Electron scattering from light nuclei

Ingo Sick

Interest

can calculate wave function exactly from V_{NN}
Faddeev, VMC, GFMC, Nocore SM, ...
(e,e) ± only probe sensitive to short range
rich set of form factors, including spin observables
→ excellent testing ground for understanding of nuclei

Questions

standard model of nuclear physics valid?
relativistic effects important?
role of mesonic degrees of freedom?
role of quark-structure of N?

Special emphasis of talk: radii

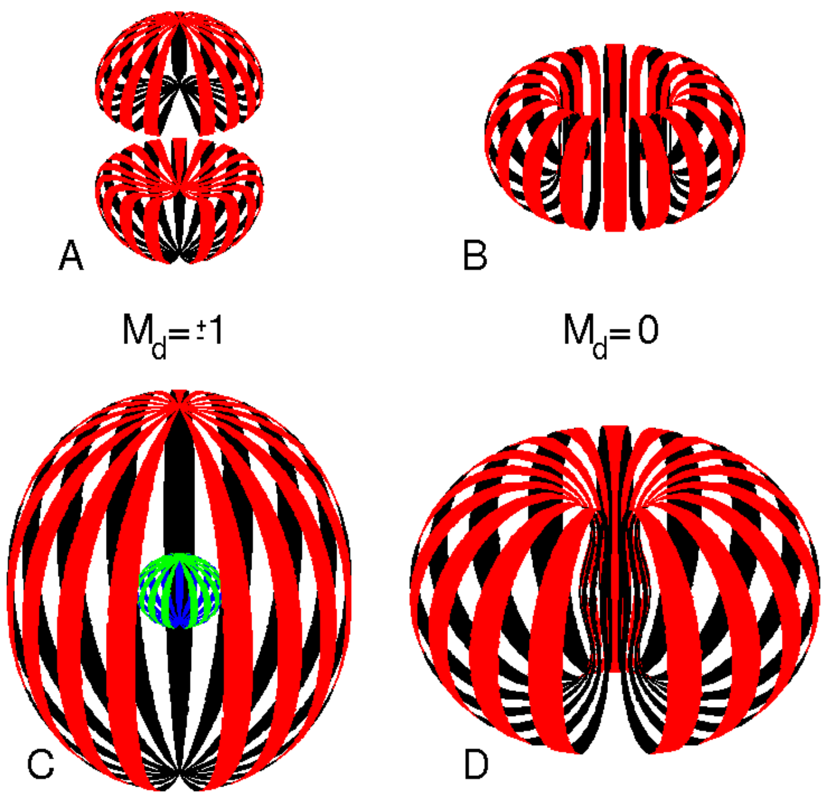
absolute radii, useful for isotope shifts from atomic physics
comparison with radii from μ atoms (Z=1,2)
reference for matter radii of unstable nuclei
    radii ± only practical observable (scattering in inv. kin.)
parallel aspects to proton radius
    presently not understood problems

Deuteron

fundamental system of nuclear physics
can be understood in terms of N+N?
rich set of observables: C0, M1, C2, M1 ΔT=1
$\pm$ best neutron target
leading NR π-exchange absent (T=0)

Structure

loosely bound
for $r > 1.5 fm$ dominated by asympt. tail
only at short range interesting structure
M=$\pm$1: dumbbell-like
M= 0: torus-like



Form factors

cross section complicated due to I=1 nature

3 independent form factors contribute

multipolarities C0, M1, C2

Equations in IA

$$\frac{d\sigma(E,\theta)}{d\Omega}^{PWIA} = \sigma_{Mott}(E,\theta)[A(q) + B(q)tan(\theta/2)^2]$$

$$A(q) = F_{C0}(q)^2 + (M_d^2 Q_d)^2 \frac{8}{9}\eta^2 F_{C2}^2(q) + (\frac{M_d}{M_p}\mu_d)^2 \frac{2}{3}\eta(1+\eta) F_{M1}^2(q)$$

