The value of Newton's constant of gravitation.

Clive Speake
University of Birmingham
5th February 2015
Eltville meeting on Fundamental Constants

Talk outline

- The measured values of G.
- Résumé of the BIPM G experiment

- Systematic effects and the BIPM value
  - Anelasticity
  - Electrostatic calibration
  - Calculation of G torques
  - Mass homogeneity
  - Moment of inertia
  - Uncertainties
- Work in Birmingham
- Conclusions on BIPM experiment
- Ways forward

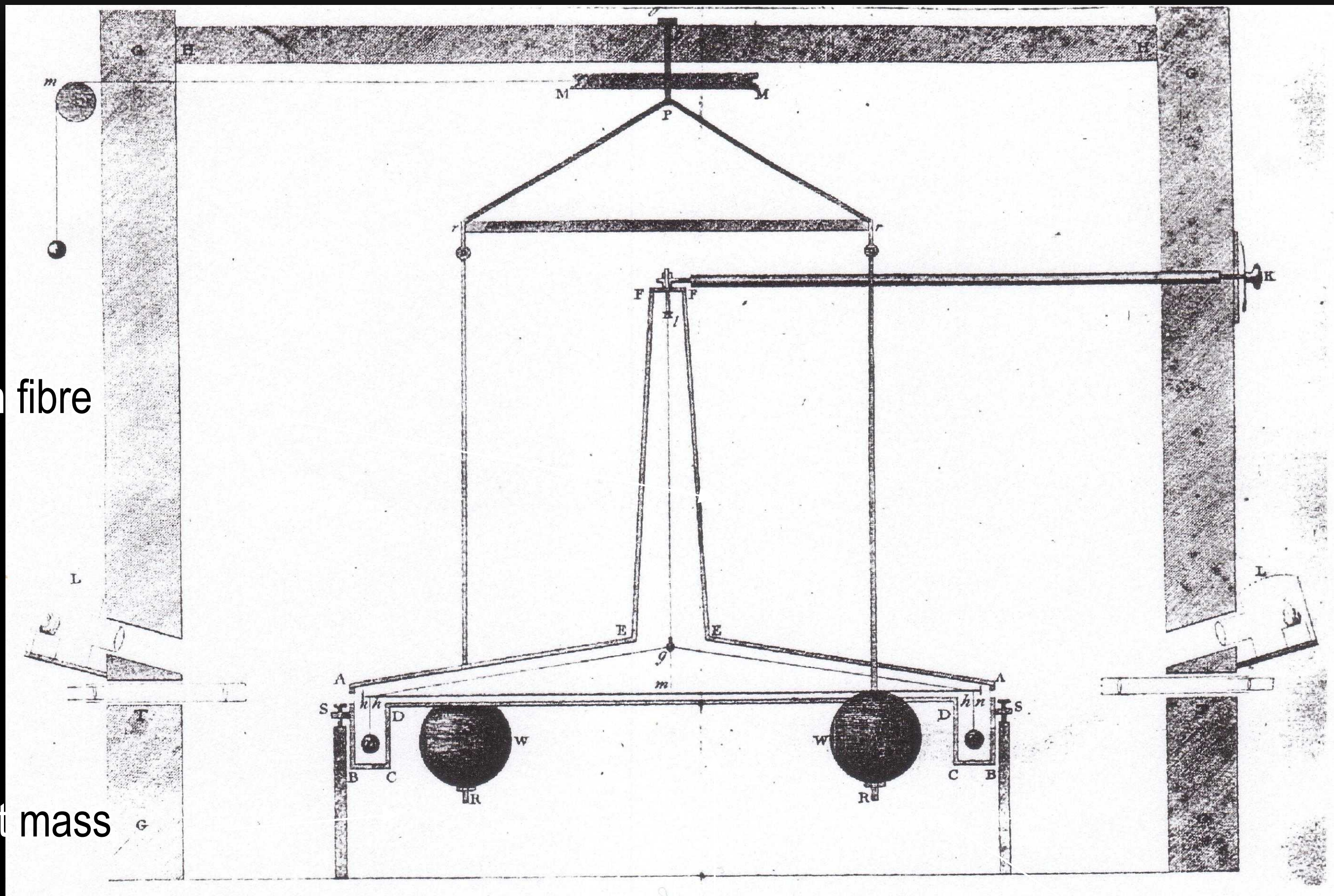
Acknowledgements

- Terry Quinn
- Richard Davis
- Harold Parks
- Sam Richman
- Alain Picard
- Wes Tew
- BIPM workshop

Richard Davis and Harold Parks in contemplative mood.

- Thanks to TJQ for the pictures of the BIPM apparatus.

- Cavendish's experiment 1798 achieved an accuracy of about 5%.



Torsion fibre

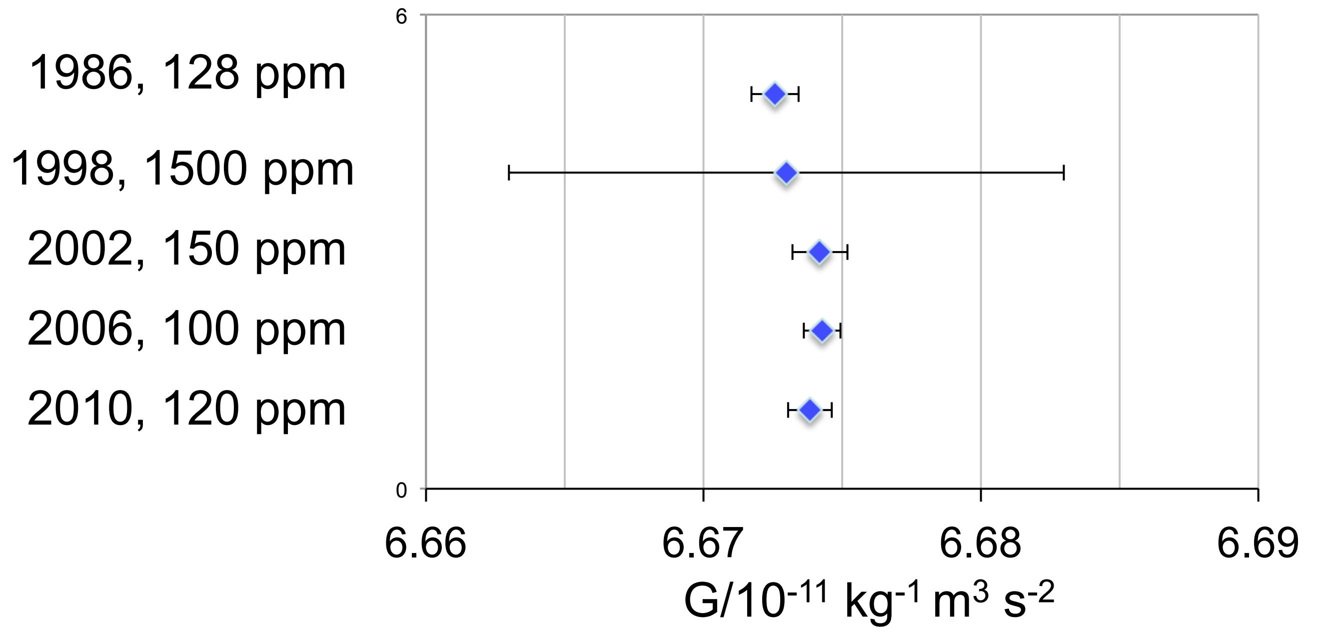
Lead test mass

0.7kg

Lead source 45kg

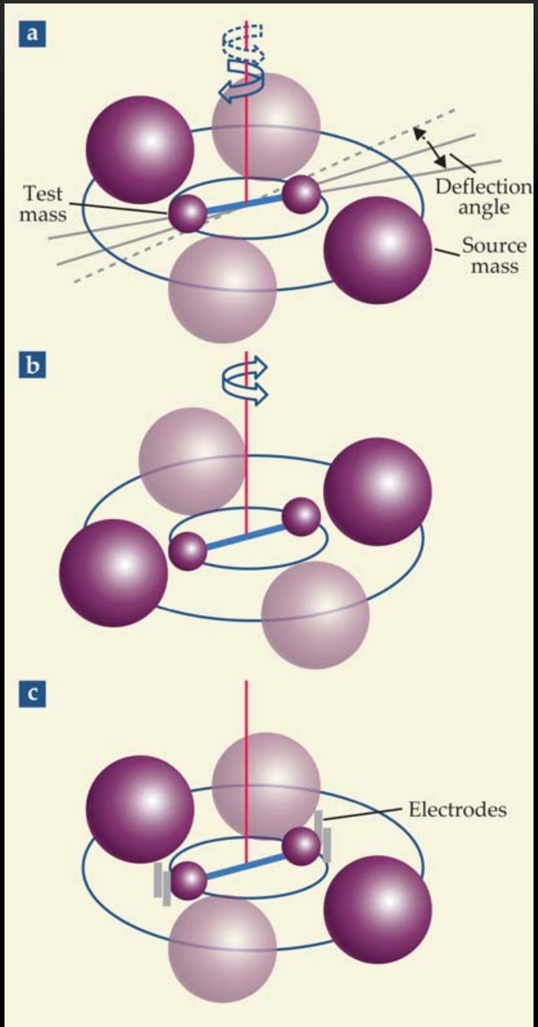
Beam ~1m long

CODATA VALUES OF G



P.J.Mohr, B.N.Taylor and D.B.Newell, Rev Mod Phys 84 Oct-Dec 2012 1527-1605

Methods of measuring G



Cavendish or Free-deflection method

Time of swing method

Torque balance method

July 2014 Physics Today

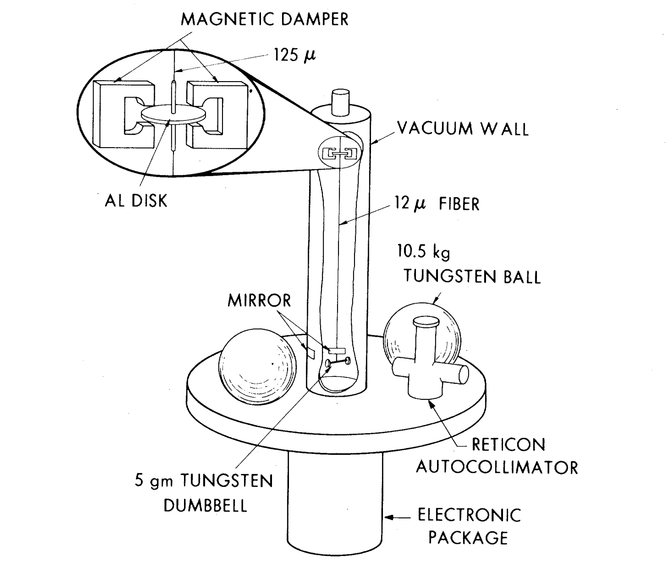
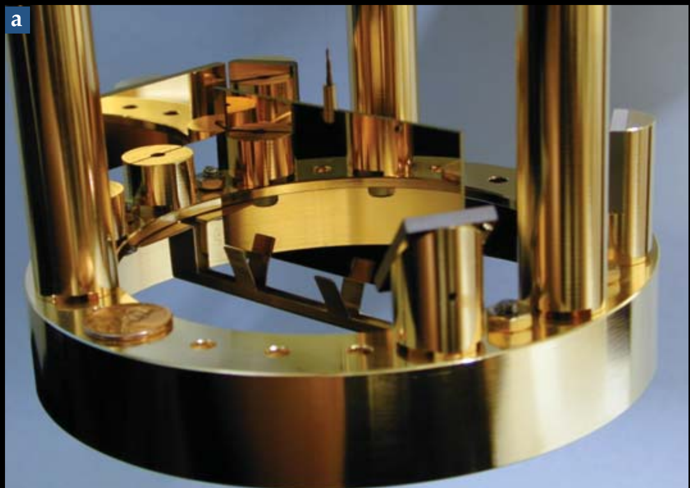
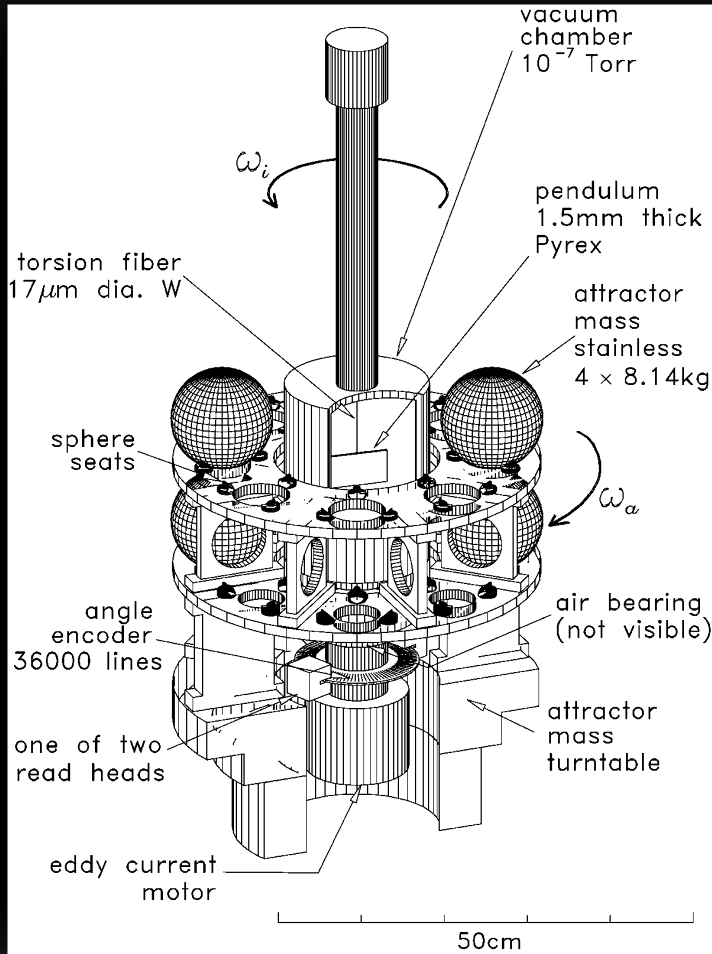


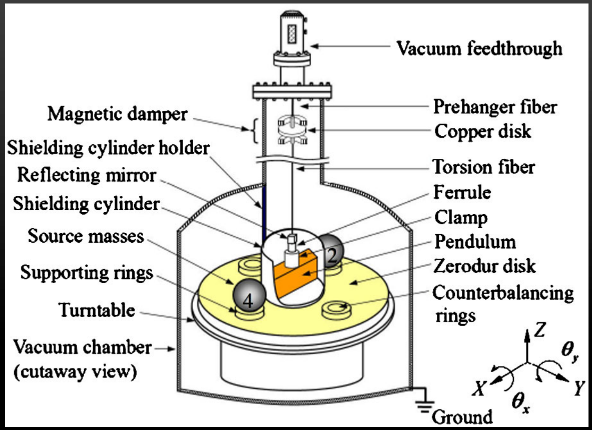
FIG. 1. Diagram of the apparatus with inset showing detail of the damper.

Luther and Towler 1982: the invention of the damper for the simple pendulum mode. Time of swing.

