Reflections on the NPL-2013 Estimate of the Boltzmann Constant

Michael de Podesta

Fundamental Constants
February 2015

EMRP
European Metrology Research Programme
Programme of EURAMET
The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

EURAMET
European Association of National Metrology Institutes

Also my wonderful NPL colleagues

Gavin Sutton, Robin Underwood,
Gordon Edwards, Graham Machin,
Richard Rusby, David Flack,
Andrew Lewis, Michael Perkin,
Stuart Davidson, Kevin Douglas,
Rob Ferguson, David Putland,
Anthony Evenden, Louise Wright,
Eric Bennett, Alan Turnbull,
Gareth Hinds, Phil Cooling,
Gergely Vargha, Martin Milton,
Michael Parfitt, Peter Harris, Leigh Stanger

and others

Also my colleagues outside NPL

Paul Morantz, Cranfield

Darren Mark and Fin Stuart, SUERC

Laurent Pitre, LNE-CNAM

Roberto Gavioso, INRiM

Inseok Yang, KRISS

Reflections on the NPL-2013 Estimate of the Boltzmann Constant

1. Background

2. History

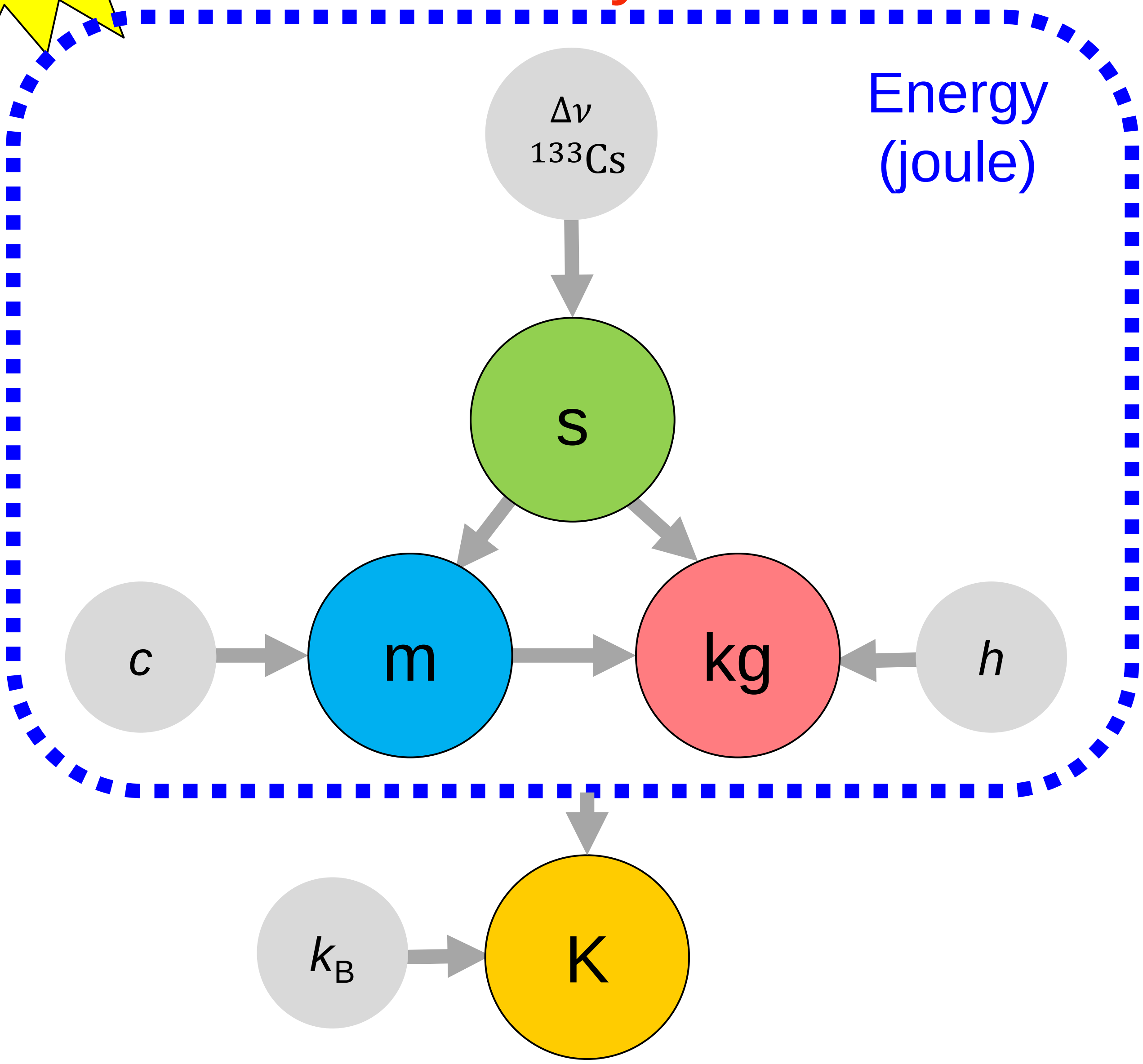
3. The NPL uncertainty estimate

4. NPL Update February 2015

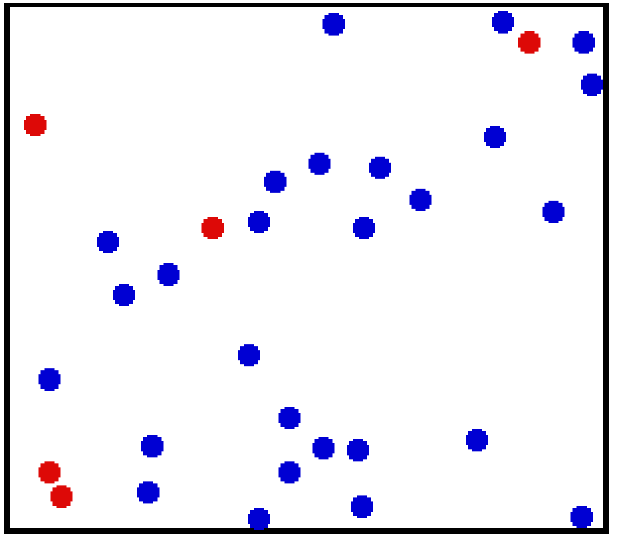
5. The NPL Analysis

6. Summary

The NEW International System of Units



The Challenge



Molecular Motion

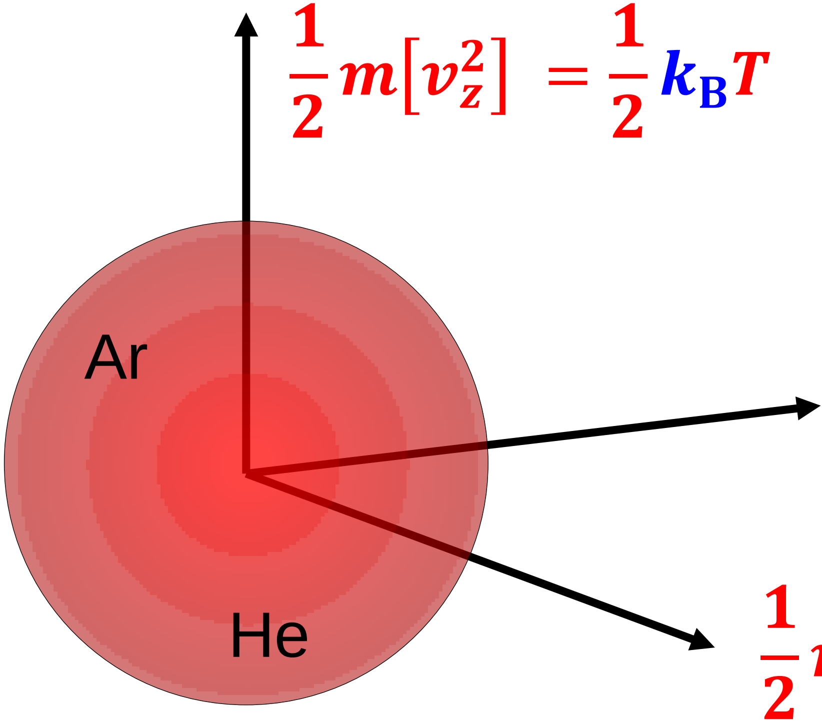
Thermometer

How do we relate the number produced by a thermometer (e.g. 20 °C) to the basic physics describing the jiggling of molecules?

Primary thermometers are based on gases

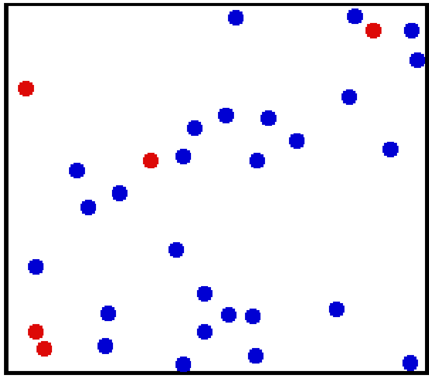
- Molecular motions are simple
- We can approach 'ideal gas' conditions at low pressure
- In an ideal gas the internal energy is just the kinetic energy of the molecules

$$\frac{1}{2}m\left[v_z^2\right] = \frac{1}{2}k_{\mathrm{B}}T$$

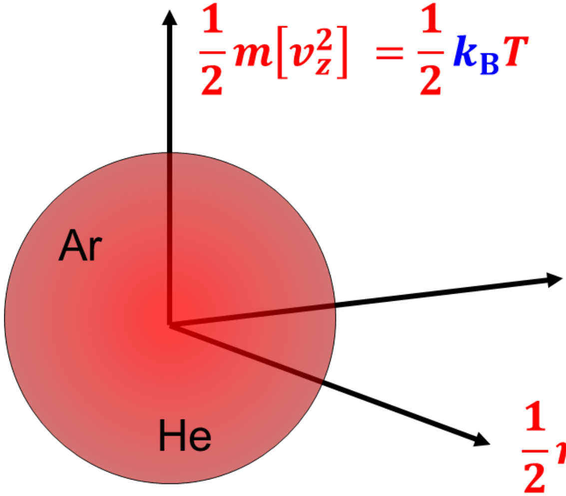


$$\frac{1}{2}m\left[v_y^2\right] = \frac{1}{2}k_{\mathrm{B}}T$$

$$\frac{1}{2}m\left[v_x^2\right] = \frac{1}{2}k_{\mathrm{B}}T$$



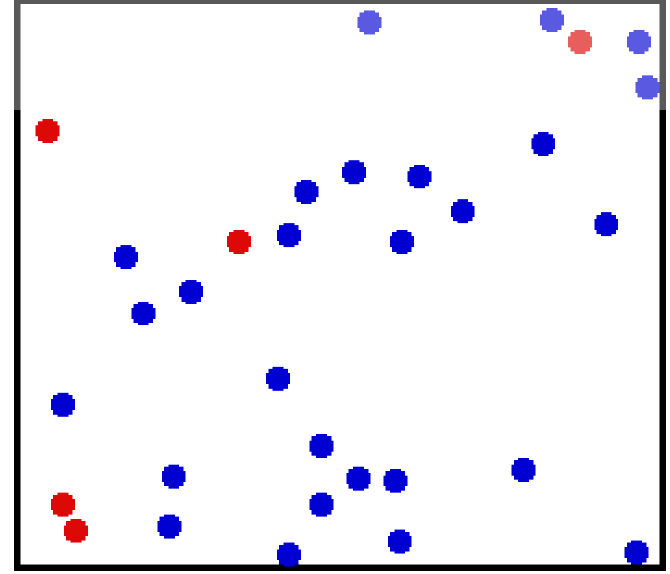
The big idea…



$$\frac{1}{2}m[v_z^2] = \frac{1}{2}k_B T$$

$$\frac{1}{2}m[v_y^2] = \frac{1}{2}k_B T$$

$$\frac{1}{2}m[v_x^2] = \frac{1}{2}k_B T$$



Find mass of molecule

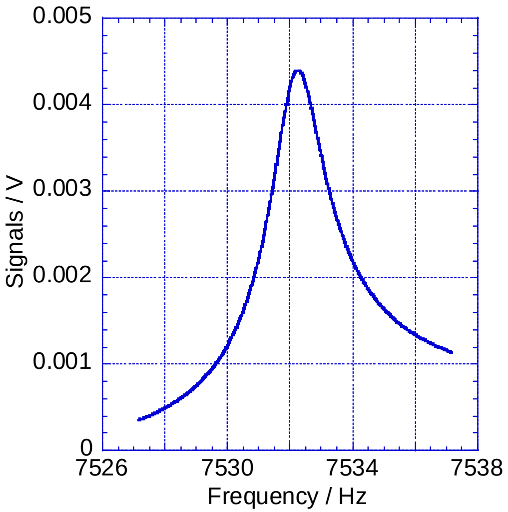
$$k_B = \frac{3m}{5T}\left[\,speed\ of\ sound\right]^2$$

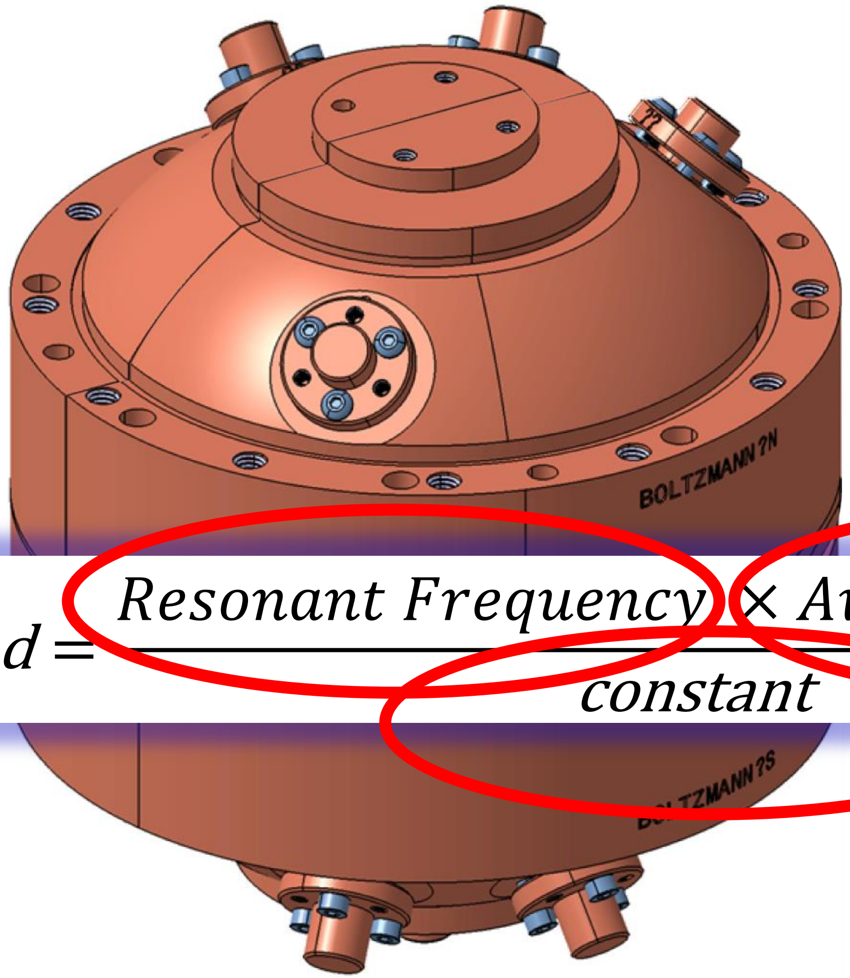
$$\frac{9}{5}(speed\ of\ sound)^2$$

Carry out experiment at T_{TPW}

Measure the speed of sound

Measure the speed of sound in a spherical resonator

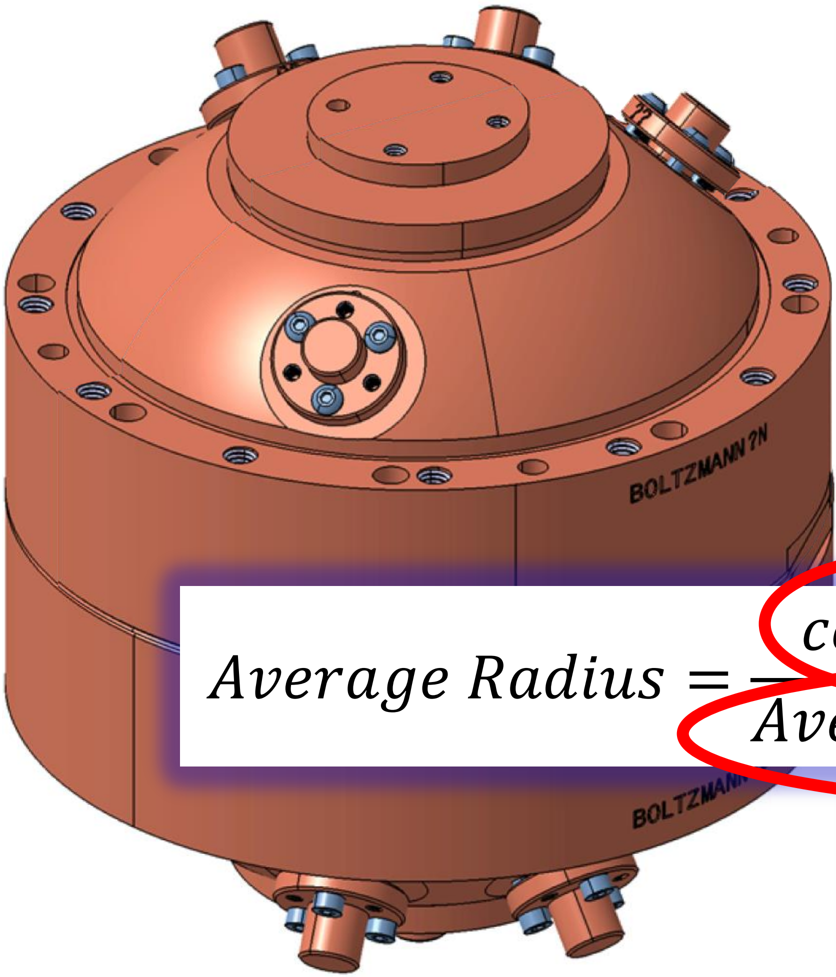




?

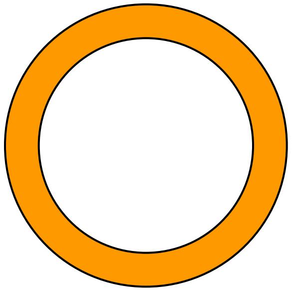
$$\textit{Speed of Sound} = \frac{\textit{Resonant Frequency} \times \textit{Average Radius}}{\textit{constant}}$$

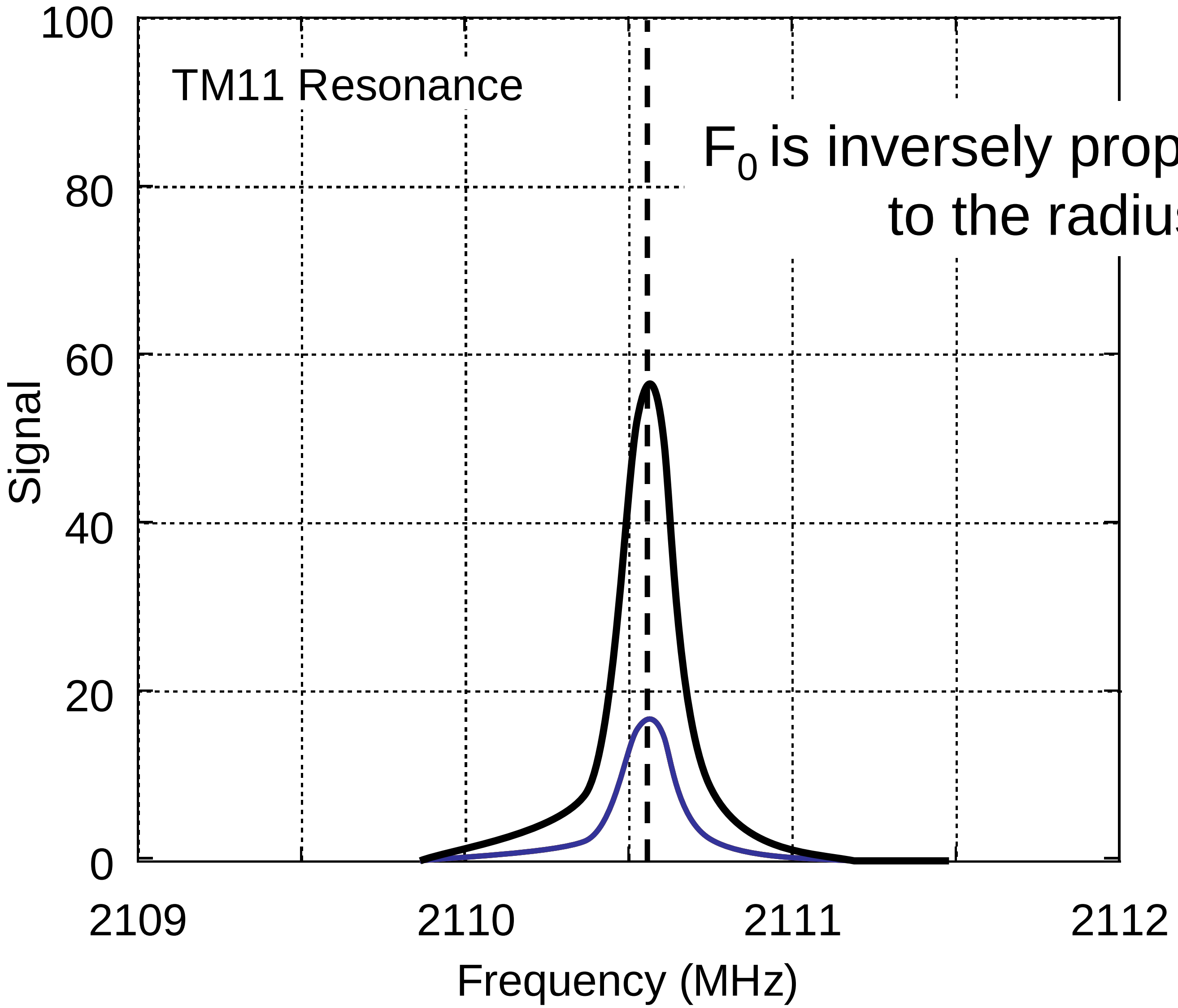
Measure the Average Radius using Microwaves



$$Average\ Radius = \frac{constant \times speed\ of\ light}{Average\ Resonant\ Frequency}$$

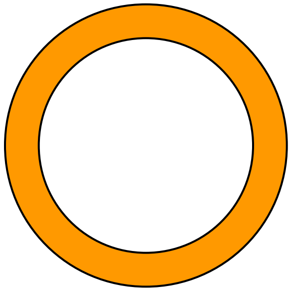
Microwave resonance
in a perfect sphere

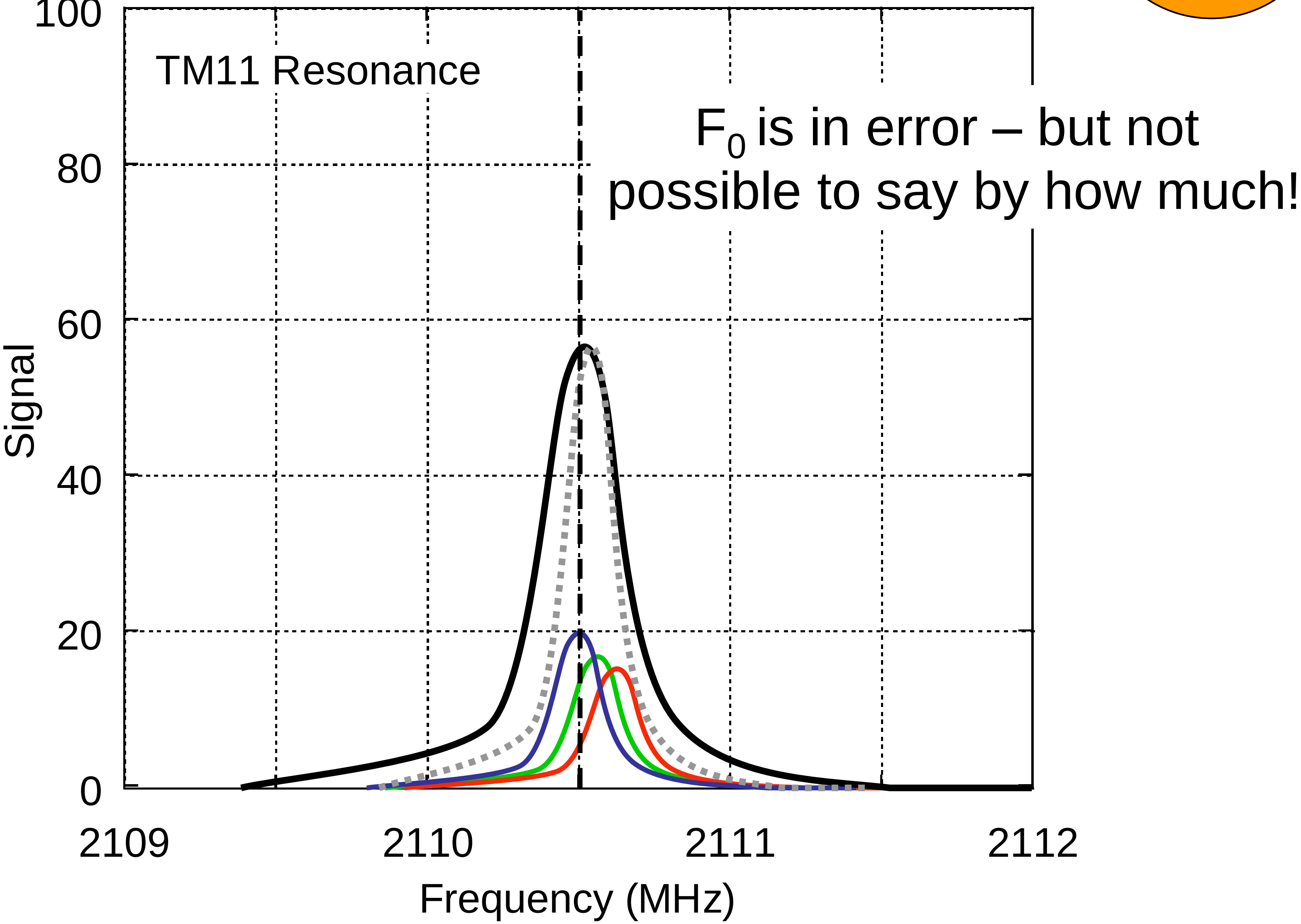




F_0 is inversely proportional to the radius

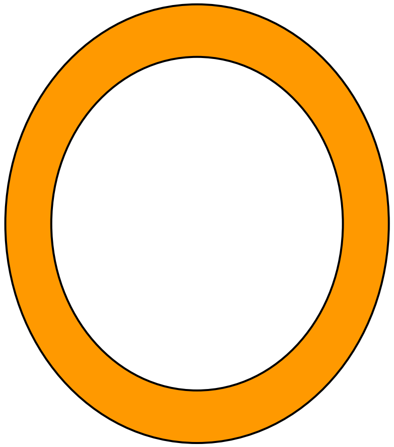
Microwave resonance
in a ***nearly*** perfect sphere

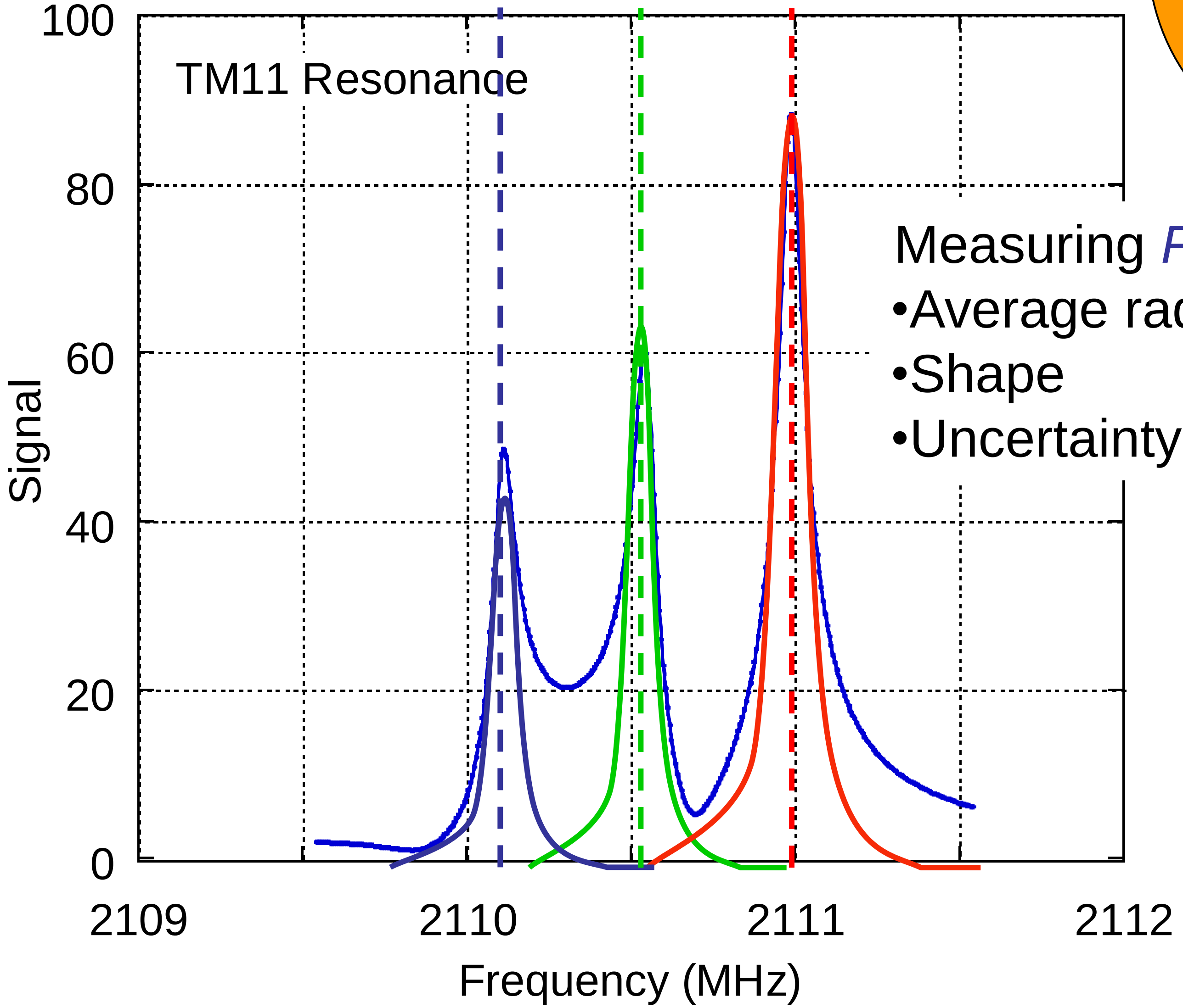




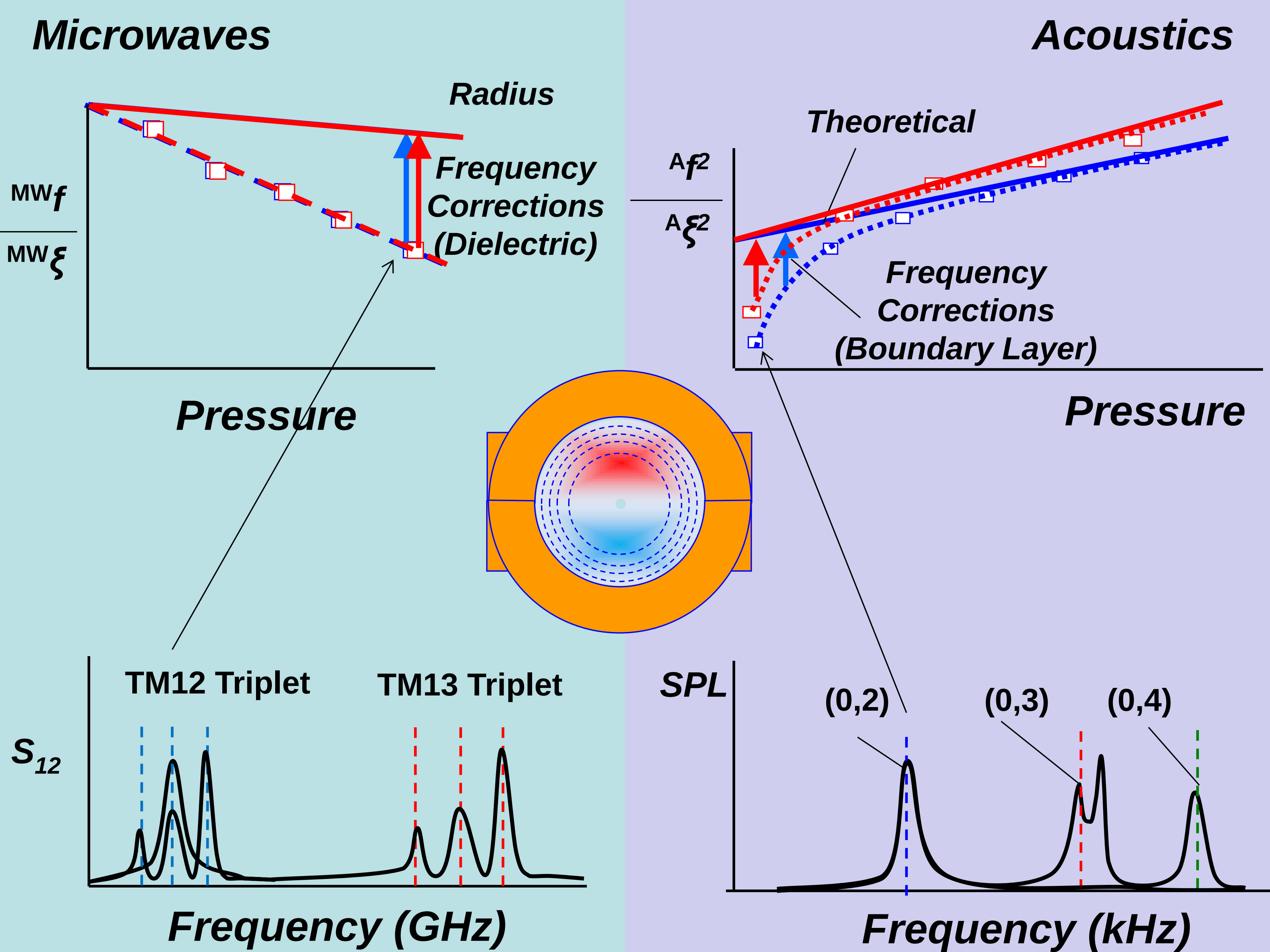
Microwave resonance
in a triaxial ellipsoid

