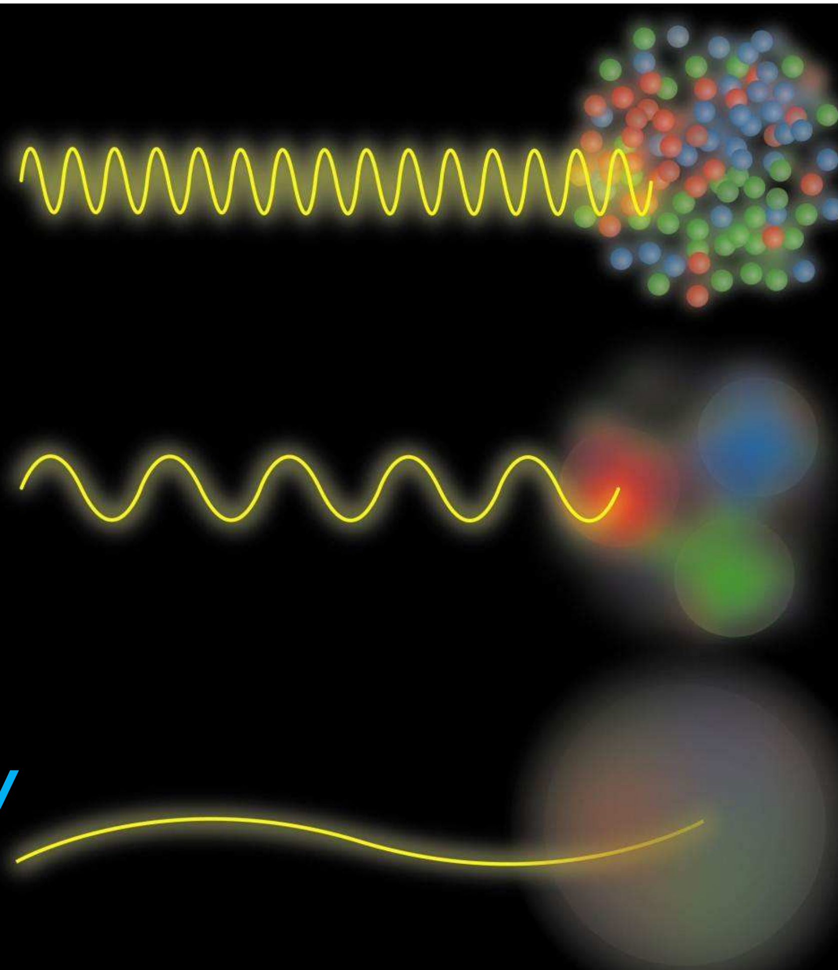
Proton charge radius extracted from unpolarized electron scattering



Graphic by Joshua Rubin, ANL

John Arrington
Argonne National Laboratory

Fundamental Constants 2015, Eltville, Germany, 2 Feb

Outline

General complications in extracting radius from e-p scattering data

Recent extractions: JLab global analysis, Mainz-A1 experiment

Possible issues/errors/systematics in these extractions

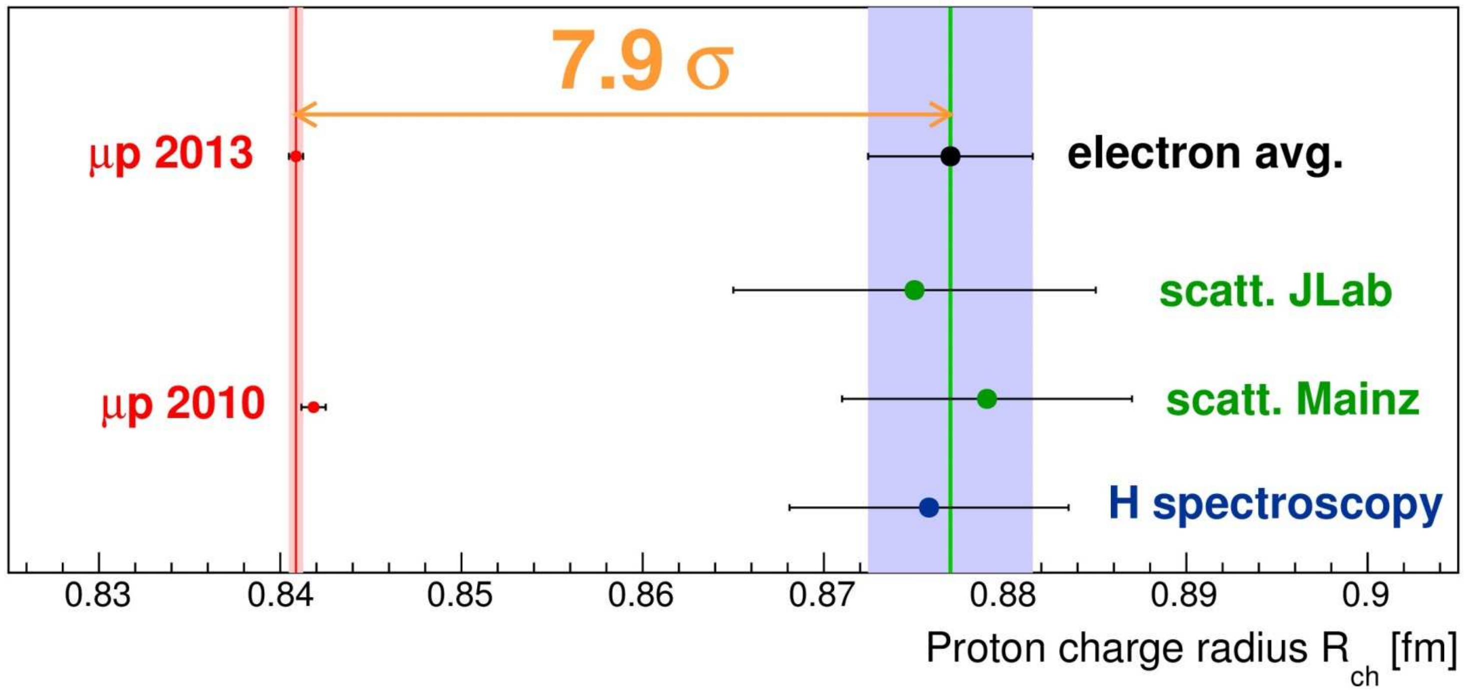
- **Under-fitting and over-fitting**
- **Discrepancy between Mainz, previous data at cross section level**

Modified analysis of Mainz data (G. Lee, JA, R. Hill - preliminary)

- **Modified approach to address convergence, over/under-fitting**
- **Broader tests of potential systematic effects**
- Can we find a way to ‘make’ the radius match muonic hydrogen?
- Separate analysis of Mainz, other world’s data to examine consistency

Future experimental plans

Proton Charge Radius Extractions



2010: Two new charge/magnetic radii extracted from electron scattering

J. Bernauer, et al., PRL 105 (2010) 242001

X. Zhan, et al., PLB 705 (2011) 59

2013: Updated results from muonic hydrogen

A. Antognini, et al., Science 339 (2013) 417

Unpolarized Elastic e-N Scattering

Nearly all of the measurements used Rosenbluth separation

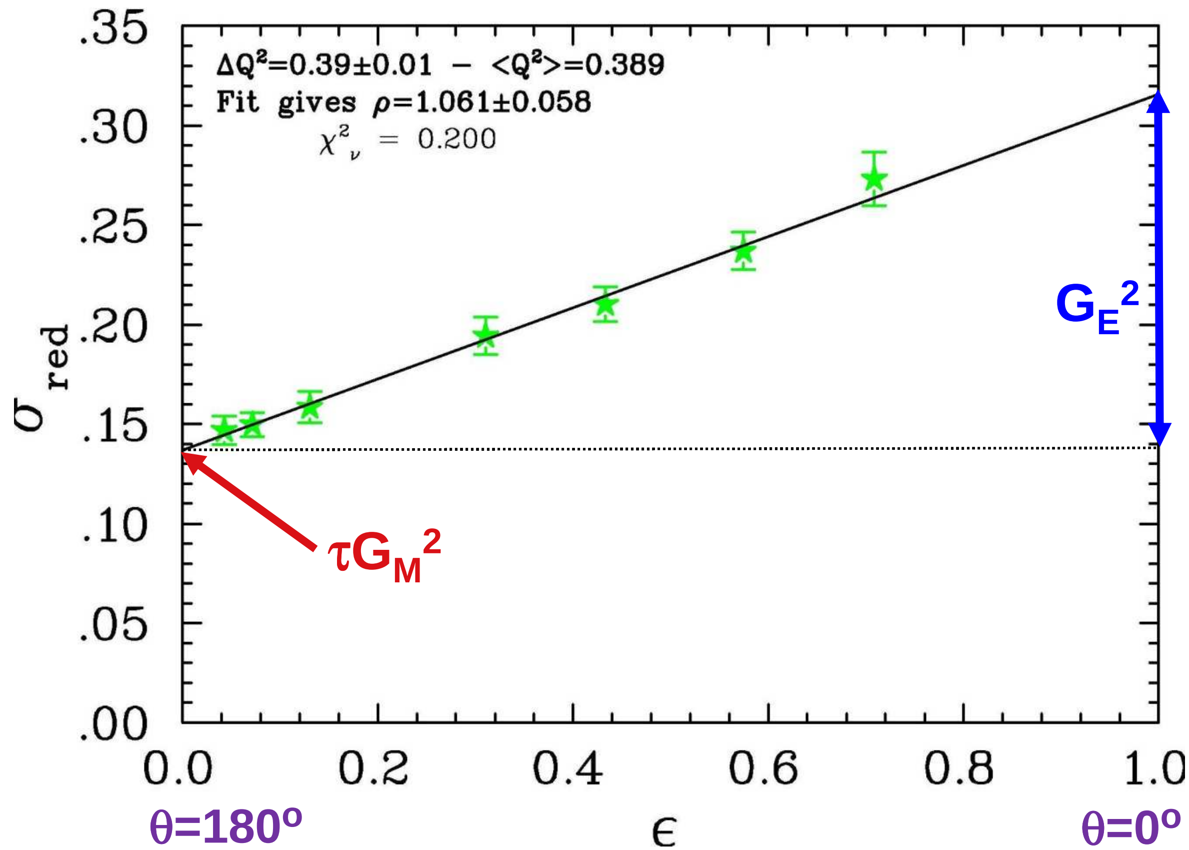
$\sigma_R = d\sigma/d\Omega\ [\varepsilon(1+\tau)/\sigma_{Mott}] = \tau G_M^2 + \varepsilon G_E^2$

$\tau = Q^2/4M^2$

$\varepsilon = [\ 1 + 2(1+\tau)\tan^2(\theta/2)\ ]^{-1}$

Reduced sensitivity when one term dominates:

- **G_M at high Q^2 ($\tau >> 1$)**
- **G_E at low Q^2 ($\tau >> 1$)**



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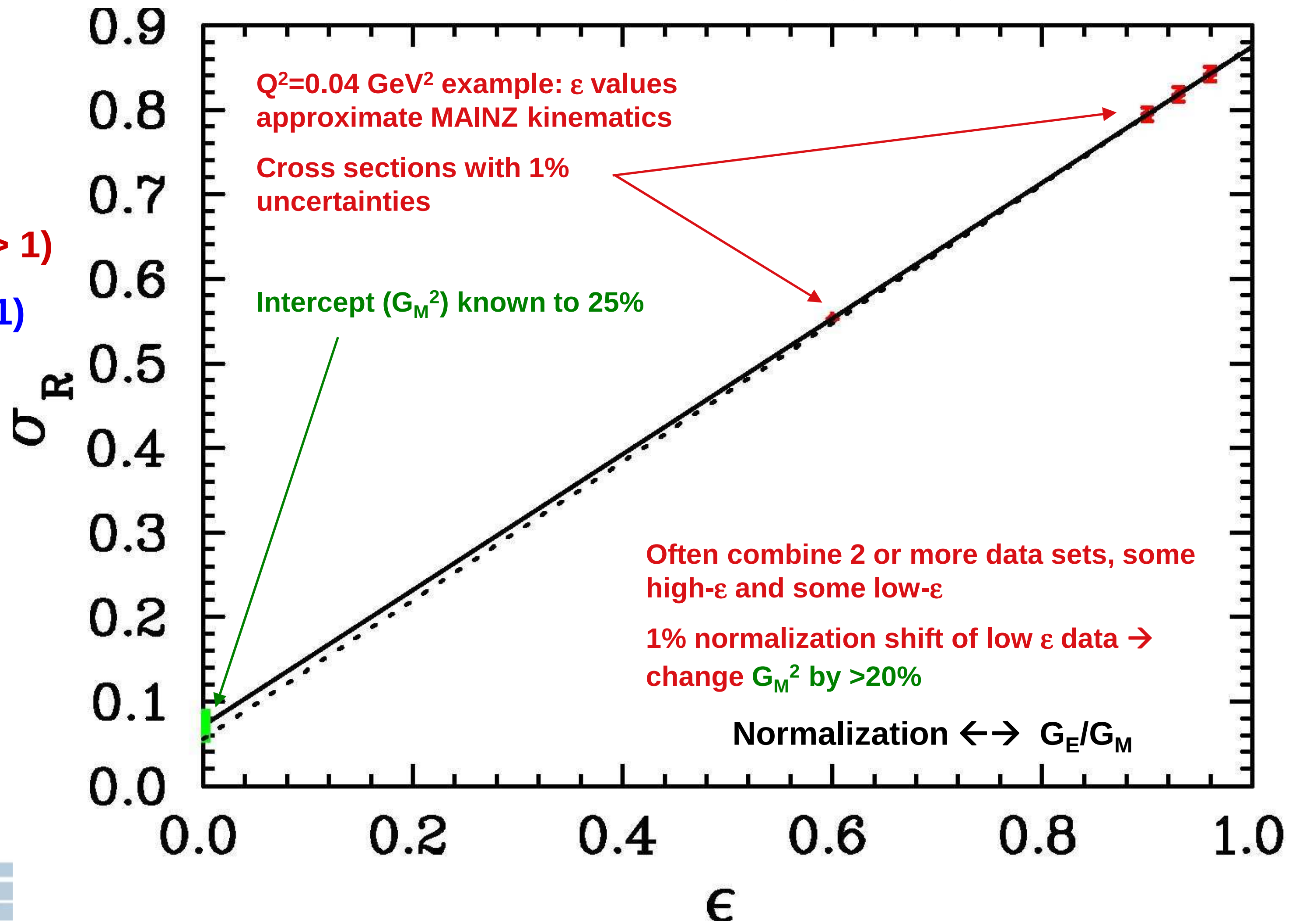
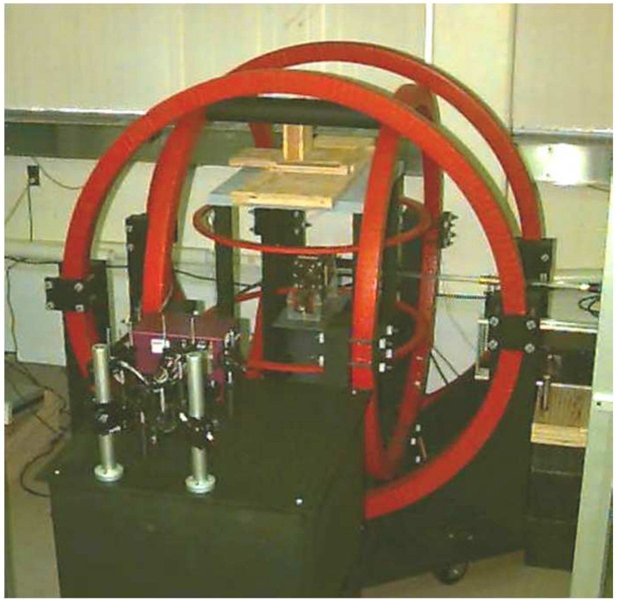


New techniques: Polarization and A(e,e'N)

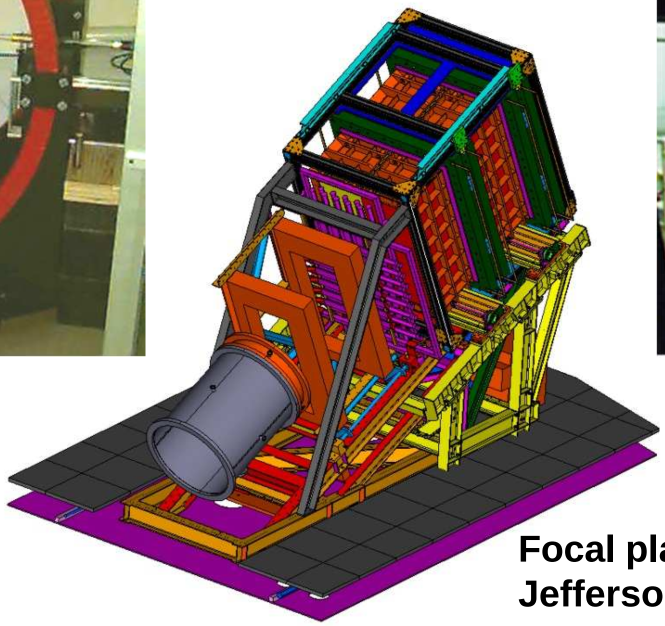
- **Mid '90s brought measurements using improved techniques**
  - **High luminosity, highly polarized electron beams**
  - **Polarized targets (^{1}H, ^{2}H, ^{3}He) or recoil polarimeters**
  - **Large, efficient neutron detectors for ^{2}H, ^{3}He(e,e'n)**



