PROGRESS IN THE DETERMINATION OF THE BOLTZMANN CONSTANT WITH DIELECTRIC-CONSTANT GAS THERMOMETRY

C. Gaiser, T.Zandt, B. Fellmuth

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- **Dielectric-Constant Gas Thermometry**
- **Previous Results**
- **Improvements and Status**
- **Polarizability of ^4He**
- **Summary & Outlook**

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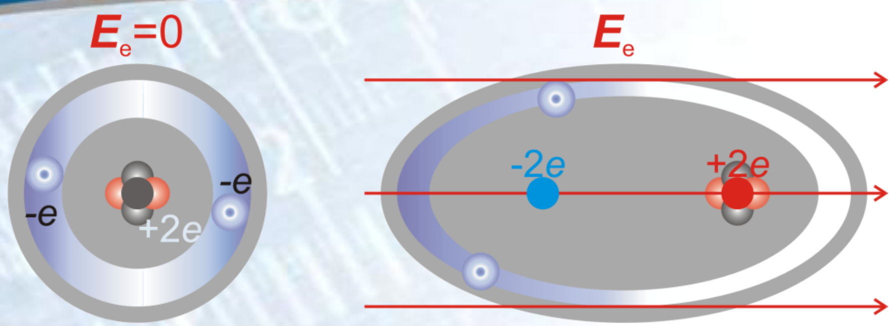
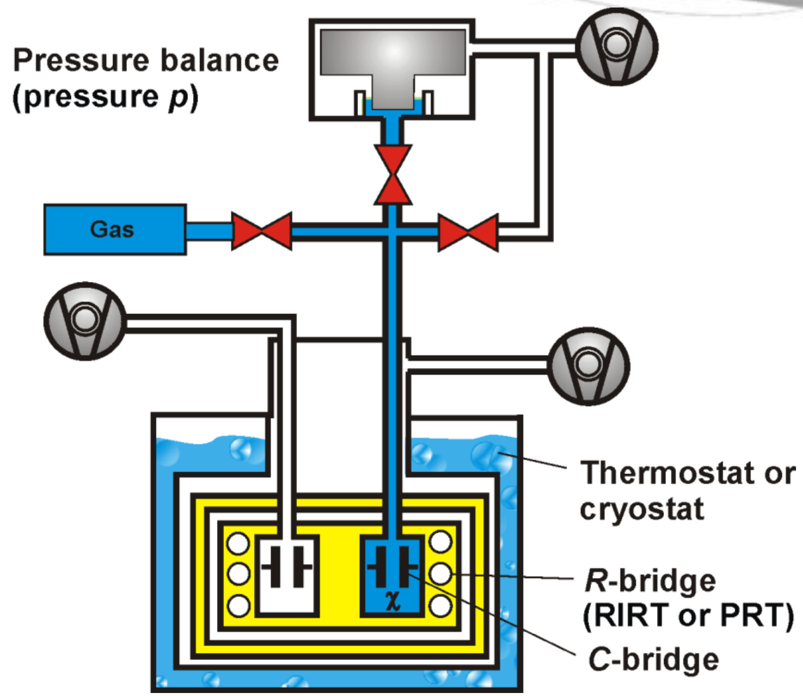
Dielectric-Constant Gas Thermometry (DCGT)





Clausius-Mossotti equation combined with the ideal-gas law:

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{p}{kT}\frac{\alpha_0}{3\varepsilon_0}$$

Measuring quantity :

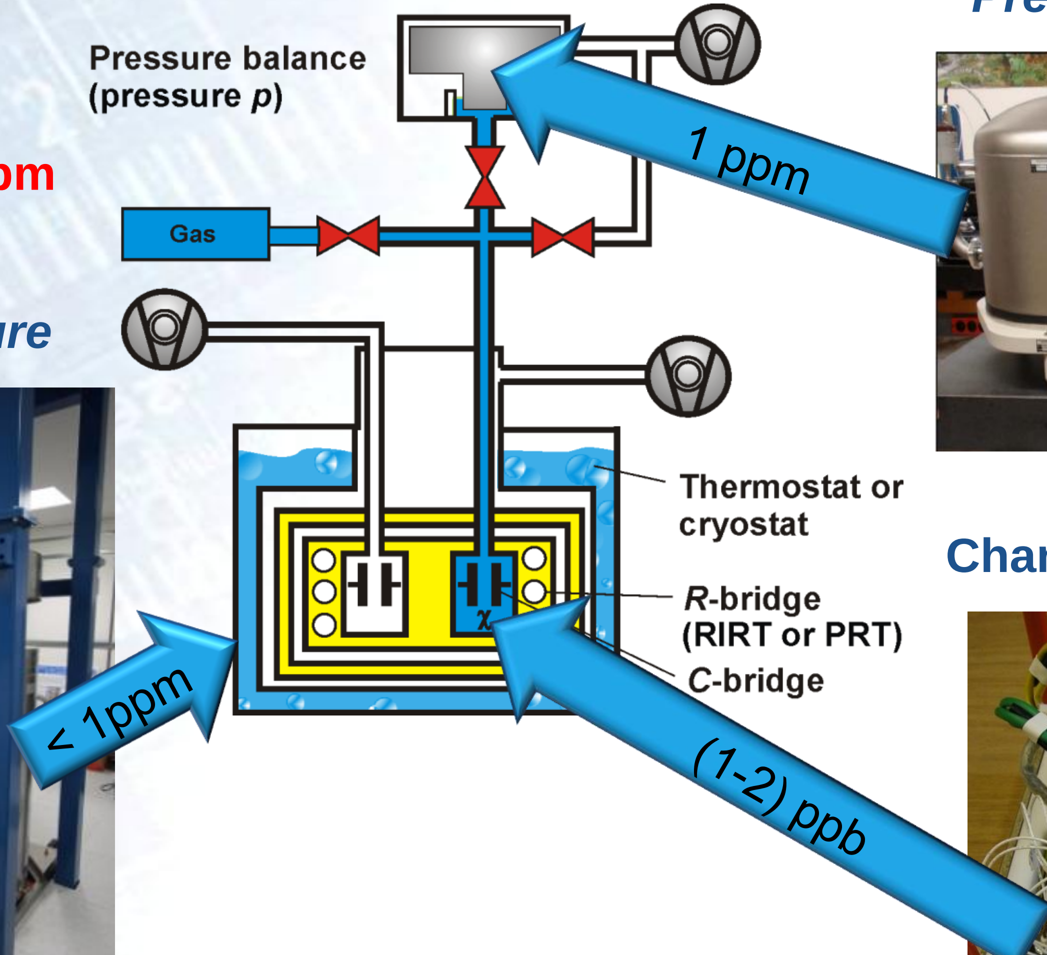
$$\frac{C(p) - C(0)}{C(0)} = \underbrace{\varepsilon_r - 1}_{\chi} + \varepsilon_r \kappa_{eff}\, p$$

ε_r dielectric constant
ε_0 electric constant
α_0 atomic polarizability
κ_{eff} effective compressibility
χ electric susceptibility
p pressure
T temperature

