THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Hadronic corrections to $(g\text{-}2)_\mu$
- status update of the Mainz workshop

Marc Vanderhaeghen

Fundamental Constants Meeting 2015, February 1- 6, 2015

Hotel Frankenbach, Eltville, Germany

MITP/SFB $(g-2)_\mu$ workshops: April 1-5, 7-11, 2014

Mainz Institute for Theoretical Physics

Organizers: T. Blum, A. Denig, S. Eidelman, F. Jegerlehner, D. Stöckinger, M. Vdh

SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Mini-proceedings: arXiv:1407.4021 [hep-ph]

Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction

April 1–5, 2014 in Waldthausen Castle, Mainz, Germany

AND

$(g-2)_\mu$: Quo vadis?

April 7–10, 2014 in Mainz, Germany

Mini Proceedings

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ABSTRACT

We present the mini-proceedings of the workshops *Hadronic contributions to the muon anomalous magnetic moment: strategies for improvements of the accuracy of the theoretical prediction* and $(g-2)_\mu$*: Quo vadis?* held in Mainz from April 1$^{\text{rst}}$ to 5$^{\text{th}}$ and from April 7$^{\text{th}}$ to 10$^{\text{th}}$, 2014, respectively.

The web page of the conferences, which contains all talks, can be found at

- *Hadronic contributions to the muon anomalous magnetic moment*: https://indico.mitp.uni-mainz.de/conferenceDisplay.py?confId=13
- $(g-2)_\mu$*: Quo vadis?*: https://indico.cern.ch/event/284012/

magnetic moment of muon: $(g\text{-}2)_\mu$

magnetic moment

$$\vec{m} = \mu_B \, g \, \vec{S}$$

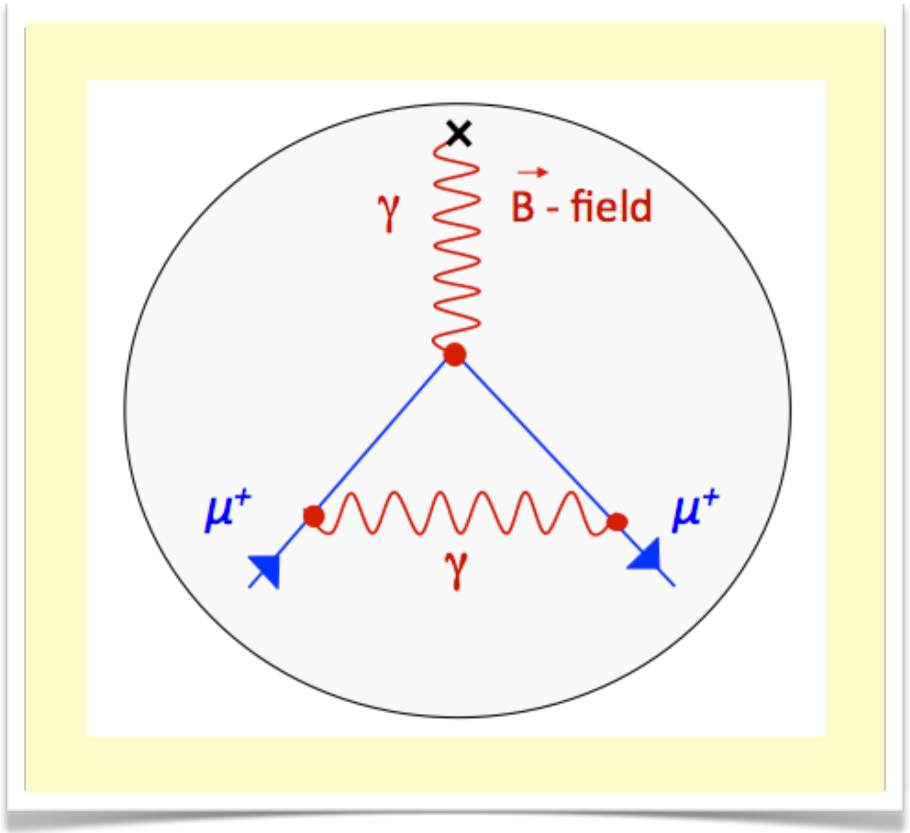
μ_B: Bohr magneton
g: gyromagnetic factor ~ 2

Dirac

anomalous part:

$a_\mu = (g\text{-}2)_\mu/2$
$= \alpha_{em}/2\pi + \ldots = 0.00116\ldots$

Schwinger



magnetic moment of muon: $(g-2)_\mu$

➡ **SM prediction for a_μ**

QED: $a_\mu^{QED} = (11\,658\,471.896 \pm 0.008) \times 10^{-10}$

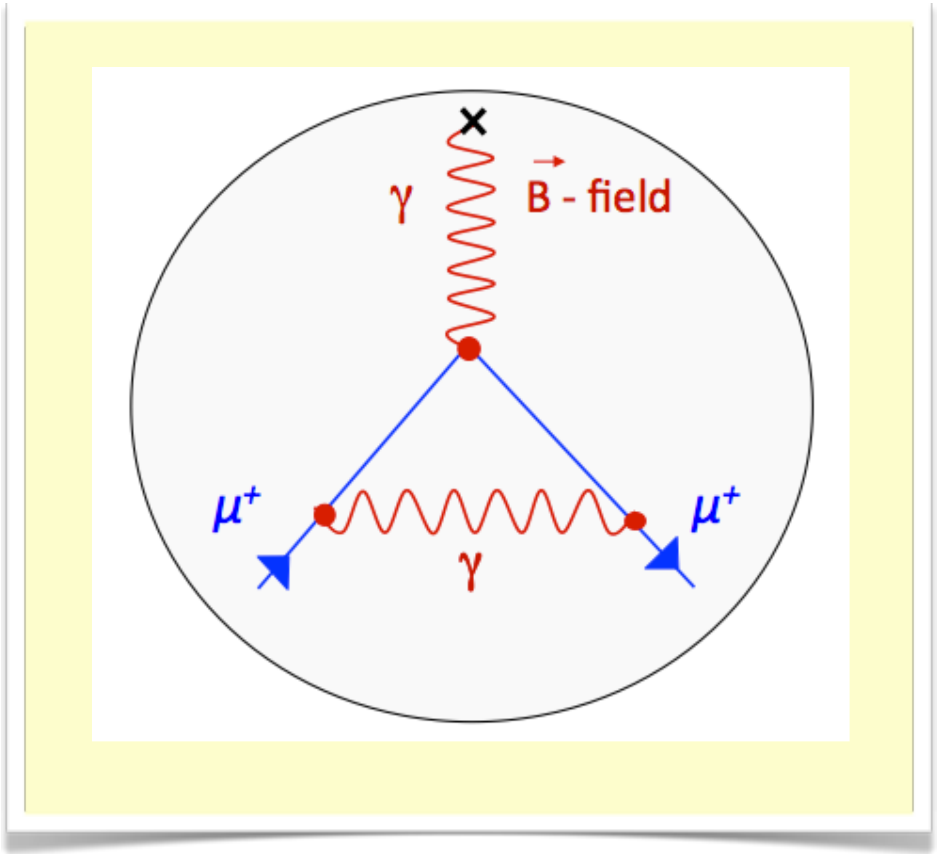
up to $O(\alpha_{em}^5)$!

Aoyama, Hayakawa, Kinoshita, Nio (2012)

weak: $a_\mu^{weak} = (15.4 \pm 0.1) \times 10^{-10}$

strong: $a_\mu^{strong} = (706.2 \pm 5.0) \times 10^{-10}$

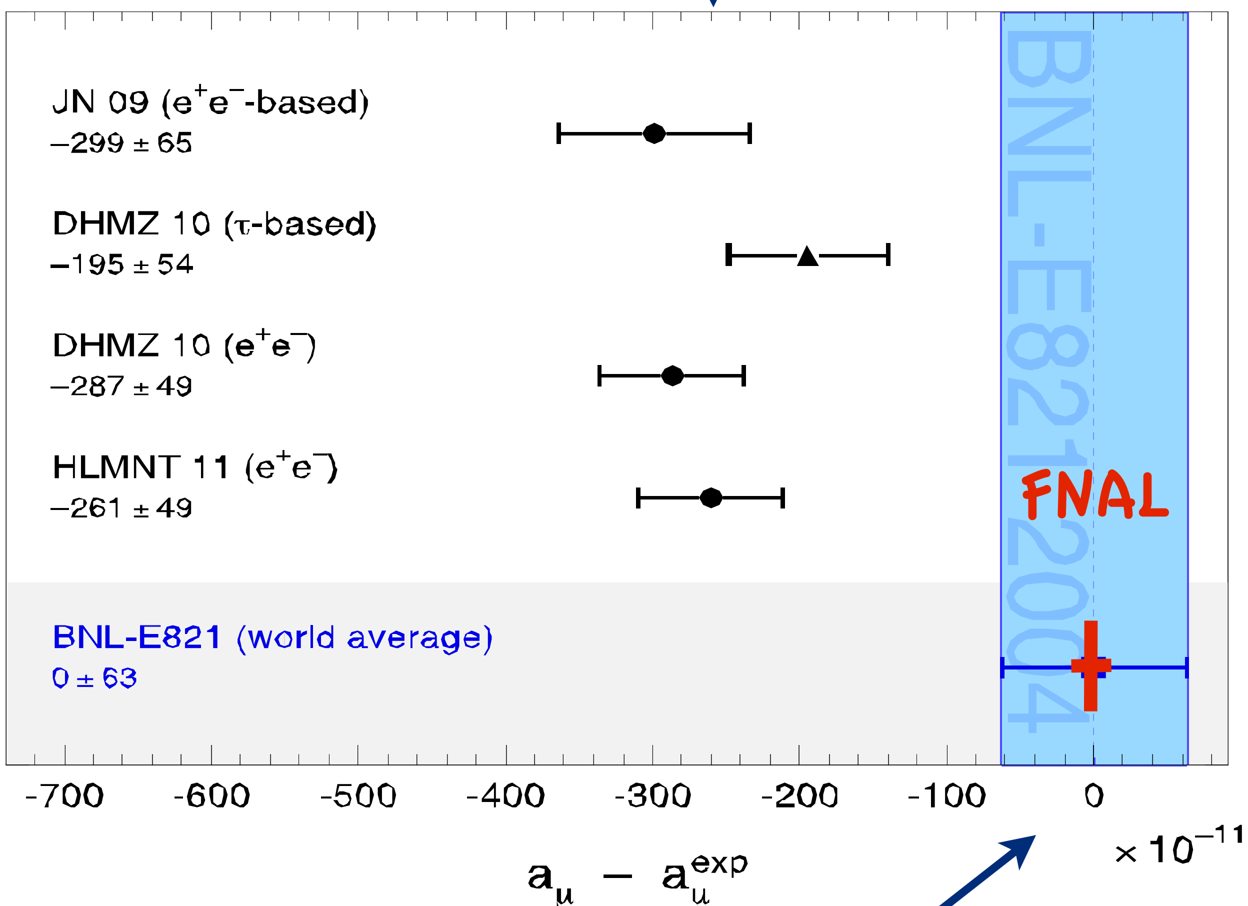
Hagiwara et al. (2011)



hadronic uncertainties completely dominate the accuracy of the SM result

$(g-2)_\mu$: theory vs experiment

SM predictions for a_μ



BNL-E821 measurement of a_μ

$$a_\mu^{exp} = (11\,659\,208.9 \pm 6.3) \times 10^{-10}$$

$$a_\mu^{exp} - a_\mu^{SM} = (26.1 \pm 5.0_{th} \pm 6.3_{exp}) \times 10^{-10}$$

Hagiwara et al. (2011)

3 - 4 σ deviation from SM value !

Errors or new physics ?

New FNAL experiment (2016)

$$\delta a_\mu^{FNAL} = 1.6 \times 10^{-10}$$

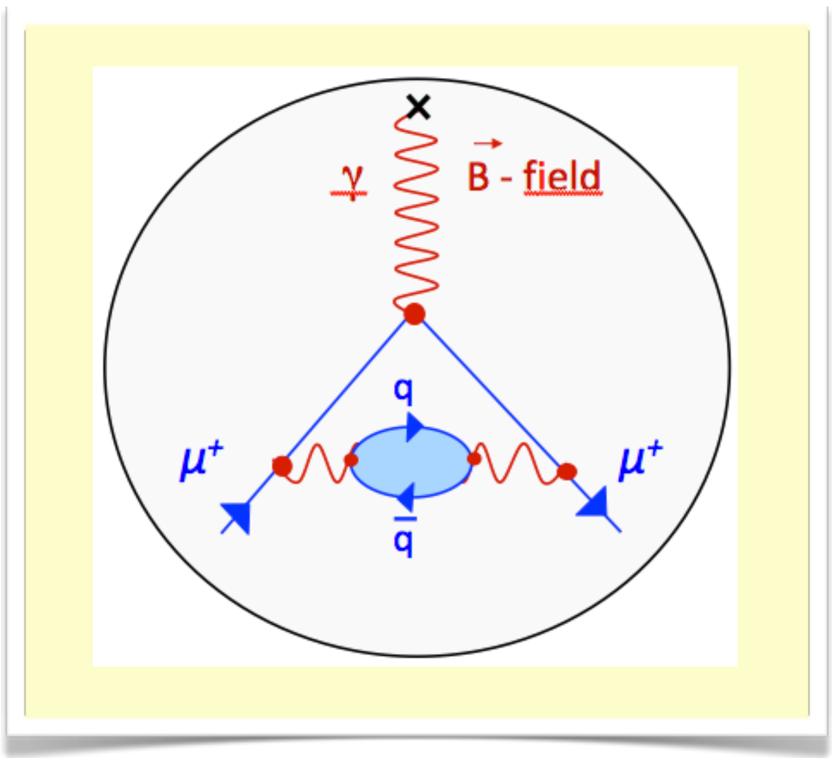
factor 4 improvement in exp. error

-> Improve theory !

strong contributions to $(g-2)_\mu$

contributions from strong interactions NOT calculable within perturbative QCD

hadronic vacuum polarization

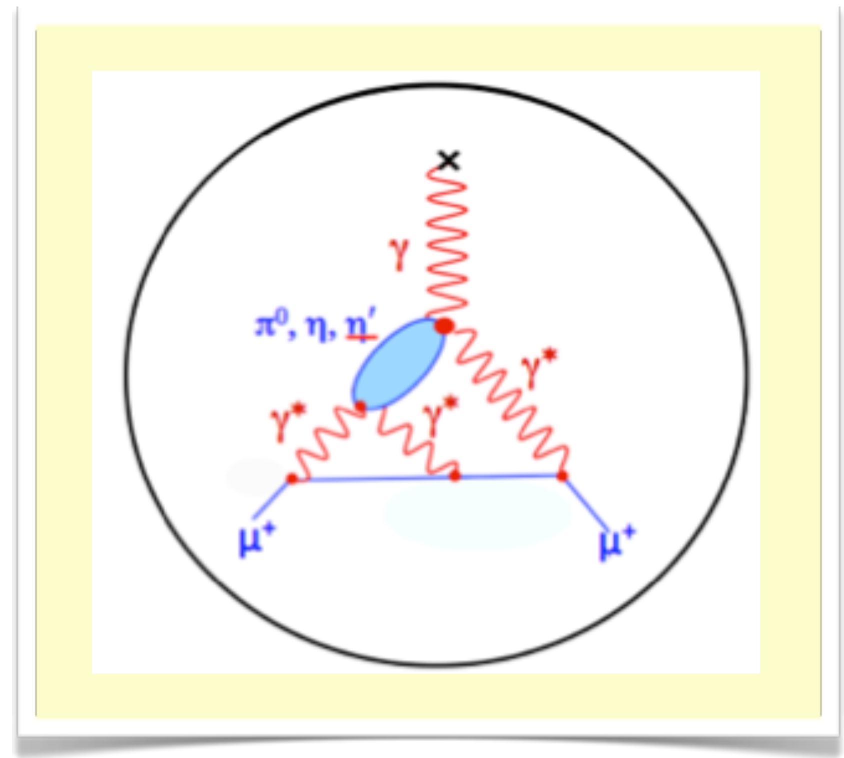


$a_\mu^{\text{l.o. had, VP}} = (694.9 \pm 4.3) \times 10^{-10}$

Hagiwara et al. (2011)

hadronic vacuum polarization determined by cross section measurements of e^+e^- -> hadrons

hadronic light-by-light scattering



$a_\mu^{\text{had, LbL}} = (11.6 \pm 4.0) \times 10^{-10}$

Jegerlehner, Nyffeler (2009)

measurements of meson transition form factors required as input to reduce uncertainty

