Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin
Nationales Metrologieinstitut

The International Temperature Scale and the new Definition of the Kelvin

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Fundamental Constants Meeting 2015
Eltville
4th February 2015

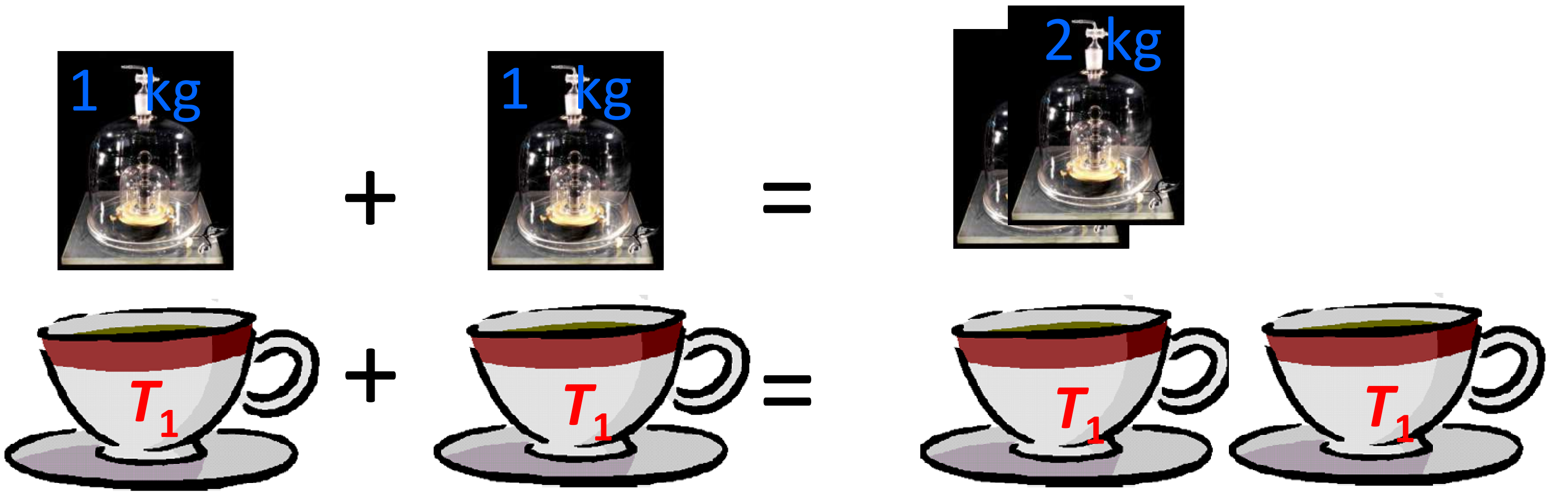
Contents:

- **Introduction**
- International Temperature Scale
- Consequences of the new definition
- Determination of the Boltzmann constant
- New definition of the kelvin

Why do we need a temperature scale ?

Temperature not additive

→ **Two systems with *T* do not yield 2*T* (in contrast to mass or length)**



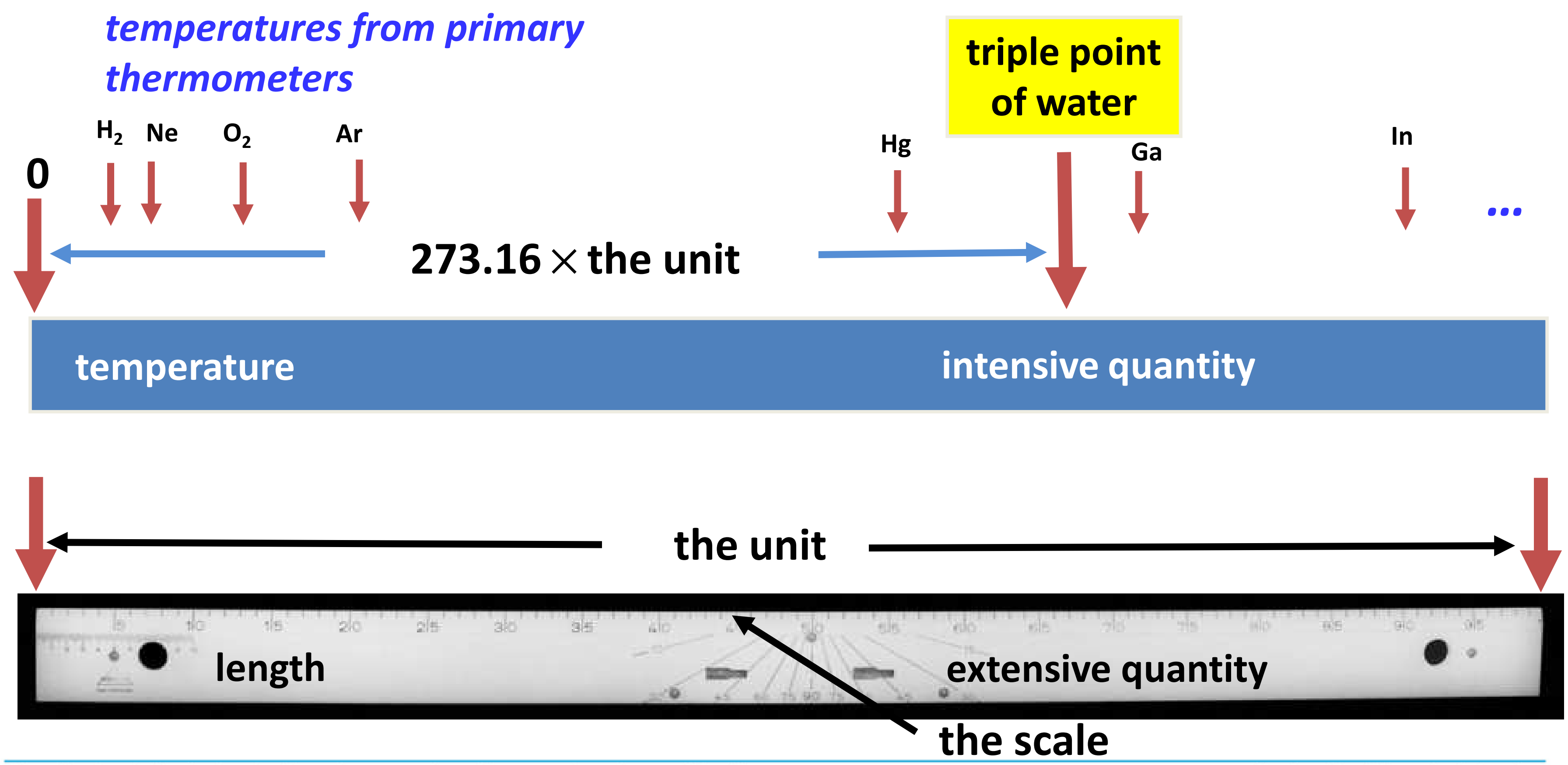
Temperature scale based on temperature fixed points realized by phase transitions of pure elements

Temperatures of fixed points determined by primary thermometers

Standards and Scales

the scale:

additional fixed points and interpolating instruments



The Meter Convention of 1875

CONVENTION DU MÈTRE

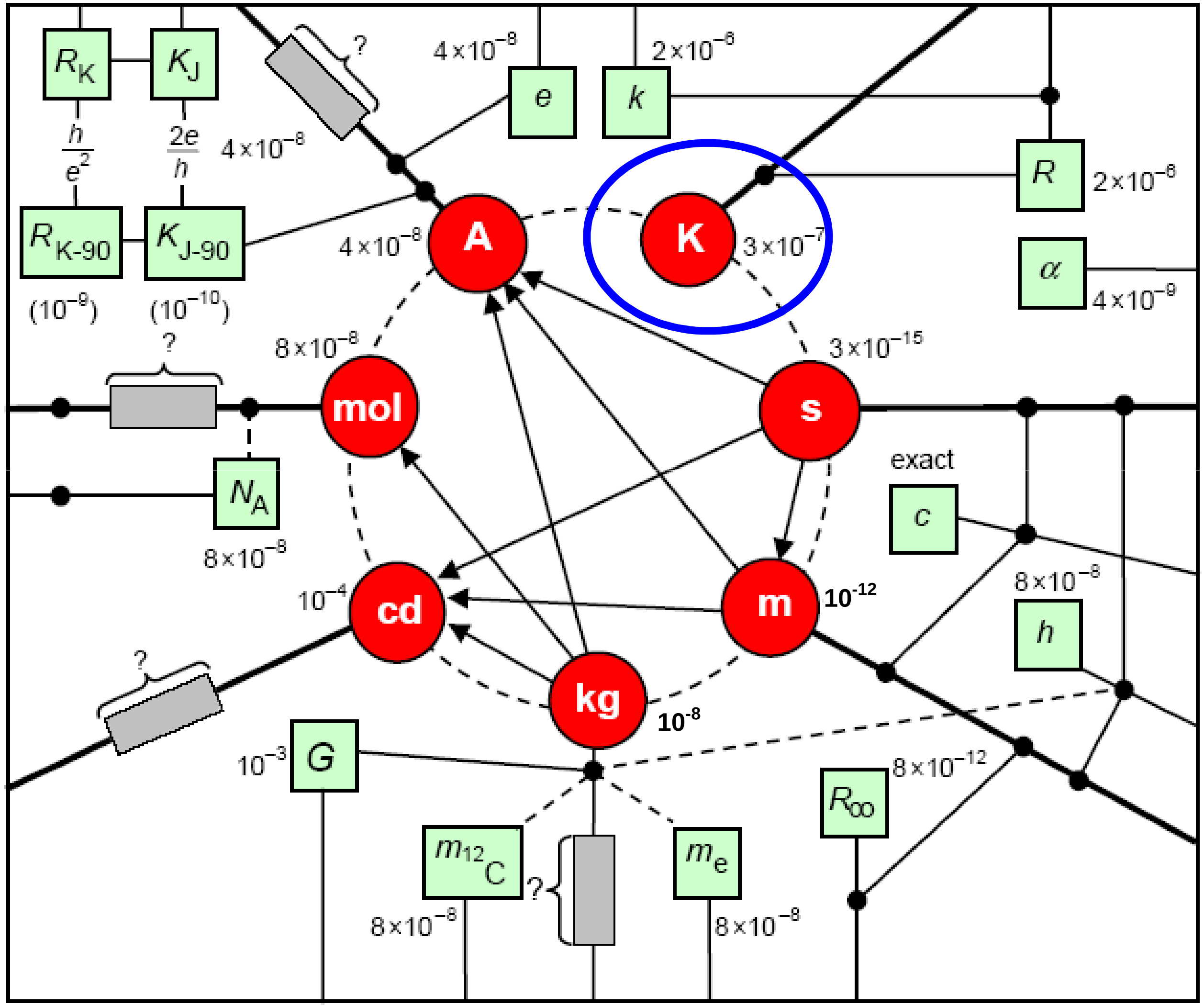
ARTICLE PREMIER (1875)

Les Hautes Parties contractantes s'engagent à fonder et entretenir, à frais communs, un *Bureau international des poids et mesures,* scientifique et permanent, dont le siège est à Paris ([1]).

ART. 3 (1875)

Le Bureau international fonctionnera sous la direction et la surveillance exclusives d'un *Comité international des poids et mesures,* placé lui-même sous l'autorité d'une *Conférence générale des poids et mesures,* formée de délégués de tous les Gouvernements contractants.

