Recent Progress in Determining Gravitational Constant *G* at HUST

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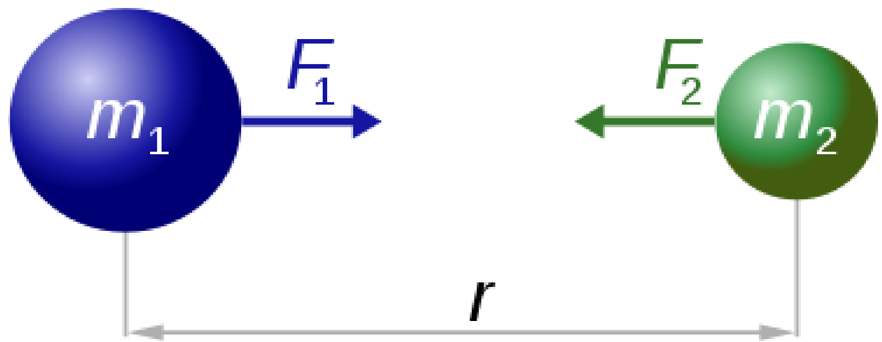
February 5th, 2015, Germany

Outline

- **Brief Review of *G***
- **HUST-09 experiment**
- **Recent progress**

About big *G*

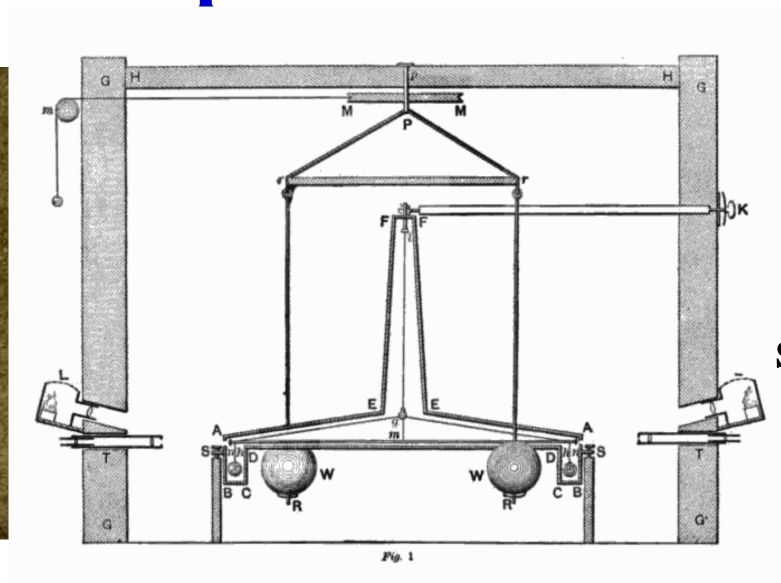
Newtonian law of universal gravitation

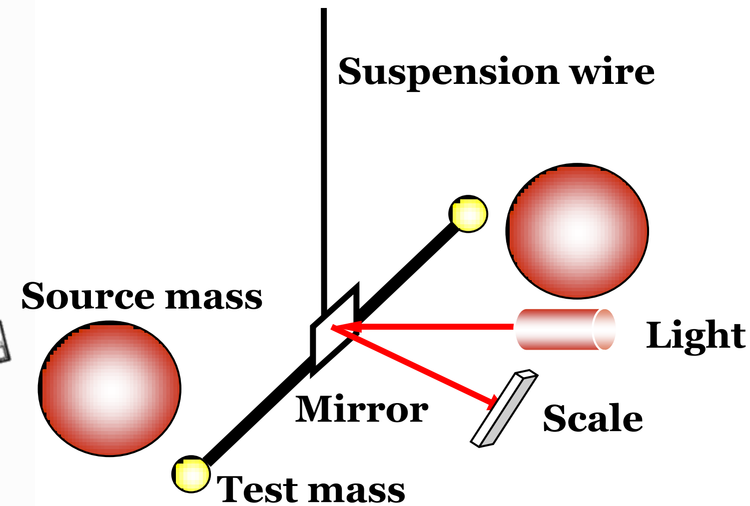


$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

The first G value

Cavendish' experiment





$$G = \frac{2\pi^2 L R^2}{M T^2}\theta$$

1798, G = (6.67±0.07)×10^{-11} $m^3kg^{-1}s^{-2}$ 1%

CODATA recommended *G* values

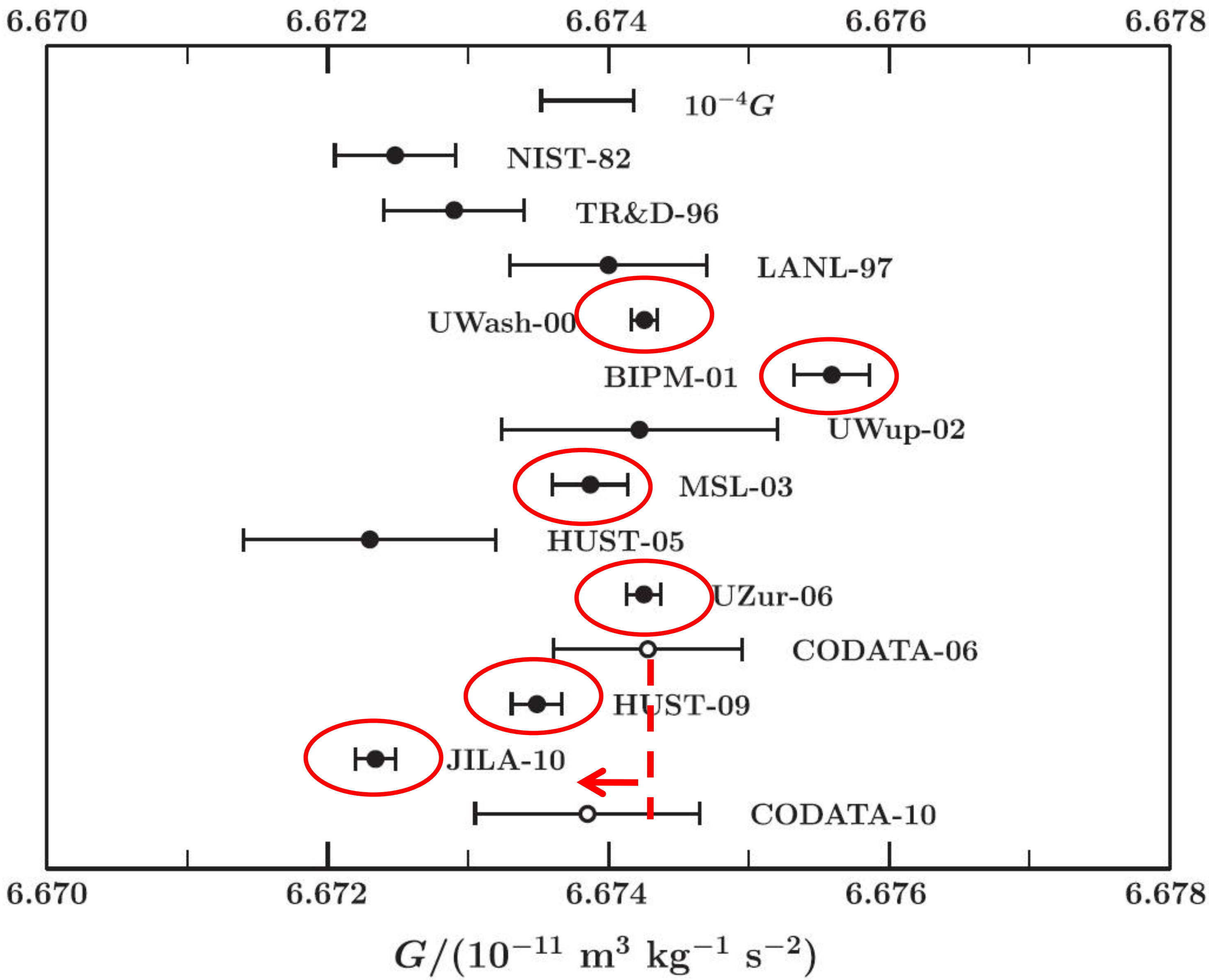
G **was known earliest, but with worst precision.**

Year	*G* value/ $\times 10^{-11} m^3 kg^{-1} s^{-2}$	Uncertainty
1798, Cavendish	6.67 ±0.07	1%
1973 CODATA	6.6720 ±0.0041	0.061%
1986 CODATA	6.67259 ±0.00085	0.013%
1998 CODATA	6.673 ±0.010	0.15%
2002 CODATA	6.6742 ±0.0010	0.015%
2006 CODATA	6.67428 ±0.00067	0.01%
2010 CODATA	6.67384 ±0.00080	0.012%

During past 200 years, there are more than 300 results

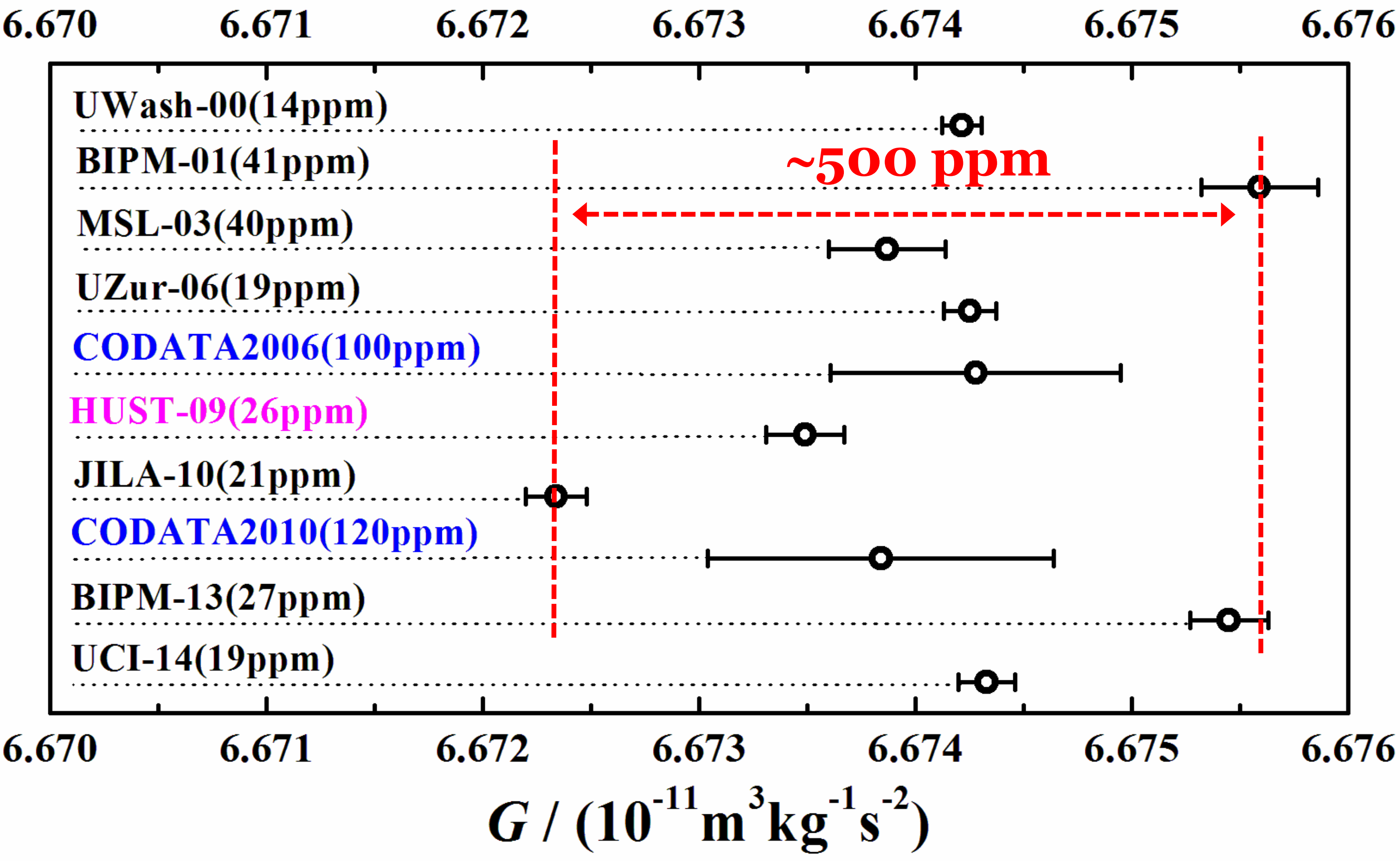
but its accuracy was improved by only two orders !