$(g\text{-}2)_\mu$: hadronic vacuum polarization

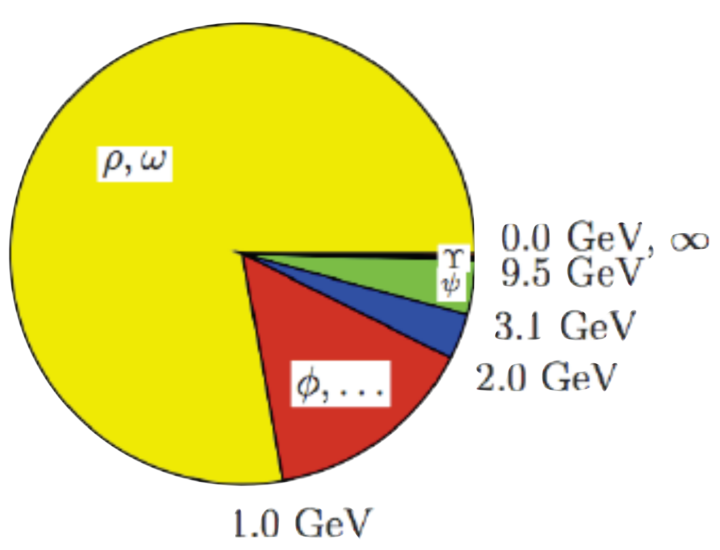
Optical theorem and analyticity allow to relate HVP contribution to $(g\text{-}2)_\mu$ with $\sigma_{had} = \sigma(e^+e^- \to hadrons)$

$$a_\mu^{had,VP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}$$

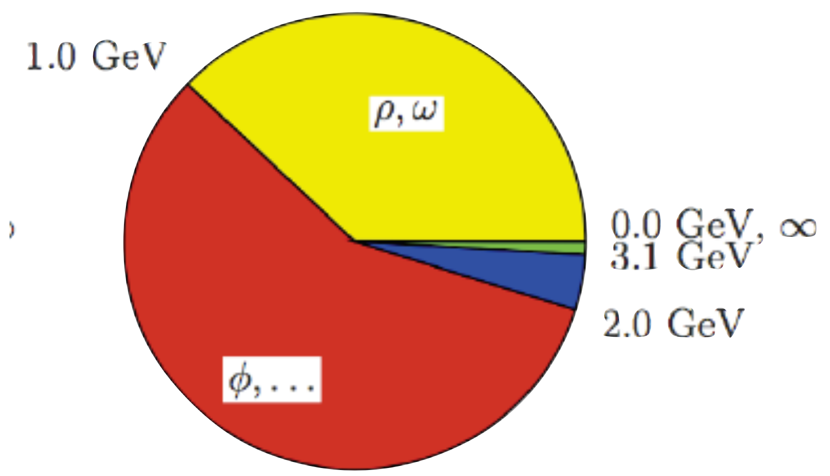
Kernel function

Hadronic cross section

a_μ^{had}

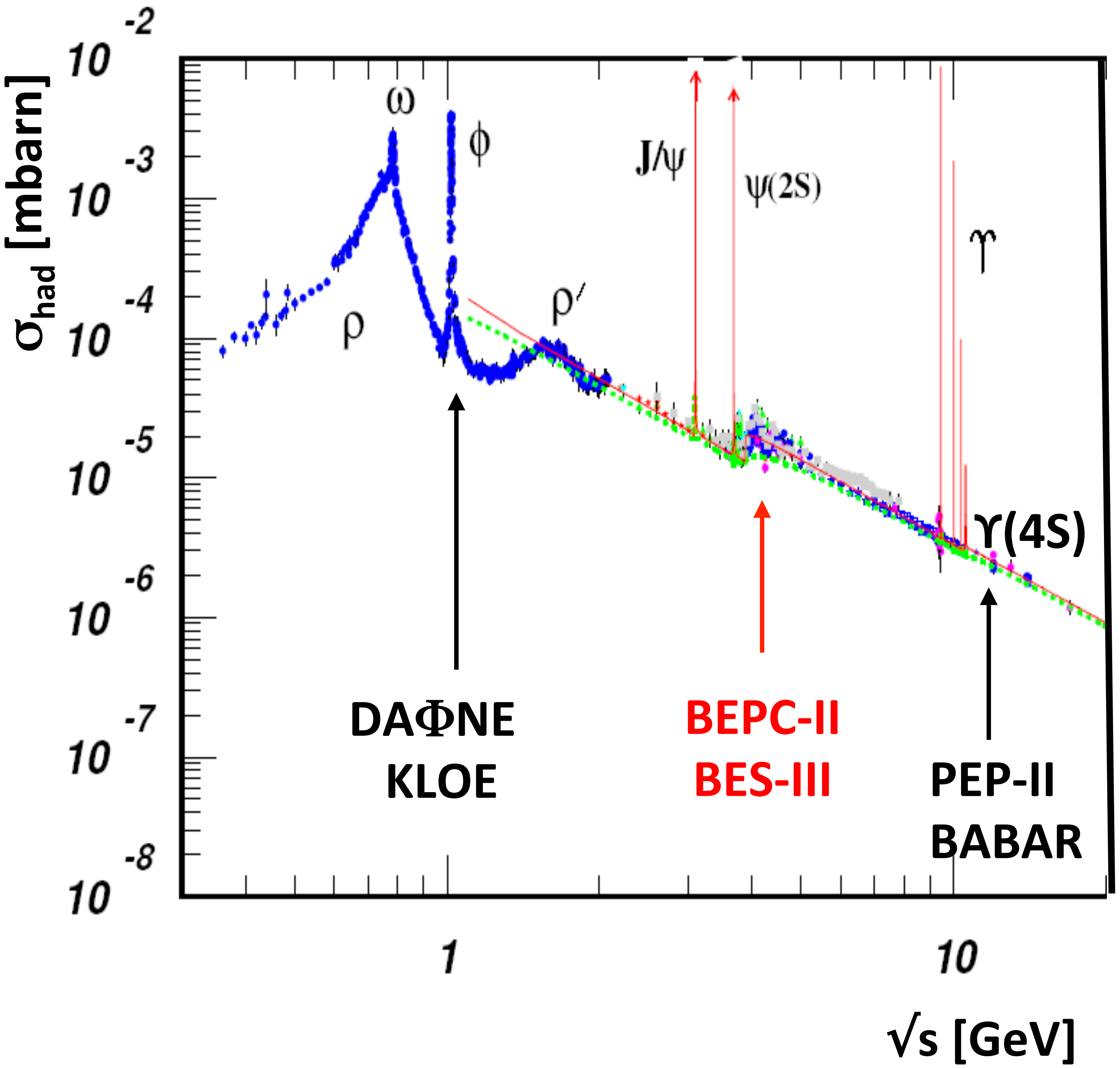


δa_μ^{had}



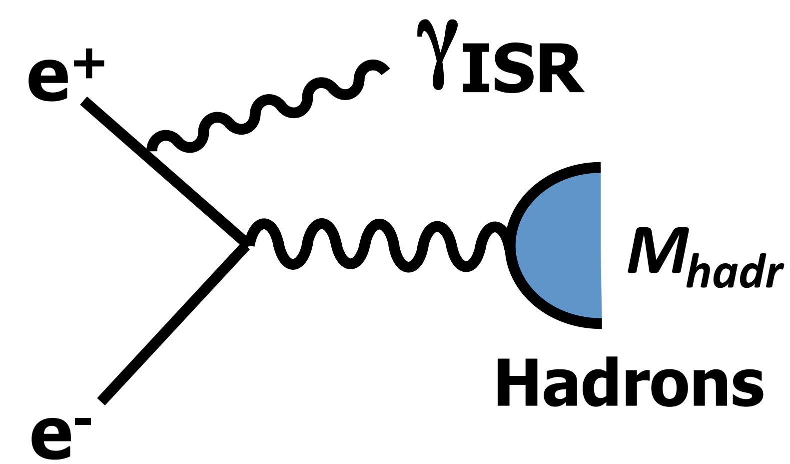
σ_{had} : energy range up to 3 GeV essential!

measure σ_{had} via ISR at BES-III



Approach for measuring hadronic cross section at modern particle factories with fixed c.m.s. energy √s:

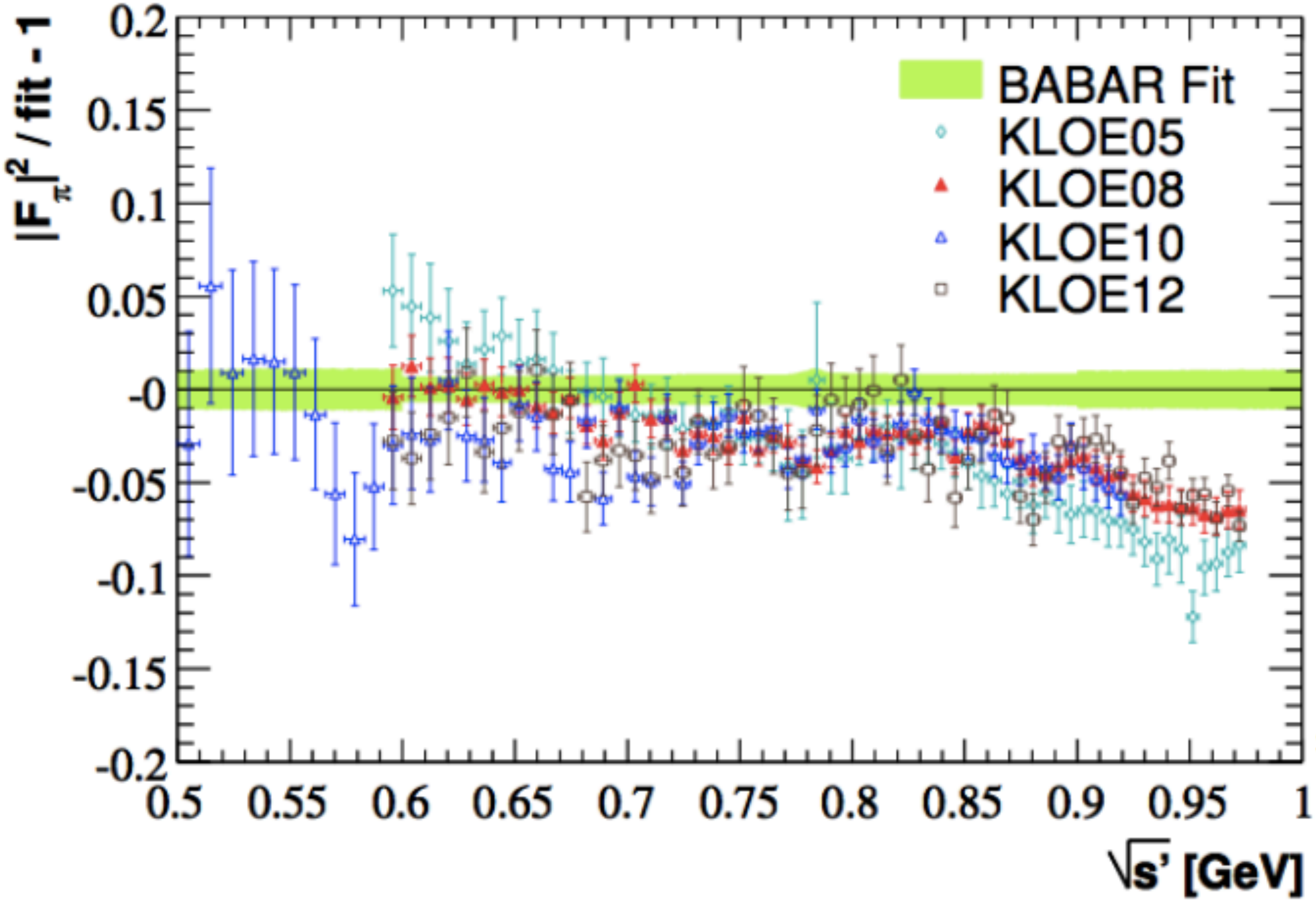
Initial State Radiation (ISR)



ISR method allows access to mass range M_{hadr}< 3 GeV at BES-III

ongoing exp. program

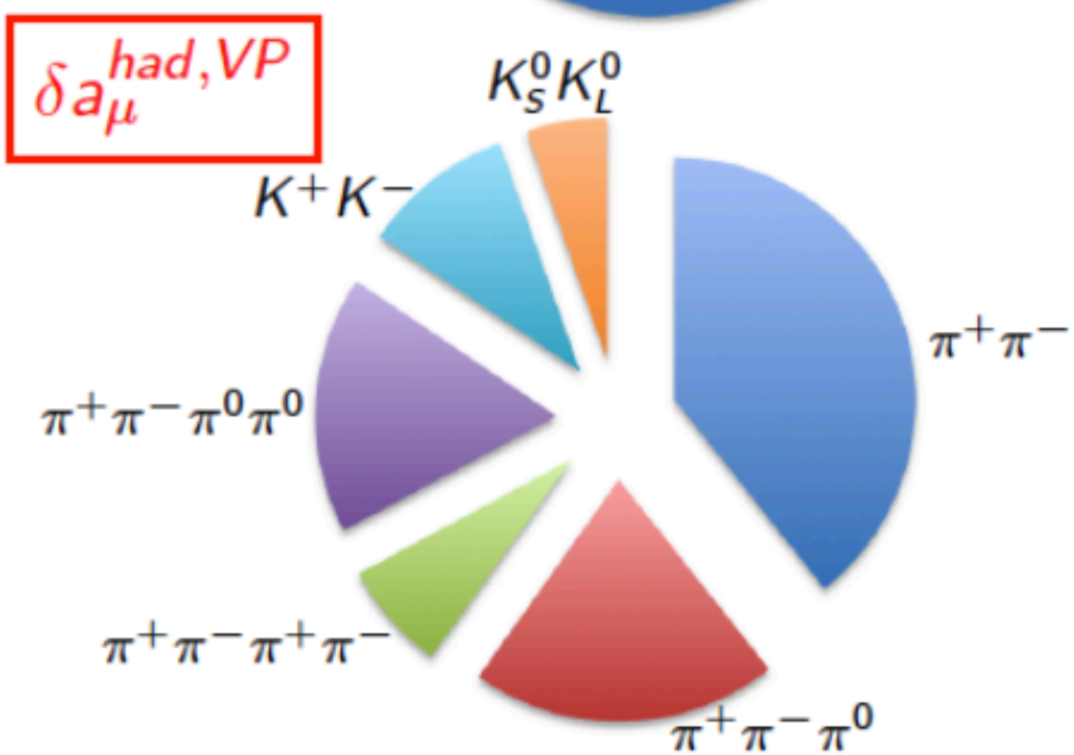
HVP: most relevant channel $e^+ e^- \rightarrow \pi^+ \pi^-$



Note: KLOE05 superseded by KLOE08

- KLOE and BABAR dominate the world average
- Relatively large systematic differences, esp. above ρ peak
- Knowledge of a_μ^{had} dramatically limited due to this difference

Status Hadronic Vacuum Polarization



Future improvement of a_μ^{had} ?