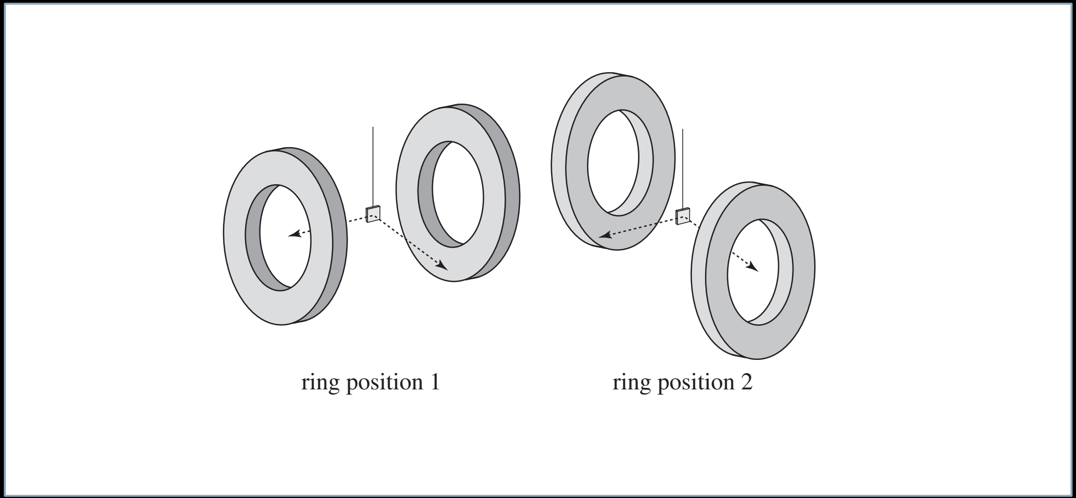


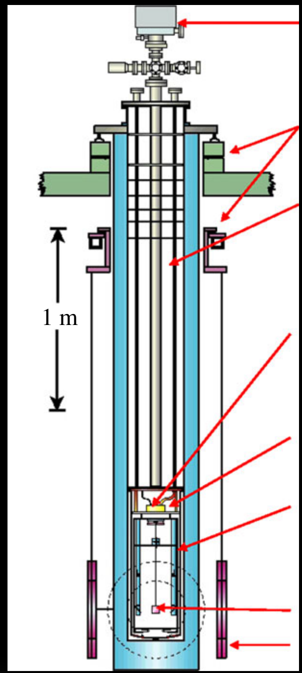


Gundlach and Merkovitz 2001: The flat plate and optimum source mass configuration, rotating table and source masses.



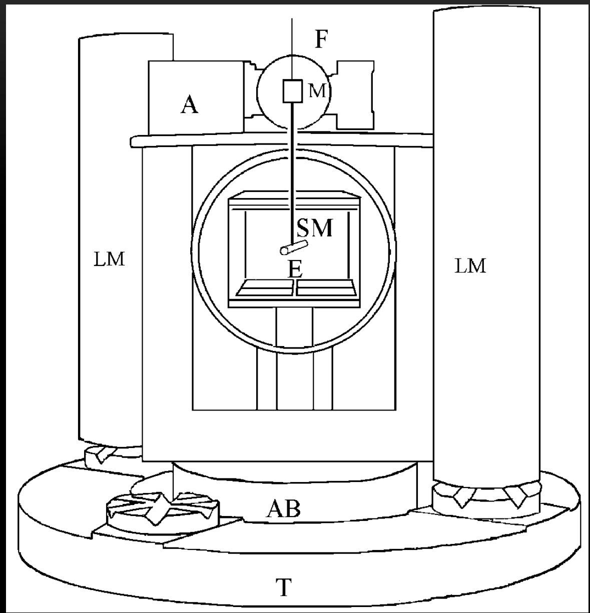
Luo and colleagues and HUST 1999, 2005. Time of swing.



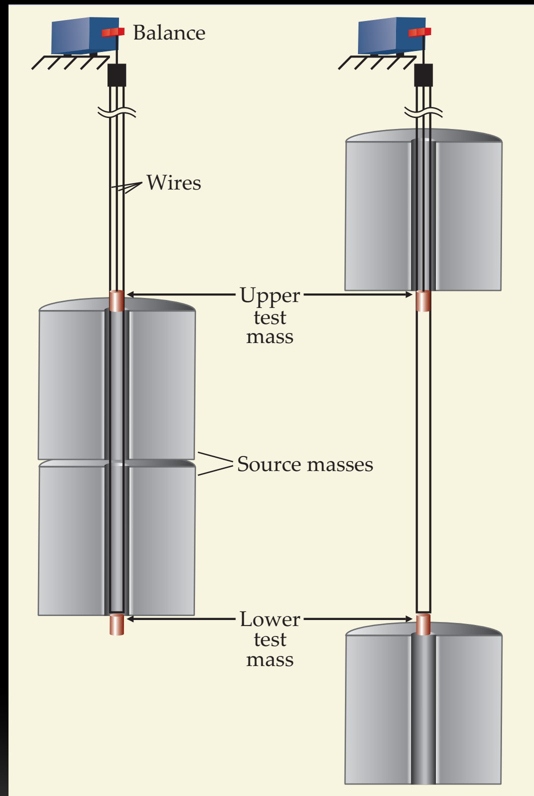


Newman and colleagues 2014. Time of swing.

Armstrong and Fitzgerald 2003: torsion strip and inertial electrostatic calibration.



Schlamminger et al 2006. Beam balance with moving mercury tanks.



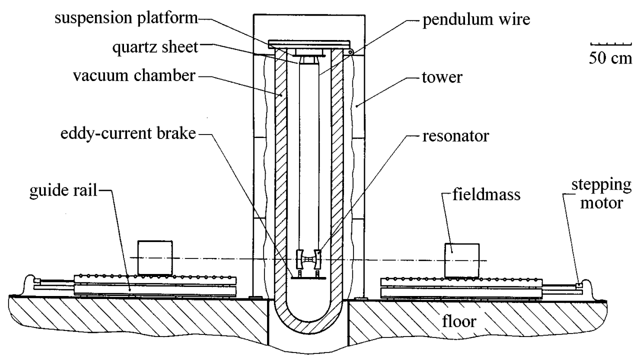
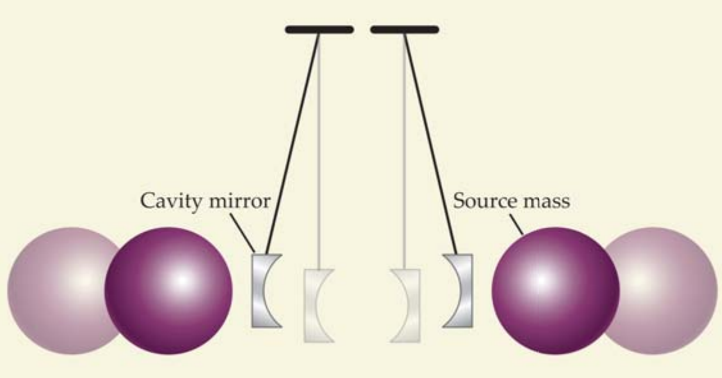
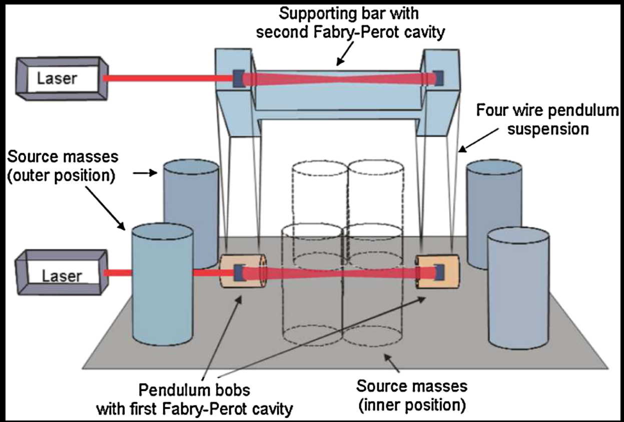


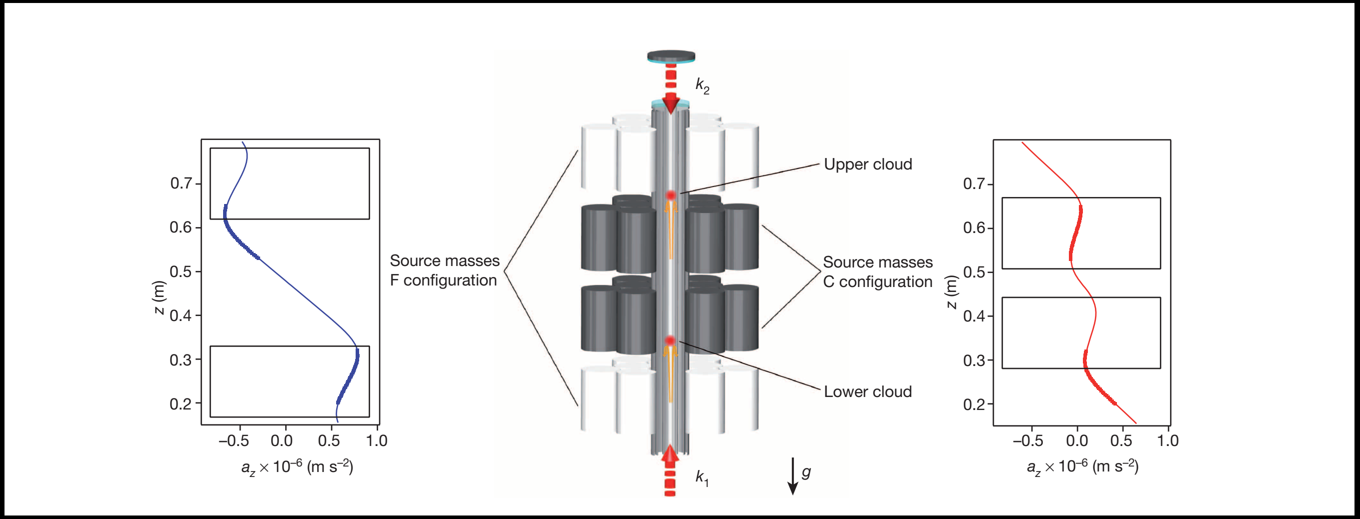
Figure 1. Schematic view of the experimental set-up with the Fabry–Pérot resonator and the two fieldmasses.



Meyer and colleagues 2002: simple pendulum gradiometer with a microwave Fabry-Perot cavity.



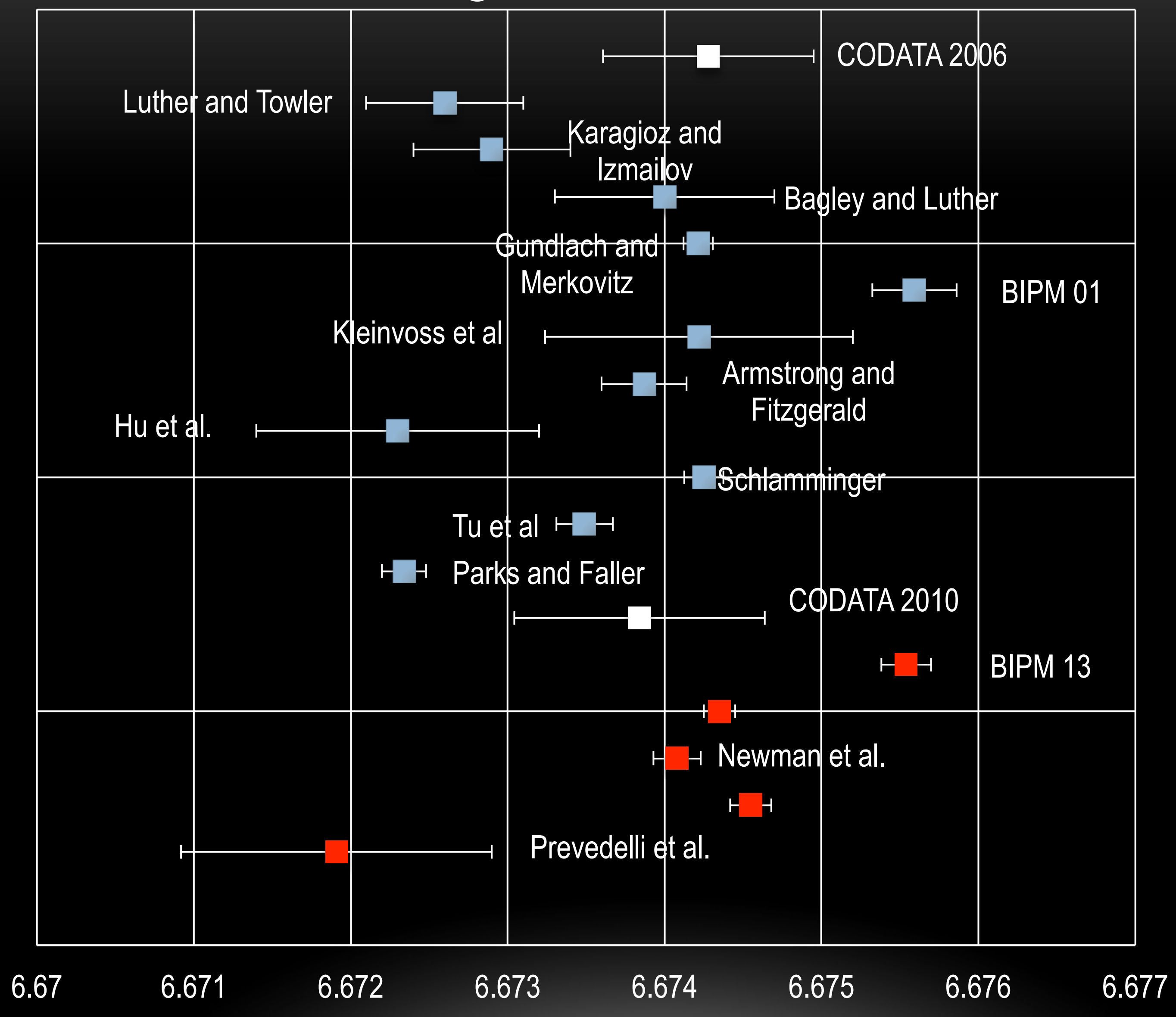
Parks and Faller simple pendulum gradiometer with an optical Fabry-Perot cavity.



Tino and colleagues 2014 Cold Rb atoms in a quantum gravity gradiometer.

Barry Wood Phil Trans 2014

Measured values of Newton's Constant $kg^{-1} m^3 s^{-1} x10^{11}$



BIPM G experiment

- Use two methods to determine G with the same apparatus in order to identify and eliminate systematic errors that affect one method independently of the other.
  - Simple ‘free-deflection’ (or Cavendish) method and
  - Electrostatic torque balance method.
- Both methods have potential sources of error that need to be addressed in the design.
- Uncorrelated errors are effectively eliminated if the values from both methods agree within their random uncertainties.
- Perhaps necessary but not necessarily sufficient!