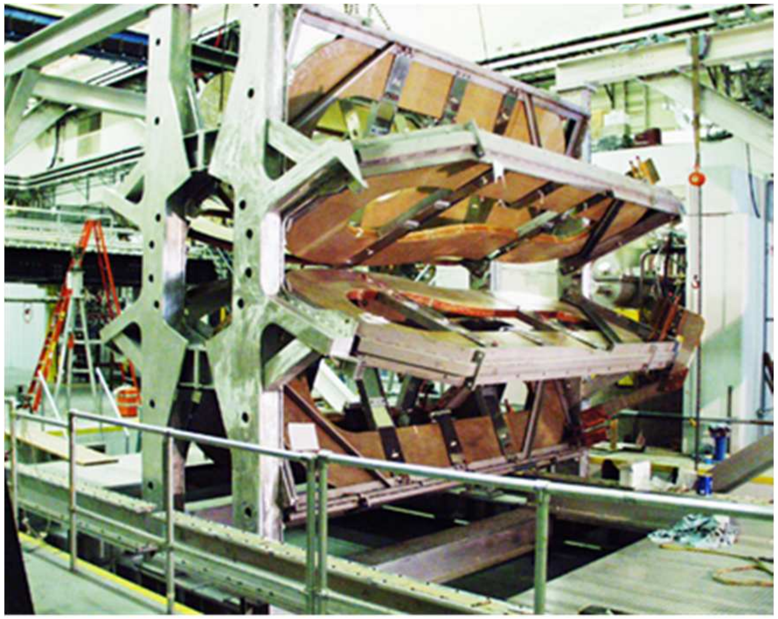
Polarized ^{3}He target

Unpol: $\tau G_M^2 + \varepsilon G_E^2$

Pol: G_E/G_M



Focal plane polarimeter – Jefferson Lab



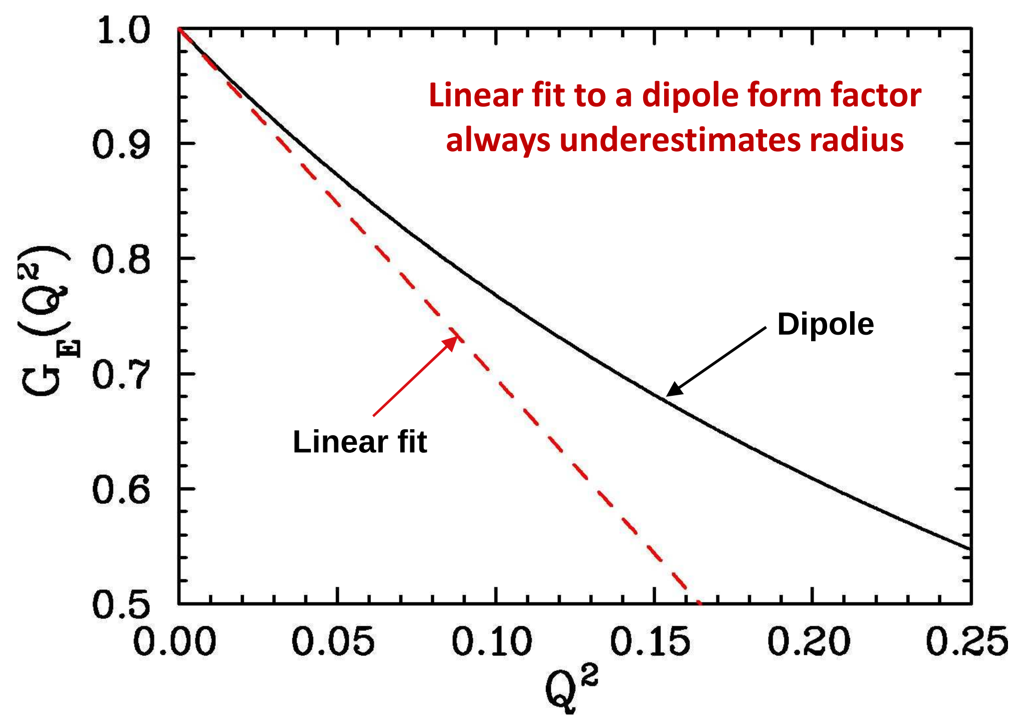
BLAST at MIT-Bates

Difficulties in extracting the radius: Underfitting

Need enough Q^2 range to have good lever-arm to measure slope

Need to limit Q^2 range so that you're including data that's sensitive to the radius

Need to have fit function with enough flexibility to match data in your Q^2 range, but not too much



Linear fit uncertainty best up to $Q^2 \approx 0.02$, where fit & "truncation error" both large (~2%)

Quadratic fit works well up to $Q^2 \approx 0.1$ before "truncation error" dominates (~1.2%)

Cubic fit works well up to $Q^2 \approx 0.3$ before truncation error dominates (~1.1%)

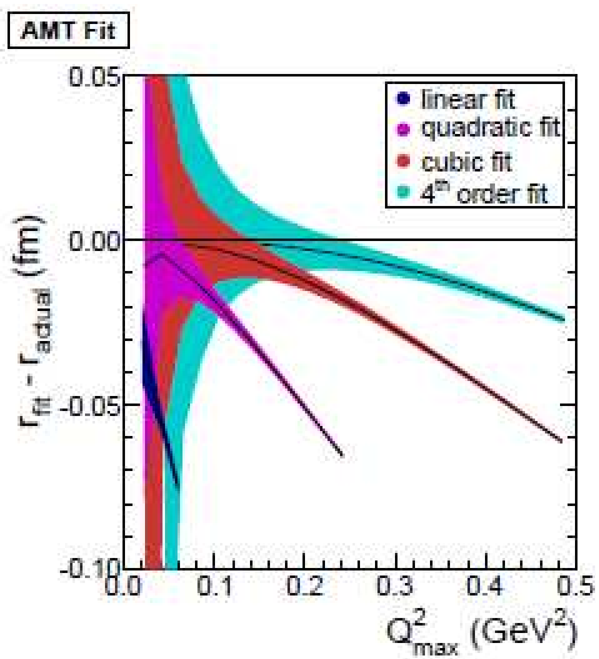
Based on assumption of dipole form, ten 1% measurements from $Q^2 = 0$ to Q^2_{max}

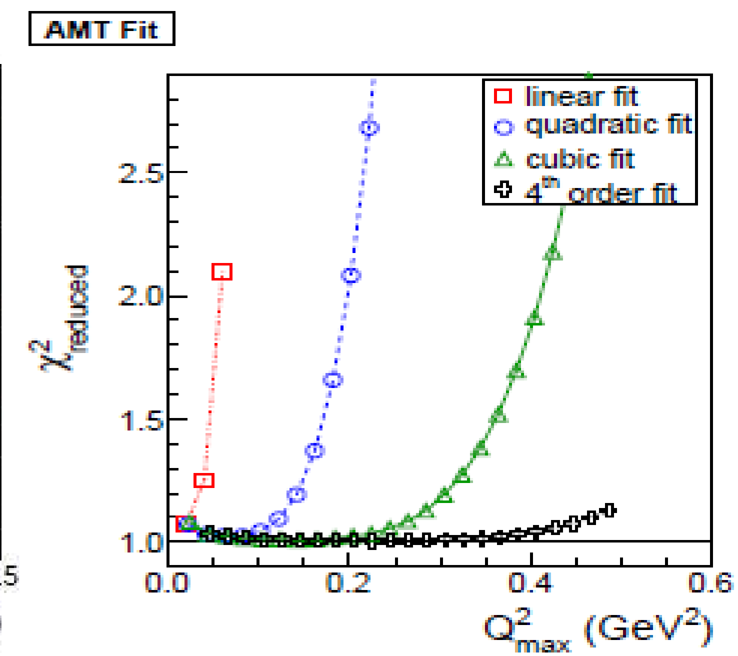
“Polynomial fits and the proton charge radius”

E. Kraus, K.E. Mesick, A. White, R. Gilman, S. Strauch, PRC 90 (2014) 045206

More detailed examination of statistical power vs. error due to imperfect functional form as one varies the Q^2 range, number of parameters, etc….

Polynomial fits tend to be the ‘worst case’ among published analyses: effects generally smaller for inverse polynomial, continued fraction, etc…





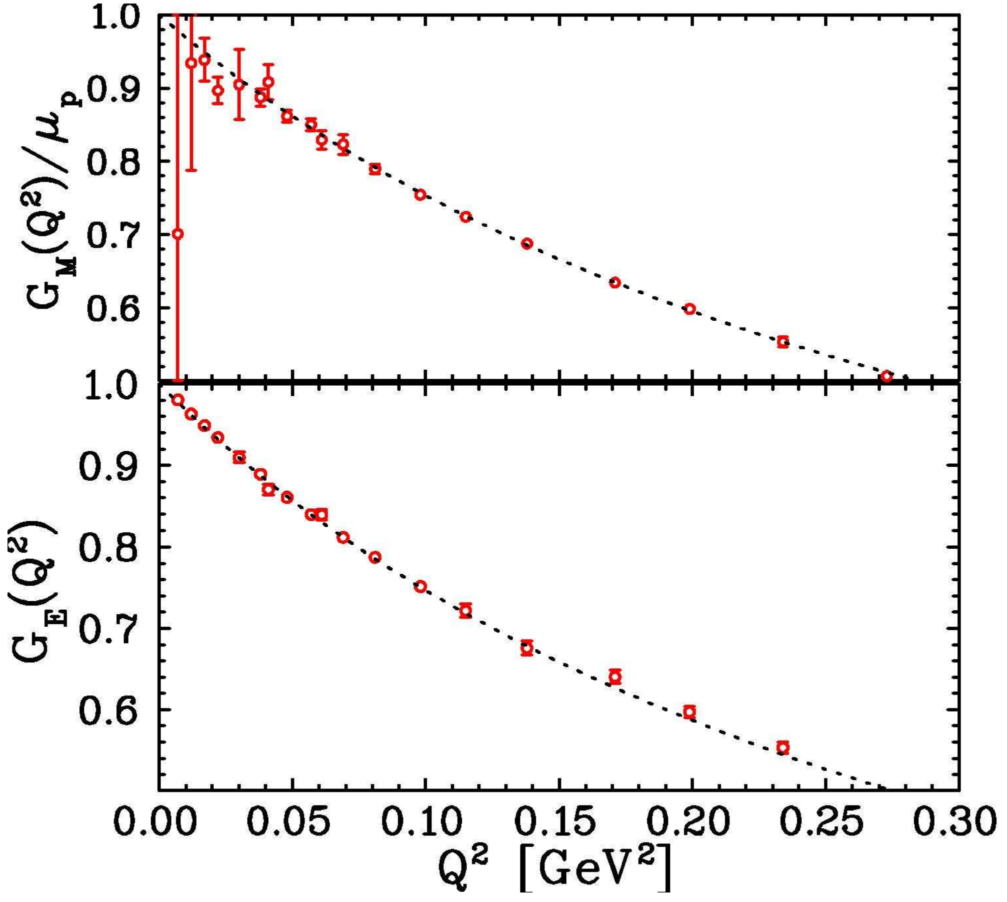
Difficulties in extracting the radius: overfitting

Need enough Q^2 range to constrain higher terms, but don't want to be dominated by high Q^2 data; Global fits almost always give poor estimates of the radii

More important for magnetic radius, where the precision on G_M gets worse at low Q^2 values

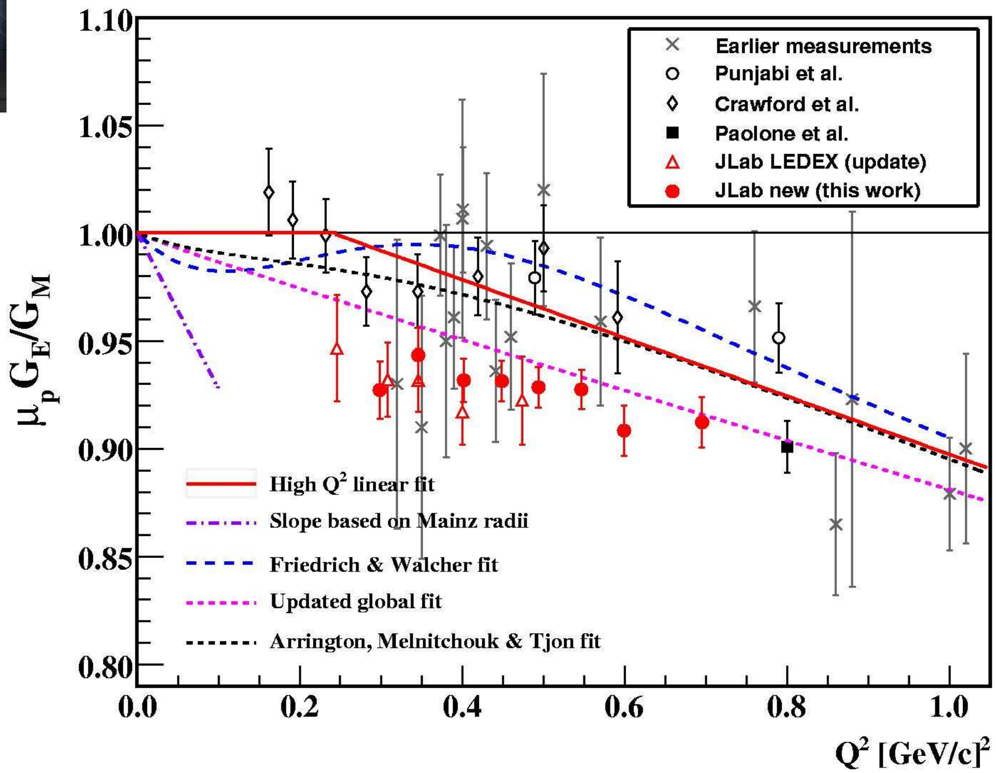
Can becomes more favorable to fit noise in high-precision data than give good fit to low Q^2 data:
Improve fit to the data, worsen extraction of the underlying physics

JA, W. Melnitchouk, J. Tjon, PRC 76, 035205 (2007)



Low Q^2 data:

- **JLab E08-007 and "LEDEX" polarization transfer data**
  - Extract ratio G_E/G_M
    - ~1% uncertainty
    - Q^2=0.3-0.7 GeV^2



- **Global fit directly to cross sections and polarization ratios**
  - Limit fit to low Q^2 data, TPE corrections applied to cross sections

- **Estimate model uncertainty by varying fit function, cutoffs**
  - Different parameterizations (continued fraction, inverse polynomial)
  - Vary number of parameters (2-5 each for G_E and G_M) [some over-fitting tests]
  - Vary Q^2 cutoff (0.3, 0.4, 0.5, 1.0 GeV^2)

Some other issues

Most older extractions dominated by Simon, et al., low Q^2 data
- 0.5% pt-to-pt systematics
- 0.5% normalization uncertainty

All other experiments quote >1-1.5% systematic, normalization uncertainties

Why is Simon, et al., so much better?