Main Challenges

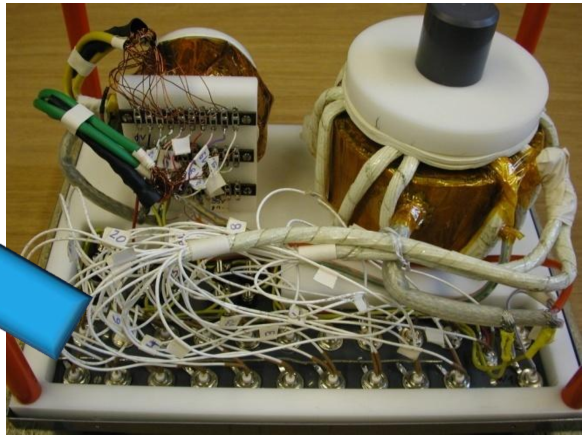
Goal:
$u_r(k) \approx 2$ ppm

Temperature



Pressure

Change of capacitance ratio



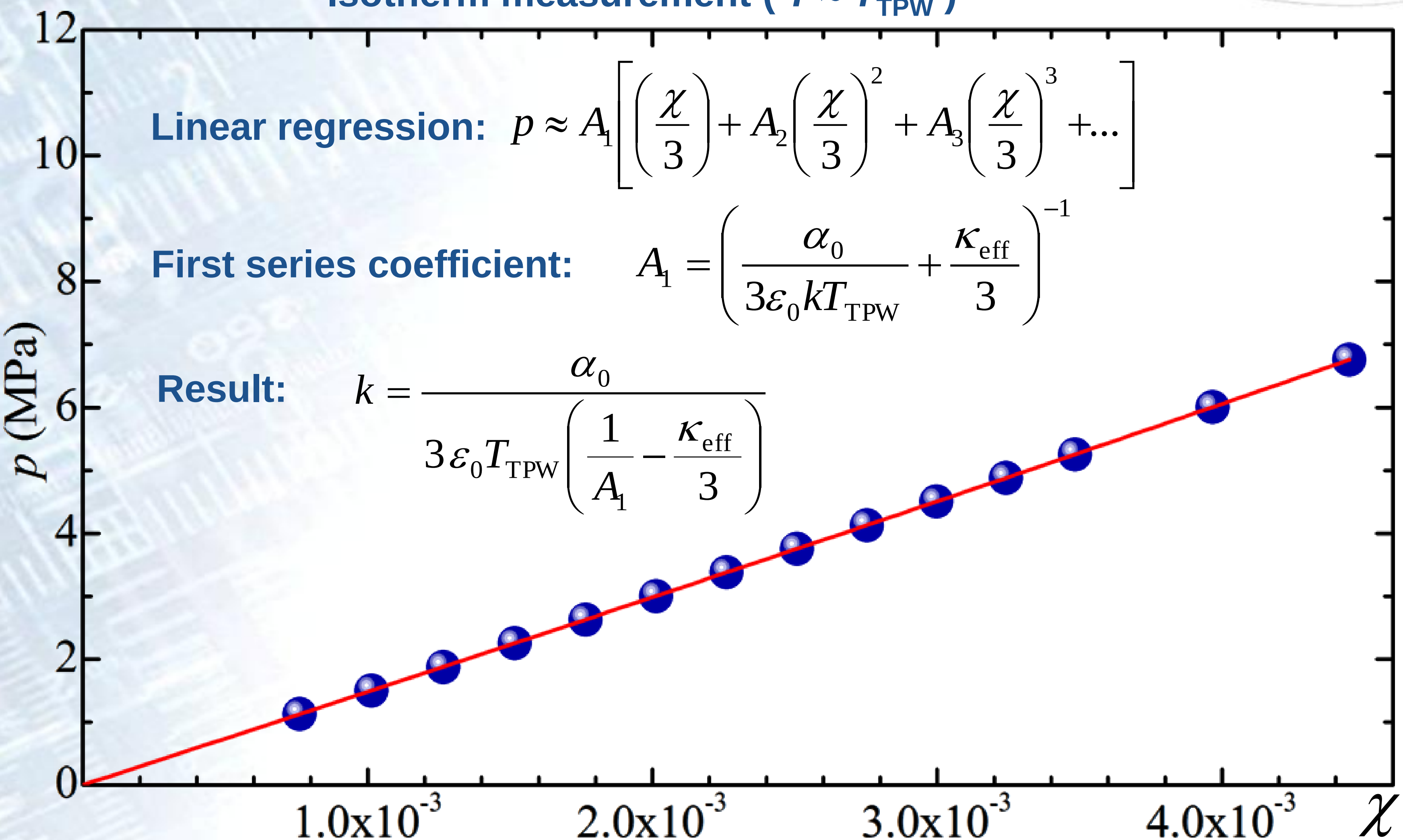
Determination of the Boltzmann constant with DCGT

Isotherm measurement ($T \approx T_{\mathrm{TPW}}$)

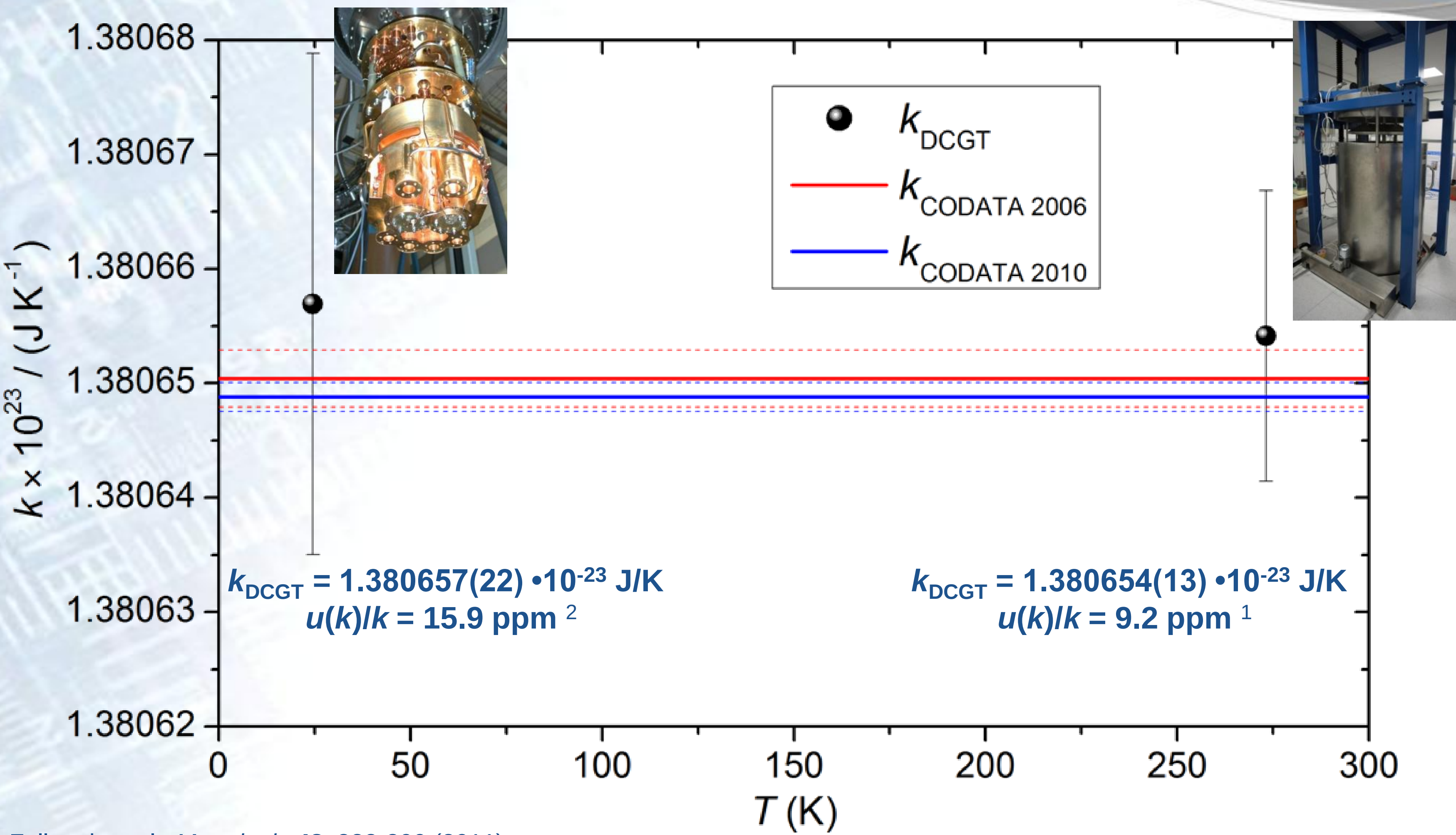
Linear regression: $$p \approx A_1 \left[\left(\frac{\chi}{3} \right) + A_2 \left(\frac{\chi}{3} \right)^2 + A_3 \left(\frac{\chi}{3} \right)^3 + \ldots \right]$$