PRL 2001, PRL 2013

The torsion strip

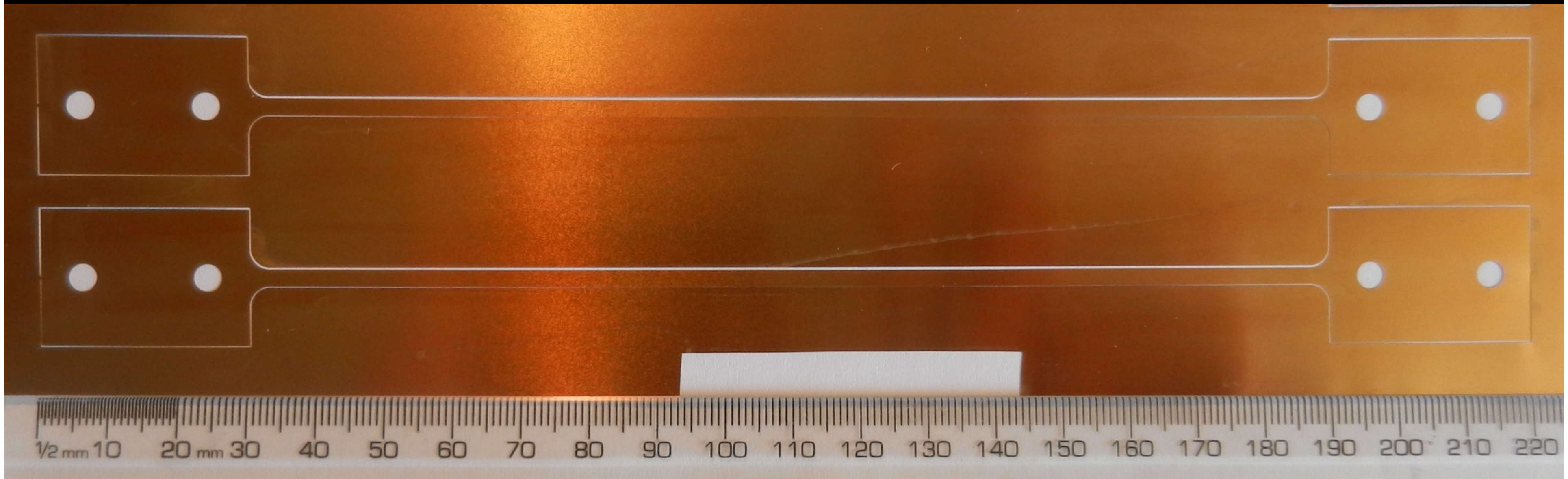
- The restoring torque of the torsion strip comprises a lossy elastic component and a lossless gravitational component:

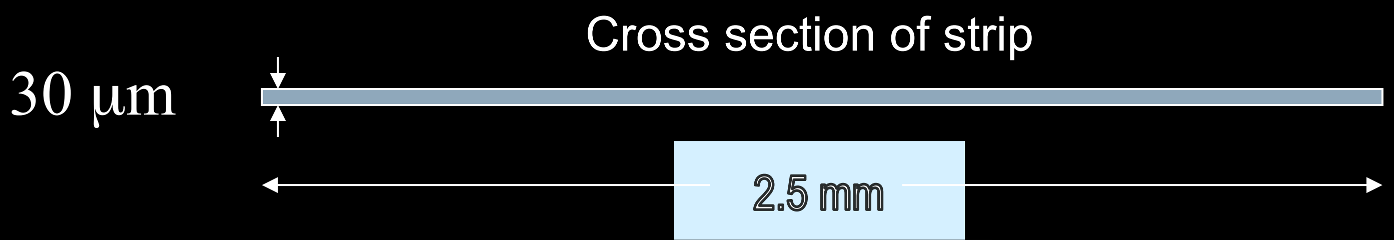
$$c = \frac{bt^3F}{3L} + \frac{Mgb^2}{12L}$$

$$c_e = \frac{bt^3F}{3L}, c_g = \frac{Mgb^2}{12L}$$

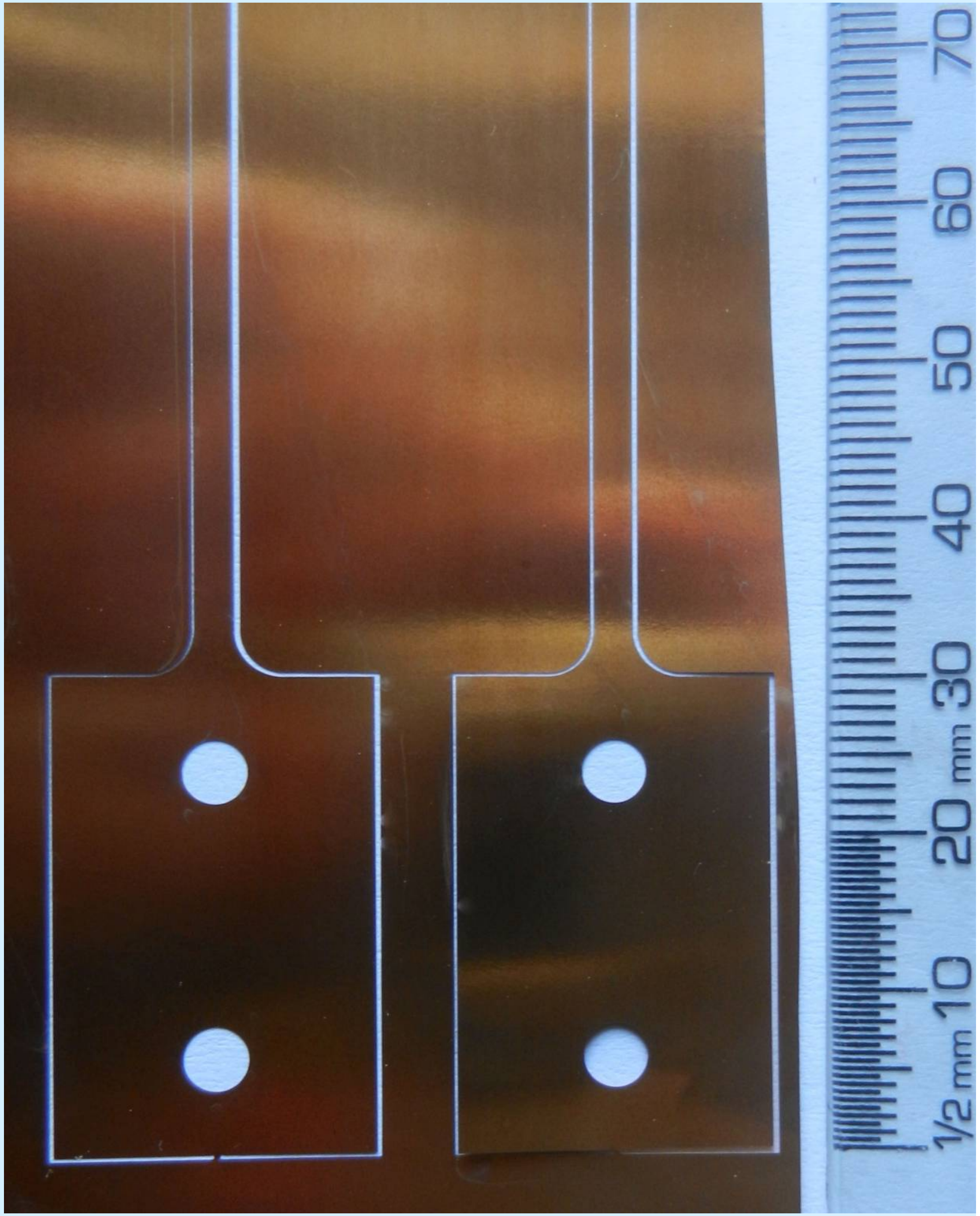
- We can achieve a Q of ~10^5 this mitigates the effects of anelasticity see later.
- Allow a large mass and gravity torque which is ~10^3 larger than round-section fibre.
- We use a test mass geometry with 4-fold symmetry. Coupling to sources drops as $1/R^5$.

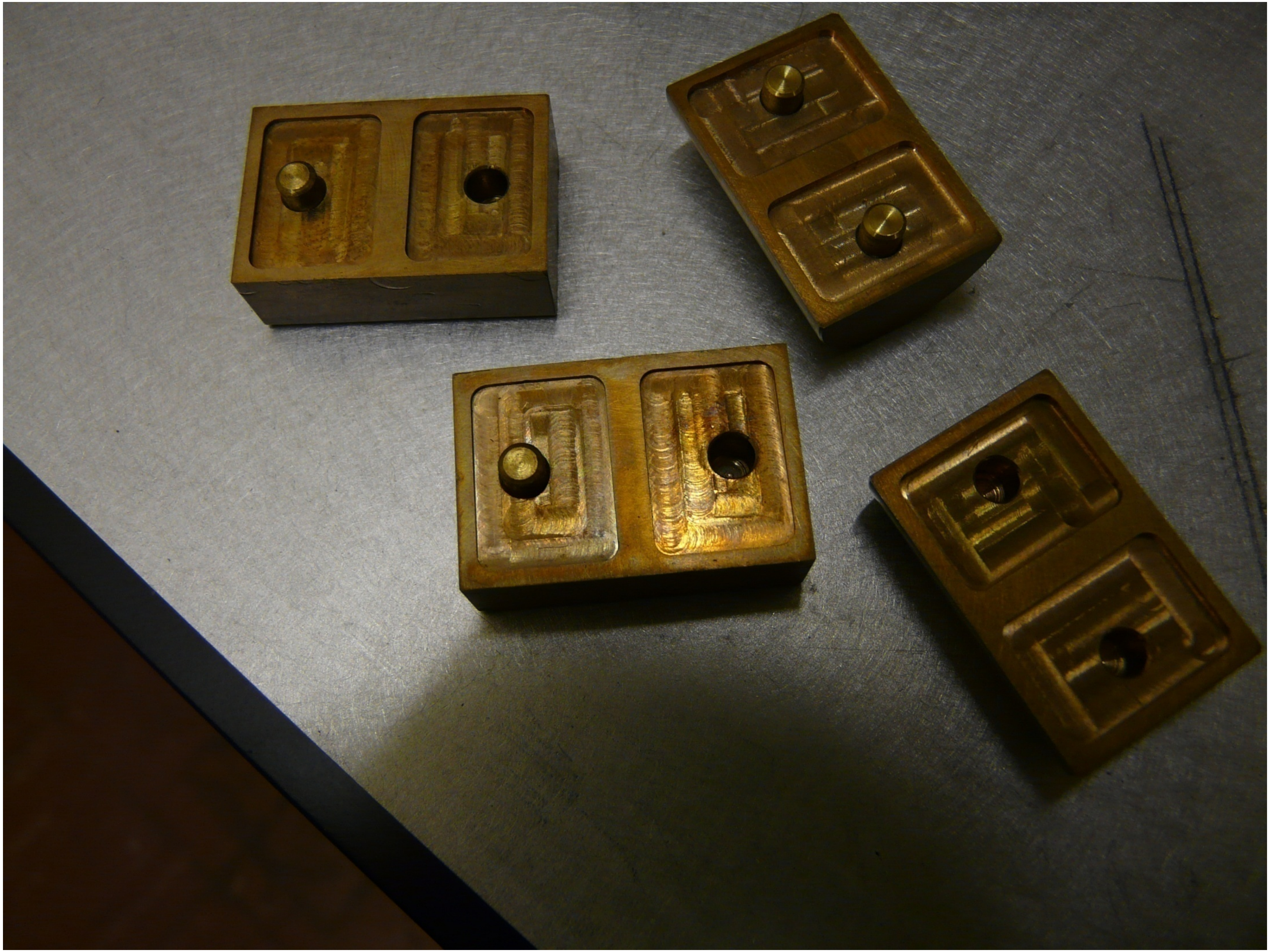
The Cu-Be torsion strip, 160 mm long, 2.5 mm wide and 30 μm thick





The strip loaded to about 2/3 of its yield stress and stretches by nearly 1 mm as the load is applied.


70
60
50
40
30
20 mm
10
1/2 mm



The torsion strip

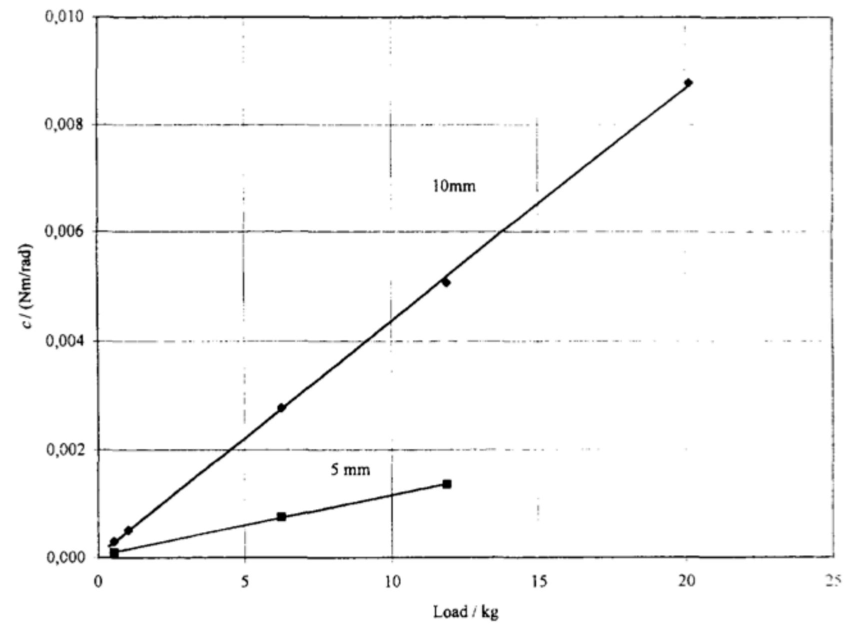
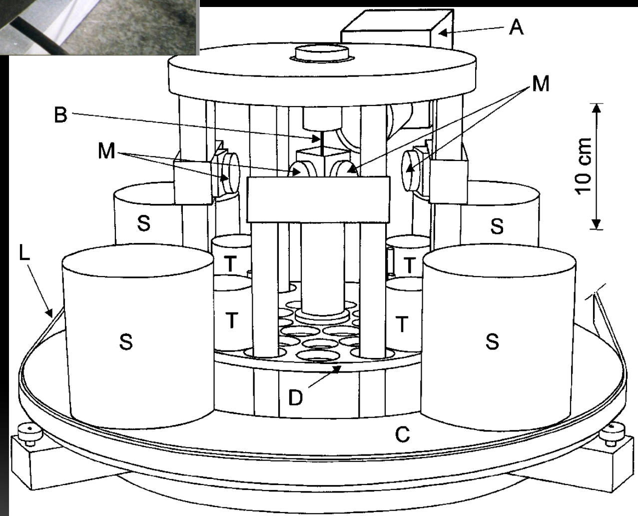


Fig. 1. The measured restoring torques (□) as a function of load for two 50 μm thick strips 180 mm long and of widths 10 mm and 5 mm. In Table 1 the coefficients of the fits to this data are compared with the predicted values from Eqs. (1) and (2).