Units and Fundamental Constants



New definitions of SI base units

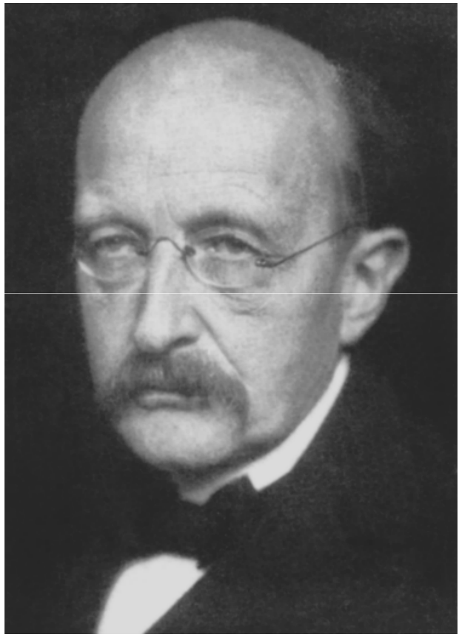
SI Base unit	Definition	Uncertainty	Related FC	Prospected FC
kilogram	1889	10^{-8}		h
ampere	1948	10^{-7}		e
second	1967	10^{-15}	^{133}Cs	
kelvin	1967	10^{-7}		k
mole	1971	10^{-7}		N_A
candela	1979	10^{-4}		
metre	1983	10^{-12}	c_0	

Annalen der Physik, 1, 69-122, 1900
Planck´s natural units

Dem gegenüber dürfte es nicht ohne Interesse sein zu bemerken, dass mit Zuhülfenahme der beiden in dem Ausdruck (41) der Strahlungsentropie auftretenden Constanten a und b die Möglichkeit gegeben ist, Einheiten für Länge, Masse, Zeit und Temperatur aufzustellen, welche, unabhängig von speciellen Körpern oder Substanzen, ihre Bedeutung für alle Zeiten und für alle, auch ausserirdische und aussermenschliche Culturen notwendig behalten und welche daher als „natürliche Maasseinheiten" bezeichnet werden können.

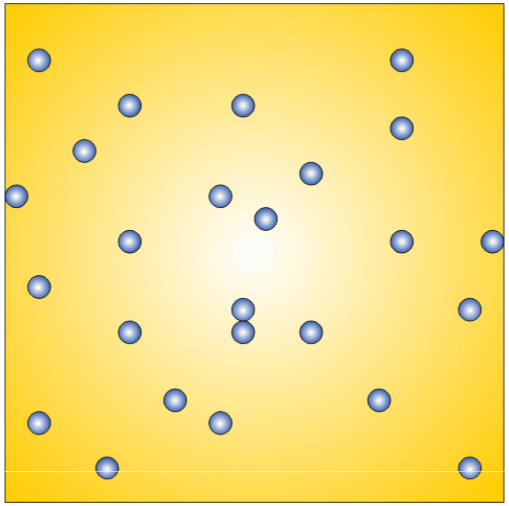
$$L_s(\lambda, T) = \frac{2hc^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right)-1\right)^{-1}$$

Die Mittel zur Festsetzung der vier Einheiten für Länge, Masse, Zeit und Temperatur werden gegeben durch die beiden erwähnten Constanten a und b, ferner durch die Grösse der Lichtfortpflanzungsgeschwindigkeit c im Vacuum und durch die der Gravitationsconstante f.



Max Planck
(Nobel price 1918)

New definition based on Boltzmann constant *k*



ideal system of "particles"

thermal energy E per degree of freedom

thermodynamic temperature T

k conversion factor

$$E = \tfrac{1}{2}kT$$

2010 CODATA value of Boltzmann constant :

Rev. Mod. Phys. 84 2012, 1527

$k = R/N_A = 1.3806488 \times 10^{-23}$ JK^{-1} $u_r = 9.1 \times 10^{-7}$

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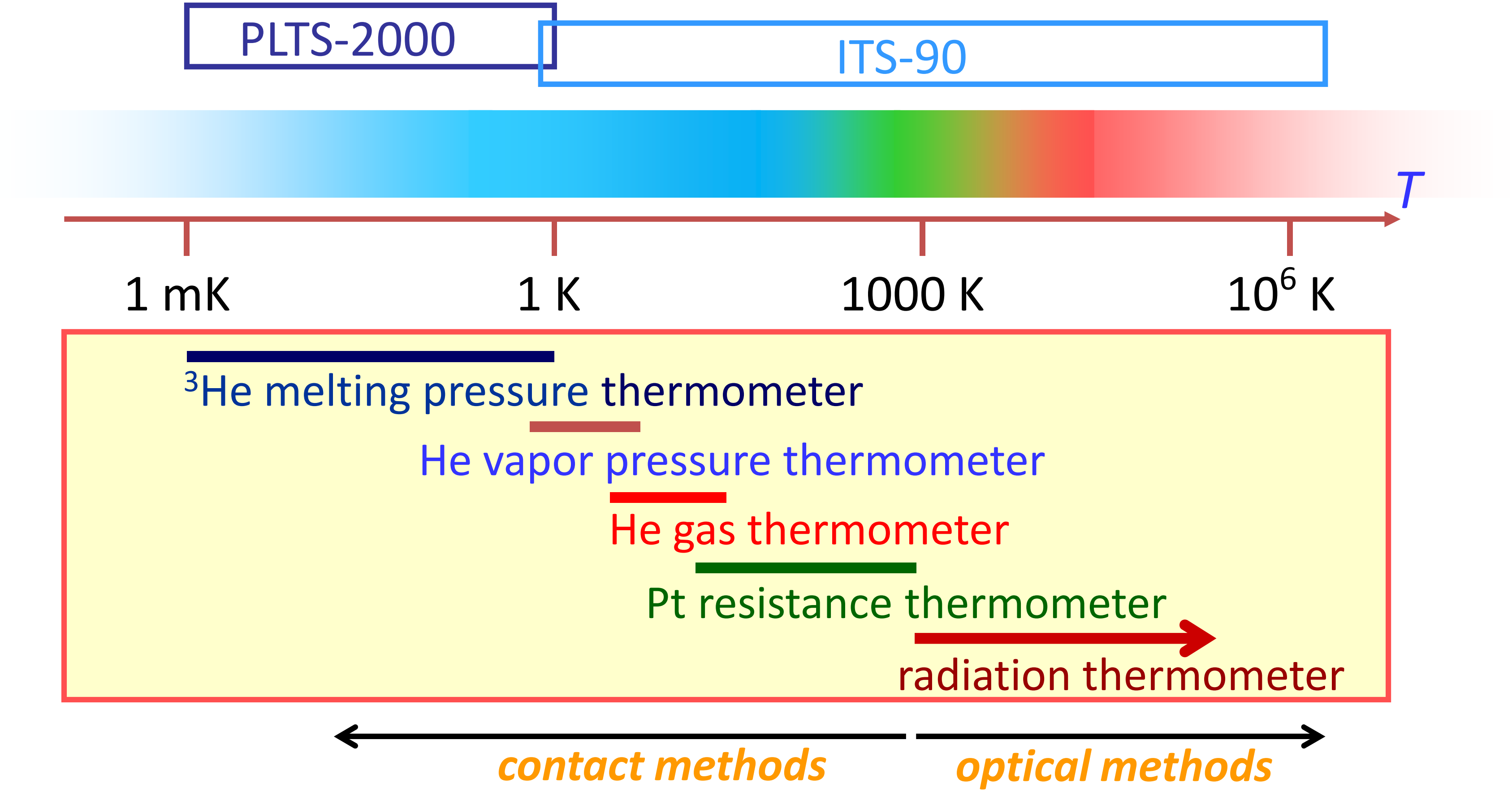
Temperature scales ITS-90 and PLTS-2000



The International Temperature Scale ITS-90

TABLES, FIGURES AND REFERENCES

fixed points

TABLE 1.1

The defining fixed points of the ITS-90.

Number	Temperature T_{90}/K	Temperature t_{90}/°C	Sub-stance[1.7]	State[1.8]	$W_r(T_{90})$
1	3 to 5	−270,15 to −268,15	He	vp	
2	13,8033	−259,3467	e-H_2	tp	0,001 190 07
3	≈17	≈ −256,15	e-H_2 (or He)	vp (or gp)	(0,002 296 46)[1.9]
4	≈20,3	≈ −252,85	e-H_2 (or He)	vp (or gp)	(0,004 235 36)[1.9]
5	24,5561	−248,5939	Ne	tp	0,008 449 74
6	54,3584	−218,7916	O_2	tp	0,091 718 04
7	83,8058	−189,3442	Ar	tp	0,215 859 75
8	234,3156	−38,8344	Hg	tp	0,844 142 11
9	273,16	0,01	H_2O	tp	1,000 000 00
10	302,9146	29,7646	Ga	mp	1,118 138 89
11	429,7485	156,5985	In	fp	1,609 801 85
12	505,078	231,928	Sn	fp	1,892 797 68
13	692,677	419,527	Zn	fp	2,568 917 30
14	933,473	660,323	Al	fp	3,376 008 60
15	1234,93	961,78	Ag	fp	4,286 420 53
16	1337,33	1064,18	Au	fp	
17	1357,77	1084,62	Cu	fp	

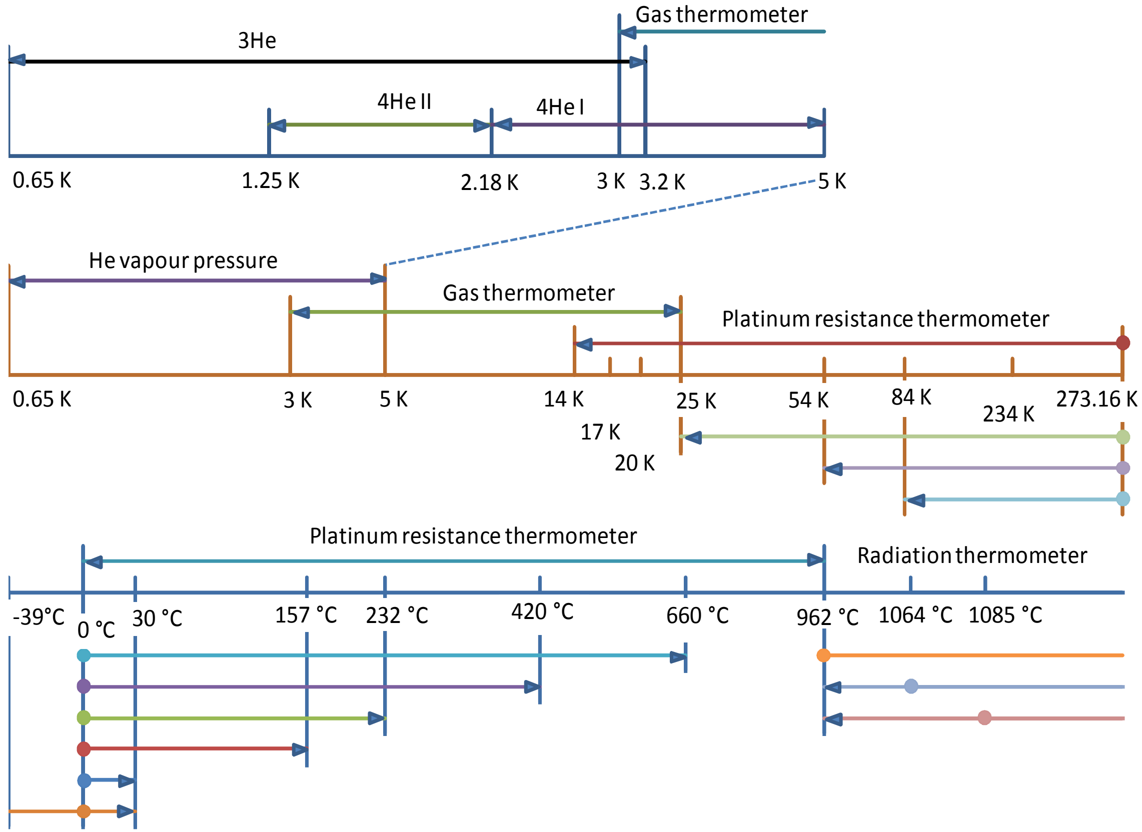
1.7 All substances except helium (both ^{3}He and ^{4}He are used) are of natural isotopic composition, e-H_2 is hydrogen at the equilibrium concentration of the ortho- and para-molecular forms.