Neglects dominant uncertainty in Radiative Corrections (TPE)

- Global analysis applies uncertainty consistent with other data sets

<u>Relative normalization of experiments:</u>

- Typical approach: fit normalizations and then neglect uncertainty (wrong)

- Ingo Sick: Get uncertainty by varying based on quoted uncertainties (conservative)

- Our approach: Fit normalization factors, vary based on remaining uncertainty
    - Systematics → hard to tell how well we can REALLY determine normalization
    - We set minimum uncertainty to 0.5%

Low Q^2 data: Mainz high-precision cross sections

1422 measurements at 658 different kinematics (ε,Q^2 values)

- **~0.2% statistics**
- **0.02-0.9% uncorrelated syst.**
- **5+% normalization**

$Q^2 \approx 0.01$ to 1 GeV^2

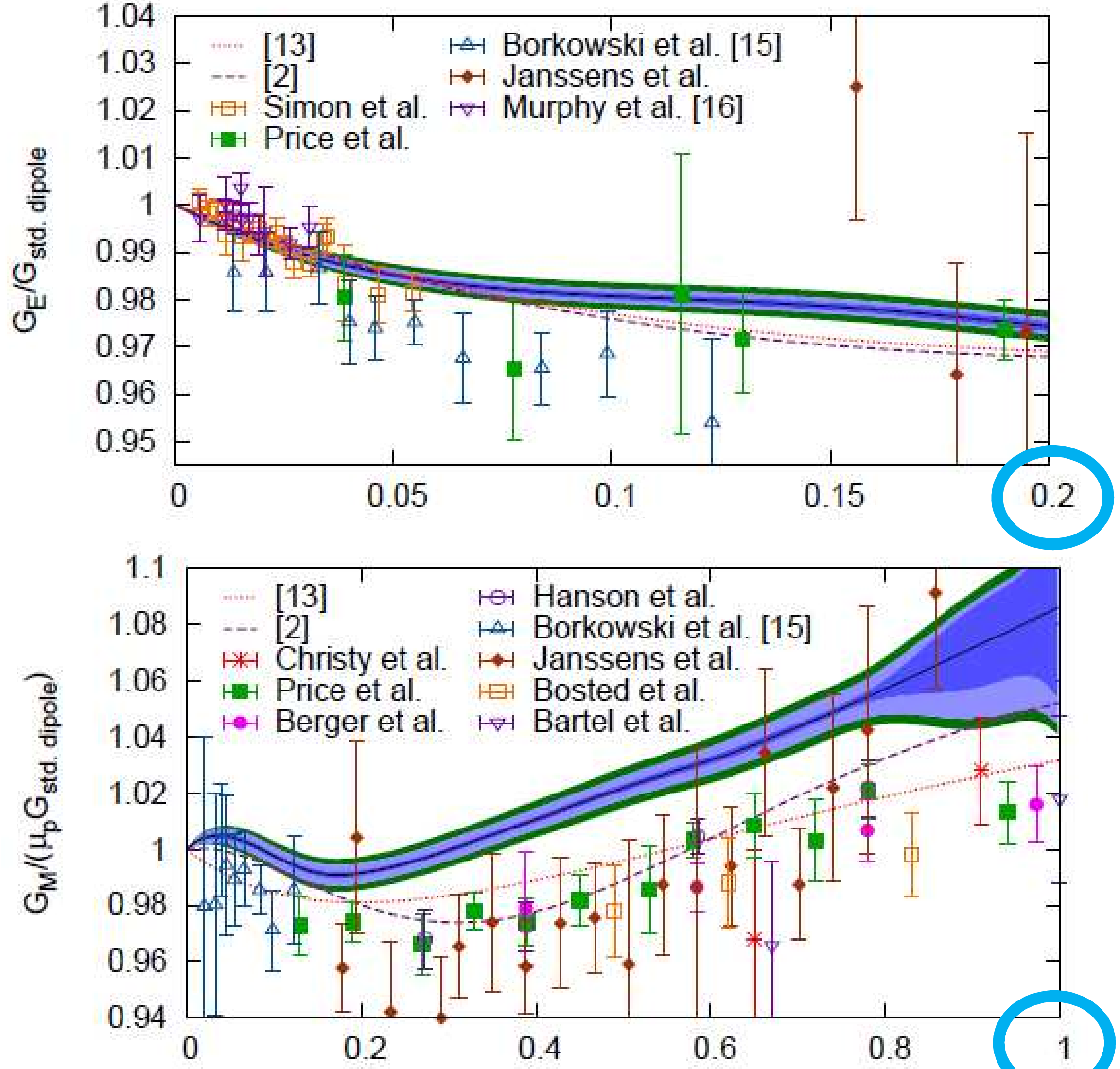
Wide range in θ (ε)

G_E, G_M obtained in global fit

- **Tension with previous G_E, G_M**

Bands include uncertainty based on small (<0.5%) θ-dependent correlated systematic

Details in previous talk



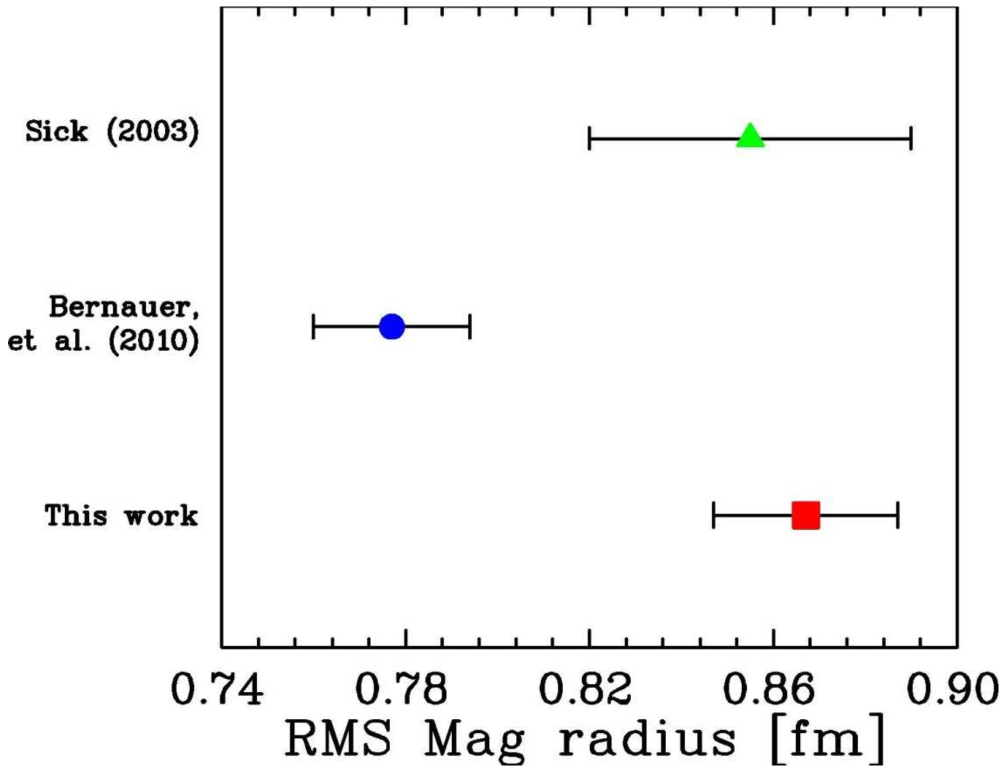
J. Bernauer, *et al.*, PRL 105, 242001 (2010)

Radius extractions from 2010

- Mainz and JLab charge radii agree:
  - 0.879(08) fm or 0.879(15) ?
  - 0.875(10) fm

- Significant (3.4σ) difference between Mainz and JLab results
  - 0.777(17) fm or 0.777(28)?
  - 0.867(20) fm

- Need to understand impact before we can reliably combine values for charge radius



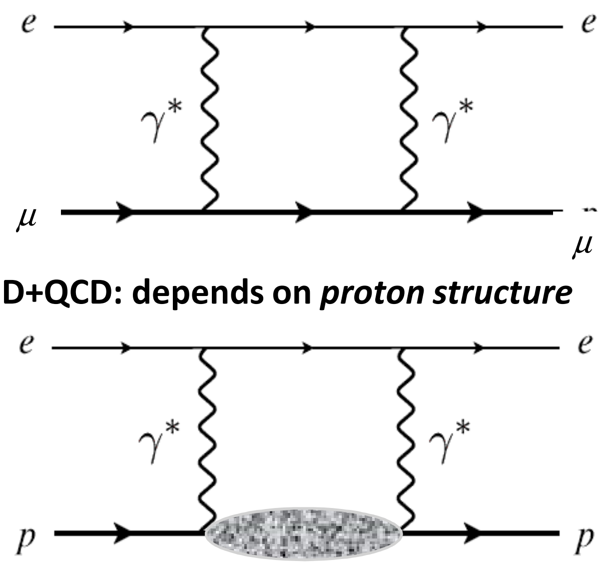
Next several slides: I take liberties with the Mainz data to test robustness of R_E extraction given R_M disagreements: Start with early estimates (2012)

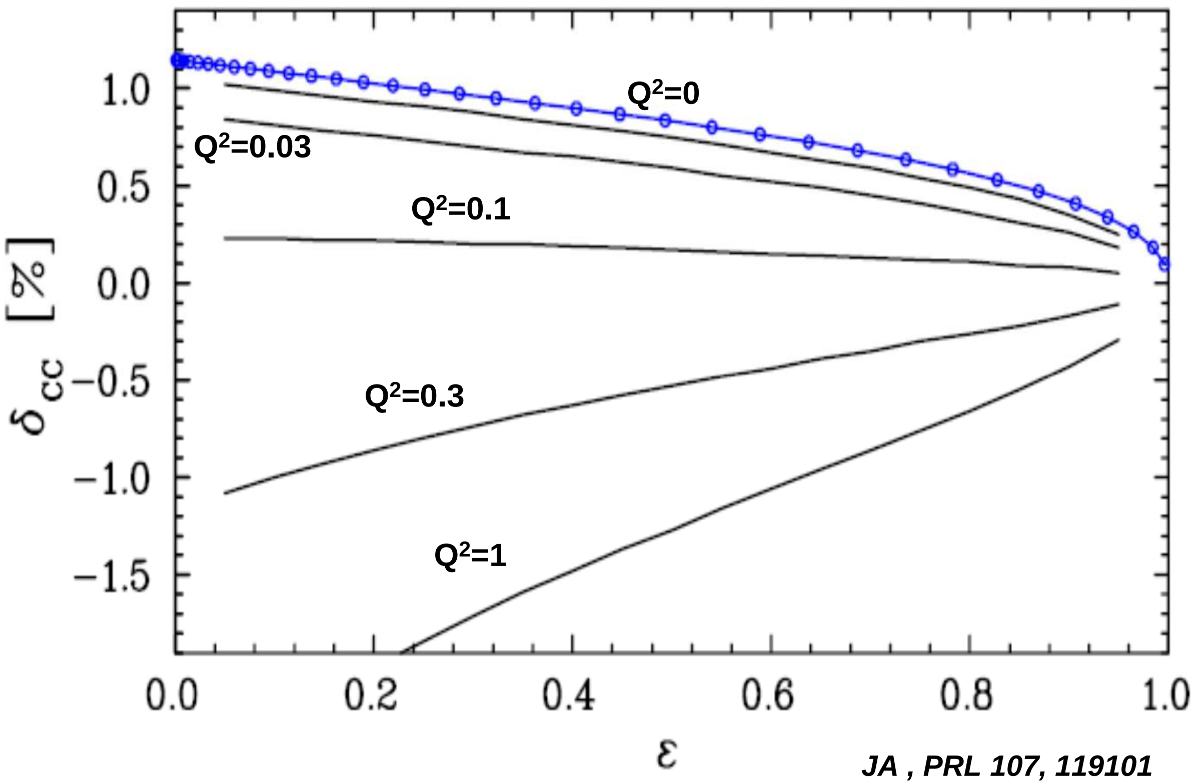
1) Two-photon exchange corrections

- **Mainz analysis applied Q^2=0 (point-proton) limit of TPE/Coulomb corrections**
  - **Correction has no Q^2 dependence, clearly important in extracting radii**
- **No uncertainty included in quoted charge, magnetic radii**

QED: straightforward to calculate

QED+QCD: depends on *proton structure*





JA , PRL 107, 119101

J.Bernauer, et al., PRL 107, 119102

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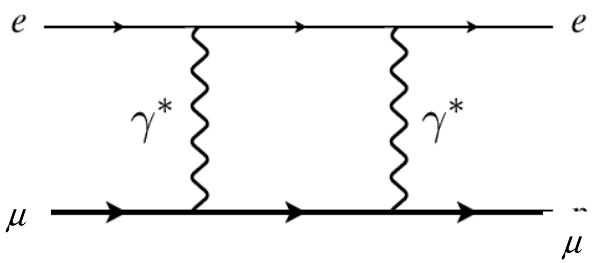
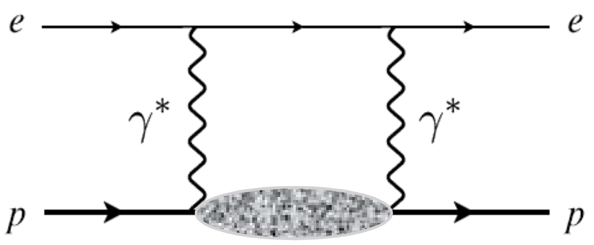
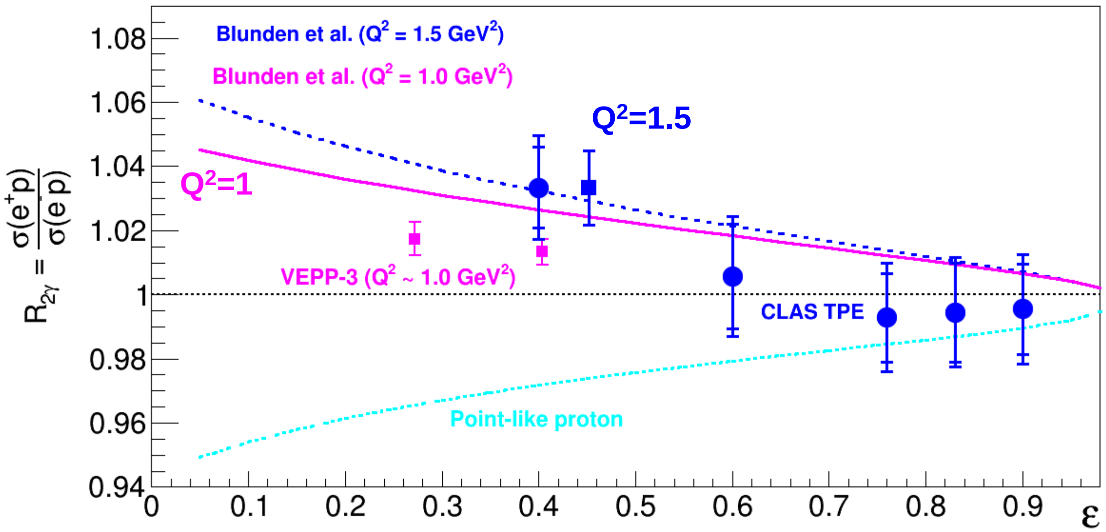


QED+QCD: depends on *proton structure*





Excellent agreement between TPE calculations for $Q^2 \leq$ 0.2-0.3 GeV^2; 0.002-0.003 fm change in radius

JA, JPG 40 (2013) 115003; G. Lee, JA, R. Hill, In preparation

Two new e+/e- comparisons show that sign of TPE corr changes by 1 GeV^2: Q^2=0 limit is off by >100%

D. Adikaram, et al., arXiv:1411.6908 (in press) [JLab-CLAS]

I.A.Rachek, et al., arXiv:1411.7372 (in press) [VEPP-3]

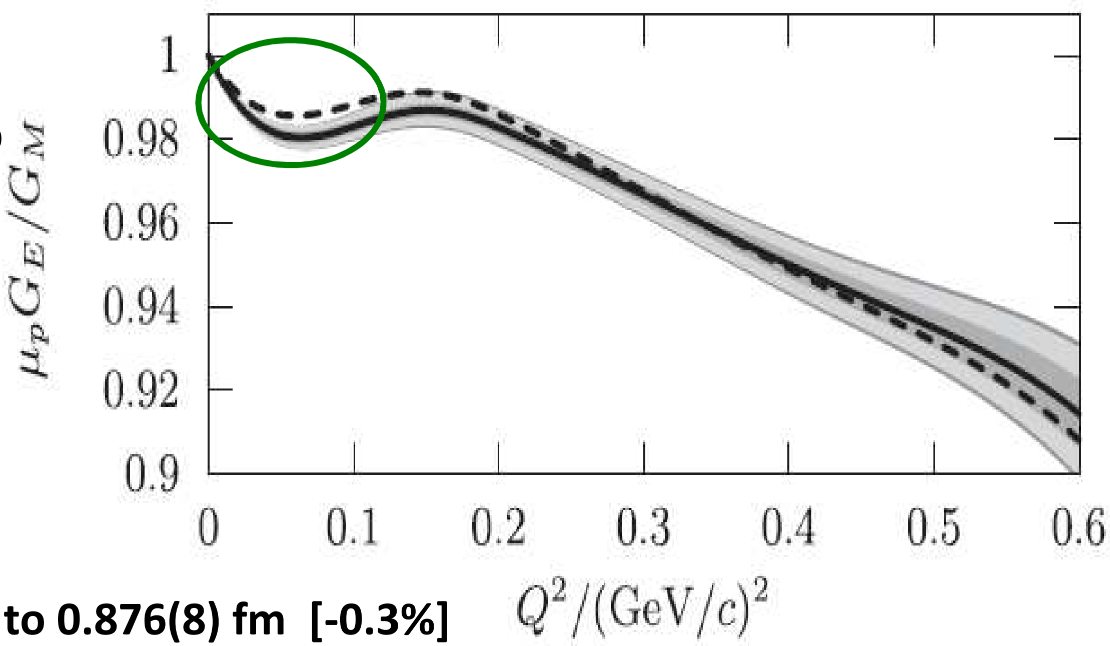
Impact of TPE

JA , PRL 107, 119101; J.Bernauer, et al., PRL 107, 119102

Apply low-Q^2 TPE expansion, valid up to Q^2=0.1 GeV^2

Borisyuk/Kobushkin, PRC 75, 028203 (2007)