Radius 62 mm
0.031 mm eccentrcity

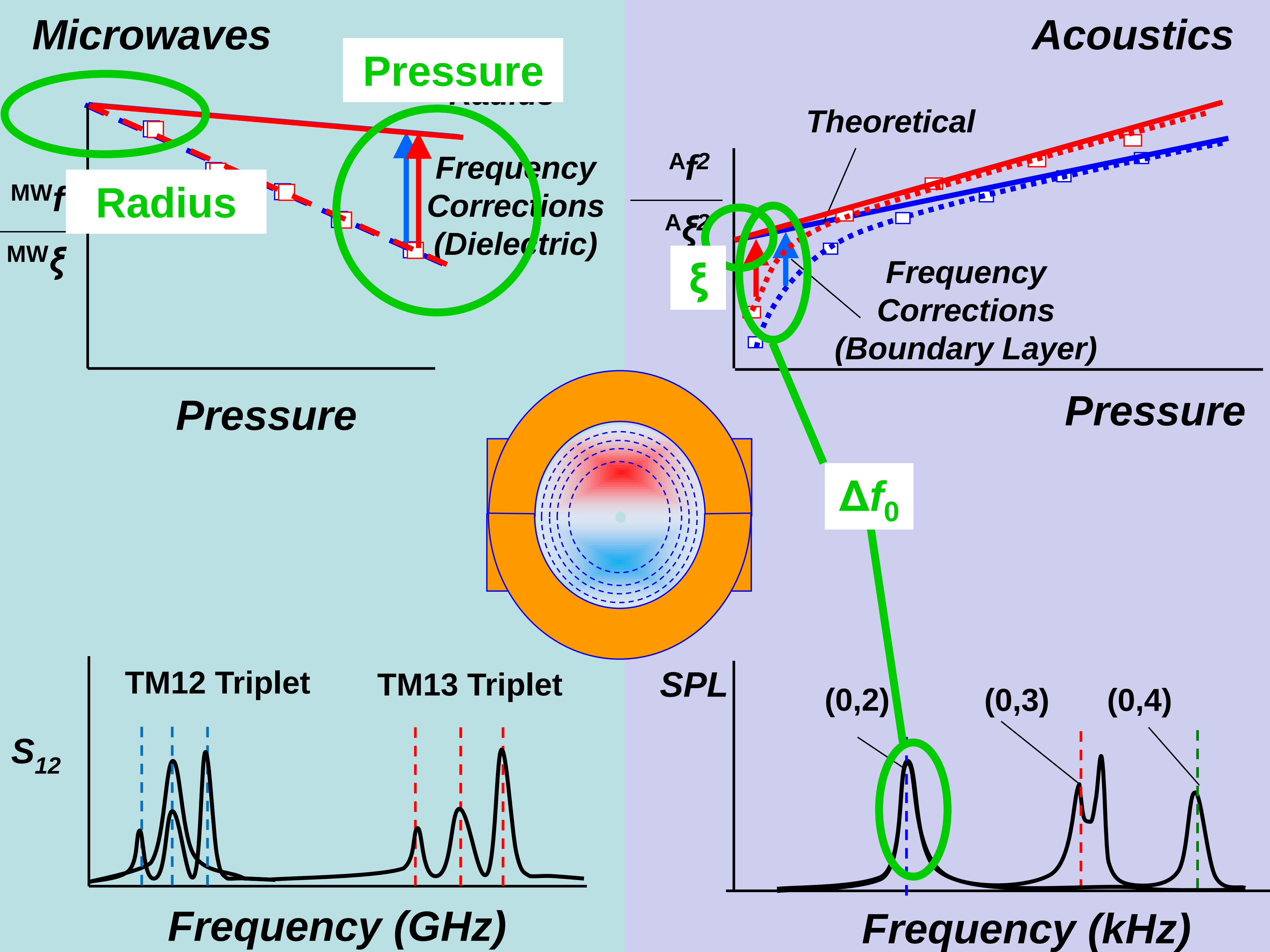




Measuring F_0, F_1 and F_2
- Average radius
- Shape
- Uncertainty



Microwaves
Acoustics
Radius
Frequency
Corrections
(Dielectric)
MWf
MWξ
Pressure
Theoretical
Af2
Aξ2
Frequency
Corrections
(Boundary Layer)
Pressure
TM12 Triplet
TM13 Triplet
S12
Frequency (GHz)
SPL
(0,2)
(0,3)
(0,4)
Frequency (kHz)

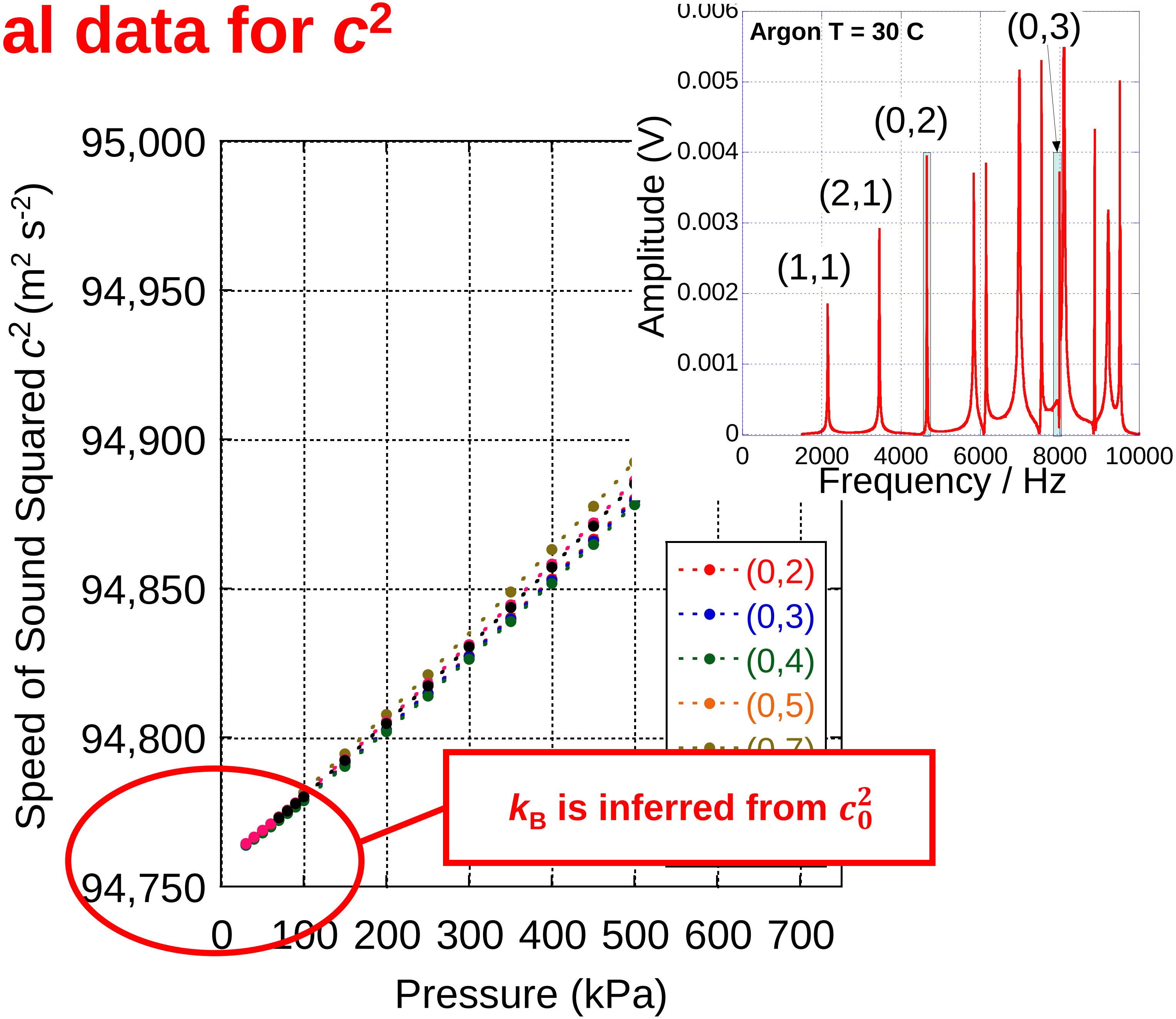


Microwaves
Acoustics
Pressure
Radius
Frequency Corrections (Dielectric)
Theoretical
Frequency Corrections (Boundary Layer)
ξ
Pressure
Pressure
Δf0
TM12 Triplet
TM13 Triplet
S12
Frequency (GHz)
SPL
(0,2)
(0,3)
(0,4)
Frequency (kHz)

Isothermal data for c^2

c^2_{EXP}

c^2_0



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2005:

NPL REPORT
DEPC TH 006

Review of methods for

ACOUSTIC GAS THERMOMETRY

A Report for the DTI
Quantum Programme

scheduled for 2011.

the change is made.

The need for a better value of the Boltzmann constant was highlighted by B N Taylor of NIST,

New data was needed for the redefinition of the kelvin, and Dr Taylor considered this to be next in significance to the need for data from watt-balances in supporting the proposed revision of the SI.

R RUSBY, M de PODESTA
and J WILLIAMS

NOT RESTRICTED

NOVEMBER 2005

EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

iMERA PLUS

EURAMET
European Association of National Metrology Institutes

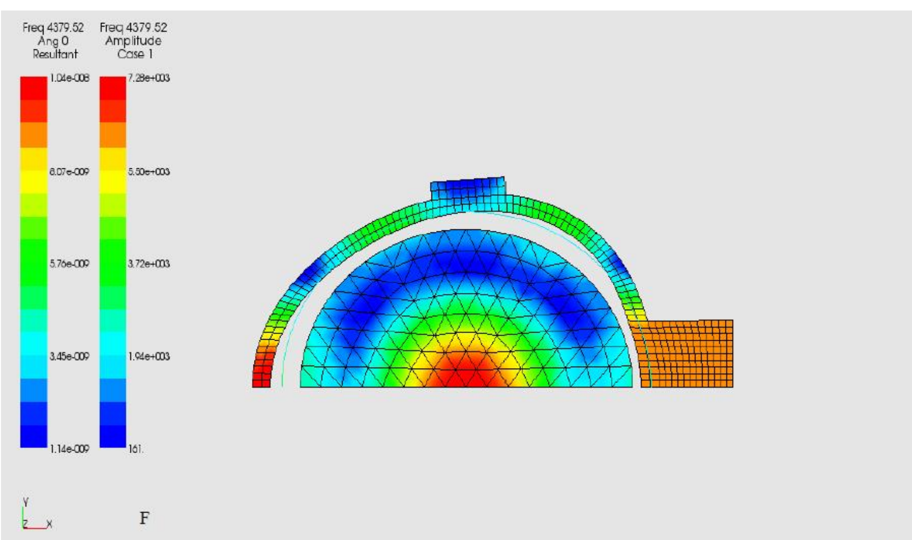
…and just 4 years later…

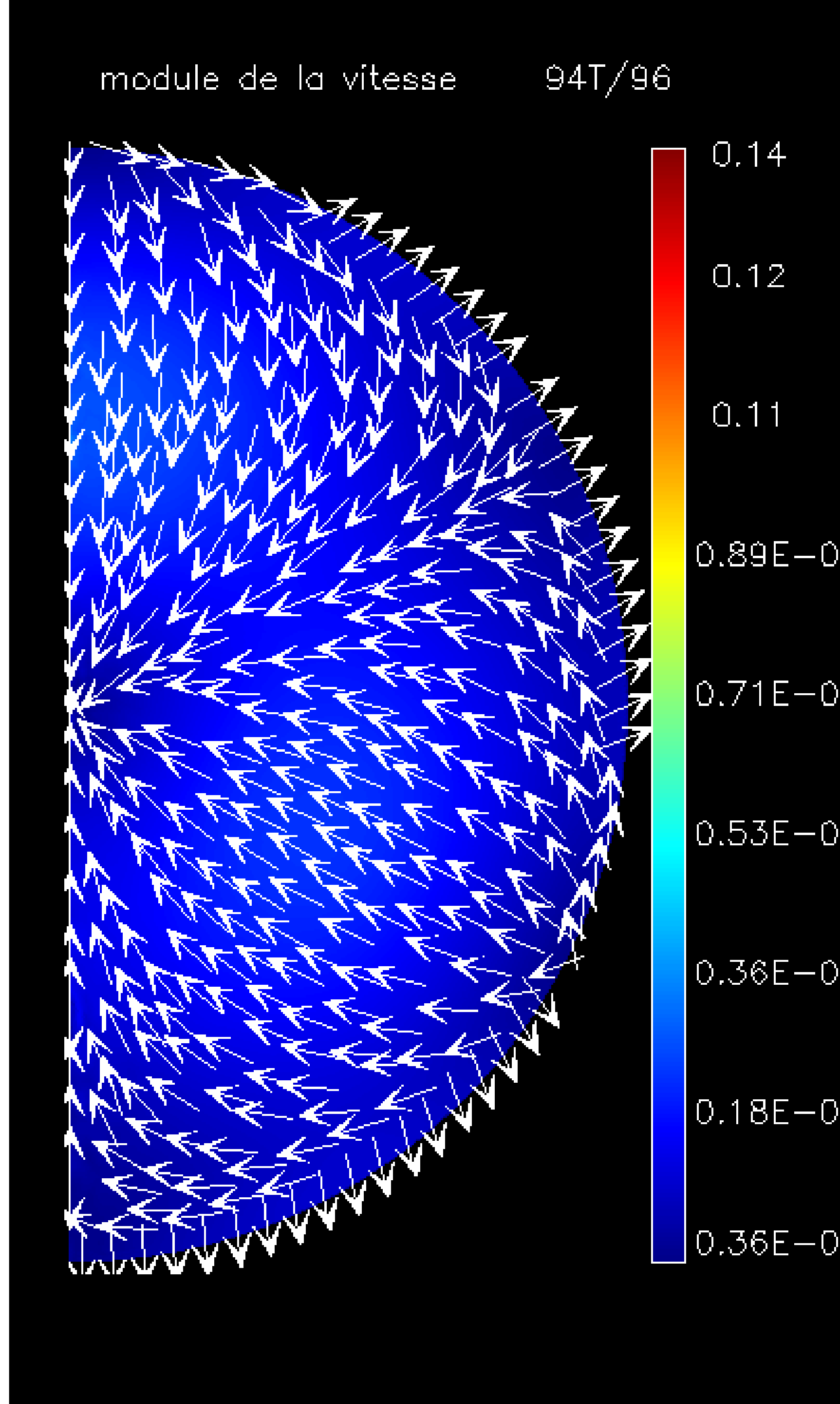
2009:

Anglo–French Physical Acoustics Conference 2009 IOP Publishing
Journal of Physics: Conference Series **195** (2009) 012002 doi:10.1088/1742-6596/195/1/012002

Acoustic modelling in view of a determination of the Boltzmann constant within 1 ppm for the redefinition of the kelvin

Pierre Gélat[1], Nicolas Joly[2], Michael de Podesta[1,†], Gavin Sutton[1], Robin Underwood[1]





2010:

IOP Publishing — Measurement Science and Technology

Meas. Sci. Technol. **21** (2010) 075103 (11pp) — doi:10.1088/0957-0233/21/7/075103

Waveguide effects on quasispherical microwave cavity resonators

R J Underwood[1], J B Mehl[2], L Pitre[3], G Edwards[1], G Sutton[1] and M de Podesta[1]

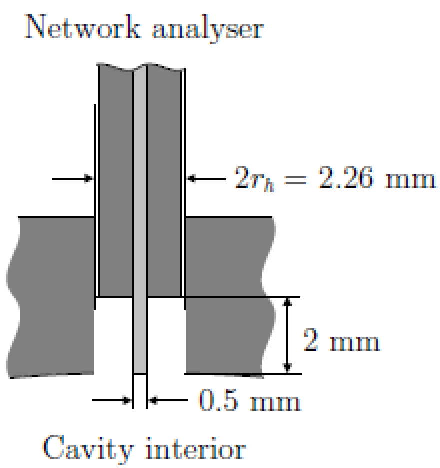


Figure 1. Diagram of a waveguide in TCU2v2, formed from a cylindrical hole and stripped coaxial cable. Although a looped antenna would offer the possibility of additionally detecting the TE1n modes, such antennas are more complex to model and were not considered in this study.

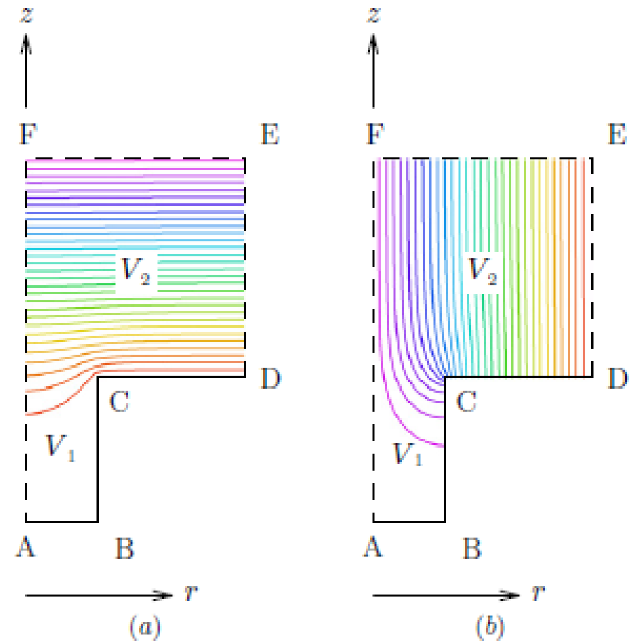


Figure 4. Potential fields (*a*) Φ_E and (*b*) Φ_H near the entrance of a cylindrical waveguide of depth $2r_h$. Adjacent contours represent equal increments in Φ.

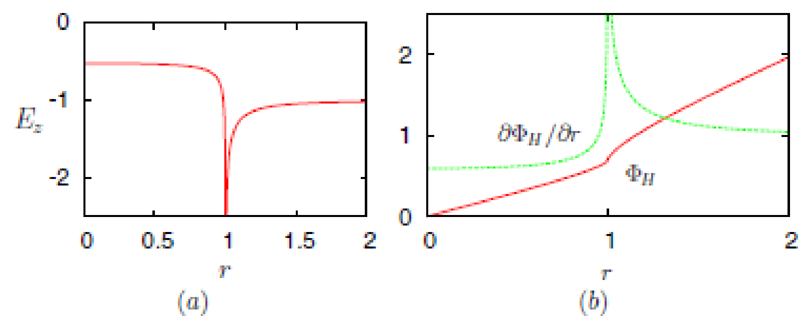


Figure 5. (*a*) Normal electric field and (*b*) magnetic potential and its radial derivative near the waveguide entrance in the static limit. The fields and lengths are all dimensionless, and were computed for unit incident values of E_0 and H_0. The value of $|E_z|$ approaches 0.53 as r is reduced below 1; similarly $|\partial\Phi_H/\partial r|$ approaches 0.59 as r is reduced below 1.

2010:

Int J Thermophys (2010) 31:1310–1346
DOI 10.1007/s10765-010-0722-z

Acoustic Resonator Experiments at the Triple Point of Water: First Results for the Boltzmann Constant and Remaining Challenges

Gavin Sutton · Robin Underwood · Laurent Pitre · Michael de Podesta · Staf Valkiers

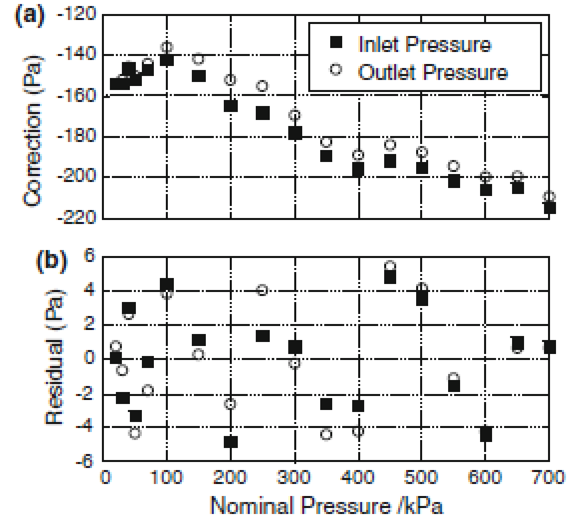


Fig. 5 (a) The calibration correction required for the pressure meters and (b) The residuals of a fit to the calibration data

Table 1 Isotopic composition of the argon gas used in the two isotherms

Molar mass[Ar] $(\mathrm{g \cdot mol^{-1}})$
39.947 706 21
39.947 857 10

$$u_R = 3.1 \times 10^{-6}$$

Highlighted Issues

- Variability of Molar Mass
- Need for precise pressure measurement
- 'Negative' Excess Half-Widths
  - *Line narrowing*

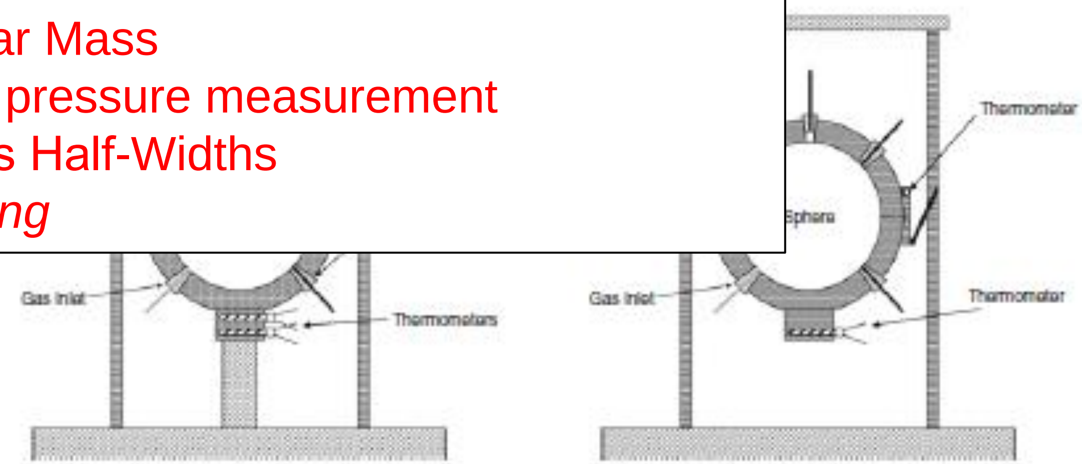
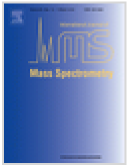


Fig. 2 The sphere in the two measurement configurations 'Fixed' and 'Hung'

2010:



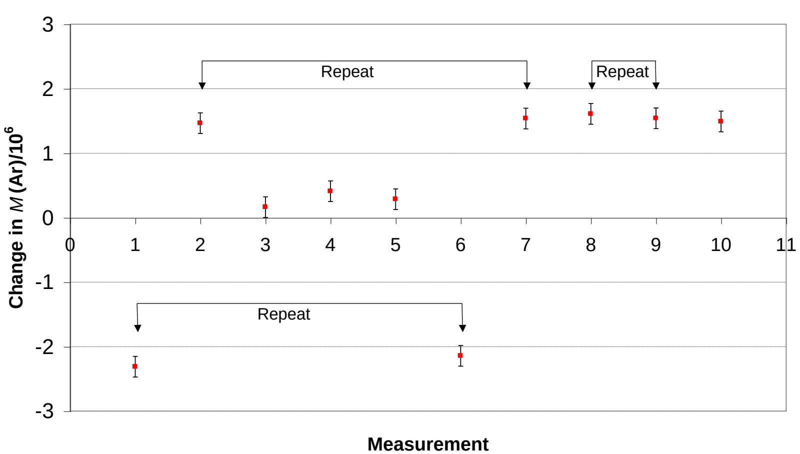
International Journal of Mass Spectrometry

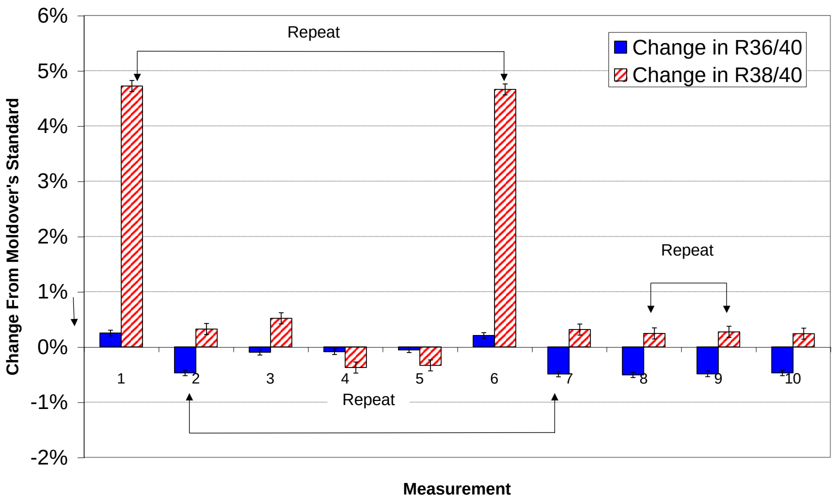
Volume 291, Issues 1–2, 15 March 2010, Pages 41–47

Preparation of argon Primary Measurement Standards for the calibration of ion current ratios measured in argon

S. Valkiers[a], D. Vendelbo[a], M. Berglund[a], M. de Podesta[b]

Received 16 December 2009, Revised 7 January 2010, Accepted 8 January 2010, Available online 15 January 2010





2011:

IOP Publishing Metrologia
Metrologia 46 (2009) 554–559 doi:10.1088/0026-1394/46/5/020

Second-order electromagnetic eigenfrequencies of a triaxial ellipsoid

James B Mehl

IOP Publishing Metrologia
Metrologia 48 (2011) 114–122 doi:10.1088/0026-1394/48/3/005

The electromagnetic fields of a triaxial ellipsoid calculated by modal superposition

Gordon Edwards and Robin Underwood

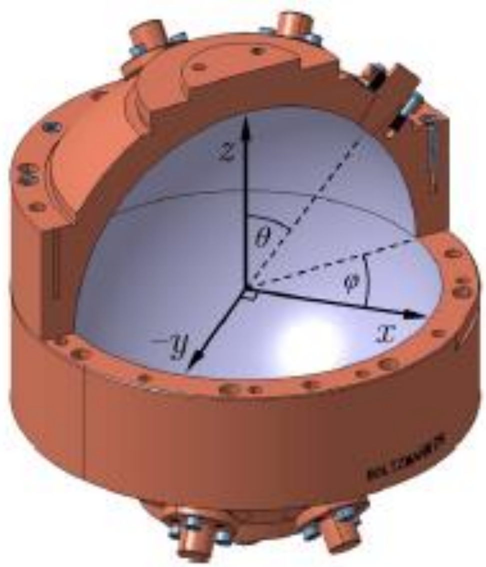


Figure 1. Cut-away diagram of NPL-C2 showing the coordinate axes. Notice the −y-axis is shown. The outer cylinder and inner

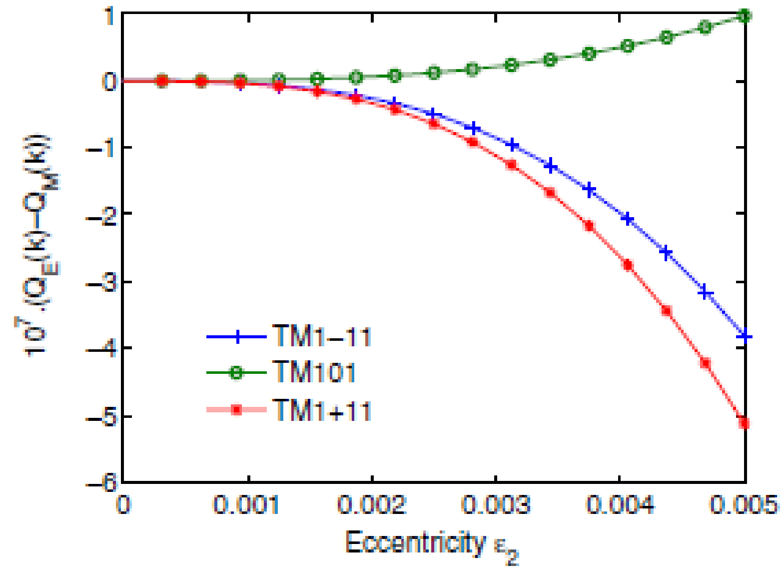


Figure 11. The difference between $Q(k)$ values calculated by modal analysis, $Q_E(k)$, and Mehl's second-order analysis, $Q_M(k)$, for $TM_{1,p,1}$ states.

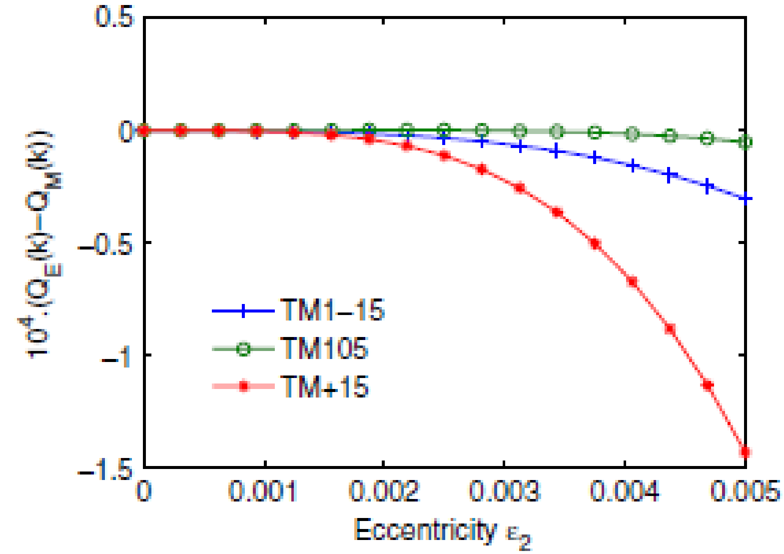


Figure 12. The difference between $Q(k)$ values calculated by modal analysis, $Q_E(k)$, and Mehl's second-order analysis, $Q_M(k)$, for $TM_{1,p,5}$ states.