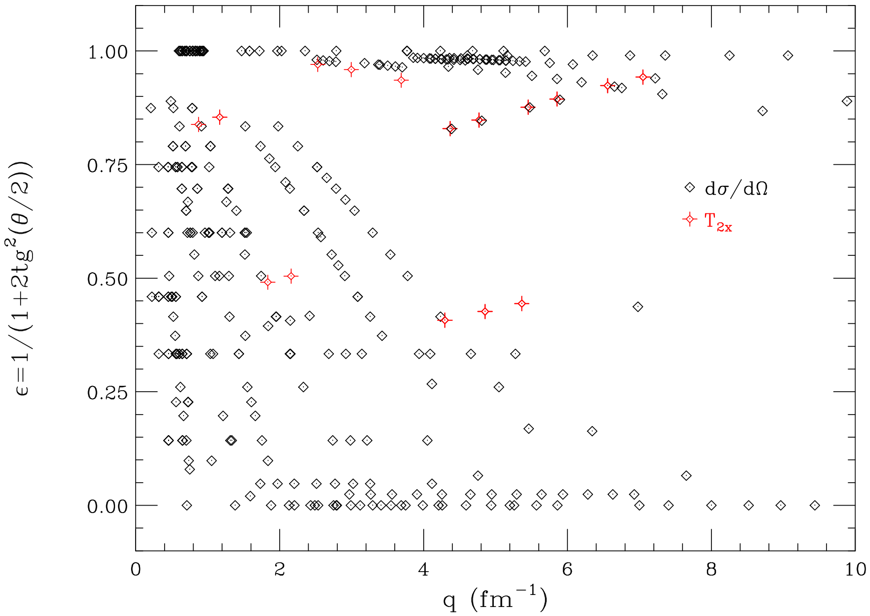
$$B(q) = (\frac{M_d}{M_p}\mu_d)^2 \frac{4}{3}\eta(1+\eta)^2 F_{M1}^2(q) \qquad \eta = q^2/(4M_d^2)$$

$$t_{20}F^2 = \frac{-1}{\sqrt{2}}(\frac{8}{3}\eta F_{C0}F_{C2} + \frac{8}{9}\eta^2 F_{C2}^2 + \frac{1}{3}\eta[1 + 2(1+\eta)tg^2\frac{\theta}{2}]F_{M1}^2)$$

To separate C0, C2 need polarization observables

Available data

Cross sections: many experiments, large q, θ-range, very different accuracies
some 512 data points



Experimental form factors (much more sensitive than σ's)

Usual determination

experiments measure some σ, t_{2x}
observable dominated by one of the form factors F_i
use other F_j's from some other data to extract F_i in PWIA
publish F_i

Non-optimal

- involves inter/extrapolation of F's
- does not use all info on F_i, F_j today available
- ignores Coulomb distortion

Optimal determination

- use *all* primary data σ, $t_{2x}, ...$
- parameterize 3 F's using flexible parameterization
- apply Coulomb corrections
- fit simultaneously to *all* data

Get

- L/T-separation during fit
- C0/C2-separation during fit
- statistical errors (error matrix)
- systematic errors (conservative estimation)
  - change every data set by error
  - refit
  - add quadratically changes
- total error: quadratic sum

Same procedure as in N-N scattering

use cross section data in (energy-dependent) phase-shift analysis
then discuss only phase shifts
see e.g. Stoks *et al.*, PRC48(93)792

Main difference

do not "prune" the data set
- in N-N ∼30% of data eliminated to get $\chi^2 \sim 1$

do not float normalization
- largest effort of experimentalists has gone into normalization
- take seriously

do not use theoretical NN potential as in phase-shift analysis
- no bias from parametrization (energy dependence)