1st priority:
Clarify situation regarding $\pi+\pi-$
(KLOE vs. BABAR puzzle)

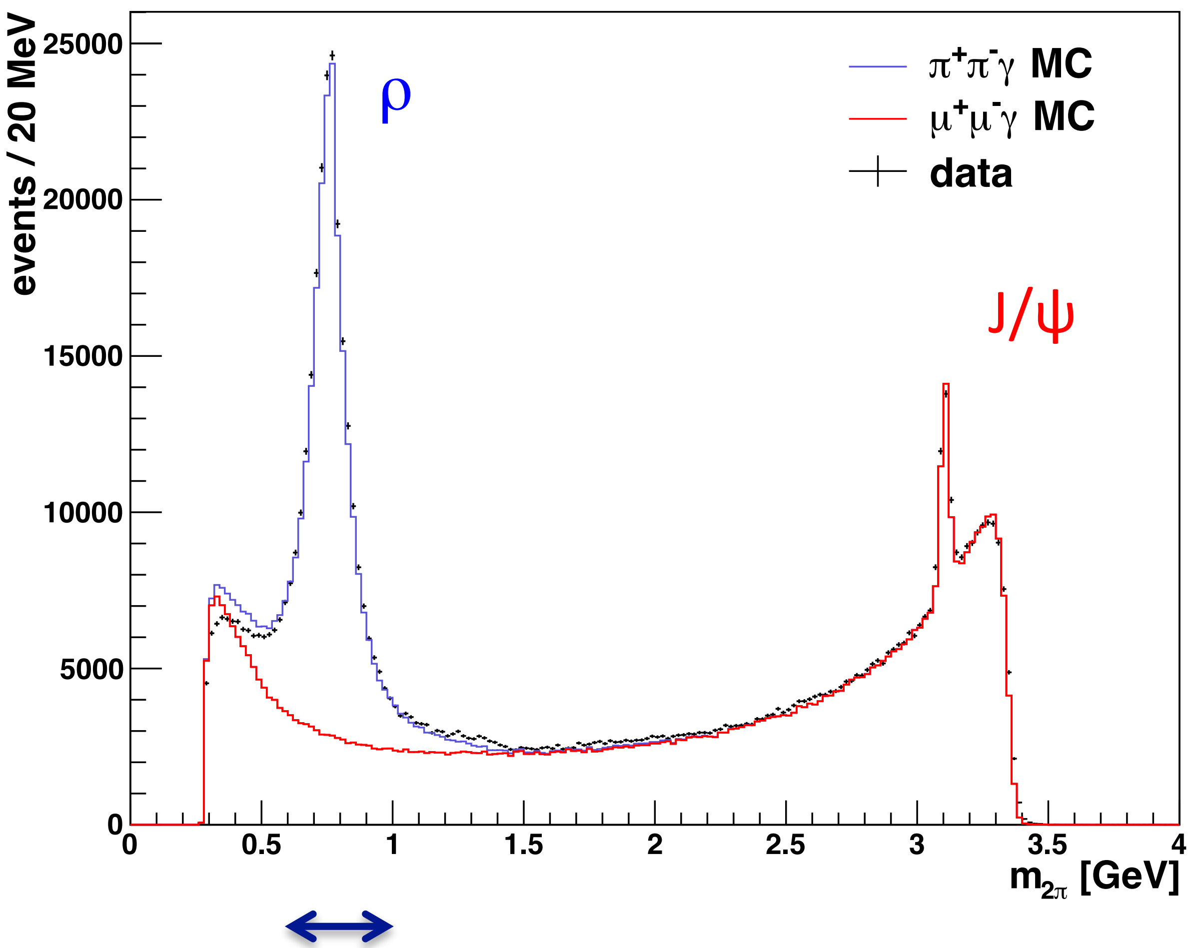
2nd priority:
Measure 3π, 4π channels

Ongoing ISR analyses
BESIII, BEPC-II collider

3rd priority:
$K\bar{K}$ and higher multiplicities

HVP: BES-III ISR results $e^+e^- \rightarrow \pi^+\pi^-\gamma$

Event yield after acceptance cuts **only**

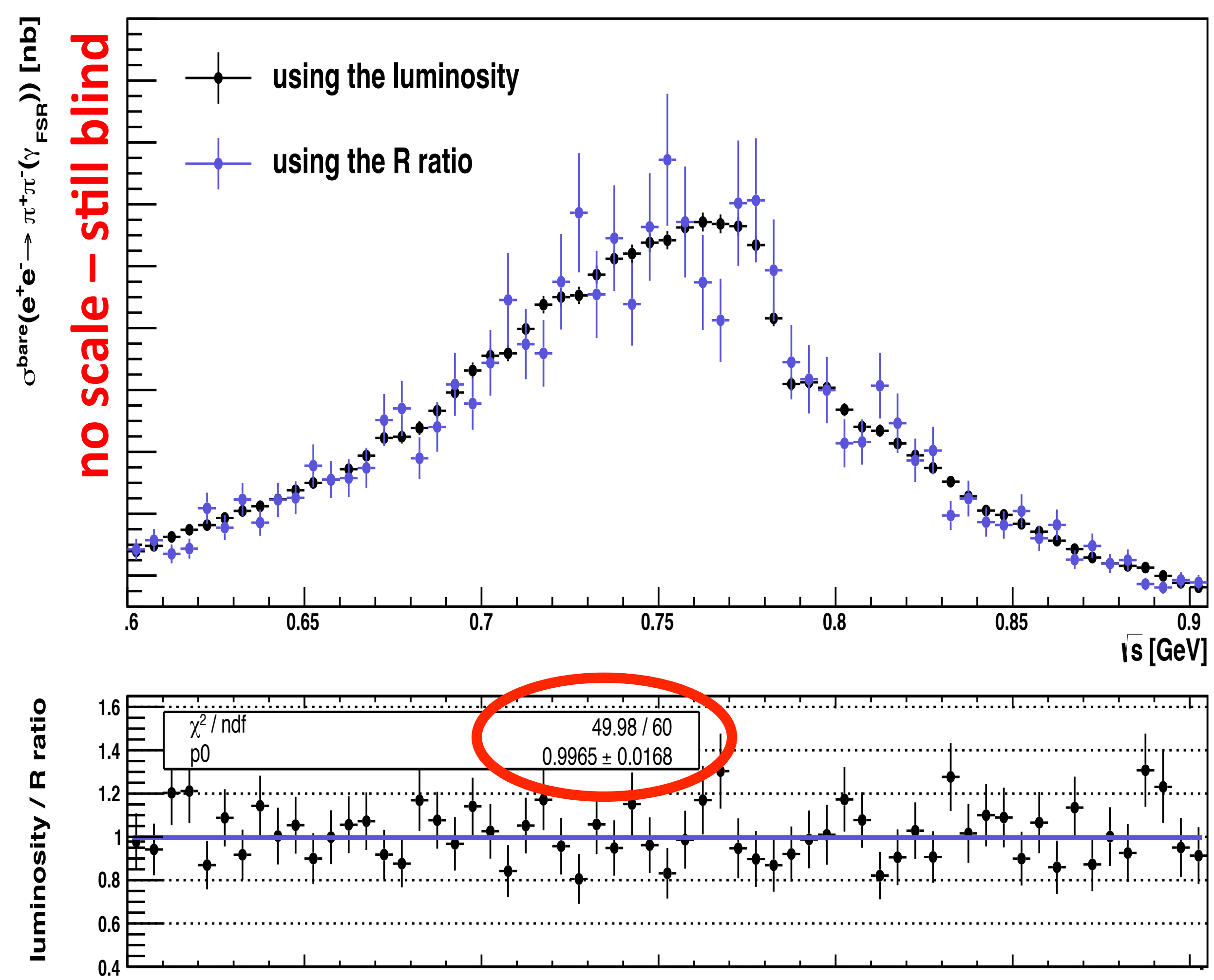


Absolute cross section σ(e+e-→π+π-)

2 normalization methods:

1) normalization to L_{int} and Radiator-Function

2) normalization to μμγ, i.e. R ratio (ππγ/μμγ)



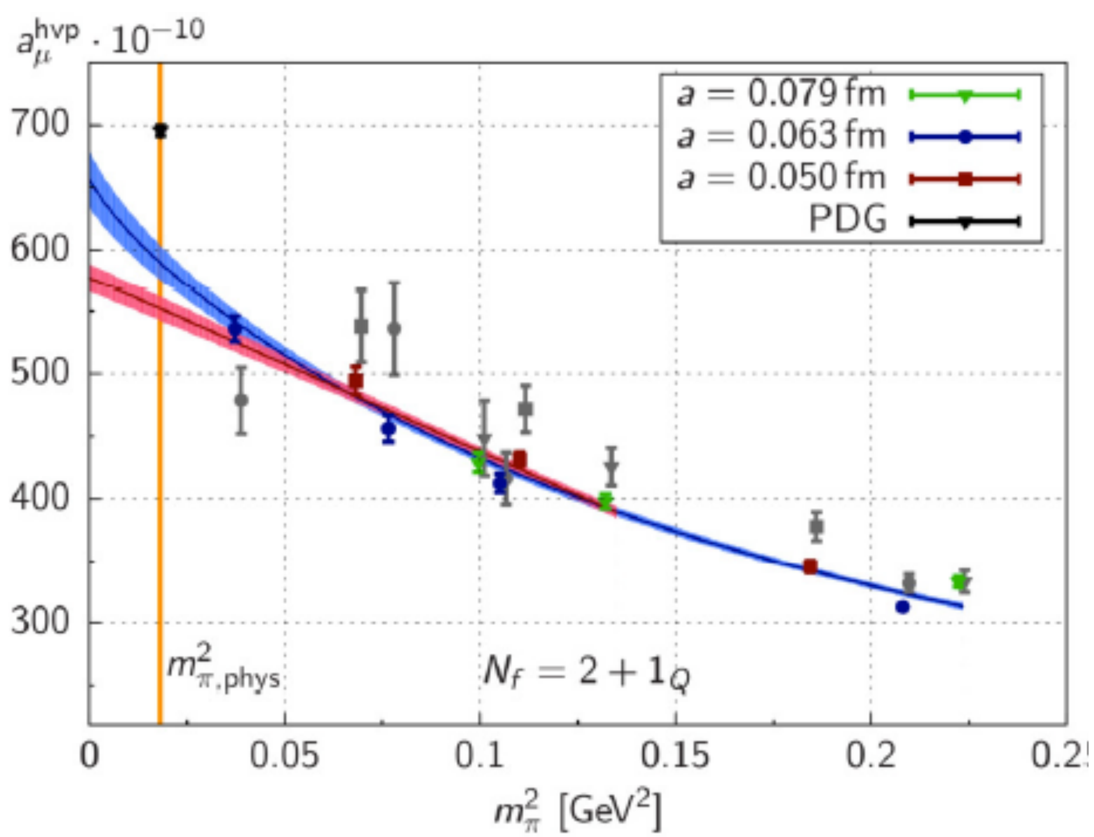
Cross section Precision 1.3 % -> 1.0 %

BESIII preliminary

luminosity / R ratio -1 = (0.35 ± 1.68) %

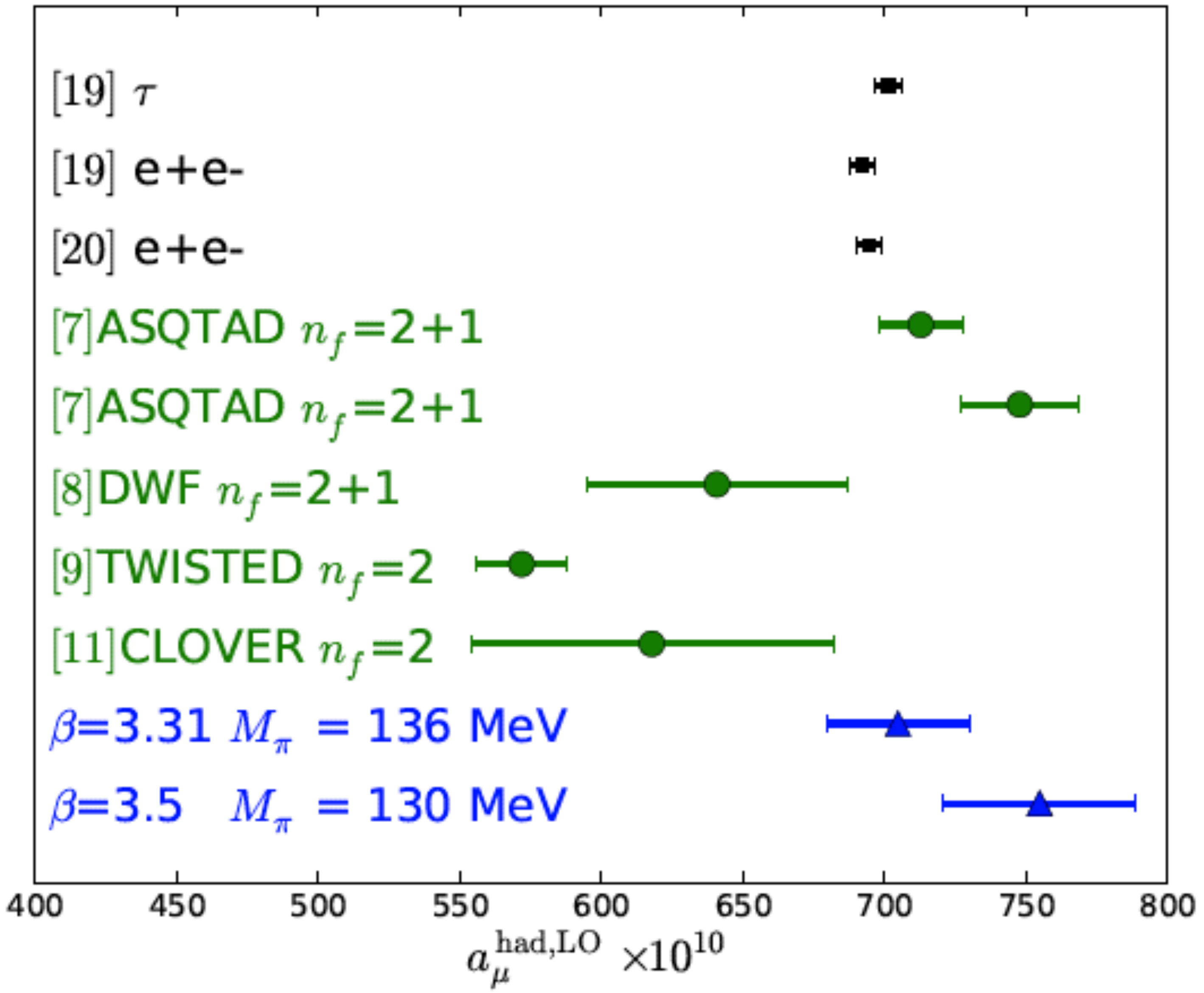
limited by low μμγ statistics

HVP: progress in lattice QCD



Della Morte, Jäger, Jüttner, Wittig (2012)

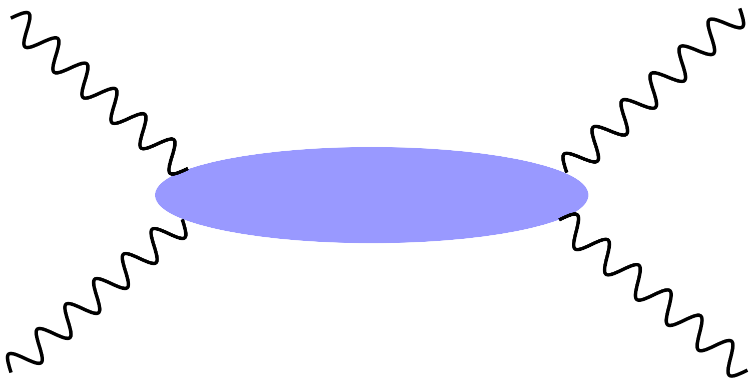
status plot by L. Lellouche



lattice : 3-10 % quoted errors

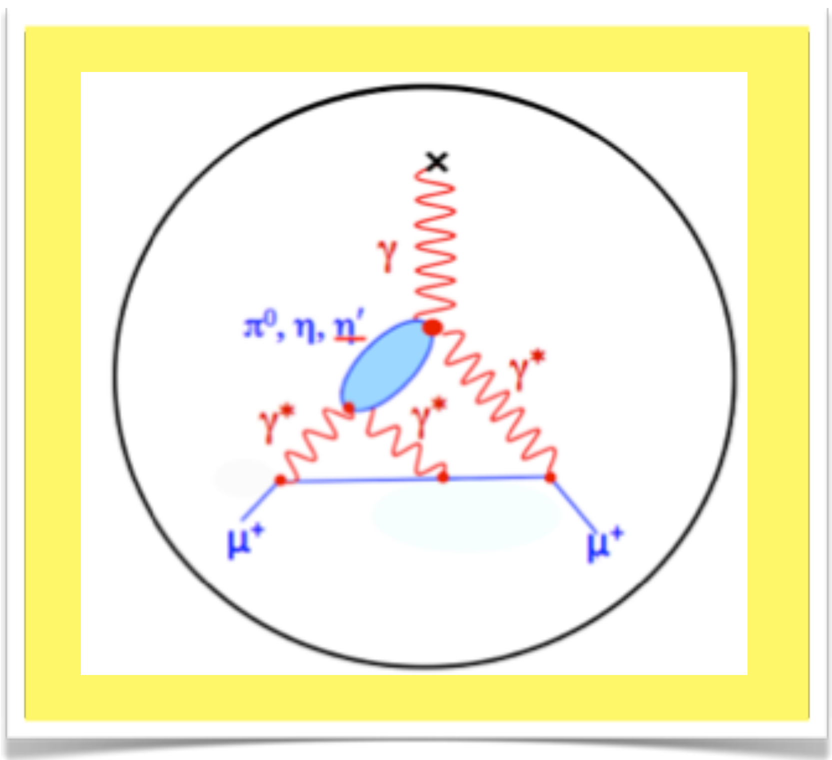
experiment: 0.6 % errors

hadronic LbL scattering



hadronic LbL corrections to $(g\text{-}2)_\mu$

New FNAL and J-Parc $(g\text{-}2)_\mu$ expt. : $\delta a_\mu^{exp} = 1.6 \times 10^{-10}$