2011:

IOP Publishing Metrologia

Metrologia 48 (2011) L1–L6 doi:10.1088/0026-1394/48/1/L01

SHORT COMMUNICATION

Outgassing of water vapour, and its significance in experiments to determine the Boltzmann constant

Michael de Podesta, Gavin Sutton, Robin Underwood, Stephanie Bell, Mark Stevens, Thomas Byrne and Patrick Josephs-Franks

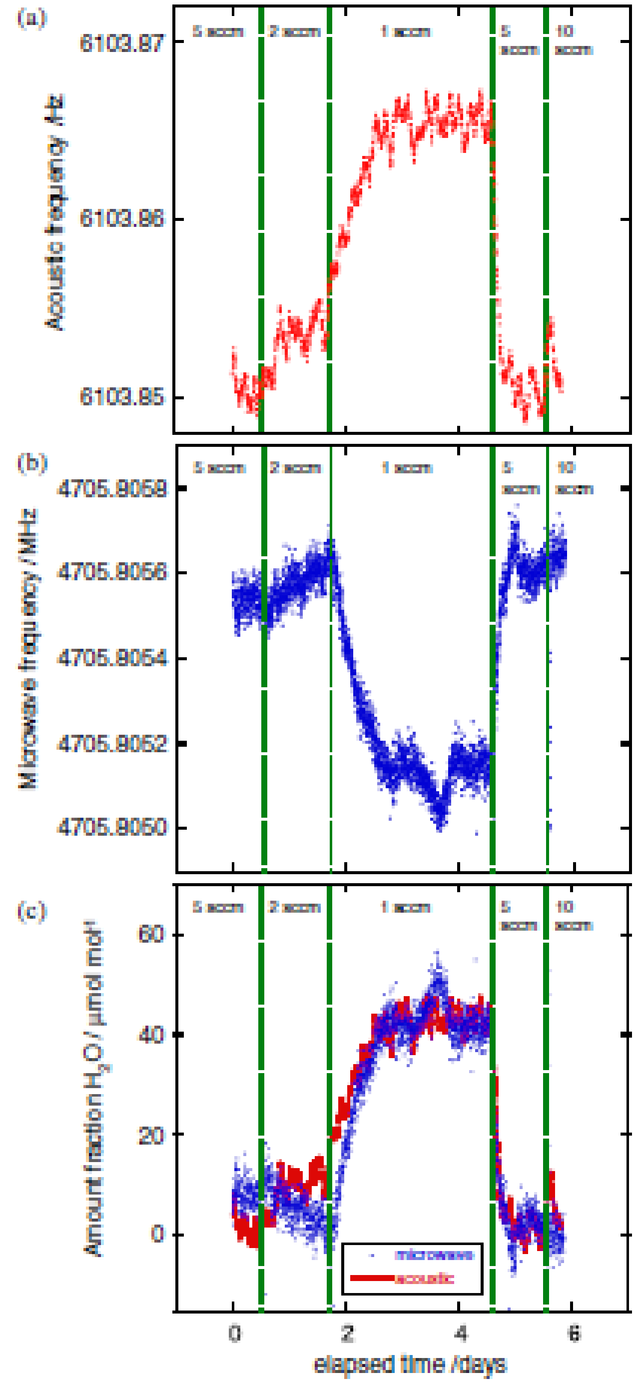


Figure 3. (*a*) Acoustic resonant frequency of the (0,3) resonance

2010:

IOP PUBLISHING METROLO
Metrologia 47 (2010) 588–604 doi:10.1088/0026-1394/47/5/

Characterization of the volume and shap of quasi-spherical resonators using coordinate measurement machines

M de Podesta[1], E F May[2], J B Mehl[3], L Pitre[4], R M Gavioso[5], G Benedetto[5], P A Giuliano Albo[5], D Truong[4] and D Flack[1]

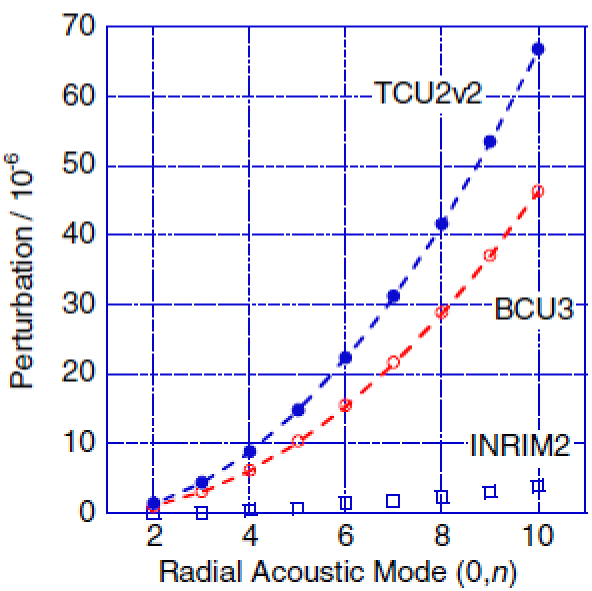


Figure 14. Relative shape perturbations $[((ka)^2 - \xi_{0n}^2)/\xi_{0n}^2] \times 10^6$ to the eigenvalues of the first nine purely radial acoustic modes of INRiM2, BCU3 and TCU2v2; BCU3 and TCU2v2 were alternatively modelled as triaxial ellipsoids (shown as dotted lines) or as more complex shapes, taking into account the complete results of the spherical harmonic expansion.

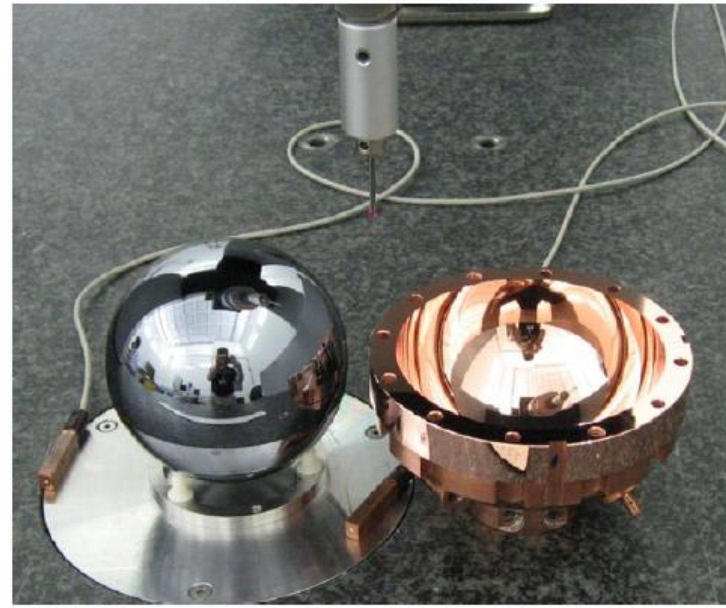
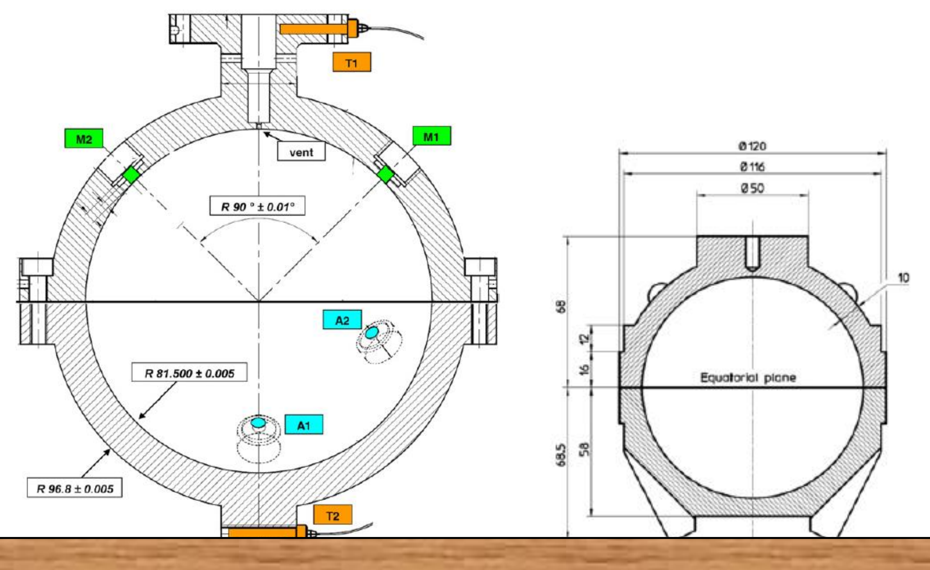
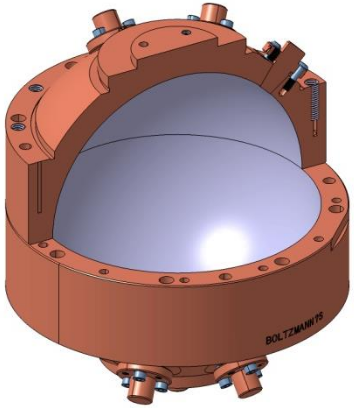


Figure 3. CMM measurements conducted at NPL probing the inner surface of a copper quasi-sphere alongside a spherical density and volume standard. Note the two thermometers (embedded in copper blocks) positioned around the artefacts.

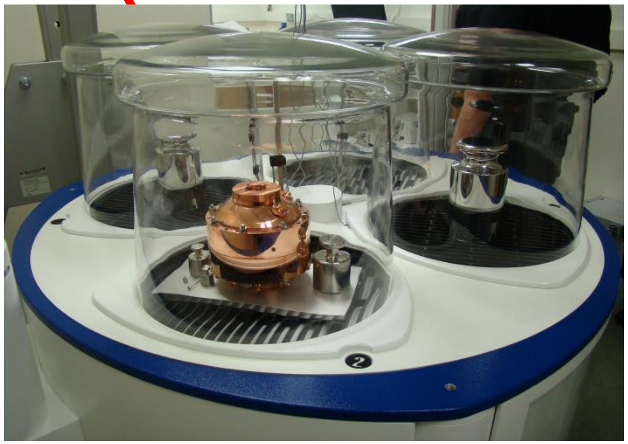


CMM



Microwaves

Pyknometry



2011:

IOP PUBLISHING METROLOGIA

Metrologia **48** (2011) 1–15 doi:10.1088/0026-1394/48/1/001

Dimensional characterization of a quasispherical resonator by microwave and coordinate measurement techniques

R Underwood[1], D Flack[1], P Morantz[2], G Sutton[1], P Shore[2] and M de Podesta[1]

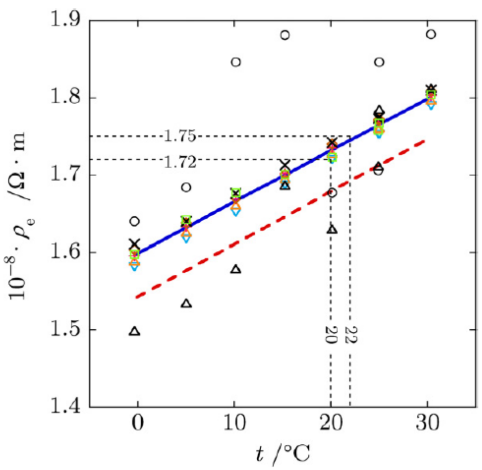


Figure 7. Measured resistivity of NPL-C2 copper, calculated from the mean half-widths of the TM_{1nx} and TM_{1ny} modes. The solid line is the result of a linear fit to all the data except the outlying TM_{16} (△) and TM_{17} (○) modes. The dashed line indicates ρ_{Cu} from Matula's data. Also marked is the dc conductivity measured by Bussey at 22 °C, and the value of 100% IACS at 20 °C.

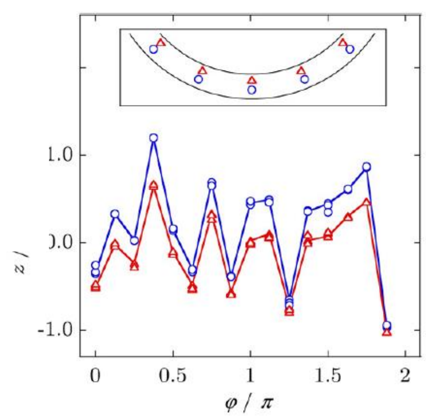


Figure 5. Typical deviations in the height of the equatorial flange from the fitted $z = 0$ plane. The inset shows the position of the probing points relative to the flange. Three data sets are plotted to highlight the repeatability of these measurements.

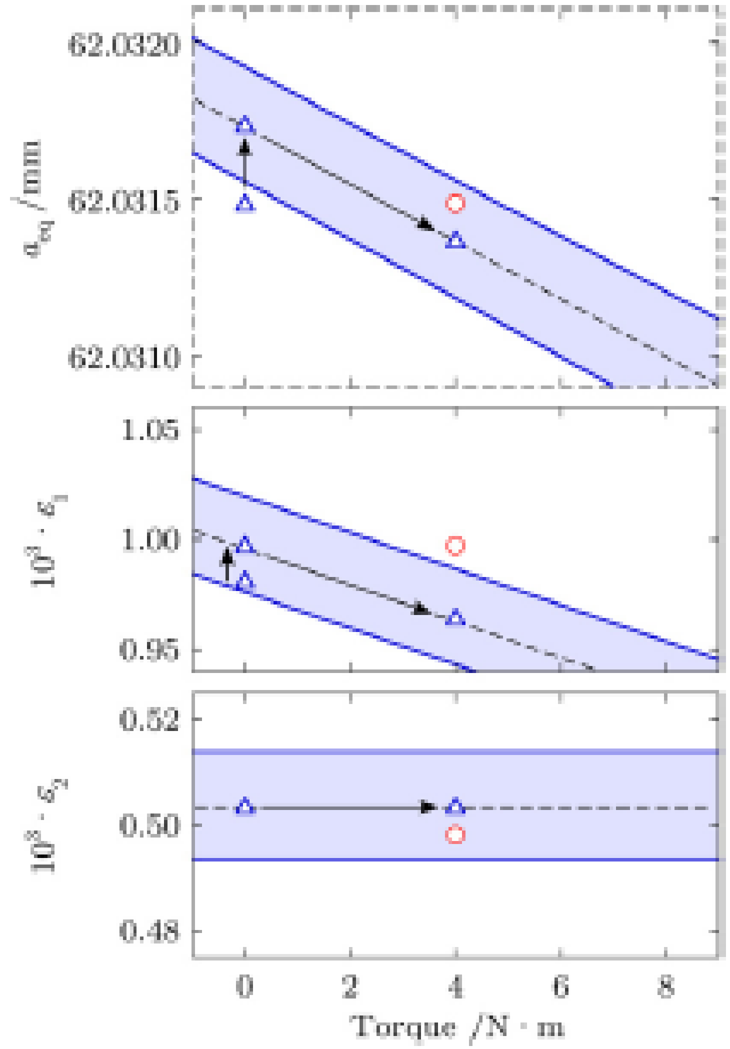
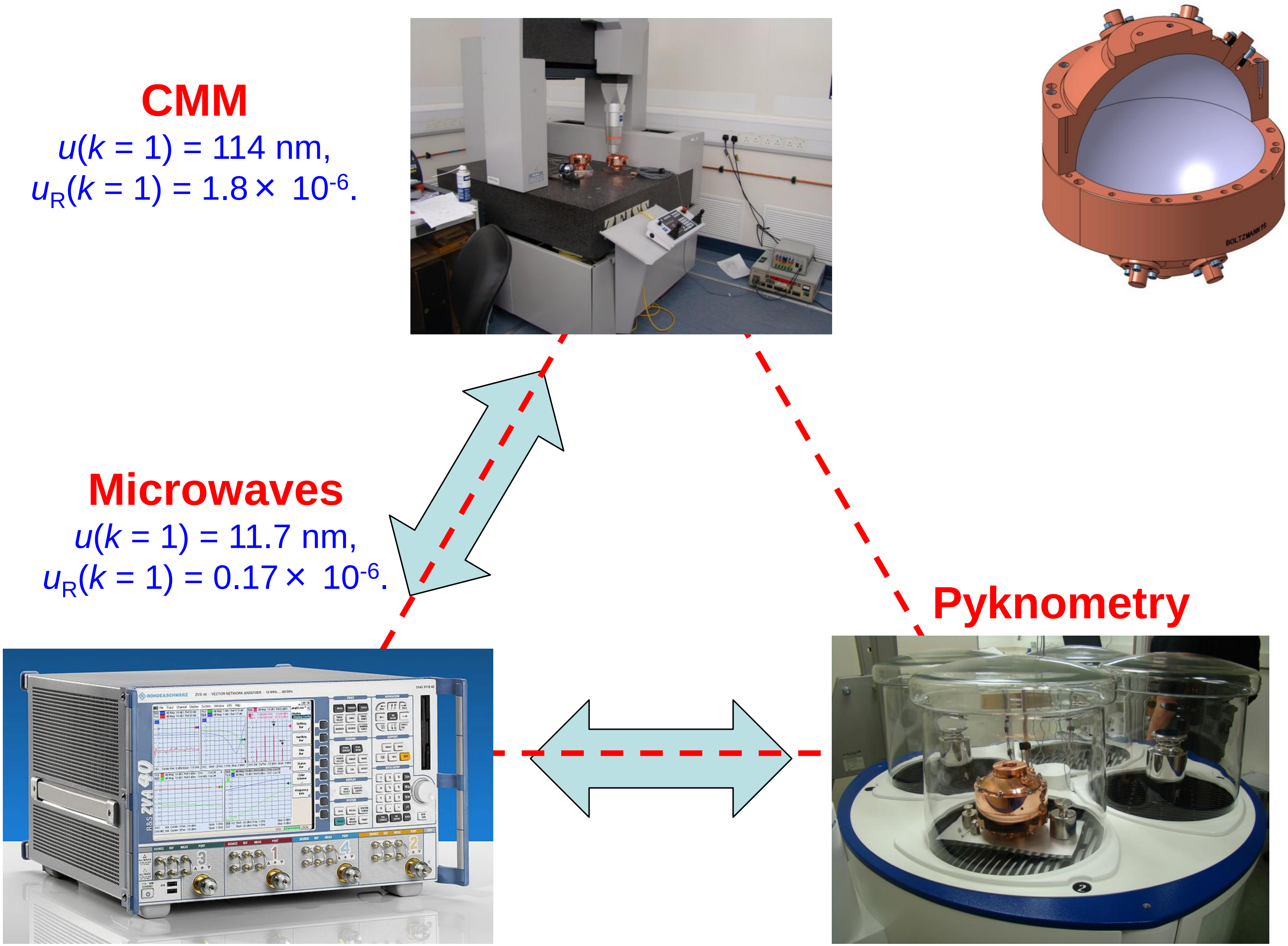


Figure 9. Comparison of CMM (△) and microwave (○) measurements of a_{eq}, ϵ_1 and ϵ_2 at 20 °C. The arrows indicate adjustments to the CMM data for a 1 µm equatorial gap and 4 N m bolt torque. The $k = 1$ uncertainty bounds (table 8) are shown as a shaded band. The uncertainty in $a_{eq}^{\mu w}$ is smaller than the marker itself.


CMM
u(k = 1) = 114 nm,
uR(k = 1) = 1.8 × 10-6.
Microwaves
u(k = 1) = 11.7 nm,
uR(k = 1) = 0.17 × 10-6.
Pyknometry

2012:

IOP PUBLISHING METROLOGIA
Metrologia **49** (2012) 245–256 doi:10.1088/0026-1394/49/3/245

Pyknometric volume measurement of a quasispherical resonator

R Underwood[1], S Davidson[1], M Perkin[1], P Morantz[2], G Sutton[1] and M de Podesta[1]

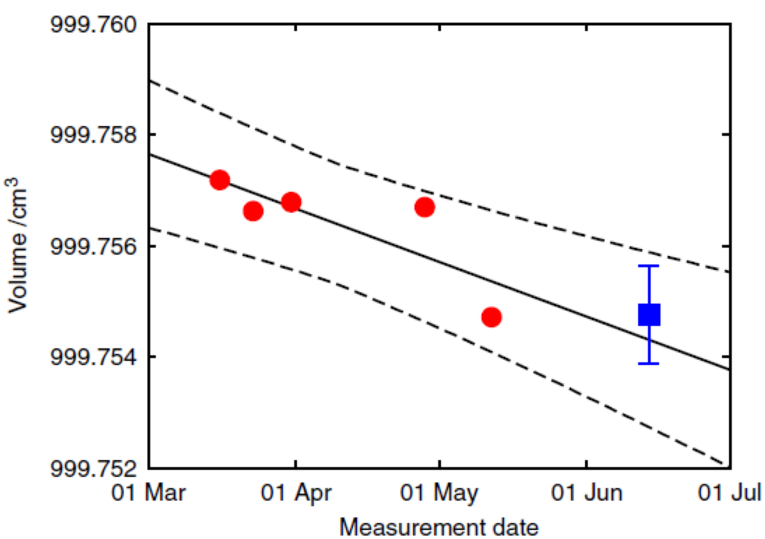


Figure 8. Volume estimates from the pyknometric (●) and microwave (■) measurements, corrected to a reference temperature of 20 °C and pressure of 101 325 Pa. The solid line is a linear fit to the five pyknometric measurements, and the dashed lines indicate the uncertainty in the pyknometric volume. The uncertainty bars indicate the microwave uncertainty. All measurements were performed in the year 2010.

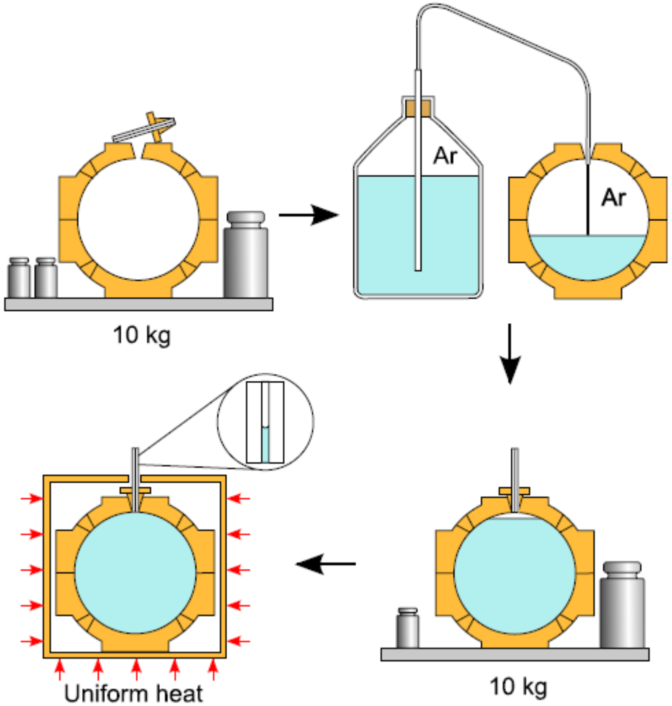
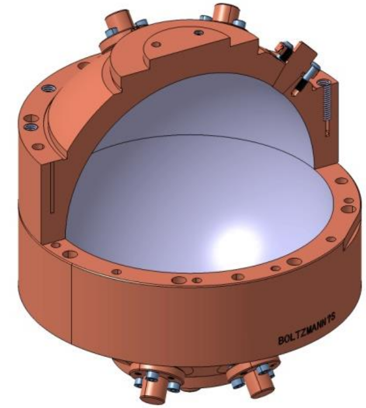


Figure 2. The five-step pyknometry procedure: empty weighing, filling, full weighing and temperature stabilization. The final step (emptying and re-weighing) is not shown.

CMM

$u(k = 1) = 114$ nm,

$u_R(k = 1) = 1.8 \times 10^{-6}$.



Microwaves

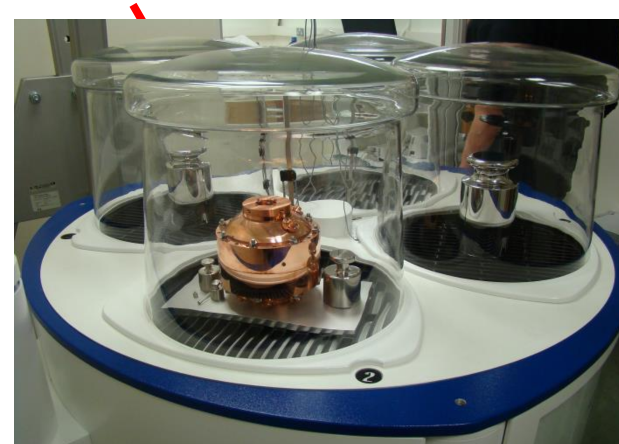
$u(k = 1) = 11.7$ nm,

$u_R(k = 1) = 0.17 \times 10^{-6}$.

Pyknometry

$u(k = 1) = 37$ nm,

$u_R(k = 1) = 0.60 \times 10^{-6}$.



2013:

IOP PUBLISHING METROLOGIA

Metrologia **50** (2013) 354–376 doi:10.1088/0026-1394/50/4/354

A low-uncertainty measurement of the Boltzmann constant

Michael de Podesta[1], Robin Underwood[1], Gavin Sutton[1], Paul Morantz[2], Peter Harris[1], Darren F Mark[3], Finlay M Stuart[3], Gergely Vargha[4] and Graham Machin[1]

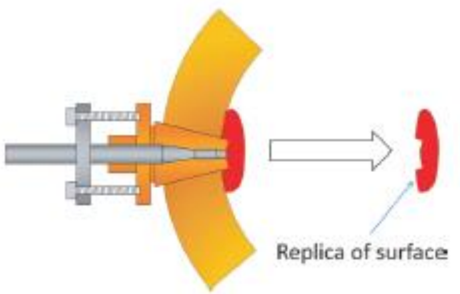
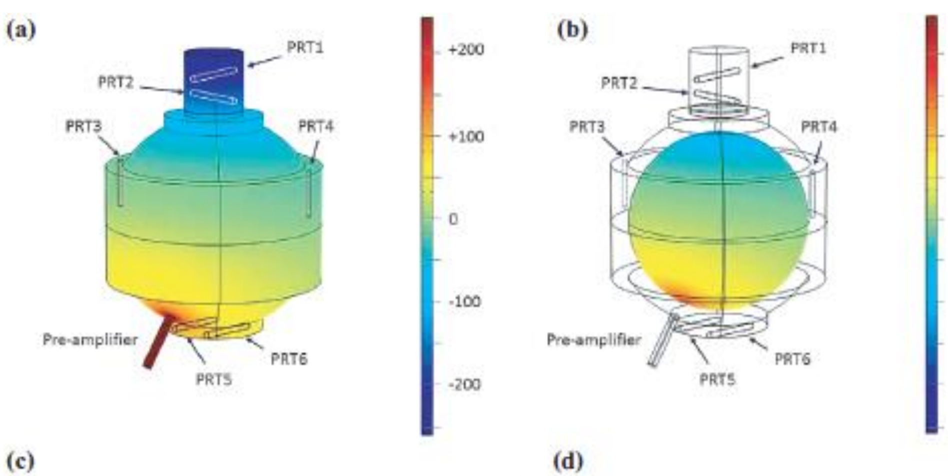
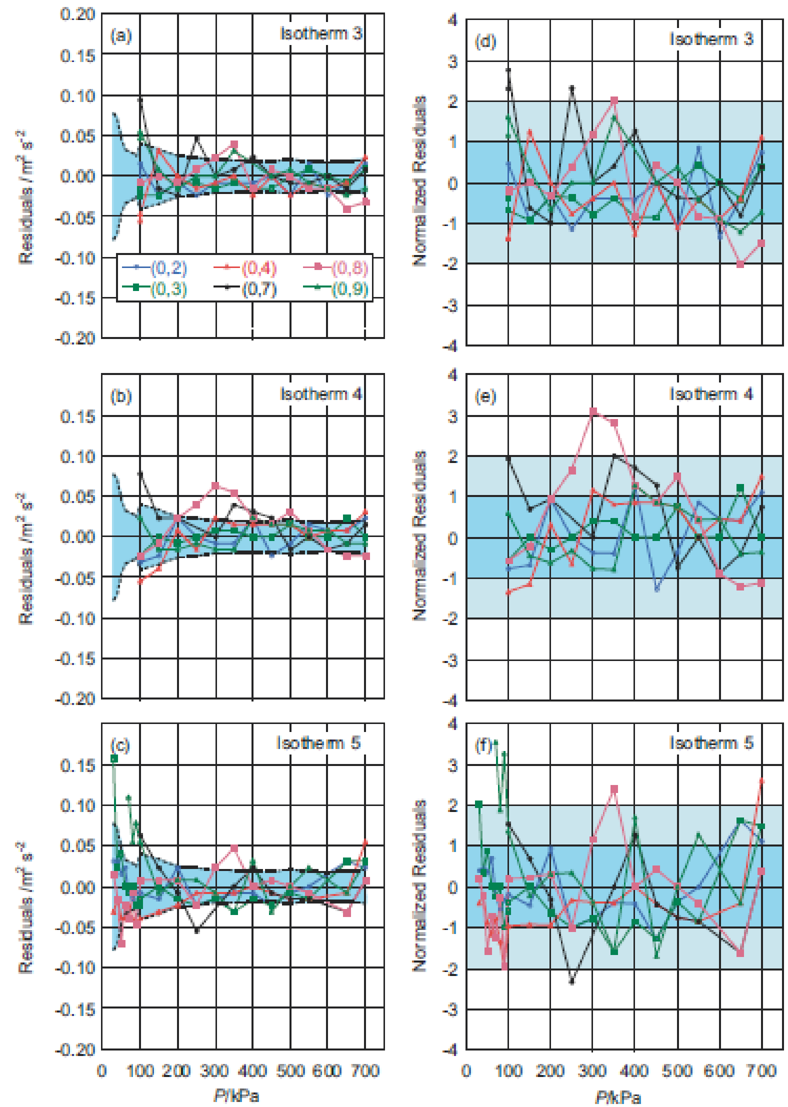


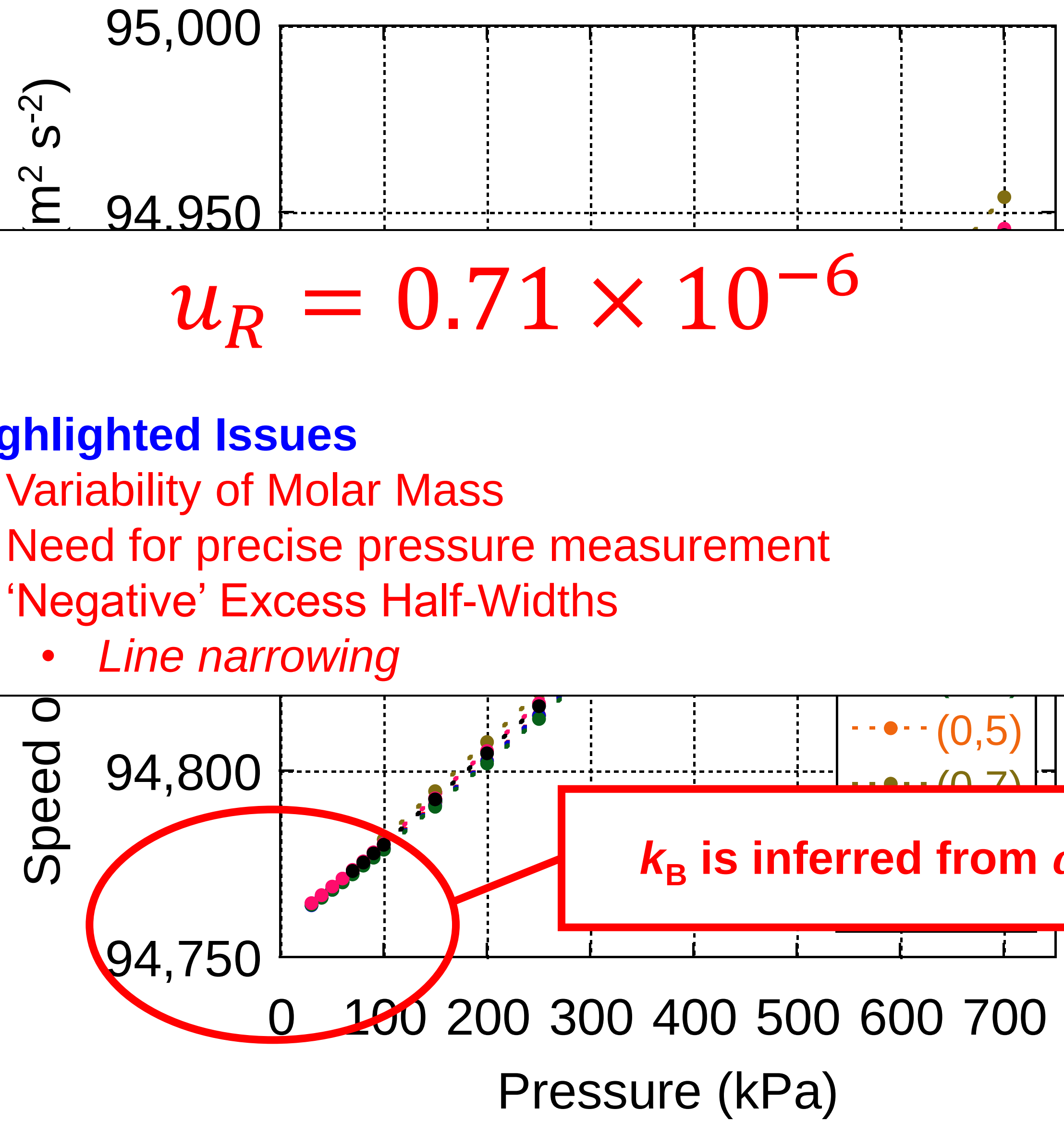
Figure 10. Illustration of how the perturbation caused by protrusion of the acoustic transducers was measured using a quick-setting silicone compound.