Change 'small', but larger than linear sum of all quoted uncertainties, including 50% variation on their TPE



RADII: $\langle r_E^2\rangle^{1/2}$ goes from 0.879(8) to 0.876(8) fm [-0.3%]

$\langle r_M^2\rangle^{1/2}$ goes from 0.777(17) to 0.803(17) fm [+3.0%]

Note: uncertainties *do not* include any TPE contribution

Excellent agreement between TPE calculations for $Q^2 \leq$ 0.2-0.3 GeV^2
Range of results yields 0.002-0.003 fm range in radius

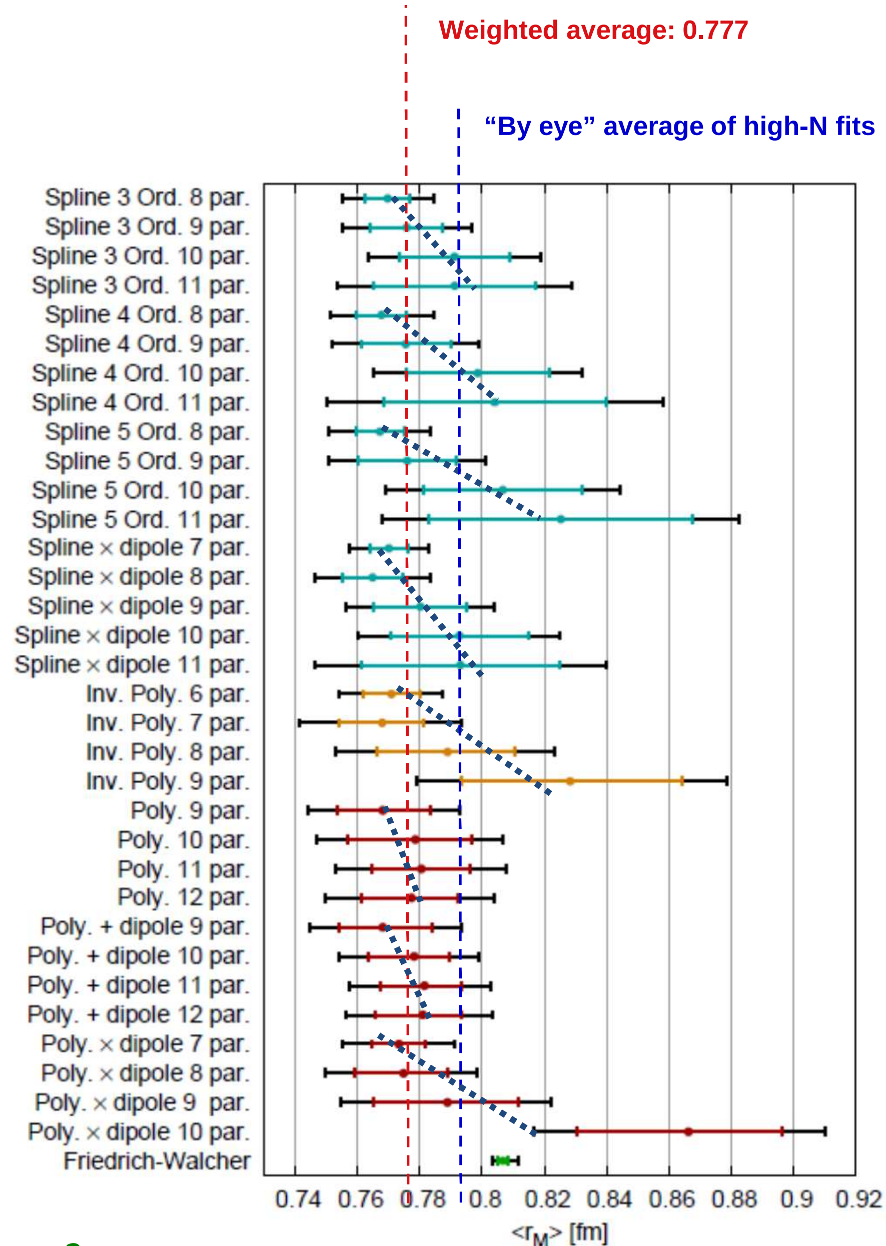
JA, JPG 40 (2013) 115003; G. Lee, JA, R. Hill, In preparation

An important correction, source of uncertainty

At this point, no reason to quote results that don't include more realistic TPE

2) Averaging of fits?

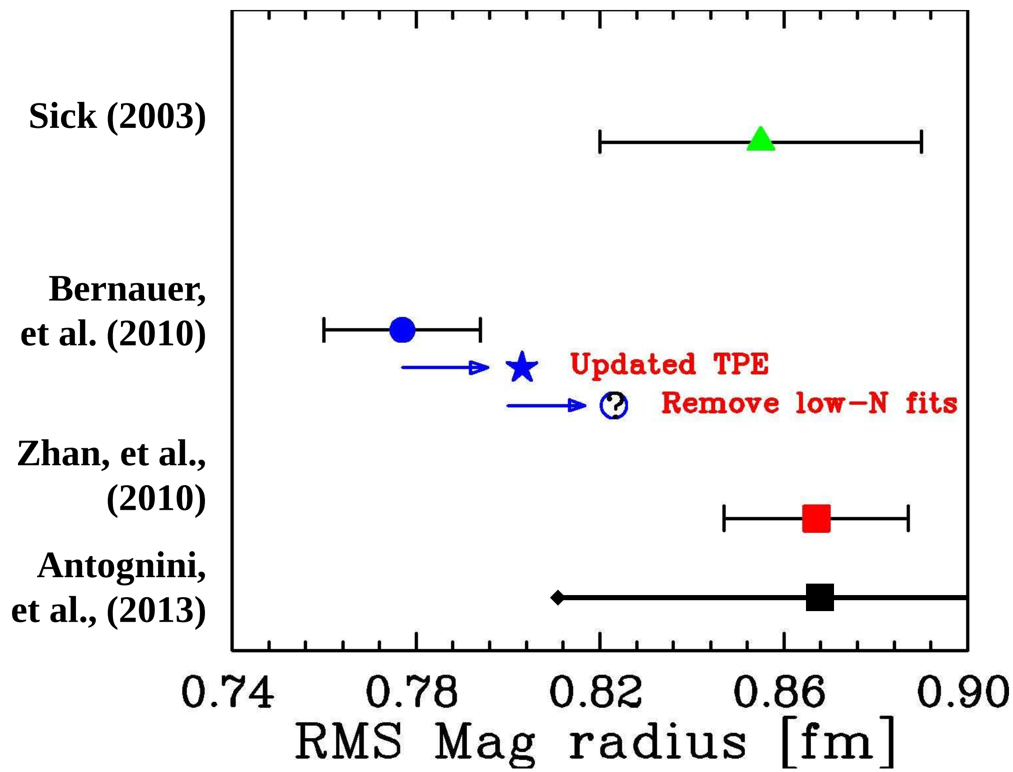
- Limited precision on G_M at low Q^2 means that more parameters are needed to reproduce low Q^2 data
  - → Low N_{par} fits may underfit data
- Greater statistical power at high Q^2 could also yield overfitting
- Statistics-weighted average of fits biases results towards small N_{par}
- If fits with more parameters to be more reliable
  - – Increase $\langle r_M \rangle^2$ by ~0.020
  - – Increase "statistical" uncertainty



No such systematic variation in $\langle r_E \rangle^2$ extraction

Proton magnetic radius

- **Updated TPE yields ΔR_M=0.026 fm 0.777(17) → 0.803(17)**
- **Remove fits that may not have sufficient flexibility: $\Delta R \approx 0.02$ fm?**
- **Mainz/JLab difference goes from 3.4σ to 1.7σ, less if include TPE uncertainty**
- **R_E value has small change: 0.879(8) → 0.876(8)**



These are important issues that clearly matter for R_M extraction, but impact on R_E is much smaller

More Detailed Examination of Mainz data

- **Looking at anything which *might* be an issue; doesn't imply that they do cause any problems**

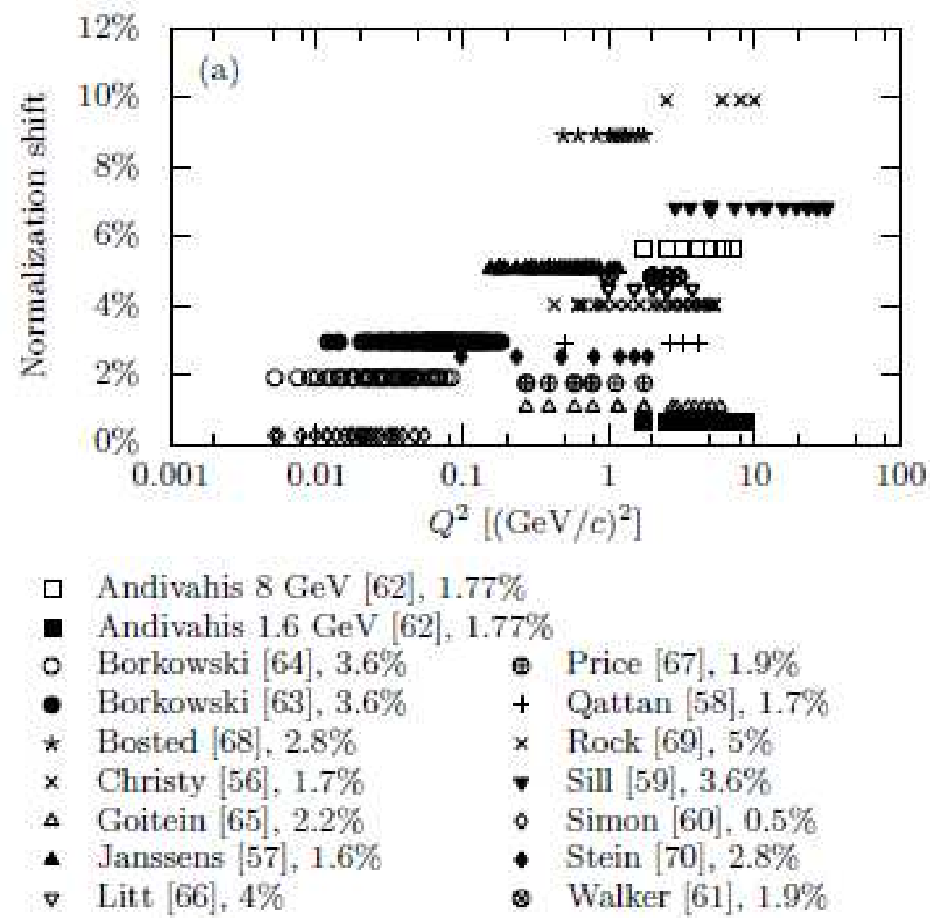
- Need to worry about under-fitting
  - Large 'truncation' errors if not enough parameters for the data
  - Limiting Q^2 range helps but limits lever arm for the fit
  - Linear fit always underestimates the radius

- Need to worry about over-fitting the data: G_M in particular
  - Uncertainties in extracted radius grow with number of parameters
  - Fit sometimes optimized by fitting high-Q^2 fluctuations rather than low Q^2 data
  - Important to have realistic systematic uncertainty estimates

- Other issues
  - Unusual analysis of statistical/systematic uncertainties
  - Uncertainty dominated over large kinematic range by possible unknown correlated error
    - One particular model chosen to evaluate this, what about other possibilities?
  - TPE (correction and uncertainties)

Potential issues

- Mainz global analysis shifts normalization up for all 16 previous data sets, several by 2-3 times the quoted uncertainties
- Data as released includes normalization from fit – floated without constraint. No information on original normalizations
- Uncertainty neglects dominant radiative correction contribution: TPE
- Always fit full data set, up to ~1 GeV^2
- Unusual procedure to extract systematics yields range of uncorrelated systematic uncertainties of 0.02%-0.9%



z expansion

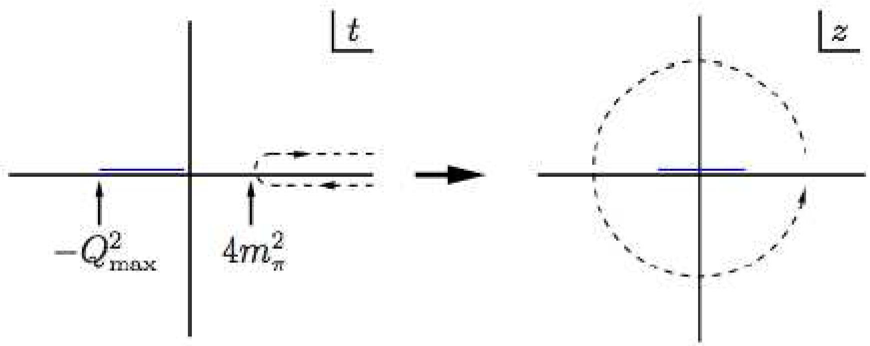
- Inverse polynomial, Pade, continued fraction, etc… are arbitrary expansions
- Analytic structure suggests better approach: z expansion

R. Hill and G. Paz, PRD 82 (2010) 113005; Z. Epstein, G. Paz, J. Roy, ???? (2014) ????

We can map the domain of analyticity onto the unit circle

$$z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}}$$

where $t_{\text{cut}} = 4m_\pi^2$, $z(t_0, t_{\text{cut}}, t_0) = 0$



- Parameterize form factors as Taylor expansion in z
- Unitarity implies bounds on the coefficients of the expansion
  - Allows fit to very high order with natural way to avoid fitting noise
  - Addresses over-fitting and under-fitting issues very naturally

Initial tests: unbounded z expansion

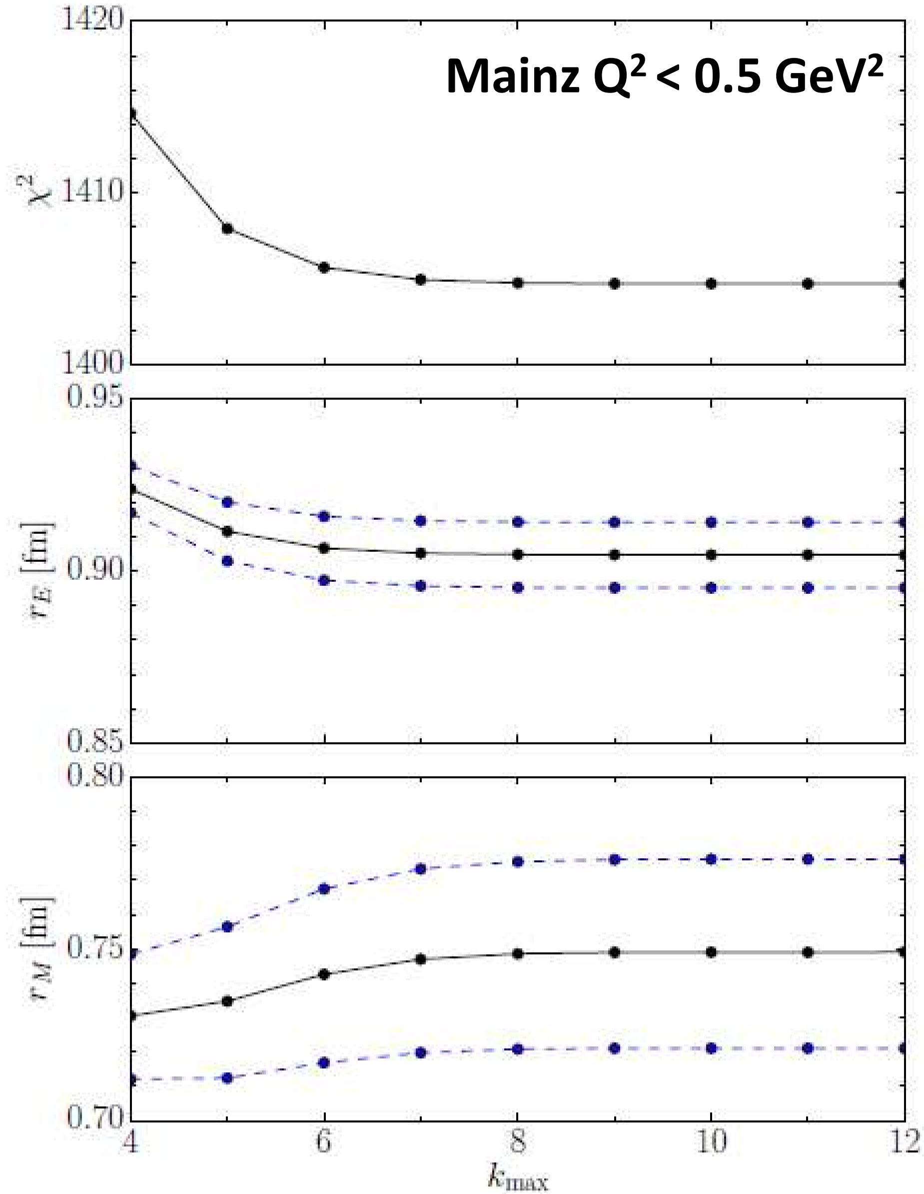
- Results similar to Lorenz, et al. [from Mainz proton radius workshop presentation]