1.8 For complete definitions and advice on the realization of these various states, *see* Section 2. The symbols have the following meaning: vp: vapour pressure point; tp: triple point (temperature at which the solid, liquid and vapour phases are in equilibrium); gp: gas thermometer point; mp, fp: melting point, freezing point (temperature, at a pressure of 101 325 Pa, at which the solid and liquid phases are in equilibrium)

1.9 The values corresponding to fixed points numbers 3 and 4 are calculated for T_{90} = 17,035 K and T_{90} = 20,27 K respectively (*see* Section 2.3.4).

Figure 1. Schematic representation of the ranges, sub-ranges and interpolation instruments of ITS-90. The temperatures shown are approximate only.

interpolation



Definition of the Kelvin (1968) and triple point of water

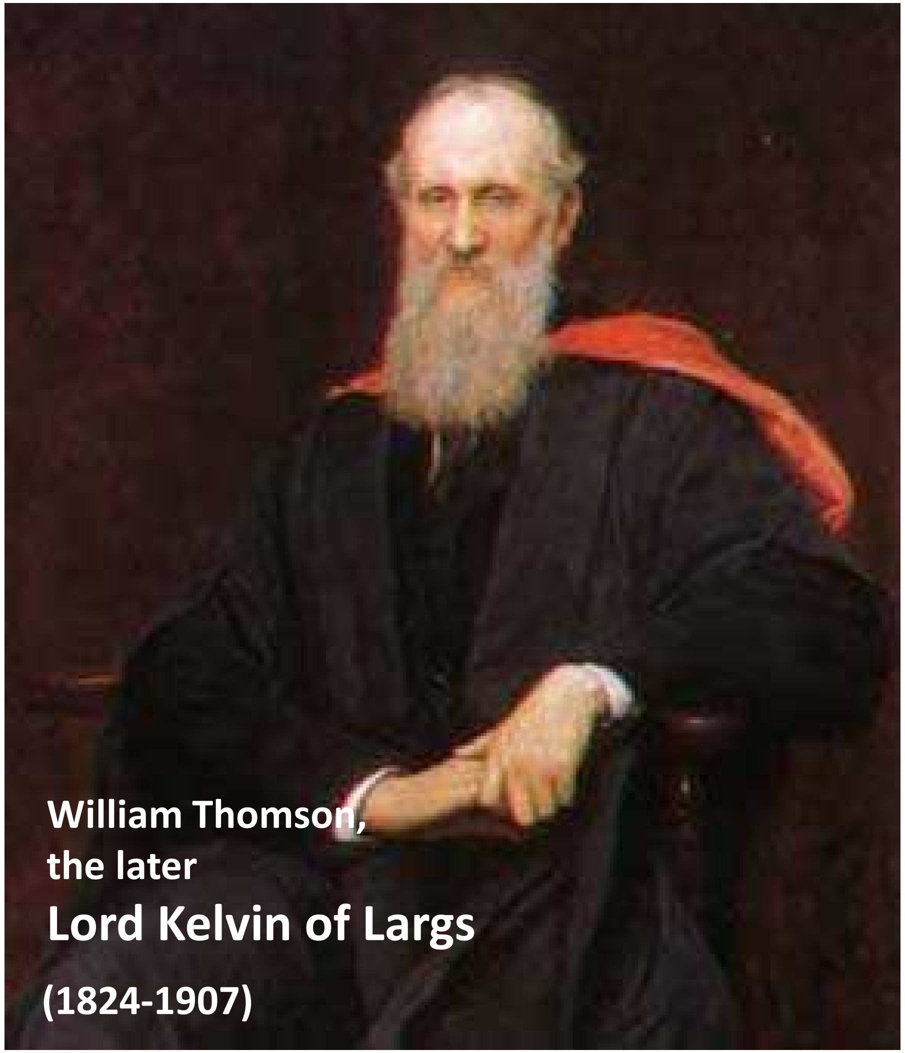
"The Kelvin : fraction 1/273.16 of the thermodynamic temperature of triple point of water"

(13. CGPM: Metrologia, 1968, **4**, 43)