First series coefficient: $$A_1 = \left(\frac{\alpha_0}{3\varepsilon_0 k T_{\mathrm{TPW}}} + \frac{\kappa_{\mathrm{eff}}}{3} \right)^{-1}$$

Result: $$k = \frac{\alpha_0}{3\varepsilon_0 T_{\mathrm{TPW}} \left(\frac{1}{A_1} - \frac{\kappa_{\mathrm{eff}}}{3} \right)}$$



Determination of the Boltzmann constant with DCGT 2011



[1] B. Fellmuth et al., *Metrologia* **48**, 382-390 (2011)
[2] C. Gaiser and B. Fellmuth, *Metrologia*, **49**, L4-L7 (2012)

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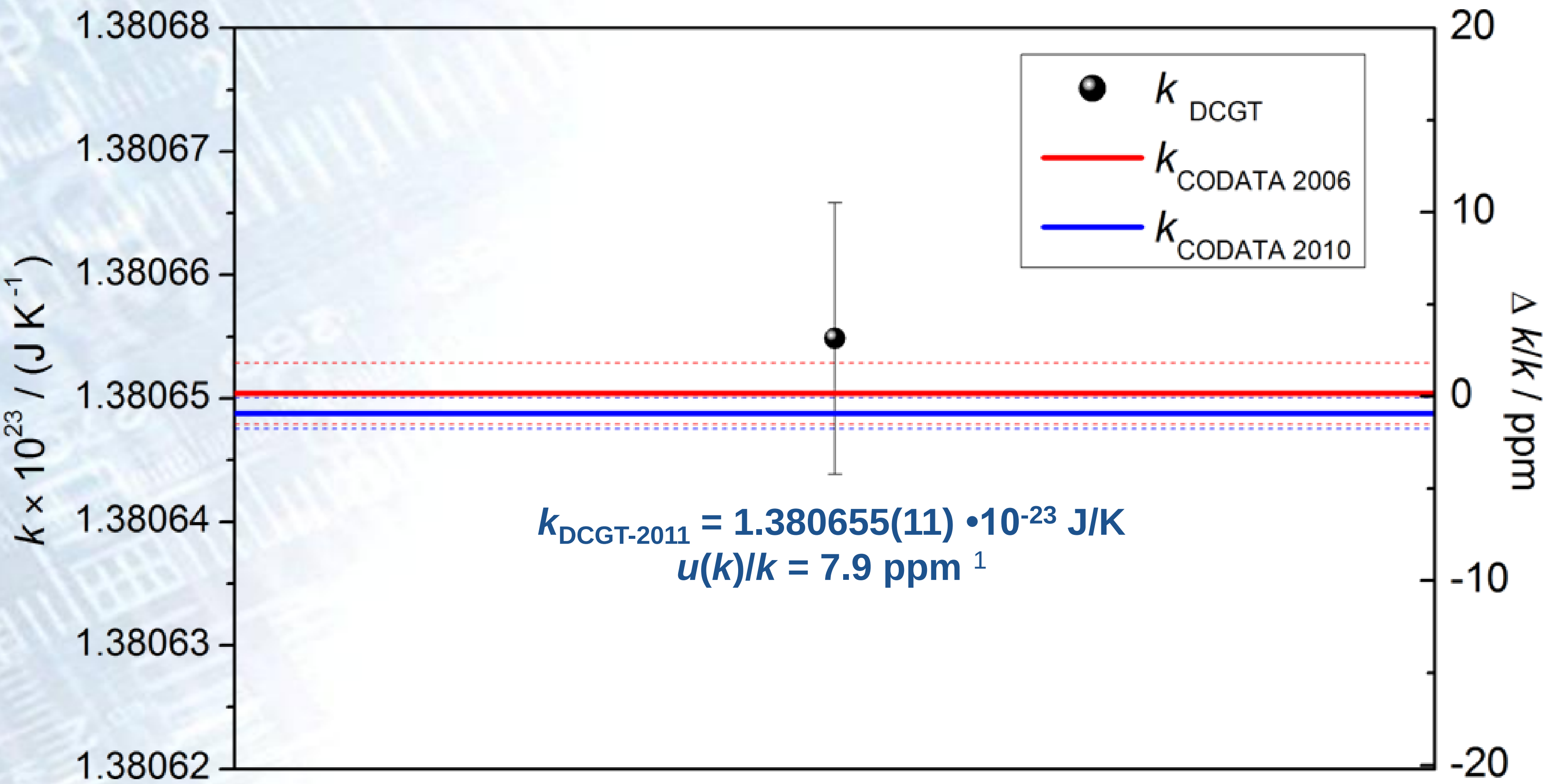
Determination of the Boltzmann constant with DCGT 2011



$k_{DCGT-2011}$ = 1.380655(11) •10^{-23} J/K

$u(k)/k$ = 7.9 ppm [1]

[1] B. Fellmuth et al., *Metrologia* **48**, 382-390 (2011)

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Uncertainty budget DCGT 2011 (TPW)

Typ A

Component	$u(k)/k \cdot 10^6$
Overall weighted mean of A_1 values	6.3

Typ B

Component	$u(k)/k \cdot 10^6$
Susceptibility measurement (capacitance change)	1.0
Pressure measurement	1.9
Temperature	0.3
Determination of the effective compressibility	5.8
Head correction	0.2
Impurities (measuring gas)	2.4
Surface layers (impurities)	1.0
Polarizability ab initio calculation (theory)	0.2

Combined standard uncertainty: 9.2 ppm

B. Fellmuth et al., *Metrologia* **48**, 382-390 (2011)

Reducing the uncertainty of the effective compressibility

Improvements in RUS measurements (e.g. temperature-dependent measurements)

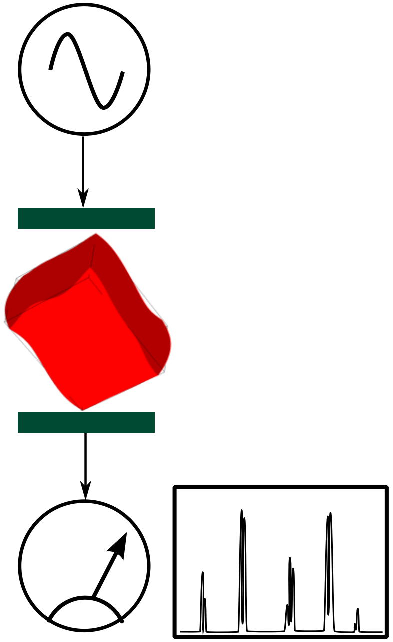
Refinement of evaluation models (FEM, Monte-Carlo simulation)

Test samples also for the insulation materials (Al_2O_3)

Determination of the thermal expansion coefficient and the molar specific heat capacity

adiabatic → isothermal

Resonant ultrasound spectroscopy (**RUS**):