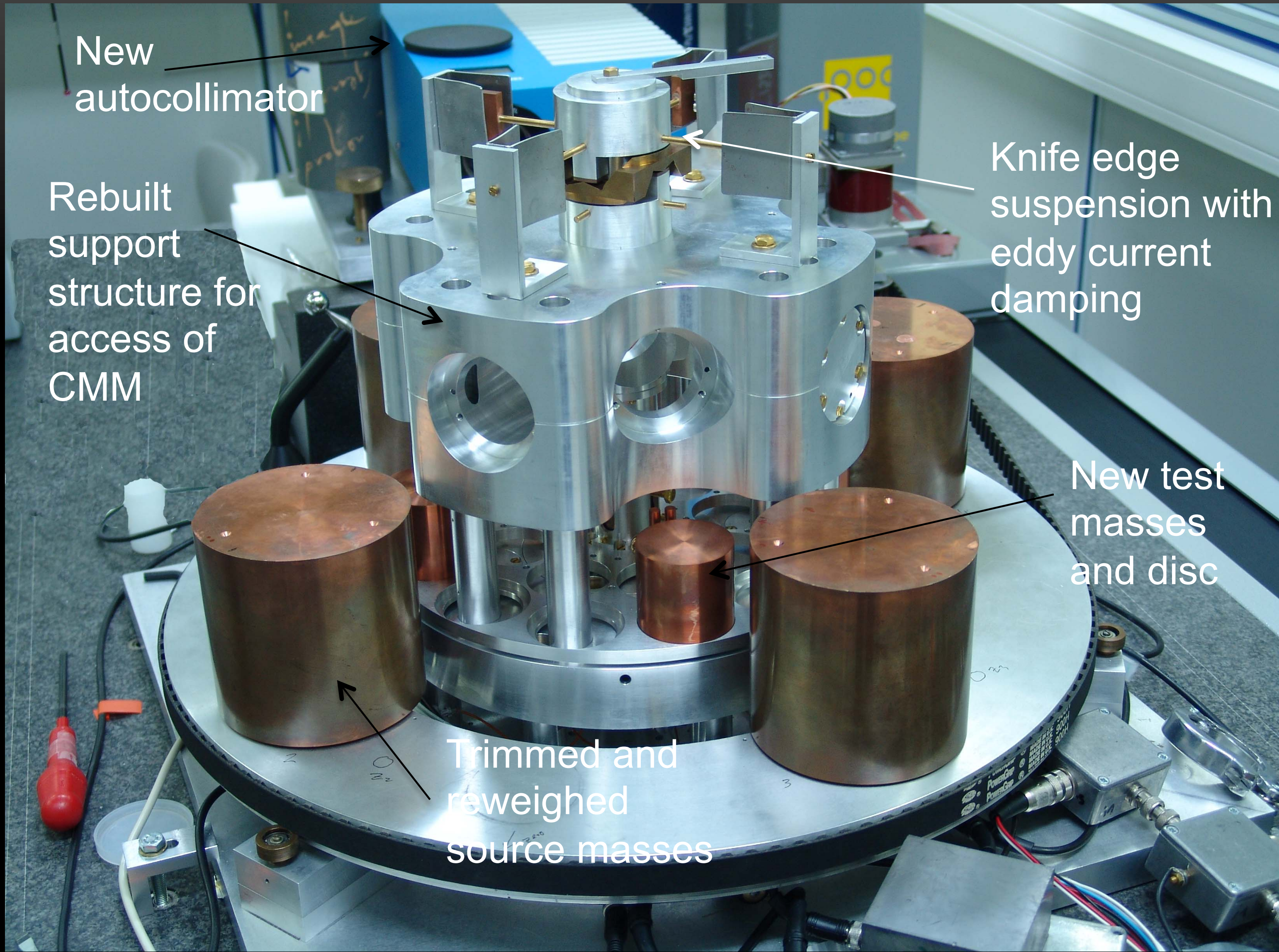
Measured restoring torque as a function of load, Phys Letts 1997. (Measurements made by RSD and TJQ

Table 1
Summary of data shown in Fig. 1. Each strip is 50 μm thick, cut from the same sheet of Cu–Be. Experimental values are the least squares coefficients of the lines shown in Fig. 1. Theoretical values are computed from (1) and (2). The effective length of each strip is taken to be 190 mm. The numbers in parentheses represent standard deviations of the least-squares fit

b [mm]	$M^{-1}c_1$ (expt.) [Nm rad^{-1} kg^{-1}]	$M^{-1}c_1$ (theo.) [Nm rad^{-1} kg^{-1}]	c_0 (expt.) [Nm rad^{-1}]	c_0 (theo.) [Nm rad^{-1}]
5	0.000111(2)	0.000108	0.000047(14)	0.000055
10	0.000431(4)	0.000430	0.000063(48)	0.000110



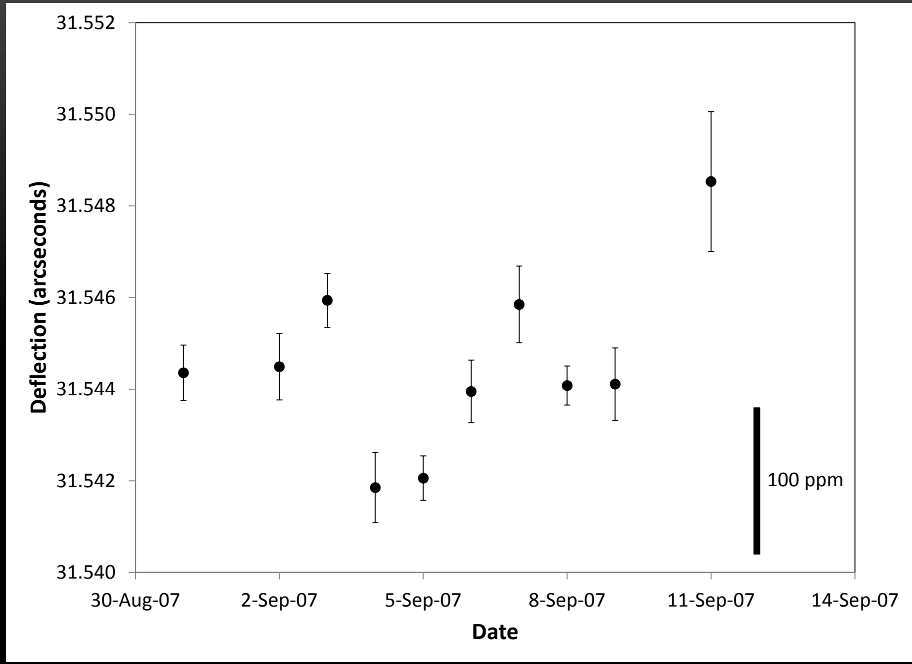
Mk 1 G machine PRL 2001

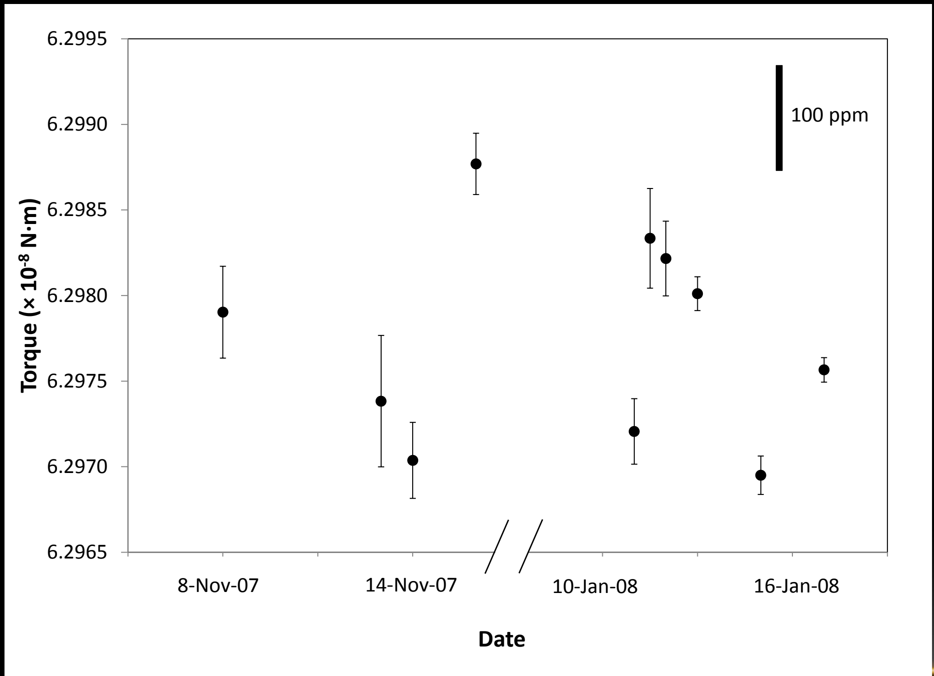


G apparatus used for 2013 value

Results from Cavendish method.

- Each data point represents 17 hrs of measurement with 34 values.





Results from servo method.

- Each data point represents 17 hrs of measurement including the calibration of $C_{ij}(\theta)$.

Systematic *Error*

- We can correct for a systematic *error* or *bias* in the result of an experiment. Our incomplete knowledge of the magnitude of this correction gives rise to its uncertainty.*

- ‘Experimental uncertainties that can be revealed by repeating the measurements are called *random* uncertainties; those that cannot be revealed in this way are called *systematic biases*.’**

*Joint Committee for Guides in Metrology 100-2008 page 5 3.2.3 BIPM website.

‘ An Inttoduction to error analysis:The study of Uncertainties in physical measurements ’ J.R.Taylor, University Science Books 1997.

Uncertainties: Type A and Type B

- **Type A evaluation**
  method of evaluation of uncertainty by the **statistical analysis** of a series of observations,
- **Type B evaluation**
  method of evaluation of uncertainty by means ***other*** **than the statistical analysis** of a series of observations.

http://physics.nist.gov/cuu/Uncertainty/basic.html

PHILOSOPHICAL MAGAZINE A, 1992, VOL. 65, NO. 2, 261–276

Materials problems in the construction of long-period pendulums

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[*Received 5 November 1990¶ and accepted 10 June 1991*]

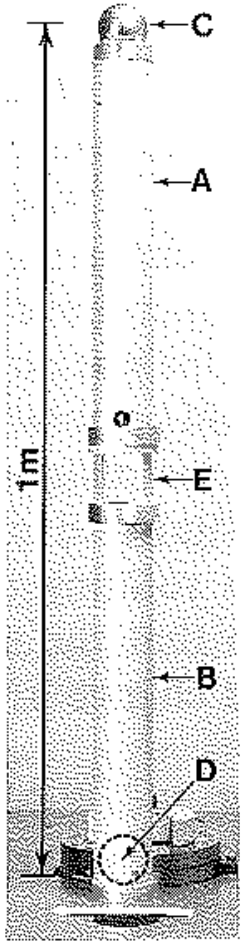


Fig. 1

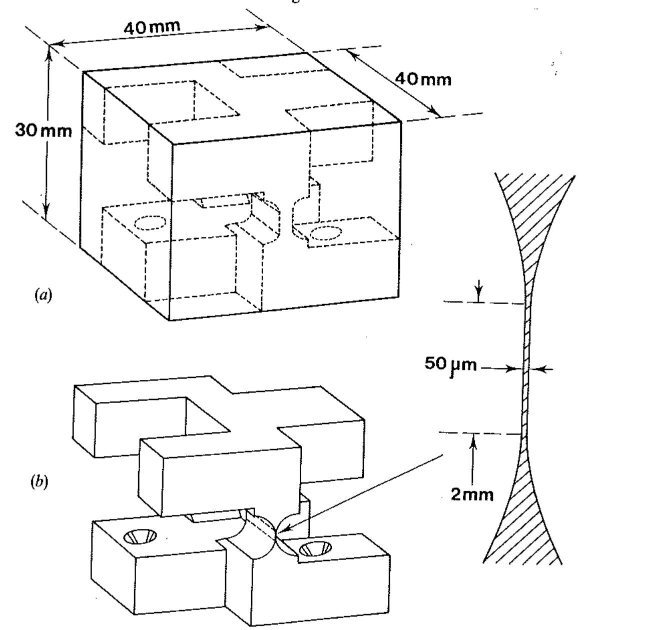
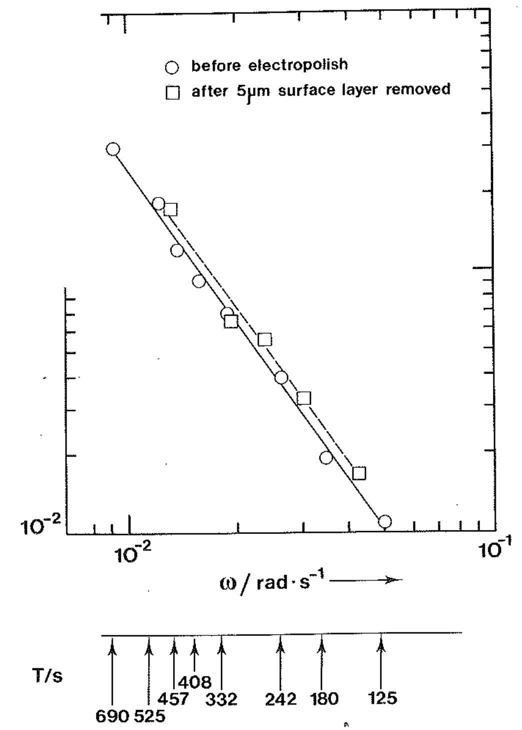
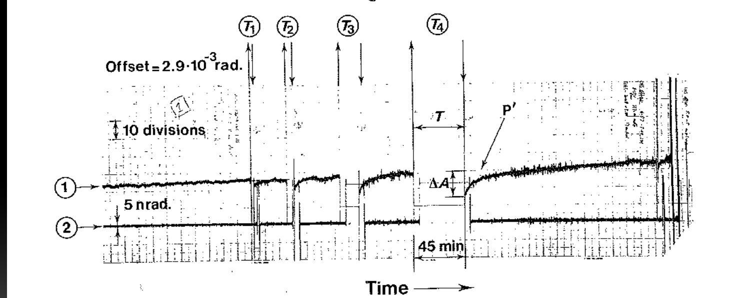


Fig. 4





Anelasticity

- The shear modulus of torsion strip will be frequency dependent due to the range of time constants determining the damping processes in the Cu-Be.

- A simple model that is consistent with observations assumes that the density of relaxation processes increases inversely proportional to the relaxation time.

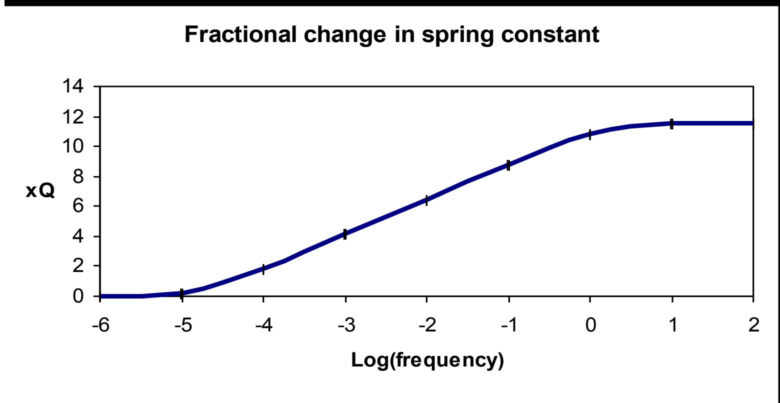
$$\Delta A(t) = \theta_0 c_0 (1 + \alpha l \coth \alpha l) \frac{\delta e}{2E \ln(\tau_\infty/\tau_0)} \{E_i[(T+t)/\tau_\infty] - E_i[(T+t)/\tau_0)] - E_i(t/\tau_\infty) + E_i(t/\tau_0)\}. \qquad (14\,a)$$

$$\gamma = \frac{\Delta}{2I\omega}\left(\tan^{-1}\omega\tau_\infty - \tan^{-1}\omega\tau_0\right) \qquad \Delta = \frac{\pi}{2}\frac{\delta e}{\ln\left(\frac{\tau_\infty}{\tau_0}\right)}$$