Fit type	r_E [fm]	Δr_E [fm]	r_M [fm]	Δr_M [fm]	χ^2	$\chi^2_{\rm red}$
z5	0.89198	0.00190	0.65808	0.01852	1734.79570	1.25619
z6	0.86812	0.00305	0.75436	0.05430	1601.45398	1.16132
z7	0.86761	0.00455	1.11954	0.08674	1582.94525	1.14956
z8	0.87632	0.00691	0.86072	0.29806	1575.82246	1.14605
z9	0.84897	0.01044	0.50819	N/A	1561.87579	1.13756
z10	0.83524	0.01430	-1.30181	N/A	1558.71133	1.13692
z11	0.83506		-1.31151	N/A	1557.88067	1.13797
z12	0.83670		0.97606	N/A	1560.15607	
z13	0.83710		0.37594	N/A	1556.66323	
z14	0.83755		0.68586	N/A	1556.26962	
z15	0.83971		0.61276	N/A	1556.20386	

- 5 parameter fit insufficient (chi-squared still decreasing)
- More parameters yields decreasing radius, but increasing uncertainties
- Do more parameters yield decreasing R_E because of extra flexibility or because of overfitting data at larger Q^2 that is not sensitive to the radius?

z expansion with bounds

- Apply bound by including chisquared contribution for each fit parameter
- Constraint on parameters prevents fitting noise to improve chisquared
- Reduces ability to trade off one parameter against another



Model dependence: z expansion

- Apply gaussian bound on a_k/a_0 – conservative bound of 5, Q^2<0.5 GeV^2

  R_E = 0.905(9), R_M = 0.749(28)

- Vary details of fit, examine impact (generally largest observed) on charge radius

ΔR_E < 0.001	**8 vs 12 parameters (uncertainty also almost unchanged)**
ΔR_E = 0.003	**Increase gaussian bounds to 10**
ΔR_E << 0.001	**Remove bound from slope parameter [avoid bias towards slope=0]**
ΔR_E = 0.002	**Choice of t_0 [=0 vs t_{opt}, which yields minimum |z| for Q^2 range]**
ΔR_E = 0.009	**Rebin to 658 independent points (no effect) and apply fixed 0.3% syst.**
ΔR_E = 0.002	**Vary TPE by approximate range of low-Q^2 calculations**

Rebinning: Where multiple runs are taken at the same kinematics with the same spectrometer, we combine them into a single point. Distribution of chisquared and confidence levels is consistent with run-to-run scattering being entirely dominated by statistics, with only handful of examples were a non-statistical scatter is at the ~0.1% level (and one point which was excluded)

Correlated systematics

- With so many data points, effects which yield a correlated shift may not be well represented with common approach of extra 0.5 or 1% applied to each point

- Mainz approach: take each angle-beam energy combination (18 total). Apply shift to data set that varies linearly from 0 at minimum angle to a value close to 0.5% at the maximum. Yields **ΔR_E = 0.004 fm, ΔR_M = 0.009 fm**

- We perform similar tests, choosing corrections varying from 0 to 0.5% for each data subset. However, we do the following:
  - Test several dependences (theta, epsilon, Q^2) for the correction on each subset
  - Vary how we break up data sets (one per spectrometer (3), one per beam-energy combo (18), every independently normalized subset (31))
  - Apply offset to all 3 spectrometers, or only to just one spectrometer

- We find many cases where larger corrections are found, as large as **ΔR_E = 0.027 fm, ΔR_M = 0.040 fm**
  - These are not 'typical' changes, but several cases yield similar effects
  - Not a 1-sigma uncertainty, but reasonable estimates of how big these effects could be without a significant change in size from what is quoted

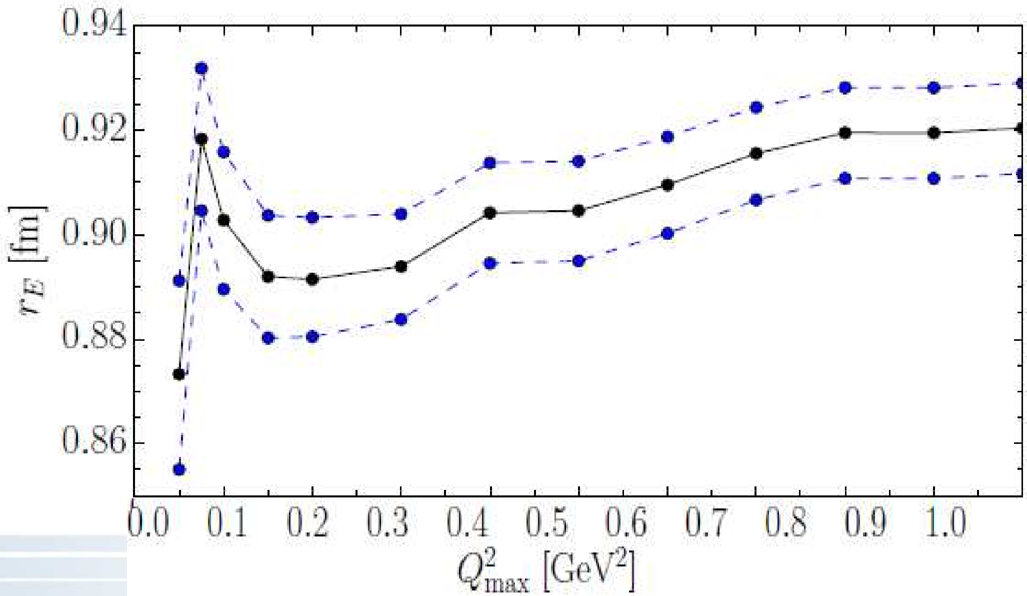
Reexamination of Mainz data: conclusions

- Primary fit [rebinned data, stat and 0.3% sys only]: **R_E = 0.908(13), R_M = 0.727(38)**
- Model dependence tests suggests uncertainties below quoted fit uncertainties
- Correlated syst. tests yield **shifts up to 0.027 fm, 0.040 fm for R_E, R_M**
  - Reasonable "one-sigma" error might be half that
- Essentially all fits/tests give R_E >= 0.90 fm
  - Only exceptions are unbounded fits [polynomial, inv. Poly, z expansion]

One possibility: The very low Q^2 data prefers smaller radius, which is not possible with bounded expansion. Also consistent with our fits looking at very low maximum Q^2(figure) and perhaps the tension with other data sets, if very low Q^2 points introduce a 'wiggle' and change higher Q^2 normalizations.

Impact of the bump/wiggle structure, as noted by A1 collaboration. Is such structure physical?

If not, bounded z-expansion should reduce impact of the these low Q^2 data on normalization at all Q^2



Reexamination of Mainz c

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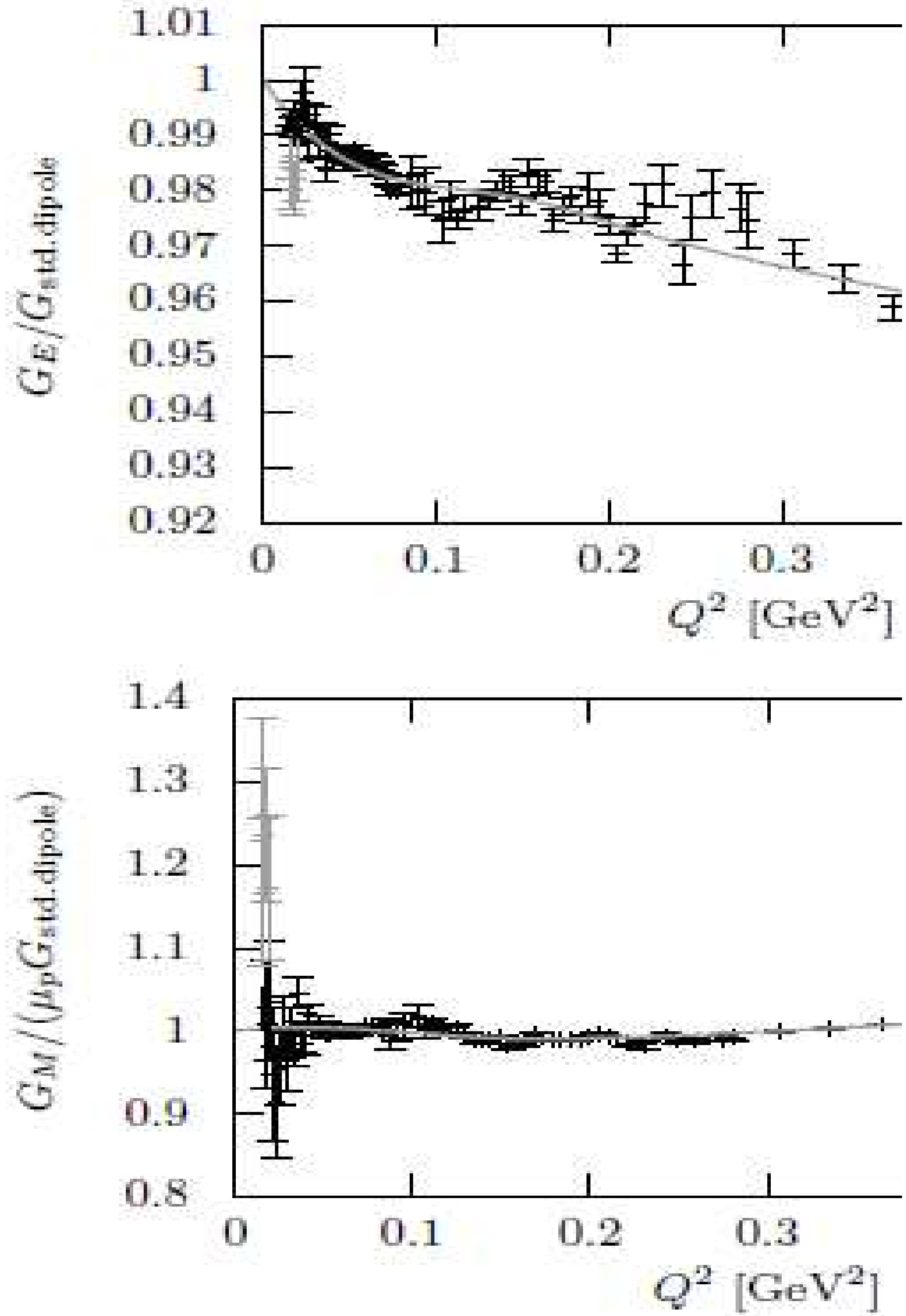

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If not, bounded z-expansion should reduce impact of the these low Q^2 data on normalization at all Q^2

Mainz vs World's data:

- Our primary fit, now with rough estimate of systematics, model-dependence:
  - **R_E = 0.908(13)(15), R_M = 0.727(38)(30)**

- Equivalent fit for World's data:
  - **R_E = 0.906(25), R_M = 0.912(39) Cross sections only**
  - **R_E = 0.922(23), R_M = 0.893(39) Cross sections + Polarization**

- Consistent charge radii, large difference in magnetic radius
- Bounded z expansion raises R_E for both Mainz and World's data

What else can be done?

- **New/stronger constraints in fit [Meissner, Sick presentations]**
- **New experiments/data**

Future low-Q^2 form factor measurements

- **Updated measurements at Mainz**
  - Measurements at lower Q^2 using Initial State Radiation (ISI)
  - Measure electron—deuteron scattering

- **Phase II of JLab polarization measurement (Hall A at JLab)**
  - Provide important constraints on low-Q^2 behavior of G_M

- **Very low Q^2 cross section measurements ("PRAD" - Hall B at JLab)**
  - Map out low-Q^2 behavior of G_E
  - Forward angle, nearly independent of TPE, G_M

- **Low Q^2 measurements of $e\pm$, $\mu\pm$ scattering cross sections ("MUSE" - PSI)**
  - Map out low-Q^2 behavior of G_E
  - Compare Two-photon exchange for leptons and muons
  - Direct e-μ comparison

Future low-Q^2 form factor measurements

- ✓ **Updated measurements at Mainz**
  - Measurements at lower Q^2 using Initial State Radiation (ISI)
  - Measure electron—deuteron scattering

- ✓ **Phase II of JLab polarization measurement (Hall A at JLab)**
  - Provide important constraints on low-Q^2 behavior of G_M

- **Very low Q^2 cross section measurements (“PRAD” - Hall B at JLab)**
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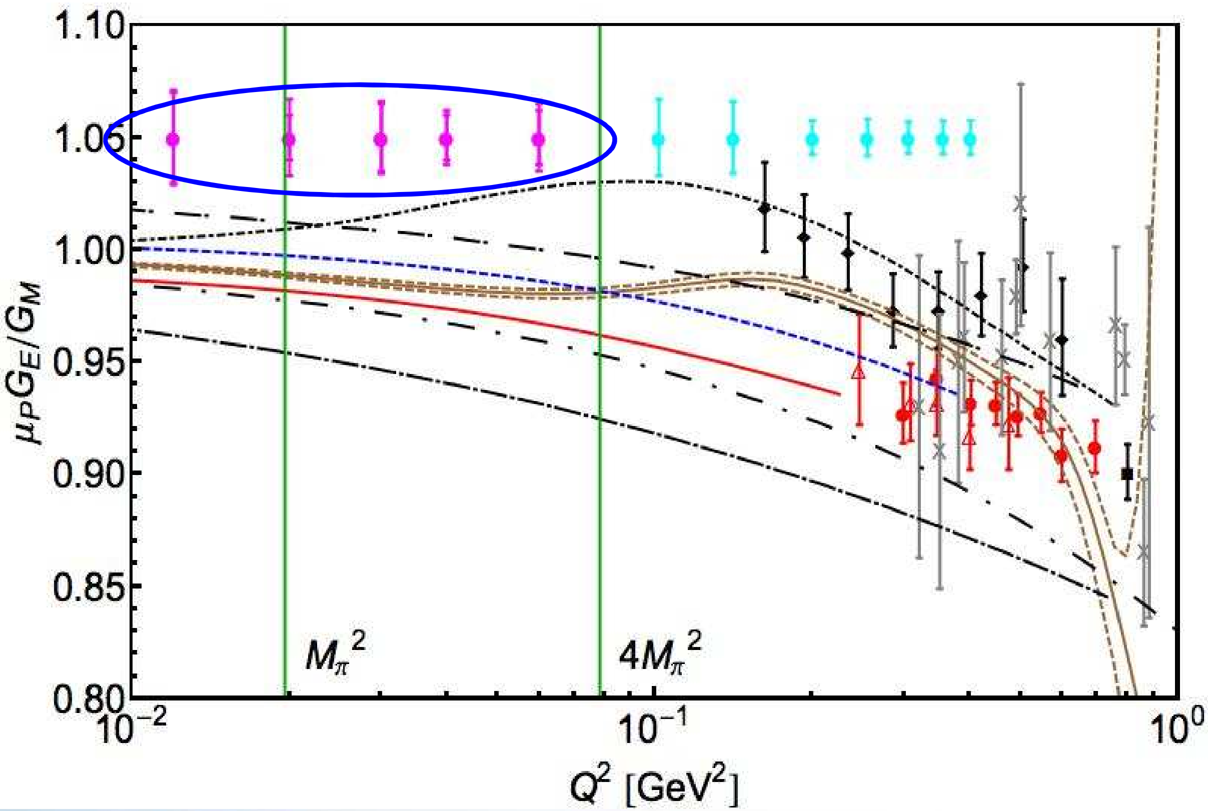
JLab: Low Q^2 Polarization measurements

Polarization transfer: X. Zhan, et al., G. Ron, et al.

