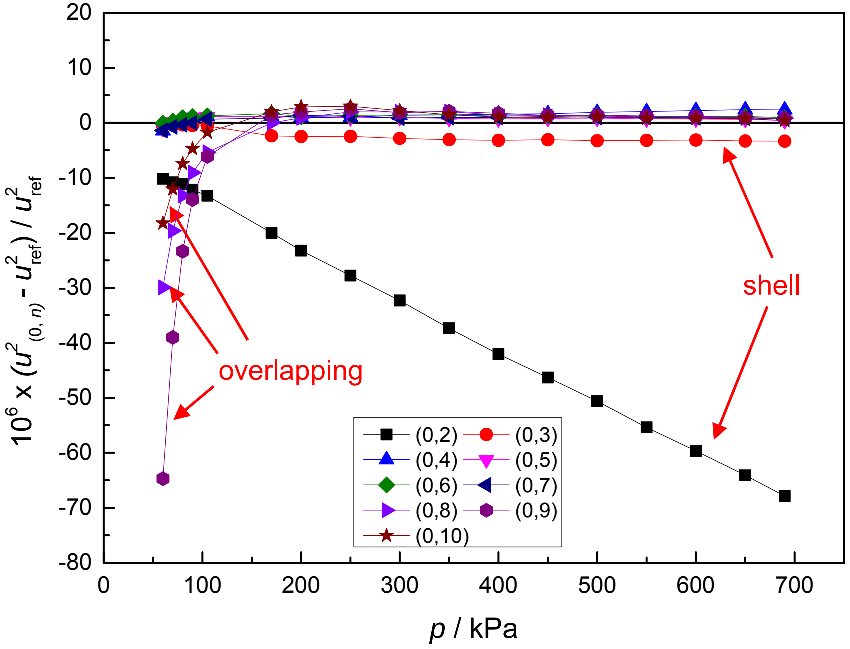
experimental datasets:

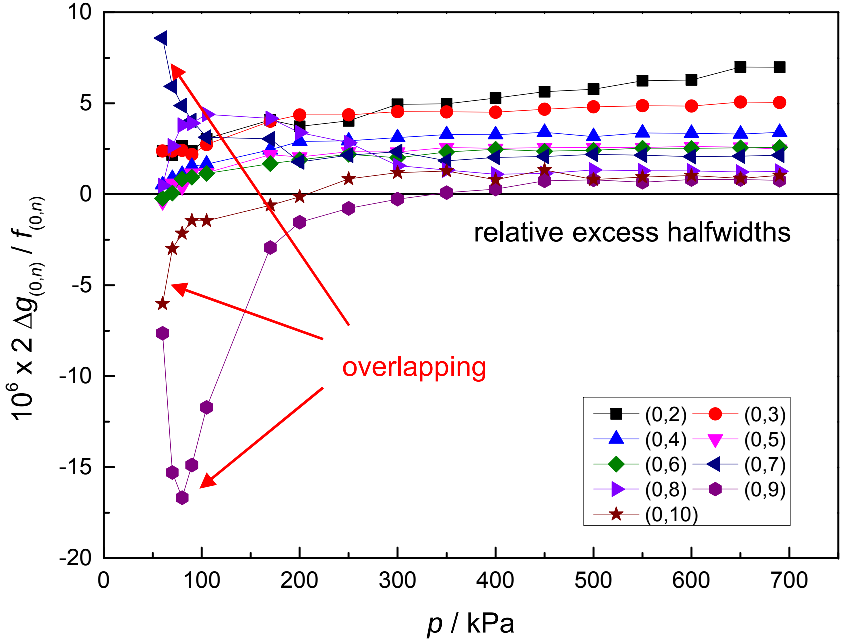
i) (high *p* - *measurement set*) nine purely radial modes (0,2) to (0,10) between 170 kPa and 690 kPa, near 273.16 K;

ii) (low *p* - *thermal accomodation set)* same modes between 60 kPa and 105 kPa to estimate *h*

perturbations and corrections: boundary layer 2nd order, shell effects, two microphones, three ducts, geometry (ellipsoidal approximation)

- mode (0,2) and (0,3) were rejected, due to evidence of large perturbations from shell modes
- mode (0,8), (0,9) and (0,10) were rejected, due to evidence of overlapping with neighbouring modes at low pressure