$a_\mu^{\text{had, LbL}} = (11.6 \pm 4.0) \times 10^{-10}$

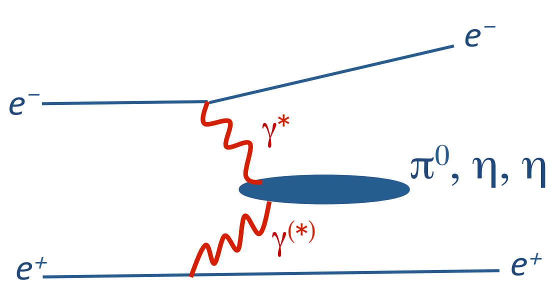
Jegerlehner, Nyffeler (2009)

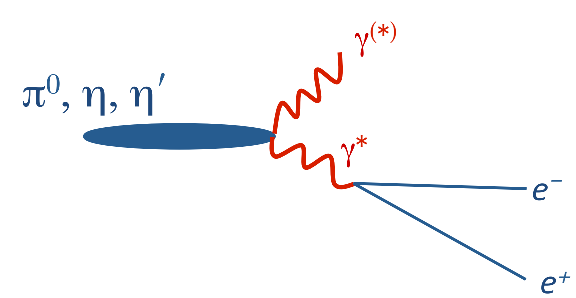
$a_\mu^{\text{had, LbL}} = (10.5 \pm 2.6) \times 10^{-10}$

Prades, de Rafael, Vainshtein (2009)

➡ **experimental input:** meson transition FFs, $\gamma^* \gamma^*$ -> multi-meson states, meson Dalitz decays

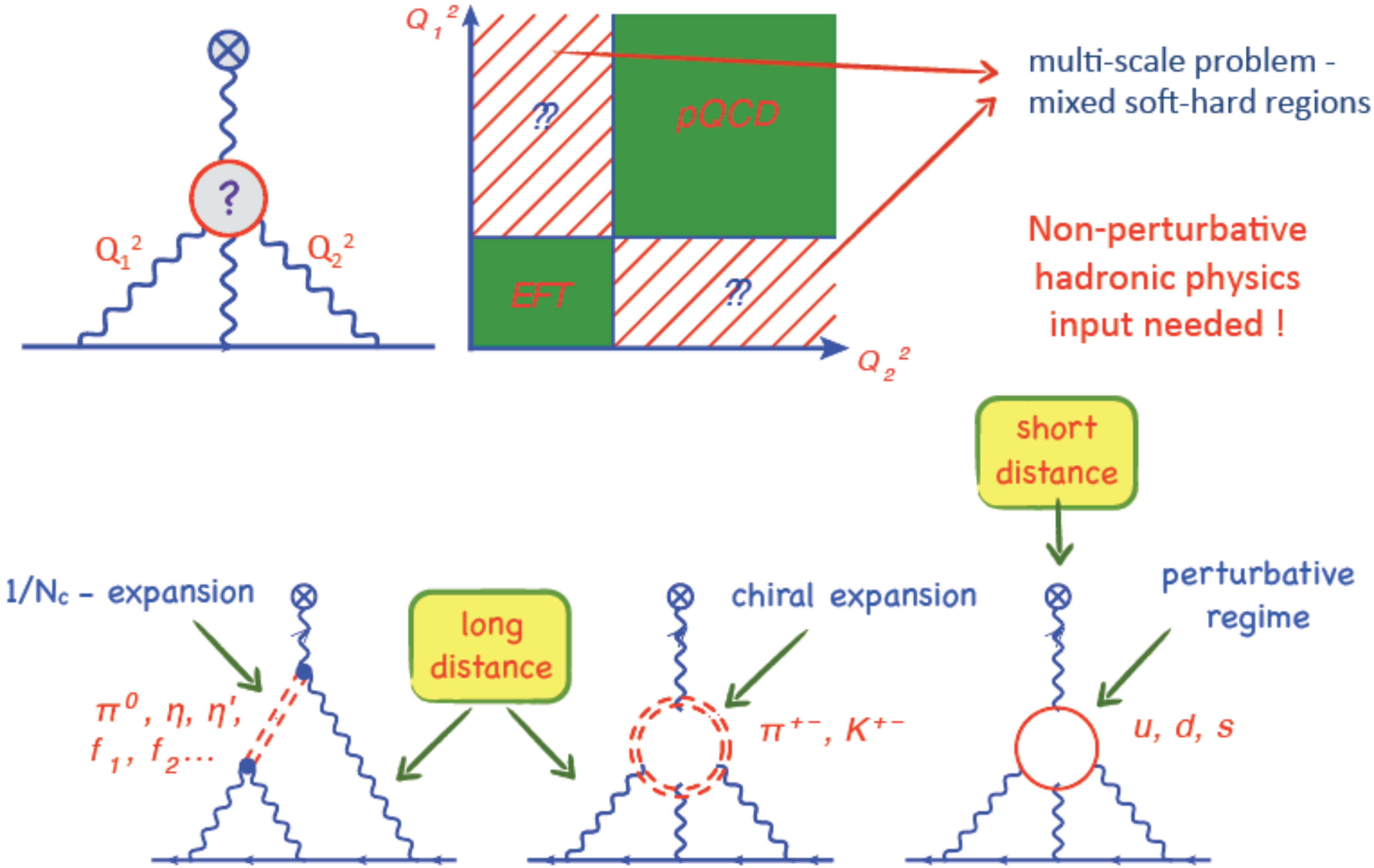
BES-III, MAMI, ...





➡ **theory deveolpments:** models, sum rules, dispersion relations, lattice, ...

hadronic LbL corrections to $(g\text{-}2)_\mu$: relevant contributions



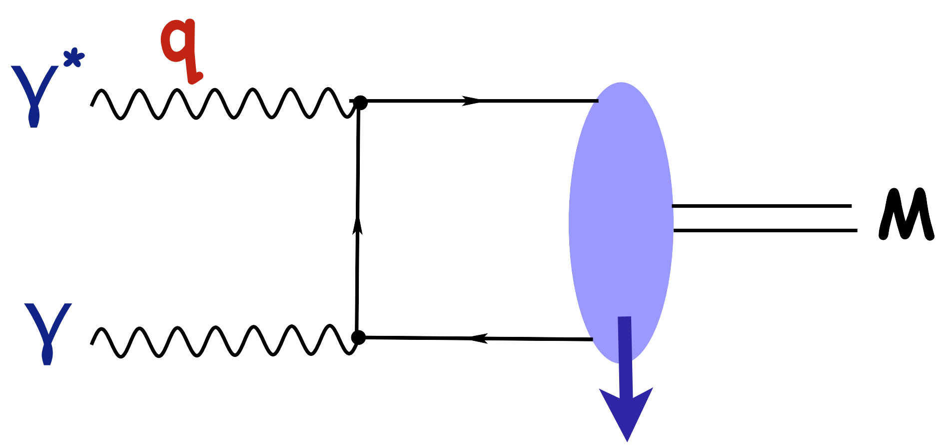
$\gamma\gamma$ physics: quark structure of mesons

spacelike

$q^2 = -Q^2 < 0$

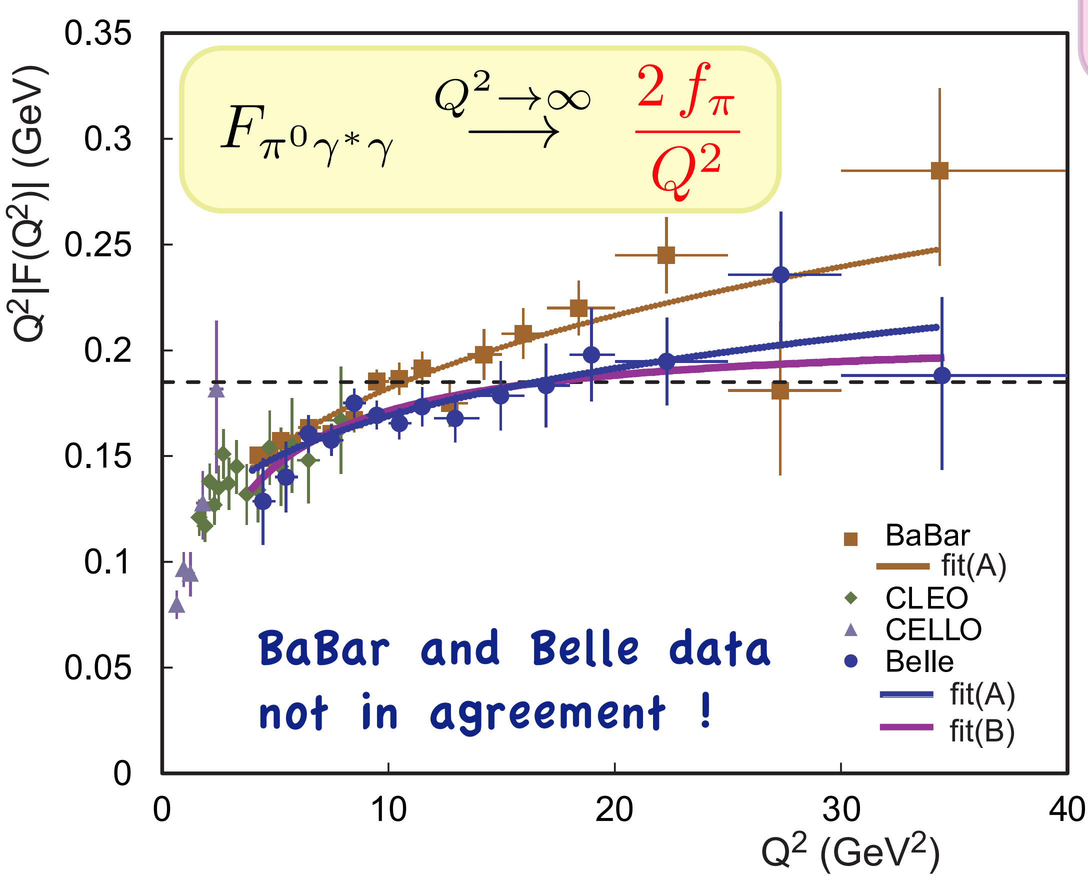
timelike

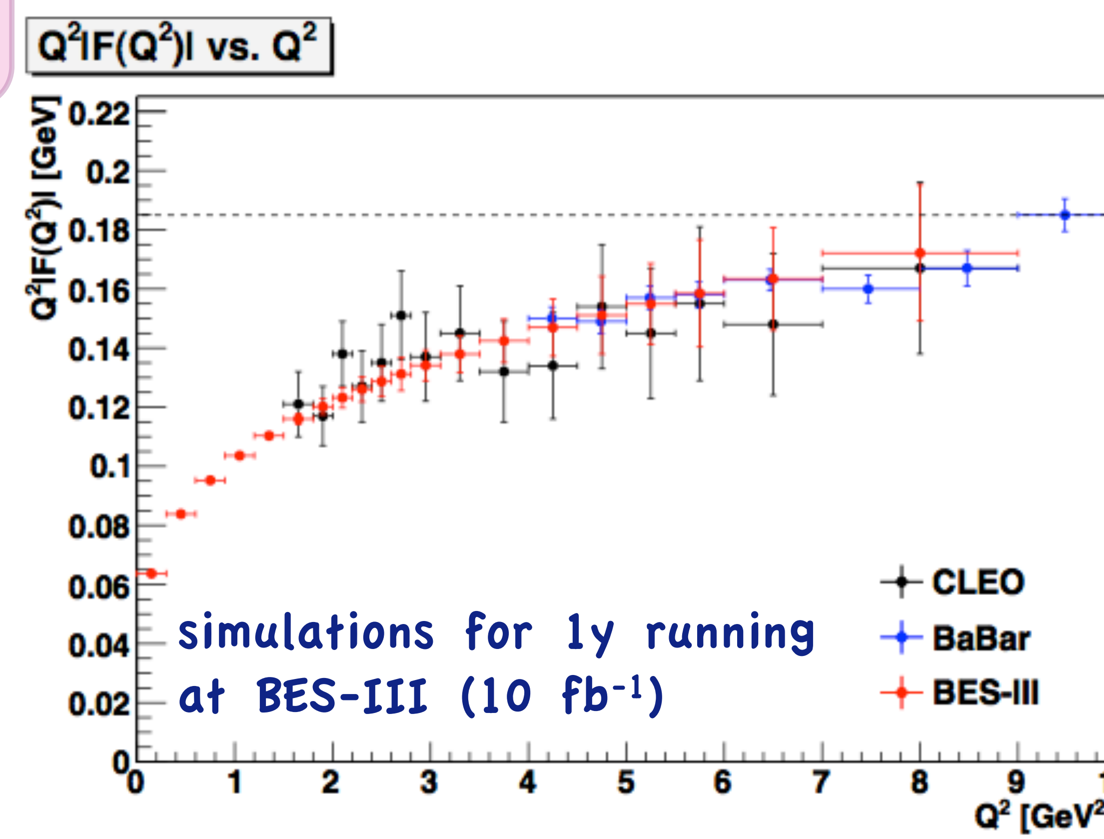
$q^2 > 0$



- paradigm for a whole program of hard processes processes planned at JLab12, Compass, EIC/ENC
- transition region to perturbative QCD remains to be understood