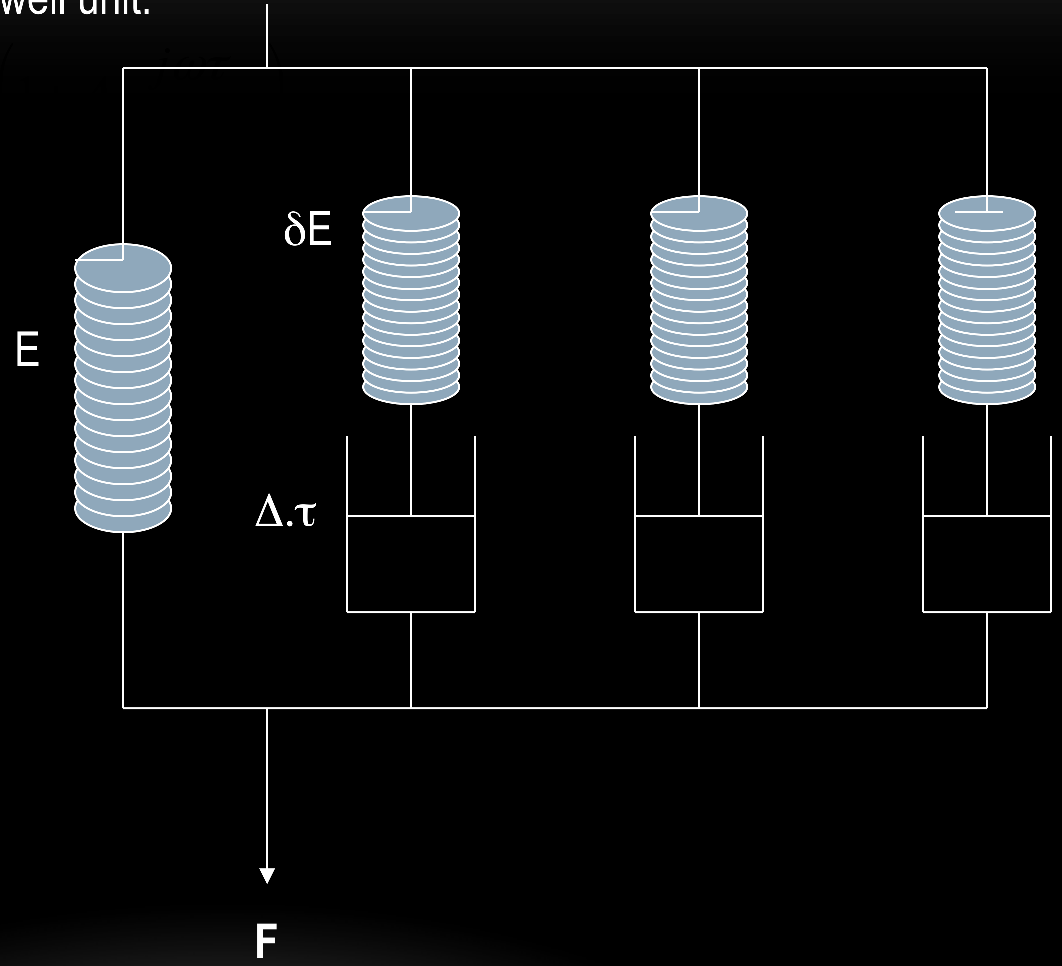
Anelasticity

- Extend to a distribution with density of states ~1/τ

$$f(\tau) = \frac{1}{\ln(\tau_\infty/\tau_0)}\,\frac{1}{\tau}.$$



Maxwell unit:



- **Leads to an *overestimate* of G**

VOLUME 75, NUMBER 15 PHYSICAL REVIEW LETTERS 9 OCTOBER 1995

Does the Time-of-Swing Method Give a Correct Value of the Newtonian Gravitational Constant?

Kazuaki Kuroda
Institute for Cosmic Ray Research, University of Tokyo, 3-2-1, Midoricho, Tanashi, Tokyo 188, Japan
(Received 12 June 1995)

A standard way of measuring the Newtonian gravitational constant has been the time-of-swing method using a torsion pendulum. A key assumption is that the spring constant of the torsion fiber is independent of frequency. This is likely to be true to a good approximation if any damping present is proportional to velocity. However, recent work on the elasticity of flexure hinges suggests that typically the damping at low frequency is best modeled by including a frequency-independent imaginary component in the spring constant. In this case, the real part of the spring constant must vary, leading to an upward bias in a measurement of G.

PACS numbers: 04.80.Cc, 06.30.Gv, 62.20.Dc

G values from time of swing method are biased to larger valus due to anelastiicity

$$\frac{\Delta G}{G} \approx \frac{\Delta}{\pi} = \frac{1}{\pi Q}$$

Anelasticity

- We have determining $\Delta \approx 1.0(2)$ x10^{-4} at a range of oscillation periods down to about 100 s.
- Damping measurements are consistent with $\tau_0 < 10$ s.
- No measurements have been made at low enough frequency to determine τ_{inf}.
- This leads to:

$$\frac{\delta G_{\mathrm{an}}}{G} = \frac{k_{\mathrm{r}}(\omega_m) - k_{\mathrm{r}}(\omega_0)}{k_{\mathrm{e}} + k_{\mathrm{g}}} = \frac{2}{\pi} \frac{k_{\mathrm{e}} \Delta \ln(T_0/T_m)}{k_{\mathrm{e}} + k_{\mathrm{g}}}.$$

- This leads to a correction on G of -6 ppm we correct and add uncertainty of 6 ppm.

Phys Letts A 1995

Calibration of electrostatic torque balance actuator.

- Start from the energy stored in the electrostatic field:

$$W = \frac{1}{2} V^t \underline{\underline{C}} V$$

- where *C* is the matrix of the self and mutual capacitances.
- *If the system is isolated by a complete electrostatic shield* we can express the self-capacitances, C_{ii}, in terms of the mutual capacitances, C_{ij}, and cross-capacitances, C_{cij}. The latter can be measured using 3-terminal methods.

$$C_{ii} = -\sum_{j \neq i} C_{ij}$$

$$C_{ij} = -C_{cij}$$

$$W = \frac{1}{2} \sum_{i,j} C_{ij} \left(V_i - V_j\right)^2$$

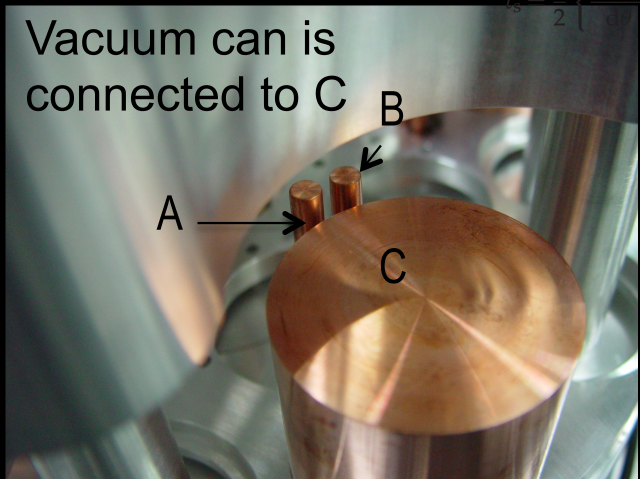
$$\Gamma = +\frac{1}{2} \sum_{i,j} \frac{dC_{ij}}{d\theta} \left(V_i - V_j\right)^2$$

Phil Trans R Soc 2005

Calibration of electrostatic torque balance actuator.

- Note that capacitances are often frequency dependent.
  - Need to apply voltages to the actuators at the same frequency at which we measure capacitance (1 kHz) .
- We have to include ALL terms in the sum.

$$\tau_s = \frac{1}{2}\left\{\frac{dC_{AC}}{d\theta}\left\langle (V_A-\delta)^2\right\rangle + \frac{dC_{BC}}{d\theta}\left\langle (V_B+\delta)^2\right\rangle + \frac{dC_{AB}}{d\theta}\left\langle (V_A-V_B)^2\right\rangle\right\},$$



$$\tau_s = \frac{1}{2}\left\{\frac{dC_{AC}}{d\theta}\left\langle (V_A-\delta)^2\right\rangle + \frac{dC_{BC}}{d\theta}\left\langle (V_B+\delta)^2\right\rangle + \frac{dC_{AB}}{d\theta}\left\langle (V_A-V_B)^2\right\rangle\right\},$$

- Finite element design in 2d to ensure that $d^2C_{AC}/d\phi^2 \approx 0$.
- Note that $dC_{AB}/d\phi$ cannot be ignored.
- Zero stiffness design.

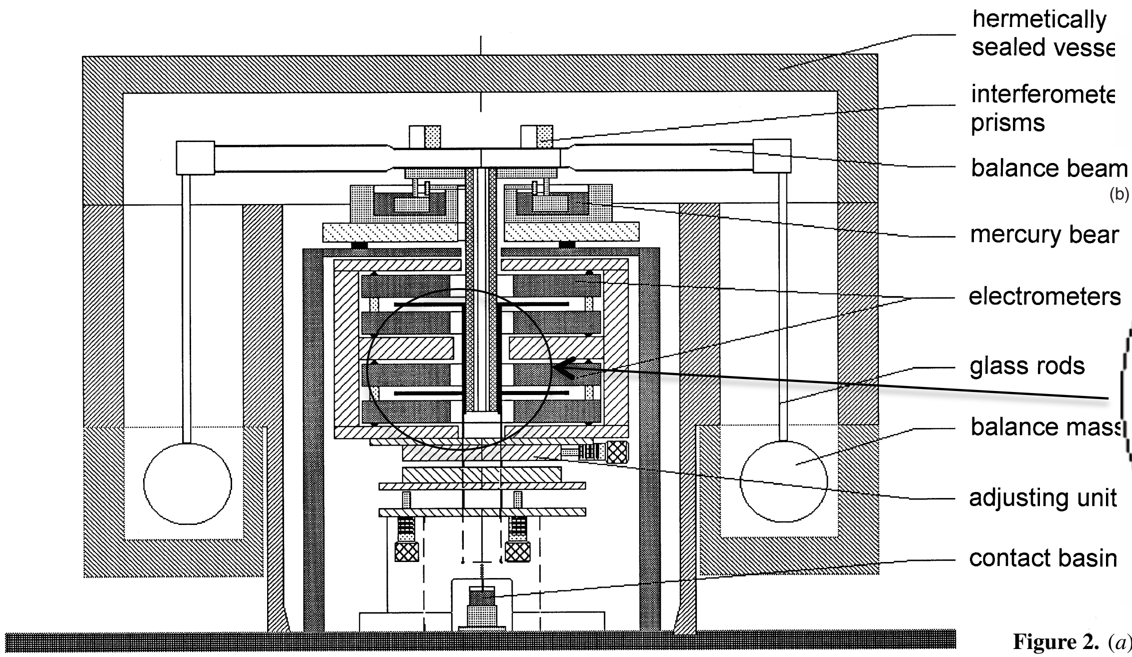
$$C_{cc} = \frac{2\pi\varepsilon_0}{\cosh^{-1} u},$$

$$u^2 = \frac{D^2 - R_1^2 - R_2^2}{2R_1R_2}.$$

Phil Trans R Soc 2005

The PTB measurement

W. Michaelis, H. Haars and R. Augustin



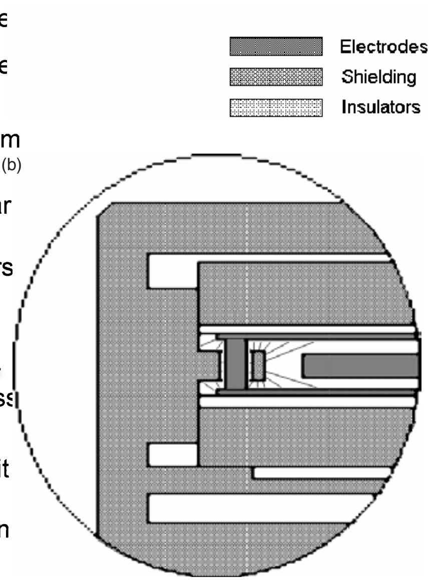


Figure 2. (*a*) Top view of the torque transmitter and a cross-section of one system. (*b*) A detail of the cross-section with some electric flux lines.

Quadrant electrometer with cylindrical grounded shield.

Metrologia 1995,2004, 2005

Calculation of the torque

- Model test objects and source objects as either cylinders or points.
- Three contributions to torque:
  - perfect source cylinders on perfect test cylinders (includes all holes in torsion disc).
  - Perfect source cylinders on points on torsion balance (includes all screws and non-uniformity of torsion disc): 150 ppm
  - source points (kinematic mounts, balls) on perfect test cylinders: 1 ppm.
- Use three methods to calculate cylinder-cylinder torques: double multipole expansion, numerical integral of elliptical integral solution and double numerical integral.

Mass homogeneity

- Hydrostatic weighing of samples of the source mass billet showed a linear gradient of approximately $\Delta = 2x10^{-4}$.

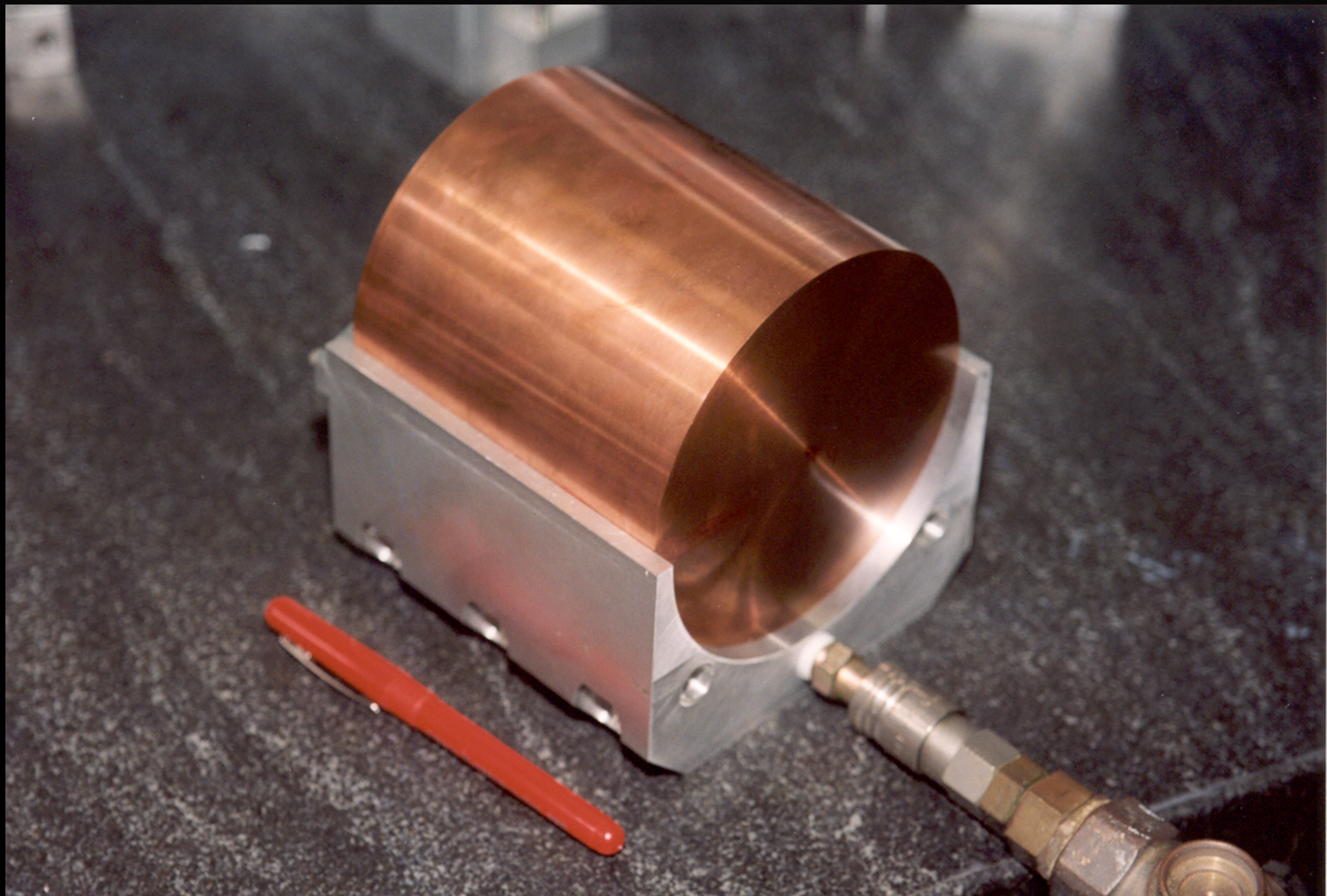
$$r = r_0\left(1 + \frac{\Delta}{R} r\cos\theta\right) \qquad \Delta = 8\pi^2 \frac{R}{T^2 g}$$