Reducing the uncertainty of the effective compressibility

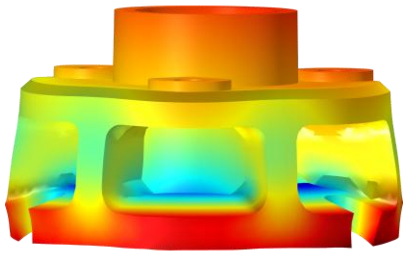
FEM calculations of effective compressibility for specific capacitor geometry

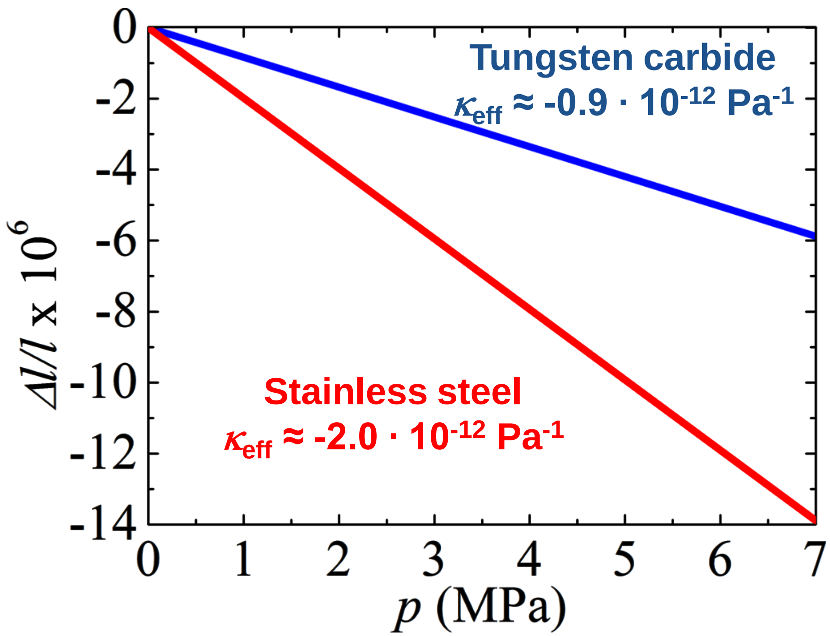
→optimization of capacitor design

Switch from stainless steel to tungsten carbide as capacitor material

Result (2013)
Composite isothermal compressibility:

-9.370×10^{-13} Pa^{-1} (u_{rel} = 0.17%)

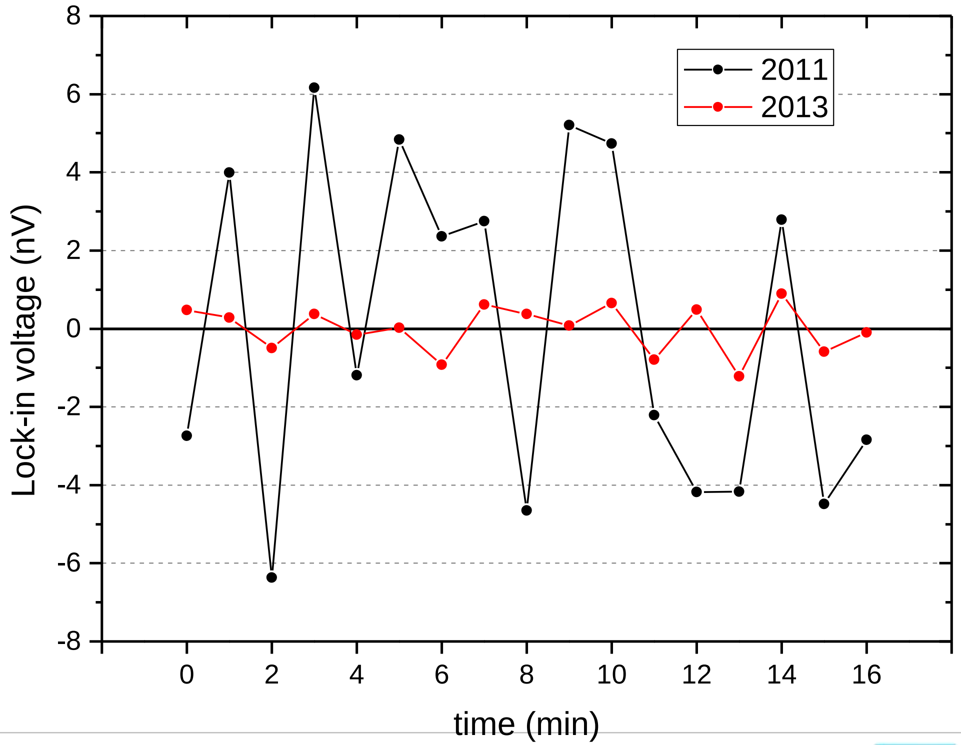




Reducing the uncertainty of the susceptibility

Careful analysis of the capacitance measuring network (chokes) and switch to low noise cable

Only one cable for the zero detector (increase of the sensitivity by nearly a factor two)



Purity of the measuring gas

99.99999% Helium
(Linde AG)

↓

Gas purifier (adsorber)
(Micro Torr SP70, SAES Pure Gase, Inc.)

↓

Helium purifier (getter)
(HP2, Valco Instruments, Co. Inc.)

Component	Certificate Gas (ppb)	Specification Getter (ppb)	Specification Adsorber (ppb)
H_2	< 30	< 10	< 0.1
H_2O	< 50	< 10	< 0.1
O_2	< 30	< 10	< 0.1
CO	< 30	< 10	< 0.1
CO_2	< 30	< 10	< 0.1
N_2		< 10	
Hydro-carbons	< 1	< 10	< 0.1
Noble gases			

Purity of the measuring gas

Mass-spectrometry measurements

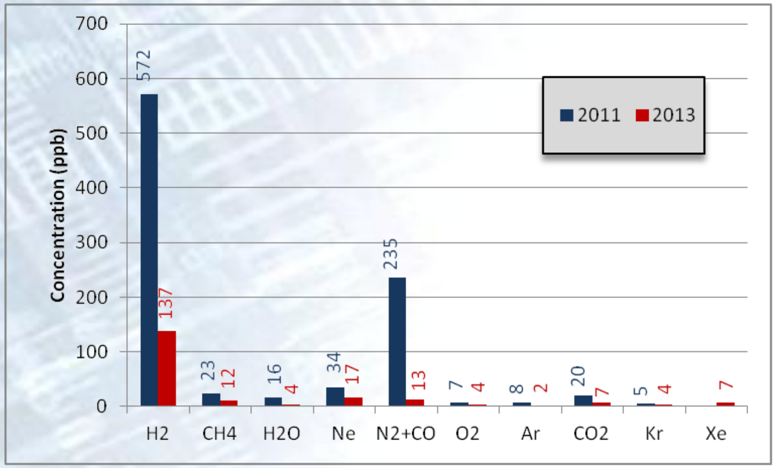
↓

Experiment

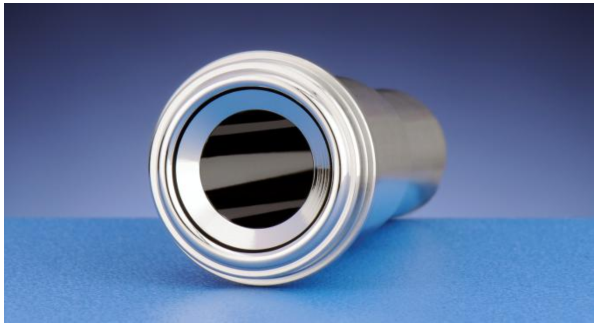
(UHP gas tubing)

↓

Mass-spectrometry measurements



Component	Dectection limit (ppb)
H_2	< 300
H_2O	< 20
O_2	< 10
CO	< 100
CO_2	< 50
N_2	< 100
CH_4	< 20
Ne	< 10
Ar	< 10
Kr	< 10
Xe	< 10