Isothermal data for c^2

c^2_{EXP}



c^2_0

$$u_R = 0.71 \times 10^{-6}$$

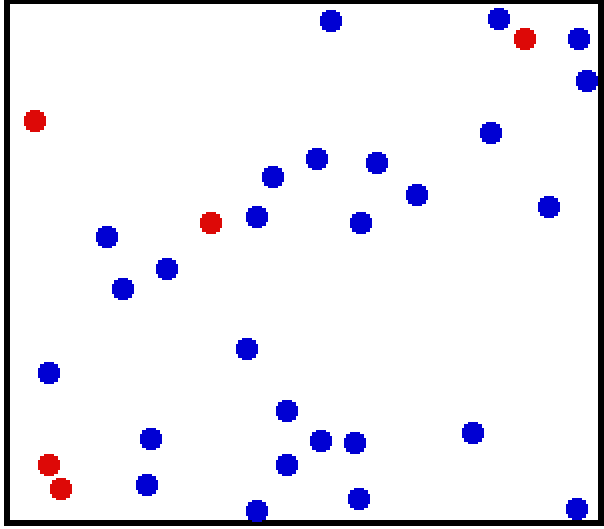
Highlighted Issues

- Variability of Molar Mass
- Need for precise pressure measurement
- ‘Negative’ Excess Half-Widths
  - *Line narrowing*

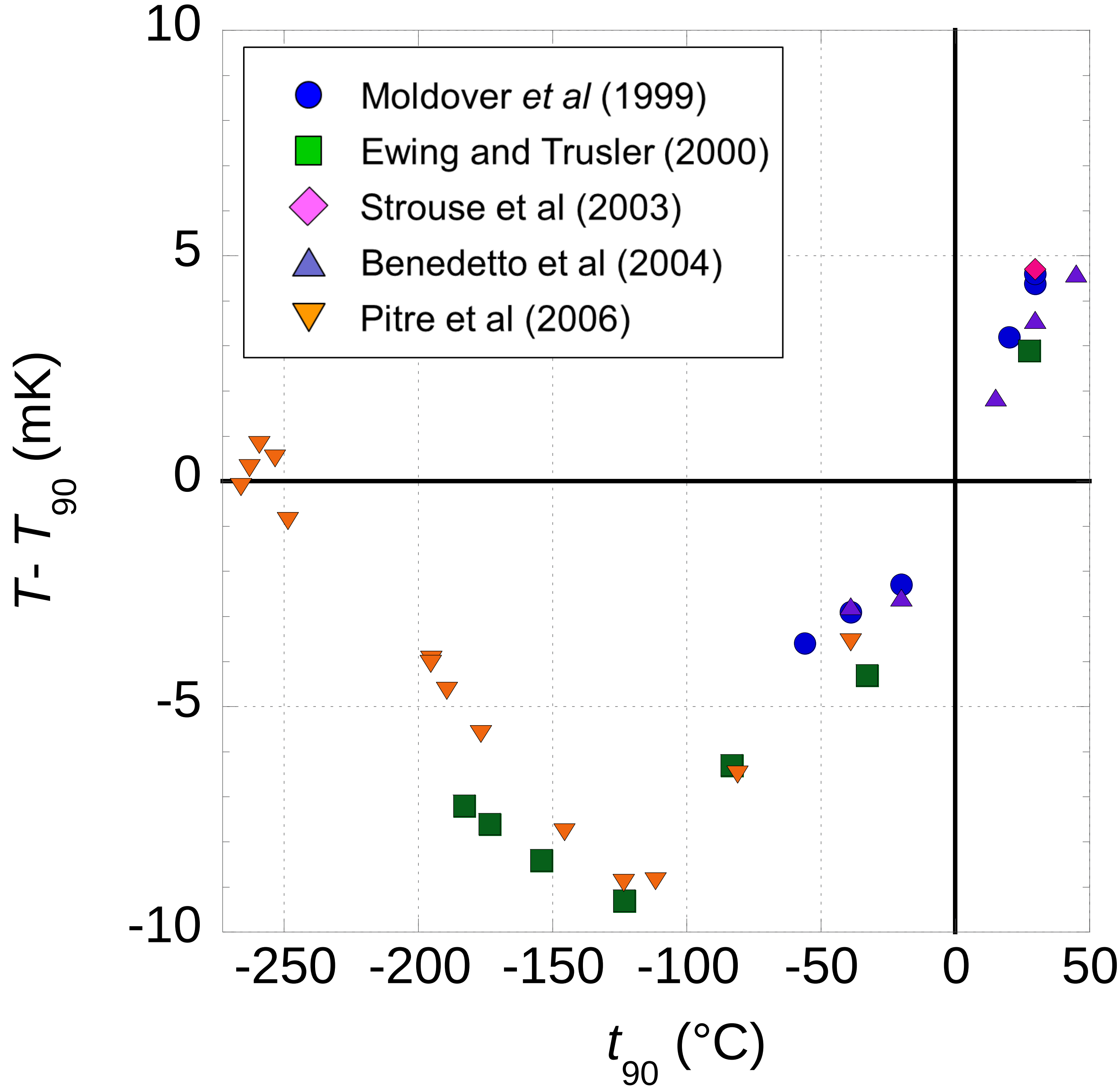
k_B is inferred from c_0^2

Since 2013

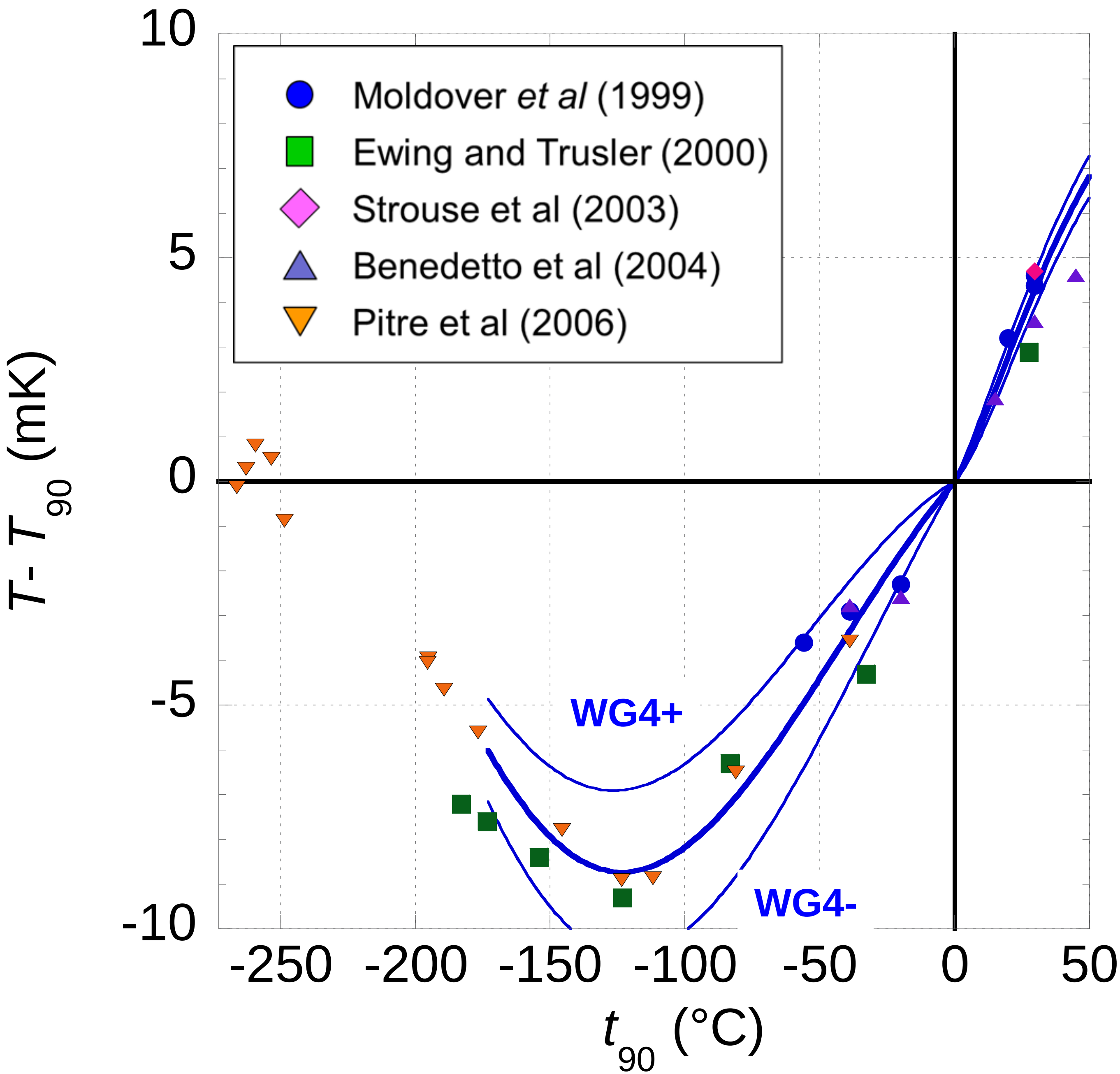
Acoustic Thermometry



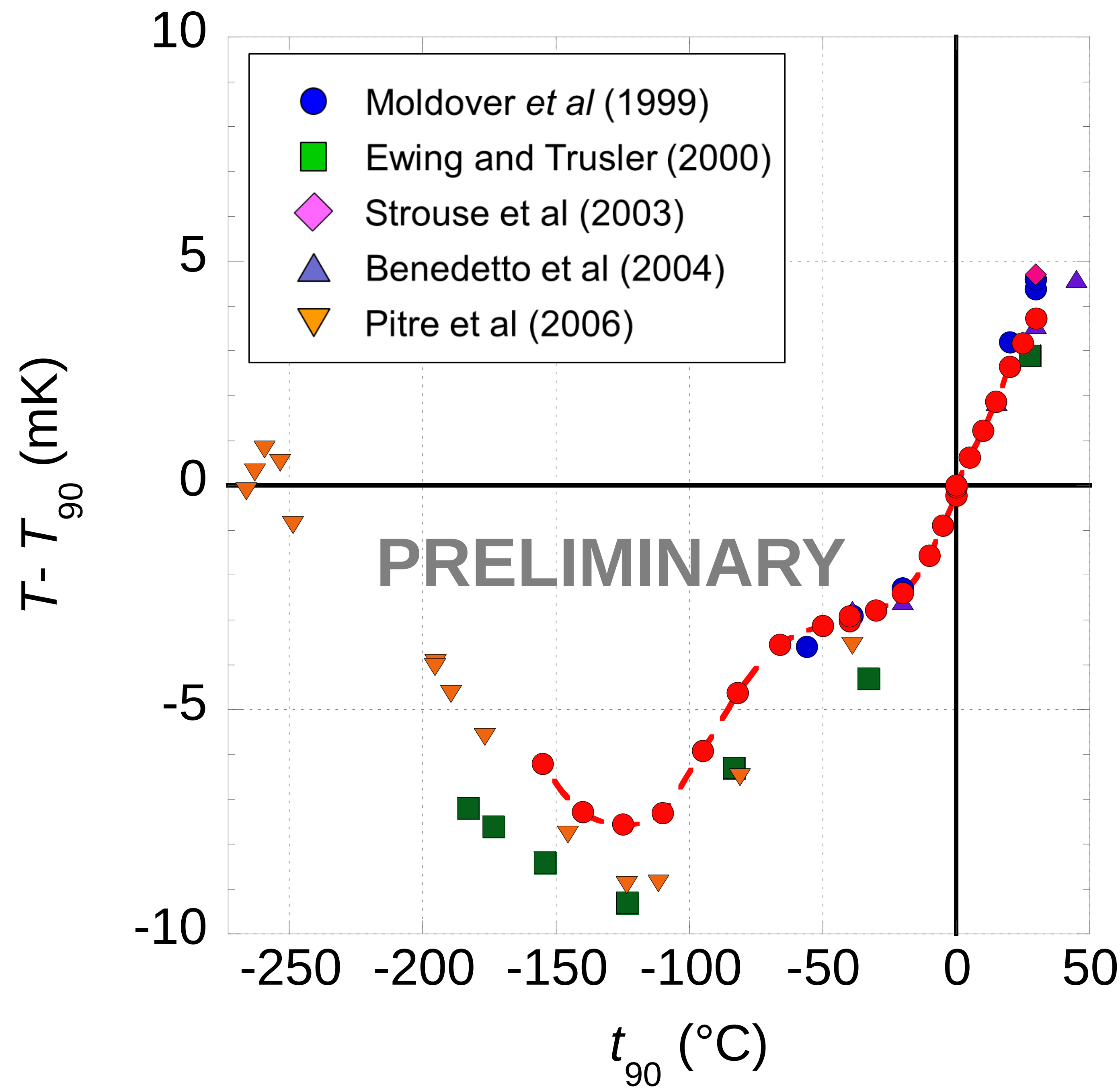
$$c^2 = \frac{\gamma k_B N_A T}{M}$$



10
5
0
-5
-10
-250
-200
-150
-100
-50
0
50
$T - T_{90}$ (mK)
t_{90} (°C)
Moldover *et al* (1999)
Ewing and Trusler (2000)
Strouse et al (2003)
Benedetto et al (2004)
Pitre et al (2006)

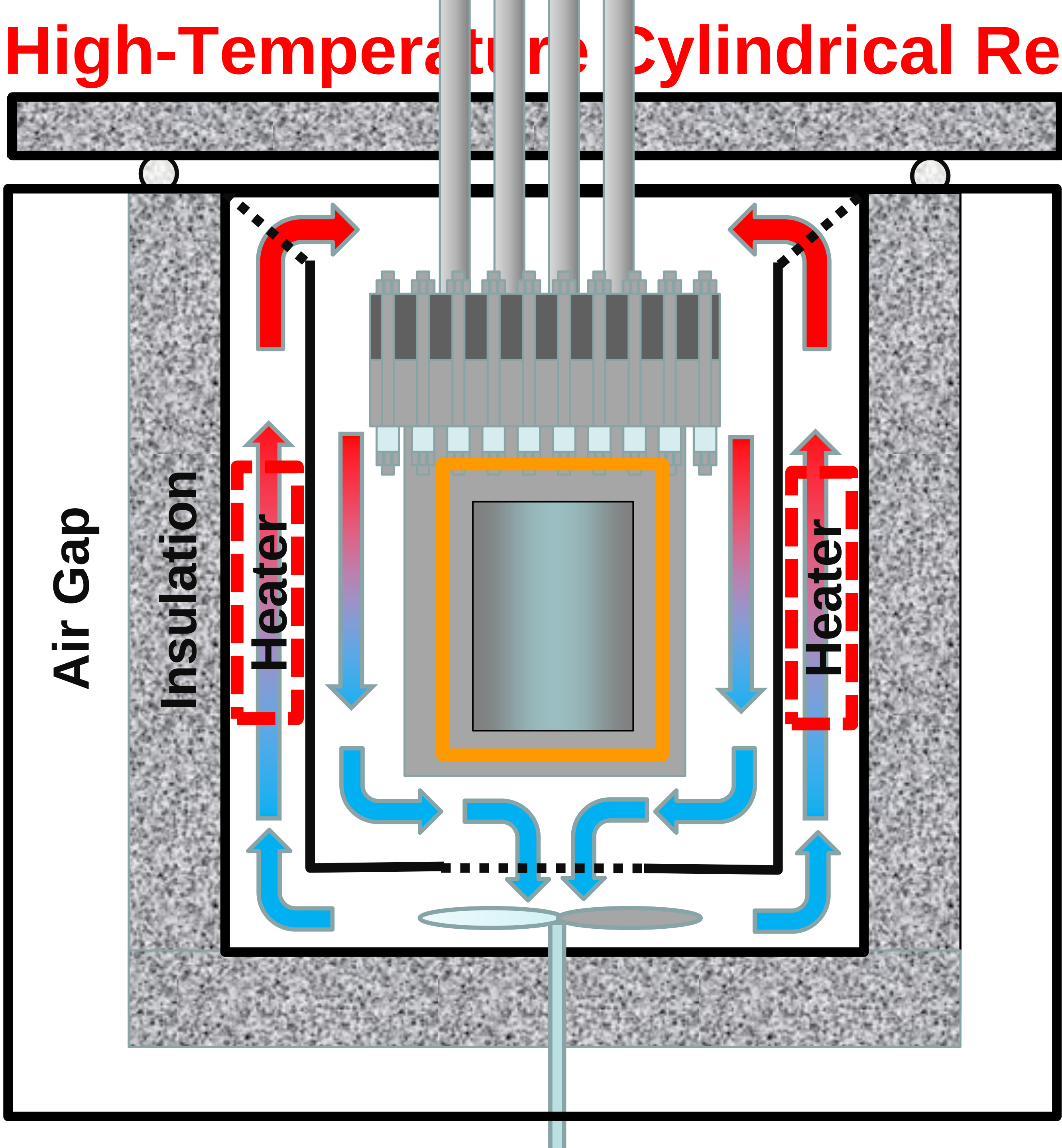


Moldover et al (1999)
Ewing and Trusler (2000)
Strouse et al (2003)
Benedetto et al (2004)
Pitre et al (2006)
WG4+
WG4-
T- T90 (mK)
t90 (°C)
10
5
0
-5
-10
-250
-200
-150
-100
-50
0
50



Moldover *et al* (1999)
Ewing and Trusler (2000)
Strouse et al (2003)
Benedetto et al (2004)
Pitre et al (2006)
PRELIMINARY
10
5
0
-5
-10
-250
-200
-150
-100
-50
0
50
$T - T_{90}$ (mK)
t_{90} (°C)

NPL High-Temperature Cylindrical Resonator



Reflections on the NPL-2013 Estimate of the Boltzmann Constant

1. Background
2. History
3. The NPL uncertainty estimate
4. NPL Update February 2015
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6. Summary

Working Equation

- Measure $f_{(0,n)}$ in the limit of low pressure

$$k_B = \frac{(M/\gamma)}{N_A T_{TPW}} \times \left[\frac{2\pi a \left(f_{(0,n)} + \Delta f_{(0,n)} \right)}{\xi_{(0,n)}} \right]^2$$

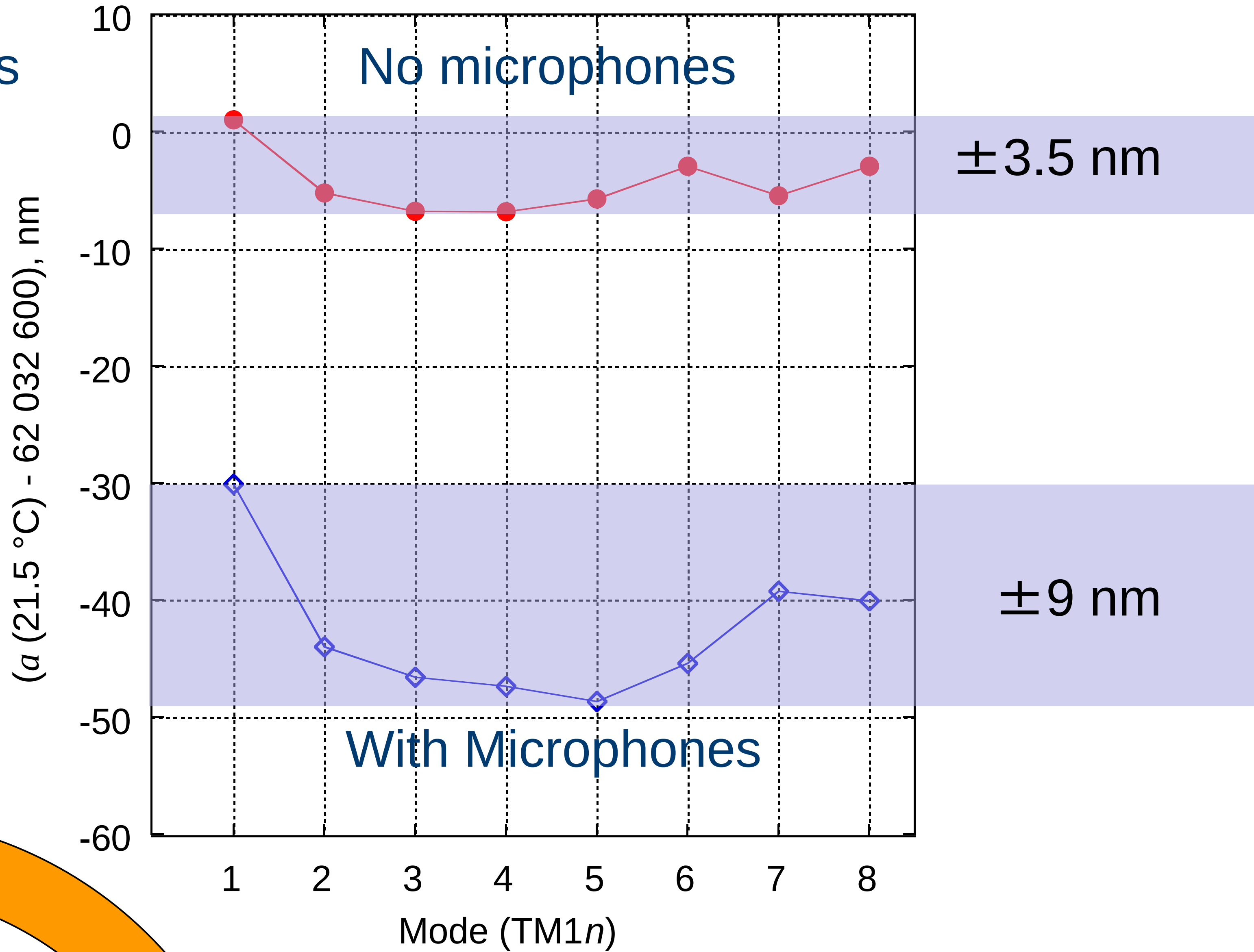
1. Resonator radius, a
2. Frequency Corrections $\Delta f_{(0,n)}$
3. Eigenvalues $\xi_{(0,n)}$
4. Pressure
5. Temperature
6. Molar Mass

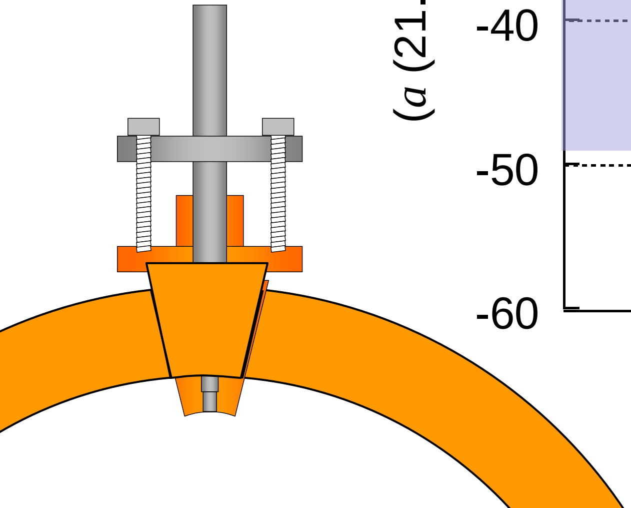
1: *How wrong could the radius estimate be?*

1. **Resonator radius, $\boldsymbol{a}$**
2. Pressure
3. Eigenvalues $\xi_{(0,n)}$
4. Frequency Corrections $\Delta f_{(0,n)}$
5. Temperature
6. Molar Mass

Microwave Radius Estimates ***in Vacuum***

nanometres



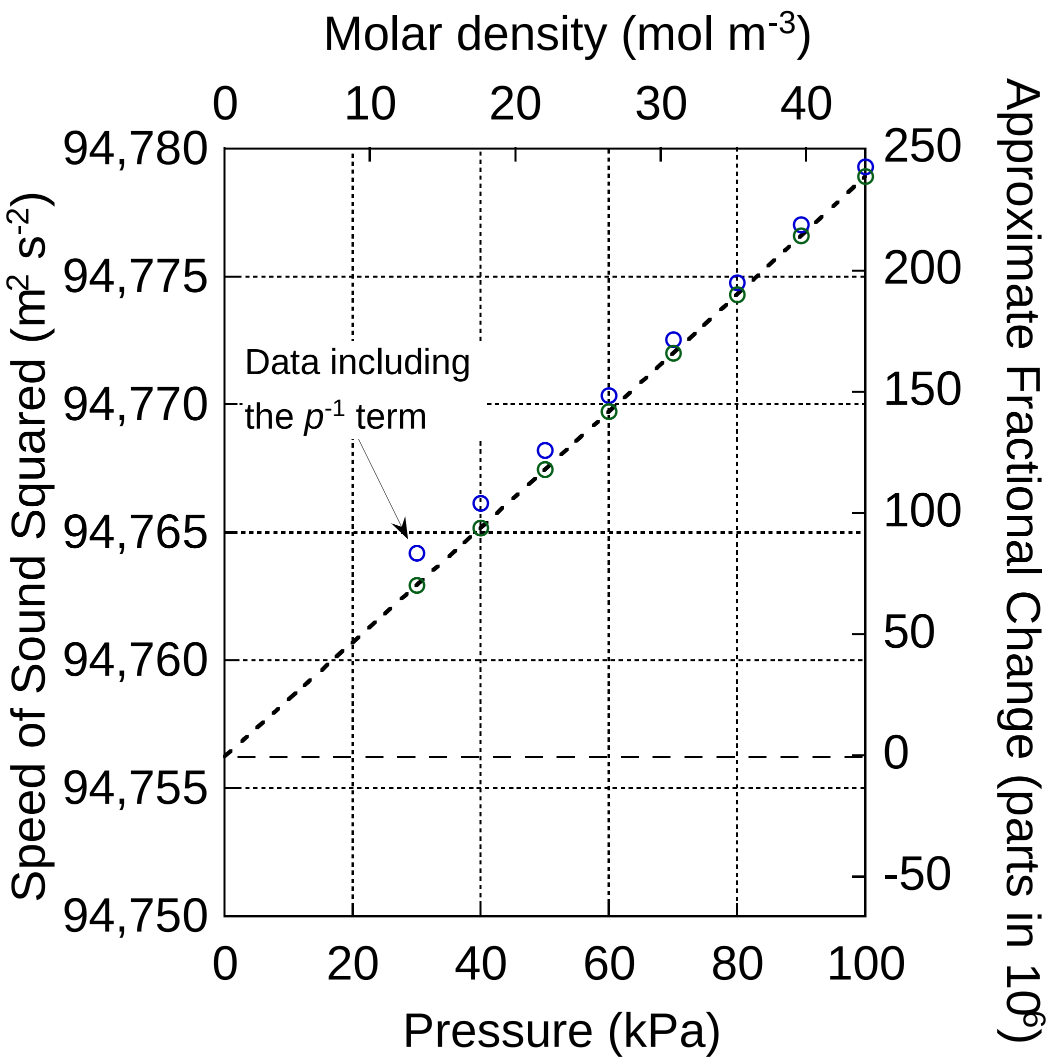


2: *How wrong could the pressure be?*

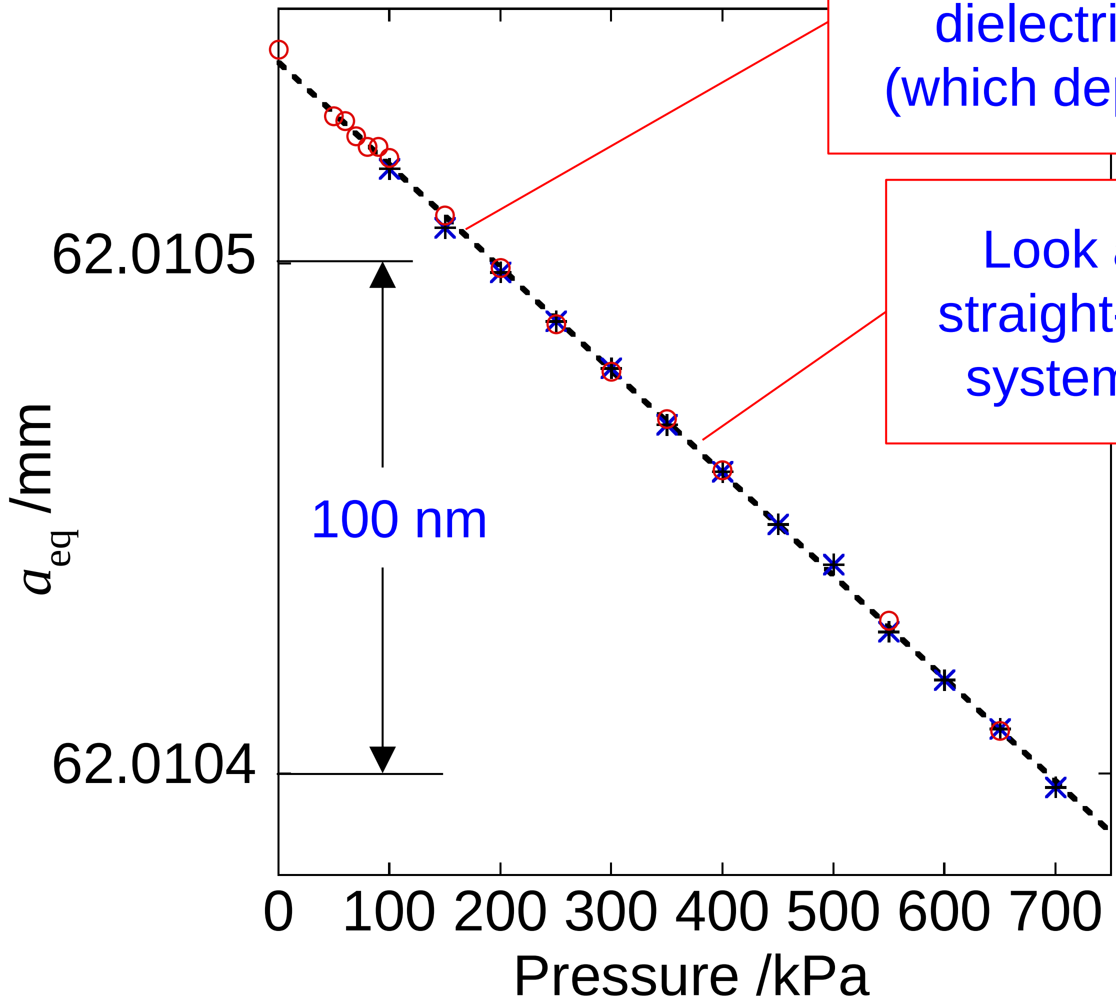
$u(k=1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a **2. Pressure** 3. Eigenvalues $\xi_{(0,n)}$ 4. Frequency Corrections $\Delta f_{(0,n)}$ 5. Temperature 6. Molar Mass	0.38 ppm