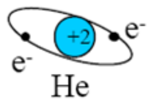


our determination of *h* fitted from low pressure data of modes (0, 4) to (0,7) is not precise and not repeatable, with two inconsistent estimates from two isotherms repeated at a distance of a few months: $h_1 = (0.393 \pm 0.009)$; $h_1 = (0.413 \pm 0.009)$; we account for such inconsistency with a relevant uncertainty contribution in our speed of sound measurement; we note that our first estimate is consistent with a recent accurate estimate obtained at LNE-CNAM: $h_1 = (0.3926 \pm 0.001)$

speed of sound in helium extrapolation to zero pressure

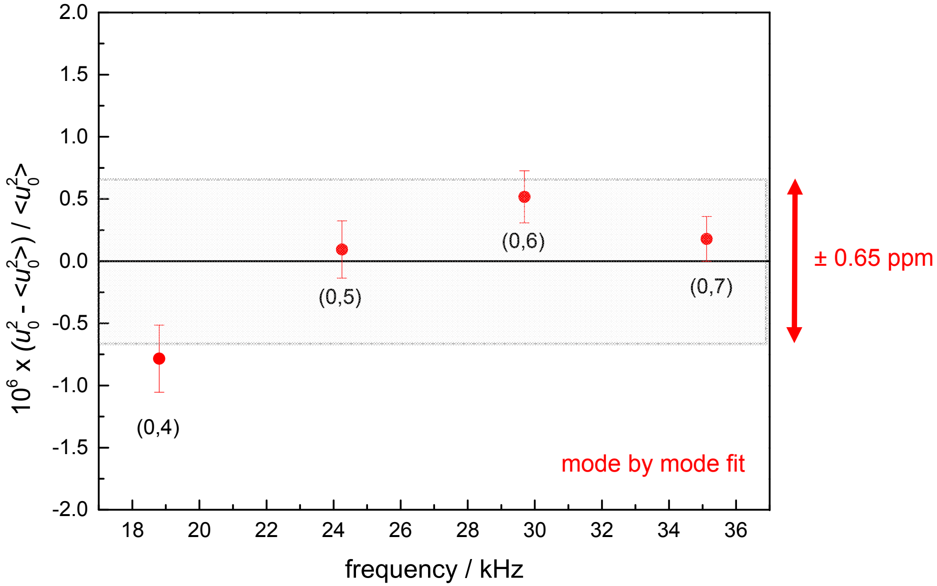
we use a single weighted linear fit with:

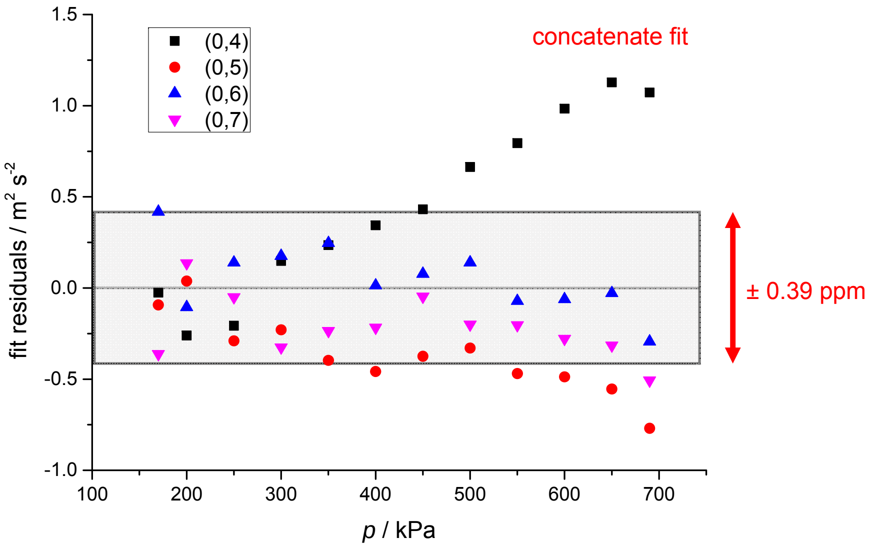
$$\boxed{u^2(p,\ 273.16) - A_{-1}p^{-1} - A_2 p^2 = A_0 + A_1 p}$$



where:

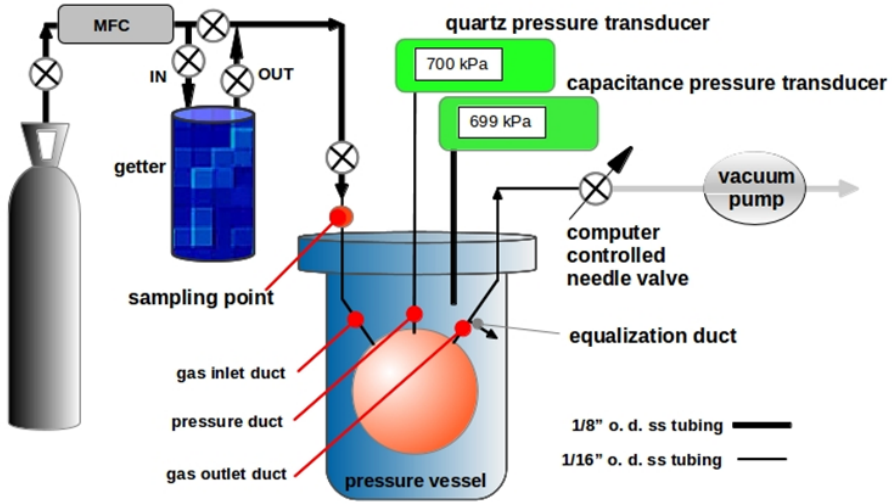
- A_{-1} is fixed to our best estimate $h = 0.393$;
- A_2 is fixed by theoretical calculated value of the acoustic virial coefficients of helium with negligible additional uncertainty
- fitting weights are equal to the standard uncertainty of repeated measurements of each datum

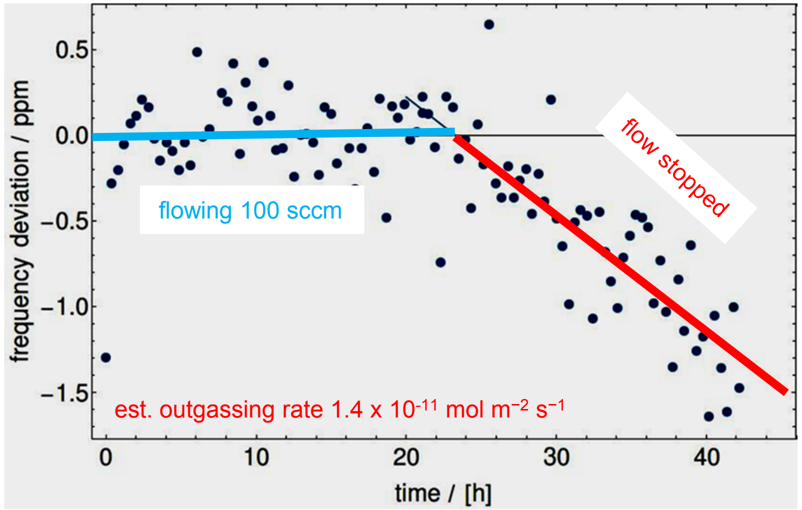




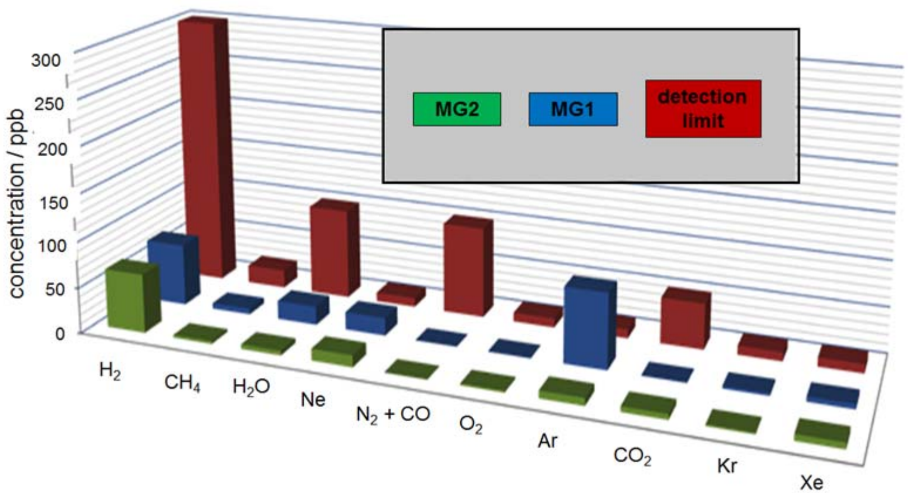
uncertainty budget for squared speed of sound at zero pressure at 273.16 K		
source	u / m^2 s^{-2}	u_r / ppm
dispersion of four radial acoustic modes	0.54	0.57
squared microwave radius	0.49	0.52
imperfect estimate of thermal accomodation coefficient h	0.41	0.43
thermal conductivity of helium	0.02	0.03
pressure	0.09	0.10
geometrical correction	0.07	0.08
ducts correction	0.10	0.11
microphone correction	0.05	0.05
total u_0^2	**0.85**	**0.90**