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↓

Mass-spectrometry measurements

↓

Experiment
(UHP gas tubing)

↓

Mass-spectrometry measurements

Component	Mass-Spec and Getter Spec. (ppb)	Sensitivity in He	Uncertainty (ppm)*
H_2	< 10	4	0.02
H_2O	< 10	160	0.9
O_2	< 10	10	0.06
CO_2	< 50	10	0.3
N_2 & CO	< 100	8	0.4
Ne	< 10	2	0.01
Ar	< 10	8	0.05
Kr	< 10	10	0.06
Xe	< 10	16	0.09
Combined uncertainty			**1.0**

*(asymmetric rectangular distribution)

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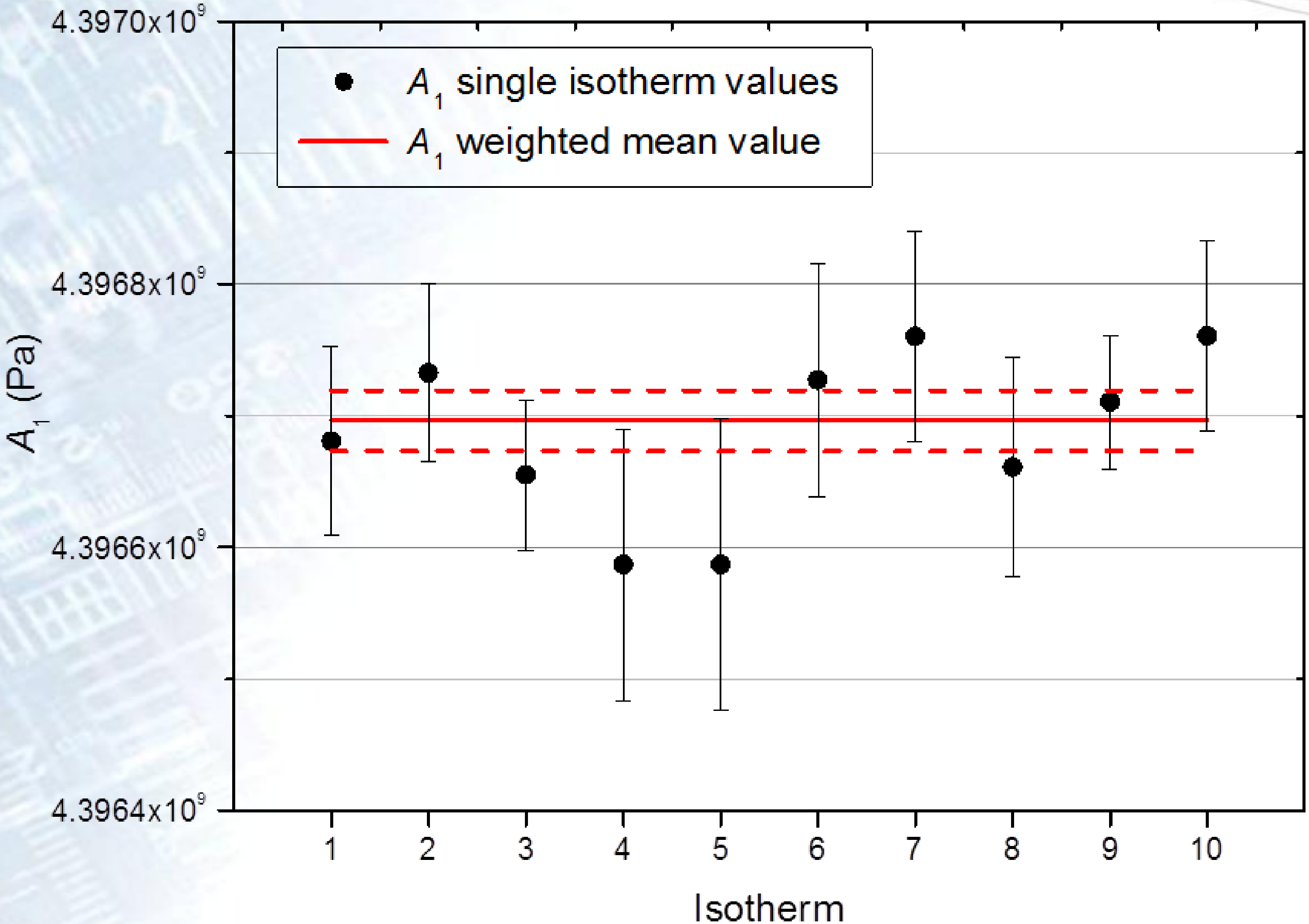
Results 2013 (10 isotherms)



Uncertainty budget DCGT 2013 (TPW)

Typ A

Component	$u(k)/k \cdot 10^6$
Overall weighted mean of A_1 values	**2.6** (6.3)