CODATA 2010



6.670
6.672
6.674
6.676
6.678
$10^{-4}G$
NIST-82
TR&D-96
LANL-97
UWash-00
BIPM-01
UWup-02
MSL-03
HUST-05
UZur-06
CODATA-06
HUST-09
JILA-10
CODATA-10
$G/(10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2})$

Uncertainty < 50 ppm



consistent with each other at only ~500 ppm

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- **Brief Review of *G***
- **HUST-09 experiment**
- **Recent progress**

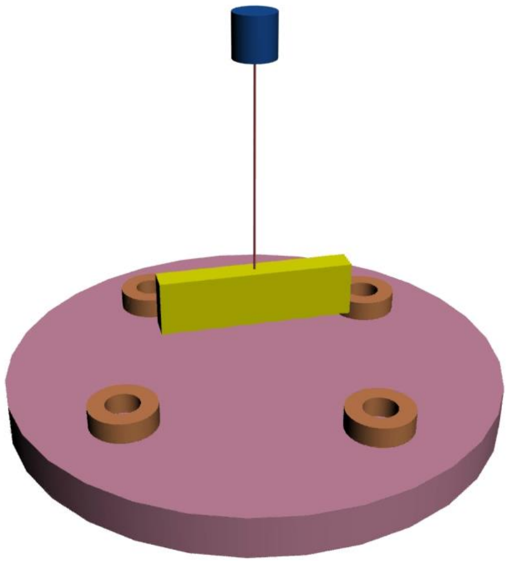
Time-of-swing method

$$I\ddot{\theta}+\gamma\dot{\theta}+K\theta=\tau_G\left(\theta\right)$$

$$\tau_G\left(\theta\right)\approx\tau_0-K_G\theta$$

$$\omega_n^2=\frac{K_n+GC_{gn}}{I} \quad \text{near}$$

$$\omega_f^2=\frac{K_f+GC_{gf}}{I} \quad \text{far}$$



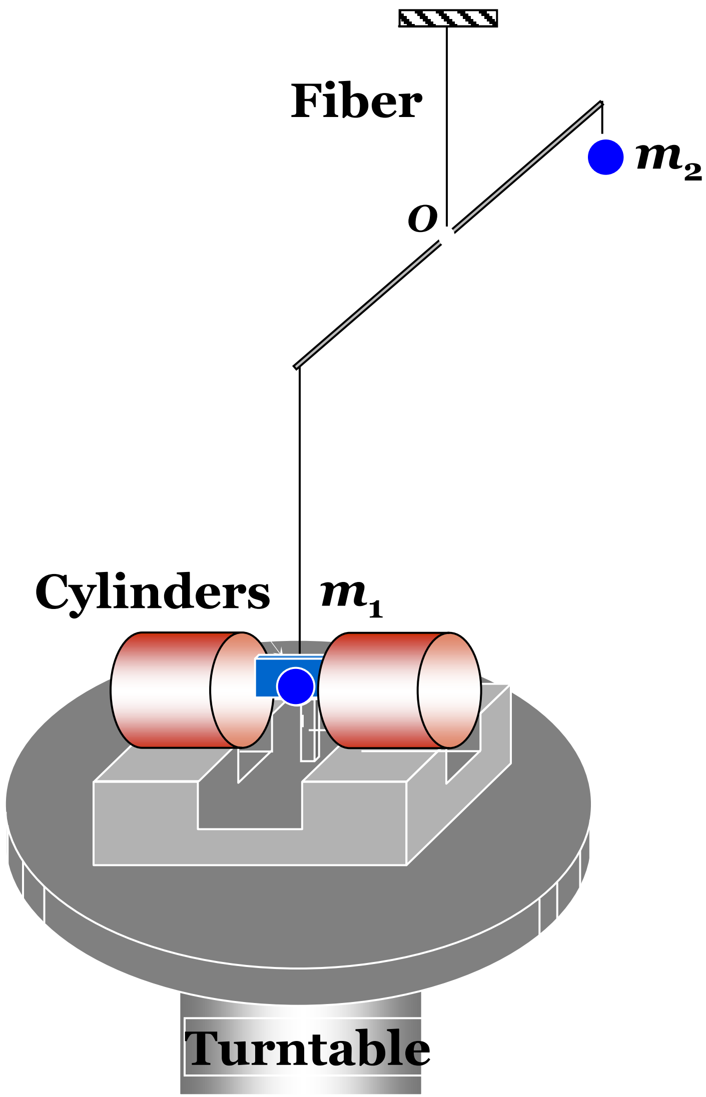
$$G=\frac{I\left(\omega_n^2-\omega_f^2\right)-\left(K_n-K_f\right)}{C_{gn}-C_{gf}}$$

$$=\frac{I\,\Delta\omega^2-\Delta K}{\Delta C_g}$$

1. Determination of $P_g=\Delta C_g/I$
2. $\Delta\omega^2$ between near and far positions
3. Anelasticity of torsion fiber ΔK

HUST-99 *G* measurement

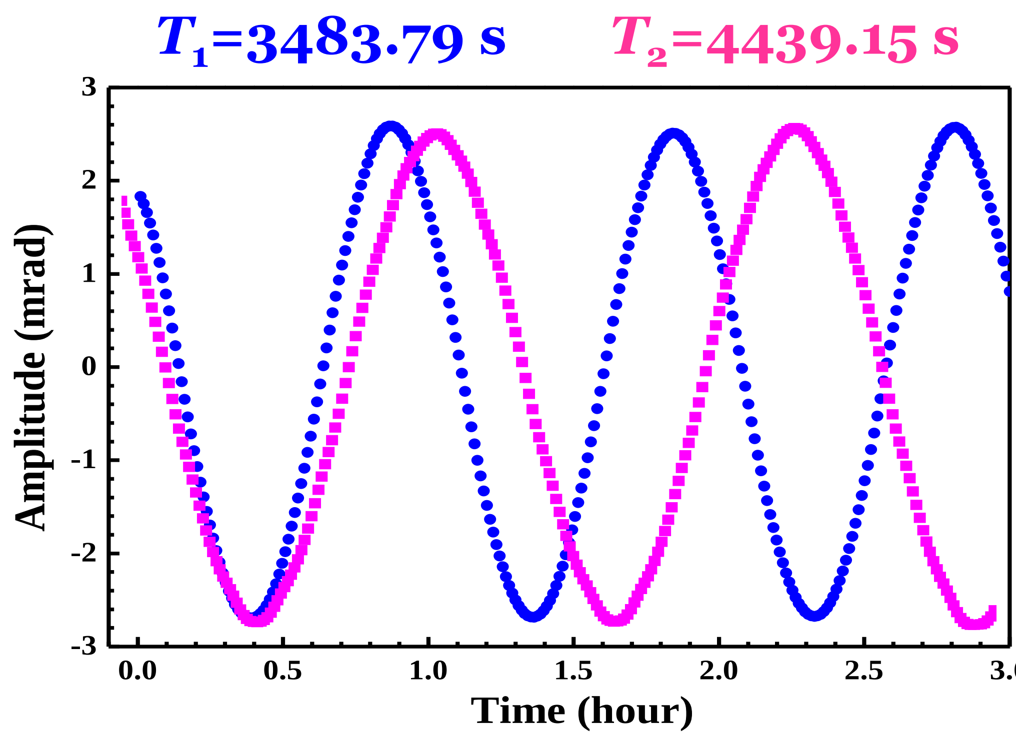
PRD 59 (1998) 042001



Merits:

The period change: 27%

High Q=36000



105 ppm

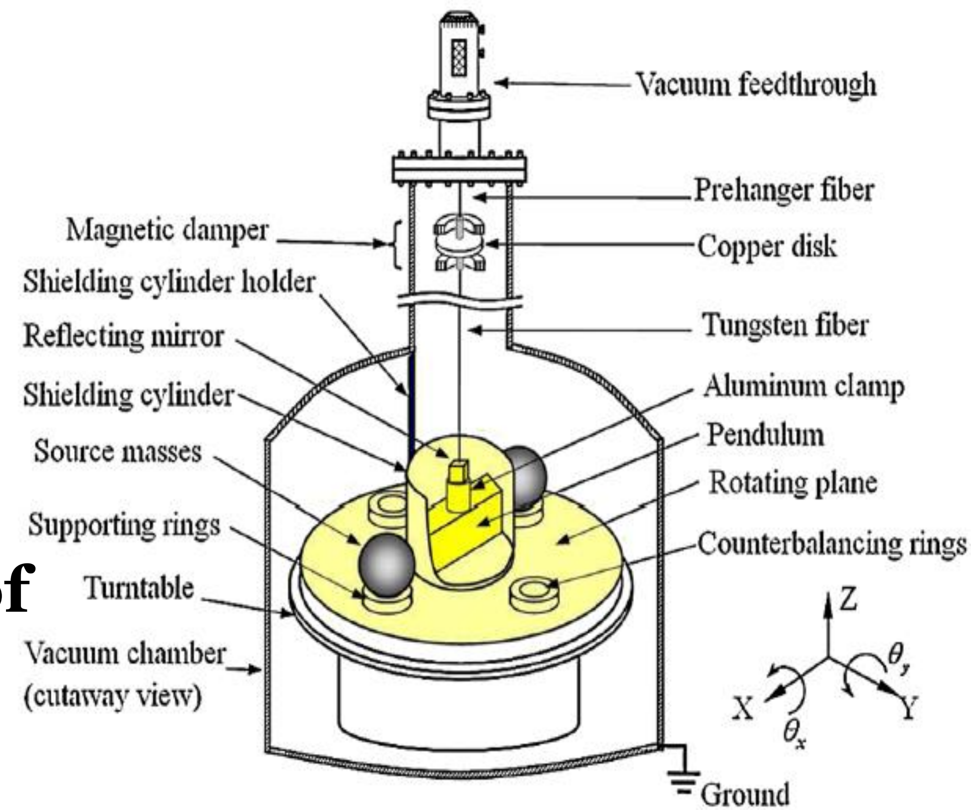
HUST-99＝(6.6699±0.0007) $\times 10^{-11}$ $m^3kg^{-1}s^{-2}$