estimate of the molar mass





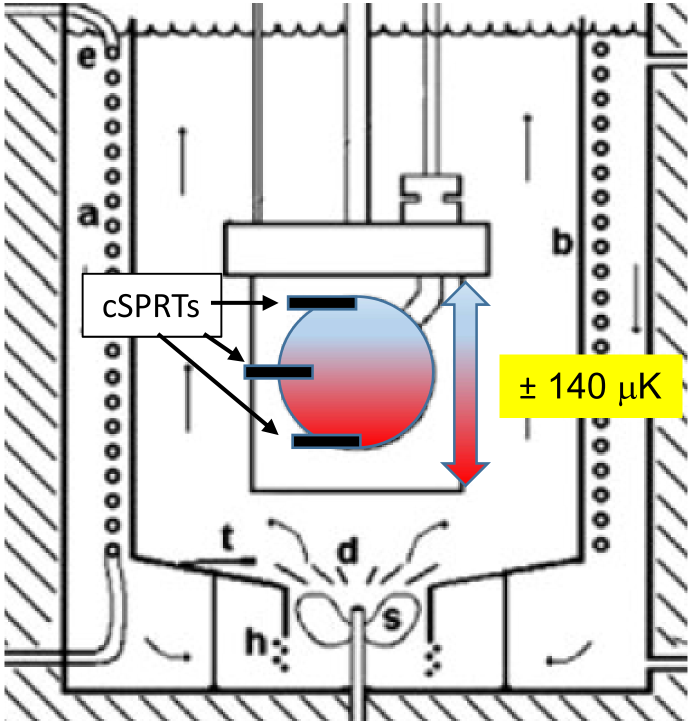
- we have a **direct measurement of pressure** from the cavity by a dedicated duct
- we **continuosly flow at 100 sccm** during measurements
- we **evaluate the rate of outgassing** and its influence on our measurements by stopping the flow
- we carefully sample just before entering the cavity and **analyse by mass spectroscopy**



uncertainty budget for molar mass to heat capacity ratio		
source	u / g mol^{-1}	u_r / ppm
impurities in He	0.000 0013	0.32
possible variation ^{3}He/^{4}He isotopic composition	0.000 0007	0.17
maximum effect water vapor degassing if H_2O or N_2	0.000 0003	0.07
total M / γ	0.000 0015	**0.37**