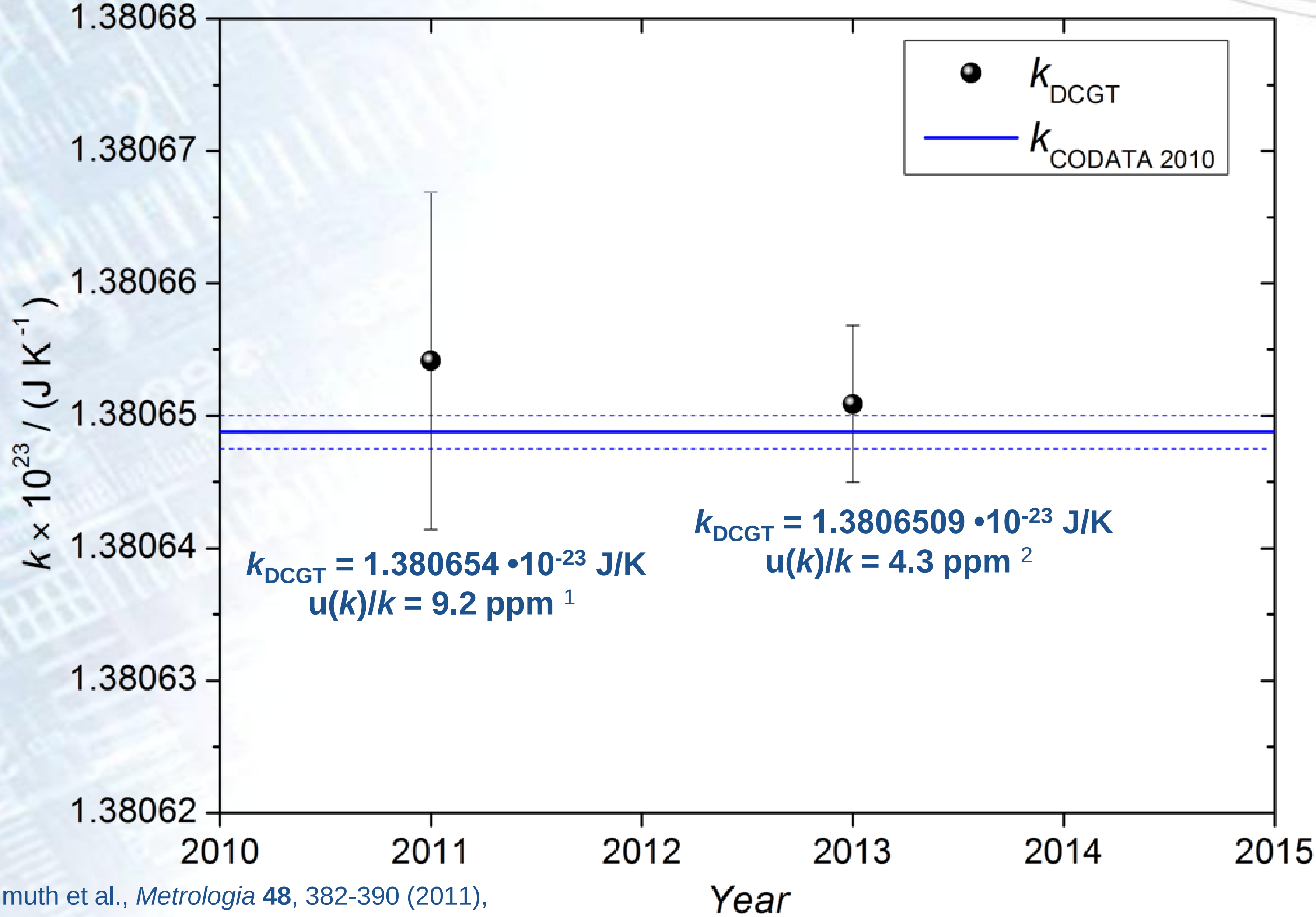
Typ B

Component	$u(k)/k \cdot 10^6$
Susceptibility measurement (capacitance change)	1.0 (1.0)
Pressure measurement	1.9 (1.9)
Temperature	0.3 (0.3)
Determination of the effective compressibility	**2.4** (5.8)
Head correction	0.2 (0.2)
Impurities (measuring gas)	**1.0** (2.4)
Surface layers (impurities)	**0.5** (1.0)
Polarizability ab initio calculation (theory)	0.2 (0.2)

Combined standard uncertainty: 4.3 ppm (9.2 ppm)

B. Fellmuth et al., *Metrologia* **48**, 382-390 (2011),
C. Gaiser et al., *Metrologia*, **50,** L7-L11 (2013)

Determination of the Boltzmann constant with DCGT (TPW)



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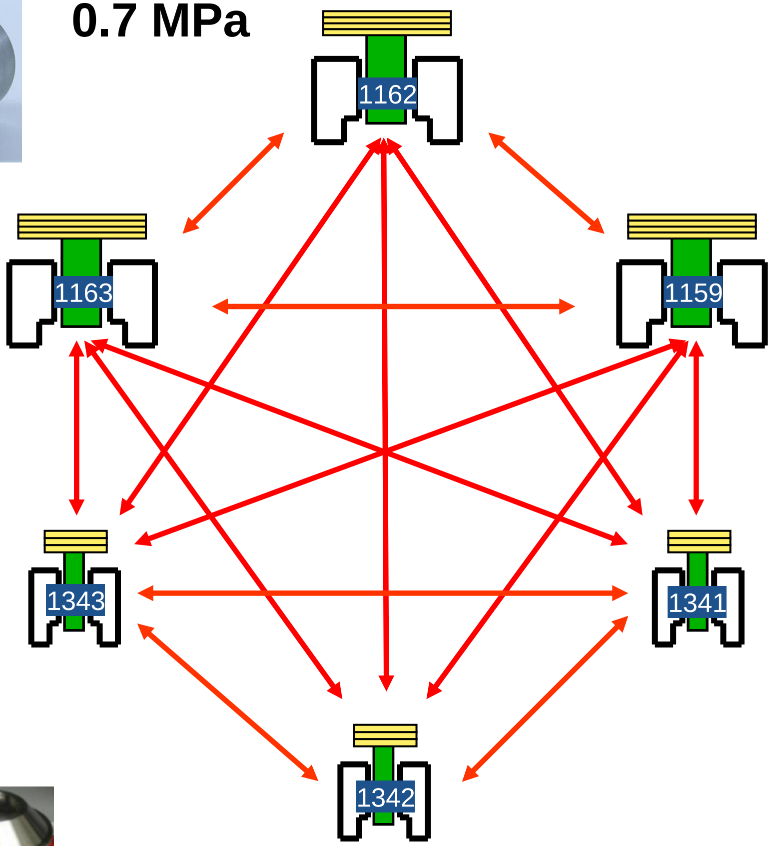
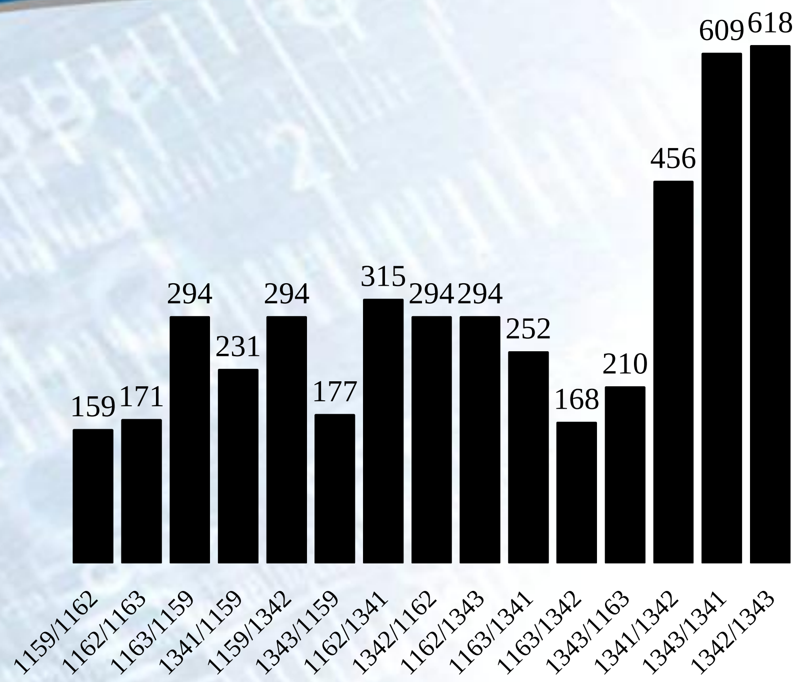
Improved pressure standard