Low Pressure Measurements

- *Affects thermal boundary layer correction*
- Affects estimate of p^{-1} term



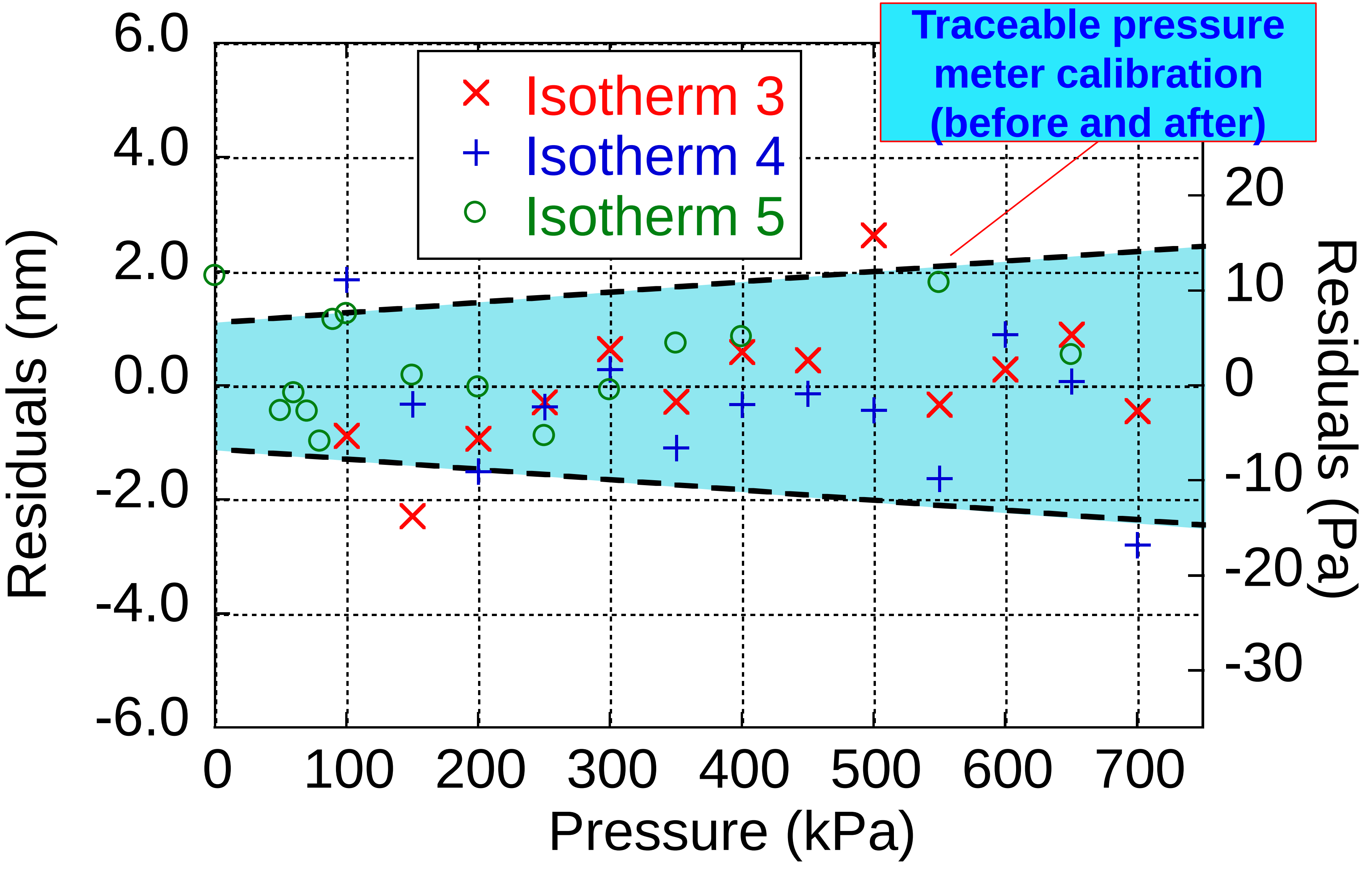
Microwave Radius Estimates ***at Pressure***



Data has been corrected for dielectric constant of gas (which depends on pressure)

Look at the residuals of a straight-line fit to this data for systematic pressure errors

Pressure



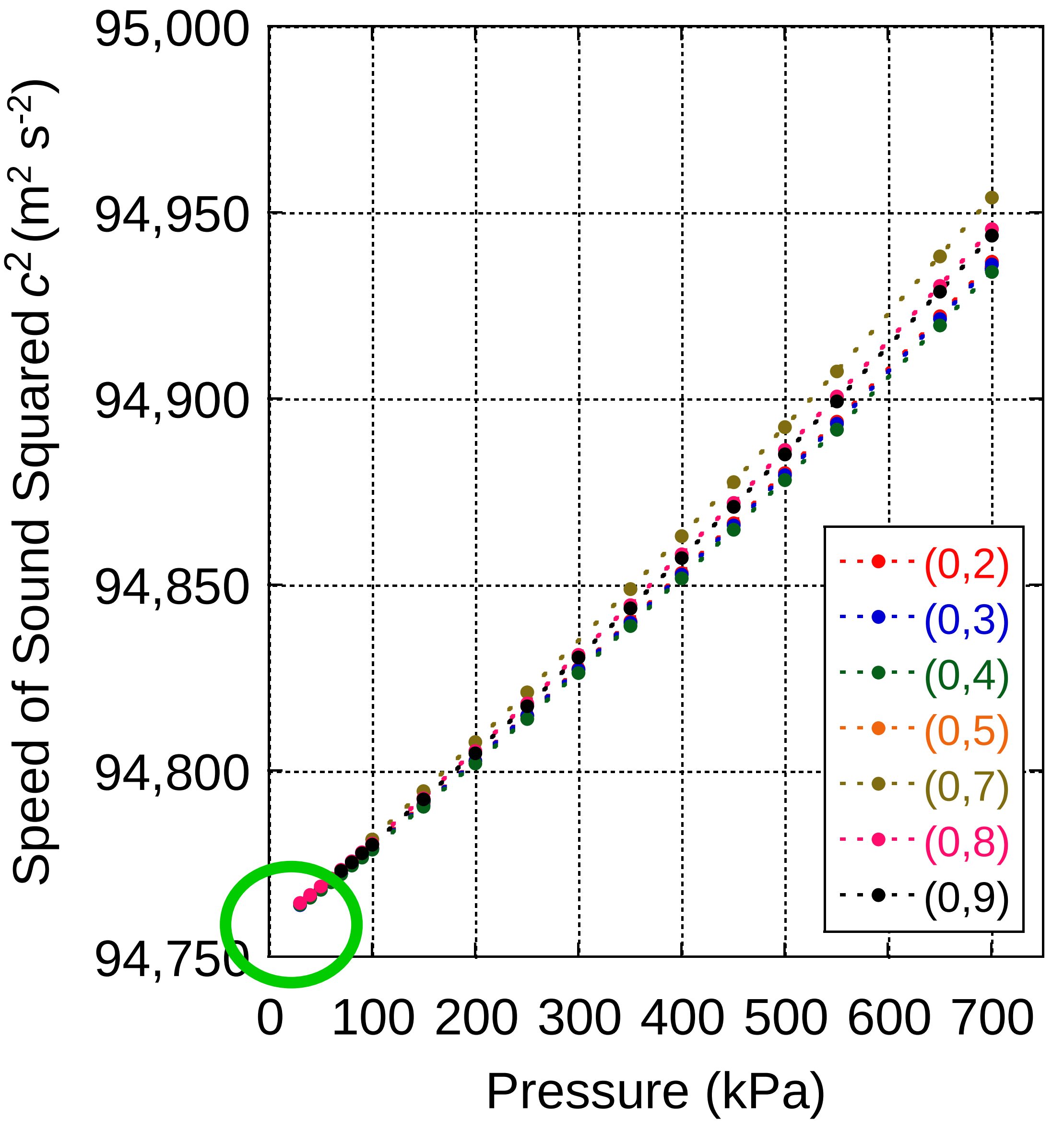
Traceable pressure meter calibration (before and after)
Isotherm 3
Isotherm 4
Isotherm 5
Residuals (nm)
Residuals (Pa)
Pressure (kPa)
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
20
10
0
-10
-20
-30
0
100
200
300
400
500
600
700

3: *How wrong could the eigenvalues be?*

$u(k=1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a	0.38 ppm
6.3 Pa	2. Pressure	0.11 ppm
	3. Eigenvalues $\boldsymbol{\xi_{(0,n)}}$	
	4. Frequency Corrections $\Delta f_{(0,n)}$	
	5. Temperature	
	6. Molar Mass	

Data for c^2

c_{EXP}^2



c_0^2

Data Model

After correction for Boundary Layer

From high pressure studies
Common to all modes

Low Pressure Speed of Sound Squared
Common to all modes

Virial Correction
Common to all modes

$$c_{EXP}^{2} - A_3 P^3 = c_0^2 + A_{-1} P^{-1} + A_1^n P + A_2 P^2$$

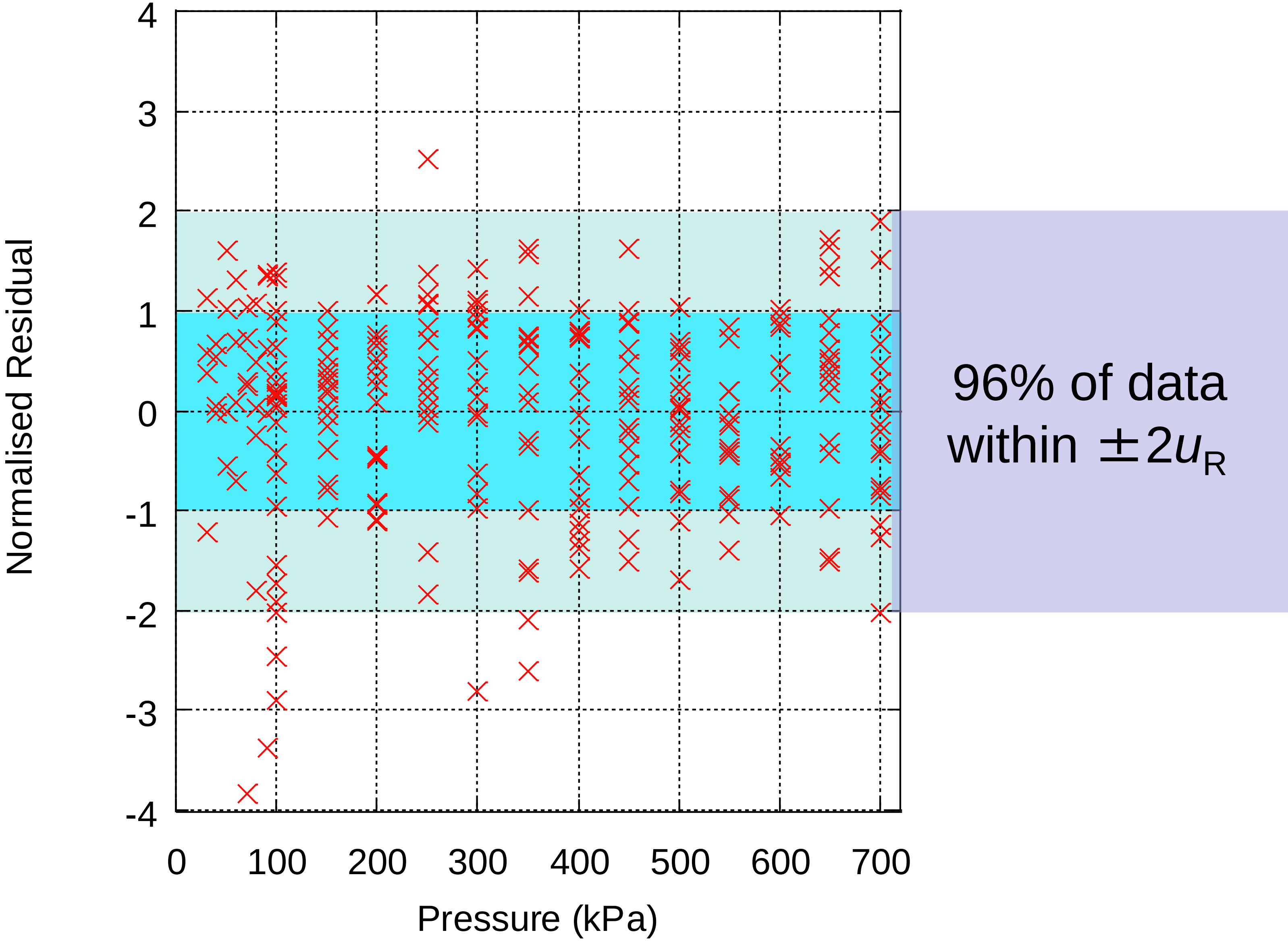
Experimental Estimates Function of pressure *P* and mode, *n*

'Accommodation' Correction to Boundary Layer
Common to all modes

Virial Correction
Common to all modes
'Shell' Correction
Varies with mode

Residuals of all data to fits

expressed in terms of standard uncertainty



Data Model

Low Pressure Speed of Sound Squared
Common to all modes

$$c_{EXP}^2 - A_3 P^3 = c_0^2 + A_{-1} P^{-1} + A_1^n P + A_2 P^2$$

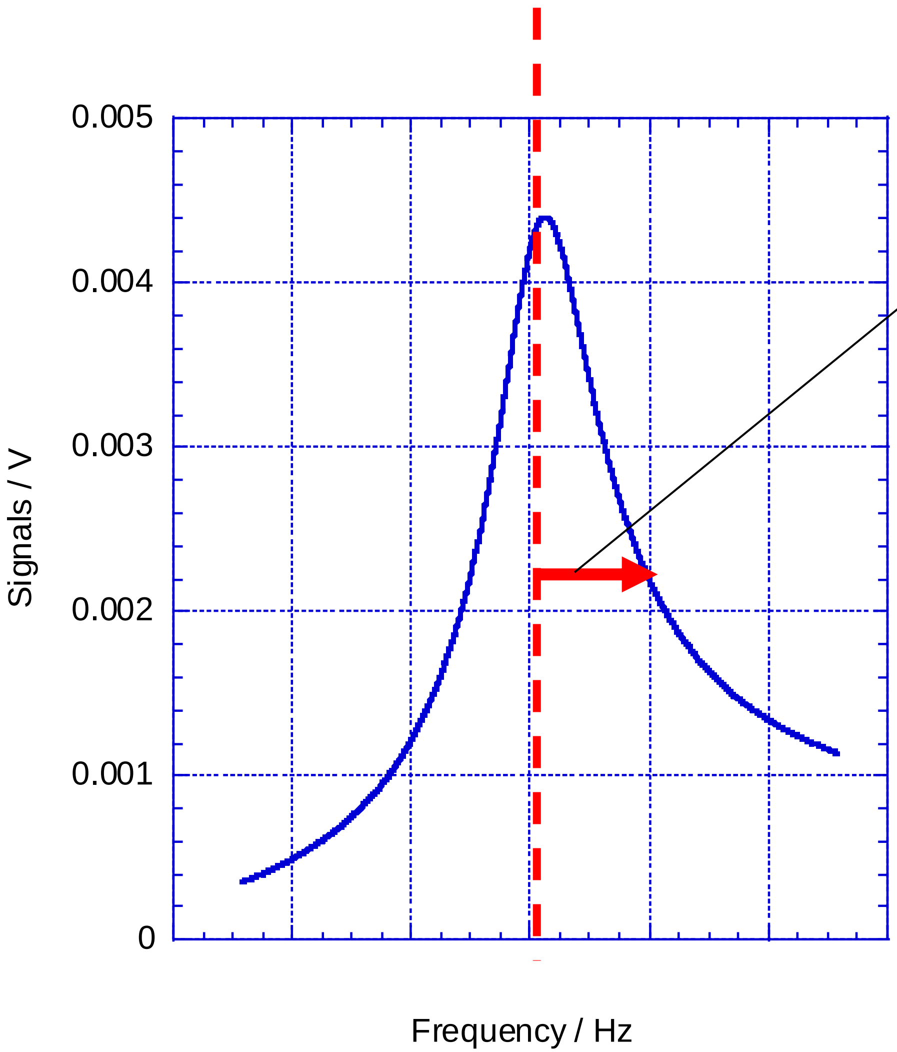
$$u(c_0^2) = 0.017\ \mathrm{m^2\,s^{-2}}$$

$$u_R(c_0^2) = 0.18 \times 10^{-6}$$

4: *How wrong could the Frequency Corrections be?*

$u(k=1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a	0.38 ppm
6.3 Pa	2. Pressure	0.11 ppm
	3. Eigenvalues $\xi_{(0,n)}$	0.18 ppm
	4. Frequency Corrections $\Delta f_{(0,n)}$	
	5. Temperature	
	6. Molar Mass	

Central frequency



Half-Width should be *exactly* what we expect

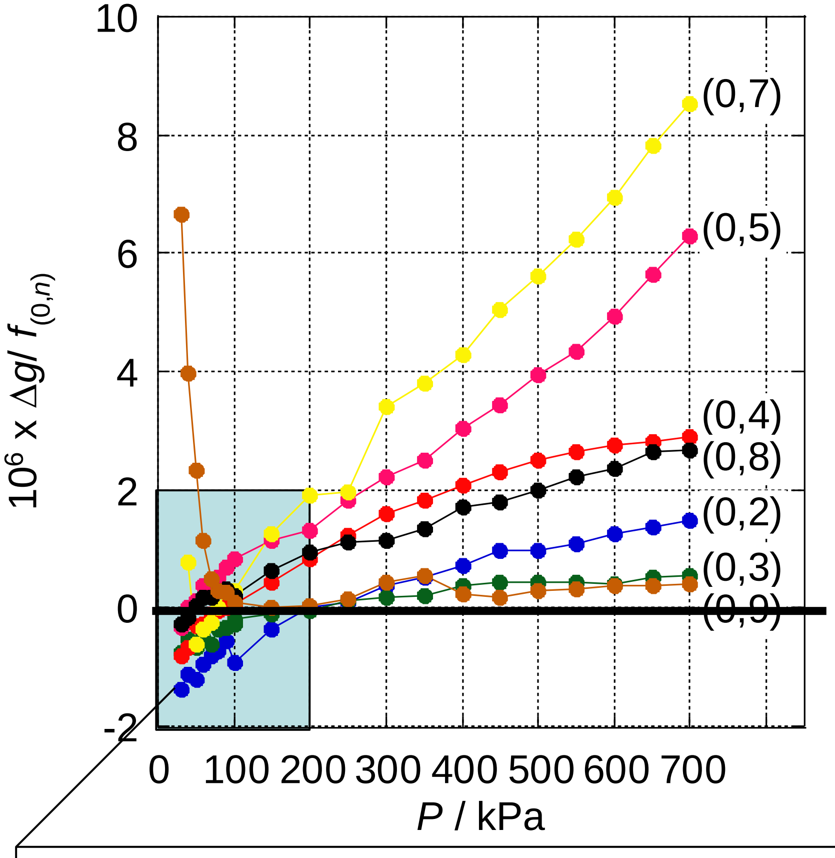
When f_0 =3548.8095 Hz

expected width = 2.864 Hz

measured width = 2.858 Hz

Half-Width (*Experiment – Theory*)

Parts per million of resonance frequency



f_0 =3548.8095 Hz

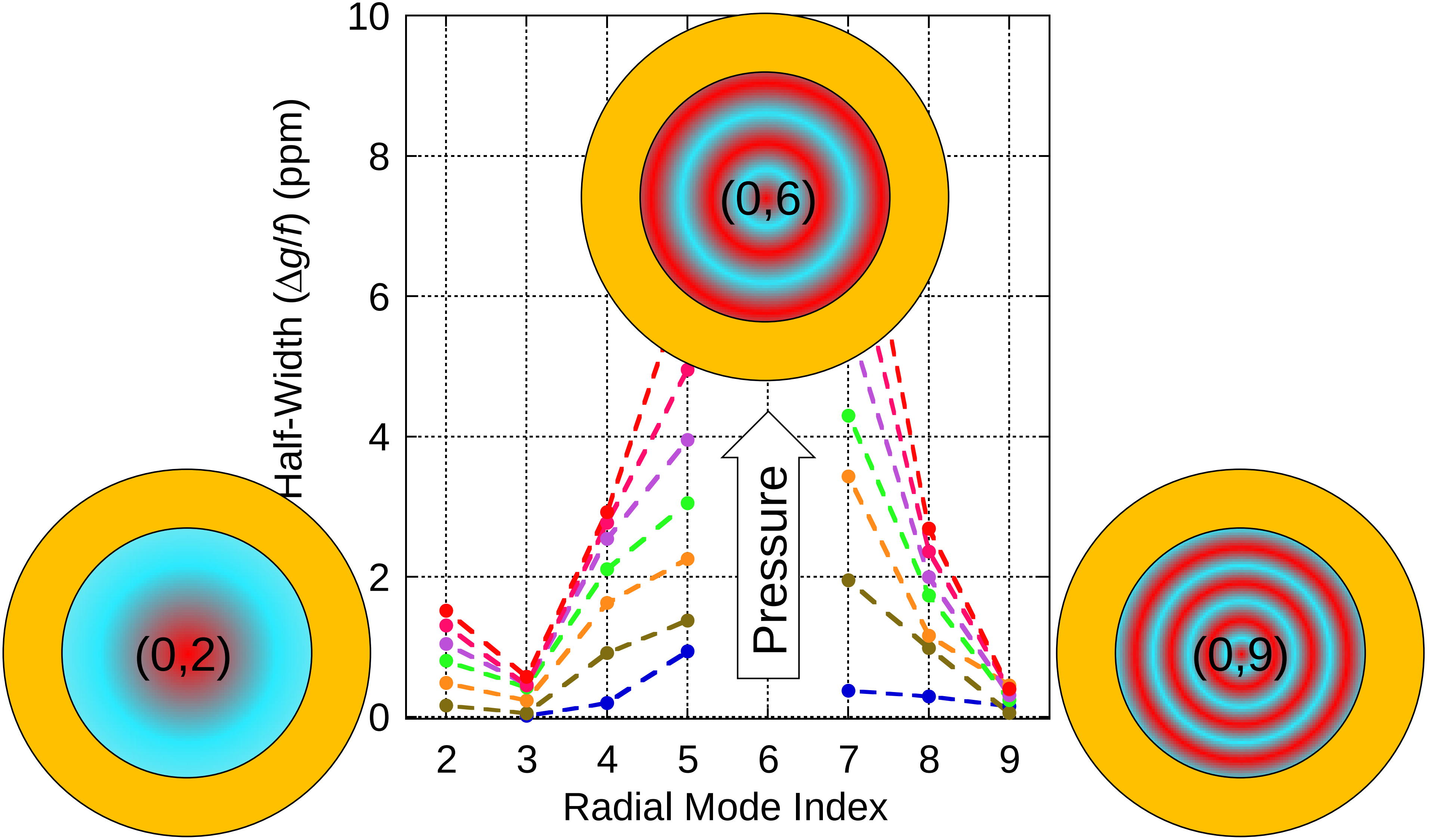
expected width = 2.864 Hz

measured width = 2.858 Hz

Half-Width (*Experiment – Theory*)

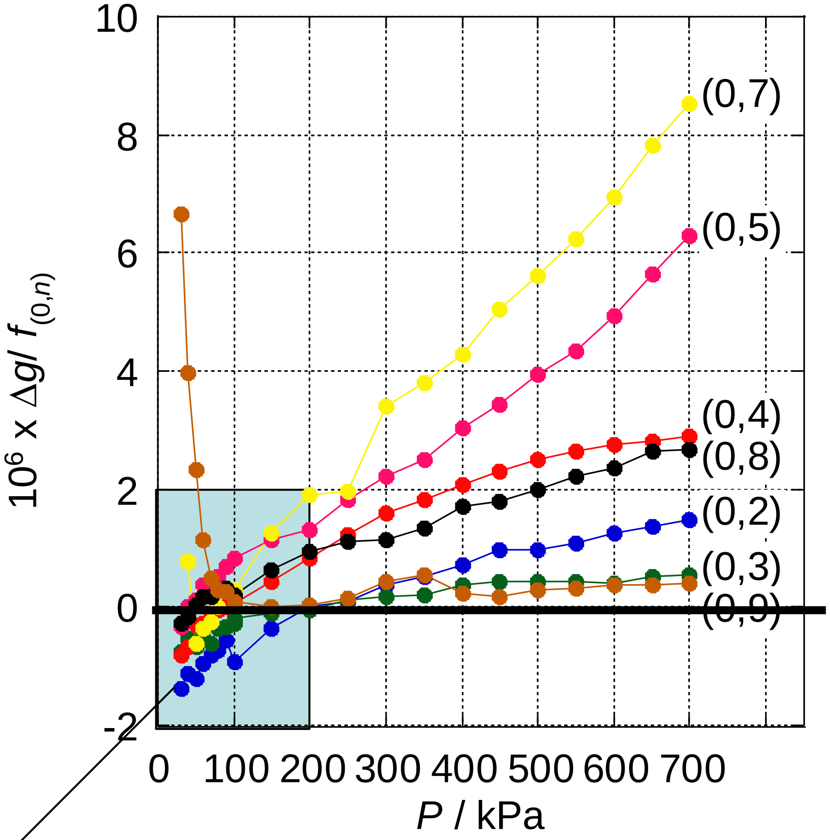
Shell Interaction

Shows the effect of mode and pressure



Half-Width (*Experiment – Theory*)