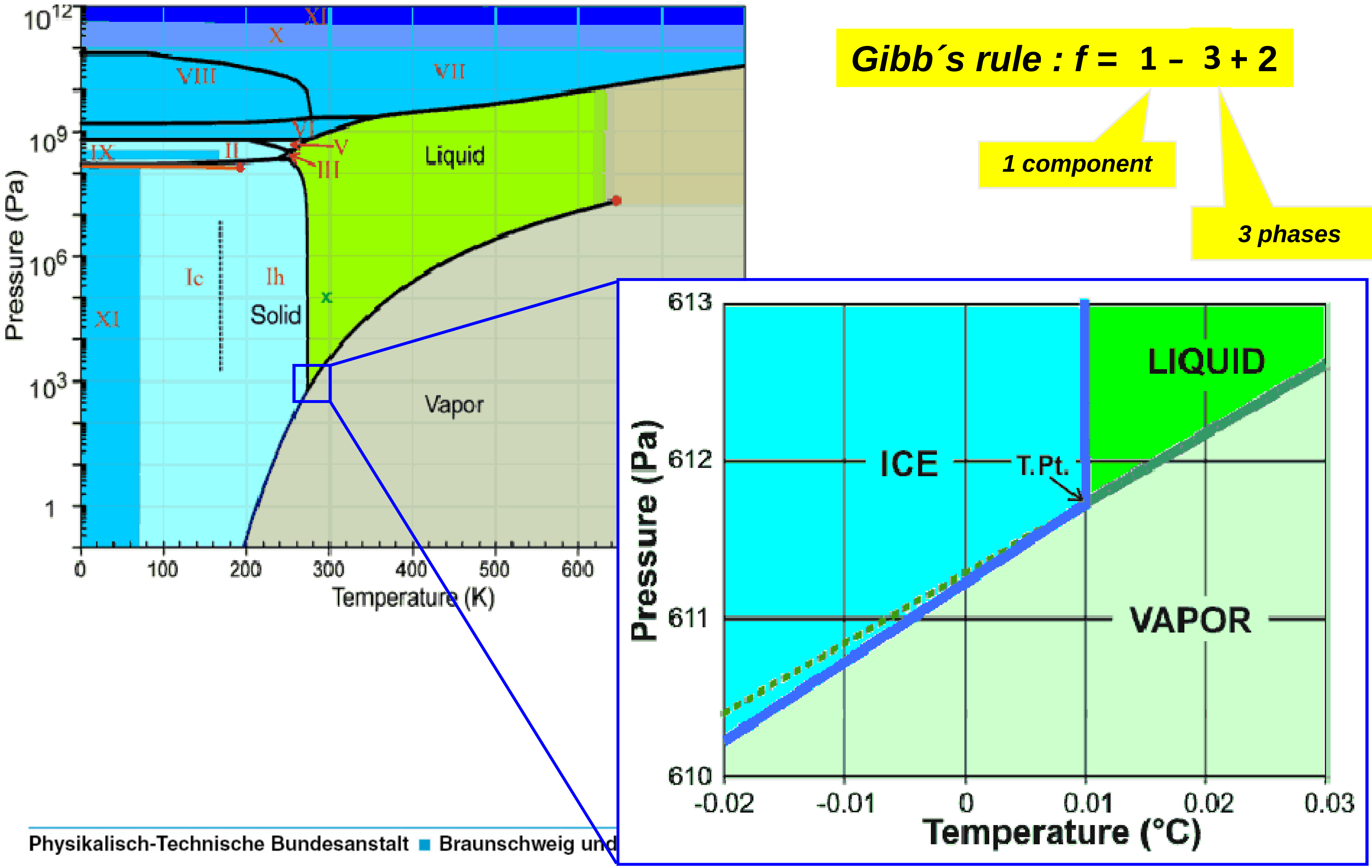
T_{tpw} = 273.16 K

definition = no uncertainty

p_{tpw} = 611.66 Pa



Phase diagram



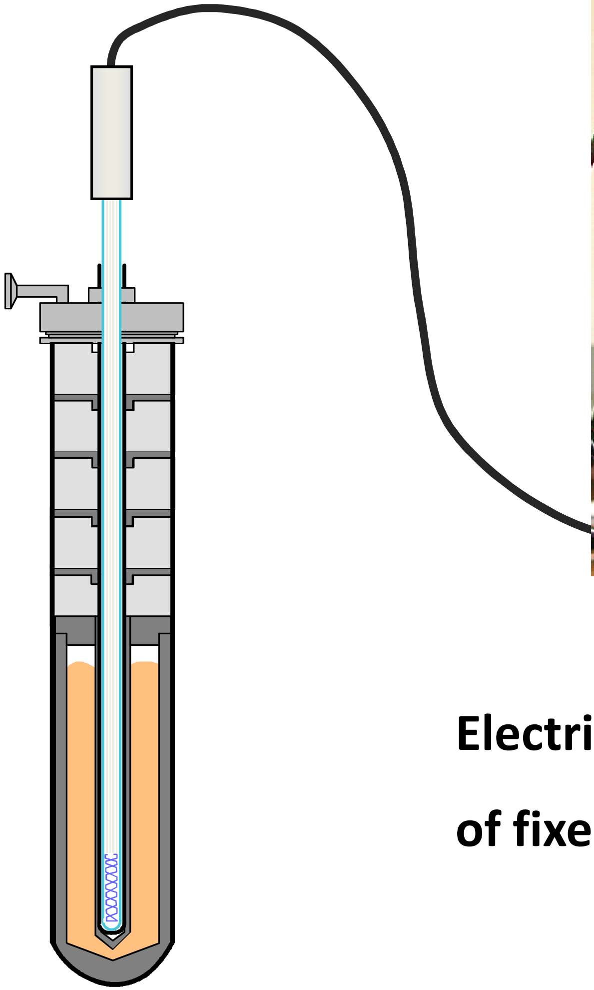
Gibb´s rule : f = 1 – 3 + 2

1 component

3 phases

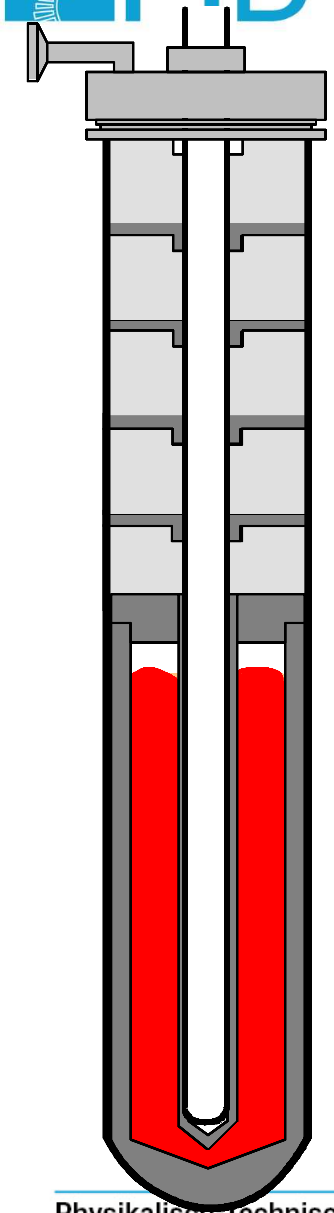
Undercooling

The measurement

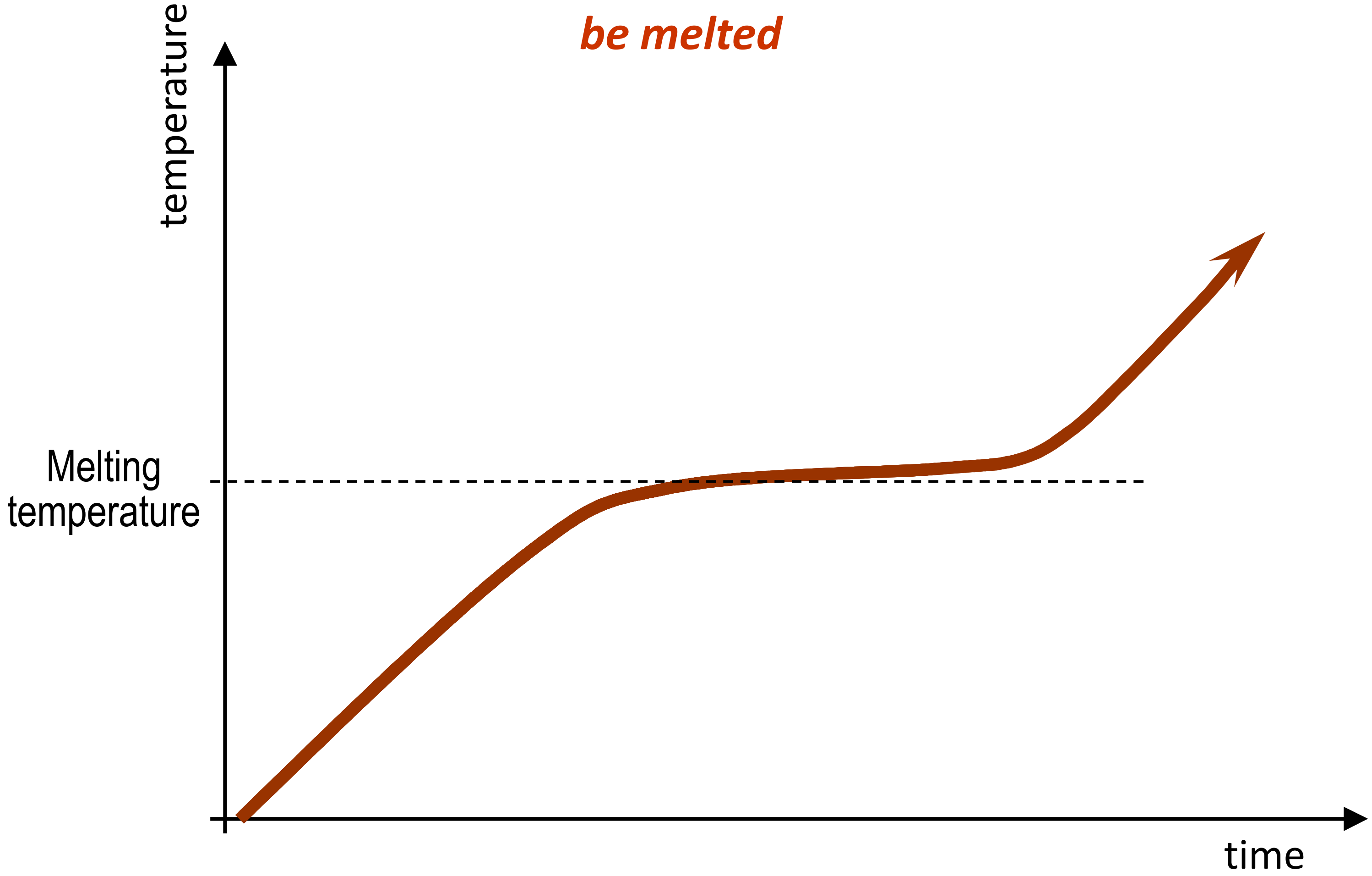


Electrical sensor measures temperature of fixed point cell

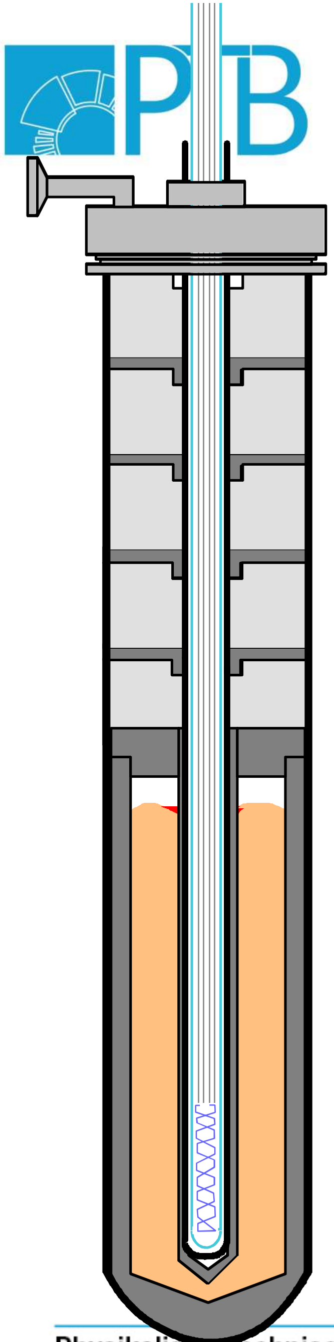
Melting of fixed point



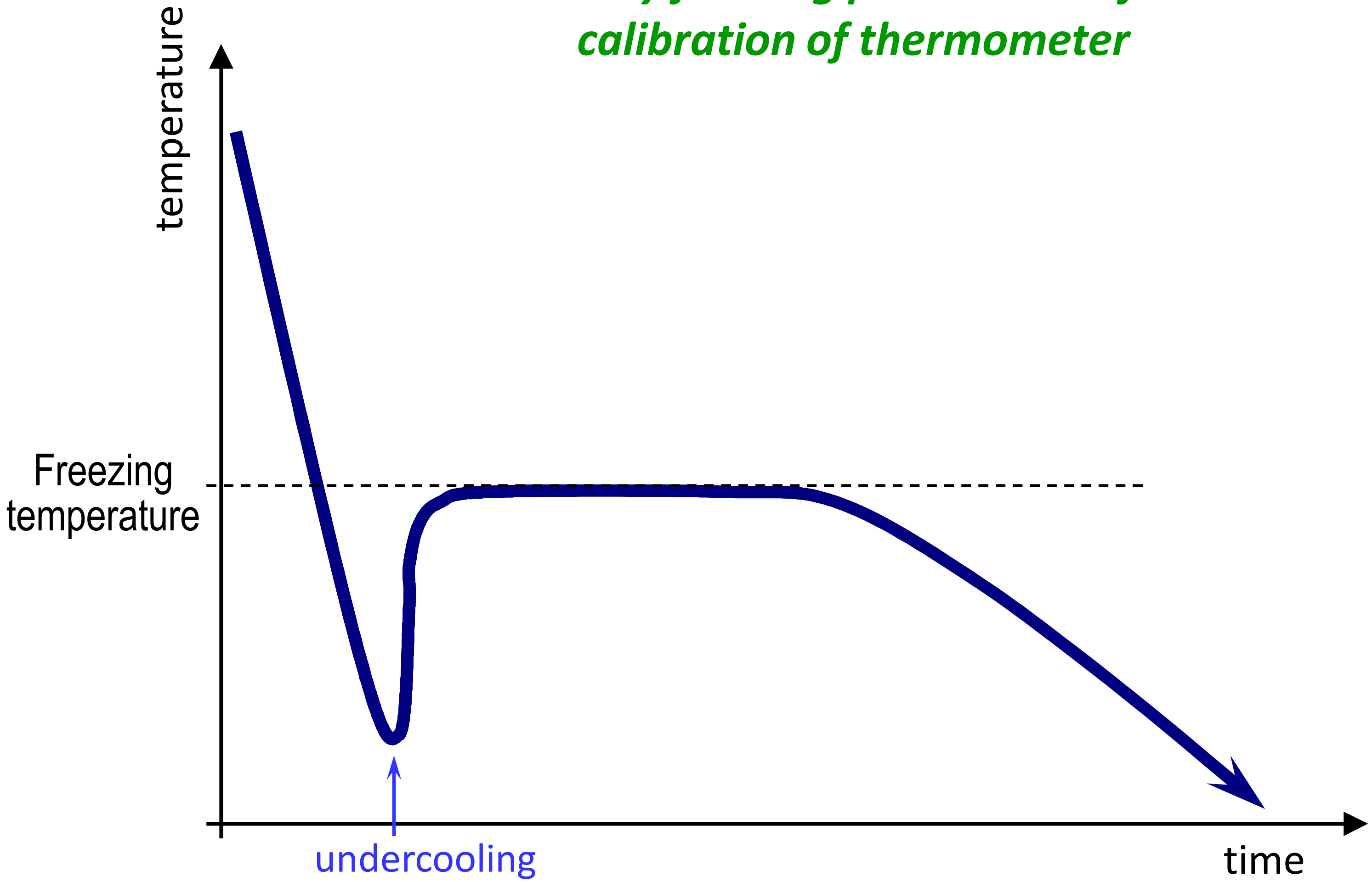
before measurement metal is to be melted



Freezing of fixed point



only freezing plateau used for calibration of thermometer



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For the foreseeable future :

Most temperature measurements in core temperature range (~ - 200 °C … 960 °C) with SPRTs calibrated accord. to ITS-90