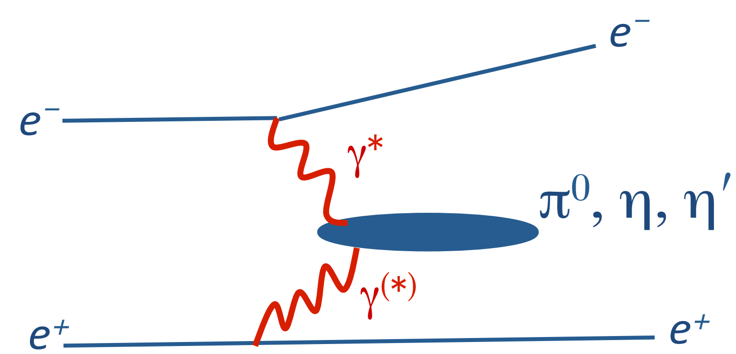
$\gamma^* \gamma \to \pi^0$

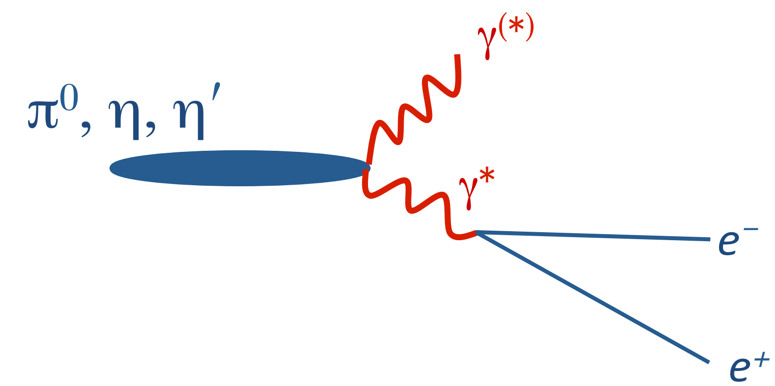




B. Kloss (2011)

η and η′ transition form factors

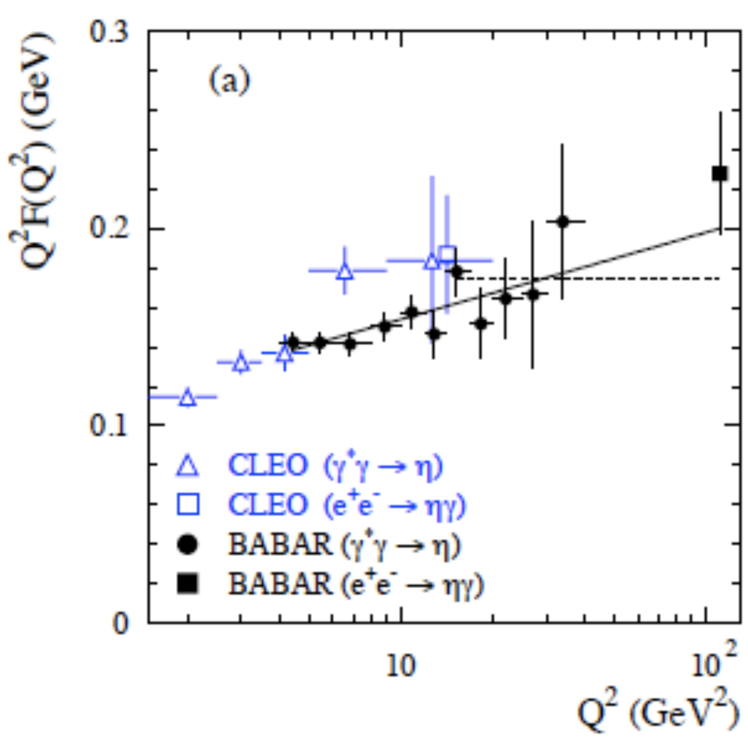




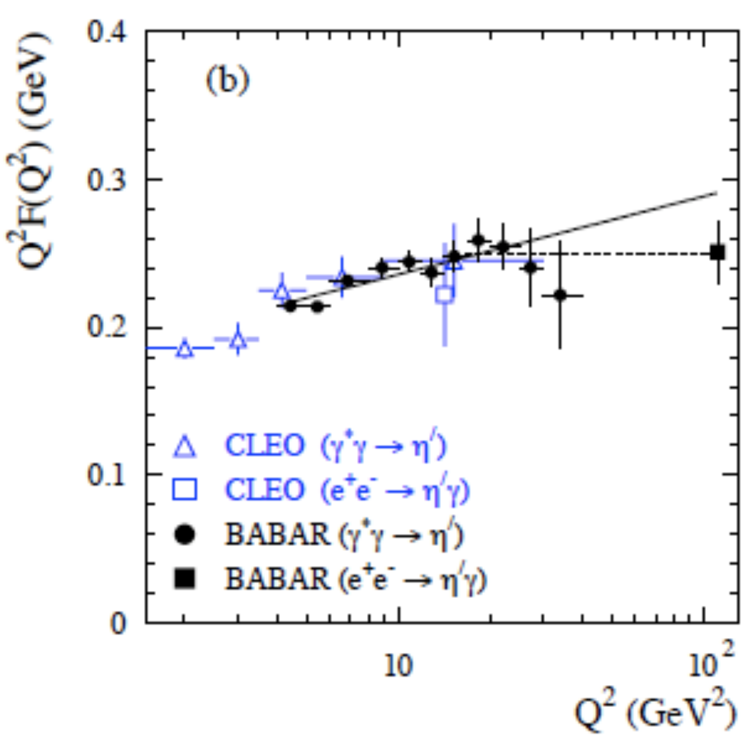
Spacelike ($q^2 < 0$): e^+e^- colliders

Timelike ($q^2 > 0$): meson decays

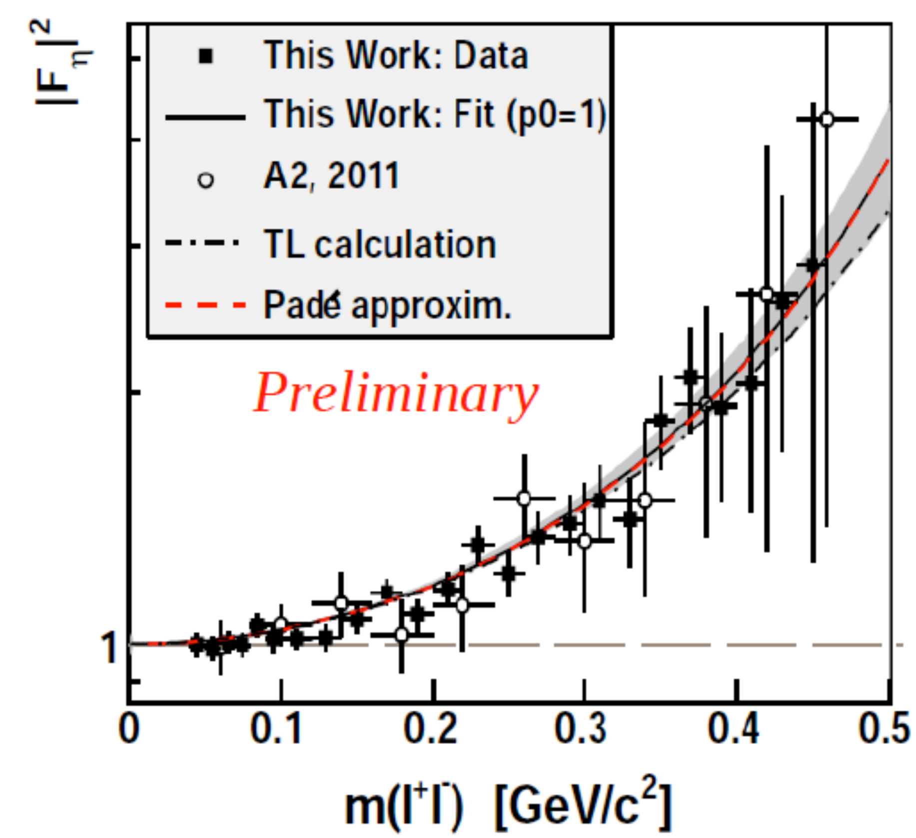
$\gamma^* \gamma$ -> η



$\gamma^* \gamma$ -> η'



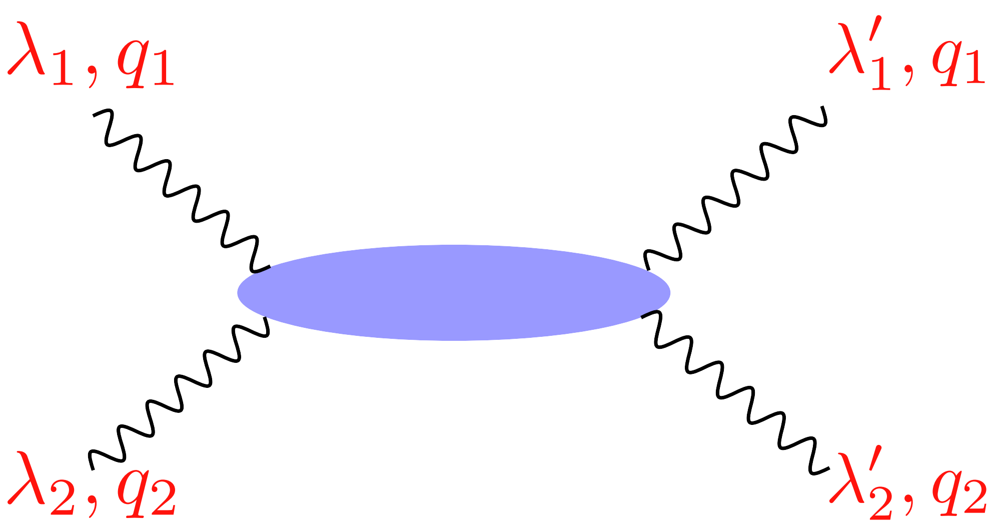
η -> γ $e^+ e^-$



new data from BES-III forthcoming

new MAMI/A2 data

Theory: sum rules for LbL scattering (I)



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda'_1, q_1) + \gamma^*(\lambda'_2, q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \qquad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes: $M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2}(\nu, Q_1^2, Q_2^2)$ $\qquad \lambda = 0, \pm 1$

discrete symmetries: → 8 independent amplitudes:

$$P: \quad M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2} = M_{-\lambda'_1-\lambda'_2,-\lambda_1-\lambda_2}$$

$$T: \quad M_{\lambda'_1\lambda'_2,\lambda_1\lambda_2} = M_{\lambda_1\lambda_2,\lambda'_1\lambda'_2}$$

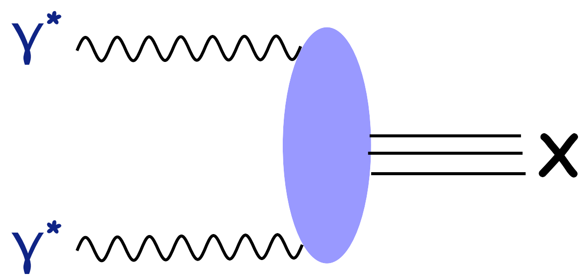
$M_{++,++},\ M_{+-,+-},\ M_{++,--},$ **T**

$M_{00,00},\ M_{+0,+0},\ M_{0+,0+},\ M_{++,00},\ M_{0+,-0}$ **T and L**

sum rules for LbL scattering (II)

Unitarity: link to $\gamma^* \gamma^* \to X$ cross sections

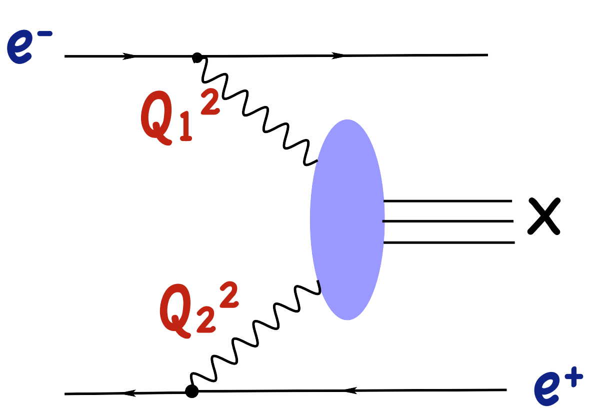
$$W_{\lambda_1'\lambda_2',\lambda_1\lambda_2} \equiv \mathrm{Im}\, M_{\lambda_1'\lambda_2',\lambda_1\lambda_2}$$



$$\begin{aligned}
W_{++,++} + W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0+\sigma_2) = 2\sqrt{X}\,(\sigma_\parallel+\sigma_\perp) \equiv 4\sqrt{X}\,\sigma_{TT},\\
W_{++,++} - W_{+-,+-} &\equiv 2\sqrt{X}\,(\sigma_0-\sigma_2) \equiv 4\sqrt{X}\,\tau^a_{TT},\\
W_{++,--} &\equiv 2\sqrt{X}\,(\sigma_\parallel-\sigma_\perp) \equiv 2\sqrt{X}\,\tau_{TT},\\
W_{00,00} &\equiv 2\sqrt{X}\,\sigma_{LL},\\
W_{+0,+0} &\equiv 2\sqrt{X}\,\sigma_{TL},\\
W_{0+,0+} &\equiv 2\sqrt{X}\,\sigma_{LT},\\
W_{++,00} + W_{0+,-0} &\equiv 4\sqrt{X}\,\tau_{TL},\\
W_{++,00} - W_{0+,-0} &\equiv 4\sqrt{X}\,\tau^a_{TL}.
\end{aligned}$$

$$X \equiv \nu^2 - Q_1^2 Q_2^2$$

Experiment: $e^- e^+ \to e^- e^+ X$ cross sections

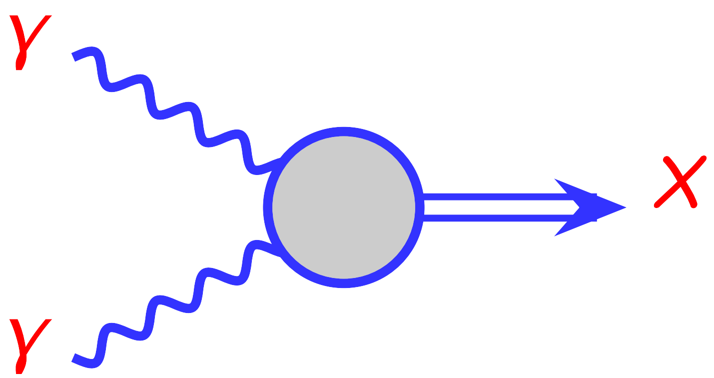


$$\begin{aligned}
d\sigma = &\frac{\alpha^2}{16\pi^4 Q_1^2 Q_2^2}\,\frac{2\sqrt{X}}{s(1-4m^2/s)}\cdot\frac{d^3\vec{p}_1{}'}{E_1'}\cdot\frac{d^3\vec{p}_2{}'}{E_2'}\\
&\times\Big\{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT}\\
&+2\,(\rho_1^{++}-1)\,(\rho_2^{++}-1)\,\left(\cos 2\tilde{\phi}\right)\,\tau_{TT} + 8\left[\frac{(\rho_1^{00}+1)\,(\rho_2^{00}+1)}{(\rho_1^{++}-1)\,(\rho_2^{++}-1)}\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau_{TL}\\
&+h_1 h_2\, 4\left[(\rho_1^{00}+1)\,(\rho_2^{00}+1)\right]^{1/2}\tau^a_{TT} + h_1 h_2\, 8\left[(\rho_1^{++}-1)\,(\rho_2^{++}-1)\right]^{1/2}\left(\cos\tilde{\phi}\right)\,\tau^a_{TL}\Big\}
\end{aligned}$$

lepton beam polarization

ρ's, ϕ : kinematical quantities

sum rules for LbL scattering (III)

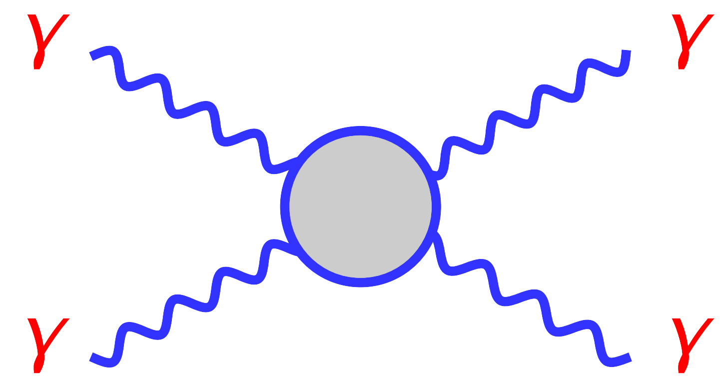


micro-causality

unitarity

LETs

dispersion theory



imaginary part of the amplitude - photon-photon fusion into leptons and hadrons:

real part of the amplitude - low-energy structure of the elastic LbL scattering:

Sum rules

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}\left[\sigma_2(s) - \sigma_0(s)\right]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}\left[\sigma_{tot}(s)\right]$$

$$\mathcal{L}^{(8)} = c_1 (F_{\mu\nu}F^{\mu\nu})^2 + c_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

$$\int_{s_0}^{\infty} \frac{ds}{s}\left[\sigma_2(s) - \sigma_0(s)\right] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi}\int_{s_0}^{\infty} \frac{ds}{s^2}\left[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)\right]$$

sum rules for LbL scattering (IV)

3 superconvergent relations:

Pascalutsa, Pauk, Vdh (2012)

helicity difference sum rule

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

for $Q^2 = 0$: GDH sum rule

Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\parallel} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^a \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2} \right]_{Q_2^2=0}$$

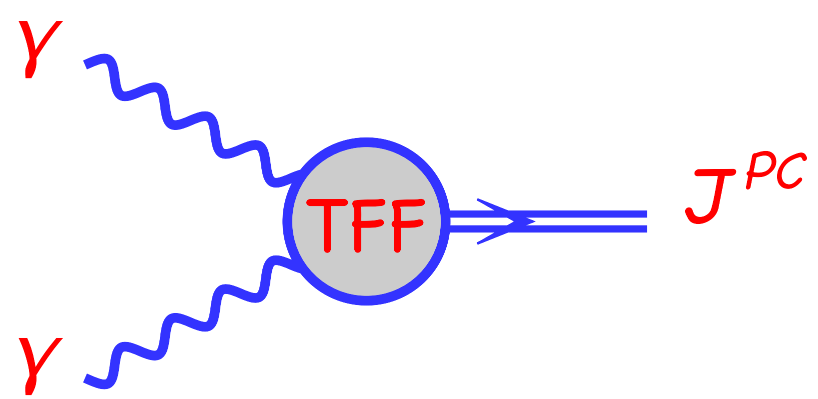
SRs involving LbL low-energy constants:

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s) \right]$$

+ 6 new LECs at next order

sum rules have been tested in perturbative QFT both at tree-level and 1-loop level

single meson production in $\gamma\gamma$ collisions (I)



- two-photon state: produced meson has C=+1

- both photons are real: J=1 final state is forbidden (Landau-Yang theorem);

the main contribution comes from

J=0: 0^{-+} (pseudoscalar) and 0^{++} (scalar)

and J=2: 2^{++} (tensor)

– the SRs hold separately for channels of given intrinsic quantum numbers: isoscalar and isovector mesons, $c\bar{c}$ states

– input for the absorptive part of the SRs: γγ-hadrons response functions, can be expressed in terms of γγ→M transition form factors

$$\sigma_{\Lambda}^{\gamma\gamma\to M}(s) \approx (2J+1)16\pi^2 \frac{\Gamma_{\gamma\gamma}}{m_M}\delta(s-m_M^2)$$

meson contribution to the cross-section in the narrow-resonance approximation

$$\Gamma_{\gamma\gamma}(\mathcal{P}) = \frac{\pi\alpha^2}{4} m^3 \, |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2$$

two-photons decay rate for the meson

single meson production in $\gamma\gamma$ collisions (II)

the I=0 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
η	-191 ± 10	0	0.65 ± 0.03
η'	-300 ± 10	0	0.33 ± 0.01
$f_0(980)$	-19 ± 5	0.020 ± 0.005	0
$f_0'(1370)$	-91 ± 36	0.049 ± 0.019	0
$f_2(1270)$	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
$f_2'(1525)$	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
$f_2(1565)$	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
Sum	-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

dominant contribution to c_2 comes from η, η' and $f_2(1270)$

dominant contribution to c_1 comes from $f_2(1270)$

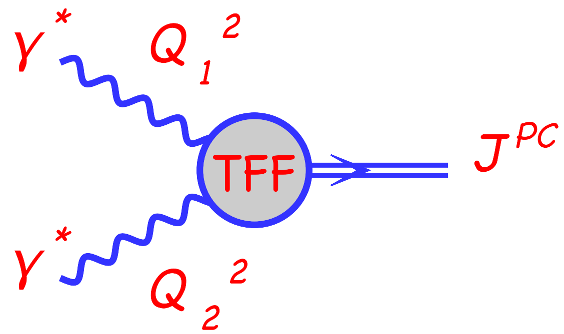
the I=1 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4} GeV^{-4}]	c_2 [10^{-4} GeV^{-4}]
π^0	-195 ± 13	0	10.94 ± 0.70
$a_0(980)$	-20 ± 8	0.021 ± 0.007	0
$a_2(1320)$	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
$a_2(1700)$	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
Sum	-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

dominant contribution to c_2 comes from π^0

Pascalutsa, Pauk, Vdh (2012)

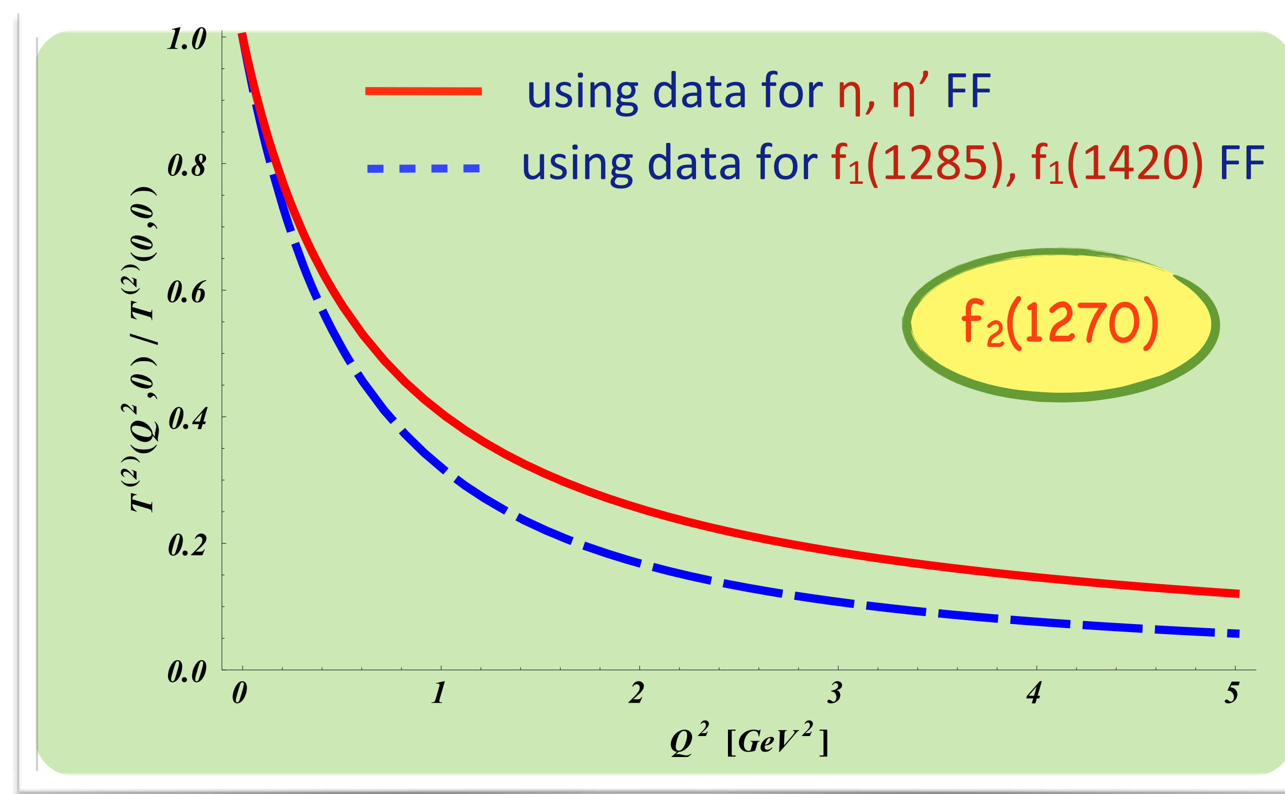
single meson production in $\gamma\gamma$ collisions (III)



- one photon is virtual Q_1^2, second is quasi-real $Q_2^2 \simeq 0$:
- axial-vector mesons 1^{++} are allowed
- $f_1(1285)$, $f_1(1420)$ transition FFs constrained from LEP (L3) data

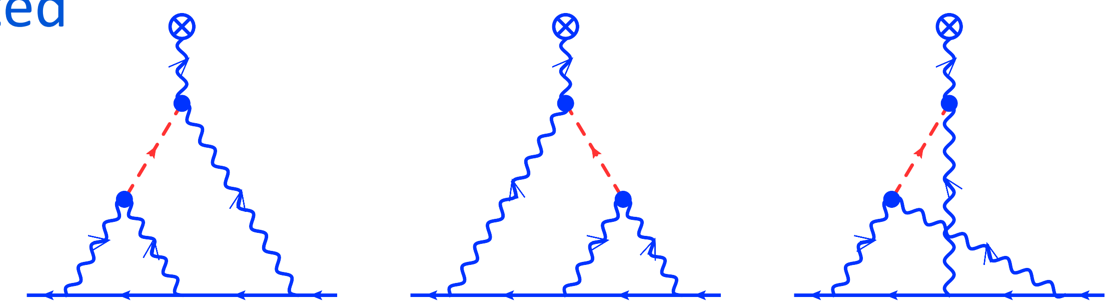
	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2}\sigma_\parallel(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s}\frac{\tau_{TL}^a}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2}\sigma_\parallel + \frac{1}{s}\frac{\tau_{TL}^a}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

- sum rules allow to constrain so far unmeasured contributions, e.g. $\gamma^* \gamma^* \rightarrow$ tensor mesons



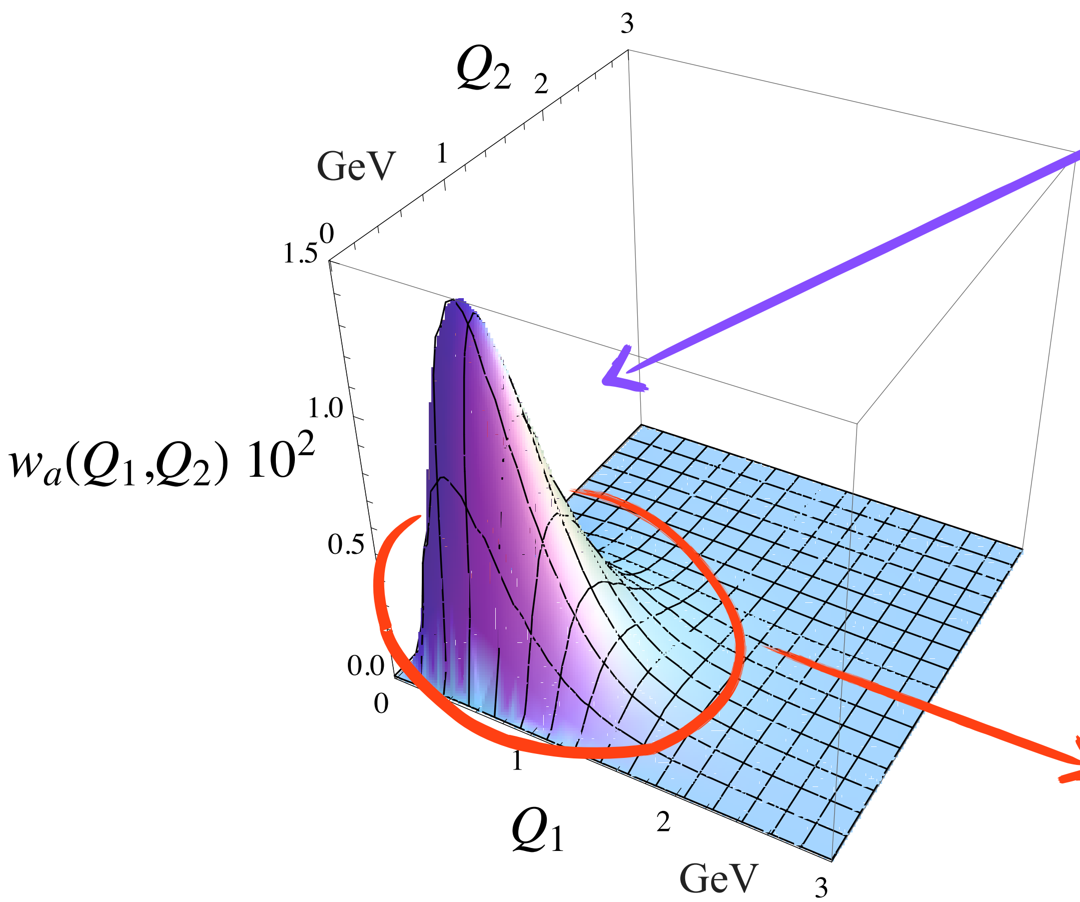
single meson contributions to a_μ (I)

- **axial-vector meson** contribution to a_μ re-evaluated
  - Landau-Yang theorem constraint implemented
  - $f_1(1285)$, $f_1(1420)$ transition FFs from L3 data



Pauk, Vdh (2013)

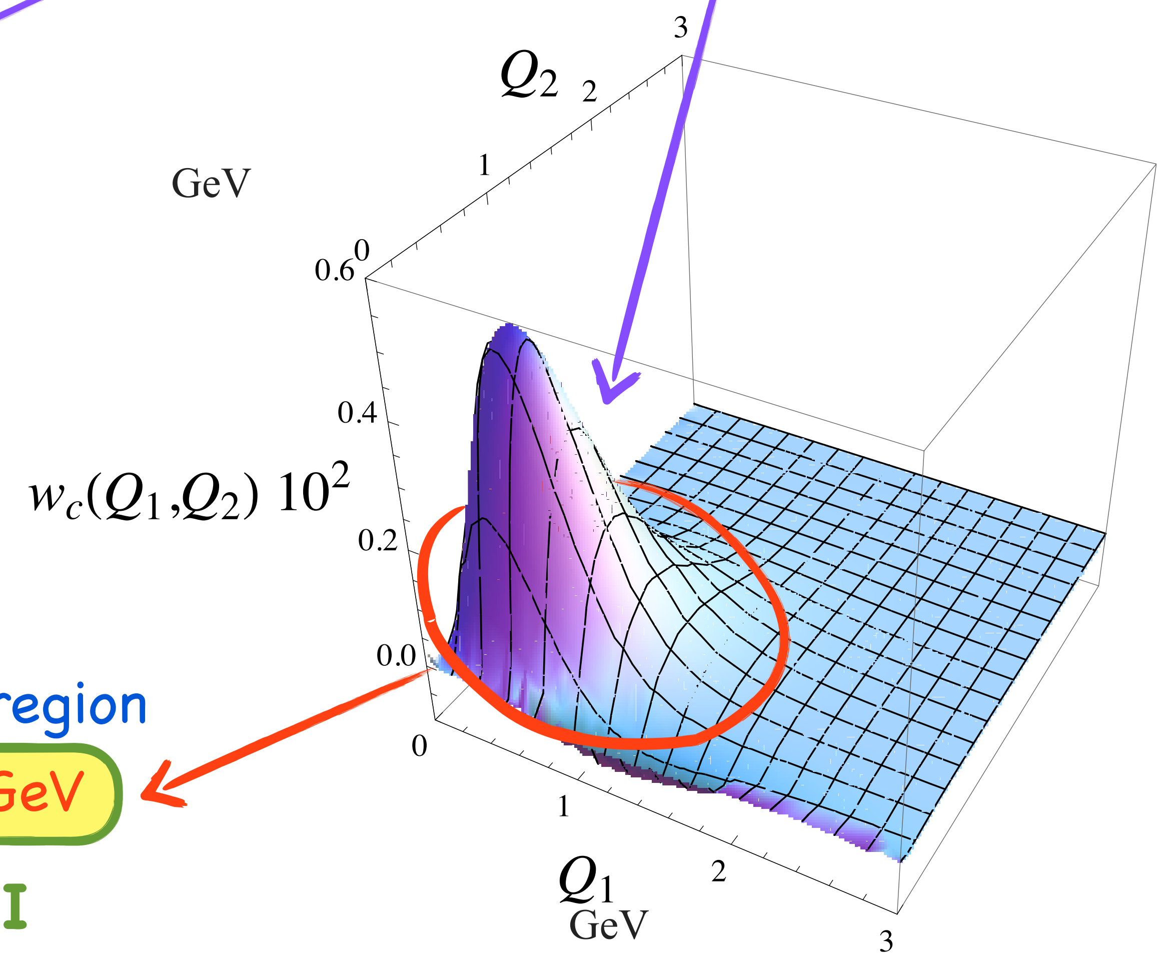
$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int dQ_1 \int dQ_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$



dominating region

$Q_1 \sim Q_2 \sim 1$ GeV

BES III

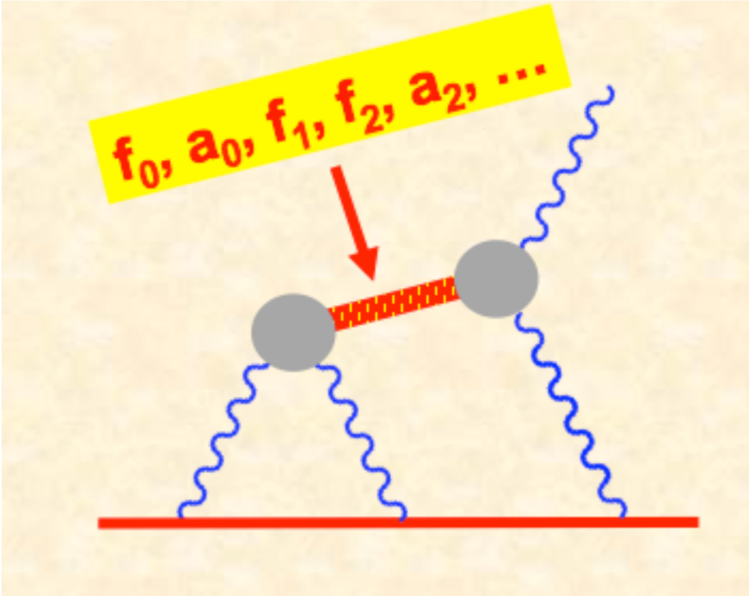


single meson contributions to a_μ (II)

- **axial-vector meson** re-evaluation was reported in 2 works

  - implementation of Landau-Yang theorem constraint leads to difference with previous results