Parts per million of resonance frequency

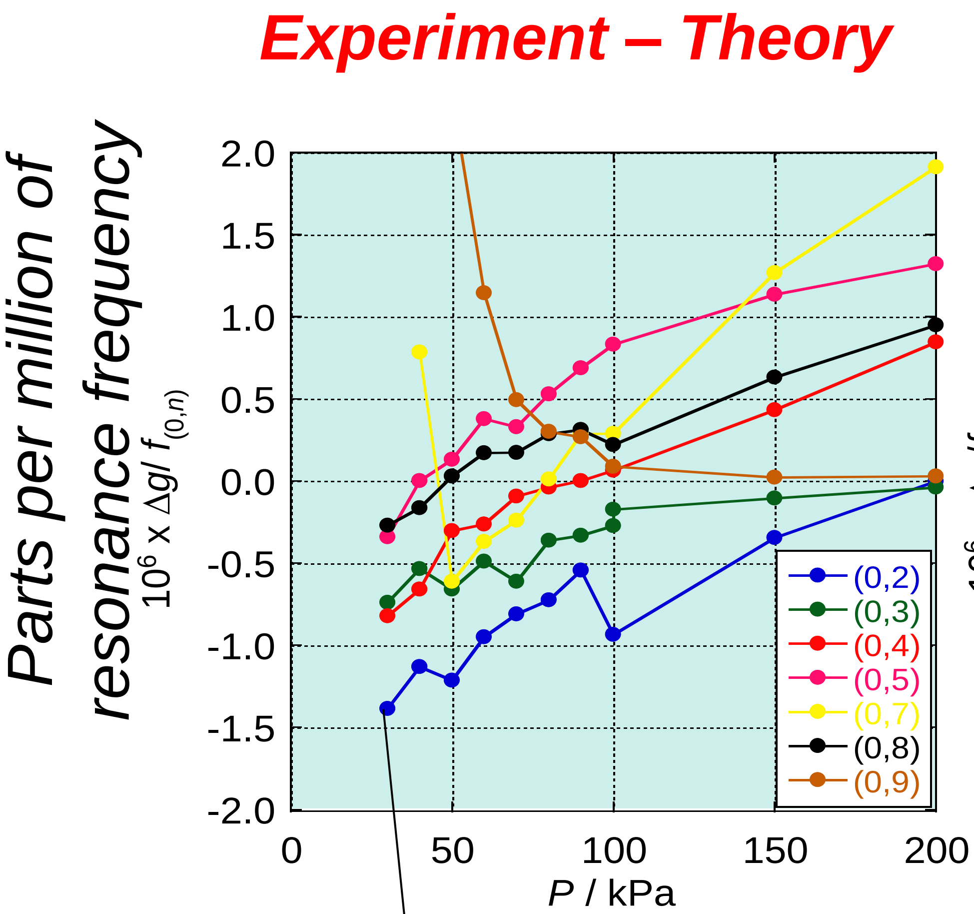


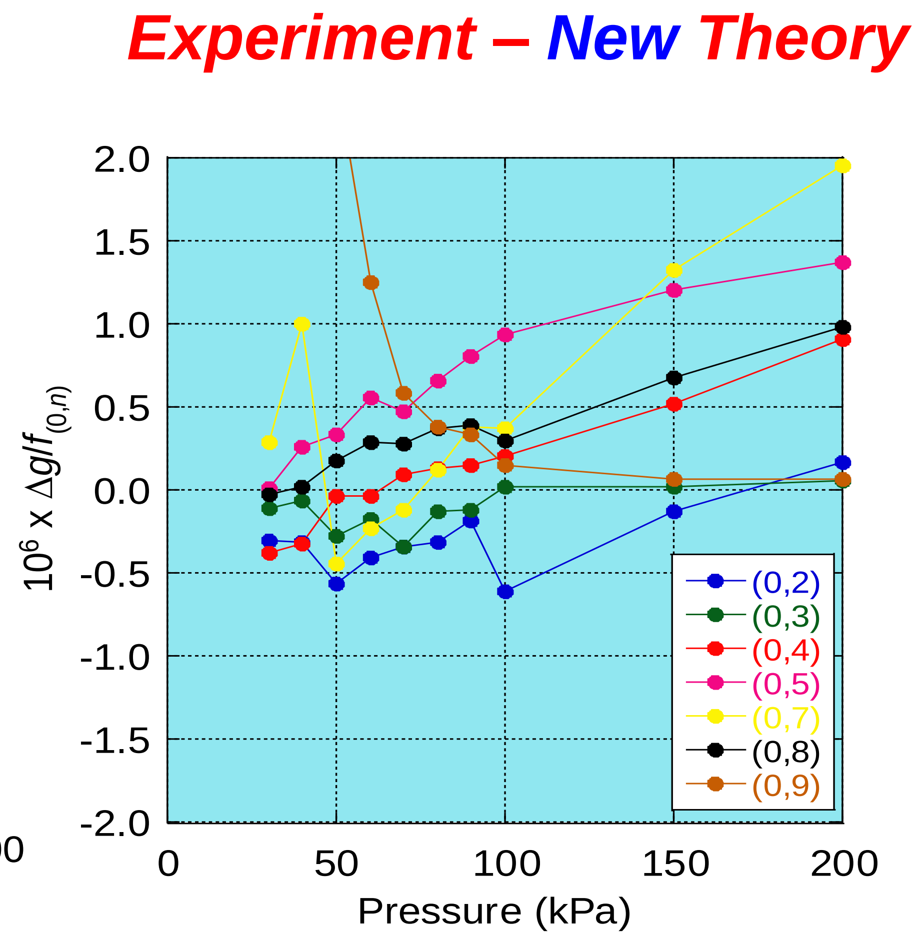
f_0 =3548.8095 Hz

expected width = 2.864 Hz

measured width = 2.858 Hz

Half-Width



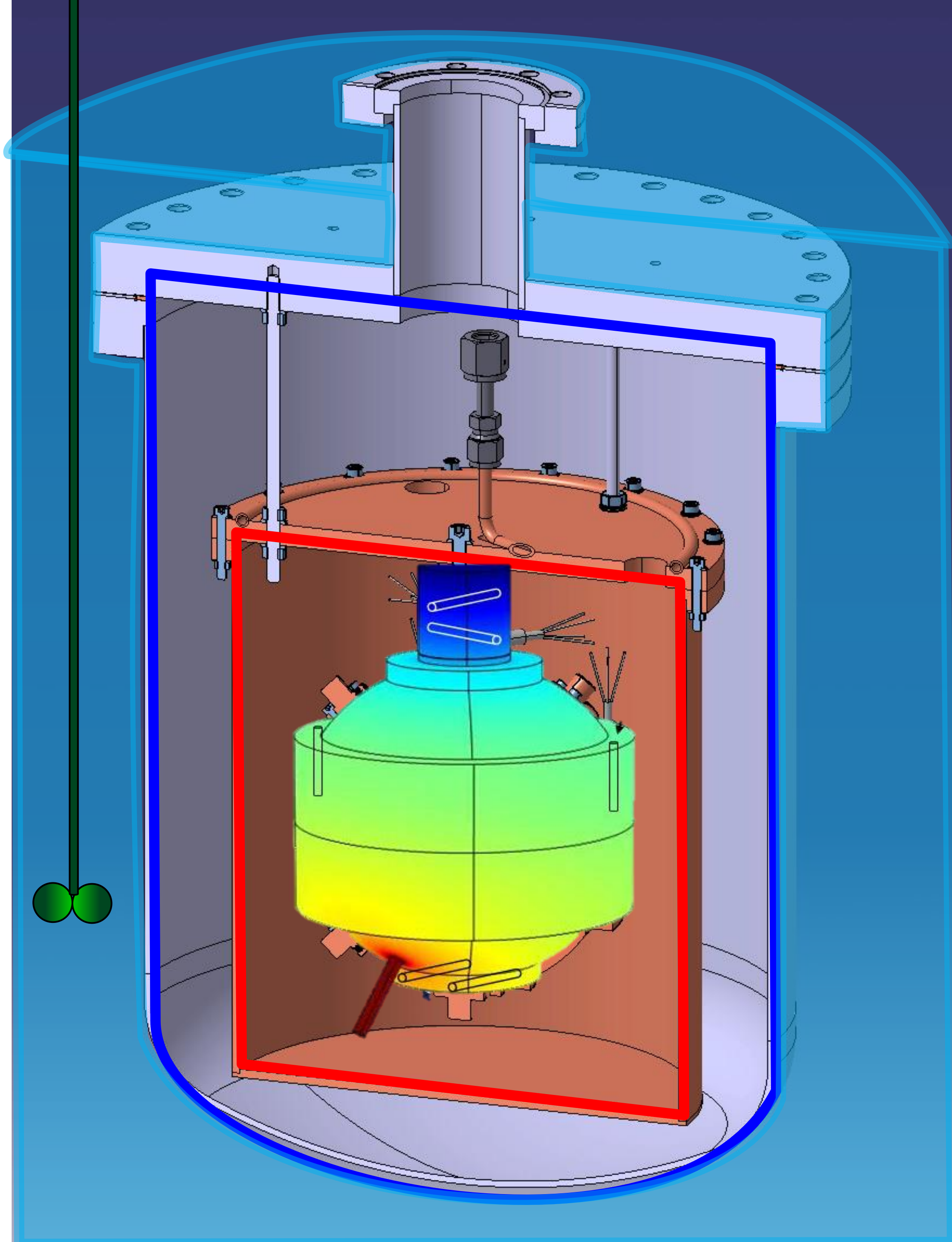


f_0 =3548.8095 Hz
expected width = 2.864 Hz
measured width = 2.858 Hz

5: *How wrong could the Temperature be?*

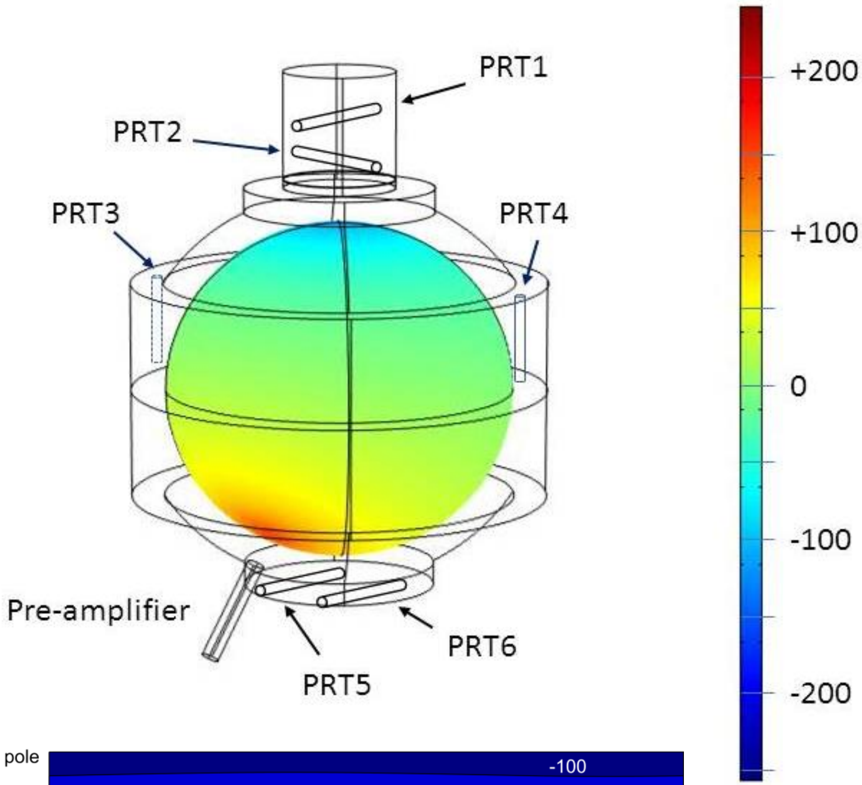
$u(k=1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a	0.38 ppm
6.3 Pa	2. Pressure	0.11 ppm
	3. Eigenvalues $\xi_{(0,n)}$	0.18 ppm
~0	4. Frequency Corrections $\Delta f_{(0,n)}$	
	5. Temperature	
	6. Molar Mass	

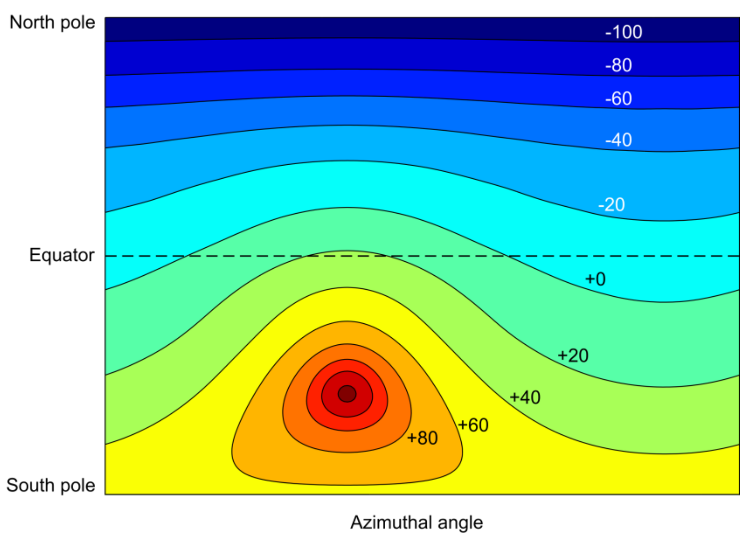
Temperature Gradient



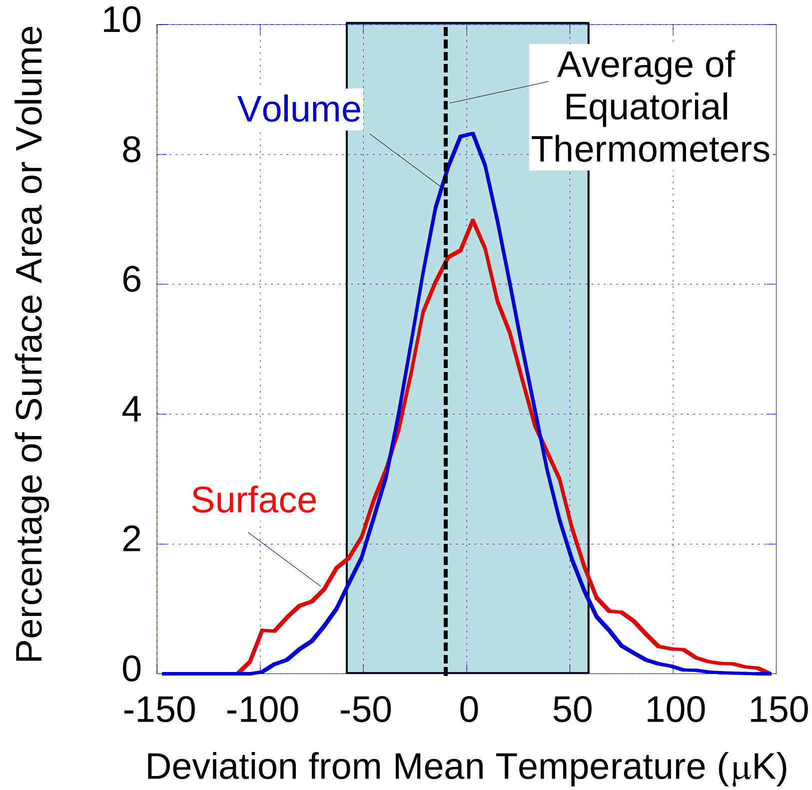
Temperature gradient was
±91 μK about equator

Temperature





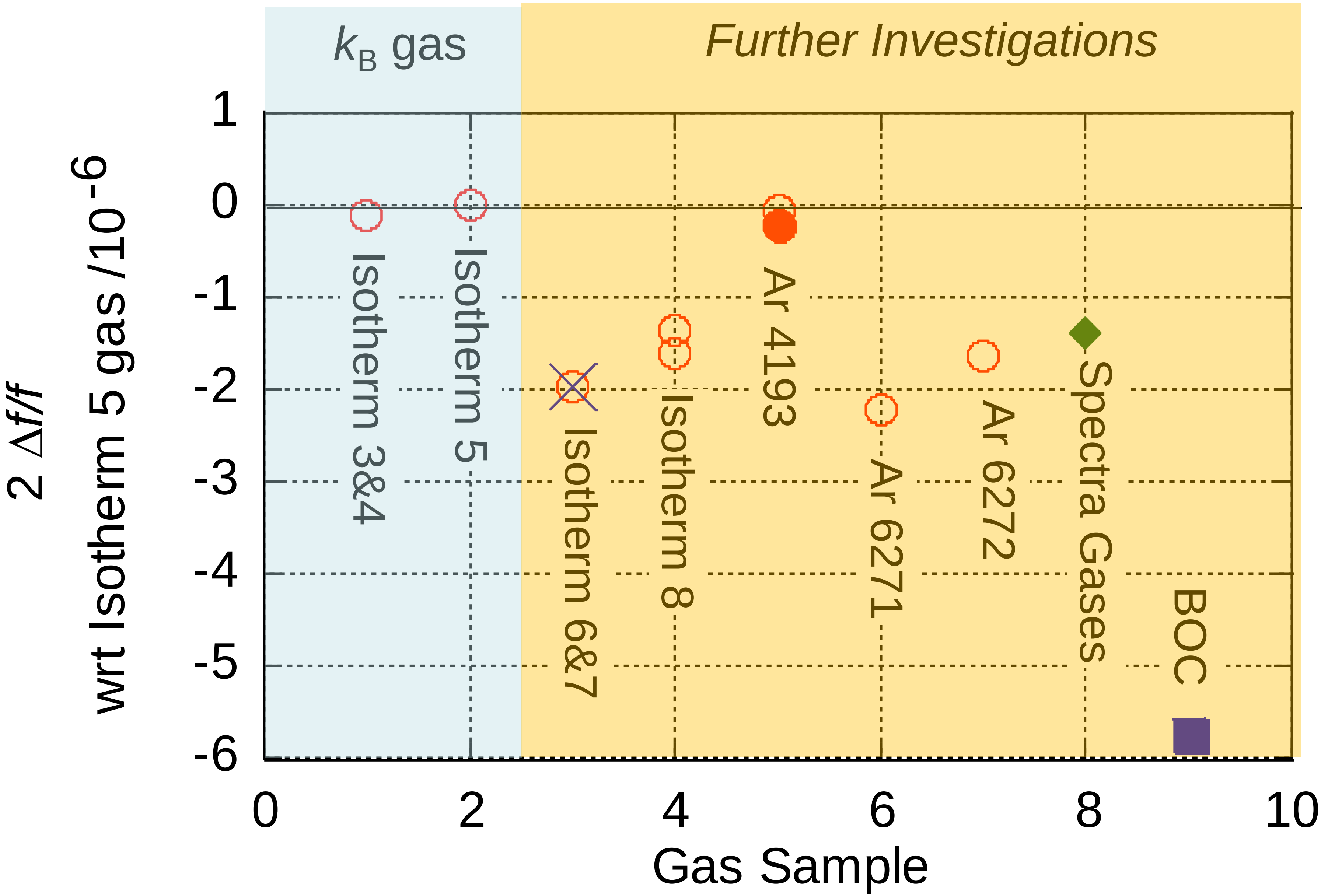
We modelled the temperature at the *inner* surface of the sphere, and then in the gas.



6: *How wrong could the Molar Mass be?*

$u(k=1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a	0.38 ppm
6.3 Pa	2. Pressure	0.11 ppm
	3. Eigenvalues $\xi_{(0,n)}$	0.18 ppm
~0	4. Frequency Corrections $\Delta f_{(0,n)}$	
0.099 mK	5. Temperature	0.364 ppm
	6. Molar Mass	

Molar Mass Differences from Isotherm 5 Gas

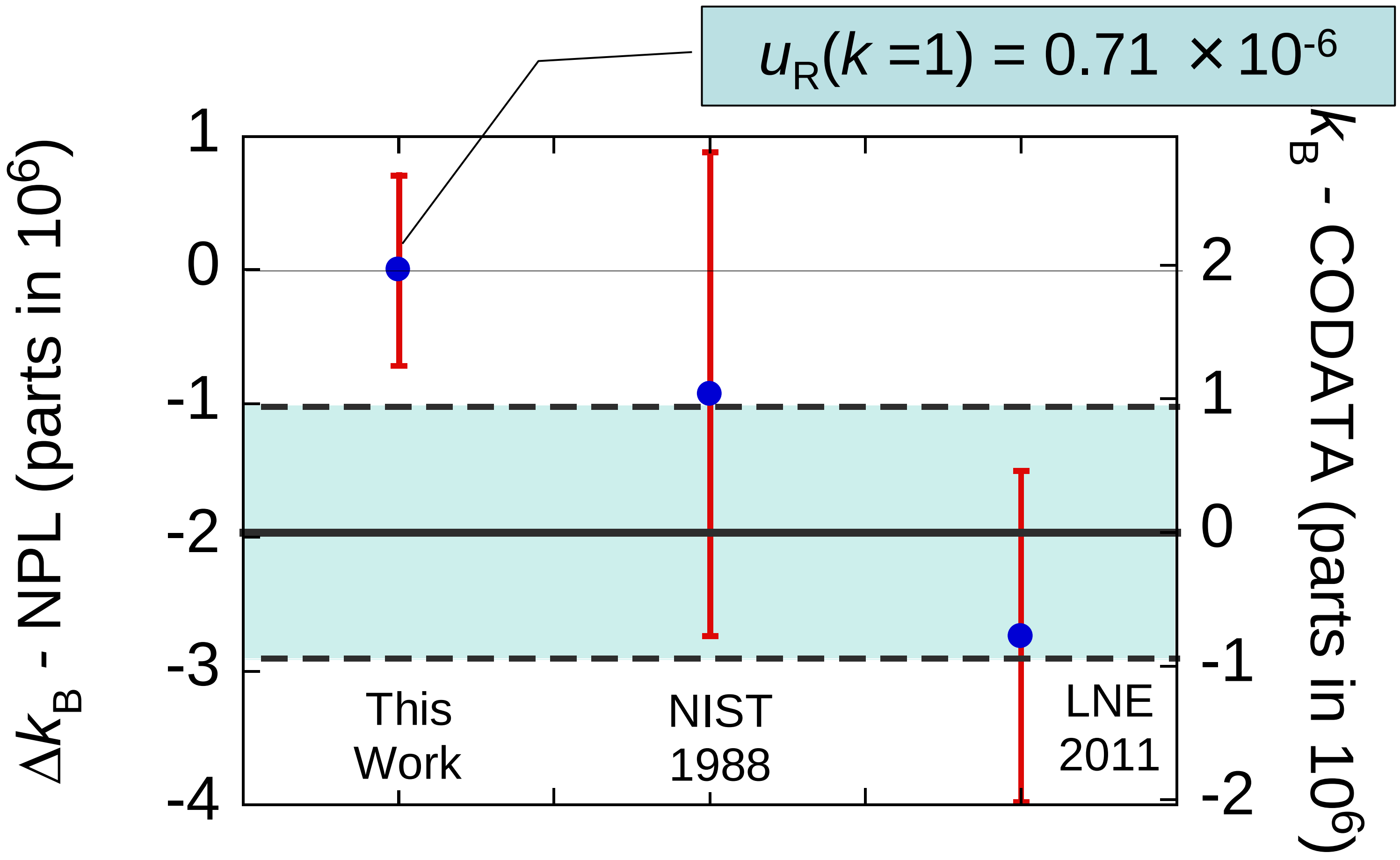


6: *How wrong could the Molar Mass be?*

$u(k{=}1)$		$u(k_B)$
11.7 nm	1. Resonator radius, a	0.38 ppm
6.3 Pa	2. Pressure	0.11 ppm
	3. Eigenvalues $\xi_{(0,n)}$	0.18 ppm
~0	4. Frequency Corrections $\Delta f_{(0,n)}$	
0.099 mK	5. Temperature	0.364 ppm
0.390 ppm	**6. Molar Mass**	0.390 ppm

Traceable to isotopic composition of atmospheric argon

Uncertainty



Have we learned anything since 2013 that could shed light on the LNE-CNAM-NPL discrepancy?

Reflections on the NPL-2013 Estimate of the Boltzmann Constant

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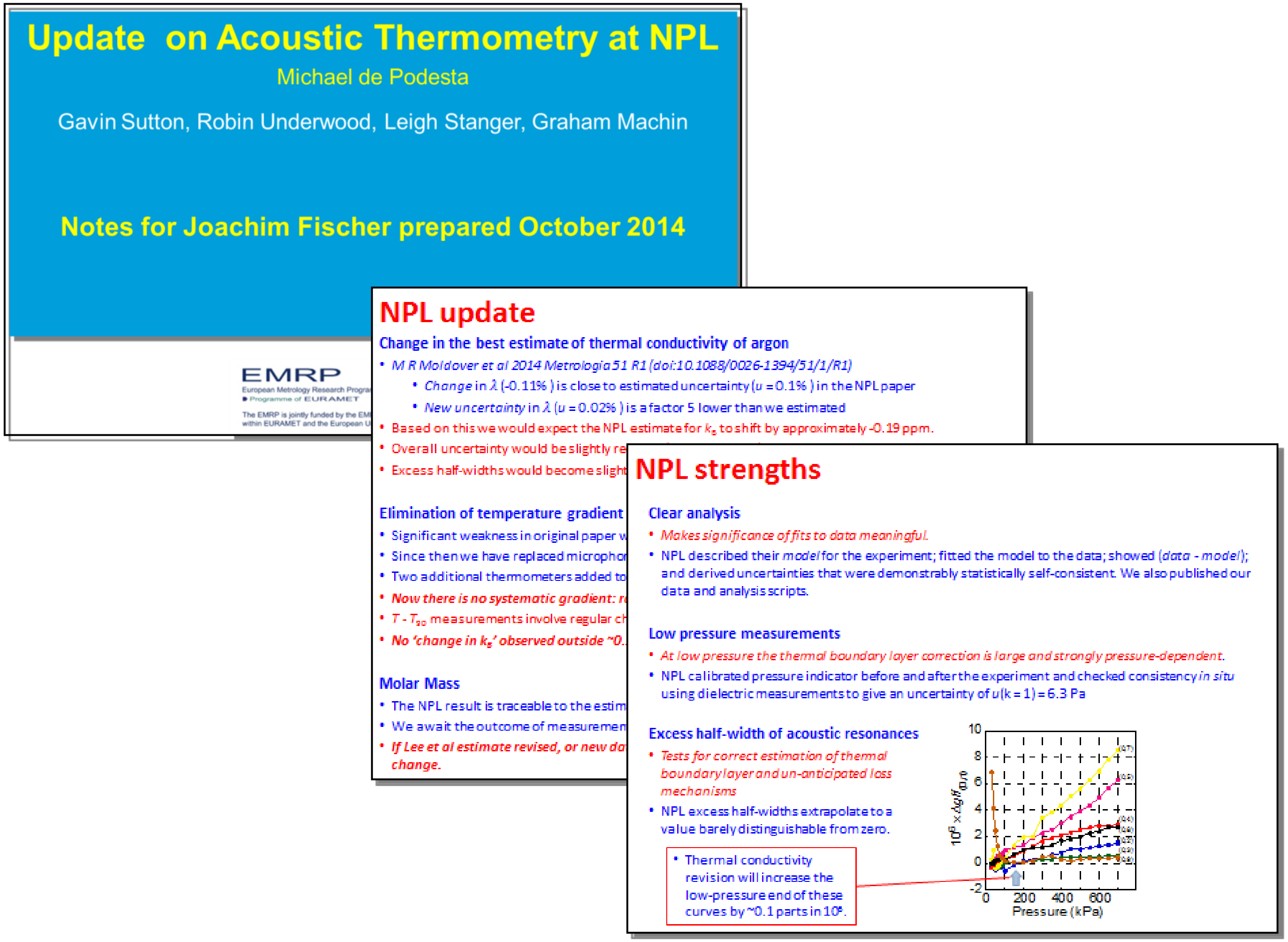
CODATA November 2015



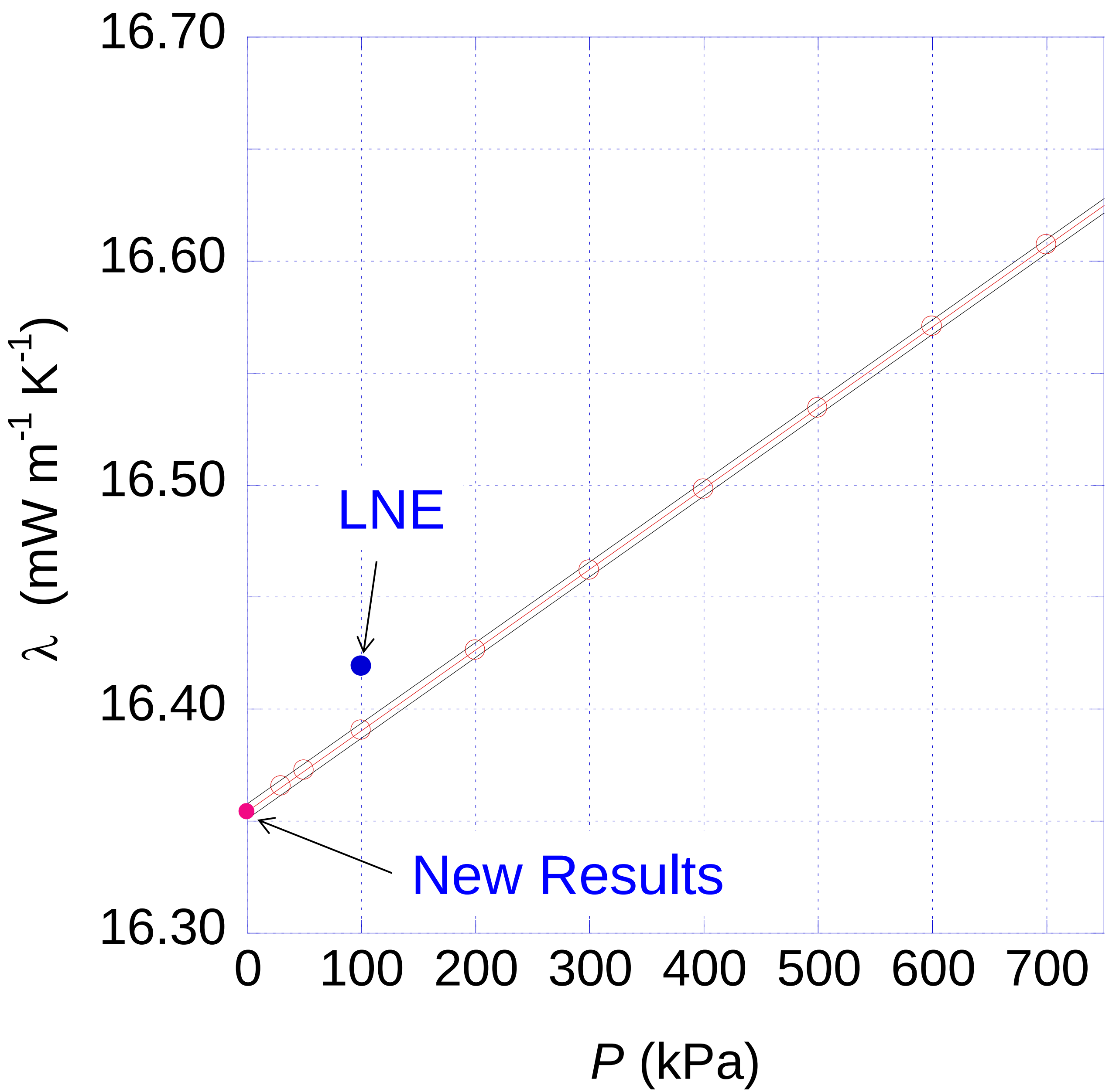
NPL Update
February 2015

1. *New estimate for thermal conductivity*
2. *Temperature Gradients*
3. *Argon Isotopic Molar Mass*

New estimate for thermal conductivity of argon

- $\Delta\lambda$ (-0.11%) is close to estimated uncertainty
- (u = 0.1%) NPL paper

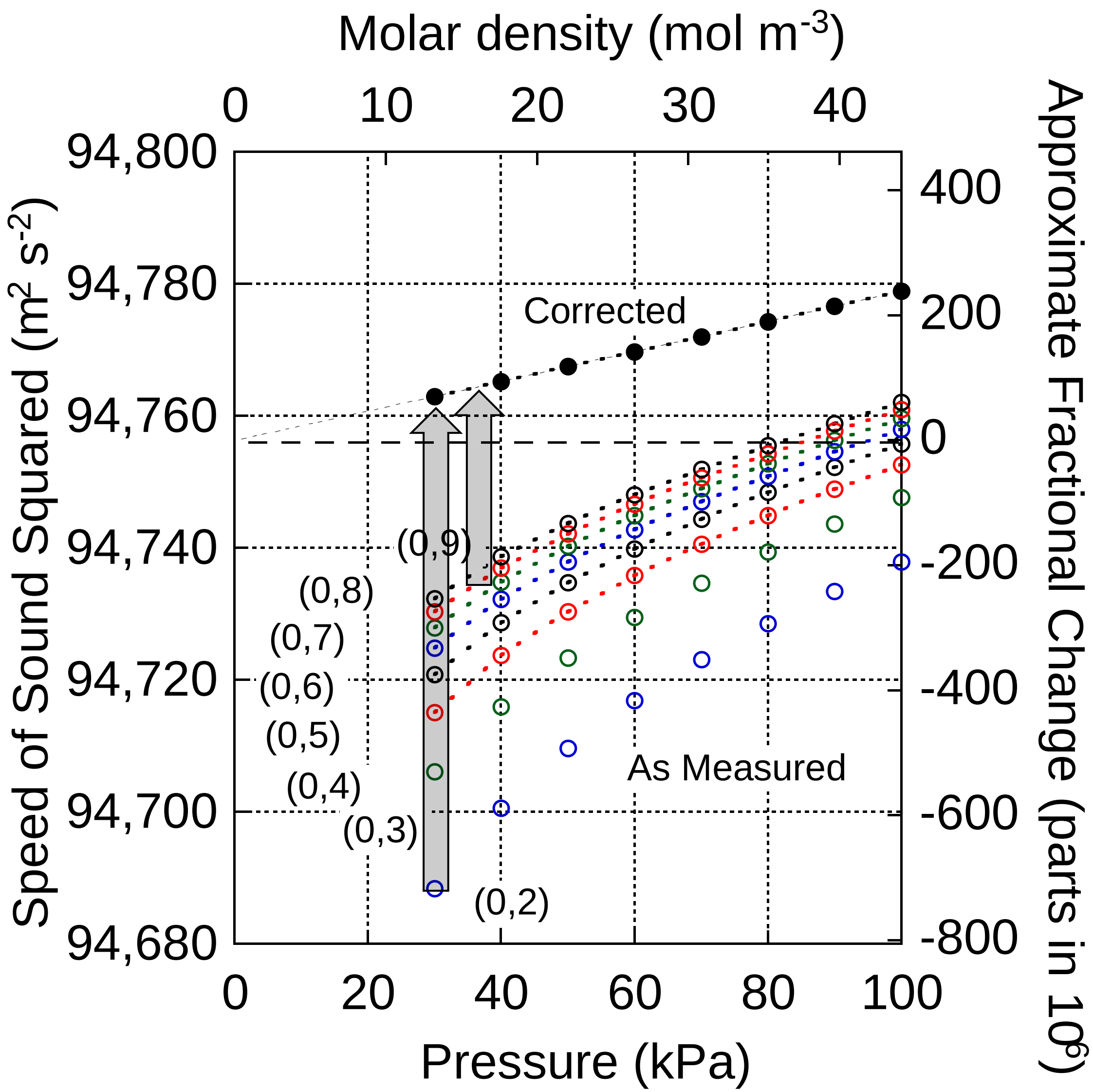
New uncertainty in λ (u = 0.02%) is a factor 5 lower than we estimated



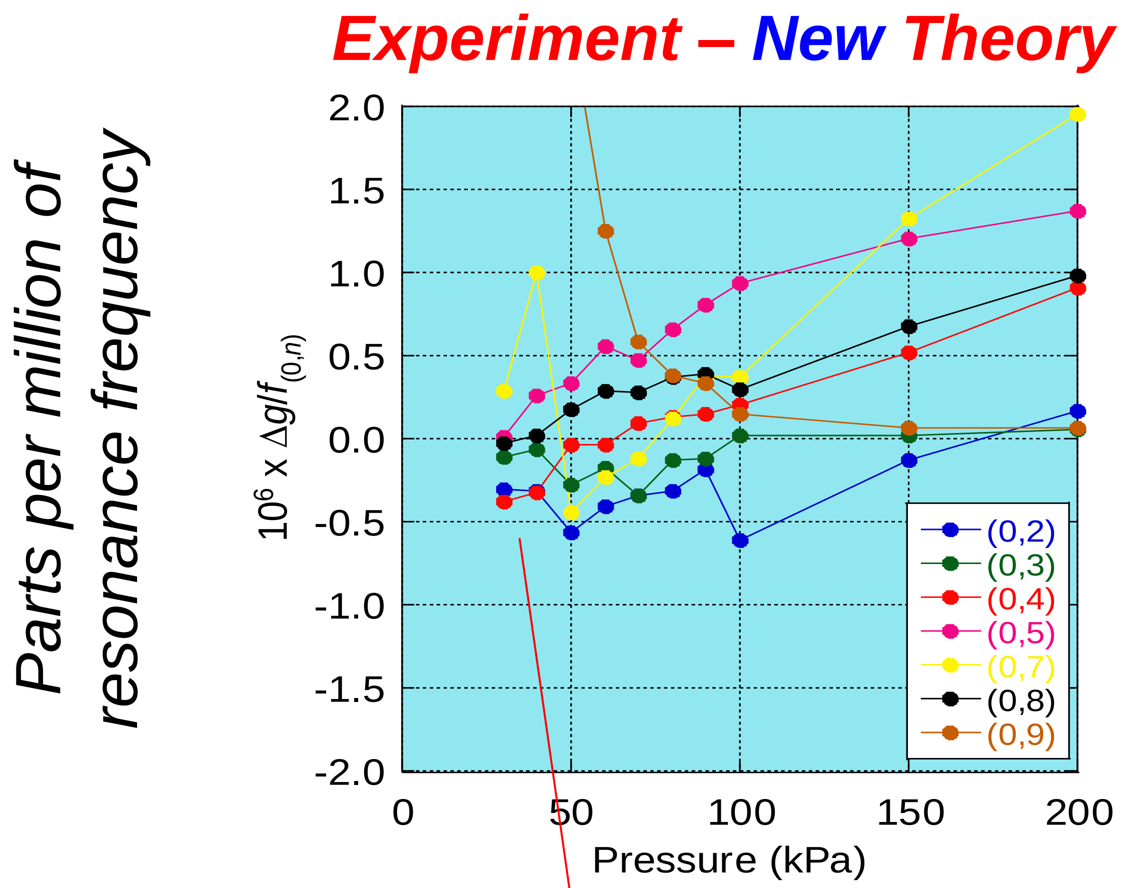
New estimate for thermal conductivity of argon

- Δk_B = -0.19 ppm.
- u_R ~ 0.69 x 10^{-6}

- Excess half-widths increased
  - $\Delta g/f$ ~ + 0.1 x 10^{-6} @100 kPa



New estimate for thermal conductivity of argon



- Thermal conductivity revision will increase the low-pressure end of these curves by ~0.1 parts in 10^6.