HUST-09 experimental design

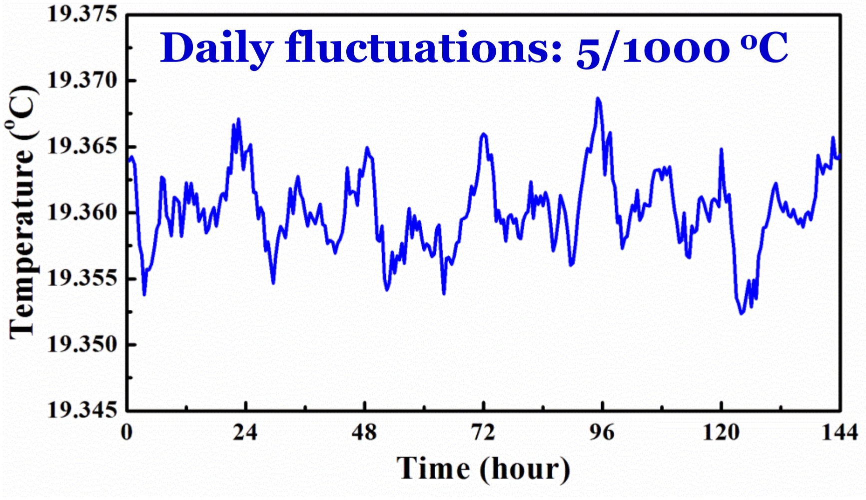
Compare with HUST-05

Merits：

1. Spherical source mass
2. Simple pendulum
3. All in vacuum
4. Direct Measurement of anelasticity



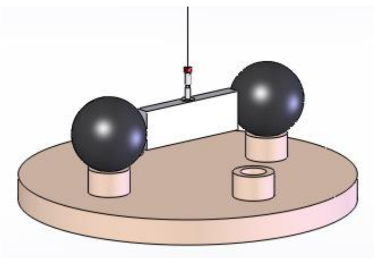
Experimental environment

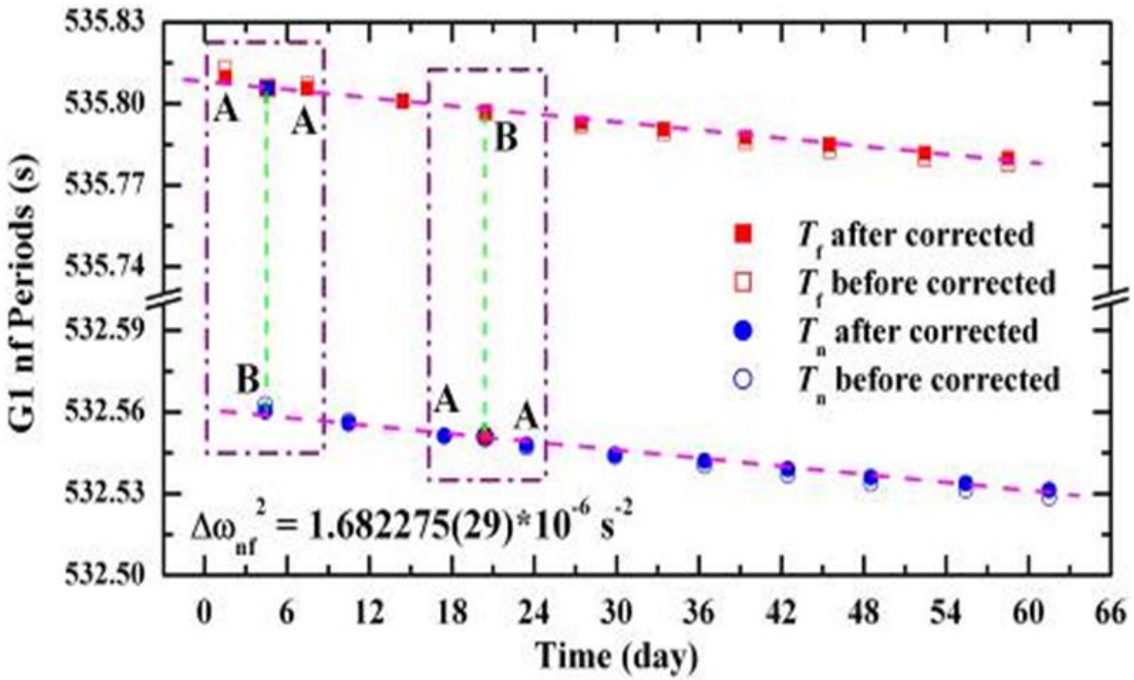


Determination of $\Delta\omega^2{}_{nf}$

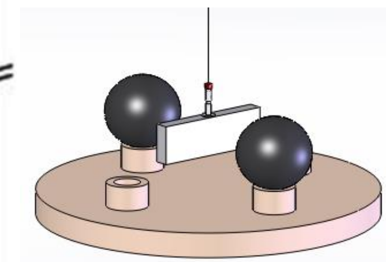
10 sets of data (6 days per set) with the spheres in near and far positions alternately

Near





Far



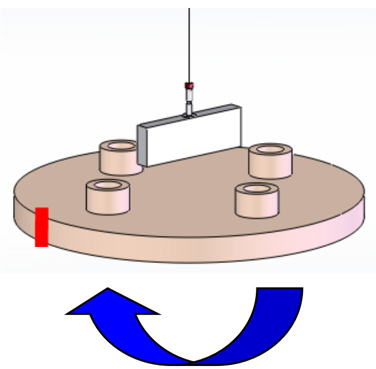
Period drift:

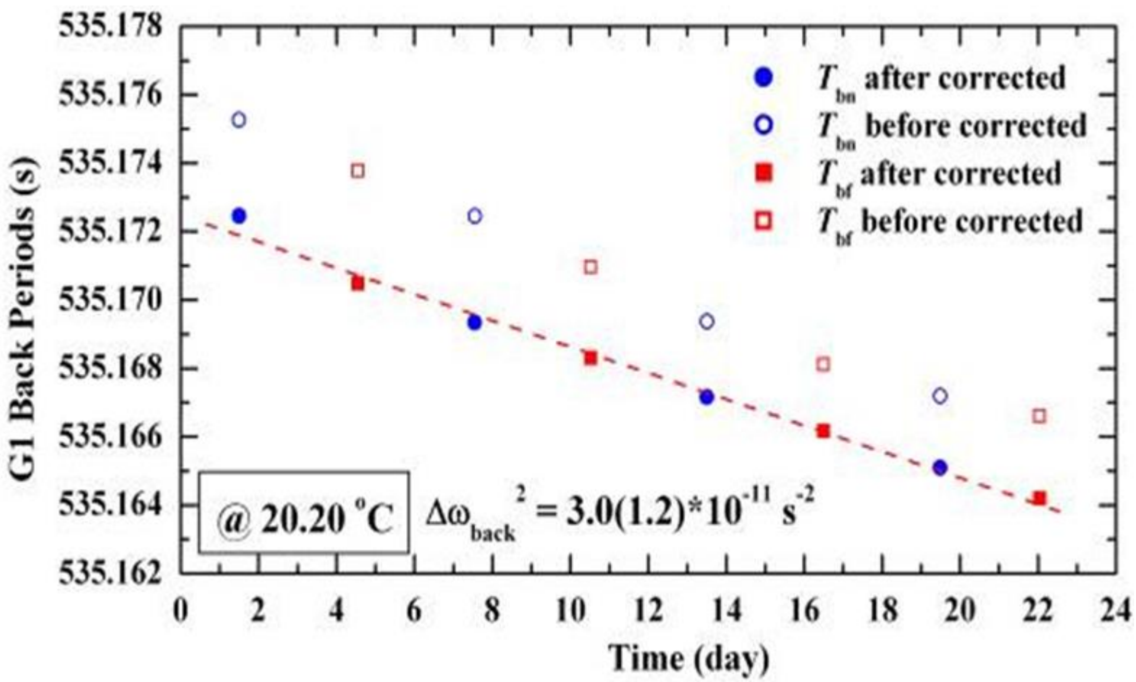
1. Aging
2. Background gravitational change caused by zero drift effect
3. A-B-A

Determination of $\Delta\omega^2_{back}$

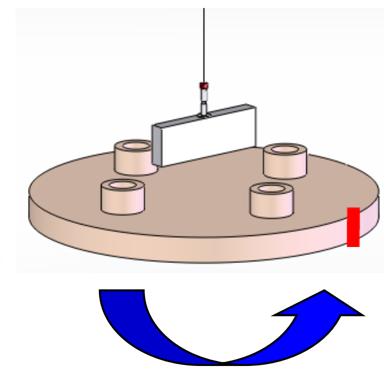
4 sets of background data (without the spheres) in near and far positions alternately

Near





Far



$$\Delta(\omega^2) = \Delta\omega^2_{nf} - \Delta\omega^2_{back} = 1.682245(31)\times 10^{-6}\ s^{-2} \quad 18.428\ \text{ppm}$$

Fiber anelasticity

$$K = K_0 + \Delta K(\omega)$$

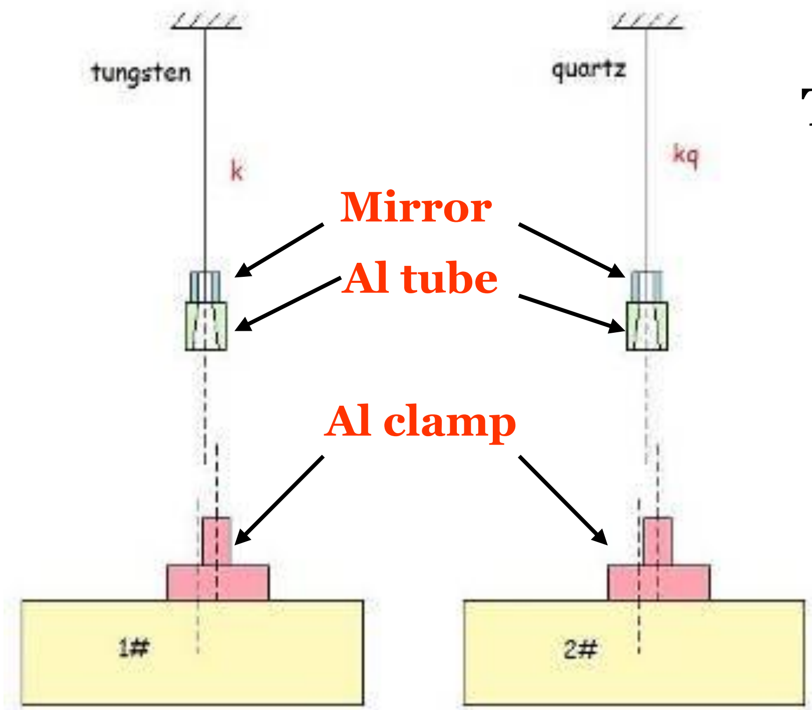
Correction for *G* value due to the fiber's anelasticity

Kuroda: $$\frac{\Delta G}{G} = \frac{1}{\pi Q}$$ PRL 75 (1995) 2796

Newman: $$0 < \frac{\Delta G}{G} < \frac{1}{2Q}$$ MST 10 (1999) 445

If $Q = 1000$, $$\frac{\Delta G}{G} = \frac{1}{\pi Q} \approx 320 \text{ ppm}$$

Direct measurement of anelasiticity



	Pendulum1	Pendulum2
Tungsten fiber	ω_1^2, I_1	ω_2^2, I_2
Silica fiber	Ω_1^2, I_1	Ω_2^2, I_2

TABLE IV. One σ uncertainty budget of the anelasticity to $\Delta G/G$.

Source	Value (error)	ppm	
I_1	$5.3318(8) \times 10^{-5}$ kgm^2		0.03
I_G	$4.505679(35) \times 10^{-5}$ kgm^2		0.00
Magnetic damper		4.78	
Statistical $(\omega_2/\omega_1)^2$	1.4171723(92)		18.41
Statistical $(\Omega_1/\Omega_2)^2$	0.7056861(5)	2.01	
ε	$-4.14(89) \times 10^{-6}$	2.52	
$\frac{I_2}{I_1} = (\frac{\Omega_1}{\Omega_2})^2(1+\varepsilon)$	0.7056832(8)		3.22
Total			18.69

$$\frac{K(\omega_n)-K(\omega_f)}{I_G(\omega_n^2-\omega_f^2)} = \frac{I_1}{I_G(\omega_2^2/\omega_1^2-1)}\left(\frac{\Omega_1^2\omega_2^2}{\Omega_2^2\omega_1^2}-1\right) = (211.80 \pm 18.69)\,\text{ppm}$$

PRD 80 (2009) 122005

Final *G* value: HUST-09

Result of *G* (10^{-11} m^{3}kg^{-1}s^{-2})

G_1 = (6.67352±0.00019)

After change the position and orientation of spheres:

G_2 = (6.67346±0.00021)

The difference: 9 ppm

Combined result:

G = (6.67349±0.00018)

26.3 ppm

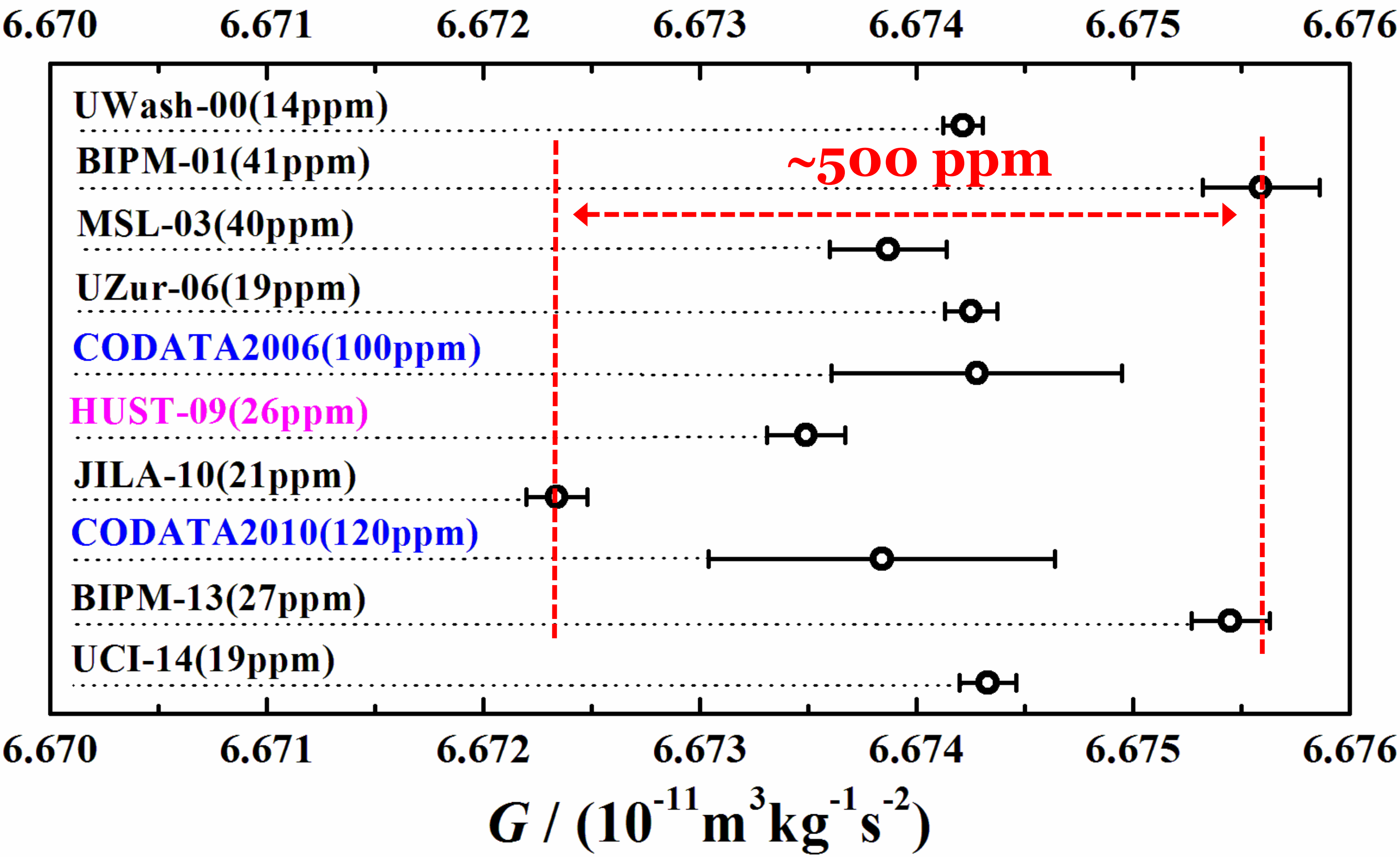
PRL 102 (2009) 240801

PRD 82 (2010) 022001

TABLE I. One σ uncertainty budget (in units of ppm).