Polarized target: Spring 2012

- **Kinematics, precision reduced due to magnet configuration change (cyan points lost)**
- **Extract R down to $Q^2 \approx 0.01$**
  - **Better constrain normalizations**
  - **Improves G_M extraction**
- **Linear approach to Q^2=0?**

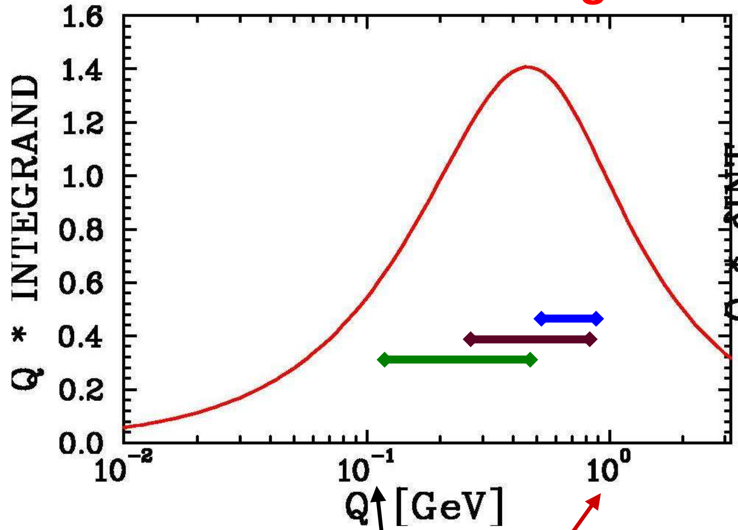


Impact of low Q^2 form factor measurements

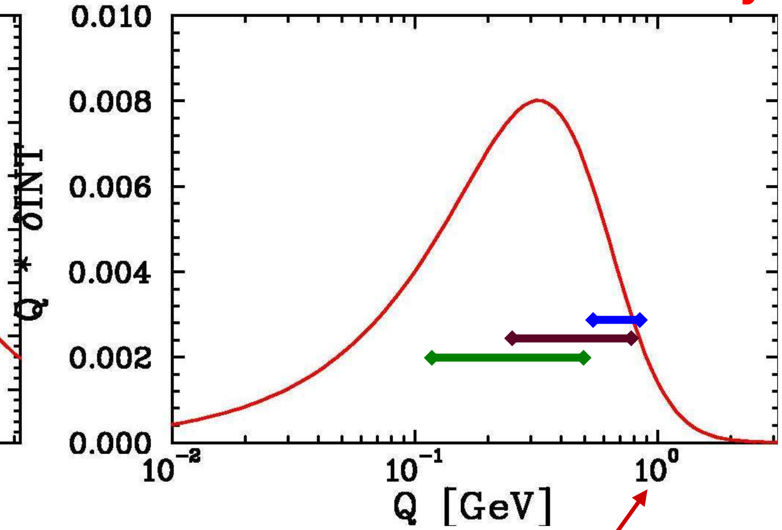
Hyperfine spitting structure correction: $[1-G_E(Q^2)G_M(Q^2)/\mu_p]$ / Q^2

- **$1/Q^2$ term suppresses high-Q^2**
- **$[1-G_E(Q^2)G_M(Q^2)/\mu_p] \rightarrow 0$ as $Q^2 \rightarrow 0$**

Zemach radius: Integrand



Zemach radius: Uncertainty

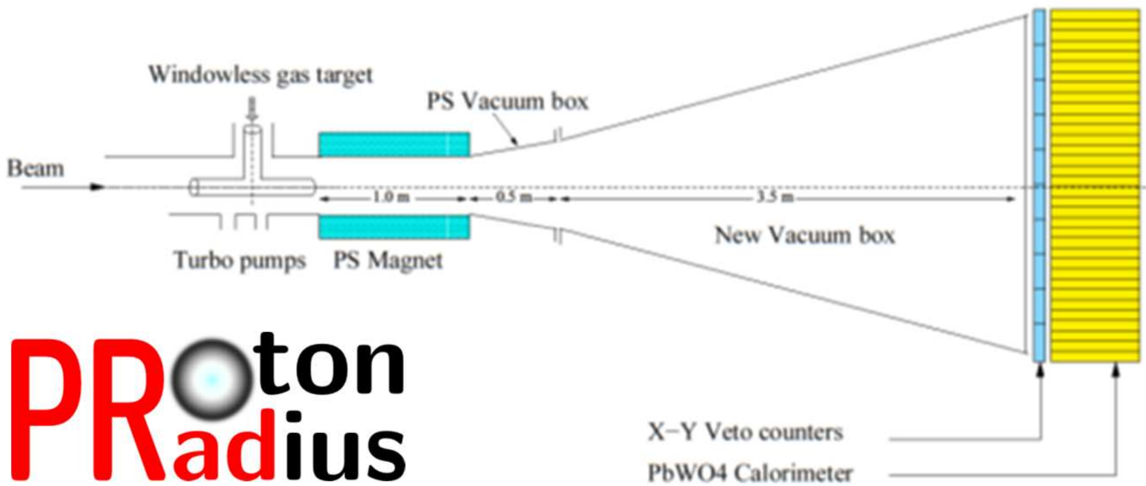


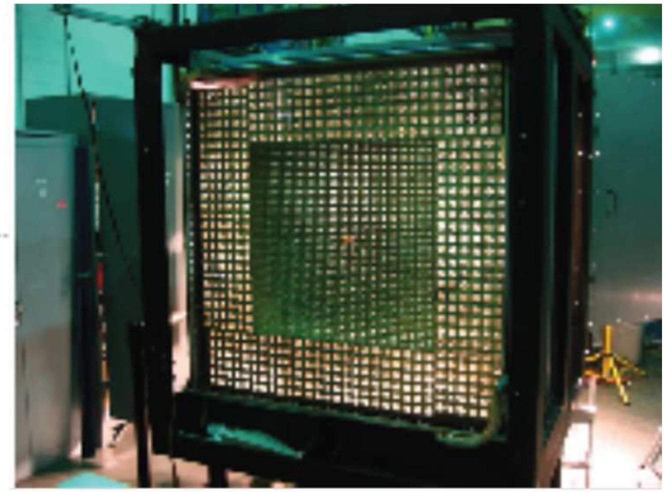
JLab Phase I
Mainz
Phase II (2012)

Significant contribution to integral above Q^2=1 GeV^2 and below Q^2=0.01 GeV^2

Negligible contribution to uncertainty above Q^2=1 GeV^2

PRAD: Hall B at JLab (very low Q^2)





Windowless target, 1-2 GeV beam

Small-θ calorimeter: magnet free

θ=0.7-4° : TPE, GMp suppressed

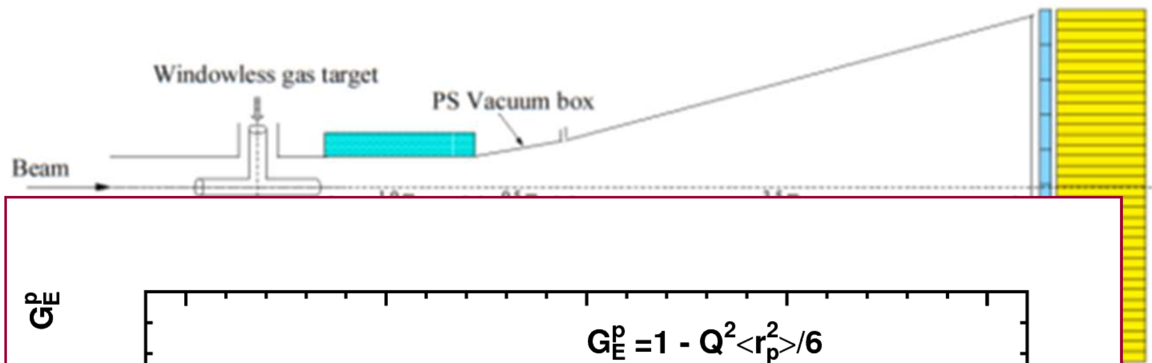
Normalize e-p to e-e (Moller)

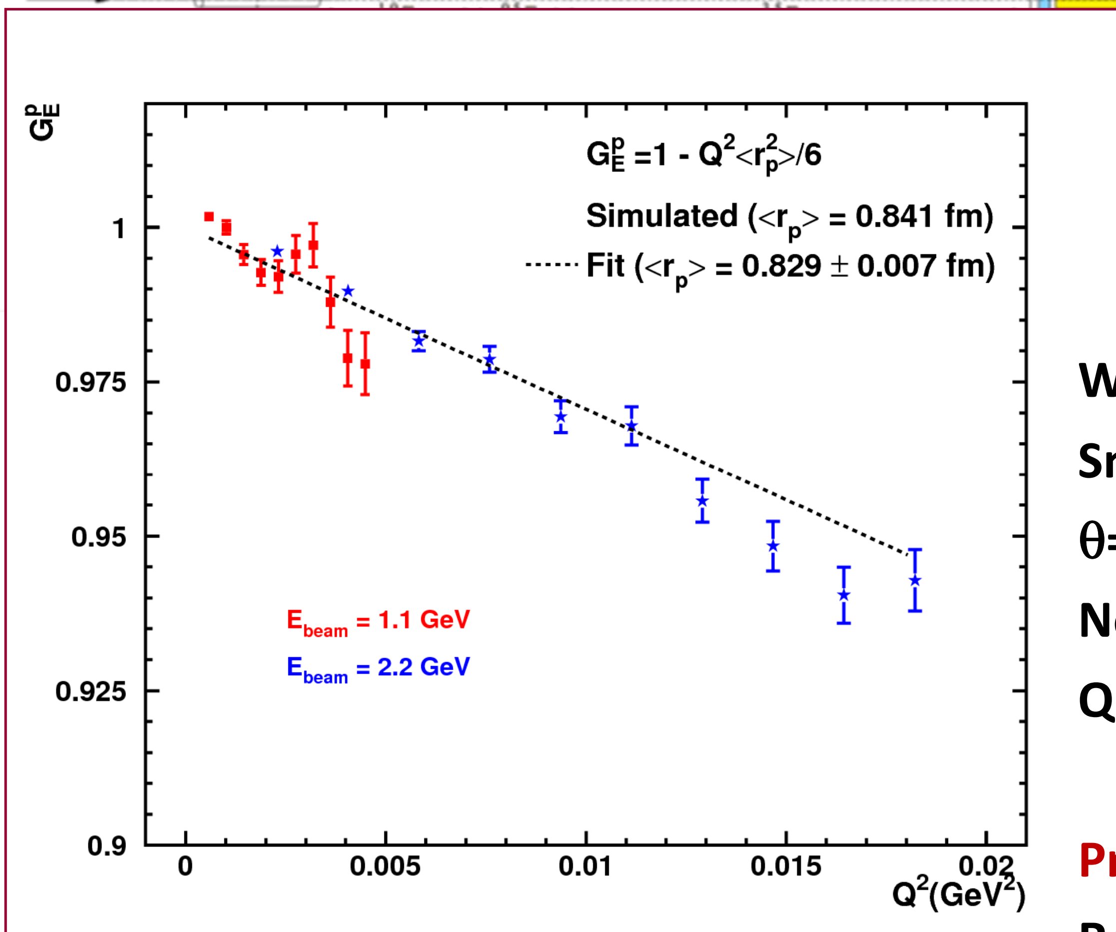
Q^2 from 0.0002-0.02 GeV^2

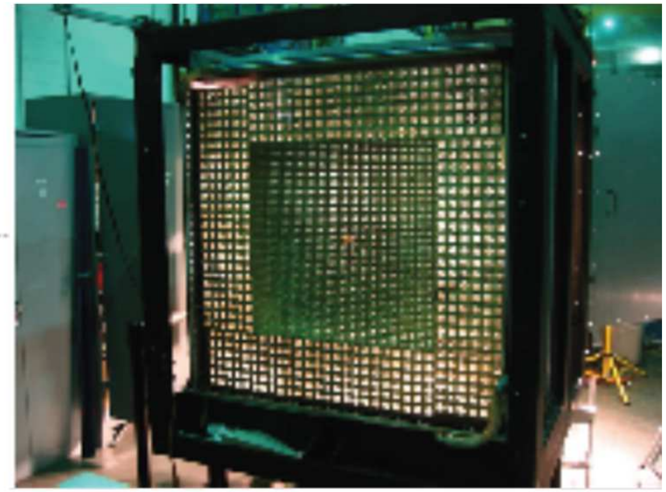
E=3 GeV → 0.14 GeV^2

Project <0.01 fm uncertainty on R_E

PRAD: Hall B at JLab (very low Q^2)







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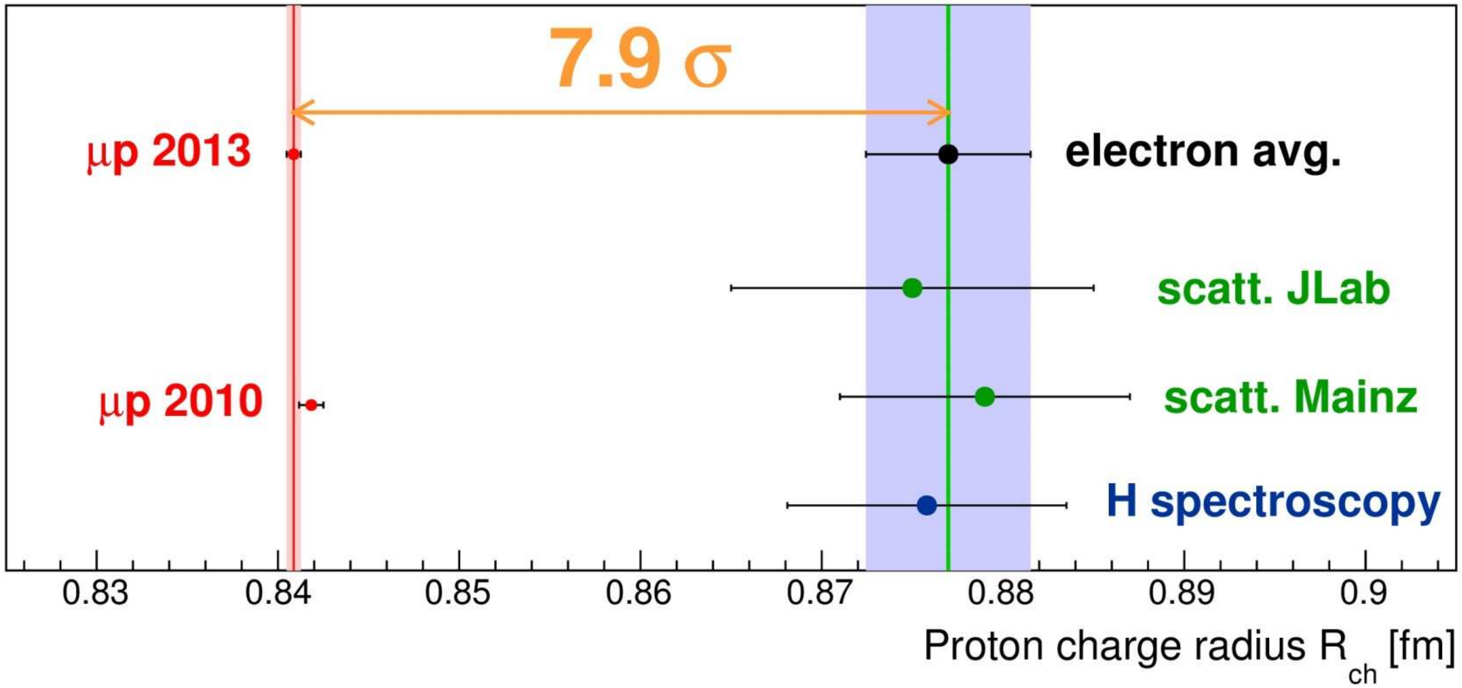
Readiness review in March

Proton Charge Radius



	Muon	Electron
Spectroscopy	0.8409(4)	0.8758(77)
Scattering	???	0.8770(60)