temperature

- we used three capsule-type cSPRTs calibrated before and after the measurements



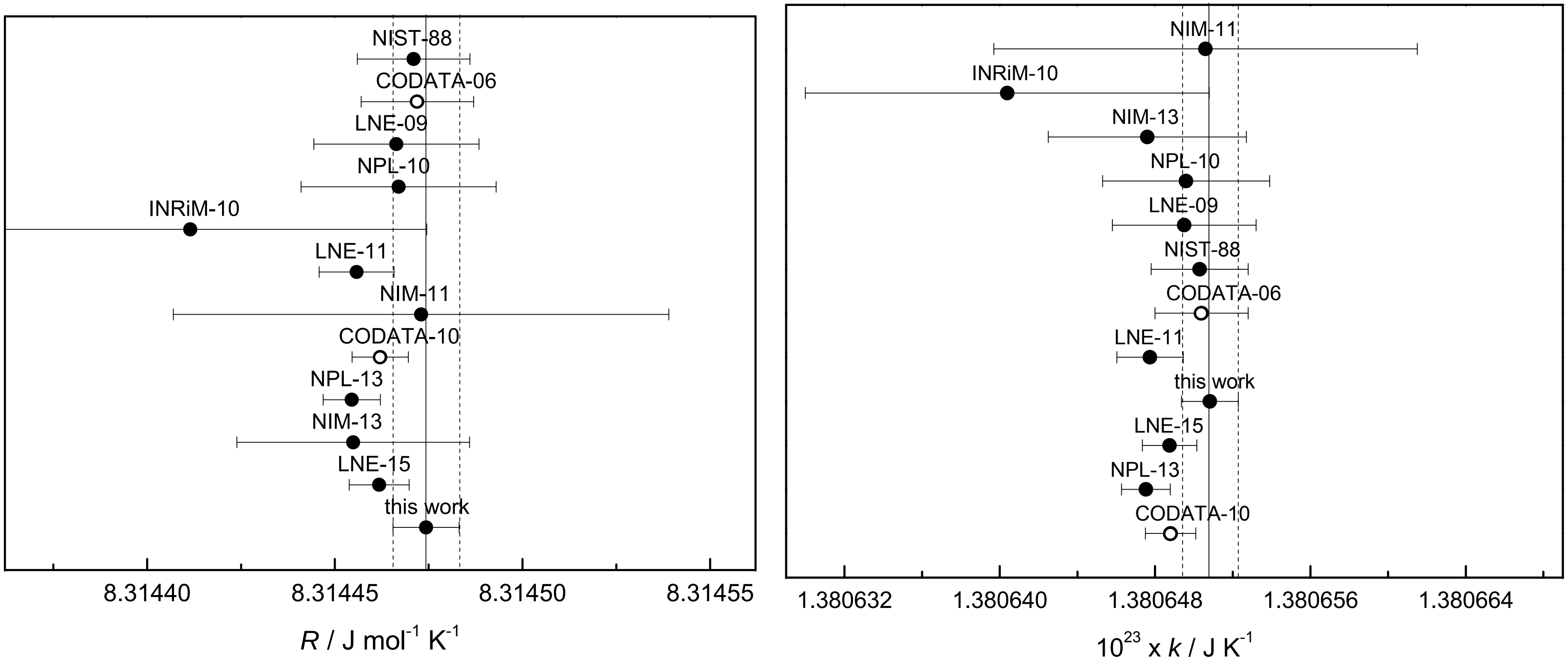
Quantity	Estimate or rel. correction	u(T) / mK	$u_r(T)$ / ppm
Thermal gradients		0.084	0.31
T_{TPW}	273.1600 K	0.020	0.07
Immersion	0.17 mK	0.007	0.03
Repeatability within calibration sequence		0.070	0.26
Self heating - calibration	2.5 mK	0.006	0.02
Short-term repeatability		0.010	0.04
Self heating - measurement	2.4 mK	0.006	0.02
Long term stability (correction)	-0.13×10^{-6}	0.030	0.11
Systematics R25/R100 bridges (correction)	0.31×10^{-6}	0.025	0.09
Total (quadrature sum) T_{Exp}	273.160 05 K	**0.115**	**0.42**

summarized budget of the uncertainty contributions to the determinations of *R* and *k*

Quantity	Estimate	u_r / ppm
u_0^2	$(945\ 710.45 \pm 0.85)\ \mathrm{m^{-2}\ s^{-2}}$	0.90
M	$(4.002\ 6032 \pm 0.000\ 0015) \times 10^{-3}\ \mathrm{kg\ mol^{-1}}$	0.37
T	$(273.160\ 05 \pm 0.000\ 12)\ \mathrm{K}$	0.42
R	$(8.314\ 4743 \pm 0.000\ 0088)\ \mathrm{J\ mol^{-1}\ K^{-1}}$	**1.06**
$k = R / N_A$	$(1.380\ 6508 \pm 0.000\ 0015) \times 10^{-23}\ \mathrm{J\ K^{-1}}$	**1.06**

- the present values of R and k are **1.47** parts in 10^6 **larger** than the corresponding 2010 CODATA values

comparison with previous determinations of *R* and *k*



(Left panel) values of the molar gas constant *R* determined by AGT in chronological order from top to bottom. (Right panel) corresponding values of the Boltzmann constant *k* in order of decreasing uncertainty from top to bottom. CODATA 2006 and CODATA2010 re-adjusted values are marked with a distinctive symbol.