- Establish axial/three-fold symmetry of source masses with CMM.
- Measurements of the free oscillation period of the source masses gave the following results

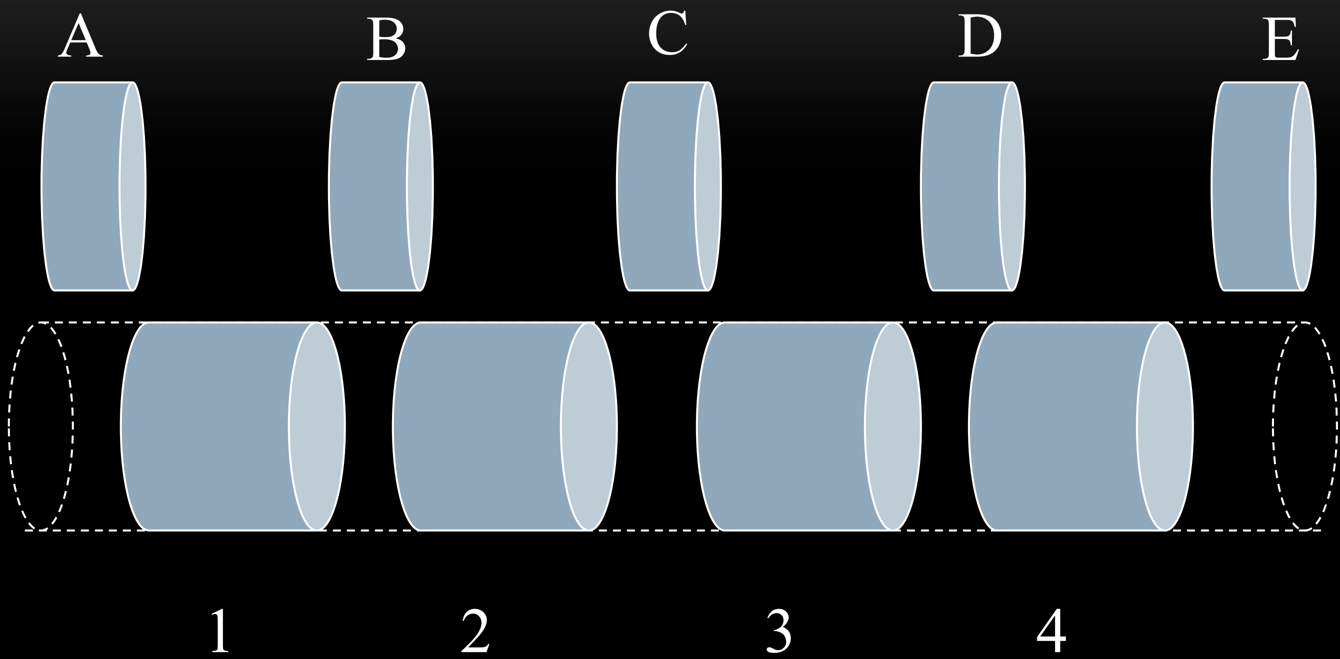
Mass #	Oscillation period, T (s).	Centre of mass offset, δ (μm)	Orientation clockwise wrt inward radial direction (degrees)	Linear density gradient, $\Delta \times 10^4$
1	68	1.6	159	1.05
2	58.8	2.1	-15	1.40
3	54.5	2.4	159	1.63
4	44.7	3.6	140	2.42

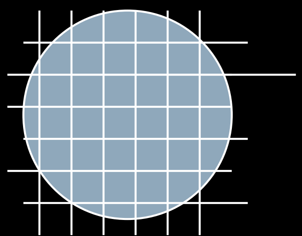
Measurements made by R.S.Davis (after Gabriel Luther, J.E.Faller)

Centre of mass determination using air bearing



Measurement of density inhomogeneities in source masses





Hydrostatic weighing of witness samples from ingot (RSD)

Mass homogeneity

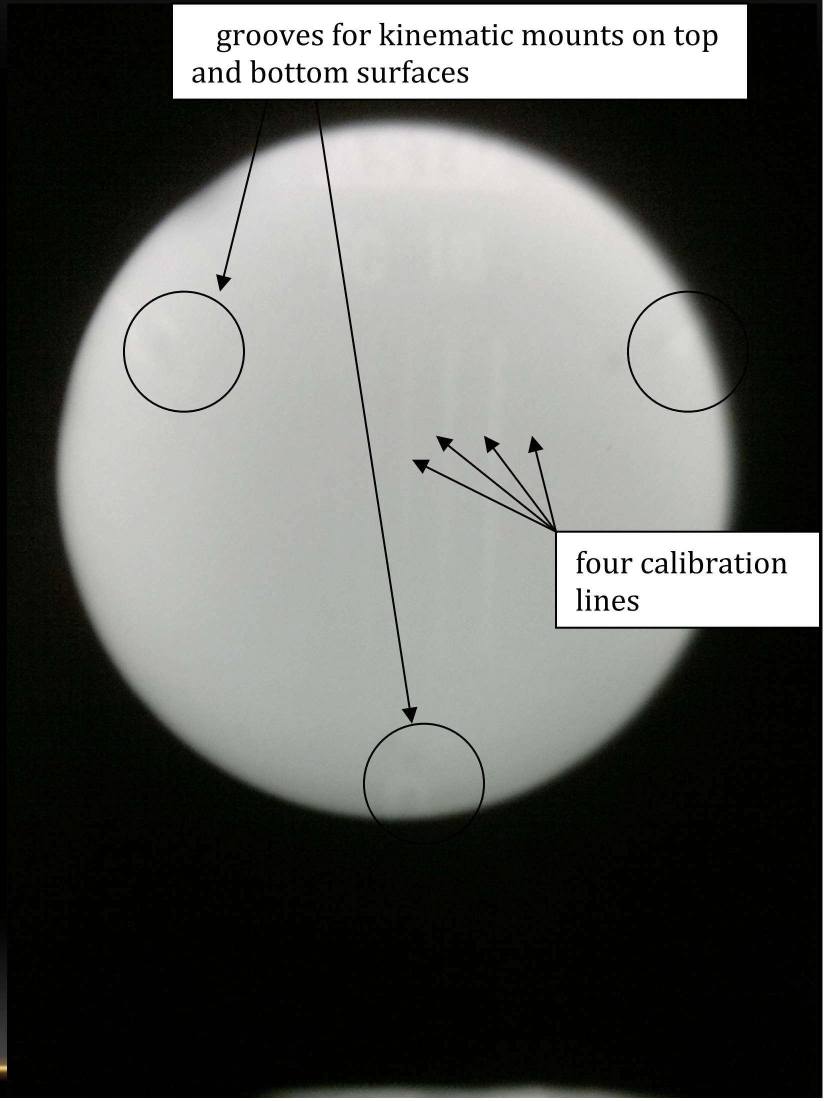
- The source masses could be placed in 3 orientations on their kinematic mounts. Measurements of torques; Γ_A, Γ_B, Γ_c , were compared with predictions assuming linear density gradients

$$\frac{\Gamma_B - \Gamma_A}{\Gamma_A} = \begin{Bmatrix} 161 \pm 86\,ppm & \mathrm{expt} \\ 121\,ppm & calc \end{Bmatrix} \qquad \frac{\Gamma_C - \Gamma_A}{\Gamma_A} = \begin{Bmatrix} -588 \pm 86\,ppm & \mathrm{expt} \\ -641\,ppm & calc \end{Bmatrix}$$

- ONLY **22 ppm of this torque difference is due to density gradient. Most is due to shifts in the mass centres as they are rotated.**
- Conclude that linear density model is consistent measurements and can only change the value of G by 22 ppm if ignored..

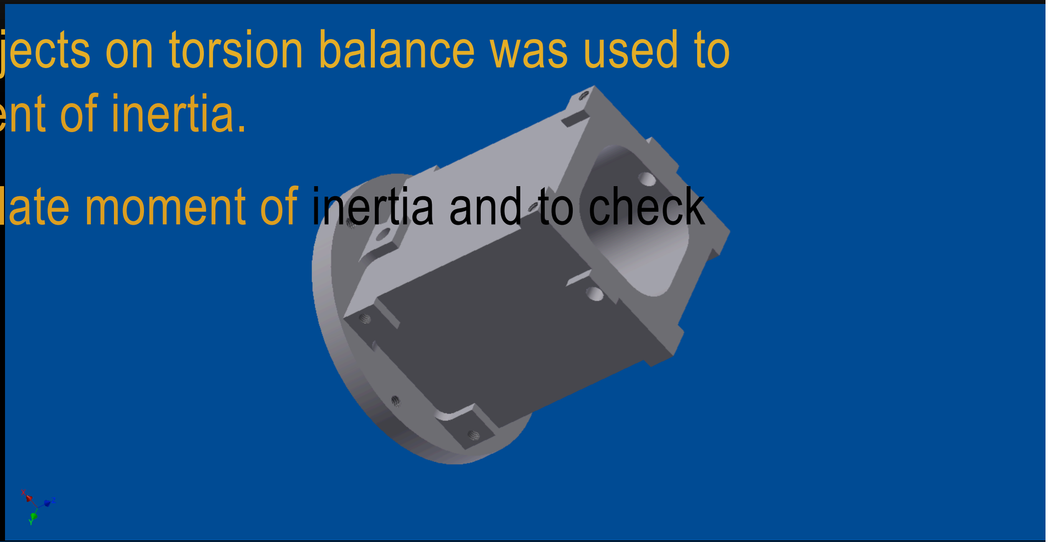
Mass homogeneity

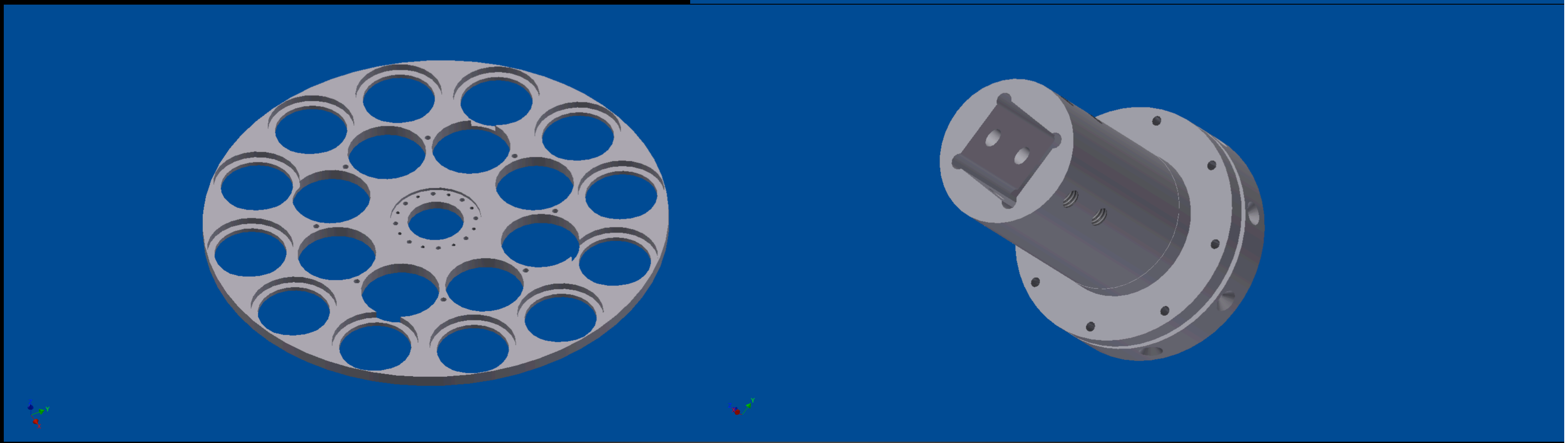
- Suppose that the source masses had identical voids located at their centres. We require a void of ***5 mm radius*** in order to produce a reduction of torque of 200 ppm.
- We took radiograms with 6 MeV X-rays.
- The grooves for the kinematic mounts can be seen as dark areas of 5 mm by 2 maximum depth.
- We can also see calibration lines of lead.
- Conclude that there are no voids that could be responsible for 200 ppm error.



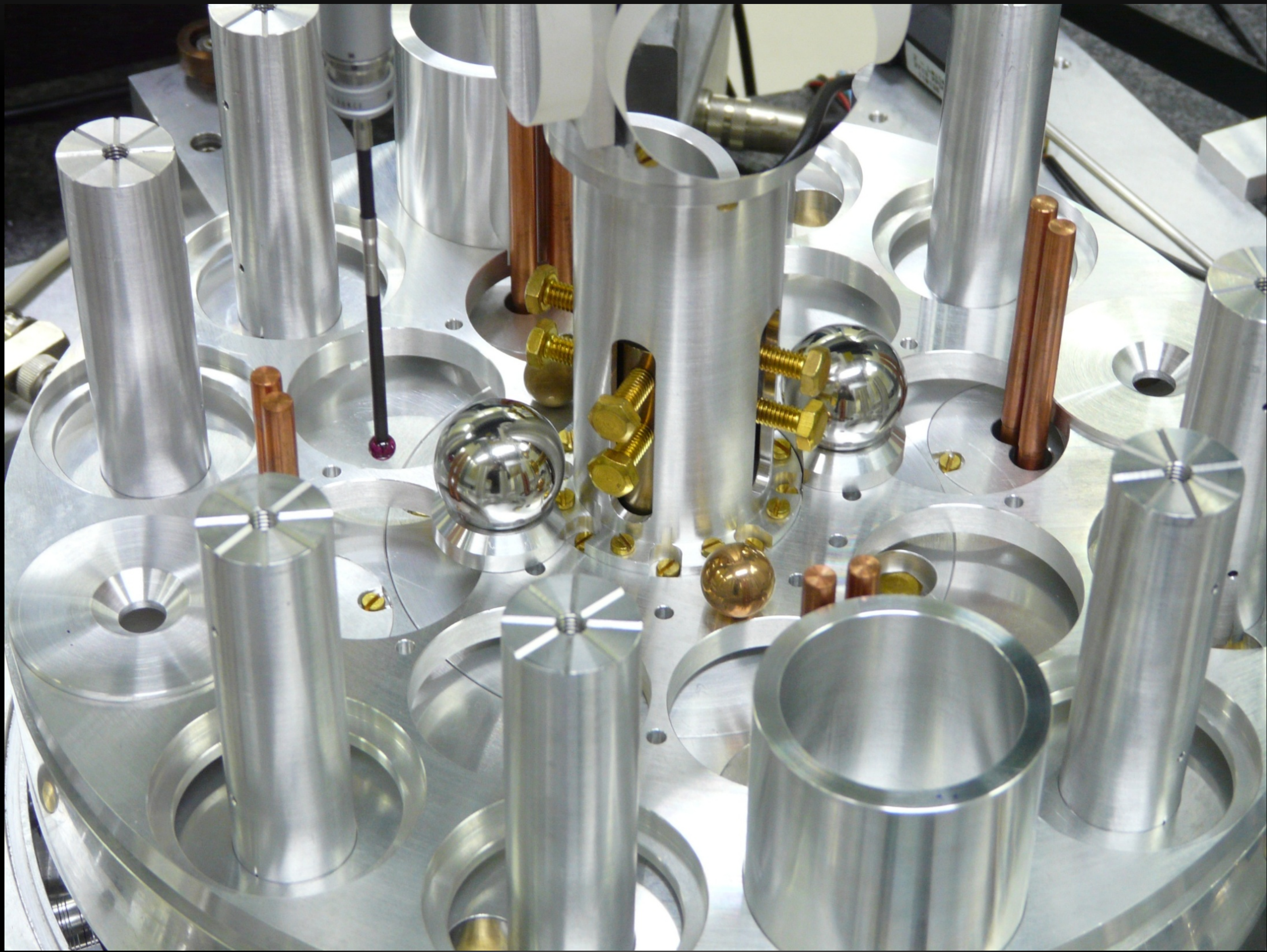
Moment of inertia calculation and measurement