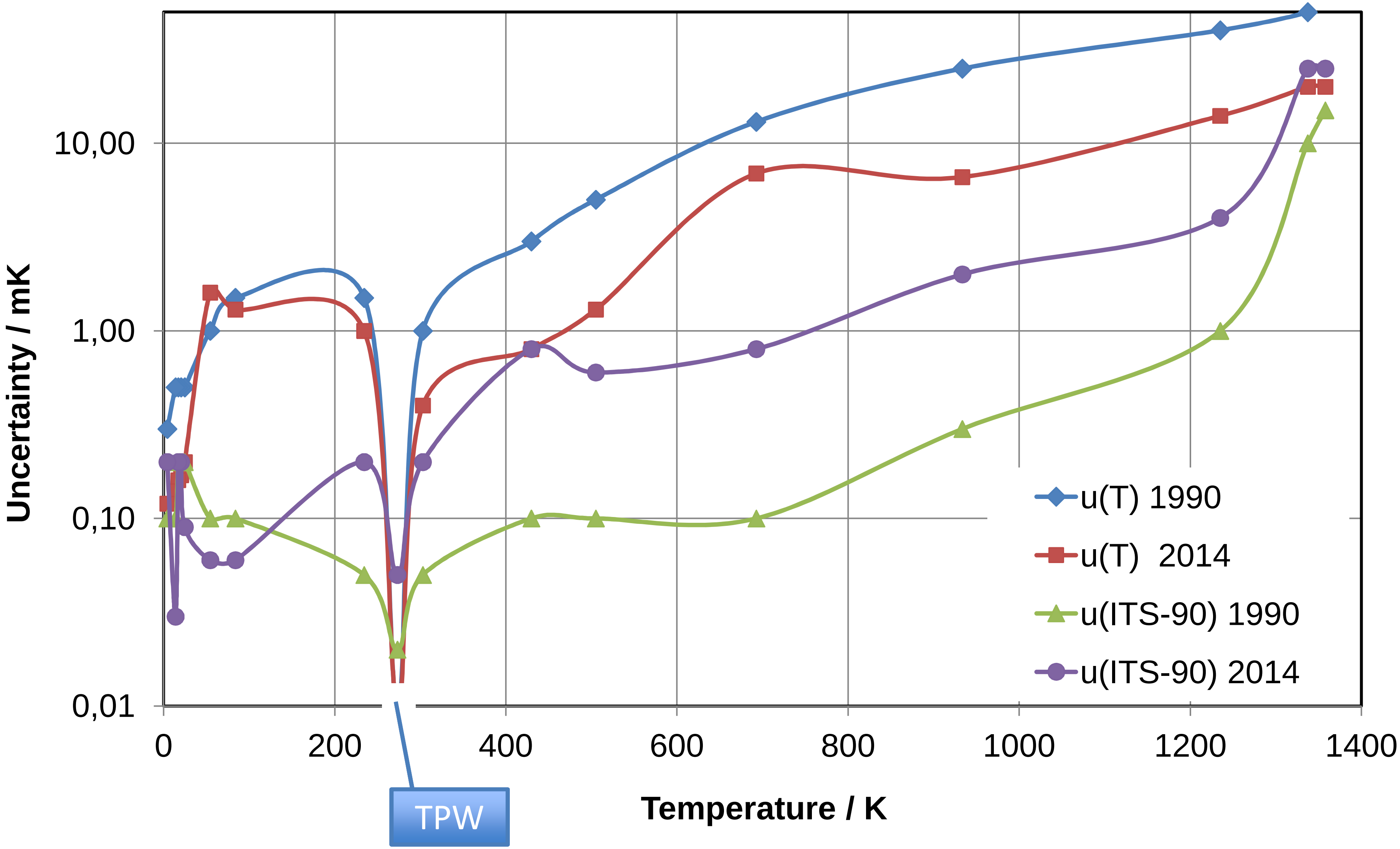
ITS-90 will remain intact, with defined values of T_{90} for all of the fixed points, including the TPW

Uncertainties in T_{90} will not change

Dominated by uncertainties in the fixed-point realizations,

and the non-uniqueness of SPRTs, typically totalling < 1 mK

Fixed points of ITS-90 (table 1 ITS guide)



Uncertainties in thermodynamic temperature

If 2010 CODATA recommended value of *k* were taken to be exact and used to define the kelvin :

Uncertainty of *k* would be transferred to the value of T_{TPW}

Best estimate of the value of T_{TPW} still 273.16 K,

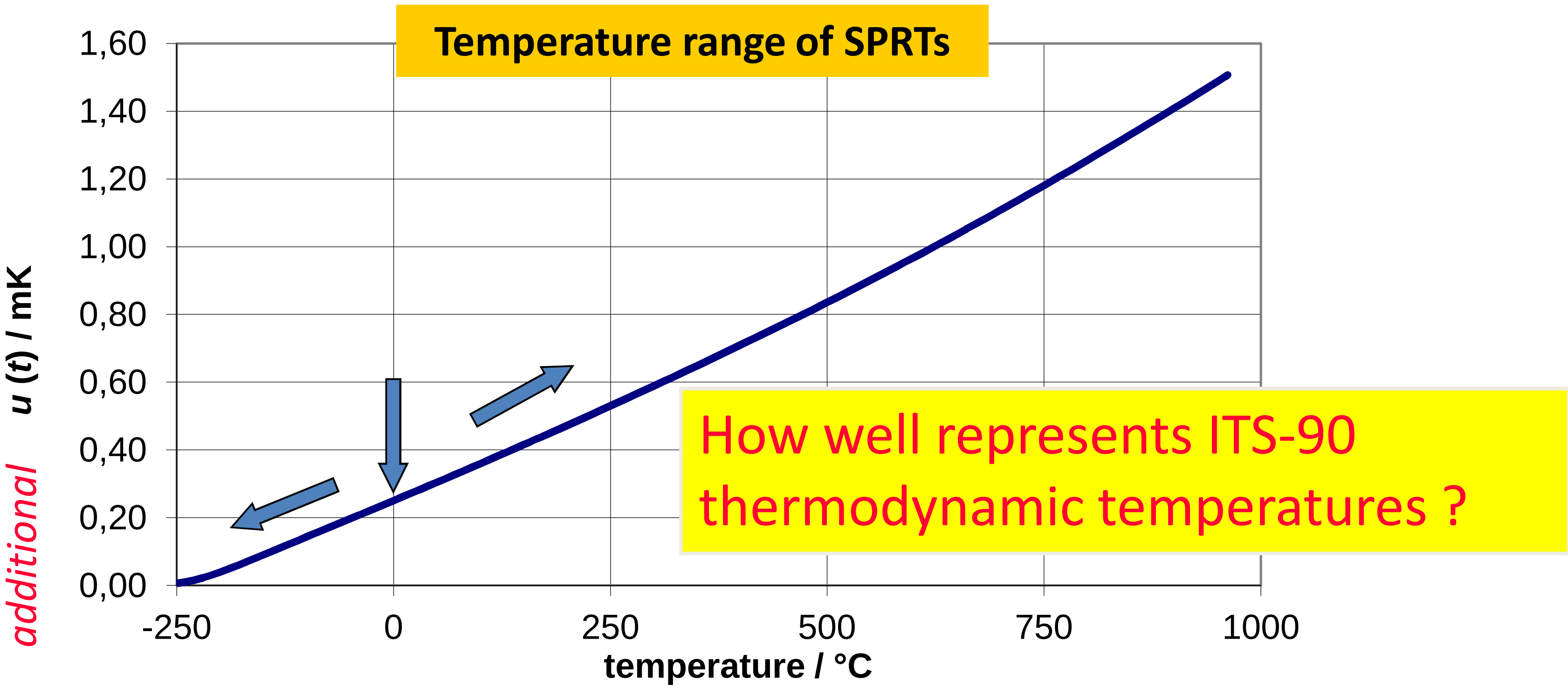
but instead of being exact as result of definition of the kelvin :

Uncertainty associated with estimate would become today :

$u_r(T_{TPW}) = 9.1 \times 10^{-7}$, corresponds to 0.25 mK

Error propagation from TPW

All thermodynamic measurements currently defined as ratios with respect to TPW :
The 0.25 mK uncertainty propagates to all historical thermodynamic temperature measurements



Uncertainties in thermodynamic temperature

⇒ TG-SI could not foresee any experiment where the slightly increased uncertainties of $u(T_{k \text{ fixed}})$ would present a problem

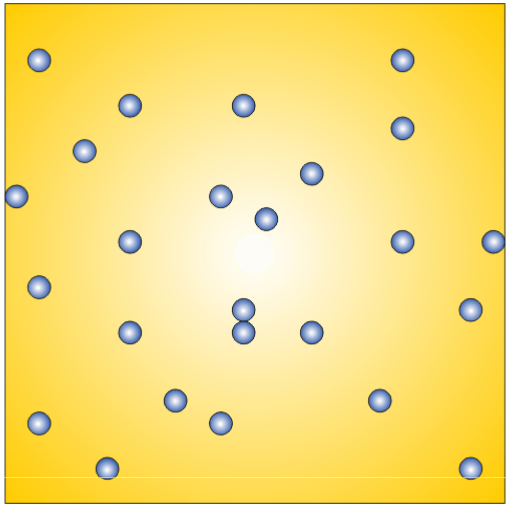
Any future changes in the temperature scale much smaller than tolerances associated with current documentary standards for thermocouples and IPRTs :
⇒ No requirement is anticipated for any future change in temperature scales to propagate to the documentary standards

Once k has been fixed in 2018 : TG-SI is not aware of any new technology for a primary thermometer providing a significantly improved uncertainty $u(T_{\text{TPW}})$

⇒ no change of the assigned value of T_{TPW} for the foreseeable future

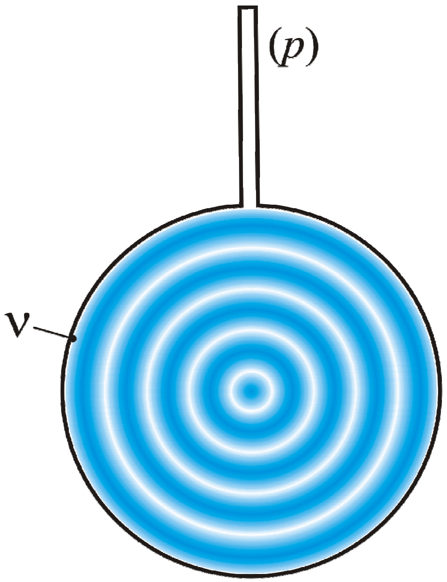
Contents:

Measurement of temperatures



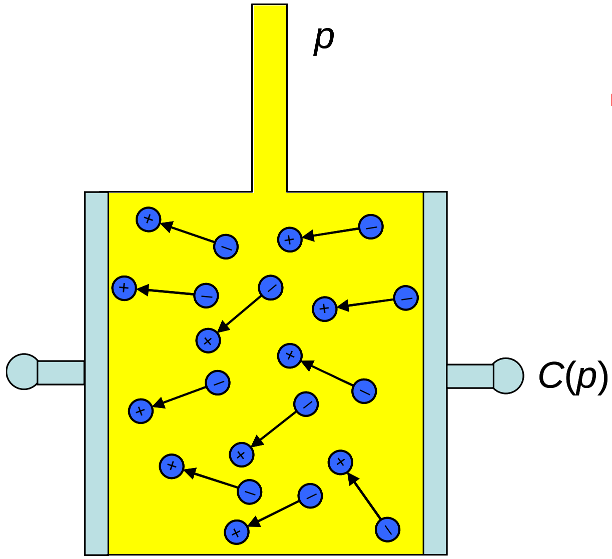
Gas molecules:	Equation of state for ideal gas
	Dielectric constant of gas
	Speed of sound in gas
	Speed of molecules: Doppler broadening
Electrons:	Thermal noise
Light quanta:	Radiation of blackbody

Determination of *k*



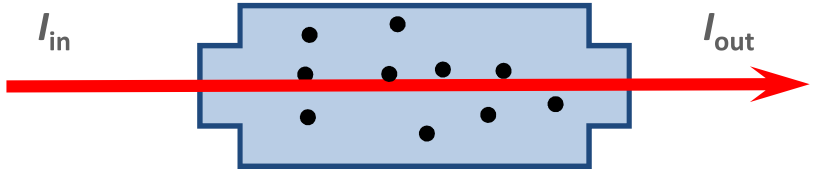
AGT

$$u_0^2 = \gamma kT / m$$
$$\gamma = c_p / c_V$$



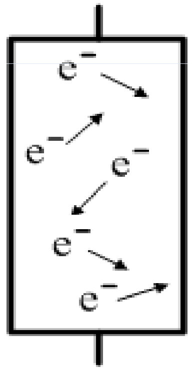
DCGT

$$p = kT \varepsilon_0 (\varepsilon_r - 1)/\alpha_0$$



$$\Delta\nu_D = [2\, kT / (mc_0^2)]^{1/2} \cdot \nu_0$$

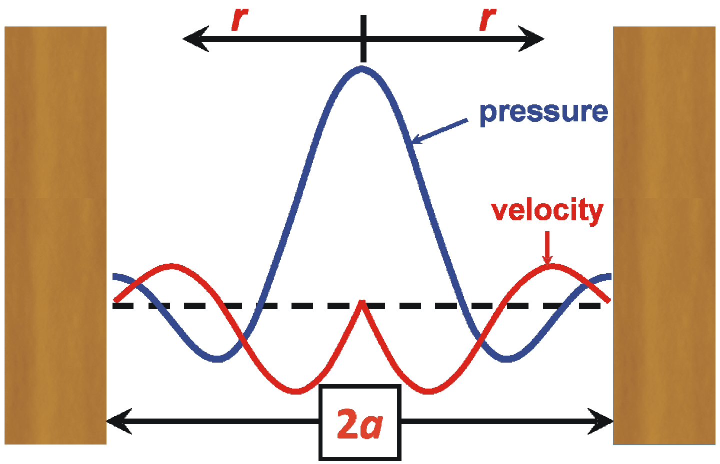
DBT



JNT

$$\langle U^2 \rangle = 4\, kT\, R\, \Delta\nu$$

Principle of AGT method



standing waves in a resonator

to be measured:

- *frequency* ν_a
- *dimensions via microwaves* ν_m *or pyknometry, CMM*

Quasi-spheres and microwaves:

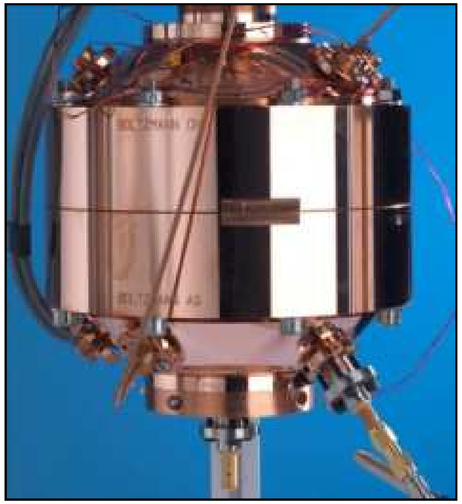
M.R. Moldover et al.
J. Res. NBS 93(2), 85-144 (1988)

$$k = \frac{M}{\gamma_0 \, T_{\mathrm{TPW}} \, N_{\mathrm{A}}} c_0^{\,2} \lim_{p\to 0} \left(\frac{\nu_{\mathrm{a}}(p)}{\langle \nu_{\mathrm{m}}(p) \rangle} \right)^2$$

2006 recommended CODATA value of k

$u_r = 1.7 \times 10^{-6} \quad (k=1)$

2013 AGT at NPL: lowest uncertainty

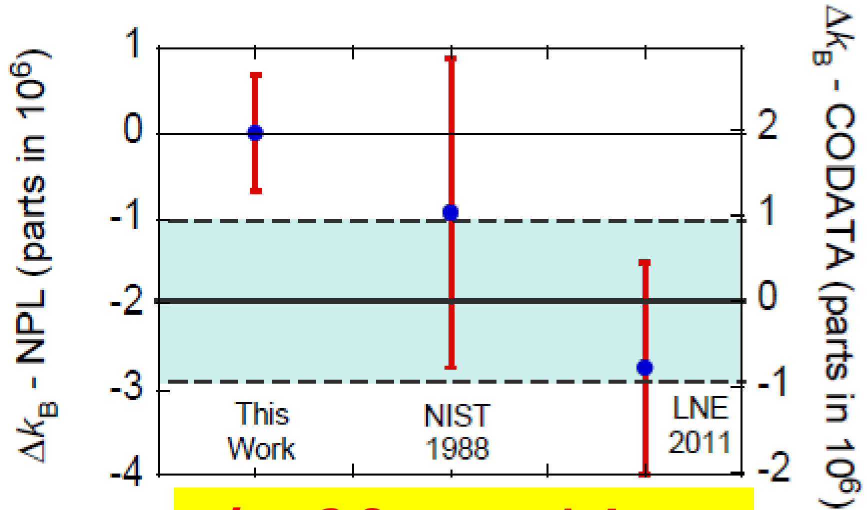


radius 62 mm, filled with Argon

Uncertainty contributions

			Estimate	$u_R/10^{-6}$	Weight
molar mass	M	$g\,mol^{-1}$	39. 947 816(16)	0.390	30.0%
temperature	T	K	273.160 000(99)	0.364	26.1%
speed of sound	c_0^2	$m^2\,s^{-2}$	94756.245(45)	0.470	43.6%
	R	$J\,K^{-1}\,mol^{-1}$	8.314 478 7 (59)	0.711	
	N_A	mol^{-1}	6.022 141 29 (27) $\times 10^{23}$	0.044	0.4%
	k_B	$J\,K^{-1}$	1.380 651 56 (98) $\times 10^{-23}$	0.712	

Explanation of discrepancy between NPL 2013 and LNE 2011 requires error of ≈ 85 nm on radius compared with NPL uncertainty estimate of 12 nm



Δk = 2.8 ppm ± 1.4 ppm

M. de Podesta, R. Underwood, G. Sutton, P. Morantz, P. Harris, D.F. Mark, F.M. Stuart, G. Vargha, G. Machin
Metrologia **50** 354-376 (2013)

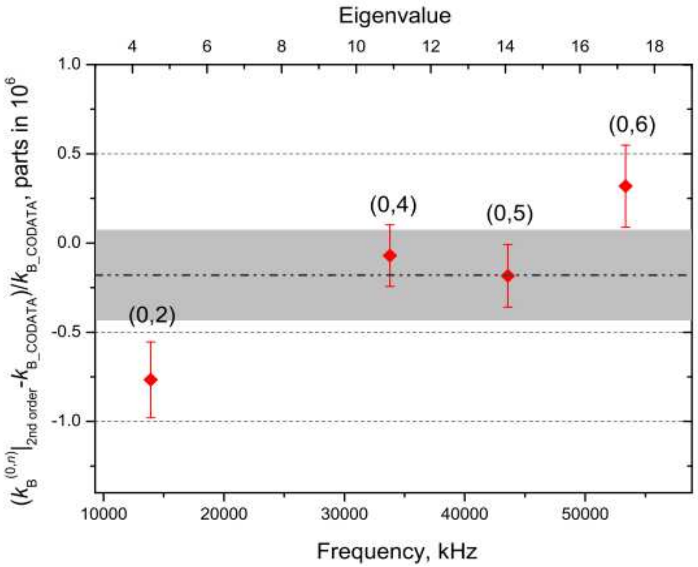
LNE AGT results: 2011 Argon 2015 Helium

Uncertainty contributions

radius 50 mm

Term	Relative uncertainty (10^{-6})	
Gas	Ar	He
Acoustic frequency	0.80	0.62
Resonator volume	0.57	0.57
Molar mass and gas purity	0.60	0.53
Thermometry	0.3	0.3
Total (square root of quadratic sum)	1.24	1.02

Scatter among modes



L. Pitre, F. Sparasci, D. Truong, A. Guillou, L. Risegari, M. E. Himbert
Int. J. Thermophys. **32** 1825-86 (2011)
$u(k)/k$ = 1.24 ppm

L. Pitre, L. Risegari, F. Sparasci, M.D. Plimmer, M. E. Himbert
Metrologia **52** (2015)
$u(k)/k$ = 1.02 ppm

Molar mass measurement

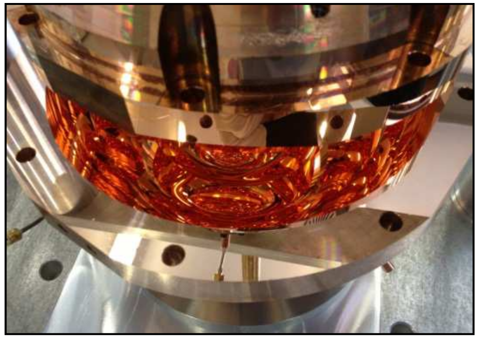
The KRISS equipment and reference gas were used for the redetermination of the isotopic abundance of the atmospheric argon (J.-Y. Lee *et al*. Geochimica et Cosmochimica Acta, 2006)

Lee was reference for NPL Boltzmann measurement

- **Largest single uncertainty in acoustic values of *k* with Ar originates in determination of relative Ar isotopic abundances**
- Investigate the consistency in molar mass measurements in various Boltzmann experiments (especially between NPL and LNE results)
- Argon samples of various origins prepared by LNE
- With source gases used in Boltzmann constant measurements of LNE, INRiM, NIM, NMIJ, and NPL new *acoustic measurements* were done at LNE
- *Mass spectroscopy analysis* at KRISS finished in February 2015
- Relative uncertainty in relative molar mass M (Ar): 0.14×10^{-6}

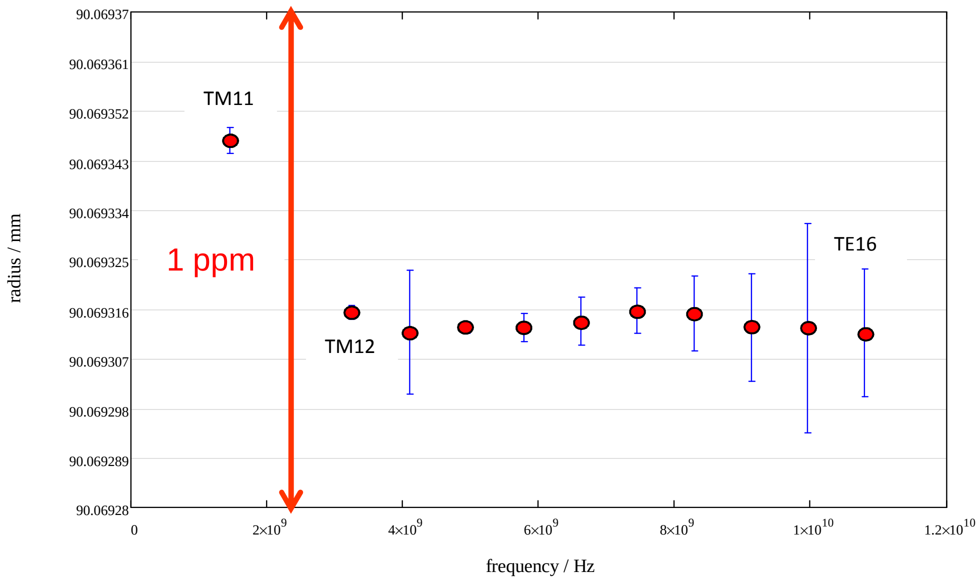
2015 AGT at INRiM

New 3-litre volume diamond turned copper spherical resonator

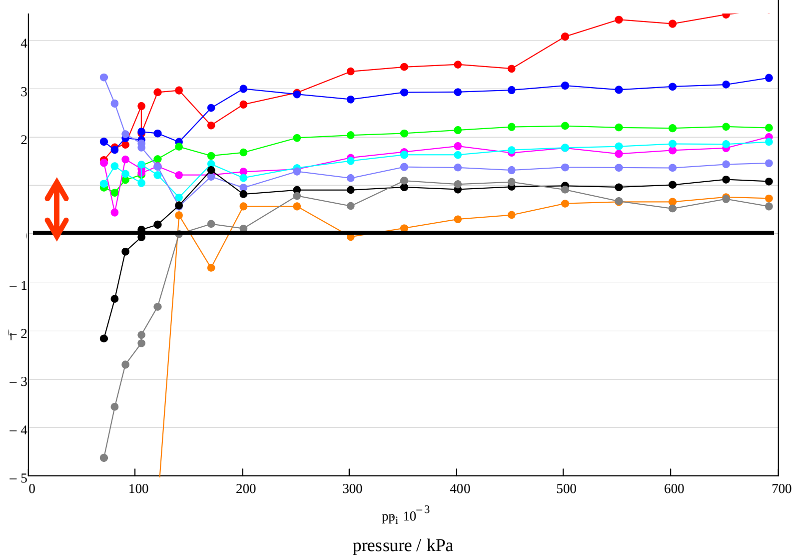


apparatus

determination of resonator radius from 11 microwave modes



relative excess halfwidths of 9 radial acoustic modes



2011 - 2015 NIM – NIST: AGT with cylindrical resonator

single cylinder arrangement

J.T. Zhang, H. Lin, X.J. Feng, J.P. Sun, K. A. Gillis, M.R. Moldover, Y.Y. Duan
Int. J. Thermophys. **32** 1297–1329 (2011)
$u(k)/k$ = 7.9 ppm