MUSE: **Further test and improve electron scattering results**

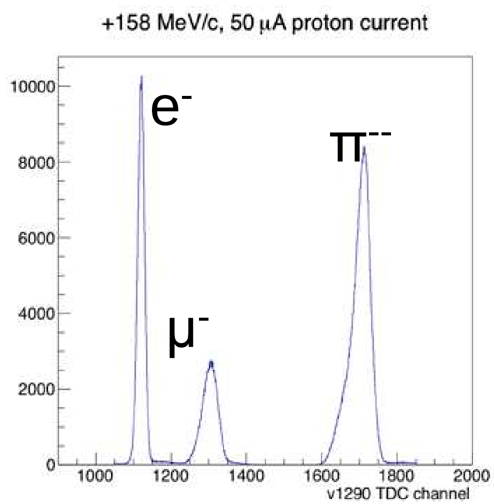
Fill in the muon scattering case

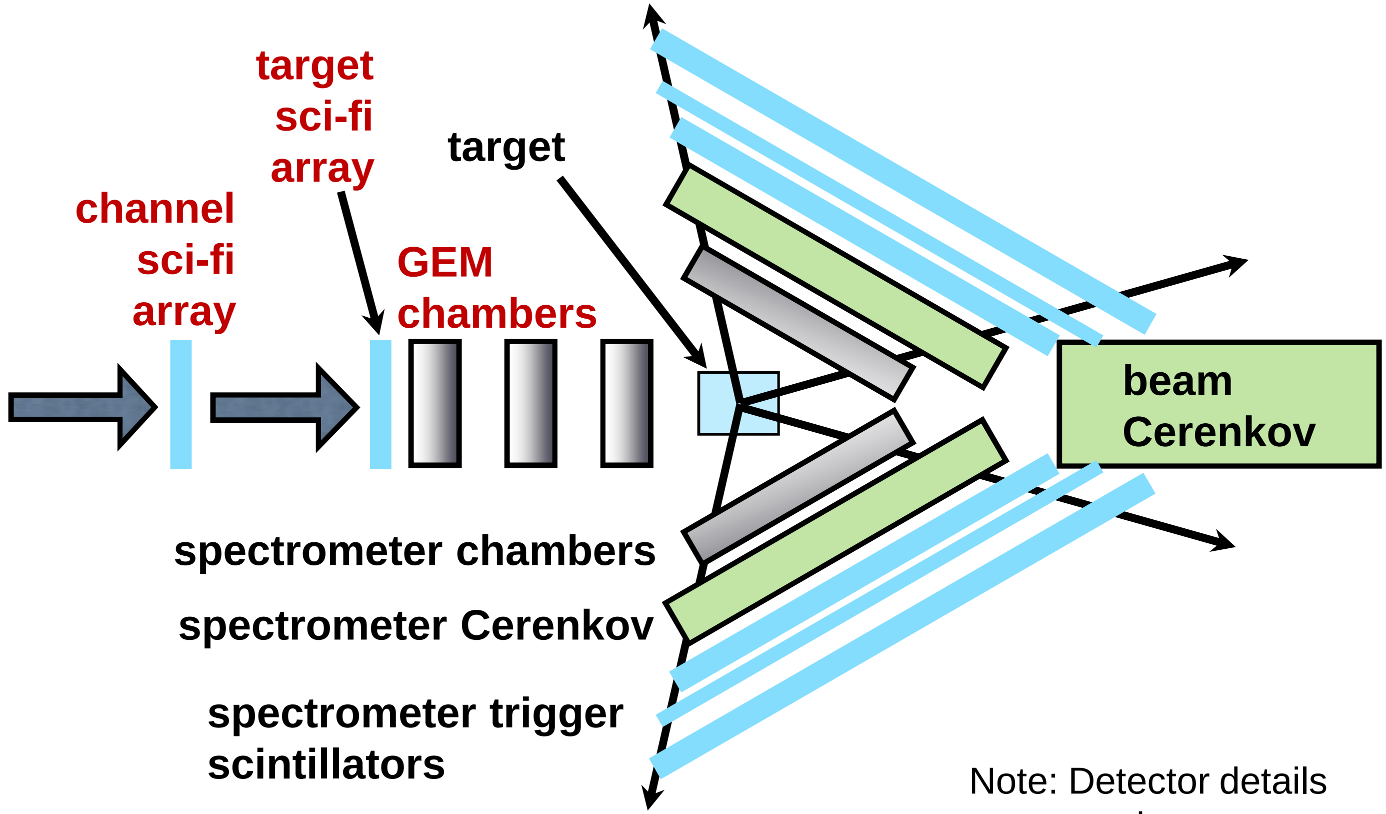
“MUSE” - MUon Scattering Experiment [PSI]

R. Gilman, et al., arXiv:1303.2160

$e^{\pm}$ / $\mu^{\pm}$ / $\pi^{\pm}$ beams

115, 153, 210 MeV/c





Note: Detector details not up to date

Beams of electrons, pions, and muons:

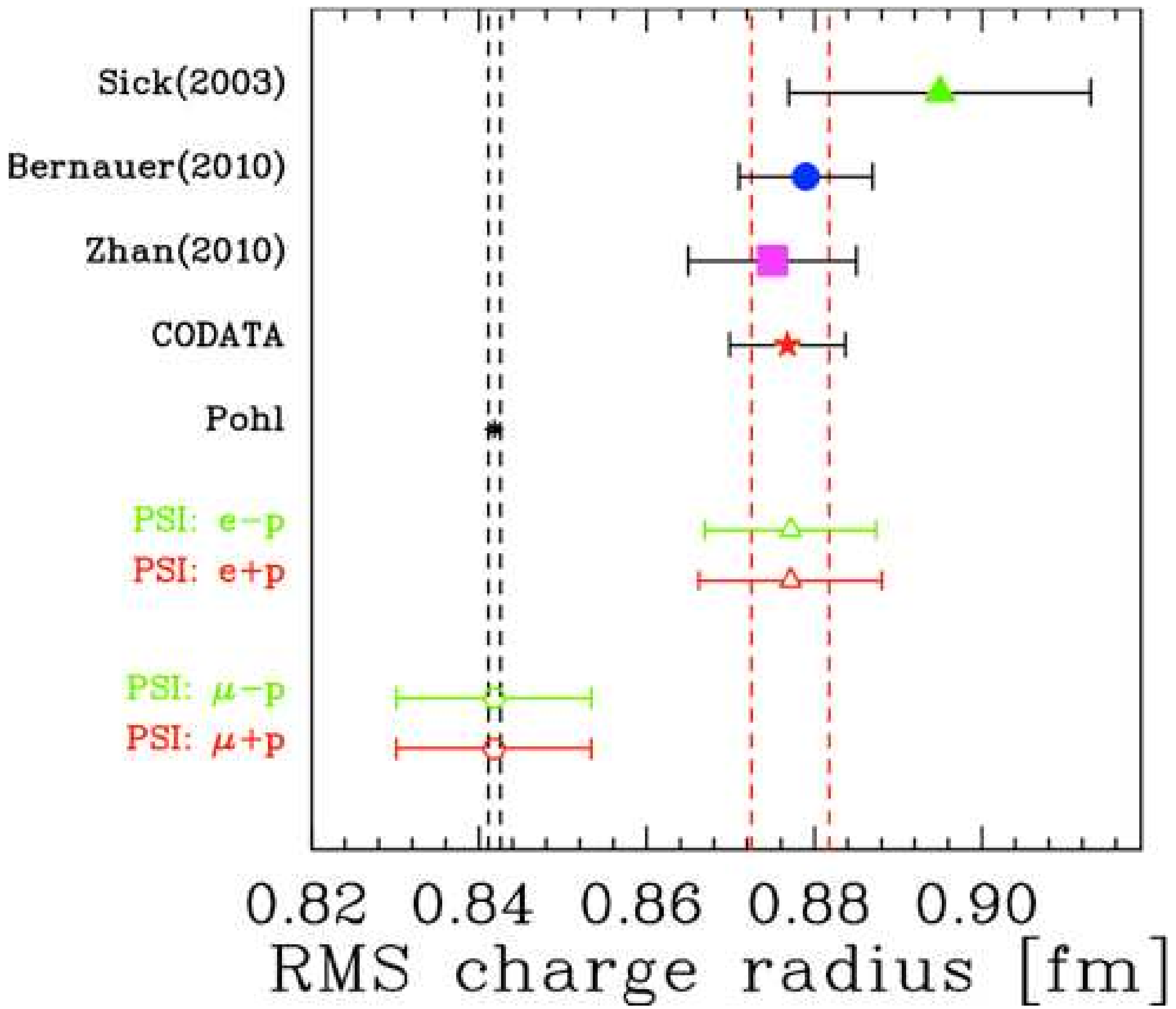
- Very low Q^2 (reduced extrapolation)
- Compare e– and e+ (opposite TPE correction)
- Compare μ– and μ+ (compare electron/muon TPE corrections)

PSI proposal: Projected results

Charge radius extraction limited by systematics, fit uncertainties

Comparable to existing e-p extractions, but not better

Many uncertainties are common to all extractions in the experiments: Cancel in e+/e-, μ+/μ-, and μ/e comparisons



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Charge radius extraction limited by systematics, fit uncertainties

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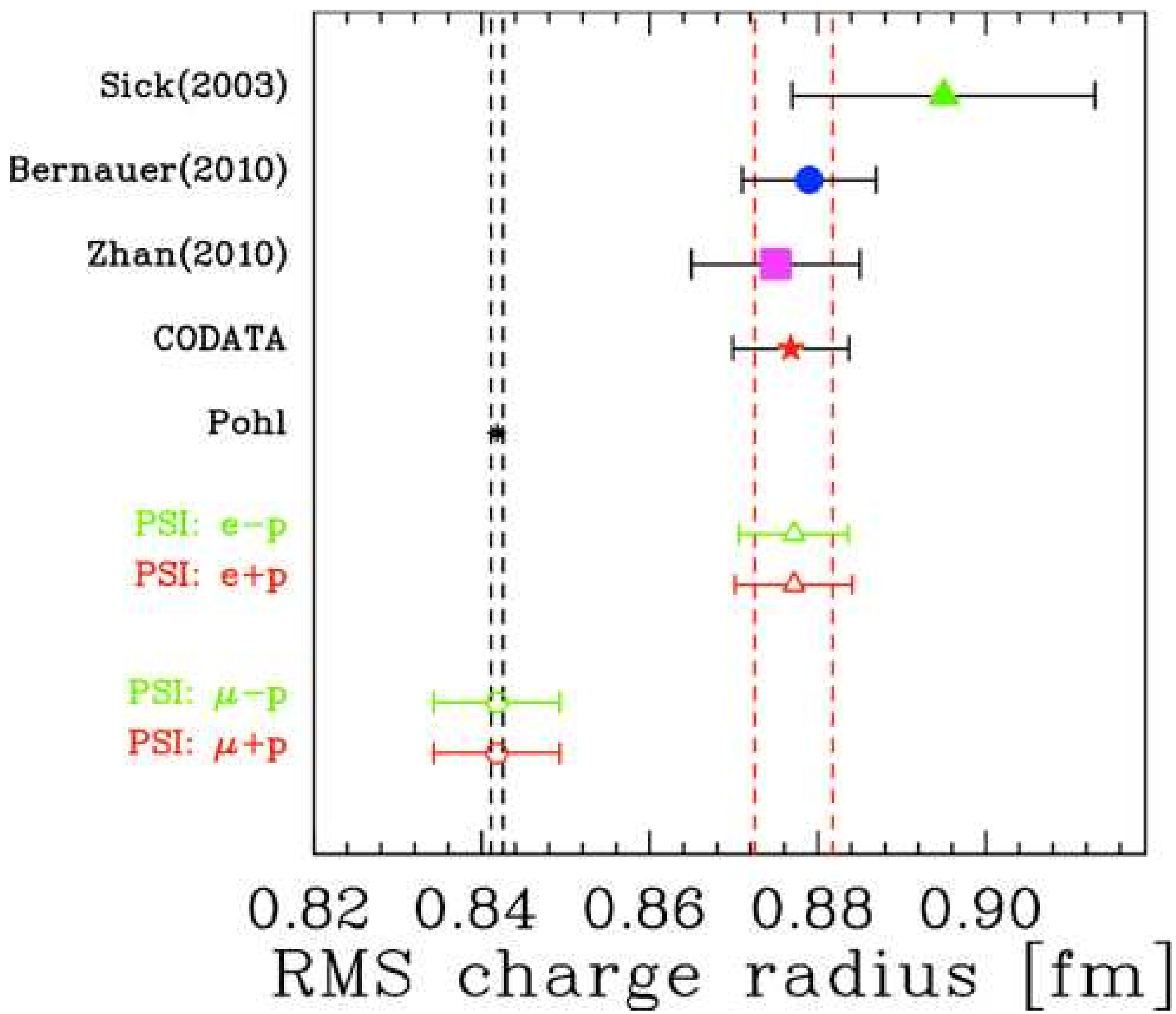
Many uncertainties are common to all extractions in the experiments: Cancel in e+/e-, μ+/μ-, and μ/e comparisons

Precise tests of TPE for e and μ as well as other electron-muon differences



Summary

- **Inconsistency between muonic hydrogen and electron scattering persists after reexamination of several aspects of the Mainz data/analysis**
  - **No indication that data, with minimal physics constraints in the fit, are consistent with R=0.84 fm (in the absence of larger or more 'extreme' systematic errors)**

- **Tensions between Mainz and other world's data in cross section and magnetic radius, but not charge radius**

- **Future experiments planned**
  - **Better constrain G_M at low Q^2**
  - **Map out structure of G_E at low Q^2**
  - **Check TPE in both electron and muon scattering**
  - **Directly compare electron and muon scattering cross sections**
  - **$^3He/^3H$ charge radius comparison in electron scattering [2016]**

Fin...

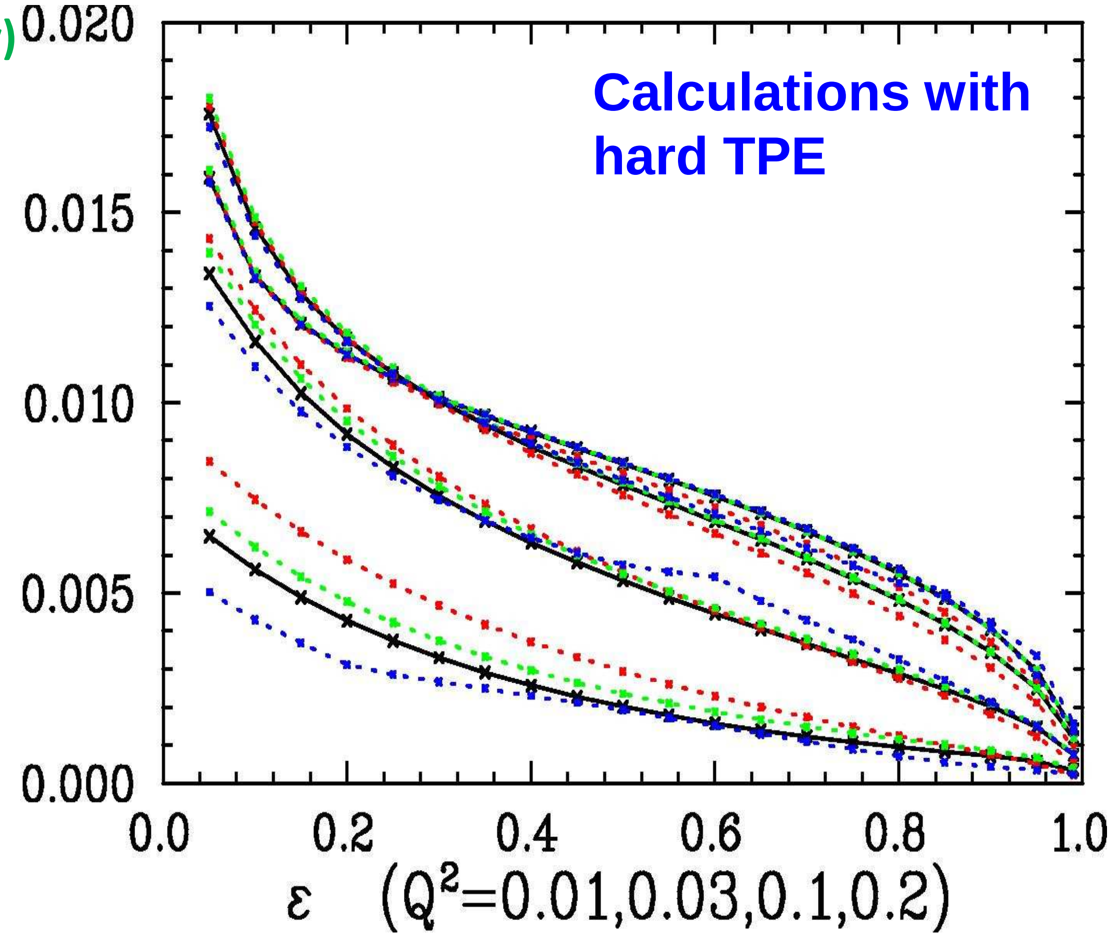
Comparison of low Q^2 TPE calculations

Blunden, et al., hadronic calculation *[PRC 72, 034612 (2005)]*

Borisyuk & Kobushkin: Low-Q^2 expansion, valid up to 0.1 GeV^2 *[PRC 75, 038202 (2007)]*

B&K: Dispersion analysis (proton only)
[PRC 78, 025208 (2008)]

B&K: proton + Δ *[arXiv:1206.0155]*



A few other calculations, not shown, fall within the range of these results

JA, JPG 40 (2013) 115003

Comparison of min vs. max TPE calculations yields change in R_E, R_M values around 0.002-0.003 fm