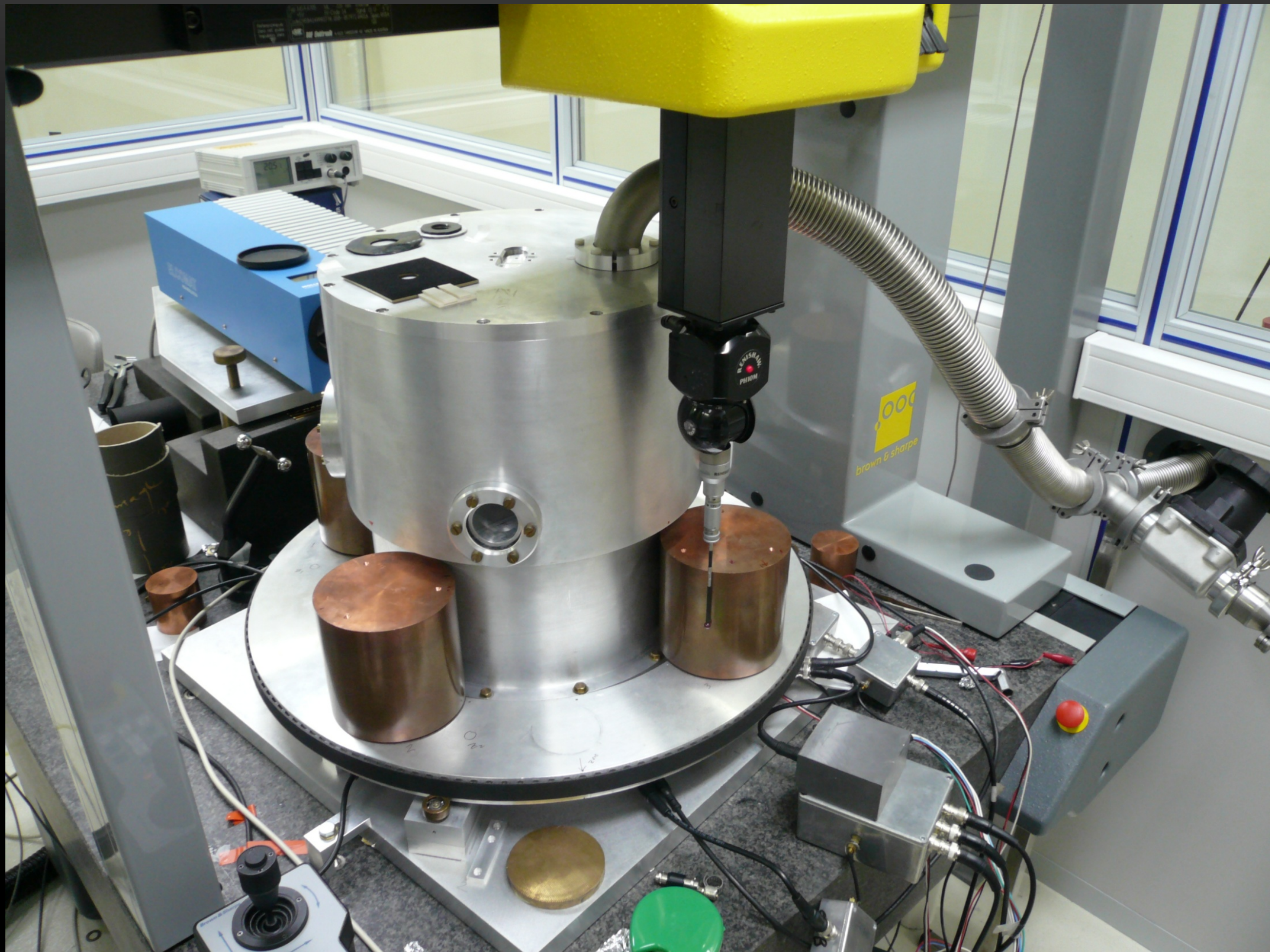
- Comprehensive model of test objects on torsion balance was used to evaluate both torques and moment of inertia.
- CAD package was used to calculate moment of inertia and to check analytical result.





Measurement of the moment of inertia of the torsion balance disk





The CMM

Correlated and uncorrelated uncertainties

- Starting point for analysis is an approximate expression for the gravity torque due to point masses from a multipole expansion .

$$\Gamma = 35Mm\frac{r^4}{R^5}\sin 4\theta,$$

- The uncertainty in the Cavendish method:

$$\frac{\Delta G_c}{G} = -\frac{\delta M}{M} - 2\frac{\delta r_a}{r} + 5\frac{\delta R_{ac}}{R} - \alpha_c - \delta\alpha_{cT} + \frac{\delta\Delta\theta_c}{\Delta\theta_c} - 2\frac{\delta T_0}{T_0} + \frac{\delta I_t}{I} + \frac{\delta k}{k_r} + \frac{\delta\tau_c}{\tau},$$

- The uncertainty in the Servo method:

$$\frac{\delta G_s}{G} = -\frac{\delta M}{M} - \frac{\delta m}{m} - 4\frac{\delta r_a}{r} + 5\frac{\delta R_{as}}{R} - \delta\alpha_s - \delta\alpha_{sT} - \frac{\delta\Delta\theta}{\Delta\theta} + \frac{\delta\Delta C}{\Delta C} + 2\frac{\delta\Delta V}{V} + \frac{\delta\tau_s}{\tau_s}.$$

Correlated and uncorrelated uncertainties

- We can express the values of G in a compact form:

$$\begin{pmatrix} \delta G_s \\ \delta G_c \end{pmatrix} = \underline{\underline{A}}\,\underline{u},$$

- where A and u are:

$$A = \begin{pmatrix} -1 & -1 & -4 & 5 & 0 & -1 & -1 & 0 & 0 & -1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 & 5 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix},$$

$$u' = \begin{pmatrix} \delta m & \delta M & \delta r_a & \delta R_{as} & \delta R_{afd} & \delta\alpha_s & \delta\alpha_{fd} & \delta\Delta\phi & \delta\Delta C & \delta V & \delta\beta & \delta T & \delta I_t & \delta F & \delta\tau_s & \delta\tau_f \end{pmatrix}$$

- We can calculate the variance covariance matrix with correlation coefficient, ρ:

$$\begin{pmatrix} \delta G_s^2 \\ \delta G_{fd}^2 \end{pmatrix} = uMM^t u^t = \begin{pmatrix} \sigma_s^2 & \kappa^2 \\ \kappa^2 & \sigma_{fd}^2 \end{pmatrix} \quad \rho = \frac{\kappa^2}{\sigma_s \sigma_{fd}}$$

Correlated and uncorrelated uncertainties

quantity	fractional uncertainty, ppm
test masses $\delta m/m$ (correlated)	1
source masses $\delta M/M$ (correlated)	1
test mass type A servo (correlated)	17
test mass type A Cavendish (correlated)	8
source mass type A for both servo and Cavendish (uncorrelated)	12
servo type B uncertainty for source and test masses $\delta\alpha_s$	4
Cavendish type B uncertainty for source and test masses $\delta\alpha_c$	−3
servo type A uncertainty for 0.1 K temperature change $\delta\alpha_{sT}$	−2
Cavendish type A uncertainty for 0.1 K temperature change $\delta\alpha_{cT}$	[illegible]
angle measurement $\delta\Delta\phi/\Delta\phi$ (anti-correlated)	47
capacitance calibration $\delta\Delta C/\Delta C$	6
voltage calibration $2\,\delta V/V$	[illegible]
timing error $2\Delta T_0/T_0$	0.5
moment of inertia of torsion disc	13
anelasticity $\delta k/k_r$	6
uncertainty in mean servo torque $\delta\tau_s/\tau$	30
uncertainty in mean Cavendish torque $\delta\tau_c/\tau$	19
net uncertainty on servo value σ_s	61
net uncertainty on Cavendish value σ_c	54
covariance κ	−2080
correlation coefficient	−0.63

Correlated and uncorrelated uncertainties

- We can choose a combination of the two values of G that minimises the variance on the combination:

$$G_F = \alpha_{fd} G_{fd} + \alpha_s G_s$$
$$\alpha_{fd} + \alpha_s = 1$$

G_{fd} = 6.67586(36) x 10^{-11} m^3 kg^{-1} s^{-2} (54 ppm)

G_s = 6.67515(41) x 10^{-11} m^3 kg^{-1} s^{-2} (61 ppm)

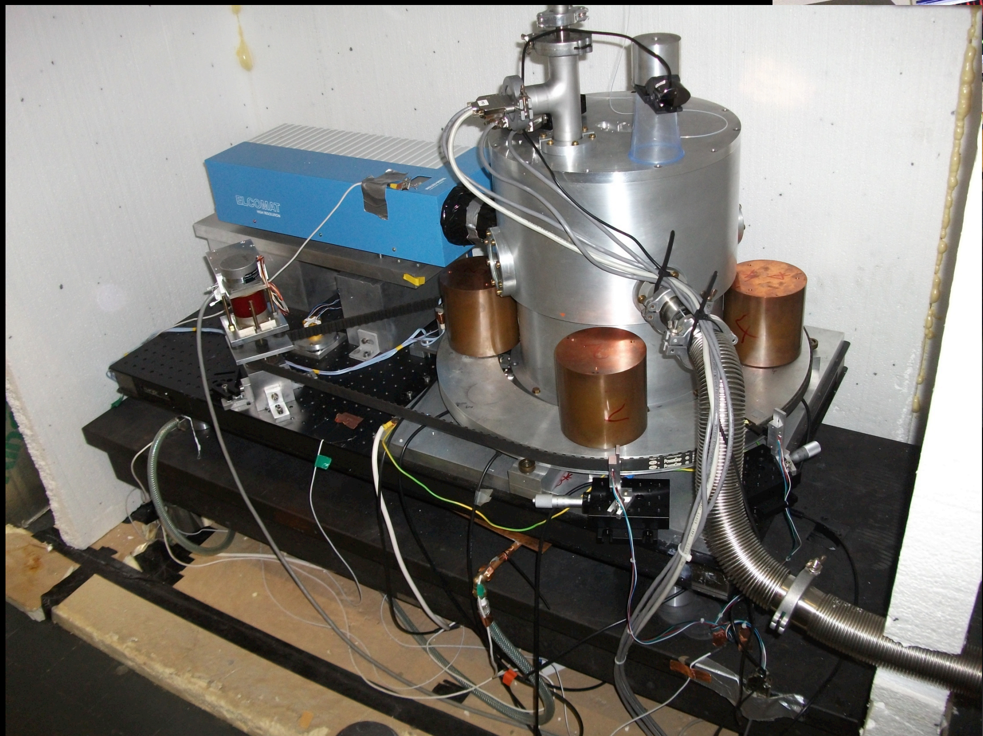
G_f = 6.67554(16) x 10^{-11} m^3 kg^{-1} s^{-2} (25 ppm)

$\Delta G/G = (G_{fd} - G_s)/G$ = (106 +/- 104) ppm

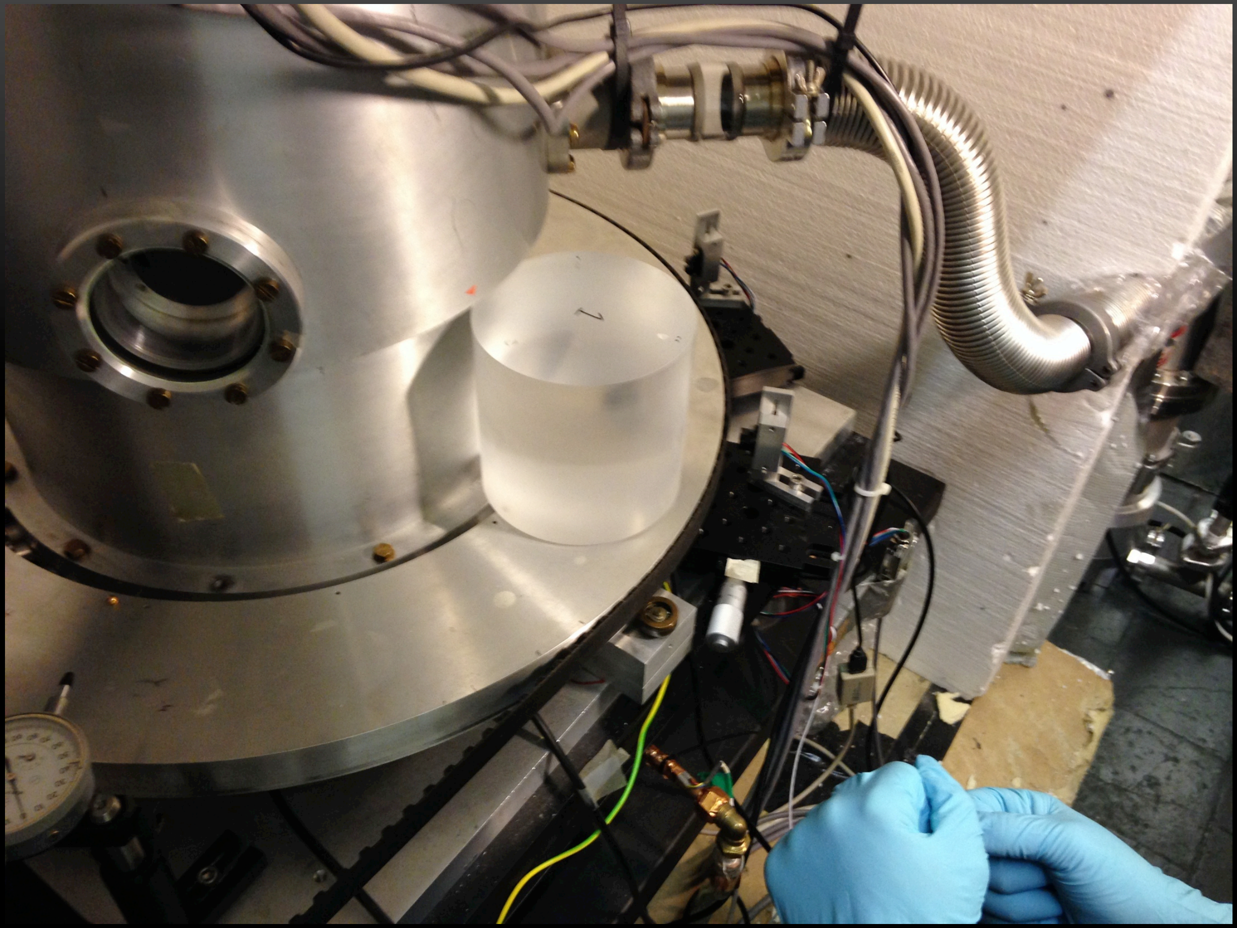
Erratum PRL 2014

G machine in Birmingham

Temperature control, magnetometers, tiltmeters and tilt servo, angle interferometers, and very flimsy table!