Error Sources	Corrections	$\Delta G/G$	
Pendulum		5.07	
Dimensions			1.95
Attitude			0.13
Nonalignment with fiber			0.45
Flatness			0.34
Clamp			1.65
Density inhomogeneity			$\leq$0.21
Coating layer	−24.28		4.33
Edge flaw	−0.12		0.17
Source masses		10.68	
Masses			0.82
Distance of GC			9.64
Density inhomogeneity			4.50
XYZ positions			0.48
Fiber		18.76	
Nonlinearity			$<$0.70
Thermoelasticity	−39.83		1.52
Anelasticity	−211.80		18.69
Aging			$<$0.01
Gravitational nonlinearity	7.73	0.30	
Magnetic damper	17.54	0.31	
Magnetic field		0.40	
Electrostatic field		0.10	
Combined statistical $\Delta(\omega^2)$		14.18	
Total		26.33	

Uncertainty < 50 ppm



consistent with each other at only ~500 ppm

A large discrepancy?

Are there any method-dependent systematic errors?

same laboratory

↓

different methods →

- **Time-of-Swing method**
- **Angular Acceleration Feedback method**

↓

same *G* value ?

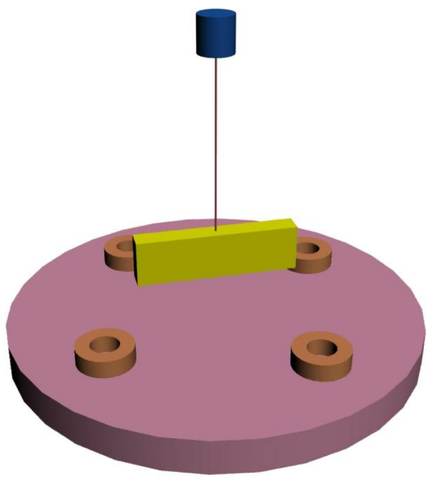
Outline

- **Brief Review of *G***
- **HUST-09 experiment**
- **Recent progress**

Two methods

Time-of-swing

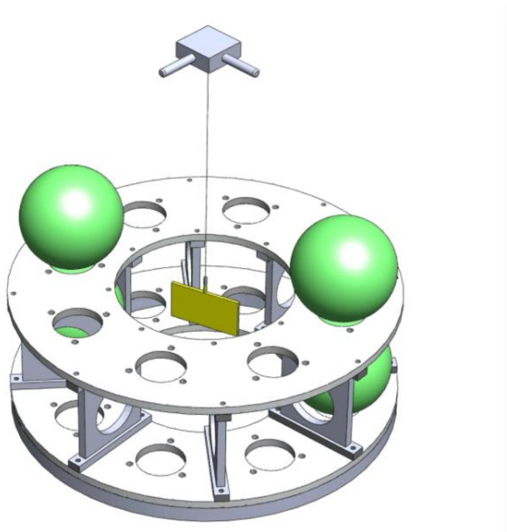
$$I\ddot{\theta}+\gamma\dot{\theta}+K\theta=\tau_g$$



$$G=\frac{I\left(\omega_n^2-\omega_f^2\right)-\left(K_n-K_f\right)}{C_{gn}-C_{gf}}$$

Angular Acceleration

$$I\ddot{\theta}+\gamma\dot{\theta}+K\theta=\tau_g-I\alpha_t$$



$$G=\frac{\alpha_t\left(\omega\right)}{P_g}$$

Time-of-swing	Angular Acceleration
• Simple device • Differential measurement	• Independent of anelasticity
• Dependence on fiber properties • Environmental influences	• Complicated device • High-performance turntable

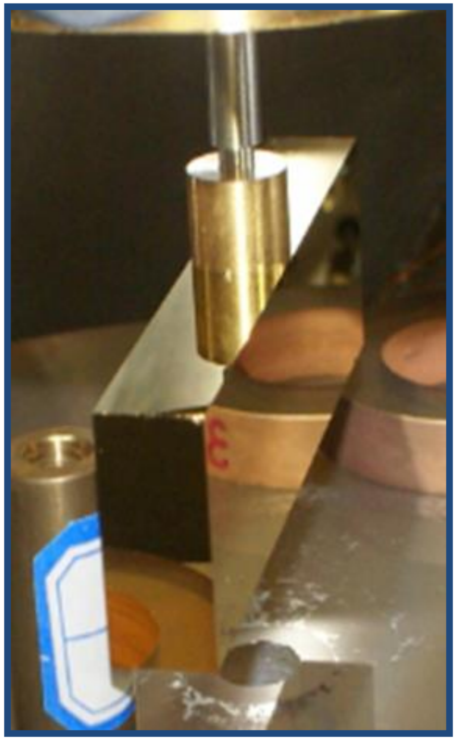
1. Improved time-of-swing method

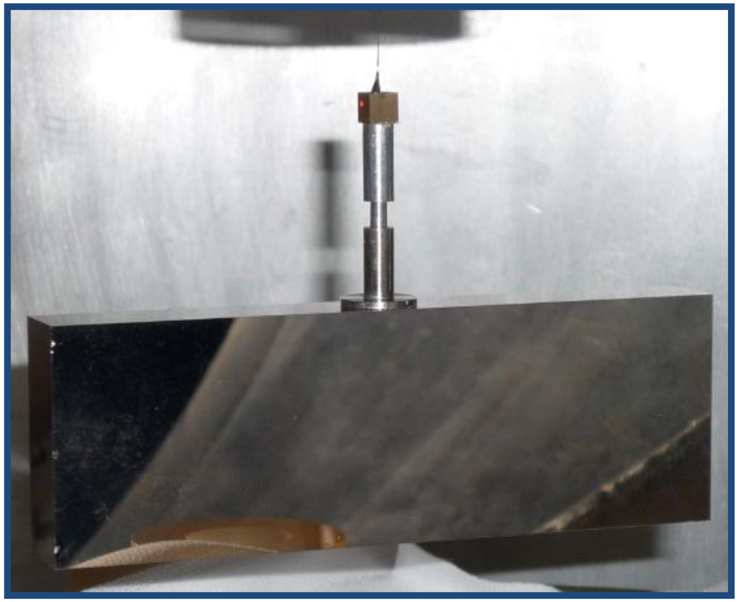
To reduce the large corrections in HUST-09:

1. **Aluminum layer**
2. **Three-point mount**
3. **Copper tube**
4. **High-Q Silica fiber** $Q>5\times10^4$
5. **Thick torsion fiber**
6. **Gravity compensation**

Error sources	Corrections (ppm)	$\delta G/G$ (ppm)
Torsion pendulum		5.05 [5.05]
Dimension		1.95
Attitude		0.13 [0.07]
Density inhomogeneity		<0.21
Chamfer property		0.34
Three chips	−0.12	0.17
Coating layer	−24.28	4.33
Clamp and ferrule		1.65
Reflecting mirror		0.03
Source masses		10.66 [10.64]
Masses		0.82
Distance of GCs		9.64 [9.61]
Density inhomogeneity		4.50
Relative positions		1.10 [1.31]
Height of pendulum		0.76 [0.40]
Height of spheres		0.48 [0.27]
Position of torsion fiber		0.63 [1.22]
Position of turntable		0.05
θ_0		0.06 [0.01]
Fiber		18.76
Nonlinearity		<0.70
Thermoelasticity	−39.83 [8.37]	1.52 [0.82]
Anelasticity	−211.80	18.69
Aging		<0.01
Gravitational nonlinearity	7.73 [4.79]	0.30 [0.20]
Magnetic damper	17.54	0.31
Magnetic field		0.40
Electrostatic field		0.10
Statistical $\Delta\omega^2$		18.43 [23.31]
Total		28.86 [32.17]
Combined		26.33

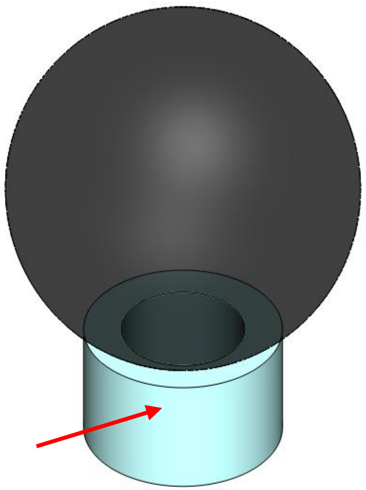
Improvement I: coating layer

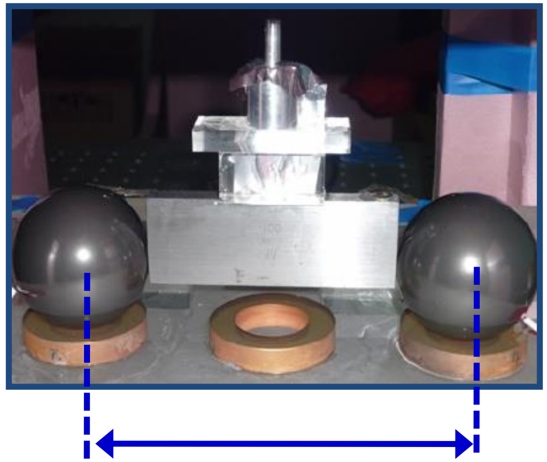




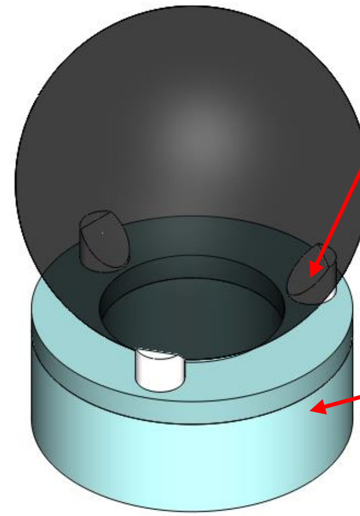
Experiment	Material	Density(g/cm³)	Correction to *G*(ppm)
HUST-09	Cu	8.96	-24.28(4.33)
	Au	19.26	
Present	Al	2.70	-1.7(8)