H. Lin, X.J. Feng, K.A. Gillis, M.R. Moldover, J.T. Zhang, J.P. Sun, Y.Y. Duan
Metrologia **50** 417-432 (2013)
$u(k)/k$ = 3.7 ppm

two cylinder arrangement
with lengths 2 *l* and *l*

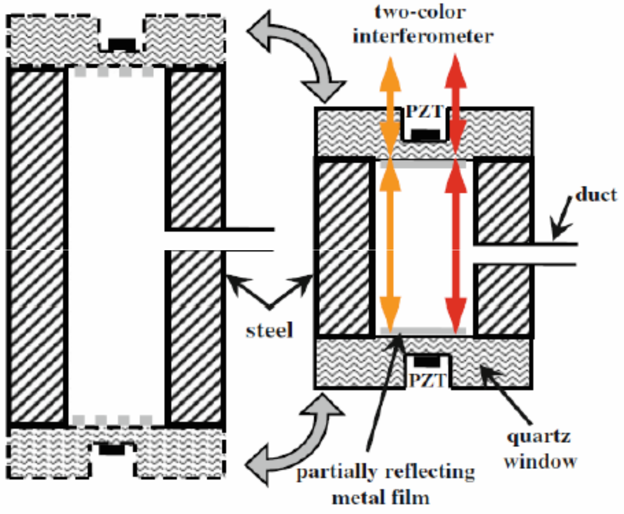
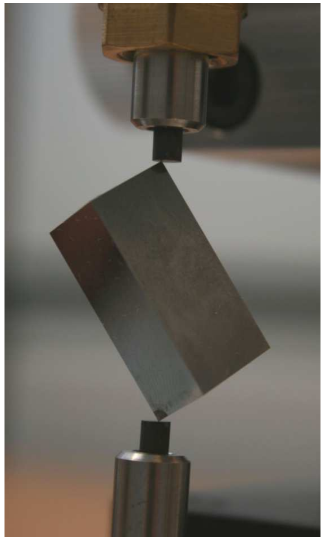


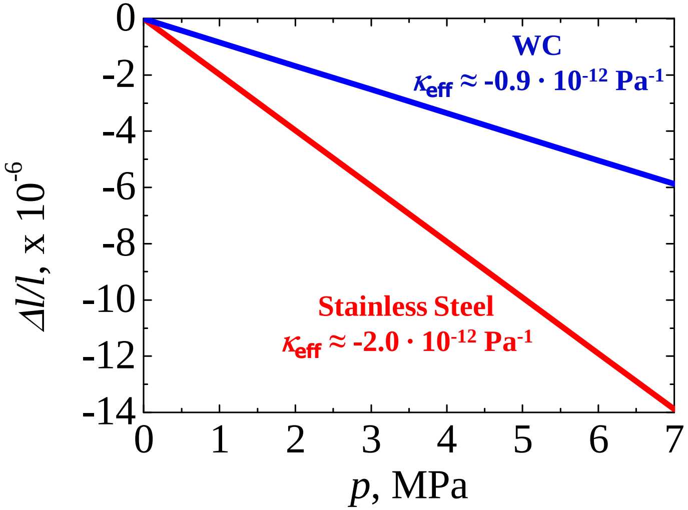
Fig. 1 Schematic diagram of two fixed-length, cylindrical, cavity resonators with interchangeable ends

Measurements free from corrections of perturbations of shell motion, transducer, gas duct

PTB 2013: DCGT with improvements in κ_{eff}
2015: pressure measurement with 1.0 ppm

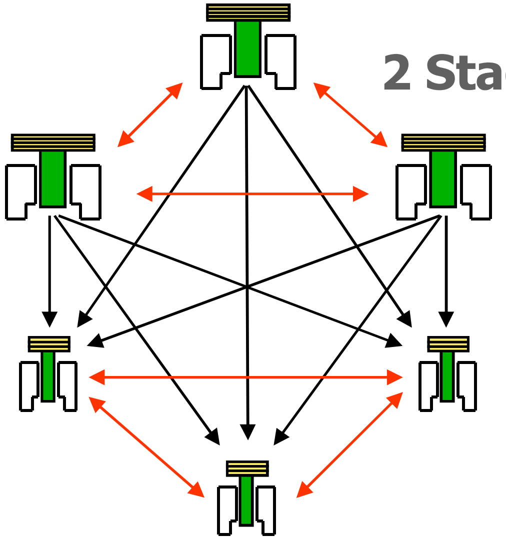
Resonant ultrasound spectroscopy **Tungsten carbide (WC) capacitors**





C. Gaiser, T. Zandt, B. Fellmuth, J. Fischer, O. Jusko, and W. Sabuga Metrologia **50** (2013) L7-L11
$u(k)/k = 4.3$ ppm

2 Stage calibration process



C. Gaiser, T. Zandt, B. Fellmuth Metrologia **52** (2015)
$u(k)/k = 4.0$ ppm

Noise thermometry at NIM/NIST

- Provides a new path to fundamental physical constants via quantum-based voltage sources
  - JNT is a **purely electronic approach** to temperature
  - Links definition of kelvin with quantum electrical standards

Spectral power density of voltage noise known :
- **AC Josephson Voltage**

$$K_J^2 = \frac{4e^2}{h^2}$$

Spectral power density of thermal noise known :
- Traceable to **Quantum-Hall Ohm**

$$R_K = \frac{h}{e^2}$$

S.P. Benz, A. Pollarolo, J. Qu, H. Rogalla, C. Urano, W.L. Tew, P.D. Dresselhaus, D.R. White
Metrologia **48** (2011) 142-153
$u(k)/k = 12.1$ ppm

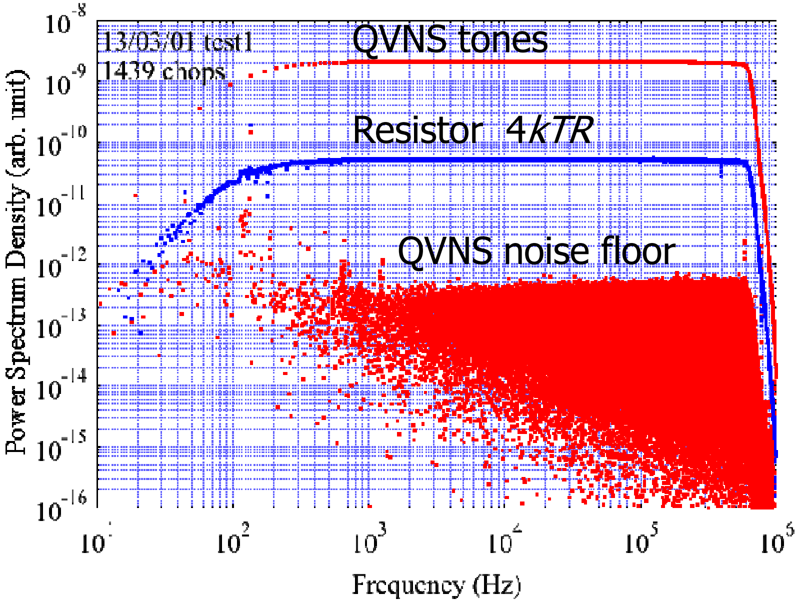
sense resistance from 100 Ω to 200 Ω
wider bandwidth electronics: 550 kHz
new ADC, new shielding : no EMI

J. Qu, S.P. Benz, A. Pollarolo, H. Rogalla, W.L. Tew, D.R. White, K. Zhou
Metrologia **52** (2015)
$u(k)/k = 4.1$ ppm

Noise thermometry at NMIJ/AIST

- 100 Ω resistor newly developed for this project
- 5 mm x 5 mm Ceramic package element
- Mounted within a stainless steel sheath
- For use directly in TPW cell

K. Yamazawa, C. Urano, T. Yamada, T. Horie, S. Yoshida, H. Yamamori, N. Kaneko, Y. Fukuyama, M. Maruyama, A. Domae, J. Tamba, S. Kiryu
Tempmeko 2013



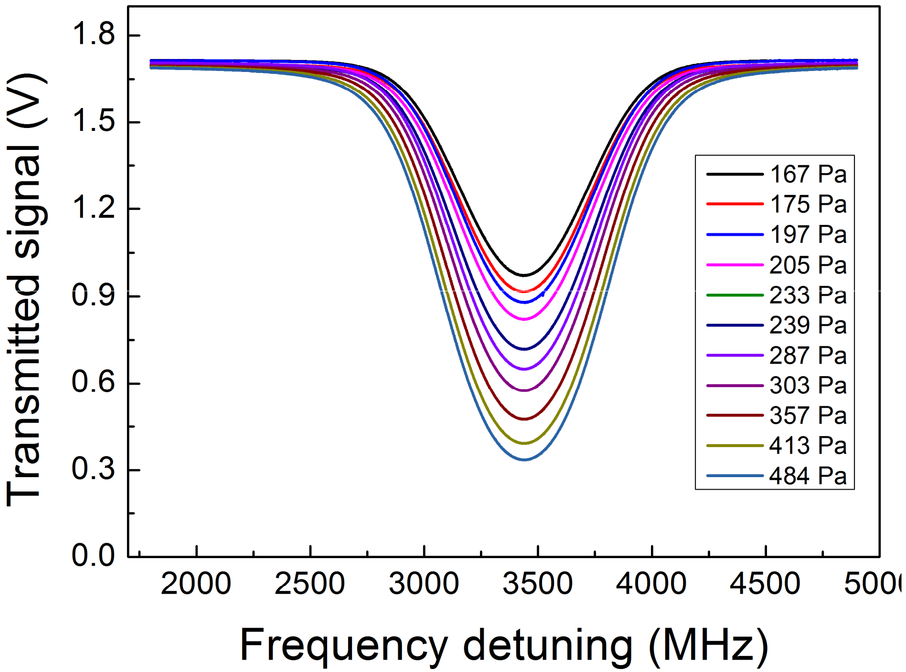
Built from scratch, independently from NIST Josephson junction array, thermometer probes, software