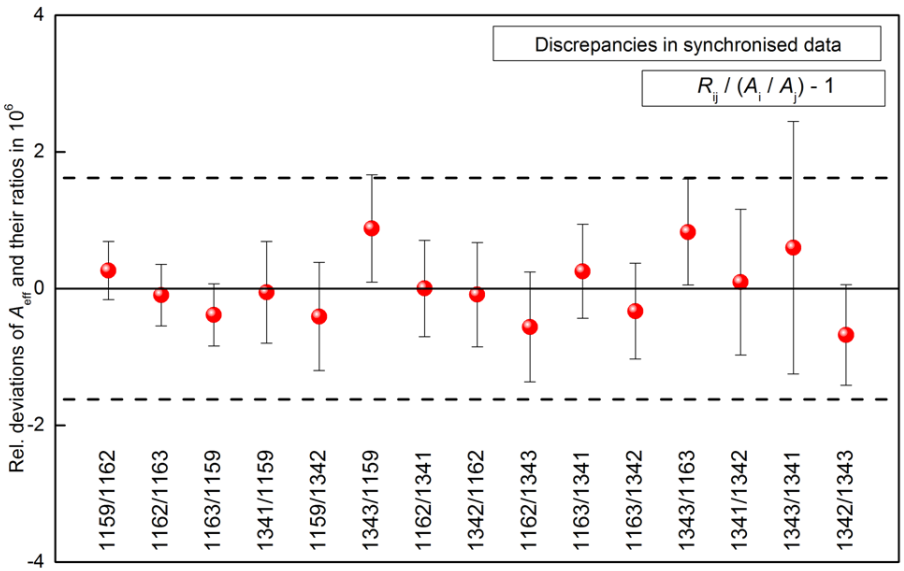
20 cm² – Systems
0.7 MPa



2 cm² – Systems
7 MPa

Improved pressure standard

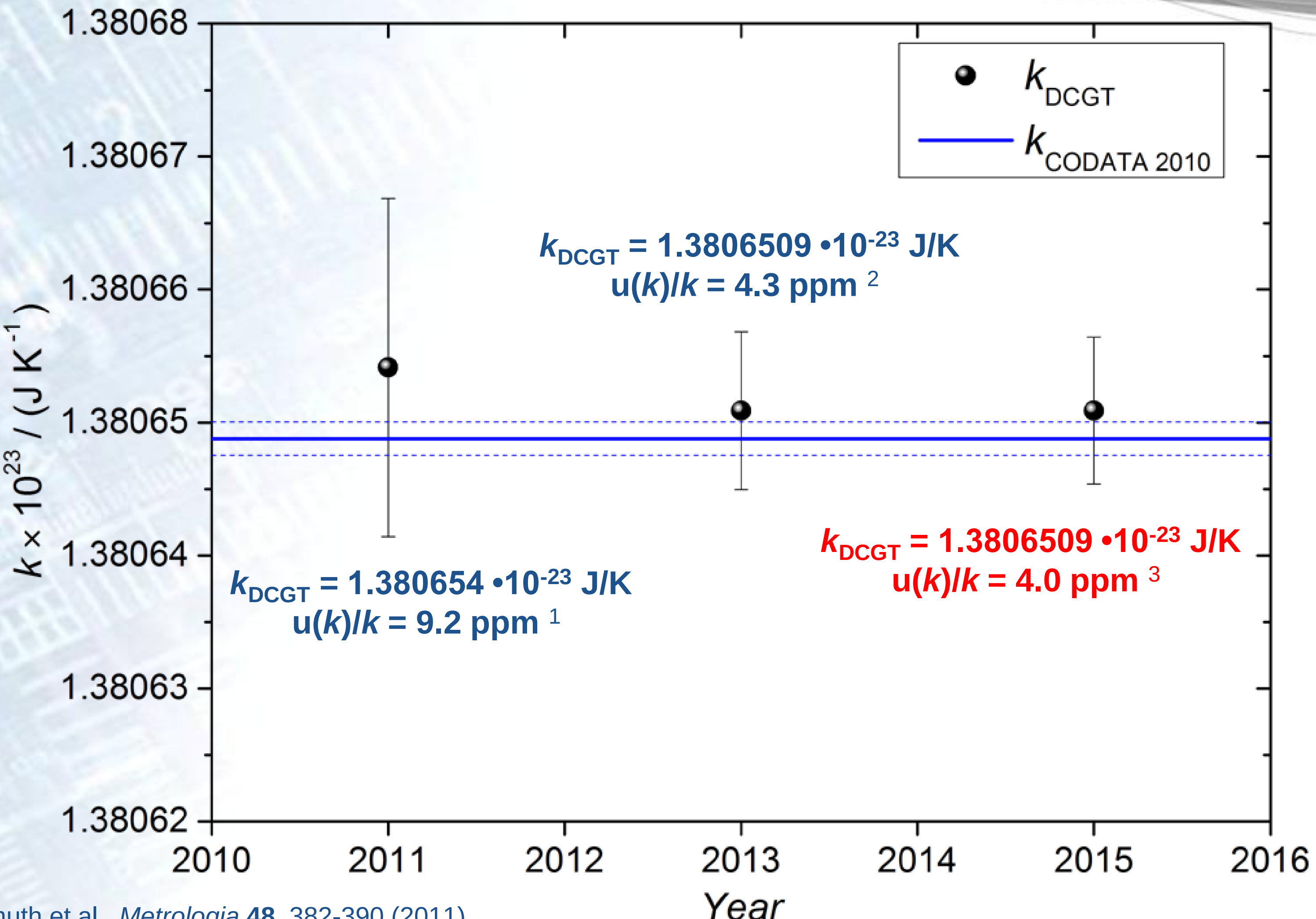




$u_{2013}(p)$=1.9 ppm→ $u_{2015}(p)$=1.0 ppm

T. Zandt et al., *Metrologia*, **Special-issue on *k*** (2015)

Determination of the Boltzmann constant with DCGT (TPW)



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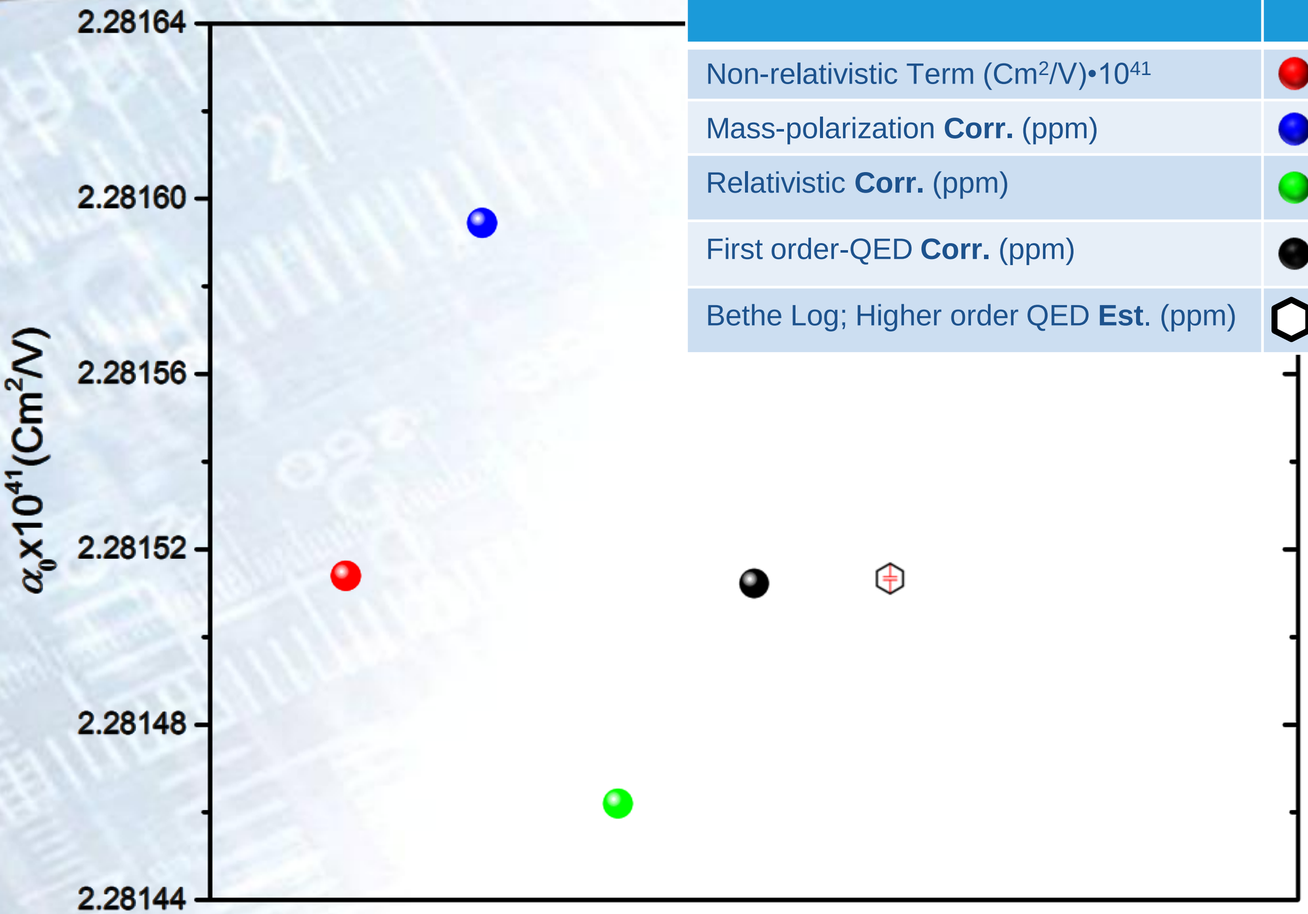
[2] C. Gaiser et al., *Metrologia*, **50,** L7-L11 (2013)
[3] C. Gaiser et al., *Metrologia*, **Special-issue on *k*** (2015)