Result

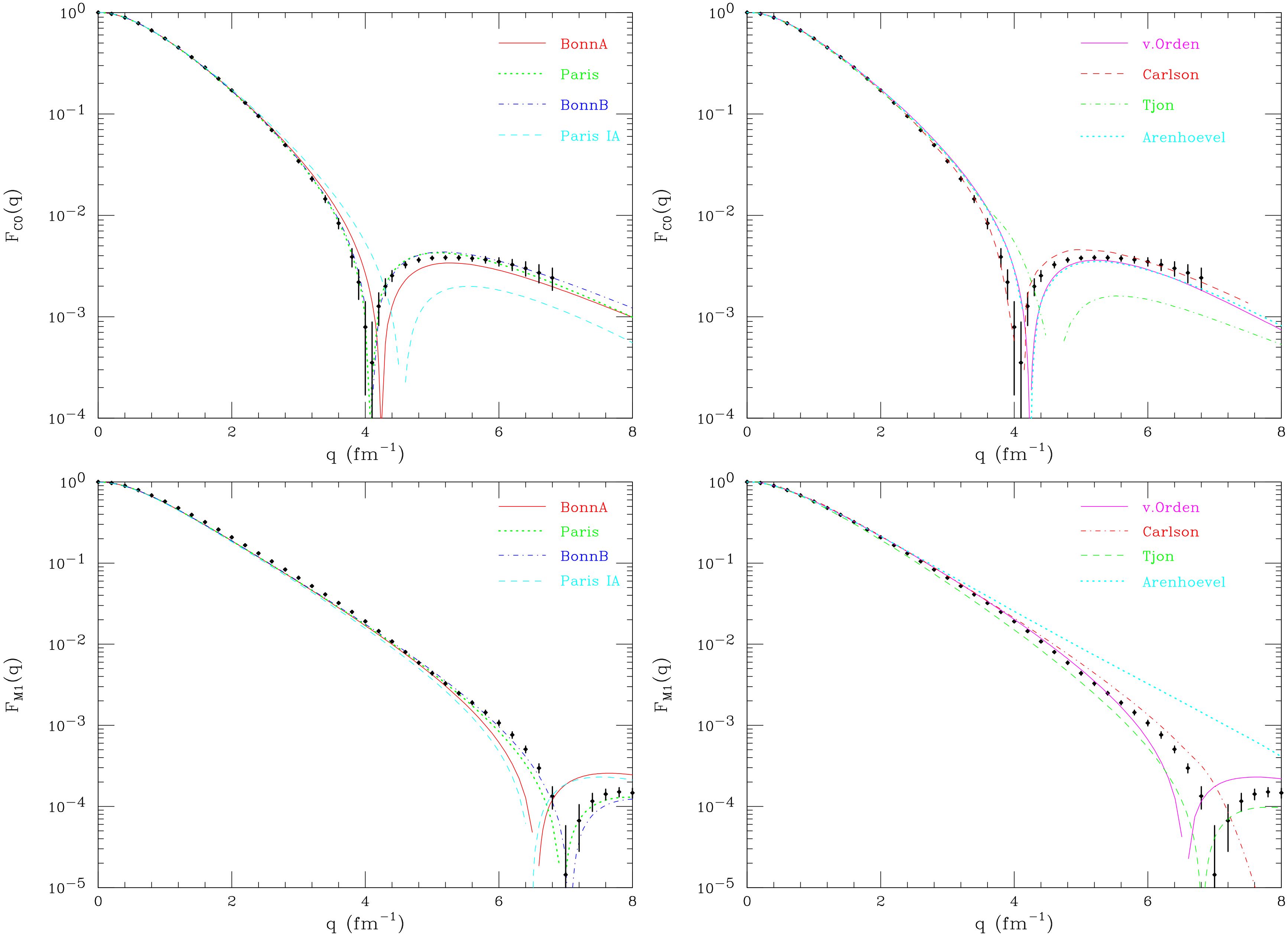
form factors with reliable error bars

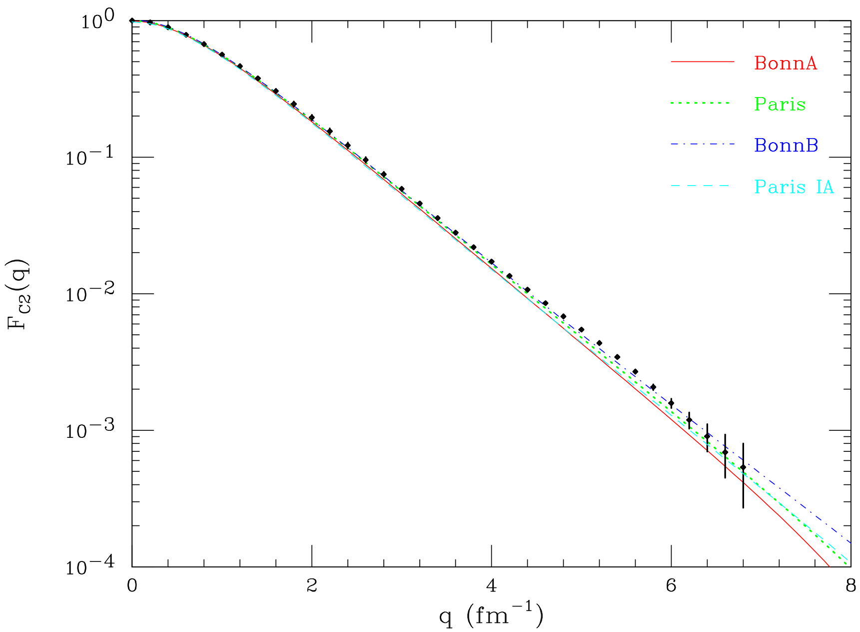
Note

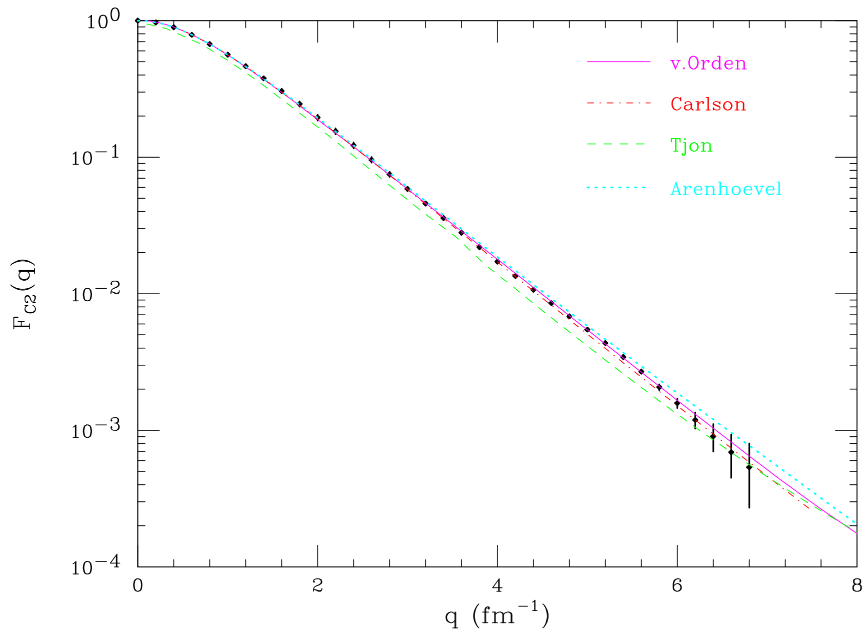
resulting $F(q)$'s correlated over interval $\Delta q \sim 1/R_{max} \sim 0.25 fm^{-1}$
(R_{max} = max. radius allowed for in r-space parametrization)