- **tensor mesons** evaluated for first time

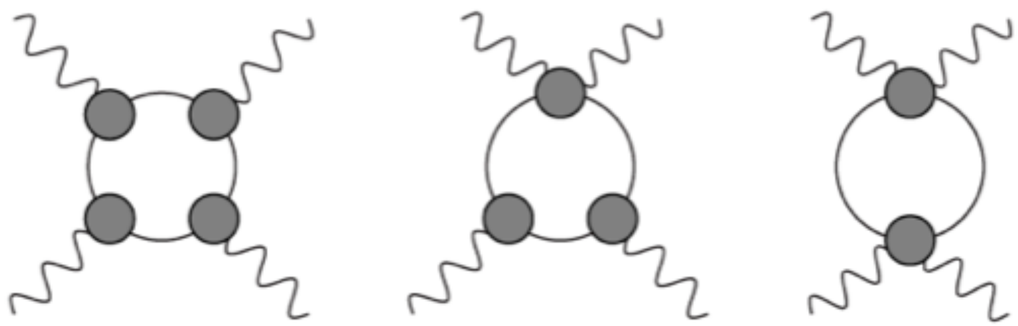


		pseudo-scalars	axial-vectors	scalars	tensors
	BPP	85 ± 13	2.5 ± 1.0	-7 ± 2	-
	HKS	82.7 ± 6.4	1.7 ± 1.7	-	-
	MV	114 ± 10	22 ± 5	-	-
	KN	83 ± 12	-	-	-
Jegerlehner	J	93.9 ± 12.4	~ 7	-6.0 ± 1.2	-
Pauk, Vdh	this work	-	6.4 ± 2.0	$-(0.9 \sim 3.1) \pm 0.8$	1.1 ± 0.1

total $(6.6 \sim 4.4) \pm 2.9 \times 10^{-11}$

multi-meson production in $\gamma\gamma$ collisions (I)

➡ new estimate for **pion loop** contribution (with full VMD FF) Bijnens (2014)

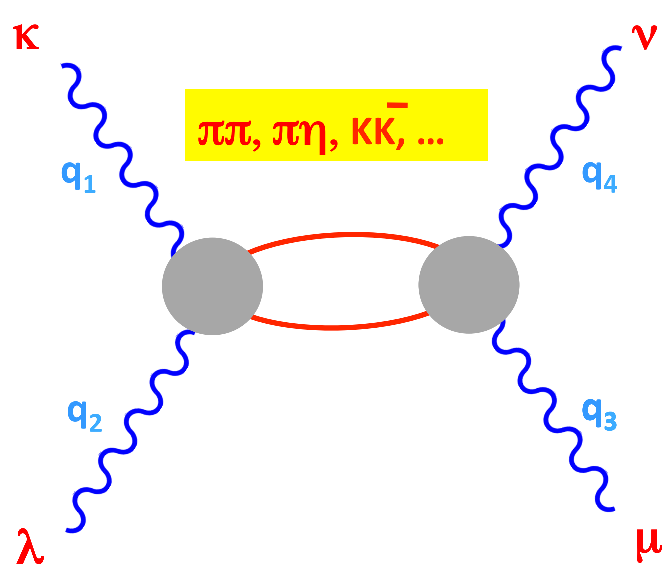


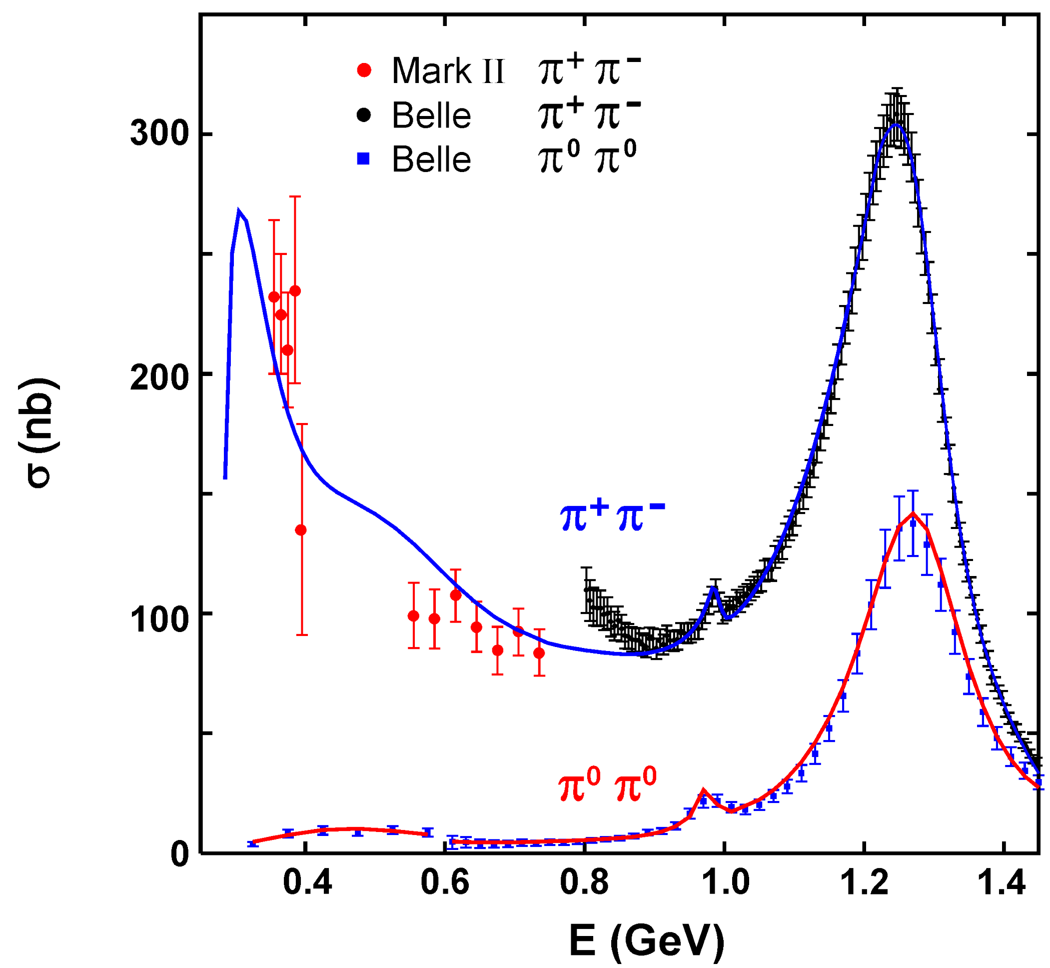
$a_\mu^{LbL\ \pi\text{-loop}} = (-\,2.0 \pm 0.5) \times 10^{-10}$

integrating momenta in loop up to 1 GeV

➡ contribution of **multi-meson channels**

Dai, Pennington (2014)

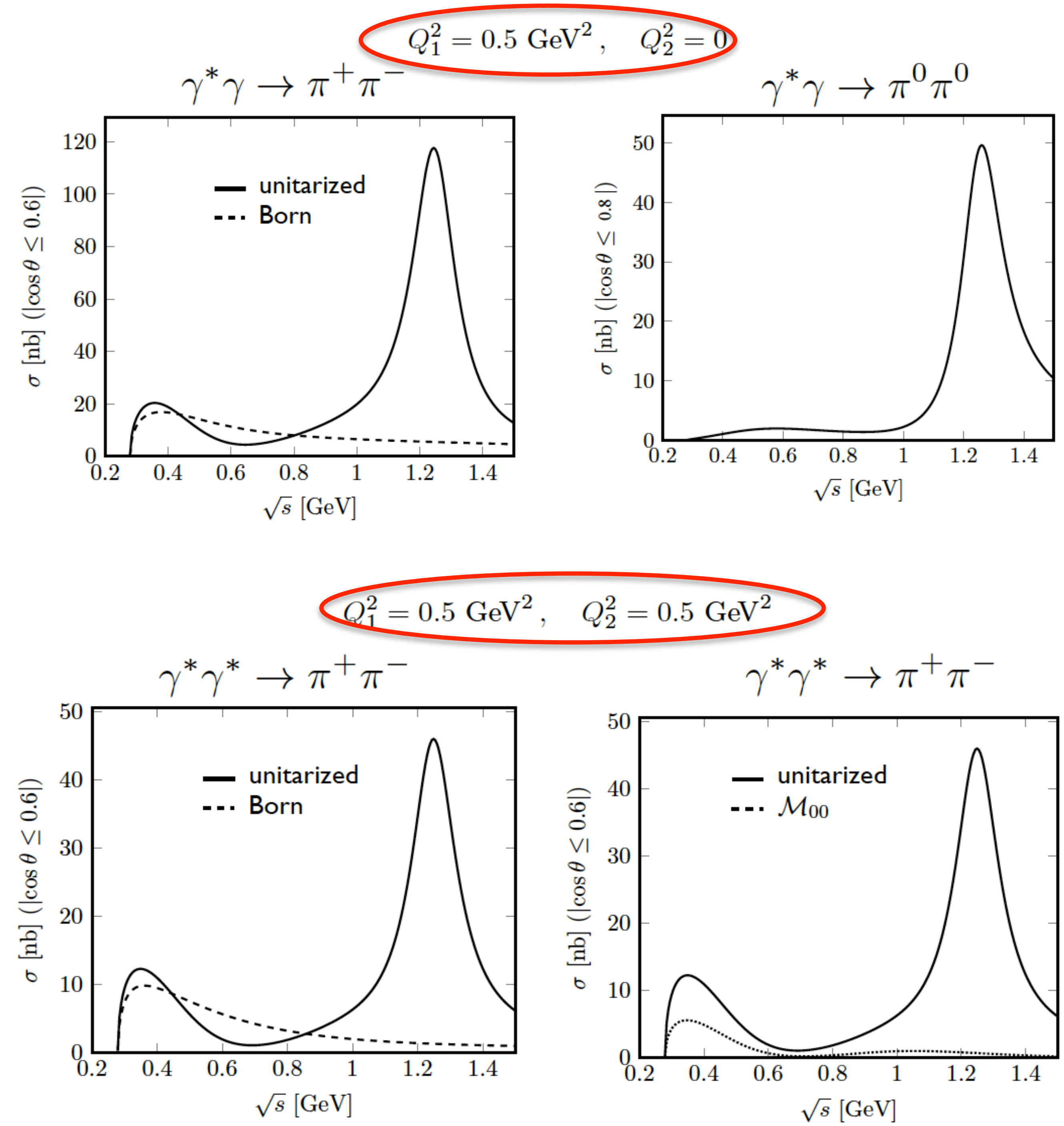




sum rules may be used as a consistency check of models

multi-meson production in $\gamma\gamma$ collisions (II)

- new **dispersion formalism** for $\gamma^* \gamma^* \to \pi\pi$



Moussallam (2013)

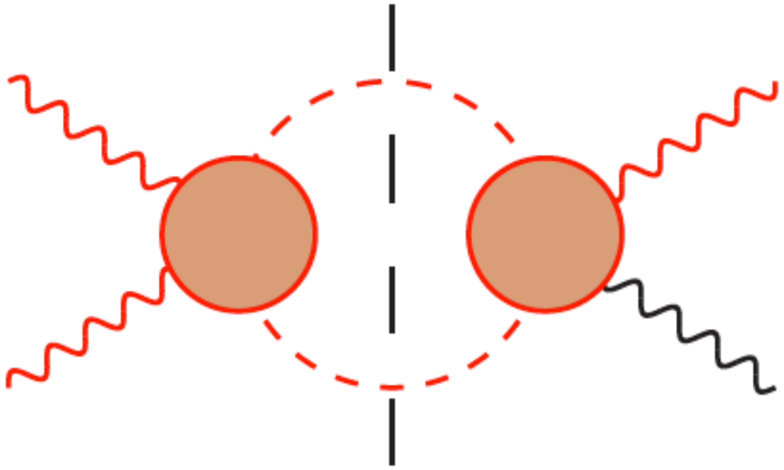
Hoferichter, Colangelo, Procura, Stoffer (2013)

Assmusen, Masjuan, Vdh (2014)

- forthcoming BES-III data for $\gamma^* \gamma^* \to \pi\pi$ Guo (2014)

dispersion relation approaches for a_μ (I)

Hoferichter, Colangelo, Procura, Stoffer (2014)



- **dispersion formalism** for $\gamma^* \gamma^* \to \gamma^* \gamma$

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots$$

- **master formula** for a_μ

$$a_\mu^{\pi\pi} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{I^{\pi\pi}}{q_1^2 q_2^2 s((p+q_1)^2 - m^2)((p-q_2)^2 - m^2)},$$

$$I^{\pi\pi} = \sum_{i\in\{1,2,3,6,14\}} \left(T_{i,s} I_{i,s} + 2 T_{i,u} I_{i,u}\right) + 2 T_{9,s} I_{9,s} + 2 T_{9,u} I_{9,u} + 2 T_{12,u} I_{12,u}$$

with $I_{i,(s,u)}$ dispersive integrals and $T_{i,(s,u)}$ integration kernels

$$I_{1,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \left(\frac{1}{s'-s} - \frac{s' - q_1^2 - q_2^2}{\lambda(s', q_1^2, q_2^2)} \right) \mathrm{Im}\bar{h}^0_{++,++}(s'; q_1^2, q_2^2; s, 0),$$

$$I_{6,s} = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{(s' - q_1^2 - q_2^2)(s'-s)^2} \mathrm{Im}\bar{h}^2_{+-,+-}(s'; q_1^2, q_2^2; s, 0) \left(\frac{75}{8}\right)$$

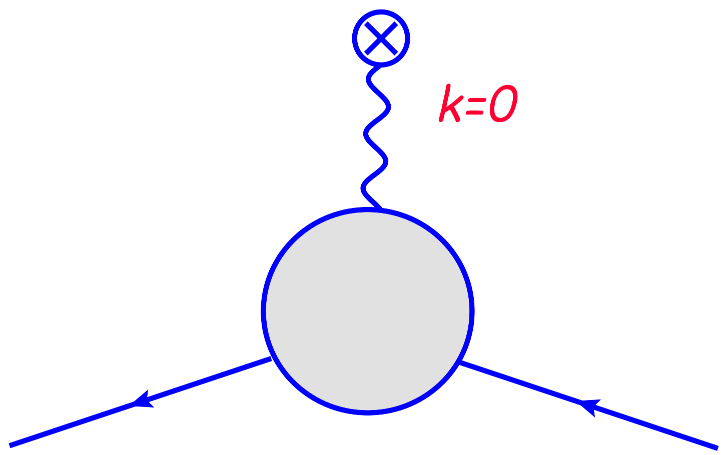
Helicity amplitudes contribute up to $J = 2$ (S and D waves)

dispersion relation approaches for a_μ (II)

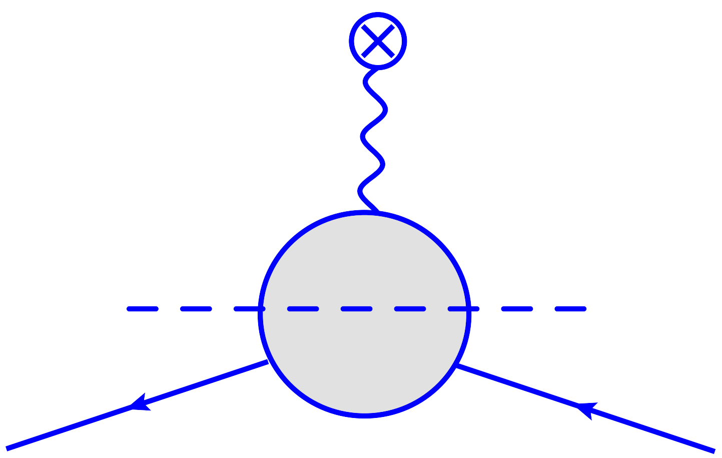
➡ **dispersion formalism** directly for a_μ Pauk, Vdh (2014)

$$a_\mu = \lim_{k \to 0} F_2(k^2, (p+k)^2, p^2)$$

$$F_2(0) = \frac{1}{2\pi i} \int \frac{dk^2}{k^2} \text{Abs}\, F_2(k^2)$$



$$a_\mu = F_2(0)$$



weighting functions (entire)

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p, p', q_1, k-q_1-q_2, q_2)$$