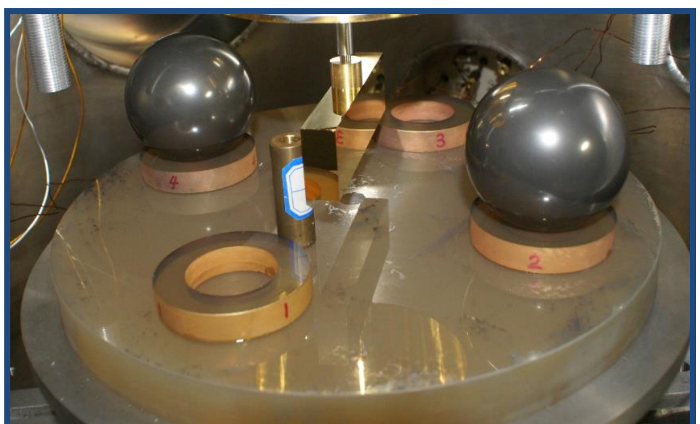
Improvement II: three-point mount



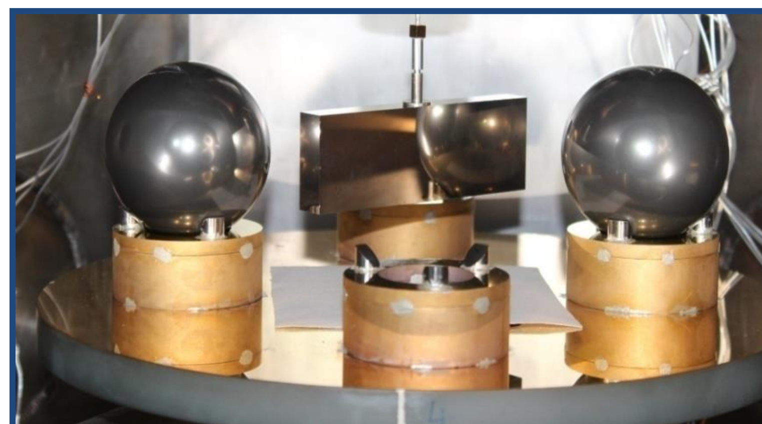


0.35 μm→9.0 ppm





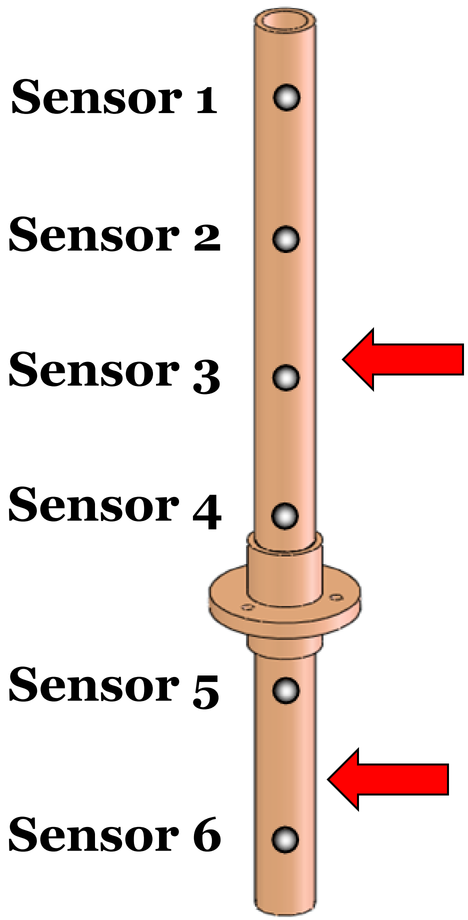
Supporting ring in HUST-09



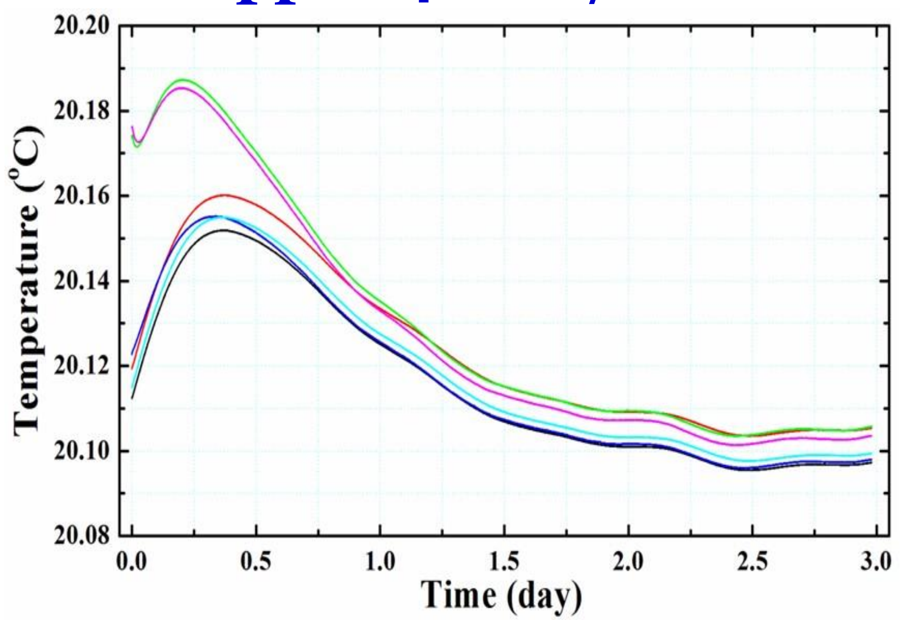
Three-point mount

The repeatability of mounting is better than 0.25 μm

Improvement III: copper tube



Thermal conductivity of copper:401 W/mK



HUST-09:<0.05 °C

Temperature difference between sensors:<0.02 °C

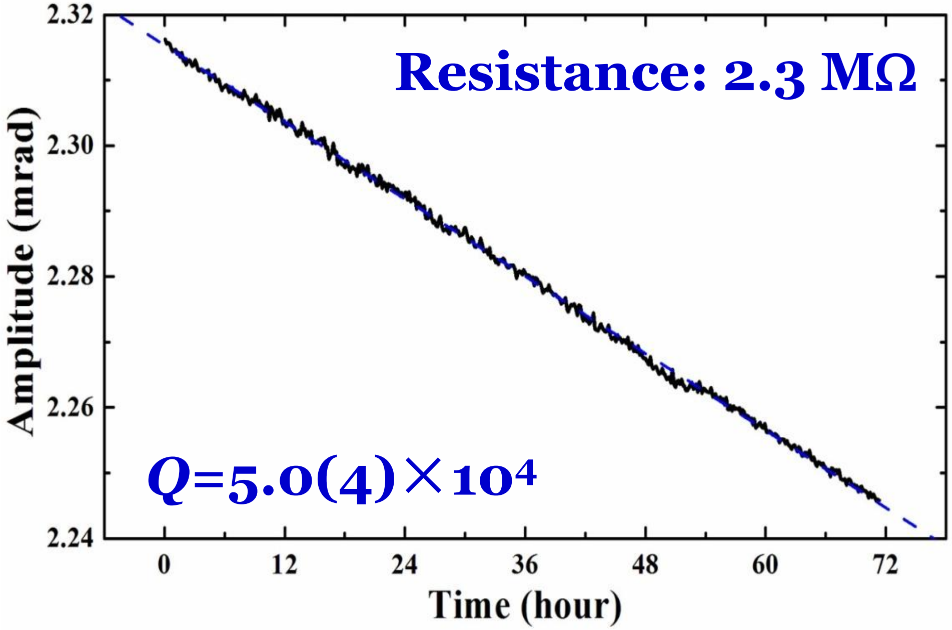
<5 ppm

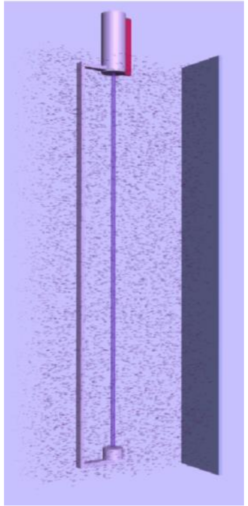
Improvement IV: silica fiber

Trade name: SuprasilR 311
Length: 900 mm
Diameter: 44 and 50 μm
Germanium: 8-nm-thickness
Bismuth: 11-nm-thickness
Anelasticity: ~6.4 ppm


3-mm rod

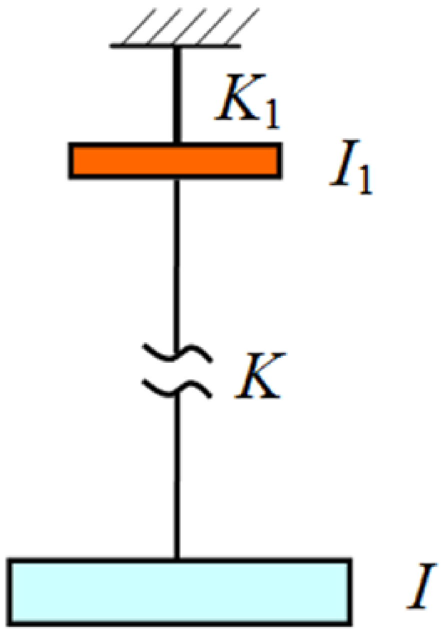

Pull fiber




Coating


Pre-loaded fibers

Improvement V: magnetic damper

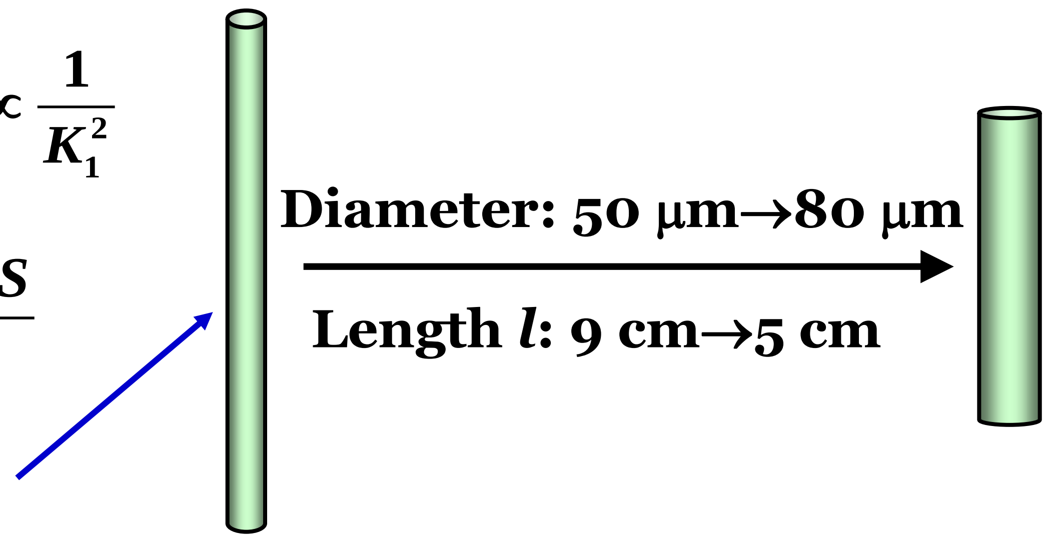


Two-stage torsion balance

$$\frac{\Delta G}{G} = \frac{I_1 K^2}{I K_1^2} \propto \frac{1}{K_1^2}$$

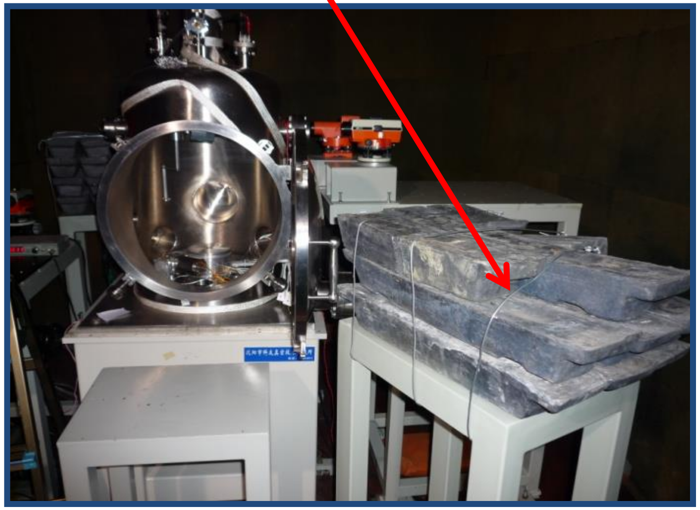
$$K_1 = \frac{\pi D^4 S}{32 l}$$

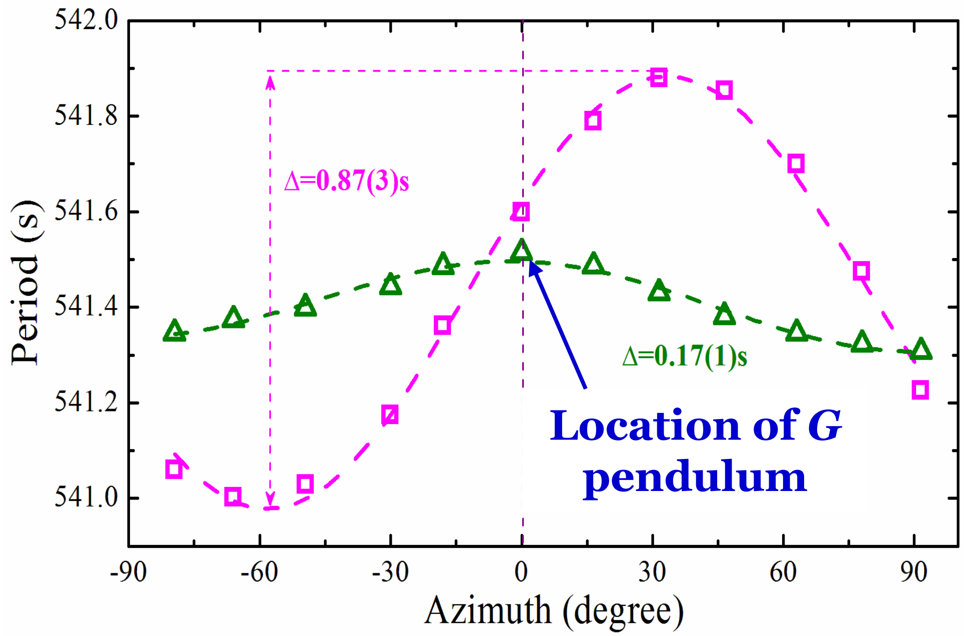
$$\frac{\Delta G}{G} \propto \frac{l^2}{D^8}$$