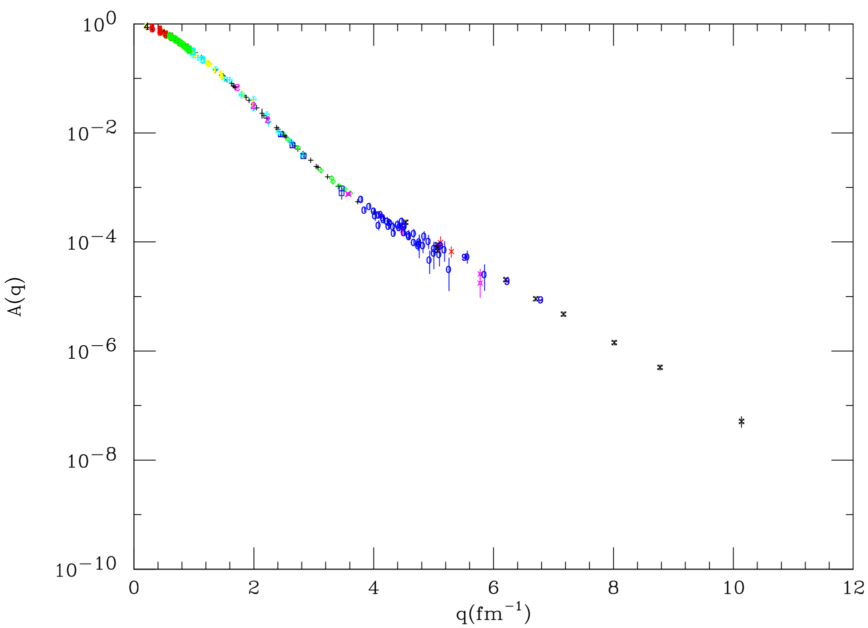
uncertainty given by δF, *not* by scatter of points

Results for deuteron









Find

- good agreement with theory
- substantial effect of MEC in C0, M1
- C2 not sensitive (short-range suppressed)
- C0 much more sensitive than A(q)
  - (A(q) = sum of 2 terms)

