Proton polarisability contribution to the Lamb shift in muonic hydrogen

Mike Birse
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Work done in collaboration with Judith McGovern

Eur. Phys. J. A **48** (2012) 120

The Lamb shift in muonic hydrogen

Much larger than in electronic hydrogen:

$$\Delta E_L = E(2p_{\frac{1}{2}}) - E(2s_{\frac{1}{2}}) \simeq +0.2 \text{ eV}$$

Dominated by vacuum polarisation

Much more sensitive to proton structure, in particular, its charge radius

$$\Delta E_L^{\text{th}} = 206.0668(25) - 5.2275(10)\langle r_E^2 \rangle \text{ meV}$$

Results of many years of effort by Borie, Pachucki, Indelicato, Jentschura and others; collated in Antognini et al, Ann Phys **331** (2013) 127

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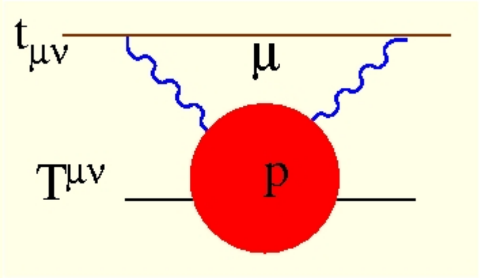
Includes contribution from two-photon exchange

$$\Delta E^{2\gamma} = 33.2 \pm 2.0\ \mu\text{eV}$$

Sensitive to polarisabilities of proton by virtual photons

Two-photon exchange



Integral over $T^{\mu\nu}(\nu, q^2)$ – doubly-virtual Compton amplitude for proton

Spin-averaged, forward scattering $\longrightarrow$ two independent tensor structures

Common choice:

$$T^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right) T_2(\nu, Q^2)$$

multiplied by scalar functions of $\nu = p\cdot q/M$ and $Q^2 = -q^2$

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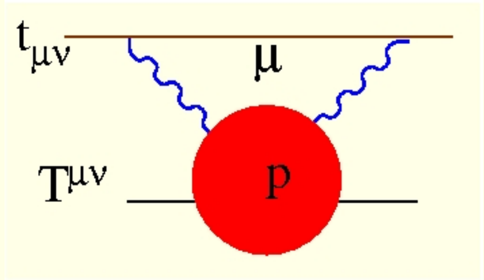

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Amplitude contains elastic (Born) and inelastic pieces: $T^{\mu\nu} = T_B^{\mu\nu} + \overline{T}^{\mu\nu}$

- elastic: photons couple independently to proton (no excitation)
- need to remove terms already accounted for in Lamb shift (iterated Coulomb, leading dependence on $\langle r_E^2 \rangle$)
- inelastic: proton excited $\longrightarrow$ polarisation effects

Doubly-virtual Compton scattering

Elastic amplitude from Dirac nucleon with Dirac and Pauli form factors

K. Pachucki, Phys. Rev. A **60** (1999) 3593

$$T_1^B(\nu, Q^2) = \frac{e^2}{M}\left[\frac{Q^4\left(F_D(Q^2)+F_P(Q^2)\right)^2}{Q^4-4M^2\nu^2} - F_D(Q^2)^2\right]$$

$$T_2^B(\nu, Q^2) = \frac{4e^2MQ^2}{Q^4-4M^2\nu^2}\left[F_D(Q^2)^2 + \frac{Q^2}{4M^2}F_P(Q^2)^2\right]$$

Other choices have been used: nonpole terms only

But depend on choice of tensor basis (energy-dependent tensors)

cf Walker-Loud et al, Phys Rev Lett **108** (2012) 232301

Low-energy theorems

V^2CS not directly measurable, but constrained by LETs

Two independent tensors of order q^2: correspond to polarisabilities

α (electric) and β (magnetic) determined from real Compton scattering

$$\begin{aligned} \overline{T}_1(\nu, Q^2) &= 4\pi Q^2\beta + 4\pi\nu^2(\alpha+\beta) + O(q^4) \\ \overline{T}_2(\nu, Q^2) &= 4\pi Q^2(\alpha+\beta) + O(q^4) \end{aligned}$$

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Other choices for elastic amplitude $\rightarrow$ LETs containing charge radius

cf Hill and Paz, Phys Rev Lett **107** (2011) 160402

Just important to use consistent definitions of elastic and inelastic amplitudes

Here: all results expressed using Pachucki's choice

Dispersion relations

Information on forward V^{2}CS away from $q = 0$ from structure functions $F_{1,2}(\nu, Q^2)$
$F_{1,2}$ well determined from electroproduction experiments eg at JLab

Dispersion relation for $\overline{T}_2$ converges since $F_2 \sim 1/\nu$ at high energies

$$\overline{T}_2(\nu, Q^2) = -\int_{\nu_{th}^2}^{\infty} \mathrm{d}\nu'^2 \, \frac{F_2(\nu', Q^2)}{\nu'^2 - \nu^2}$$