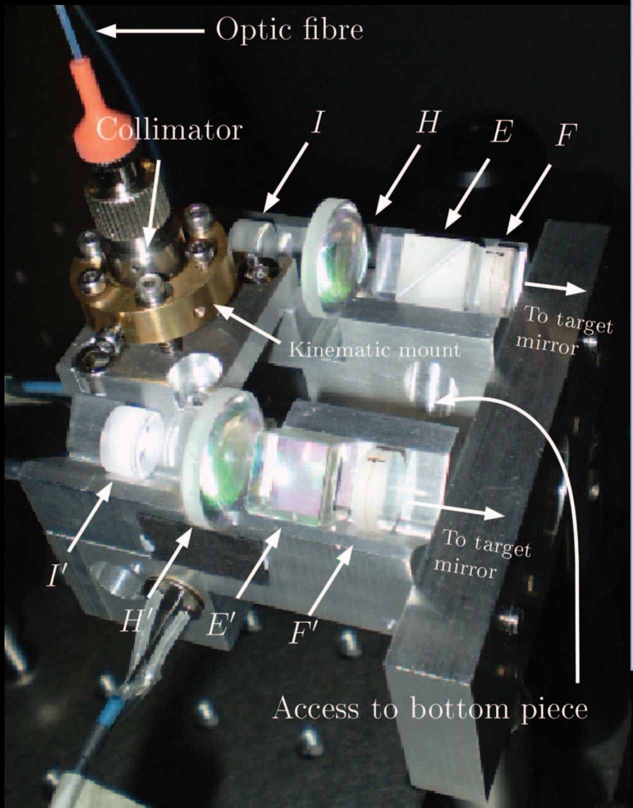


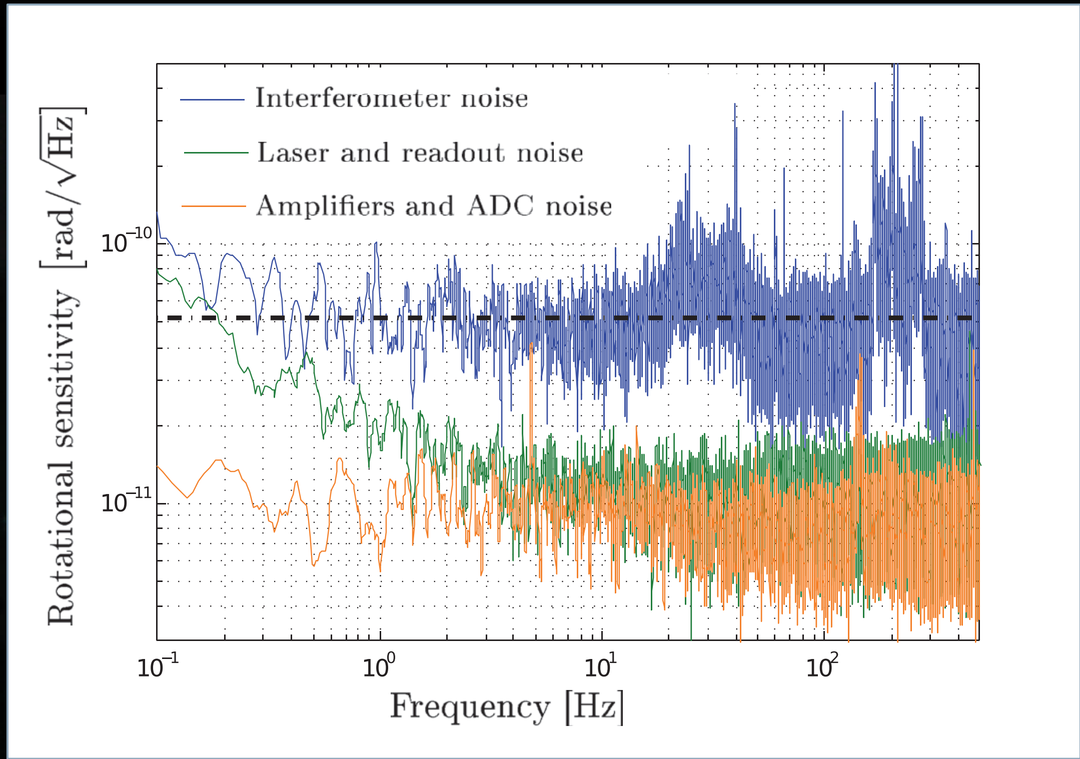
Ludovico Carbone Post Doc.
PhD Hasnain Panjwani 2012. Lindsay Lewins and Ben Rendle, Msci Project.
Courtesy John Bryant.



Checking Linearity of Cavendish method (ac magnetic bias?) with Plexiglass masses.

Homodyne interferometer with novel cat's eye configuration allows us track the mirror rotation over +/- 40 mrad with 10^{-10} radian/rtHz sensitivity.





Autocollimator resolution = 25 nrad
Range is 40 mrad.

Pena-Arellano et al 2013

Program at Birmingham

- Linearity check with plexi-glass masses
- Calibrate ILIAD interferometer versus Autocollimator. Aim to improve accuracy of capacitance calibration, eliminate calibration of autocollimator?
- Look for other systematics: AC magnetic fields, tilt, sensitivity to microseismic noise, calibration of change of period with temperature.
- Do we understand anelasticity (de Salvo). Perform anelasticity measurements of Cu-Be.

di Cintio et al (2009)

Conclusions on BIPM G experiment

- There are a number of ways that bias or systematic error can influence the value of Newton’s constant in an experimental determination.

- I have described anelasticity and biases in electrostatic actuators

- I have described how we have effectively eliminated such biases in the BIPM G determinations. I have also described how we have associated Type B uncertainties where appropriate.

- There remain other effects to look for and eliminate.

Ways forward

- We held a meeting at Chicheley Hall in February 2014 to discuss the issue of the value of G.
- The contributors wrote articles that appeared in Phil. Trans. of Royal Society A 372: ‘The Newtonian constant of gravitation, a constant too difficult to measure?’
- Our article: Quinn, CCS, Parks and Davis art 20140032
- There was a follow-up meeting at NIST in October 2014.

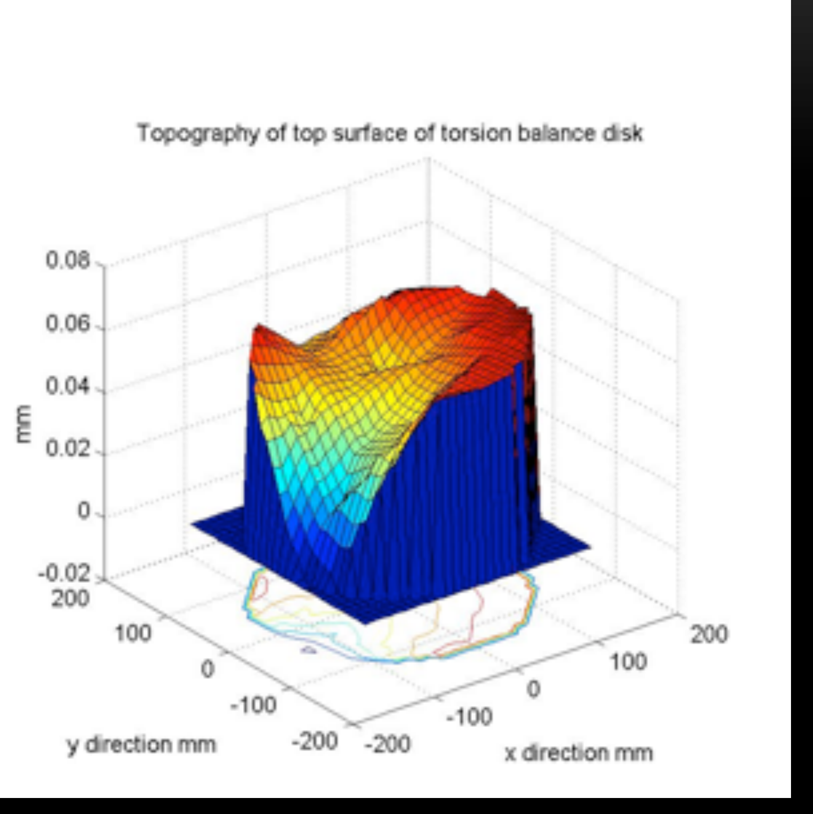
Ways forward

- IUPAP G committee Chairman: Stephan Schlamminger
- Decision CIPM/103-43
  The CIPM agreed to establish a consortium of national metrology institutes and other institutes, coordinated by the NIST, to facilitate new work aimed at resolving the present disagreement amongst measurements of the Newtonian
  constant of gravitation, *G*. The BIPM will provide facilities for meetings of those taking part in this work.
- In Spring 2016 the BIPM G apparatus will be shipped to NIST where we will repeat the measurement and hopefully improve it.

- We must proceed in a collaborative way to solve this problem!!

Thank you for your kind attention!

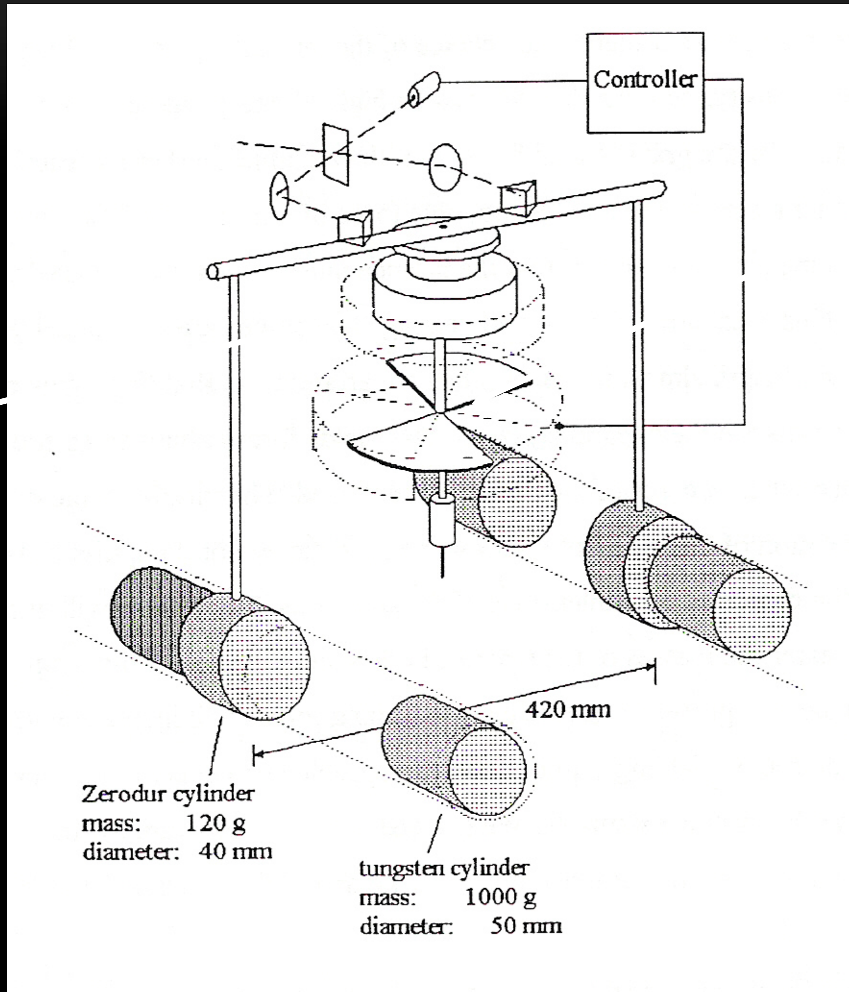
Calculation of the torque



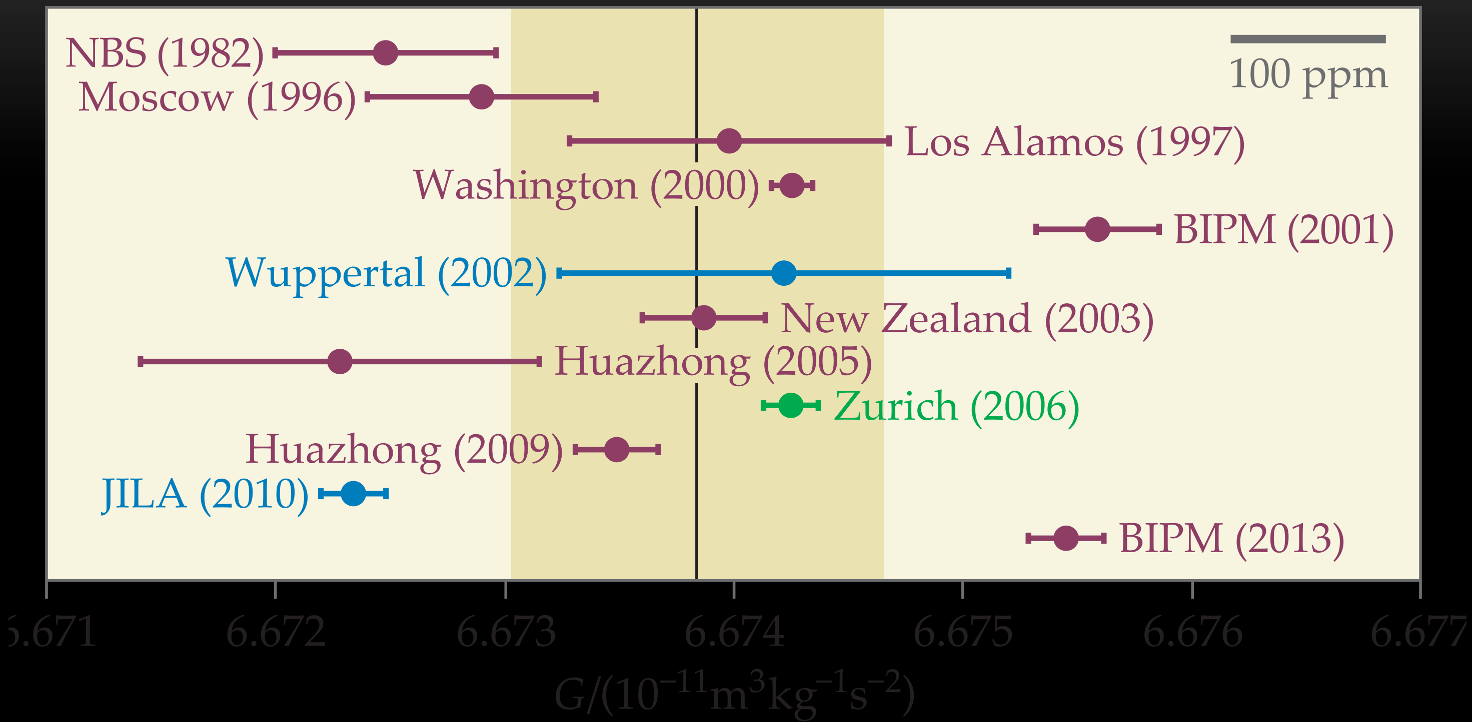
Physikalisch-Techniche Bundesantalt apparatus

Quadrant electrometer

Mercury Bearing



Michealis et al 1995/6



NBS (1982)
Moscow (1996)
Los Alamos (1997)
Washington (2000)
BIPM (2001)
Wuppertal (2002)
New Zealand (2003)
Huazhong (2005)
Zurich (2006)
Huazhong (2009)
JILA (2010)
BIPM (2013)
100 ppm
6.672
6.673
6.674
6.675
6.676