Polarizability of ^{4}He (Theory)



		Calculations [1-3]	u_r (ppm)
Non-relativistic Term (Cm^2/V)•10^{41}		**2.281513908**	**0.00015**
Mass-polarization **Corr.** (ppm)		**35.312**	**0.011**
Relativistic **Corr.** (ppm)		**-58.077**	**0.023**
First order-QED **Corr.** (ppm)		**22.023**	**0.007**
Bethe Log; Higher order QED **Est**. (ppm)		**0.68**	**0.20**

[1] Pachucki K. and Sapirstein 2000 PRA **63** 012504
[2] Cencek W et al. 2001 PRL **86** 5675
[3] Łach G et al. 2004 PRL **92** 233001
[4] Puchalski M et al. 2011 PRA 83 042508
[5]Jentschura U D et al 2011 PRA 84 064102

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Polarizability of ^{4}He (Experiment)

$$k = \frac{\alpha_0}{3\varepsilon_0 T_{\mathrm{TPW}}\left(\frac{1}{A_1} - \frac{\kappa_{\mathrm{eff}}}{3}\right)} \Rightarrow \alpha_0 = 3\varepsilon_0 T_{\mathrm{TPW}}\left(\frac{1}{A_1} - \frac{\kappa_{\mathrm{eff}}}{3}\right) k$$

$A_{1,}\kappa_{\mathrm{eff}}$

- DCGT data (same uncertainty as for *k*-determination)

k

- Boltzmann constant derived from NIST AGT value (to avoid correlations and discussions on the actual discrepancies (see you on Friday)) $k = 1.38065027 \cdot 10^{-23}$ J/K with $u_r = 1.8$ ppm

α_0

- $\alpha_{0\ \mathrm{DCGT}} = 2.2815120(98) \cdot 10^{-41}\mathrm{Cm^2/V}$ with $u_r = 4.3$ ppm

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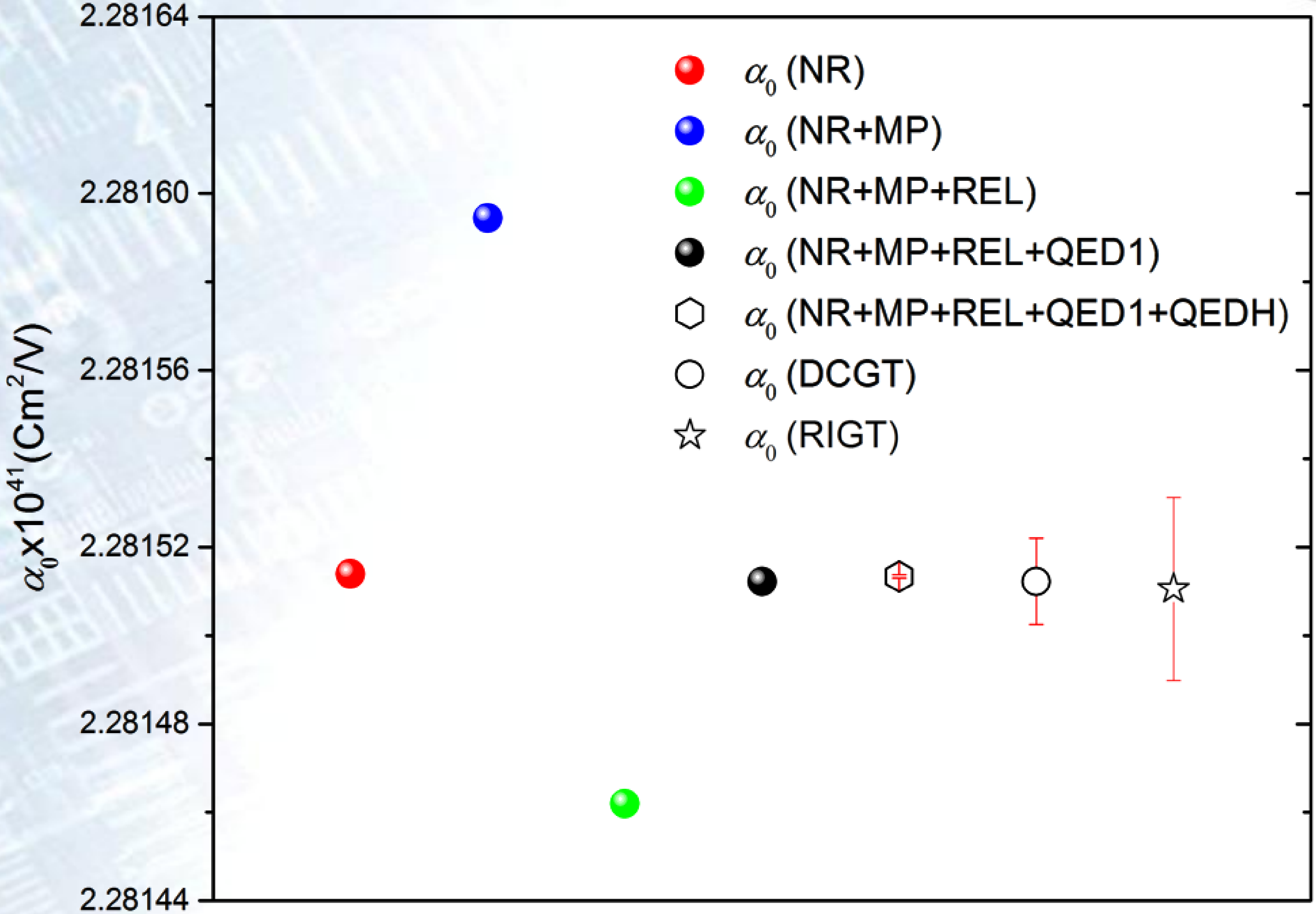
Polarizability of ^4He (Theory ↔ Experiment)



[RIGT] Schmidt J W et al. PRL 2007 **98** 254504

Outlook / Decrease of the uncertainty

Type A

Component	$u(k)/k \cdot 10^6$
Overall weighted mean of A_1 values	2.6

What is next:
DCGT measurements with two different tungsten carbide cylindrical capacitors and test measurements with a 24-ring cross-capacitor
Hope:
Extremely stable results→ reduction in the Type A uncertainty.
Consistent results for at least two or even three different capacitors
→ reduction for $u(\kappa_{\text{eff}})$

Type B

Component	$u(k)/k \cdot 10^6$
Susceptibility measurement (capacitance change)	1.0
Pressure measurement	1.0
Temperature	0.3
Determination of the effective compressibility	2.4
Head correction	0.2
Impurities (measuring gas)	1.0
Surface layers (impurities)	0.5
Polarizability ab initio calculation (theory)	0.2

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Summary

A new value for *k* has been determined applying DCGT

- k_{DCGT} = 1.3806509 x 10^{-23} J/K with a relative uncertainty of 4.0 ppm.
- 1.5 ppm larger than the 2010 CODATA value.
- Most accurate experimental value for α_0. In excellent agreement with the ab initio theory.

Improvements compared to 2011

- Improved determination of the effective compressibility κ_{eff}
- Use of tungsten carbide cylindrical capacitors
- Increase of the signal to noise ratio
- Better dealing with the purity of the measuring gas
- Extended piston gauge calibration.

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Thank for your attention!