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$$\overline{T}_1(\nu, Q^2) = \overline{T}_1(0, Q^2) - \nu^2 \int_{\nu_{th}^2}^{\infty} \frac{\mathrm{d}\nu'^2}{\nu'^2} \, \frac{F_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

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Problem: subtraction function $\overline{T}_1(0, Q^2)$ not experimentally accessible
Satisfies LET: $\overline{T}_1(0, Q^2)/Q^2 \to 4\pi\beta$ as $Q^2 \to 0$

Subtraction term

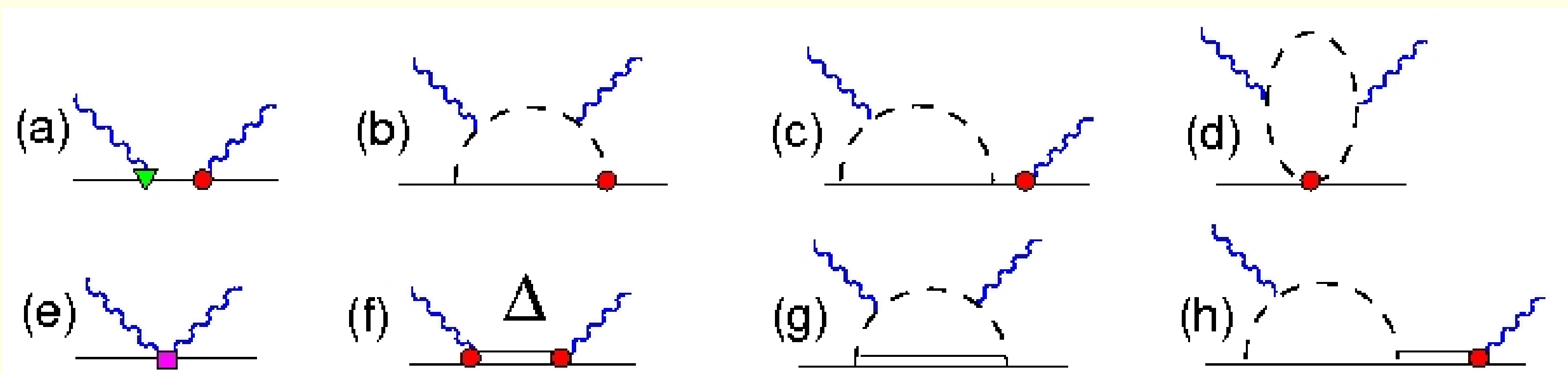
Define form factor

$$\overline{T}_1(0,Q^2) = 4\pi\beta\, Q^2\, F_\beta(Q^2)$$

Large Q^2: operator-product expansion, quark counting rules give $F_\beta(Q^2) \propto Q^{-4}$

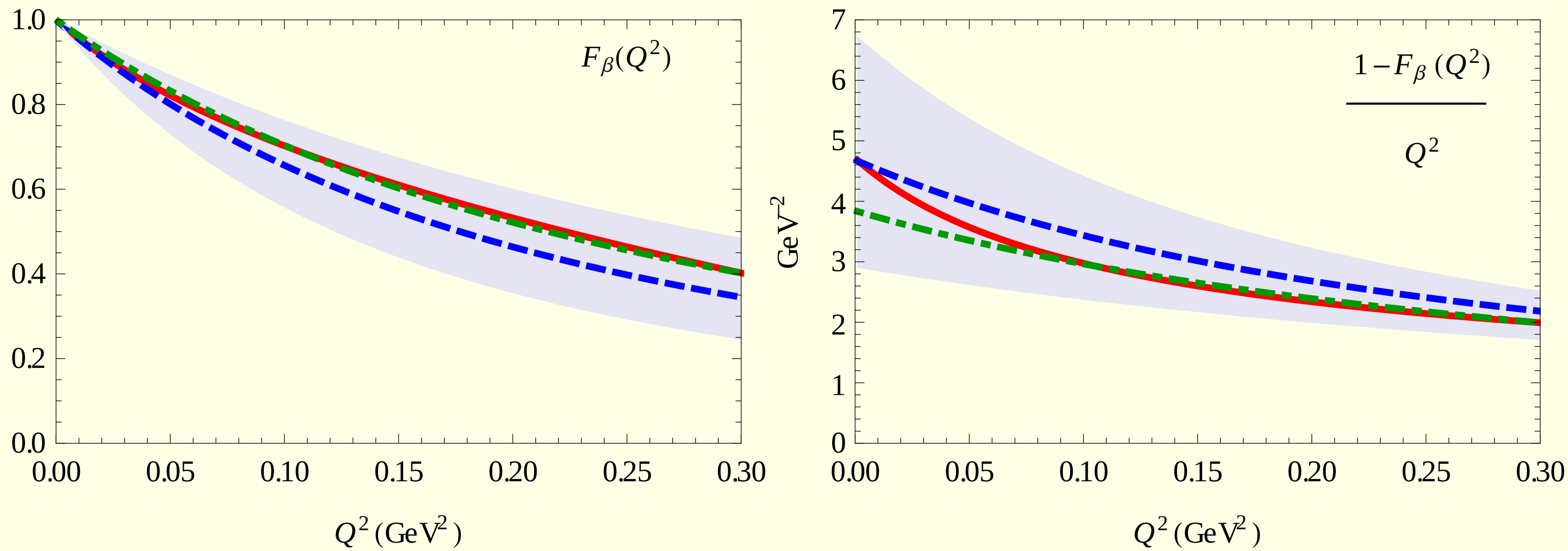
Small Q^2: use heavy-baryon chiral perturbation theory at 4th order
plus leading effect of $\gamma N\Delta$ form factor