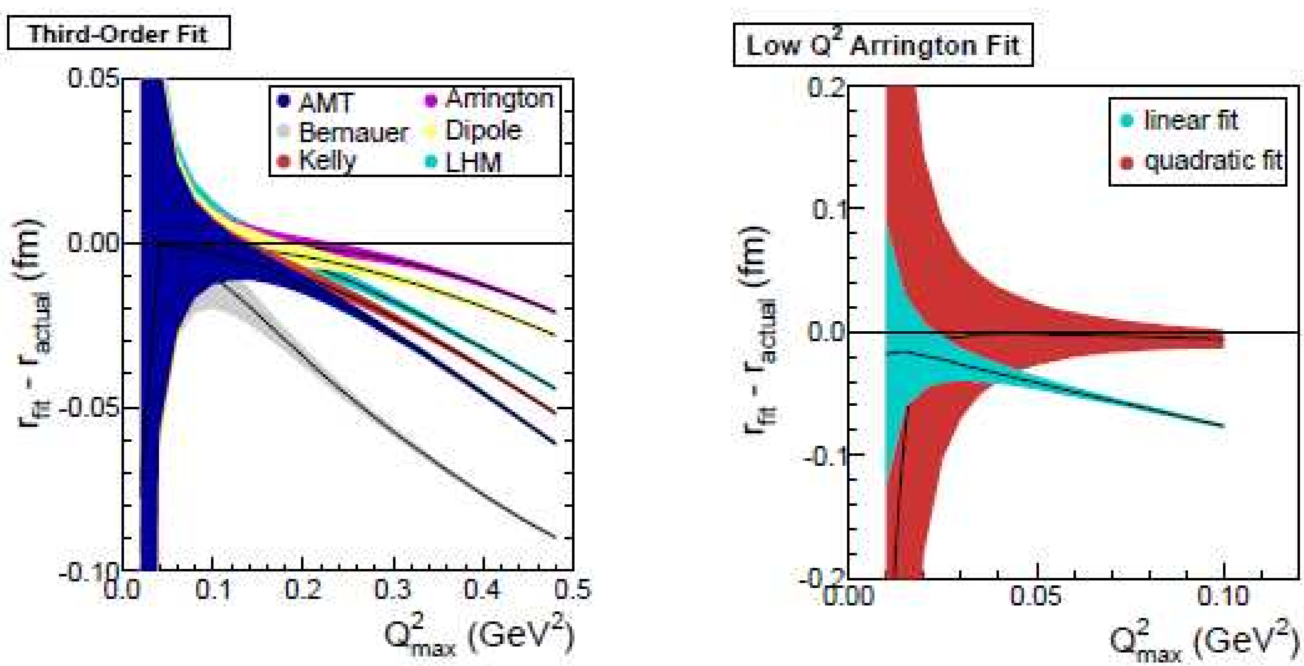
G. Lee, JA, R. Hill – in preparation

“Polynomial fits and the proton charge radius”

E. Kraus, K.E. Mesick, A. White, R. Gilman, S. Strauch, PRC 90 (2014) 045206

More detailed examination of statistical power vs. error due to imperfect functional form as one varies the Q^2 range, number of parameters, etc….

Polynomial fits tend to be the ‘worst case’ among published analyses: effects generally smaller for inverse polynomial, continued fraction, Pade expansion, etc…

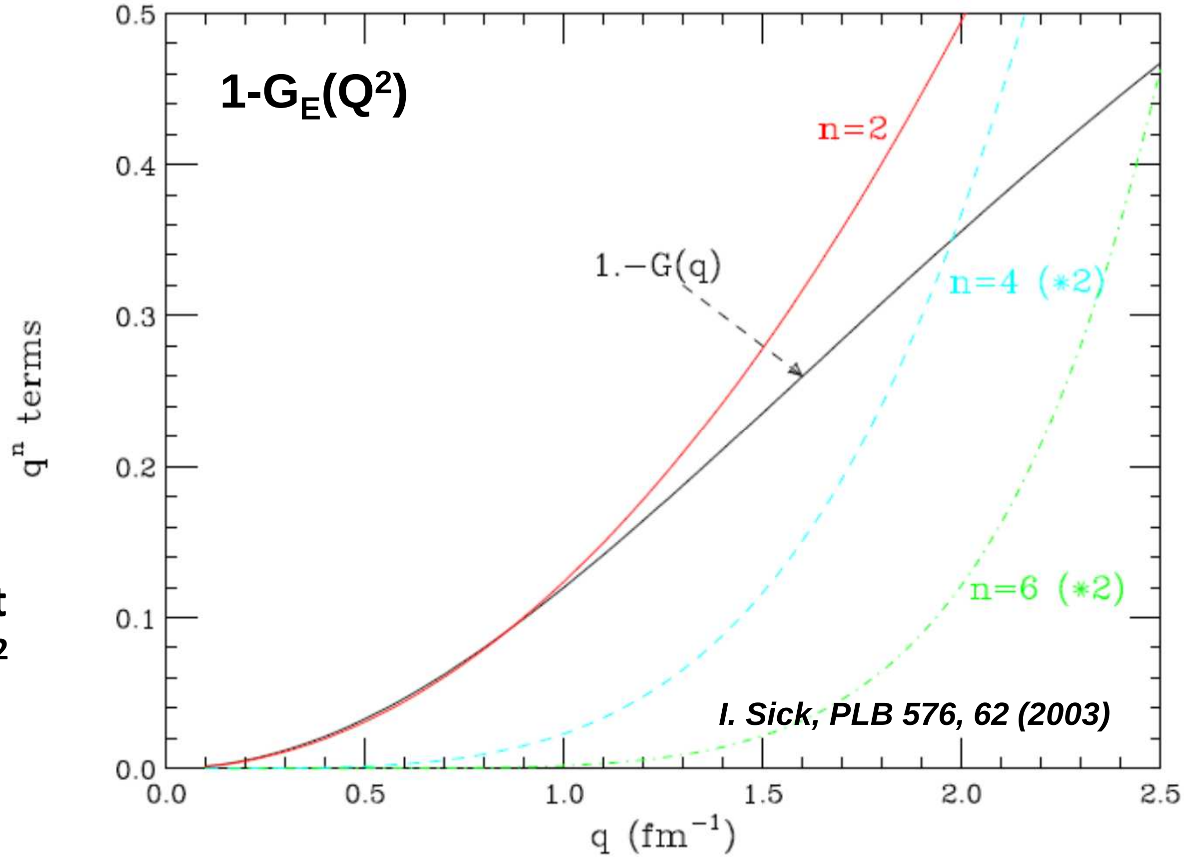


Difficulties in extracting the radius (slope)

Very low Q^2 yields slope but sensitivity to radius is low

Larger Q^2 values more sensitive, have corrections due to higher order terms in the expansion

Want enough Q^2 range to constrain higher terms, but don't want to be dominated by high Q^2 data; Global fits almost always give poor estimates of the radii



Q^2 [GeV^2] : 0	0.01	0.04	0.09	0.15	0.23
Linear fit error(stat)	4.7%	1.2%	0.5%	0.3%	0.2%
Truncation Error (G_{Dip})	0.8%	3.3%	7.5%	12%	19%
Quadratic fit	19%	4.5%	1.9%	1.1%	0.6%
Error:	0	0.1%	0.6%	1.4%	3.1%
Cubic fit	48%	11.5%	4.9%	2.8%	1.7%
Error:	0	0	0.1%	0.2%	0.5%

Fits use ten 0.5% G_E values for Q^2 from 0 to Q^2_{max}

Optimizing the extractions

Max. Q^2 [GeV^2] :	0.01	0.04	0.09	0.15	0.23	0.4
Linear fit error (stat)	4.7%	1.2%	0.5%	0.3%	0.2%	0.1%
Truncation error (G_{Dip})	0.8%	3.3%	7.5%	12%	19%	32%
Quadratic fit error	19%	4.5%	1.9%	1.1%	0.6%	0.3%
Truncation error:	0	0.1%	0.6%	1.4%	3.1%	7.5%
Cubic fit error	48%	11.5%	4.9%	2.8%	1.7%	0.8%
Truncation error:	0	0	0.1%	0.2%	0.5%	1.7%

Linear fit: Optimal Q^2=0.024 GeV^2, dR=2.0%(stat), 2.0%(truncation)

Quadratic fit: Optimal Q^2 = 0.13 GeV^2, dR=1.2%(stat), 1.2%(truncation)

Cubic fit: Optimal Q^2 = 0.33 GeV^2, dR=1.1%(stat), 1.1%(truncation)

Note: Brute force (more data points, more precision) can reduce stat. error

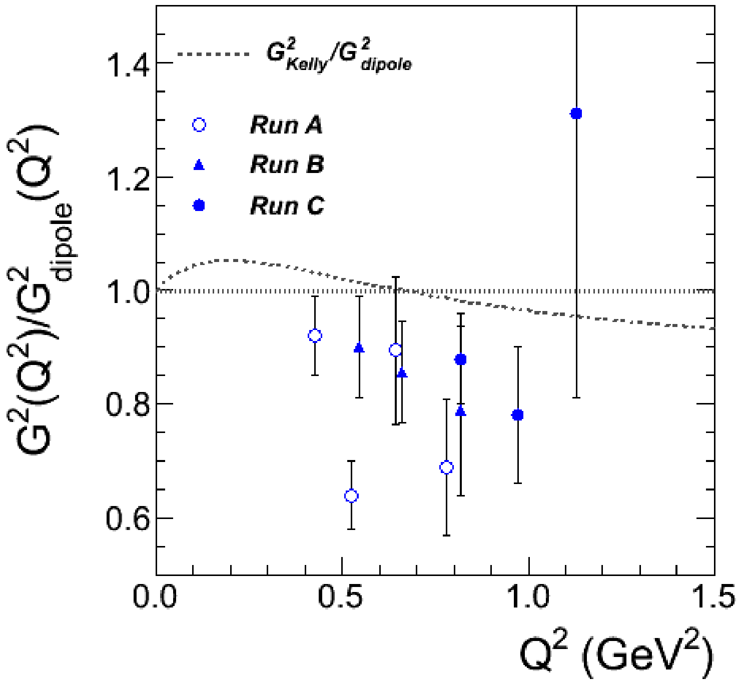
Improved fit functions (e.g. z-pole, CF form) can reduce truncation error, especially for low Q^2 extractions

"Tricks" may help further optimize: e.g. decrease data density at higher Q^2, exclude data with 'large' G_M uncertainties

e-μ Universality

Several experiments compared e-p, μ-p interactions. No convincing differences, once the μp data are renormalized up about 10%. In light of the proton ``radius'' puzzle, the experiments are not as good as one would like.

Ellsworth *et al*., form factors from elastic μp



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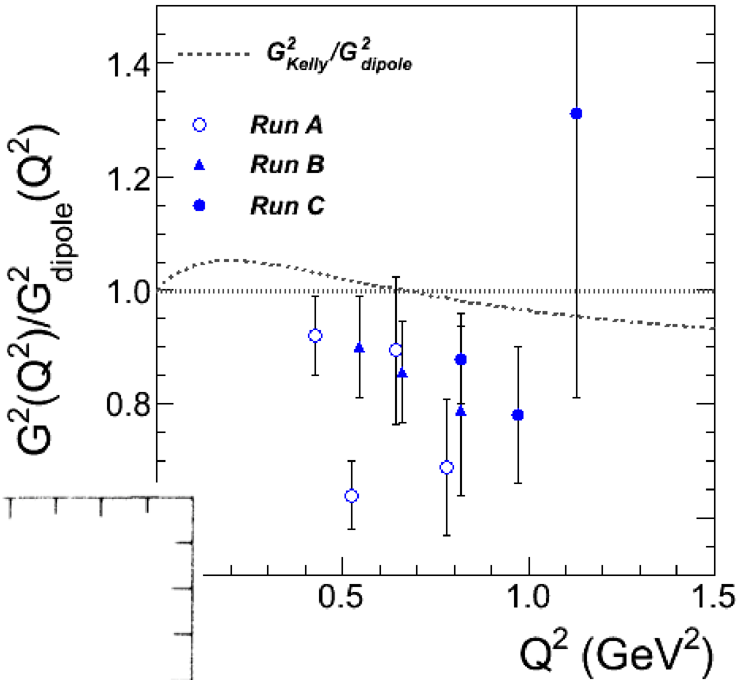

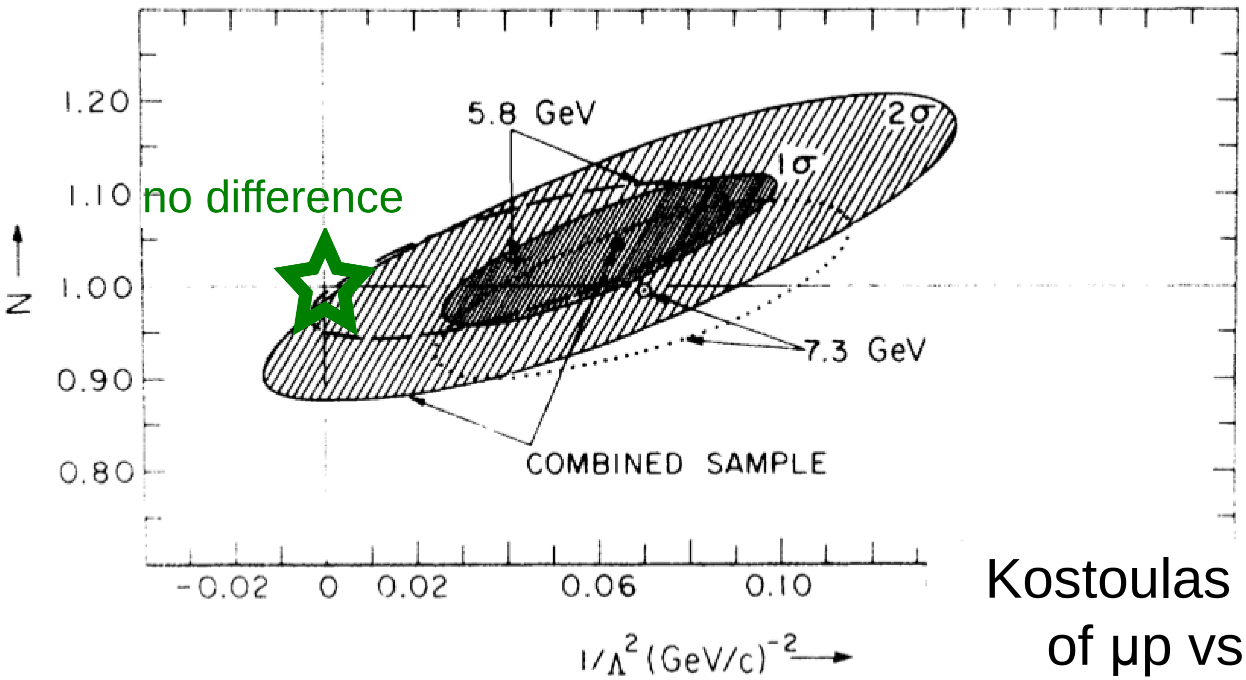

Kostoulas et al. parameterization of μp vs. ep elastic differences

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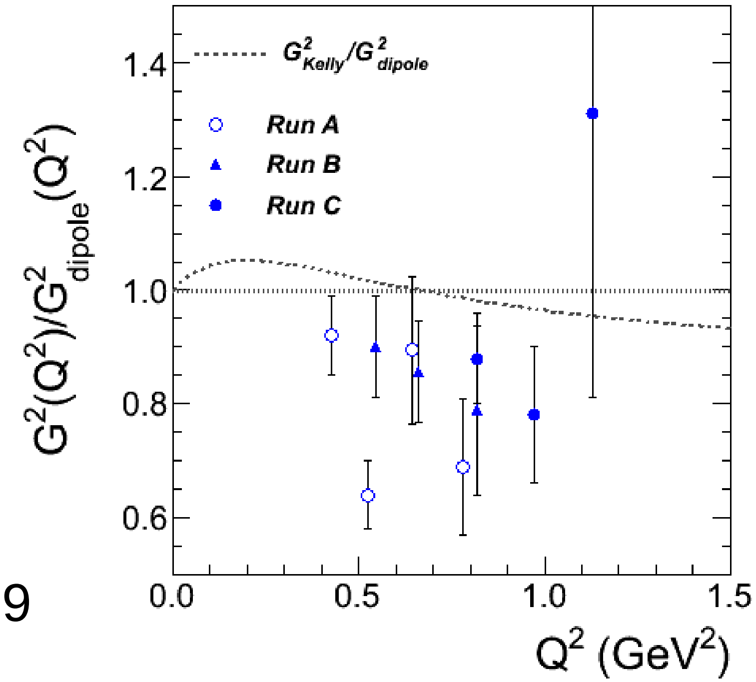

Entenberg *et al.* DIS: $\sigma_{\mu p}/\sigma_{ep} \approx 1.0 \pm 0.04 \pm 0.09$

Consistent extractions of ^{12}C radius from e-C scattering and μC atoms

Offermann *et al*. e-C: 2.478(9) fm

Ruckstuhl *et al*. μC X rays: 2.483(2) fm