Correction to *G*: 17.54(0.31) ppm→0.4(0.1) ppm

Improvement VI: gravity compensation





Lead blocks ~800 kg

1. The background gravitational gradient is reduced to 1/5 of that before compensation
2. The equilibrium position of *G* pendulum is located at the min-gradient position

For *G*: <0.4 ppm

Present result

44-μm-diameter silica fiber determine *G* (10^{-11} $m^3kg^{-1}s^{-2}$):

G_1 = (6.674±0.00011)**

~18 ppm

After change the orientation of spheres:

G_2 = (6.674±0.00013)**

~19 ppm

The difference between them is: ~10 ppm

TABLE I: The complete 1σ error budget. (Unit: ppm)

Error sources	Corrections	$\delta G/G$
Pendulum		2.2
Dimensions		2.0
Attitude		0.1
Density inhomogeneity		0.2
Chamfer property		0.1
Chips	0.6	0.2
Coating layer	-1.7	0.8
Clamp and ferrule		0.2
Reflecting mirror		0.1
Source masses		9.0
Masses		0.7
Distance of GC		9.0
Density inhomogeneity		5.0
Relative positions		1.4
Height of pendulum		0.3
Height of spheres		0.5
Position of fiber		1.2
Position of turntable		0.4
θ_0		0.1
Fiber		6.6
Nonlinearity		<0.4
Thermoelasticity	38.4	1.5
Anelasticity		6.4
Aging		<0.1
Gravitational nonlinearity	19.7	0.3
Magnetic damper	0.4	0.1
Magnetic field		0.4
Electrostatic field		0.1
Statistical $\Delta\omega^2$		11.1
Present uncertainty		<18

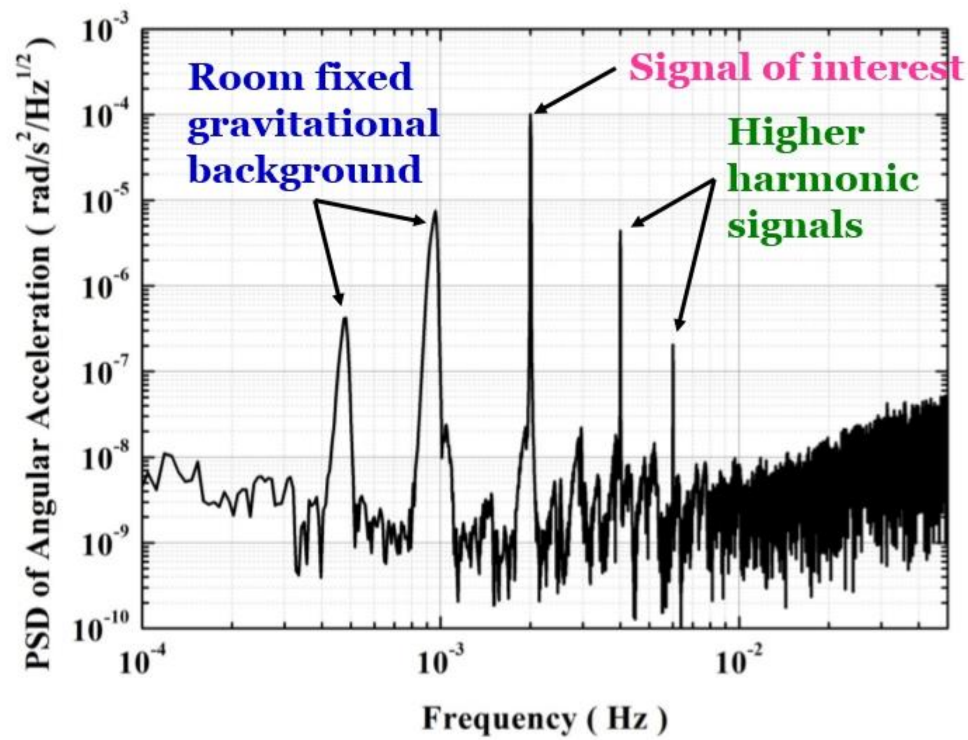
Following work

Repeat the measurements with different fibers to find the potential systematic errors, such as entangled dislocations:

Parameter	Present	Next	
Fiber material	Silica	Silica	Tungsten
Diameter(μm)	44	50	25
Natural period(s)	391	306	535
Period change(s)	1.26	0.61	3.27
Period relative change	0.32%	0.20%	0.61%
Ratio of loading	40%	31%	57%
Thickness of Ge and Bi(nm)	~8、~11	~2、~2	—
Resistance(Ω)	$\sim 2.3\times10^6$	$(8\text{-}10)\times10^6$	<50
Q	$\sim 5.0\times10^4$	$\sim 8\times10^4$	$\sim 3\times10^3$
Anelasticity(ppm)	~6.4	~4.0	~106

2. Angular acceleration feedback

Preliminary result

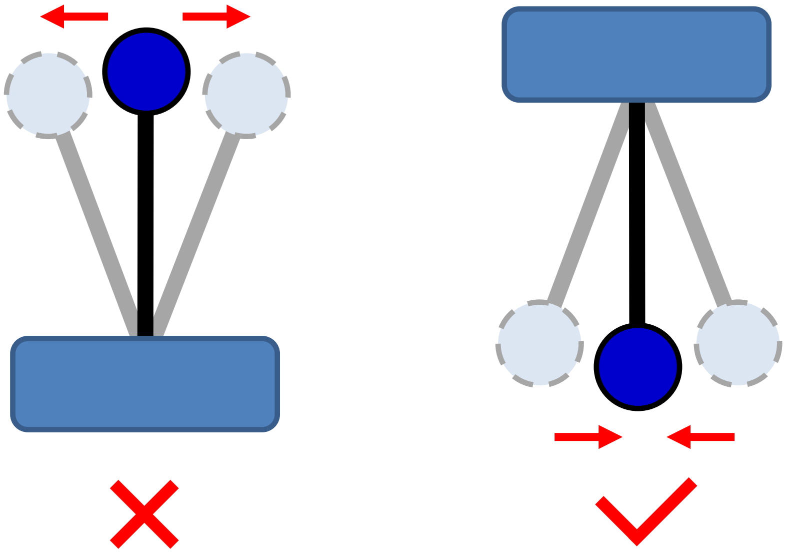


Rev. Sci. Instrum. 85, 014501(2014)

Phil. Trans. R. Soc. A 372, 20140031(2014)

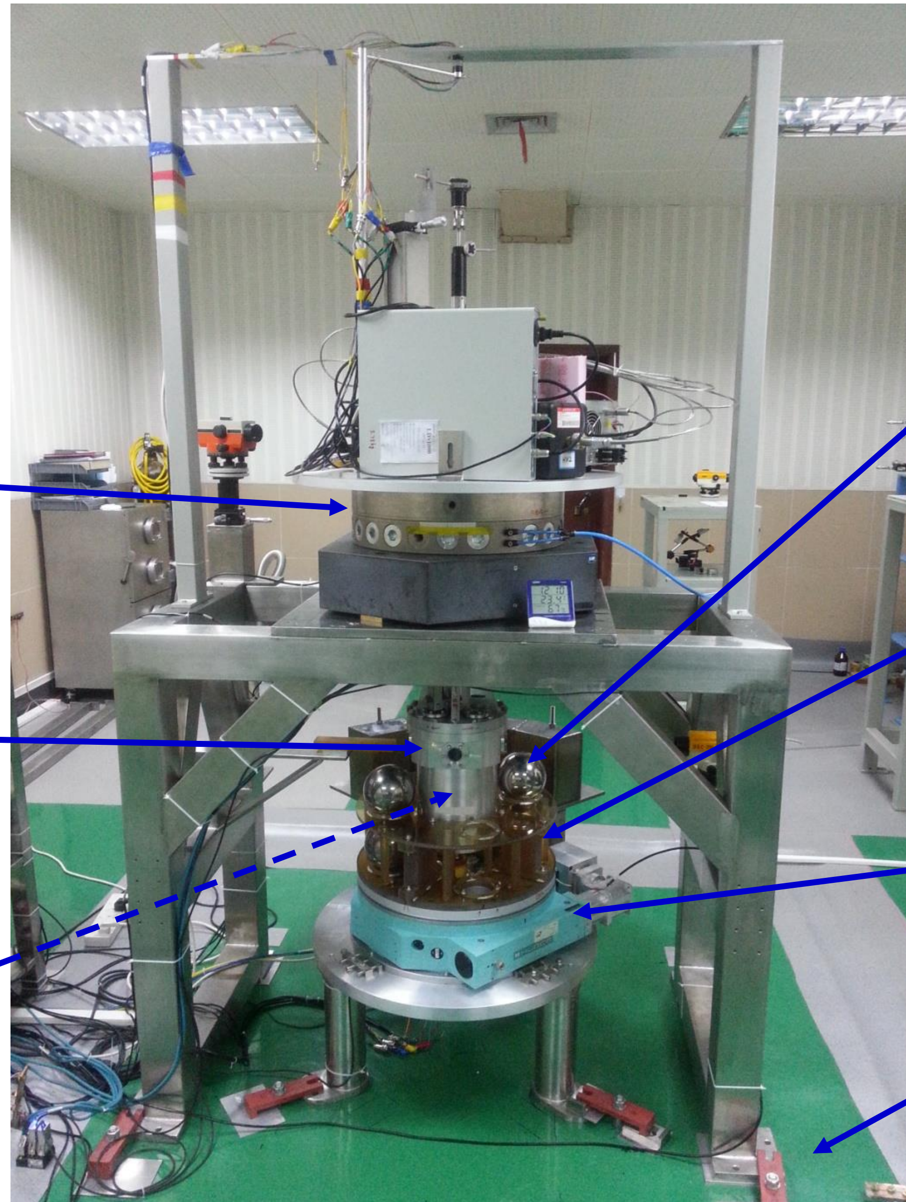
Repeatability of $\alpha_t(\omega)$: ~100ppm

Major problems



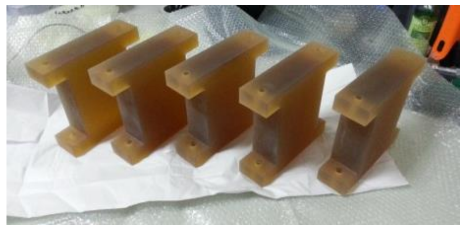
1. C.M. of apparatus is too high: inverted pendulum
2. Temperature fluctuation: distance of sphere
3. Effect of shelves