NPL Update
February 2015

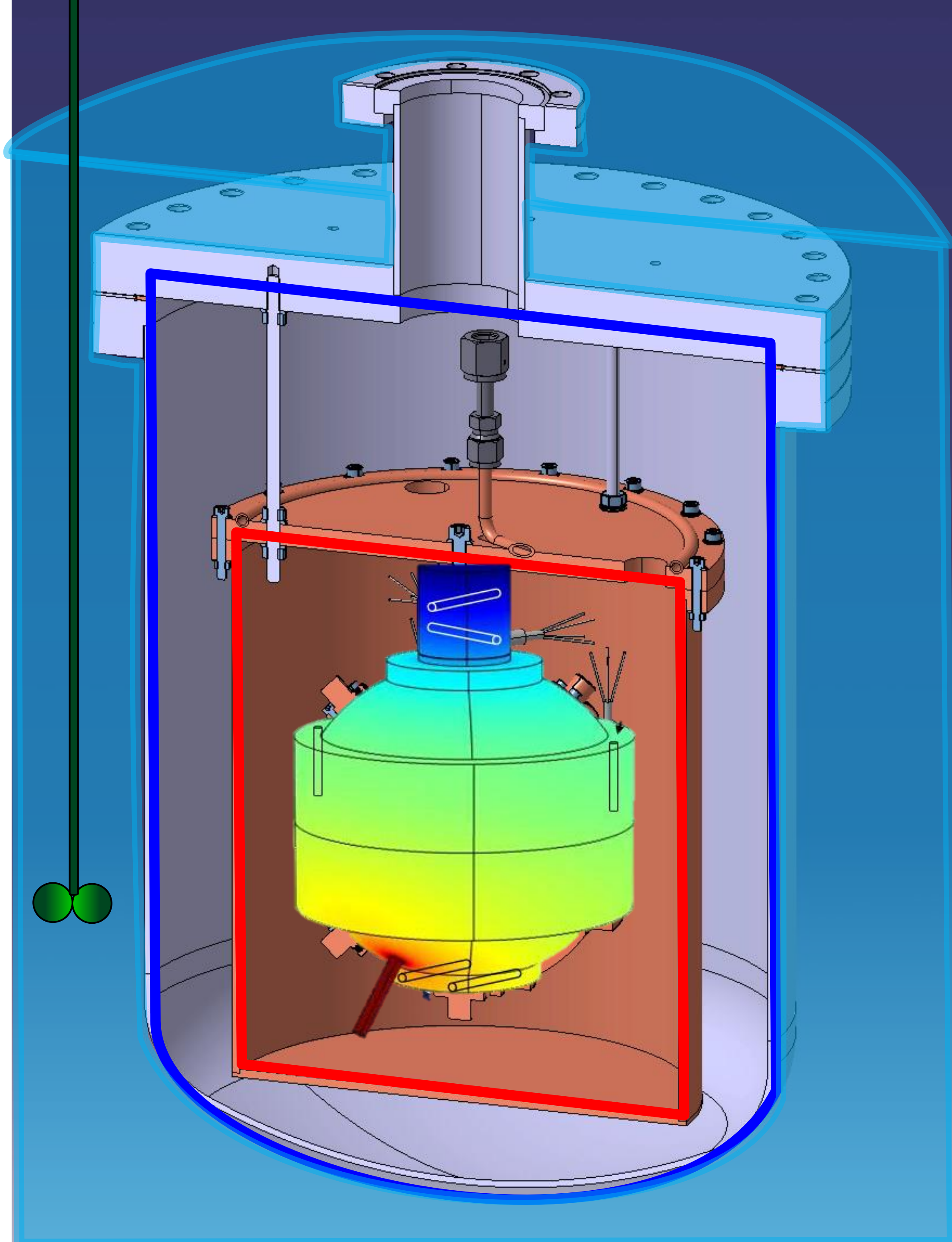
1. *New estimate for thermal conductivity*
2. *Temperature Gradients*
3. *Argon Isotopic Molar Mass*

Temperature Gradient

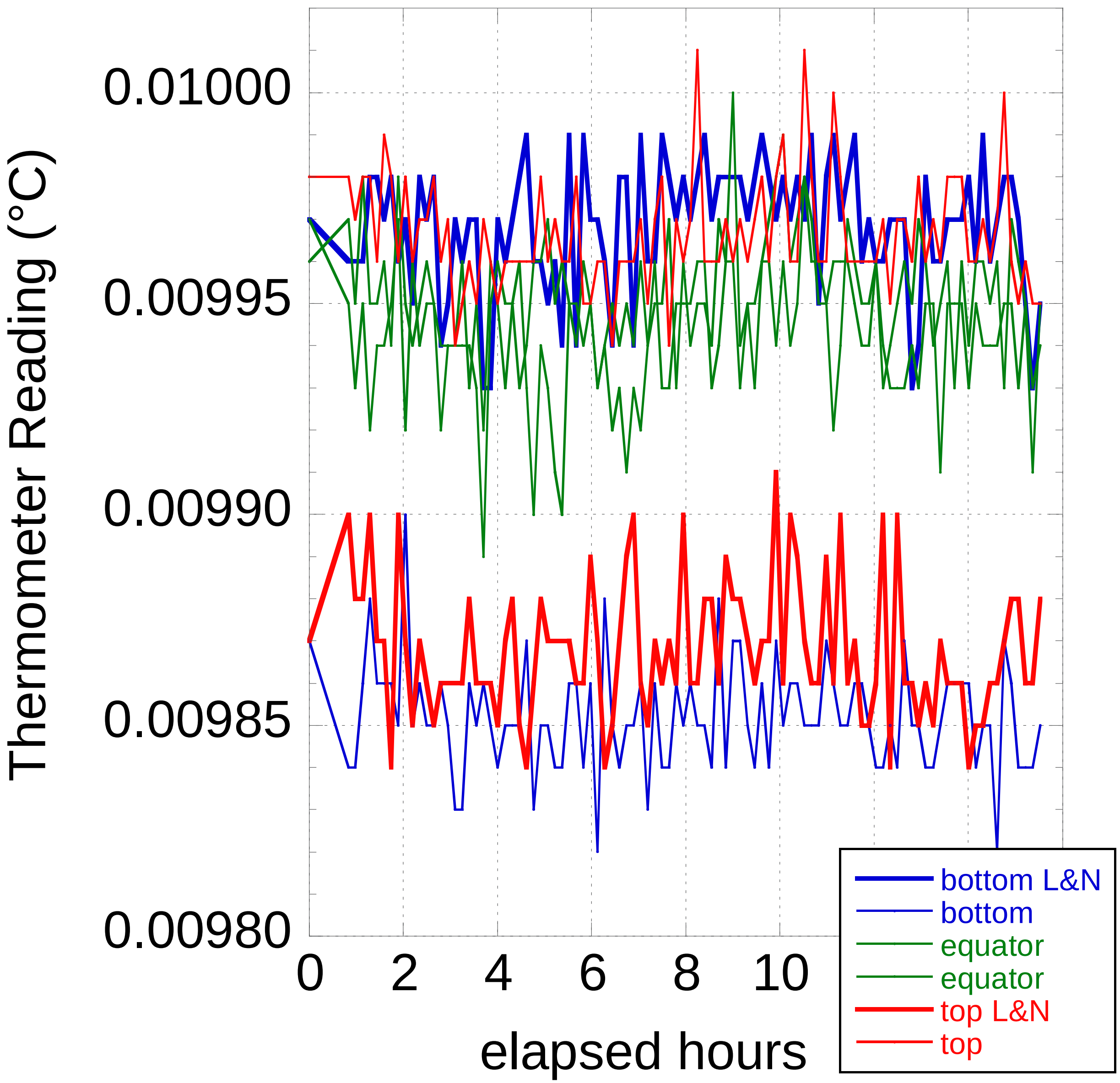
- Temperature gradient was ±91 µK about equator

Since 2013

- Replaced microphones
- Moved the pre-amplifier.
- Two additional thermometers added to sphere (6 in all).
- ***No systematic gradient: (max – min) is ±58 µK***
- ***No 'change in k_B' (u~0.3 ppm).***



Temperature Gradient

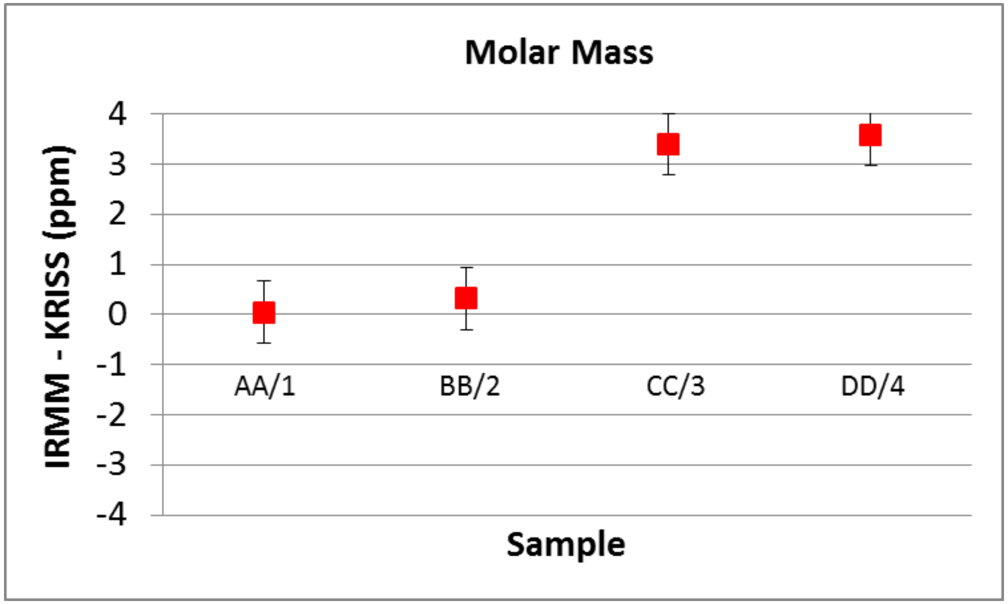


NPL Update
February 2015

1. *New estimate for thermal conductivity*
2. *Temperature Gradients*
3. *Argon Isotopic Molar Mass*

NPL update#3: Molar Mass

- IRMM and KRISS have made gravimetrically traceable isotopic analyses
- Comparison between KRISS (2014) and IRMM (2009) analysis of the same samples



- ***No clear pattern of agreement or disagreement between KRISS and IRMM***

Reflections on the NPL-2013 Estimate of the Boltzmann Constant

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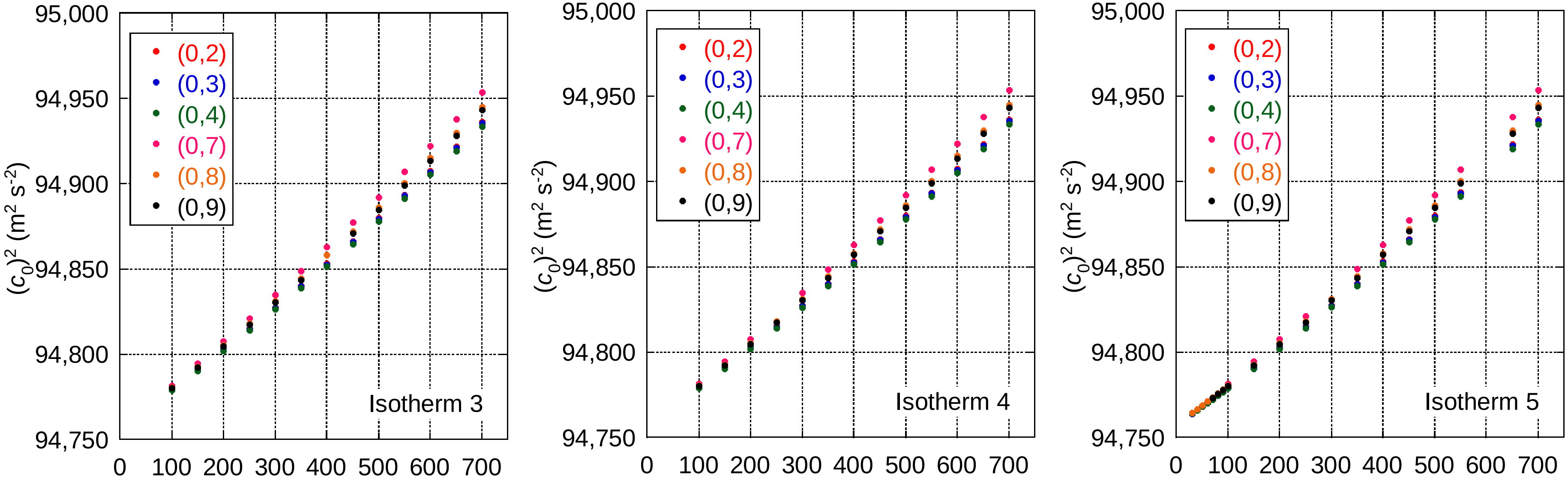
Self-Consistent Analysis

- *Makes the significance of fits to data meaningful.*
  - Data
  - Type A uncertainty of Data
  - Model
  - Fit the model to the data
  - Look at residuals (data - model)
  - Show data and model are self-consistent
- We published our data and analysis scripts.
- These have been independently checked



From high pressure studies
Common to all modes

Low Pressure Speed of Sound Squared
Common to all modes

Virial Correction
Common to all modes

$$c_{EXP}^2 - A_3 P^3 = c_0^2 + A_{-1} P^{-1} + A_1^n P + A_2 P^2$$

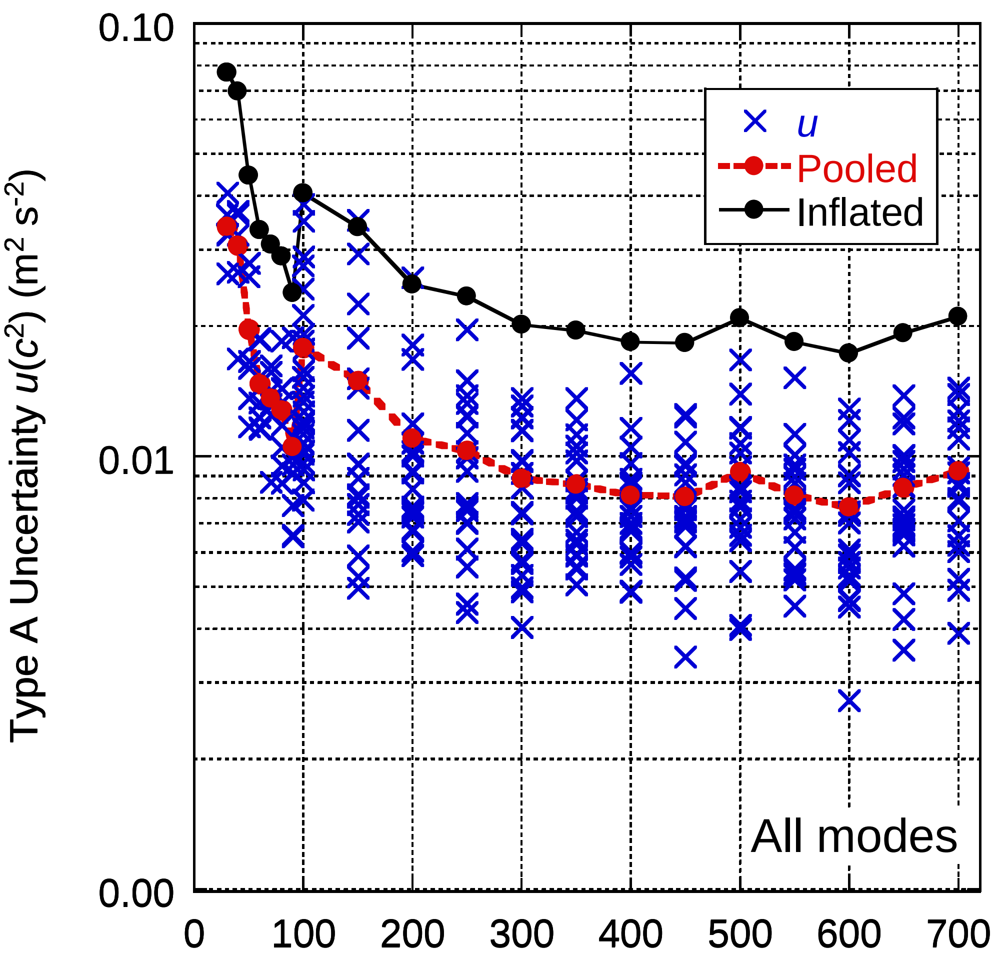
Experimental Estimates Function of pressure *P* and mode, *n*

'Accommodation' Correction to Boundary Layer
Common to all modes

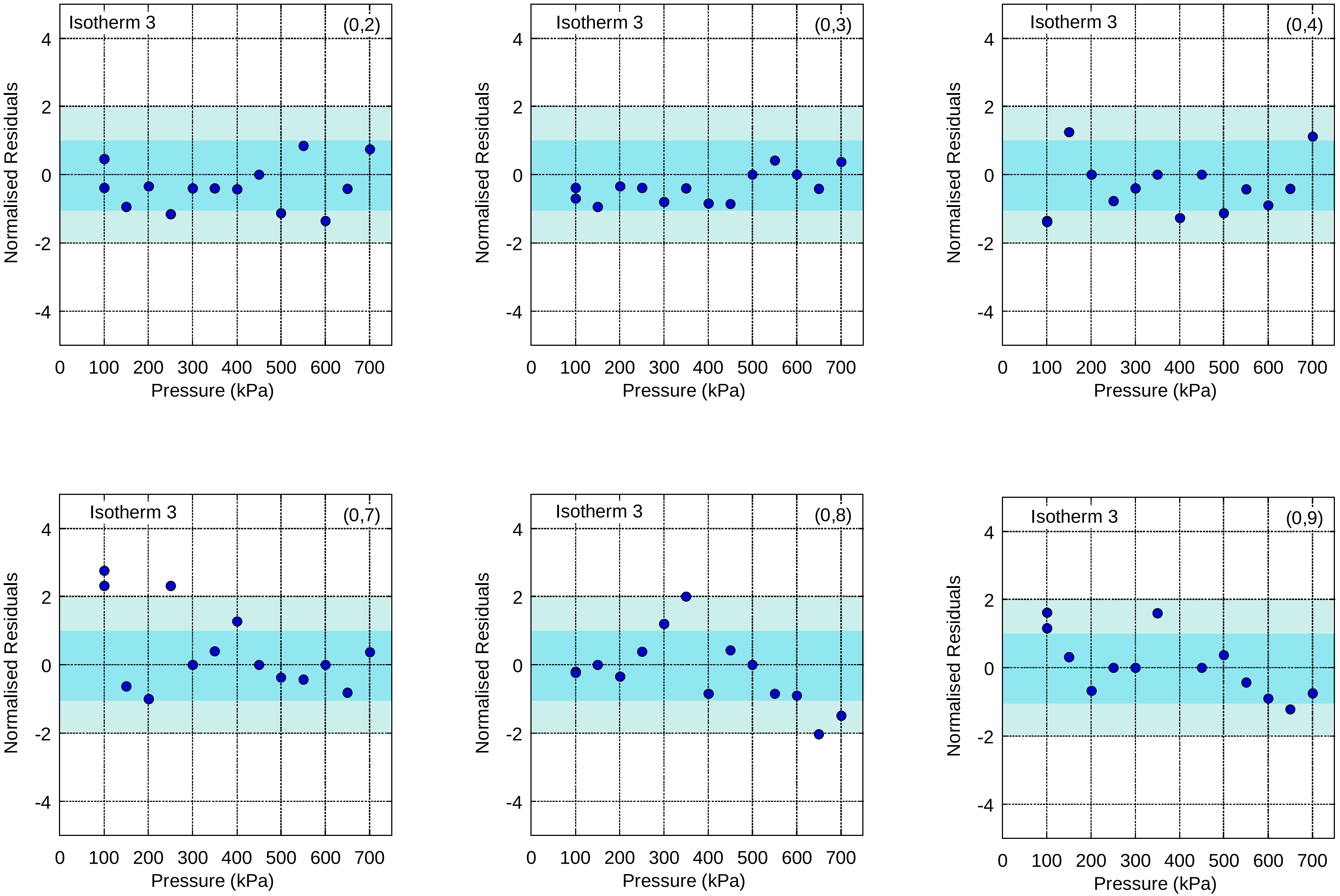
Virial Correction
Common to all modes
'Shell' Correction
Varies with mode

Type A Uncertainty

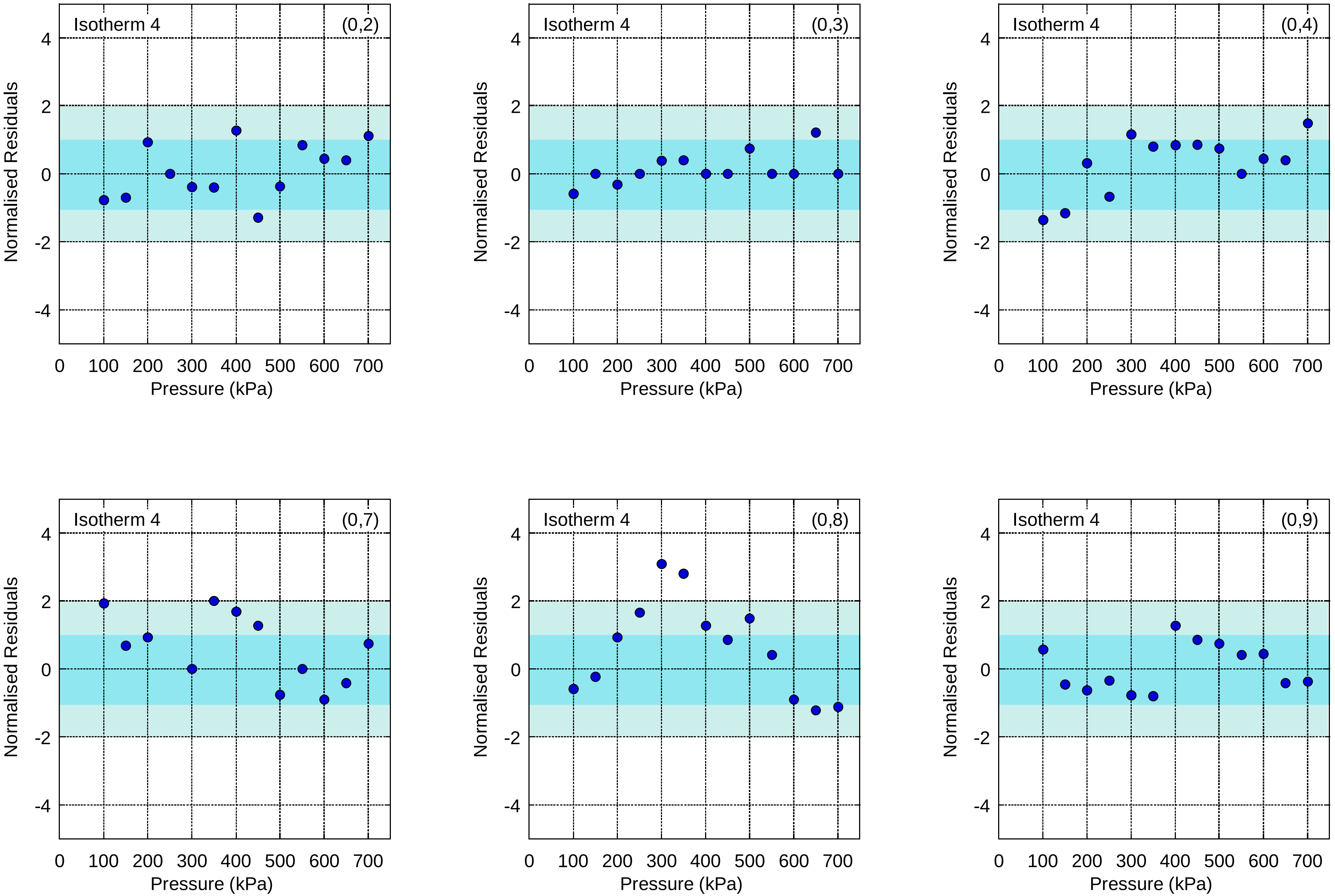
- *Estimated from repeats of a resonance acquisition.*
- *Use pooled uncertainty to weight the data used in the fit.*
- *Inflate uncertainty estimate*



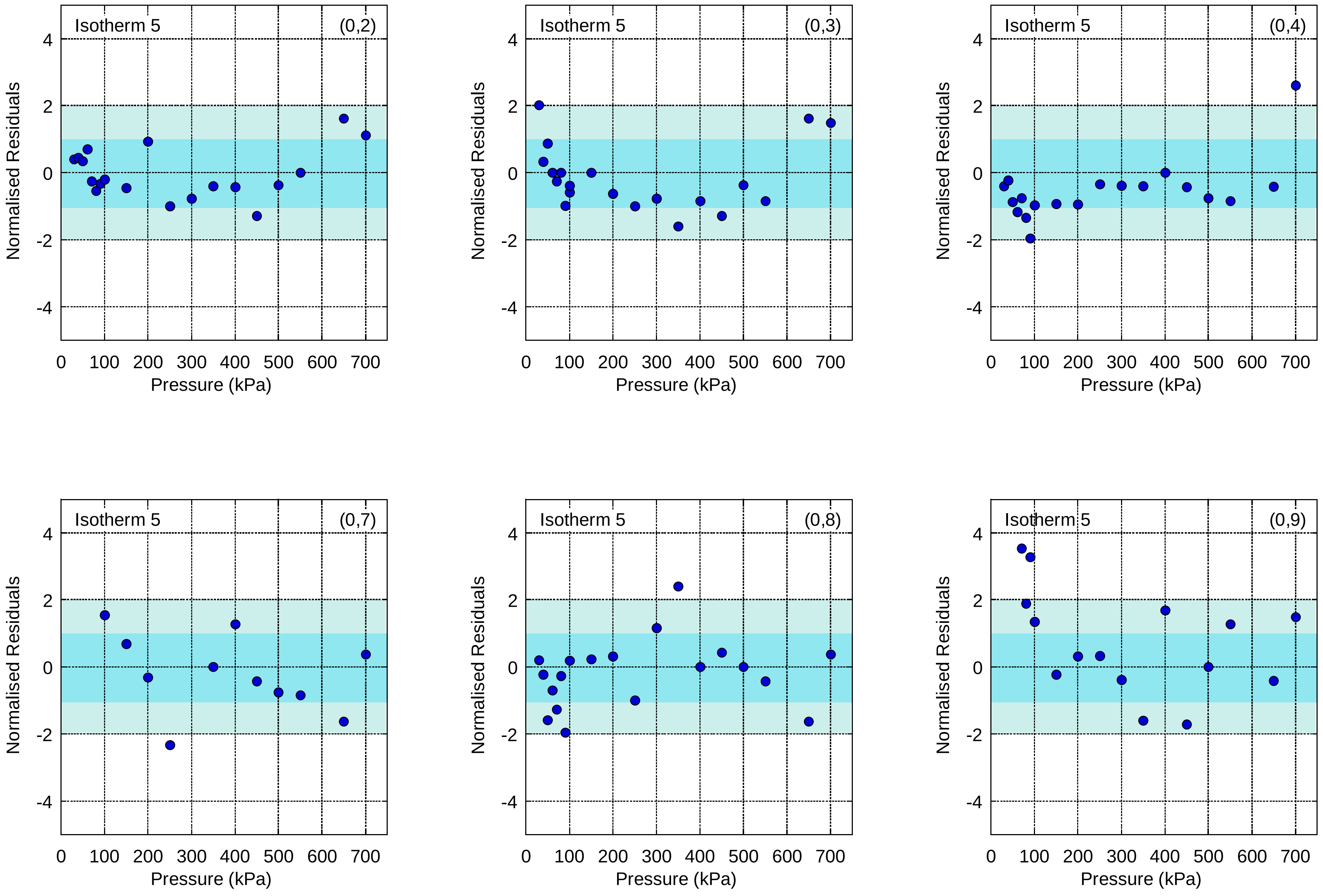
Normalised Residuals from Global Fit: Isotherm 3