- same as for real Compton scattering McGovern et al, Eur Phys J A 49 (2013) 12



- minor modifications for different kinematics
- subtract elastic (Born) contribution calculated to this order
- relevant LETs satisfied; consistent with value for β

Form factor



Extrapolate to higher Q^2 by matching ChPT form onto dipole

$$F_\beta(Q^2) \sim \frac{1}{(1+Q^2/2M_\beta^2)^2}$$

Match at $Q^2 = 0 \rightarrow M_\beta = 462$ MeV; at $Q^2 \sim m_\pi^2 \rightarrow M_\beta = 510$ MeV

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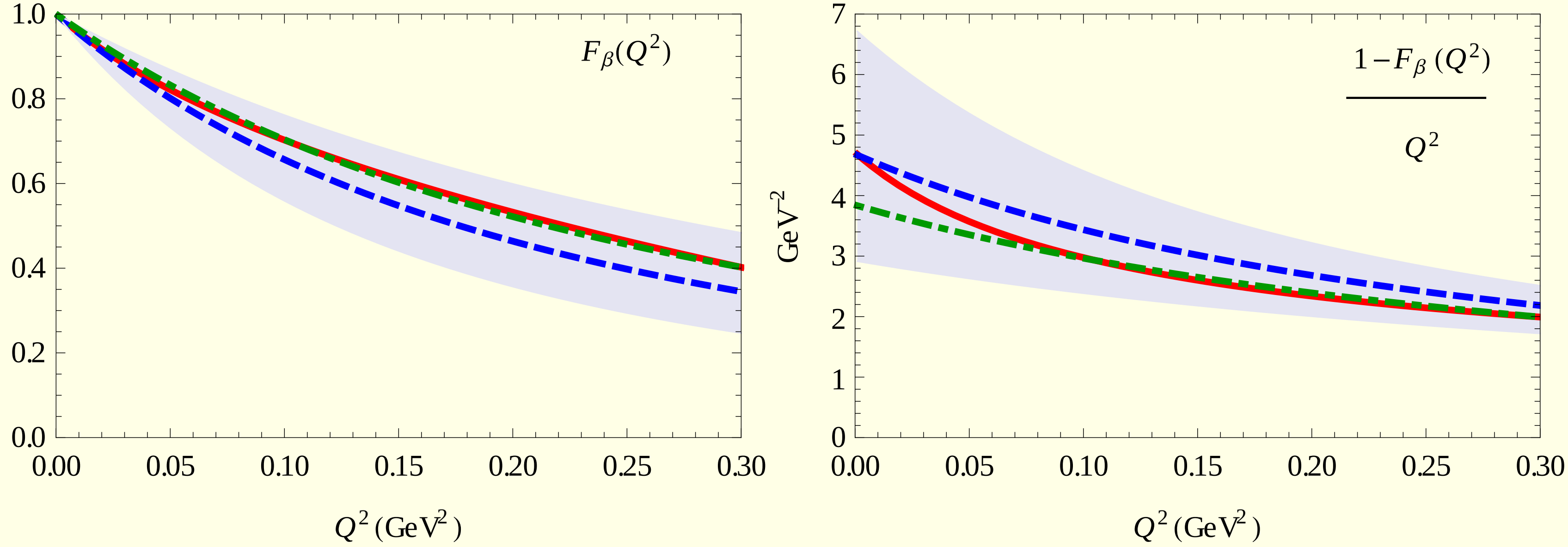

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$$M_\beta = 485 \pm 100 \pm 40 \pm 25 \text{ MeV}$$

- generous allowance for higher-order effects and uncertainties in input (shaded)
- $\beta = (3.1 \pm 0.5) \times 10^{-4}$ fm^3 Griesshammer *et al*, Prog Part Nucl Phys **67** (2012) 841
- matching uncertainty