New apparatus

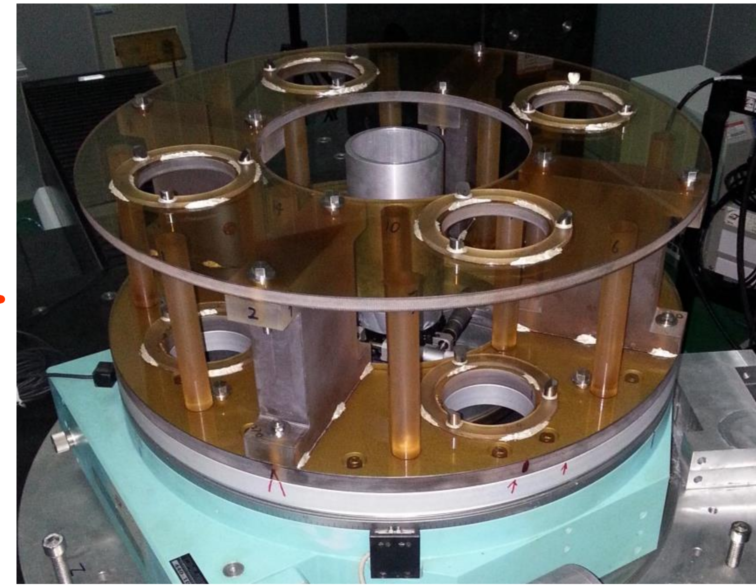


Sphere shelves

Ultra-low expansion coefficient 0.1×10^{-6}/℃



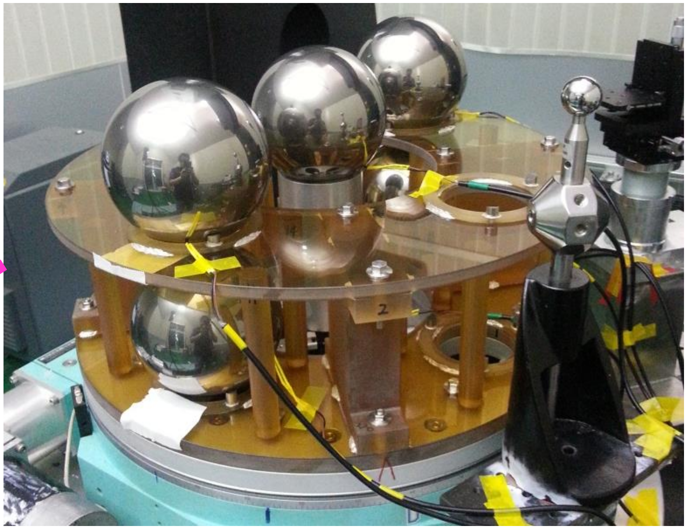
ULE Shelves

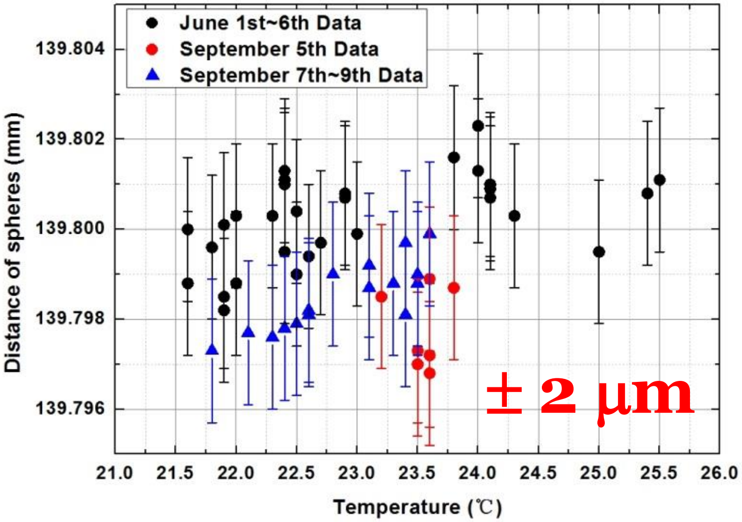


Distance (mm)	Expansion Coefficient (/℃)	Temperature Fluctuation (℃)	Distance Variation (μm)	Relative Uncertainty (ppm)
Horizontal 342.319	0.1×10^{-6}	0.7	0.02	0.10
Vertical 139.751	0.1×10^{-6}		0.01	0.04