Moments of interest

rms-radius R

R^2 defined as $\int r^4 \rho(r) 4\pi dr$

obtainable from q=0 slope of G_e: $G_e(q) = 1 - q^2R^2/6 + q^4\langle r^4\rangle/120 + ...$

Third Zemach moment

needed to get rms-radius from μ atoms data

$$\langle r^3\rangle_{(2)} = \int d^3r\ r^3\rho_{(2)}(r) \qquad with \qquad \rho_{(2)}(r) = \int d^3z\ \rho_{ch}(|\mathrm{z}-\mathrm{r}|)\ \rho_{ch}(z)$$

measurable in (e,e) via

$$\langle r^3\rangle_{(2)} = \frac{48}{\pi}\int_0^\infty \frac{dq}{q^4}\ (G_e^2(q) - 1 + q^2R^2/3)$$

First Zemach moment, needed to calculate HFS in atoms

$$\langle r\rangle_{(2)} = \int d^3r\ r\ \int d^3r'\ \rho_{ch}(|\mathrm{r}-\mathrm{r}'|)\ \rho_{mag}(r')$$

measurable in (e,e) via

$$\langle r\rangle_{(2)} = -\frac{4}{\pi}\int_0^\infty \frac{dq}{q^2}(G_e(q)\ G_m(q) - 1),$$

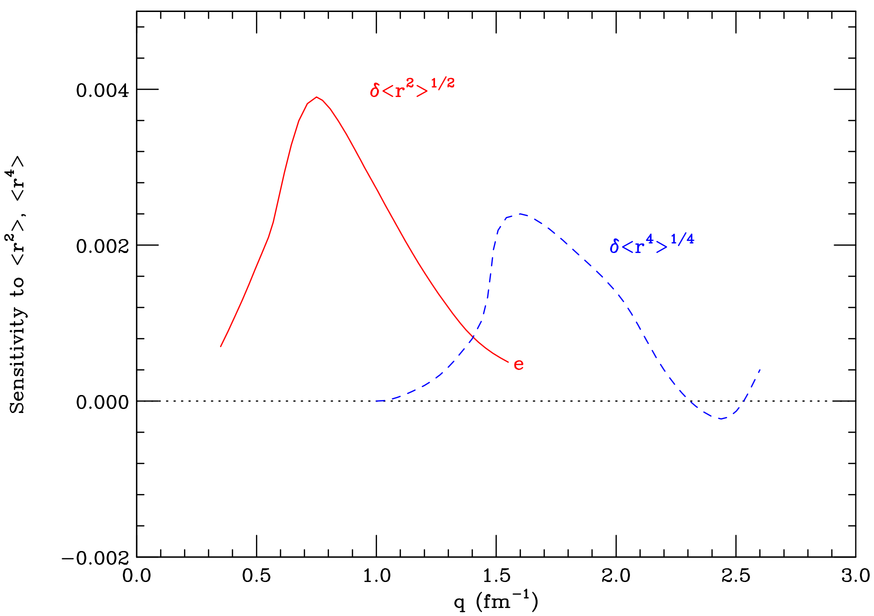
Important consideration

in which q-region are data sensitive to given moment?
diffuse answer: "small q"
when is q "small" enough? And not too small?

Quantitative answer: ±never studied

Notch test: change data in narrow interval around q_0 by 1%
refit, determine change of moment
plot this change as function of q_0

Example: for proton



Note: data above q∼ $1.1 fm^{-1}$ not useful for R-determination
Note: peak occurs at lower q if R is bigger

Deuteron rms-radius: from slope of A(q) at q=0?