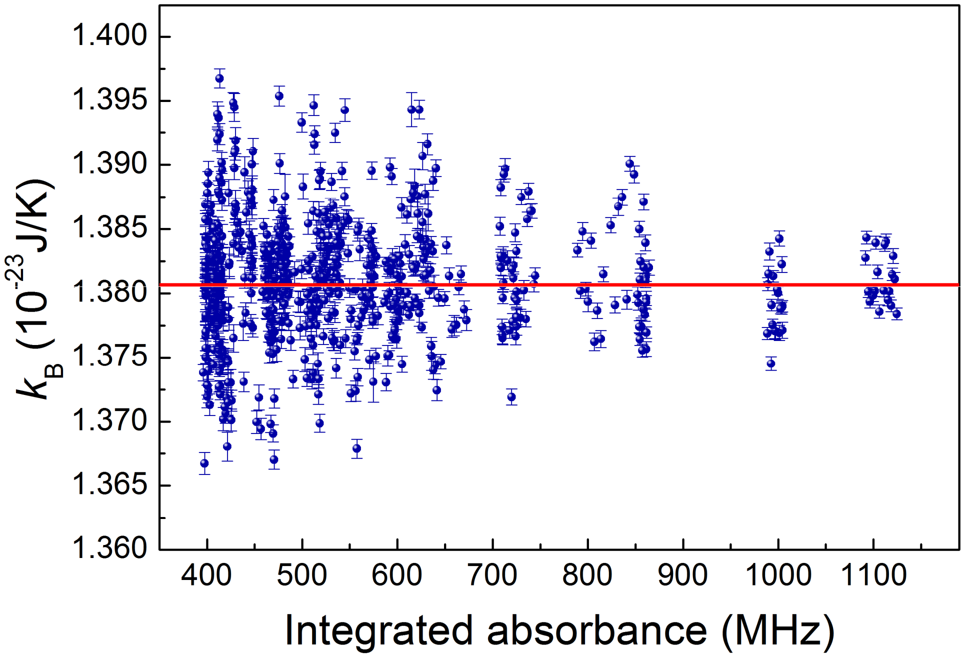
2013: DBT result at Uni Naples

Current *k* value

$H_2{}^{18}O$ spectra

$$k = (1.380631 \pm 0.000033) \times 10^{-23}\ \mathrm{J/K}$$



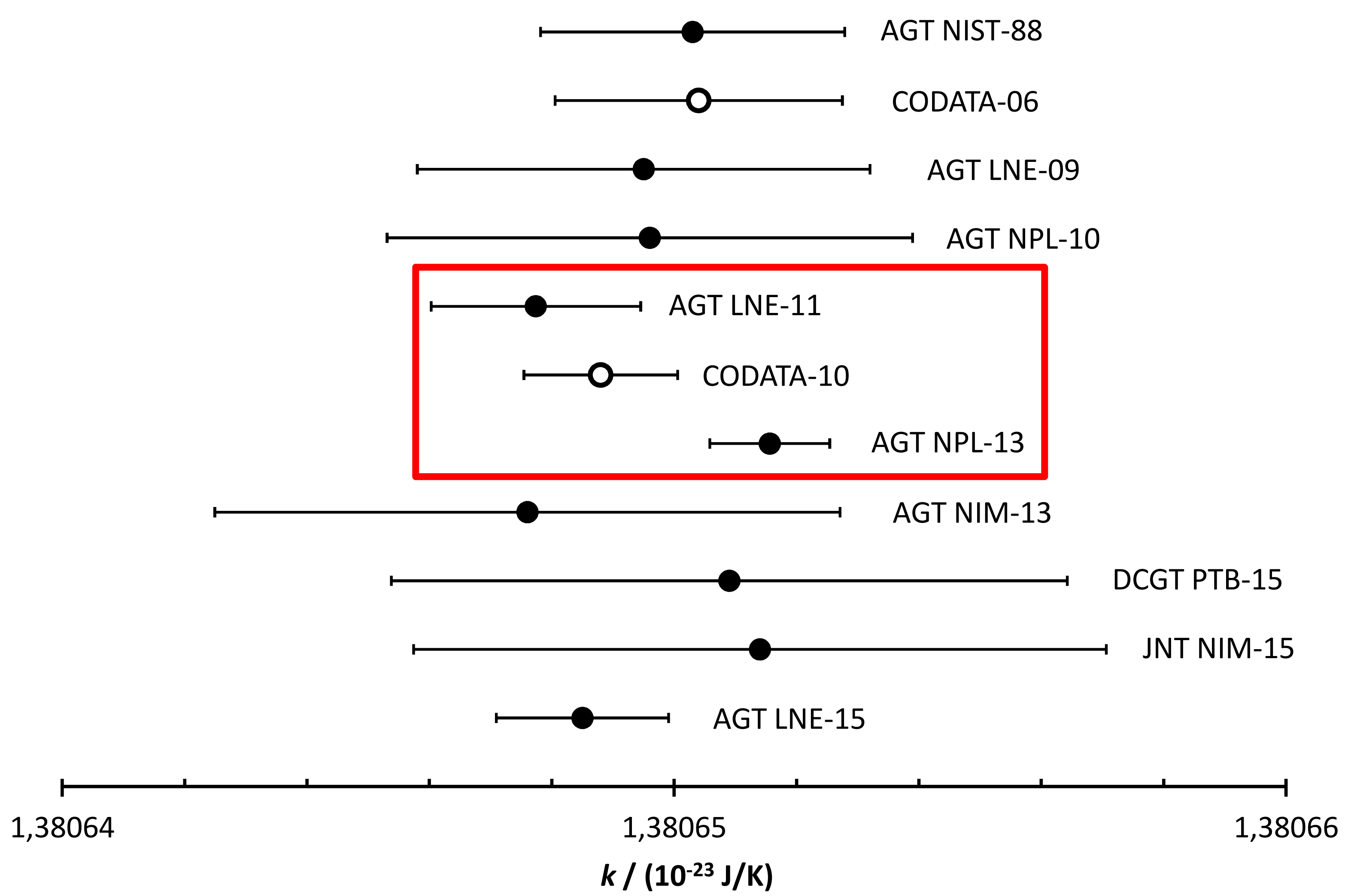


L. Moretti, A. Castrillo, E. Fasci, M. D. De Vizia, G. Casa, G. Galzerano, A. Merlone, P. Laporta, L. Gianfrani
PRL **111**, 060803 (2013) $u(k)/k$ = 24 ppm

Uncertainties $u(k)/k$ of methods

Method	gas	up to 2011	2013	2015	institute
AGT	Ar	1.7 ppm	-	-	NIST
AGT	He	7.5 ppm	-	1.2 ppm	INRiM
AGT	He	2.7 ppm	-	1.0 ppm	LNE-CNAM
AGT	Ar	1.2 ppm	-	-	LNE-CNAM
AGT	Ar	3.1 ppm	0.7 ppm	-	NPL
c-AGT	Ar	7.9 ppm	3.7 ppm	?	NIM
DCGT	He	7.9 ppm	4.3 ppm	4.0 ppm	PTB
JNT	-	-	-	4.1 ppm	NIM
JNT	-	12 ppm	-	-	NIST
JNT	-	-	-	< 20 ppm ?	NMIJ
DBT	NH_3	50 ppm	-	< 10 ppm ?	LPL+LNE-CNAM
DBT	CO_2,H_2O	160 ppm	24 ppm	< 10 ppm ?	UniNA+INRiM

Values to be considered by CODATA 2014



Contents:

- Introduction
- International Temperature Scale
- Consequences of the new definition
- Determination of the Boltzmann constant
- **New definition of the kelvin**

Recommendation of the Consultative Committee for Thermometry submitted to the International Committee for Weights and Measures

RECOMMENDATION T1 (2014)
On a new definition of the kelvin

The Consultative Committee for Thermometry (CCT)

recalling

- the CCT Report to the CIPM in 2007, "Report to the CIPM on the implications of changing the definition of the base unit kelvin";
- the CCT Recommendation to the CIPM in 2010, "Considerations for a new definition of the Kelvin", CCT T 2 (2010);

welcoming

- the Resolution 1 (2011) of the CGPM, "On the possible future revision of the International System of Units, the SI" which, when accomplished, will link the unit of temperature to the Boltzmann constant;
- the CCU Recommendation to the CIPM, "Revision of the International System of Units, the SI", CCU U 1 (2013);

recognizing

the need to confirm and clarify Recommendation CCT T 2 (2010) in the light of Resolution CCU U 1 (2013);

noting that

- experiments such as acoustic gas thermometry, dielectric constant gas thermometry, Johnson noise thermometry, and Doppler broadening thermometry represent fundamentally different methods to determine the Boltzmann constant *k*;
- the CODATA recommended a value for *k* with a relative standard uncertainty equal to 9.1 parts in 10^7 in its 2010 adjustment of fundamental constants, however based on only one experimental method;
- a relative standard uncertainty in *k* of 9.1 parts in 10^7 would correspond to a standard uncertainty of about 0.25 mK of the temperature of the triple point of water after the redefinition;

CCT Recommendation T1 (2014)

considering

- the discussions held at the 26th and 27th meetings of the CCT in 2012 and 2014;
- the considerable progress recently achieved in experimental determinations of the Boltzmann constant to improve confidence in the 2010 value, as reported at the CCT "Task Group on the SI" meetings held in 2013 and 2014;
- that additional results are anticipated before the end of 2015;
- that experimental progress has allowed the development of a *mise en pratique* for the new definition of the kelvin, which has been extended to cover direct measurement of thermodynamic temperature after the new definition of the kelvin;

recommends

that the CIPM request the CODATA to adjust the values of the fundamental physical constants, from which a fixed numerical value of the Boltzmann constant will be adopted, when the following two conditions are met:

1. the relative standard uncertainty of the adjusted value of k is less than one part in 10^6;
2. the determination of k is based on at least two fundamentally different methods, of which at least one result for each shall have a relative standard uncertainty less than 3 parts in 10^6.

The Boltzmann Constant *k* and the kelvin

P **probability**
U **internal energy**

Boltzmann: $\sigma = \ln P$

entropy

Planck: $S = k\,\sigma = k \ln P$

$1/\tau = d\sigma / dU$

measured in joule

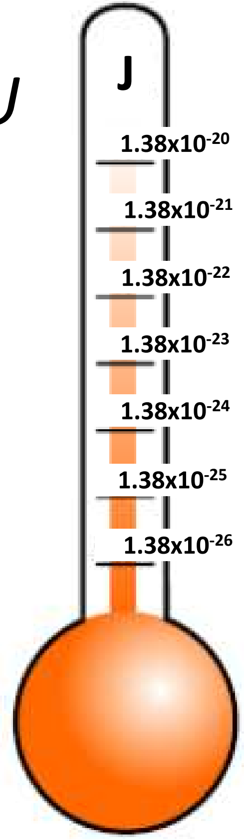
thermodynamic temperature

measured in kelvin

$1/T = dS / dU$

***k* = conversion factor between energy and temperature**

fixing the value of *k* : Boltzmann´s original intention



Mise en pratique

Mise en pratique **for the definition of the kelvin**

Adopted by the CCT in April 2006

To help users make accurate and reliable temperature measurements, the CIPM, through CCT and BIPM, will publish a collection of guidelines for temperature measurement.
Similar to the current *Supplementary Information for the International Temperature Scale of 1990,* also published by BIPM

This mise en pratique will describe recognised primary methods for measuring temperature or realizing the scale, as well as recognised approximations to thermodynamic temperature that will include ITS-90 and PLTS-2000, and the sources of uncertainty associated with the measurements.

way. Beginning in 1927, the CIPM, acting under the authority of the General Conference on Weights

Wording

1. Reliable determination of k with different methods

2. Fixing of the value of k

3. New definition of the kelvin like:

Explicit-unit definition

The kelvin is the change of thermodynamic temperature T that results in a change of thermal energy kT by exactly

1.380 6X $\times 10^{-23}$ J, where k is the Boltzmann constant

The impact of a new definition of the kelvin

Short term:

- the kelvin definition is independent of any material
- no favoured fixed point
- no favoured measurement method
- no error propagation from TPW
- thermodynamic measurements and ITS-90 are coexisting
- <20 K and >1300 K thermodynamics are superior

Long term:

- With improvement of primary thermometry thermodynamic measurements may replace ITS-90

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Thank you !

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