Energy shift

$$\Delta E^{2\gamma}_{\rm sub} = \frac{\alpha_{\rm EM}\,\phi(0)^2}{4\pi m}\int_0^\infty {\rm d}Q^2\,\frac{\overline{T}_1(0,Q^2)}{Q^2}\times\left[1+\left(1-\frac{Q^2}{2m^2}\right)\left(\sqrt{\frac{4m^2}{Q^2}+1}-1\right)\right]$$

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- rather insensitive to value of M_β
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Carlson and Vanderhaeghen, Phys Rev A **84** (2011) 020102

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Plus elastic term of Carlson and Vanderhaeghen

(converted to Pachucki's convention): $\Delta E^{2\gamma}_{\rm el} = 24.7 \pm 1.3\ \mu$eV

$\longrightarrow$ total two-photon exchange: $\Delta E^{2\gamma} = 33.2 \pm 2.0\ \mu$eV

Other treatments

	BM	P	CV	GLS	NP	PP	ALP
$\Delta E^{2\gamma}_{\rm sub}$	$-4.2(1.0)$	-1.8	$-5.3(1.9)$	$2.3(4.6)^\dagger$	-1.6	$-2.9^\ddagger$	–
$\Delta E^{2\gamma}_{\rm inel}$	$12.7(0.5)$	13.9	$12.7(0.5)^\dagger$	$13.0(0.6)$	20.1	$29^\ddagger$	–
$\Delta E^{2\gamma}_{\rm pol}$	$8.5(1.1)$	12.1	$7.4(2.4)$	$15.3(5.6)$	$19(9)$	$26(10)$	$8.2\left(^{+2.5}_{-1.2}\right)$

* all values in μeV

† converted to value corresponding to Pachucki's elastic term

‡ cannot be separated without model assumptions; could be up to 10 μeV larger

BM (our result): DR (from CV) + 4th order EFT with extrapolation for subtraction

P, CV: DR + model for subtraction Pachucki, Phys. Rev. A **60** (1999) 3593; Carlson and Vanderhaeghen, Phys Rev A **84** (2011) 020102

GLS: energy-weighted sum rules Gorchtein *et al*, Phys Rev.A **87** (2013) 052501

NP: 3rd order EFT Nevado and Pineda, Phys Rev C **77** (2008) 035202

PP: 3rd order EFT + Δ Peset and Pineda, arXiv:1403.3408, Nucl Phys B **887** (2014) 69

ALP: 3rd order covariant EFT Alarcón *et al*, Eur Phys J C **74** (2014) 2852

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Lowest (3rd) order EFTs: model-independent but with large uncertainties

- EM polarisabilities of right magnitude only
- inelastic term too large, especially when Δ included
- 3rd Zemach moment much smaller than values from empirical form factors
- ALP: large relativistic corrections compared to NP; Δ contributions cancelled due to additional low-energy expansion $\rightarrow$ errors probably underestimated

Comments 2

Better to use experimental data as far as possible

- get inelastic contribution from DRs
- empirical form factors for elastic piece (3rd Zemach moment)

(but see Karshenboim, Phys Rev D **90** (2014) 053012 for discussion)

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- needed to fit EM polarisabilities to Compton scattering
- contain leading relativistic/recoil corrections
- including charge radius piece of LET

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But γp contact terms also lead to divergences in two-photon exchange

- renormalised by unknown μp contact interactions (could fit these to Lamb shift!)

$\rightarrow$ instead calculate form factor for subtraction term for momenta $\lesssim 3\,m_\pi$ and extrapolate

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Miller, Phys Lett B **718** (2013) 1078

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Miller, Phys Lett B **718** (2013) 1078

But no evidence from related processes:

- dispersion relations for $T_2(0, Q^2)$ ($\sim \alpha + \beta$)
- proton-neutron mass difference Walker-Loud et al, Phys Rev Lett **108** (2012) 232301
- quasi-elastic electron-nucleus scattering Miller, Phys Rev C **86** (2012) 065201

Summary

Subtraction term in two-photon-exchange contribution to Lamb shift calculated using chiral EFT at 4th order, with extrapolation of form factor to $Q^2 \gtrsim 0.3$ GeV^2

$$\Delta E^{2\gamma}_{\rm sub} = -4.2 \pm 1.0\,\mu\text{eV}$$

Complete two-photon exchange contribution

$$\Delta E^{2\gamma} = 33 \pm 2\,\mu\text{eV}$$

- factor 10 too small to explain proton radius puzzle (330 μeV)

Still largest uncertainty in theoretical determination of Lamb shift in muonic H

- two main sources: β (subtraction) and form factors (elastic)

Prospects for improvement in β:

- re-analysis of world Compton data set using DRs
- upcoming experiments on (polarised) Compton scattering at HiGS, MAMI