Distance measurement

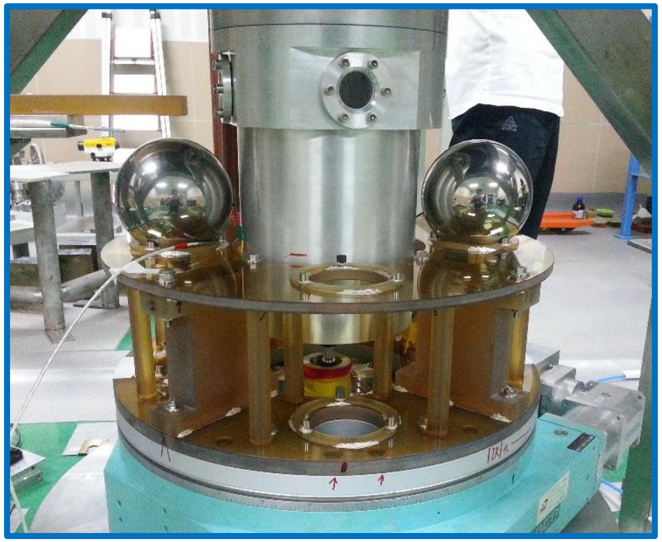
Coordinate Measuring Machine





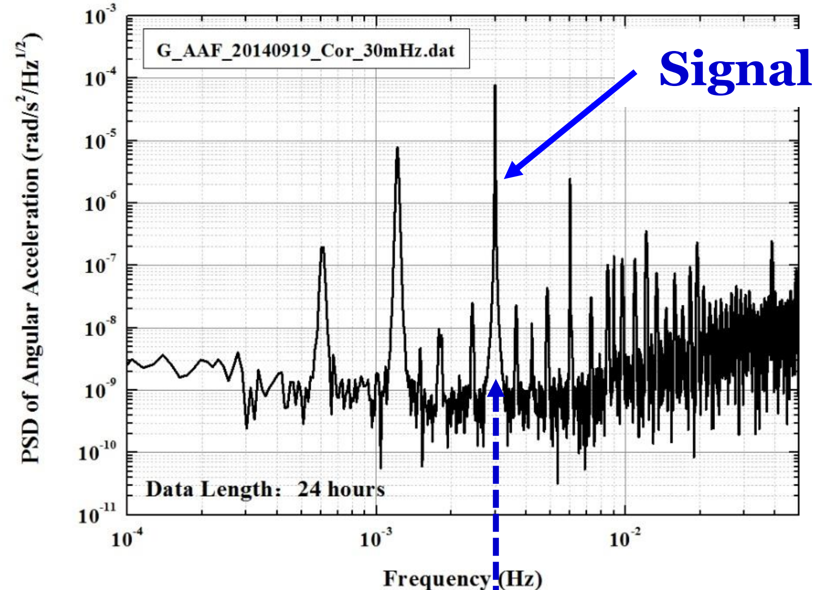
Distance vs. Temperature

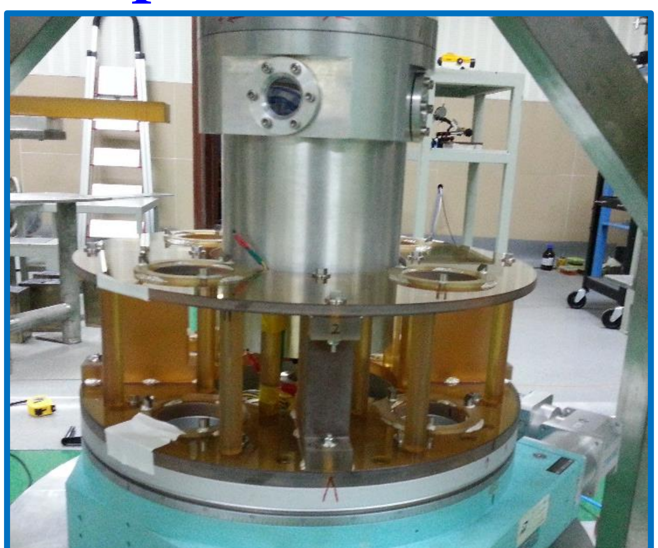
Effect of shelves



sphere + shelves

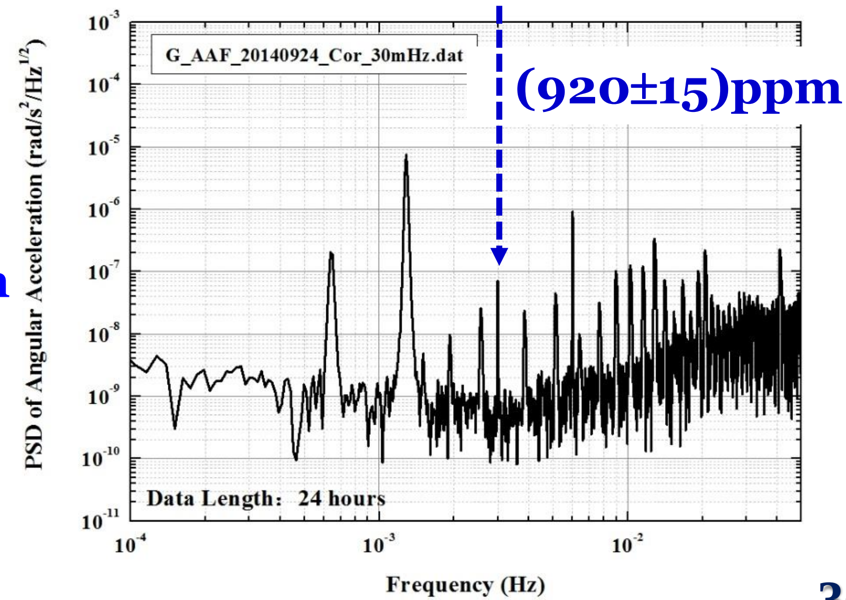
Angular Acceleration →



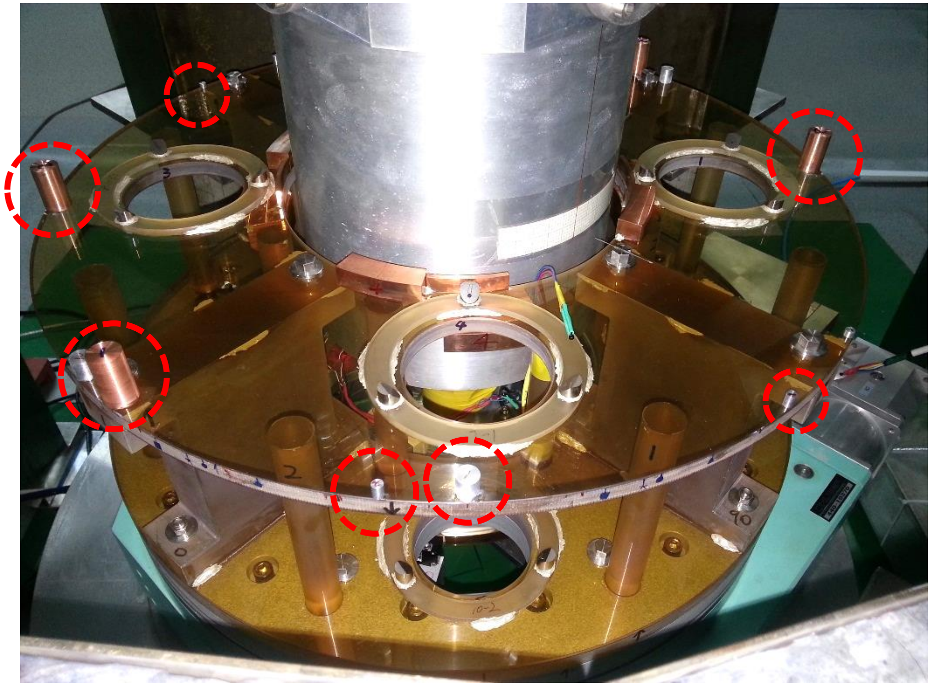


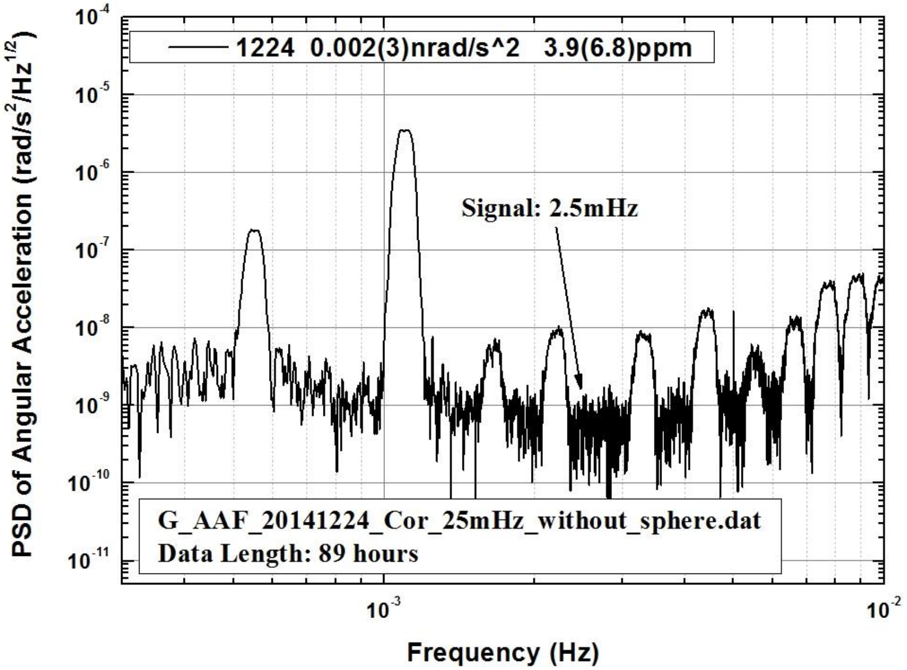
shelves

Angular Acceleration →



Effect of shelves

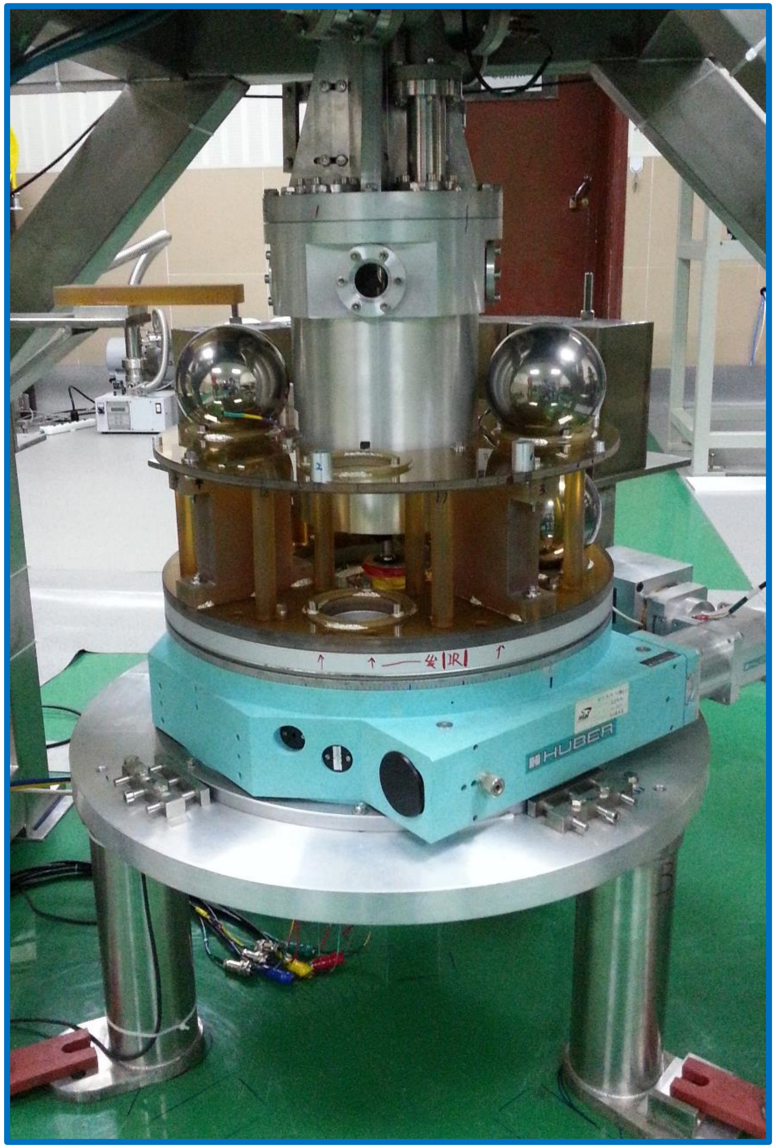


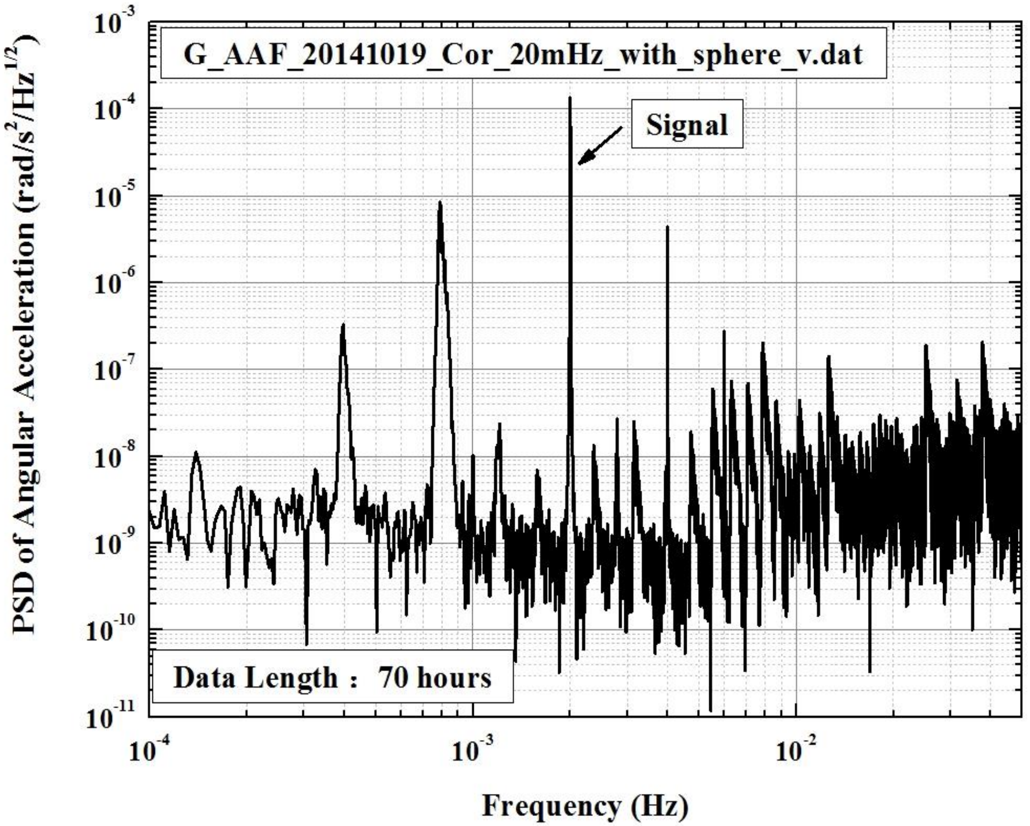


Compensate with copper and aluminum cylinders

920(15) ppm ⇒ 4(7) ppm

Result from raw data

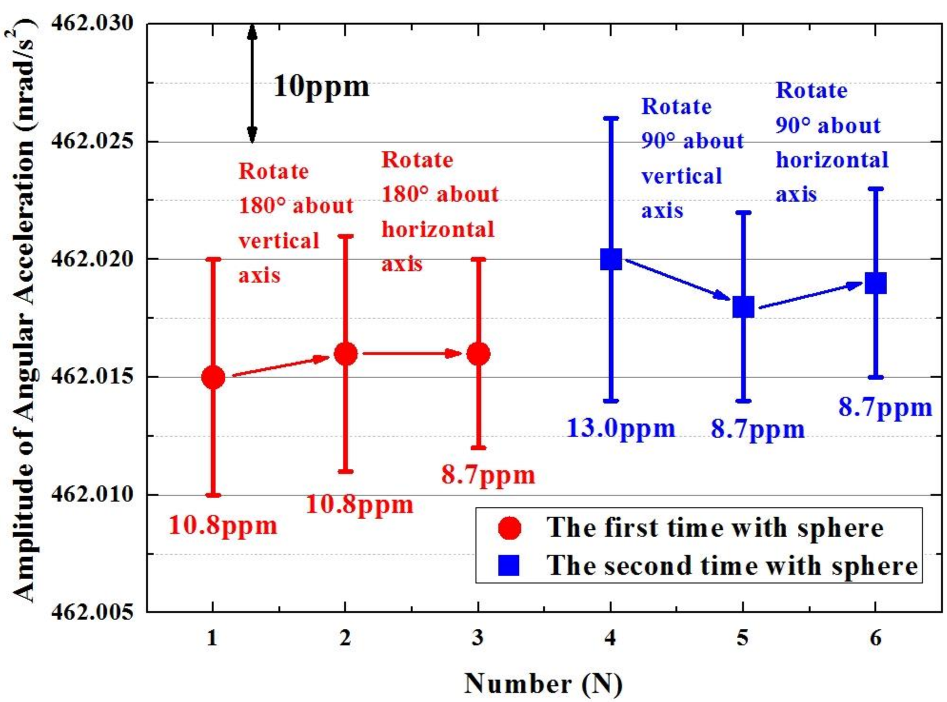




Amplitude：(462.016±0.005) nrad/s²

11ppm

Error budget (not complete)



$G = (6.674^{**} \pm 0.00011)$

$\times 10^{-11}\ m^3 kg^{-1} s^{-2}$

~16.8ppm

Error Sources	Correction (ppm)	$\delta G/G$
Torsion pendulum		**2.0**
Pendulum		2.0
Coating layer		0.1
Clamp and ferrule		0.2
Glue		0.2
Source masses		**6.5**
Masses (×4)		0.2
Distance of GCs (×4)		6.5
Relative positions		**0.9**
Residual twist angle		**0.9**
Time base		**0.1**
Magnetic damper	**455.6**	**1.6**
Air buoyancy	**149.9**	**1.5**
Amplitude of $\alpha(\omega)$		**15.2**
Total		**16.8**

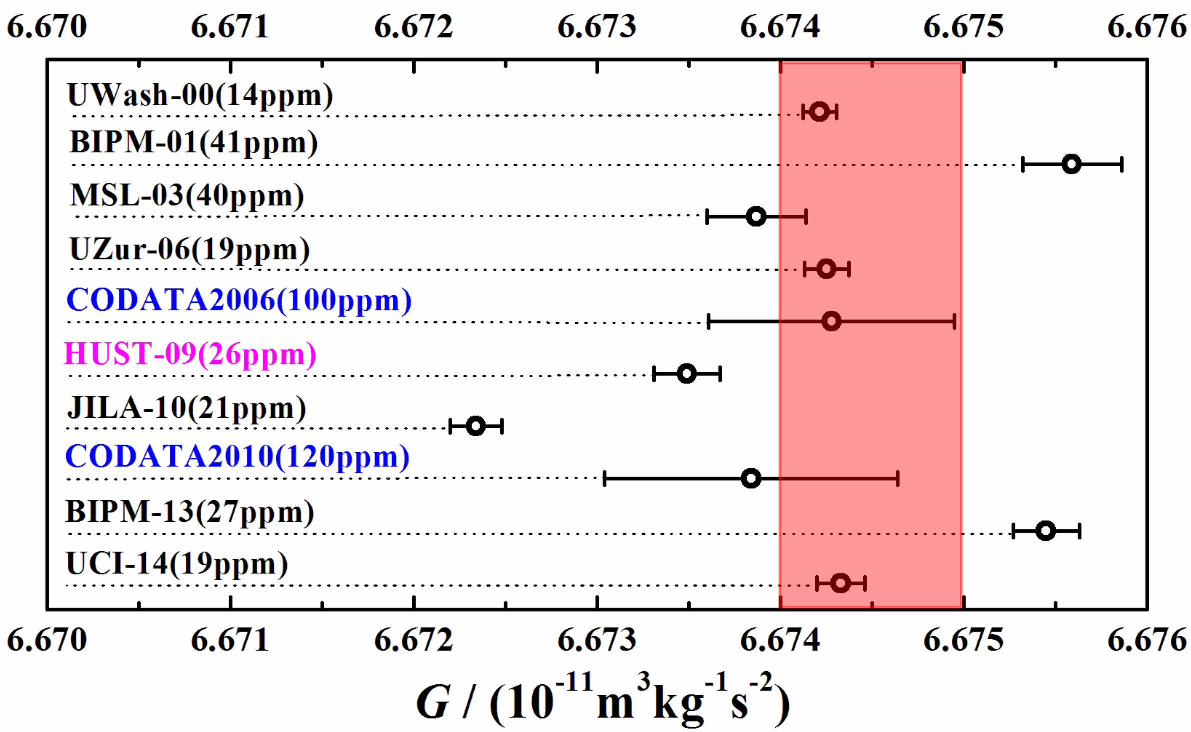
Present results of two methods

TOS : $G = (6.674^{**} \pm 0.00010) \times 10^{-11}\ m^3kg^{-1}s^{-2}$

< 20 ppm

AAF : $G = (6.674^{**} \pm 0.00011) \times 10^{-11}\ m^3kg^{-1}s^{-2}$

< 20 ppm



"Blind" experiments

We expect to get the updated G value in 2015 !

Staffs and Students of CGE 2014.09.10

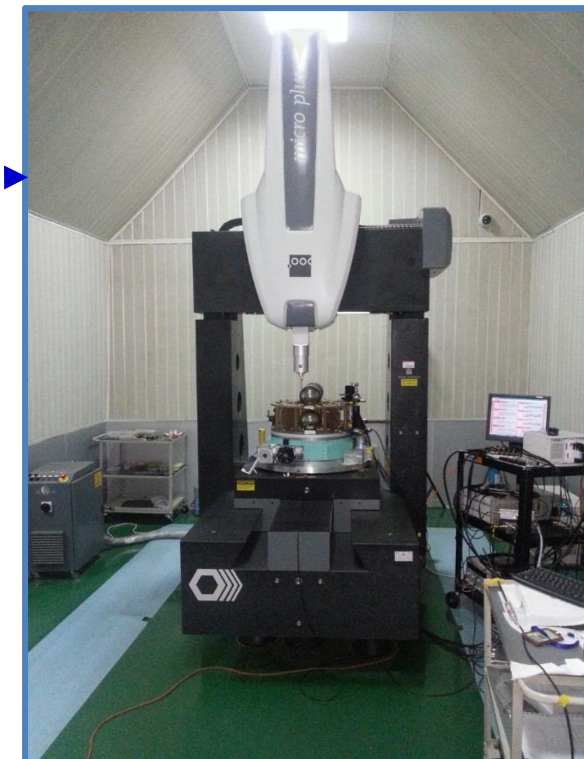
Thanks for Your Attention!

Next plan

Room 2068
Experimental site

Room 209
Measuring room

~ 80 m



Directly measure the distance of spheres in the experimental site