Normalised Residuals from Global Fit: Isotherm 4

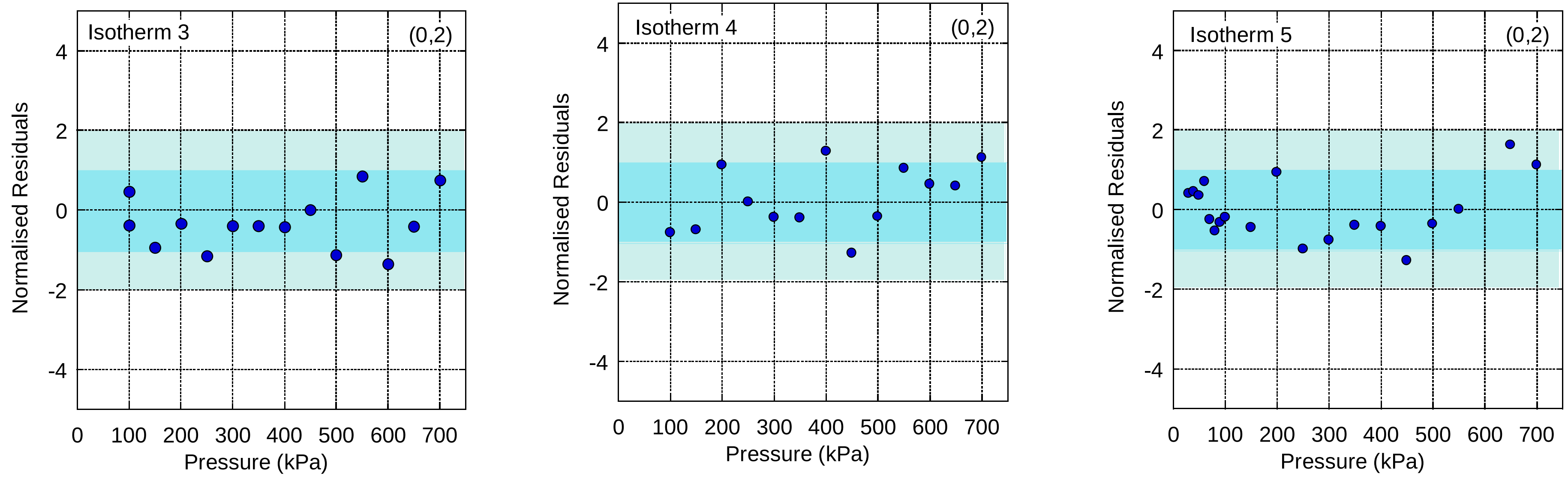


Normalised Residuals from Global Fit: Isotherm 5

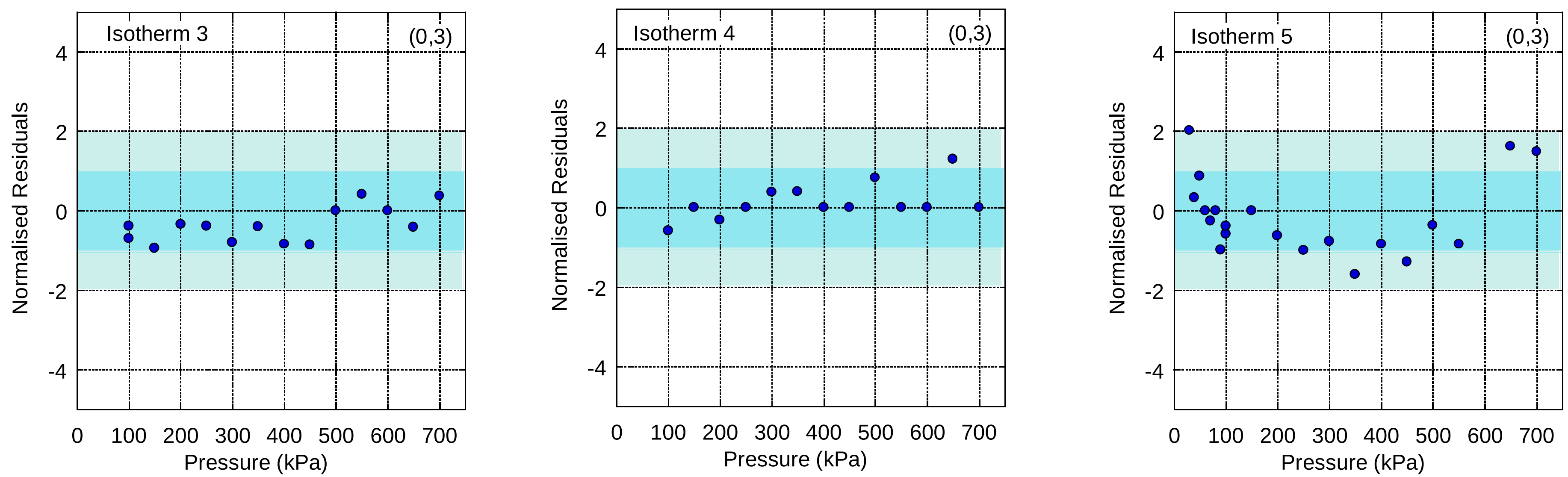


Residuals by Mode

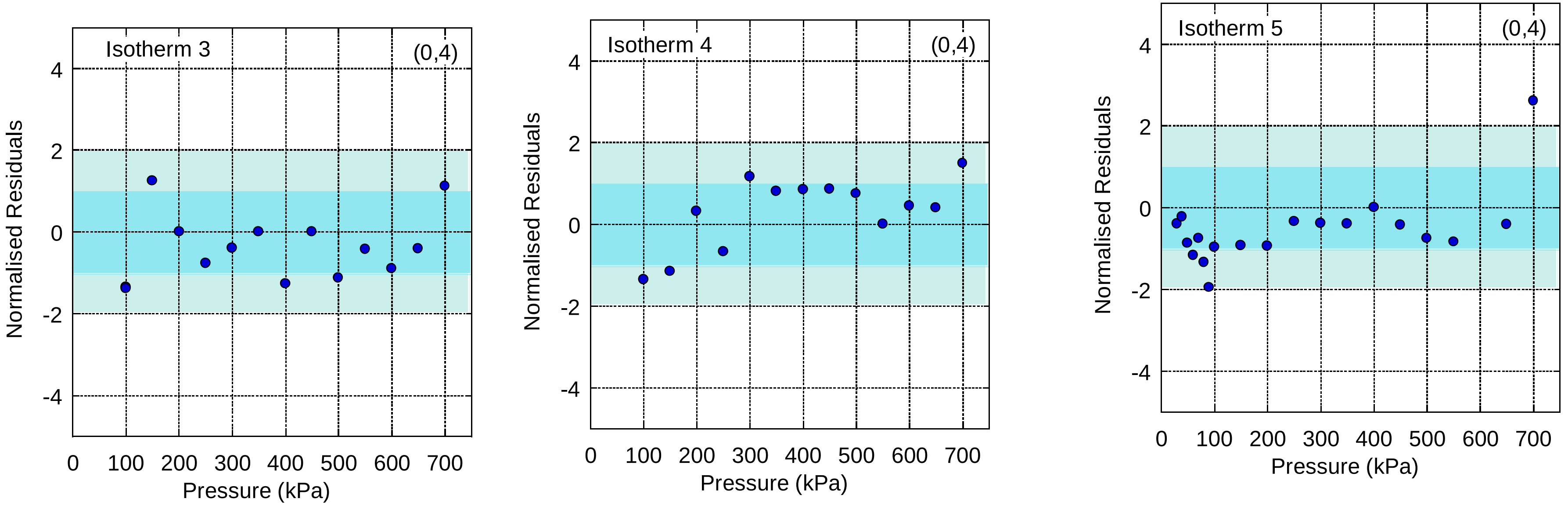
Normalised Residuals from Global Fit: (0,2)



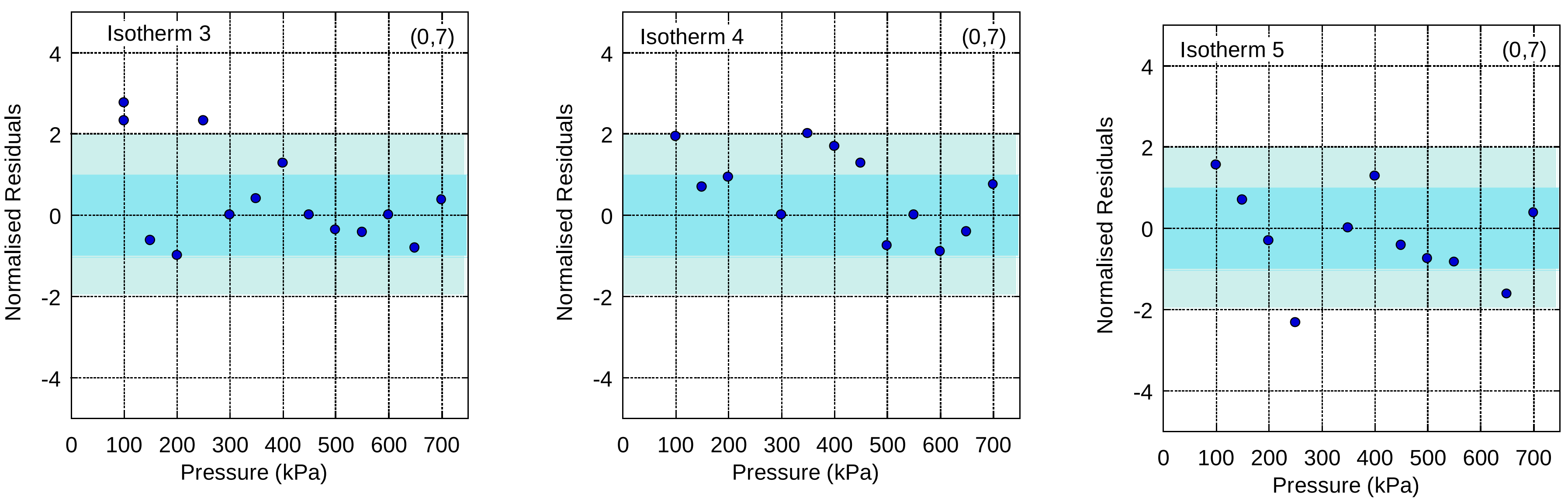
Normalised Residuals from Global Fit: (0,3)



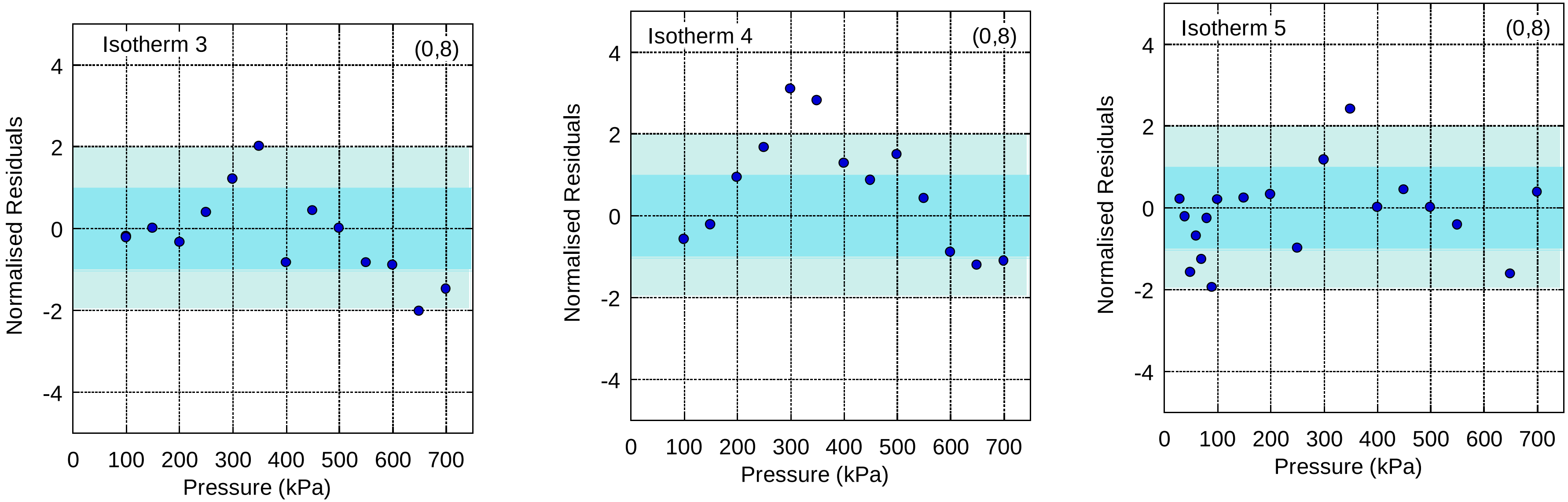
Normalised Residuals from Global Fit: (0,4)



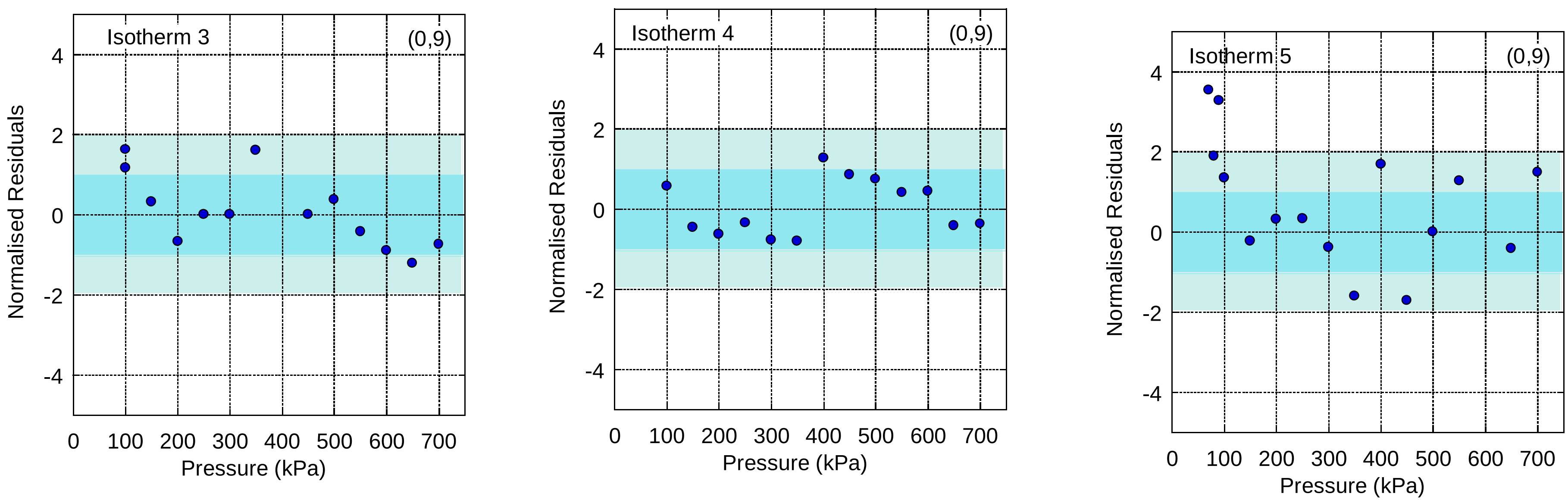
Normalised Residuals from Global Fit: (0,7)



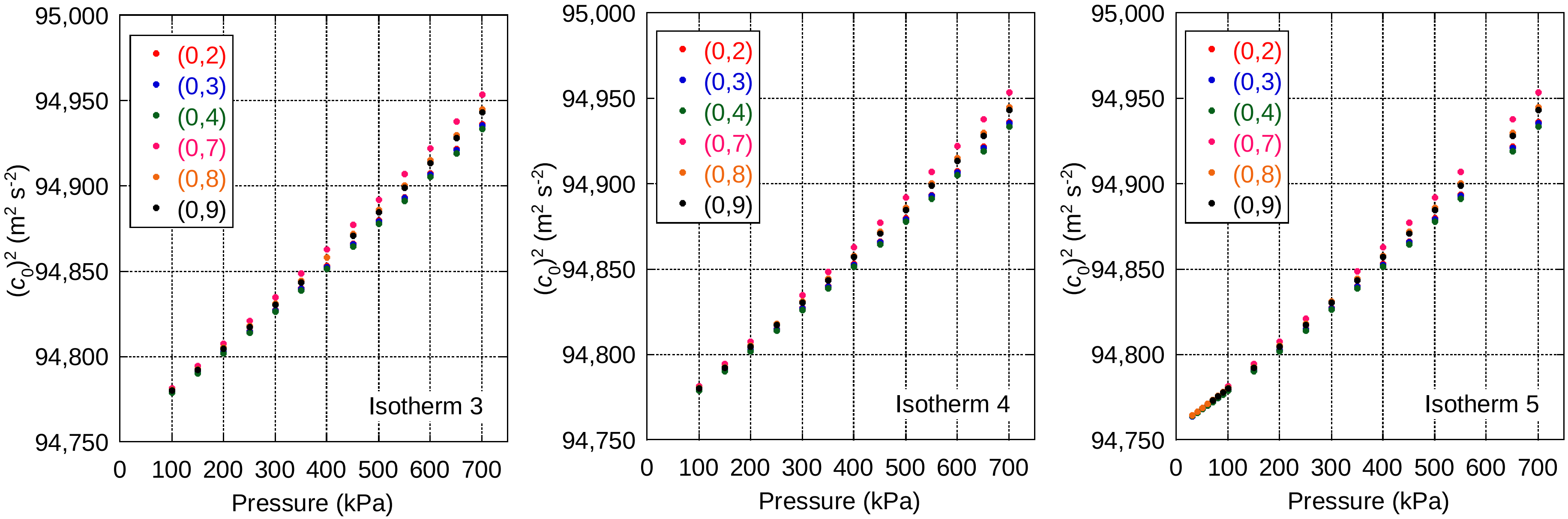
Normalised Residuals from Global Fit: (0,8)



Normalised Residuals from Global Fit: (0,9)



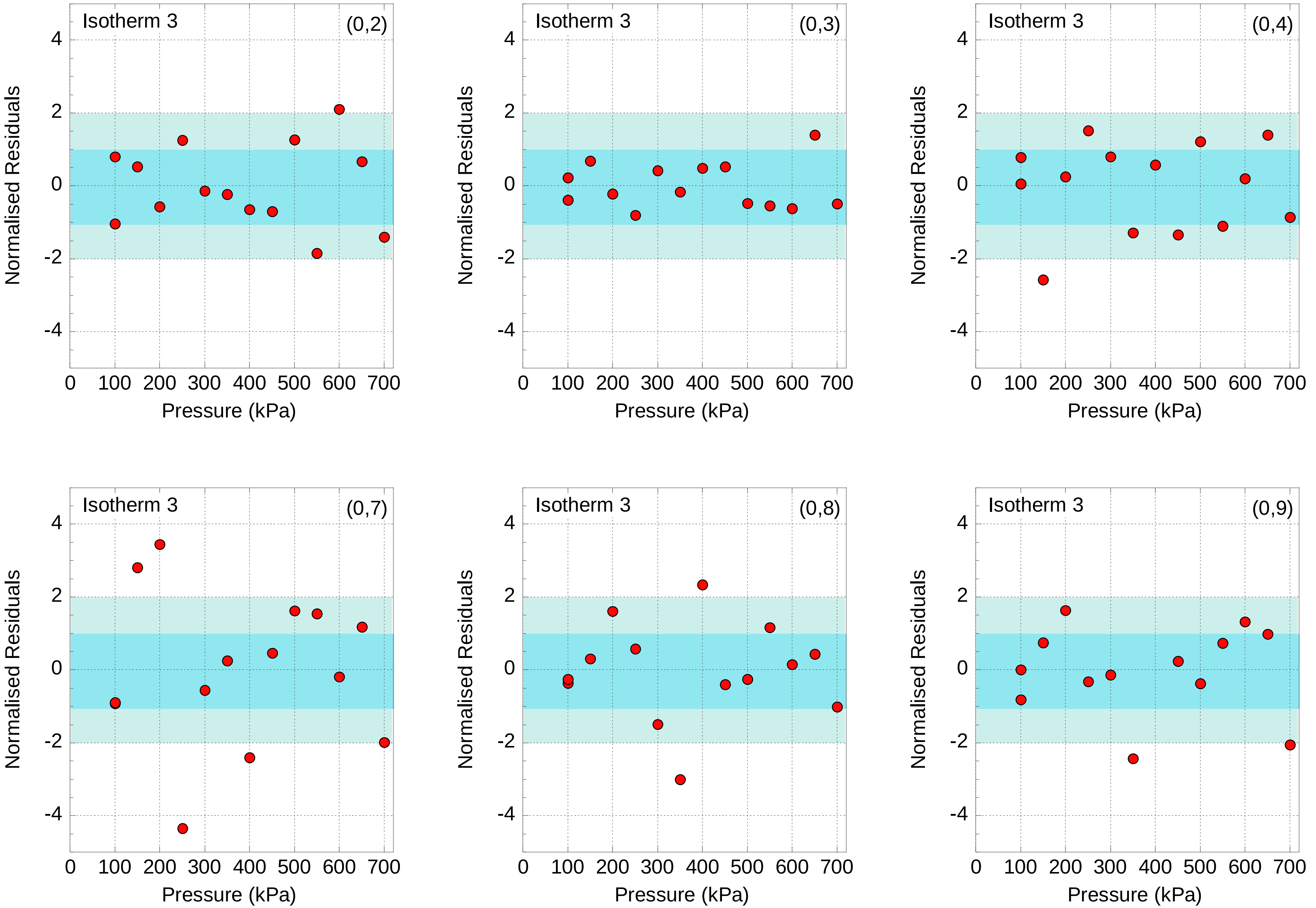
Alternative Analysis



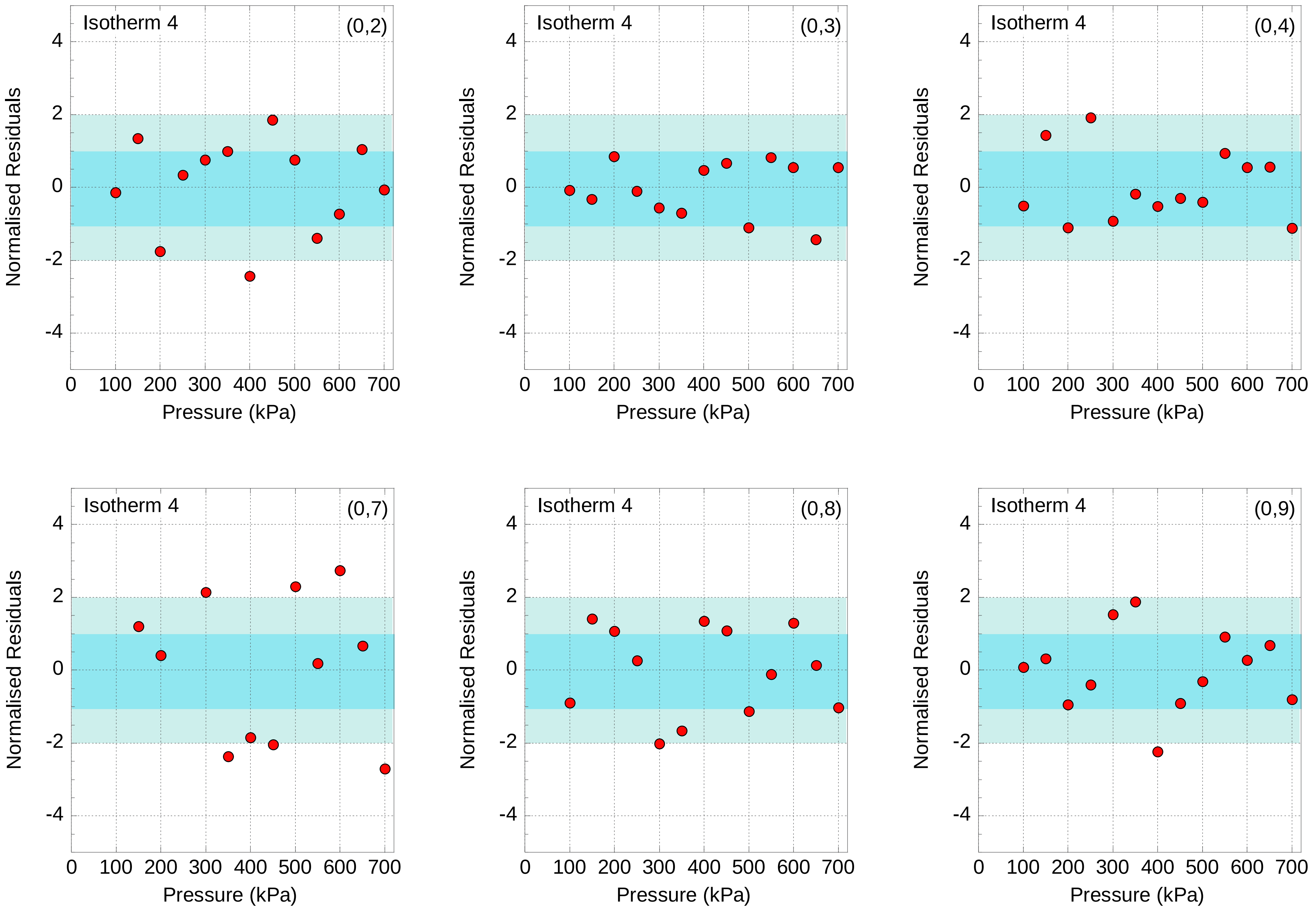
$$c_{EXP}^2 - A_3 P^3 = c_0^2 + A_{-1} P^{-1} + A_1 P + A_2 P^2$$

- Treat each of 18 isotherm/modes independently
- Gives 18 estimates for c_0^2 instead of 1
- Gives 18 estimates for A_1 instead of 6
- Gives 18 estimates for A_2 instead of 1

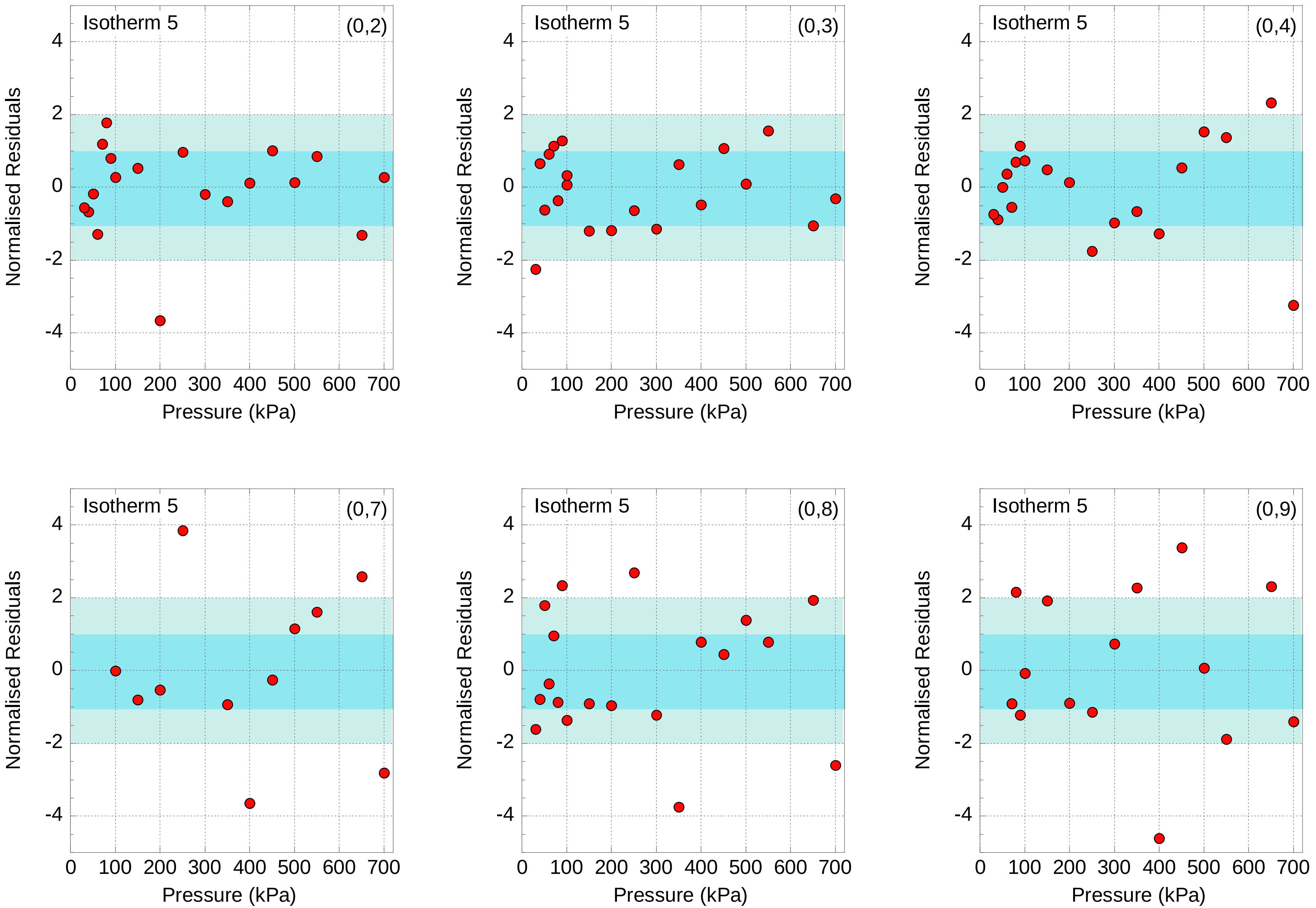
Normalised Residuals of Individual Fits: Isotherm 3



Normalised Residuals of Individual Fits: Isotherm 4

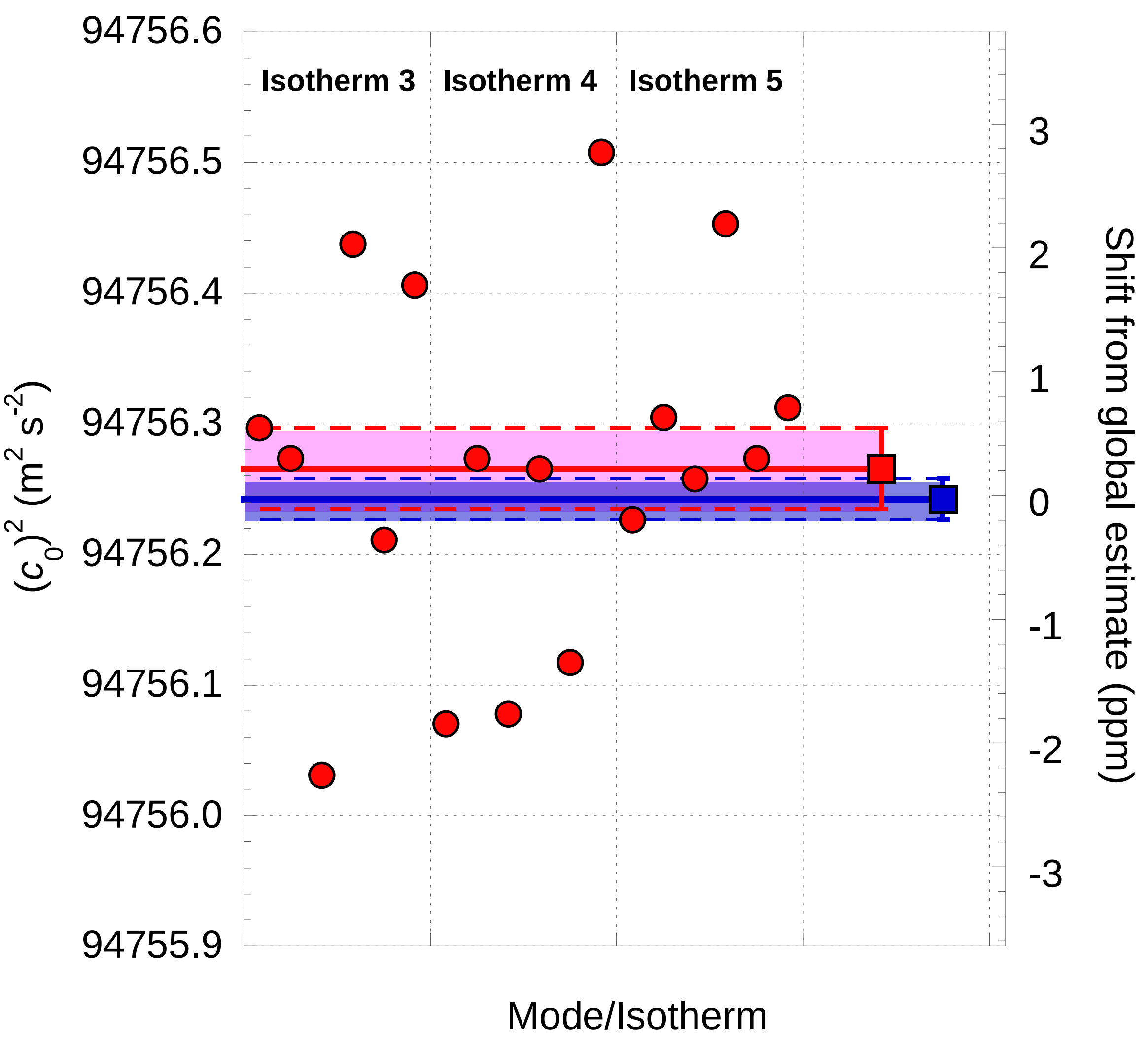


Normalised Residuals of Individual Fits: Isotherm 5



NPL Strengths#3: Alternative Analysis

- *Fit each isotherm individually and produce 18 estimates for c_0^2.*
- *Average value is ~+ 0.25 ppm higher than 'global' estimate (+1.4u)*



Reflections on the NPL-2013 Estimate of the Boltzmann Constant

1. Background

2. History

3. The NPL uncertainty estimate

4. NPL Update February 2015

5. The NPL Analysis

6. Summary

Summary

1. NPL-2013 estimate of k_B has u_R = 0.71 x 10^{-6}
2. Differs significantly from LNE-CNAM-2011
3. Significant differences in analytical assumptions and estimated sensitivity to errors
4. Possible reconciliation is through a molar mass error by either NPL or LNE-CNAM.
5. Time will tell.


BOLTZMANN DN
BOLTZMANN AS