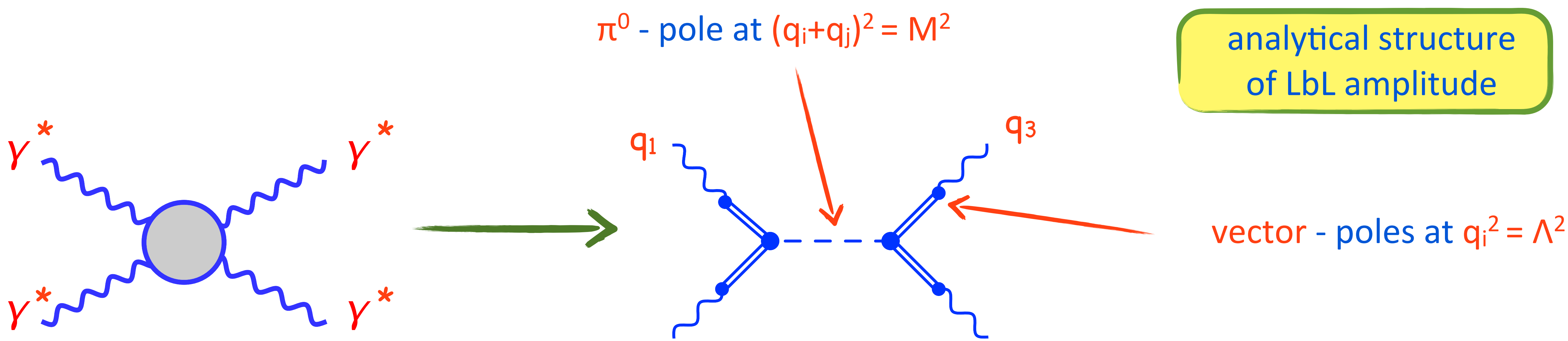
analytic structure ⟶

$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)}{q_1^2 q_2^2 (k-q_1-q_2)^2 \left[(p+q_1)^2 - m^2\right] \left[(p+k-q_2)^2 - m^2\right]}$$

$$\Pi_{\lambda_1\lambda_2\lambda_3\lambda_4}(q_1, q_2, q_3) = \epsilon^\mu(q_1,\lambda_1)\epsilon^\nu(q_2,\lambda_2)\epsilon^\lambda(q_3,\lambda_3)\epsilon^\rho(q_4,\lambda_4)\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3)$$

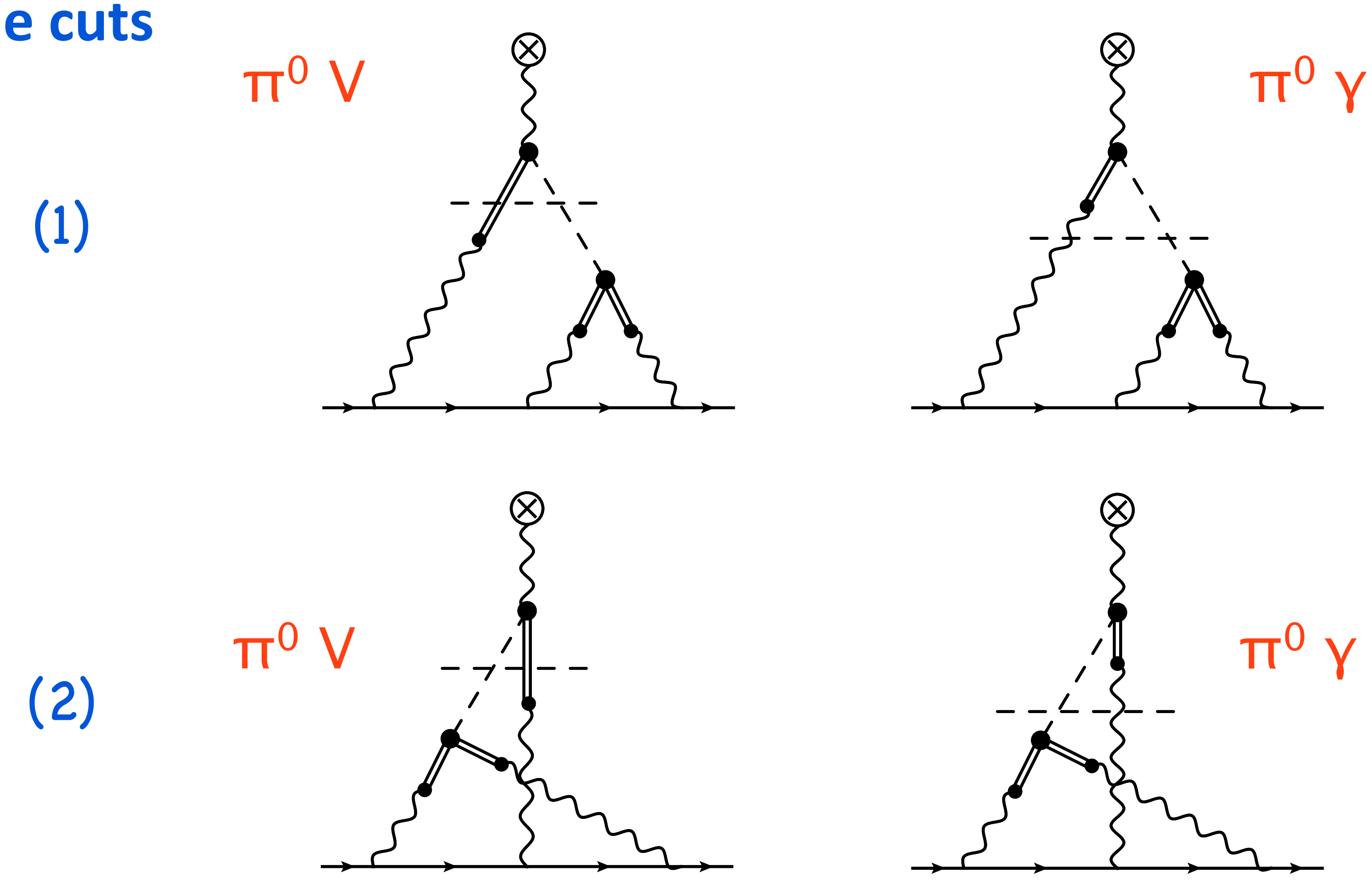
dispersion relation approaches for a_μ (III)

proof of principle: pole contributions



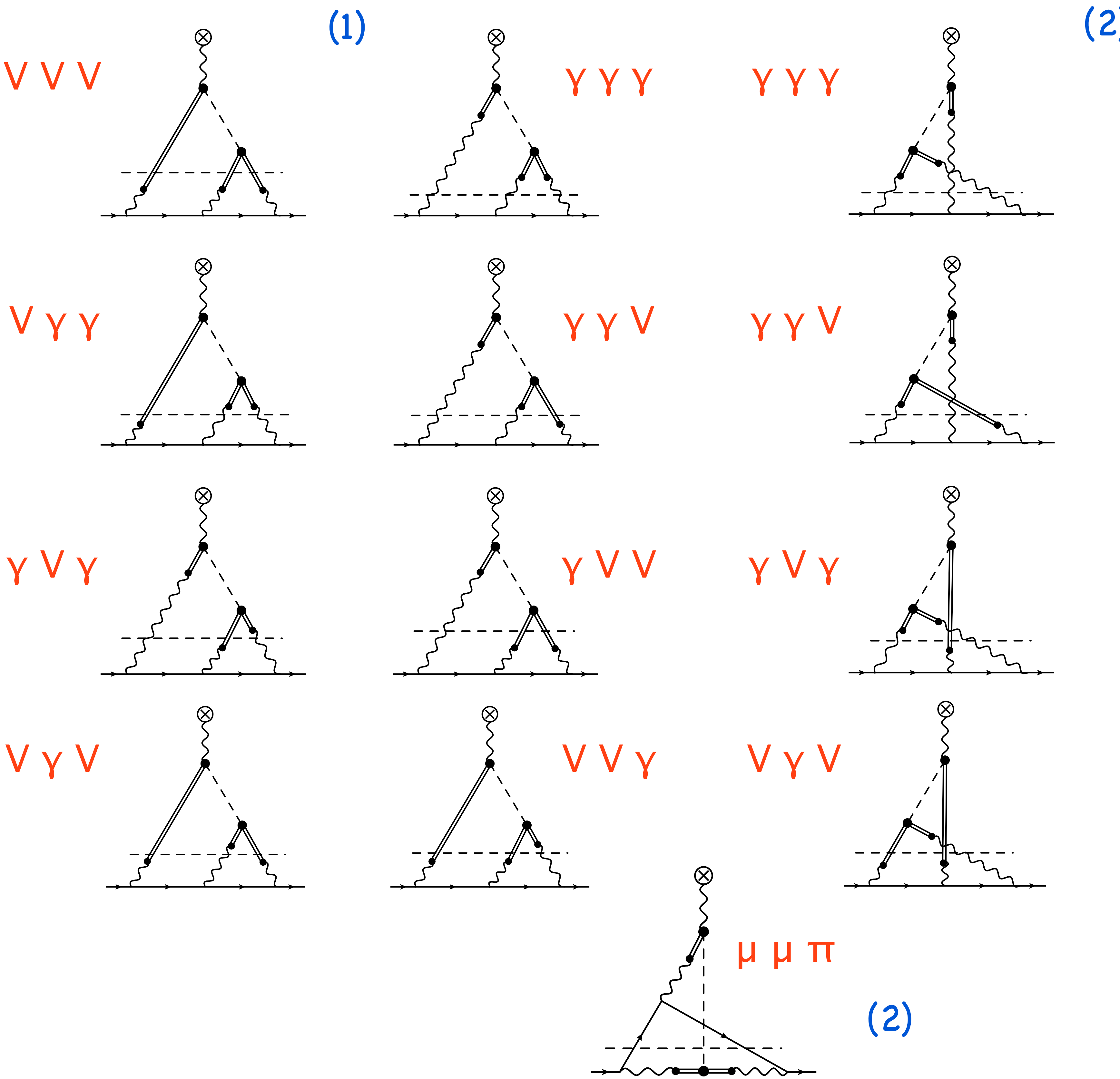
analytical structure of LbL amplitude

2-particle cuts



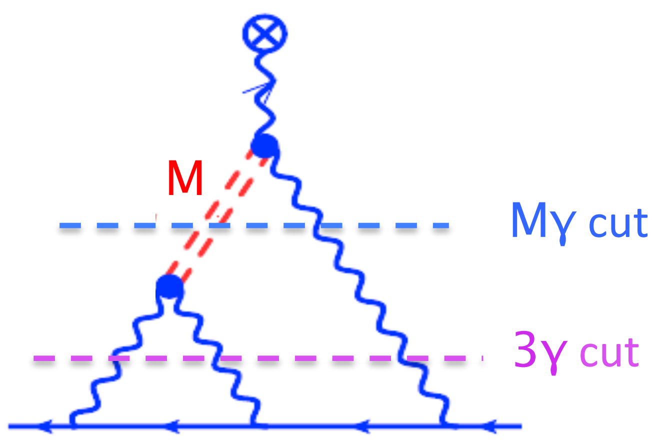
2-particle discontinuities

dispersion relation approaches for a_μ (IV)



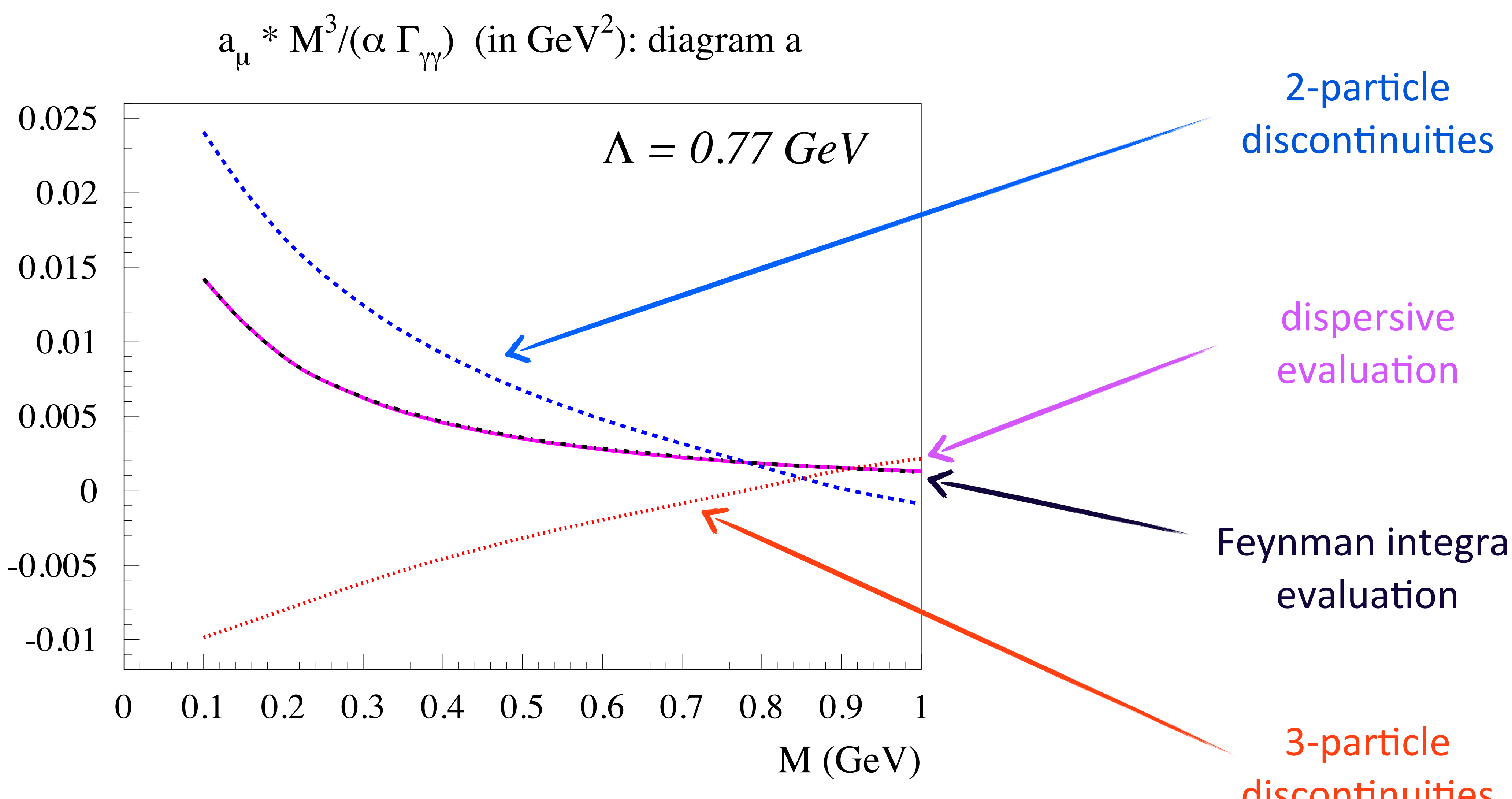
3-particle discontinuities

dispersion relation approaches for a_μ (V)



reconstruction of **a_μ** from dispersion integral: **proof of principle**

$$F_2^{(i)}(0) = \frac{1}{2\pi i}\int_{M^2}^{\infty} \frac{dt}{t}\, \mathrm{Disc}_t^{(2)} F_2^{(i)}(t) + \frac{1}{2\pi i}\int_{0}^{\infty} \frac{dt}{t}\, \mathrm{Disc}_t^{(3)} F_2^{(i)}(t)$$



Pauk, Vdh (2014)

Summary and outlook

➡ **HVP:** new experimental program at BES-III

- first results for $\mathbf{e^+ e^- \to \pi^+ \pi^-} \gamma$
- aim: hadronic cross sections to 1% accuracy
- strong lattice effort to rival experimental accuracy

➡ **HLbL: new theoretical tools** for $\gamma^* \gamma^* \to$ X

- sum rules, dispersive frameworks for transition FFs**:**
allow to include experimental constraints

- new evaluation of **heavier meson contributions:** a_μ = (6.6～4.4) ± 2.9 x 10^{-11}

- pioneering lattice efforts

➡ **new dispersion relation** frameworks for **HLbL** to $\mathbf{a_\mu}$:
-> require close collaboration with experiment (spacelike, timelike, meson decays)

➡ Outcome of Mainz workshop:
draft of roadmap for a data driven approach also in HLbL

➡ **goal: realistic error estimate on a_μ / reduce to 2 x 10^{-10} (20 % of HLbL)**