Has been problem for long time

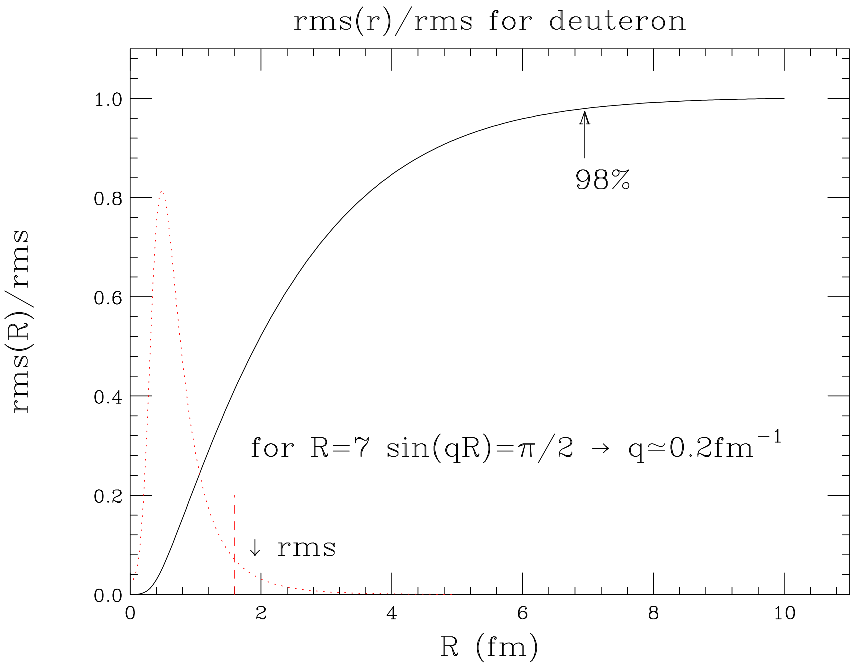
- large scatter of results
- disagreement with radius derived from n-p scattering length (Klarsfeld et al)

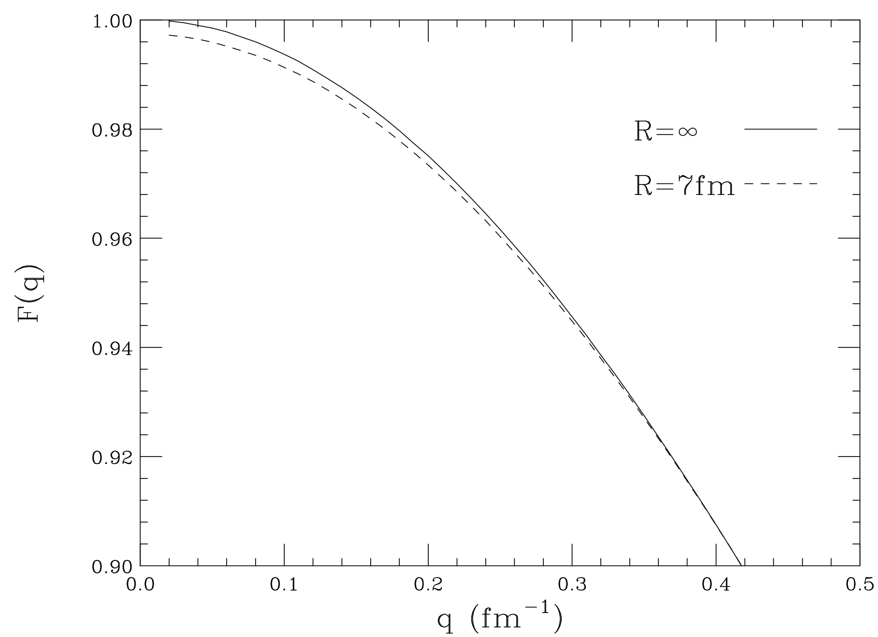
Part of problem: analysis of data in PWIA

Main difficulty: extremely long tail of $\rho(r)$

- leads to structure of A(q) at *very* low q
- complicates (implicit) extrapolation to q=0

Demonstration: study $[\int_0^{R_{max}} \rho(r) r^4\, dr]^{1/2}$ as function of $R_{max} \to \infty$





Consequence

hopeless to get radius of %-type accuracy

data at $0.5 < q < 1.$ have largest sensitivity to rms-radius

extrapolation to $q = 0$ dependent on model for A(q)

strongly dependent on tail of corresponding density (=FT[A(q)])

Analogous to situation for proton, see PRC 89(14)012201

for 98% of proton rms radius need ρ(r) out to r=3fm!

effect of remaining 2% (at $r > 3fm$) on G(q) not measurable

with q-space fit ($q_{max} = 2fm^{-1}$) can get rms-radii up to 1.5fm

Solution for both deuteron *and* proton

get away from q-space parametrization

extrapolation to q=0 too ambiguous

use r-space parametrization

with large-r tail constrained by physics

helpful: fit of data up to maximal q

such that also data constrain tail as much as possible

Shape of tail of deuteron density

well known

entirely given by BE=2.2MeV for r>1.6fm

Fit of data: see below

Floating vs. absolute σ

main purpose of floating: low χ^2

$\rightarrow$ taken as sign of "successful" data fitting

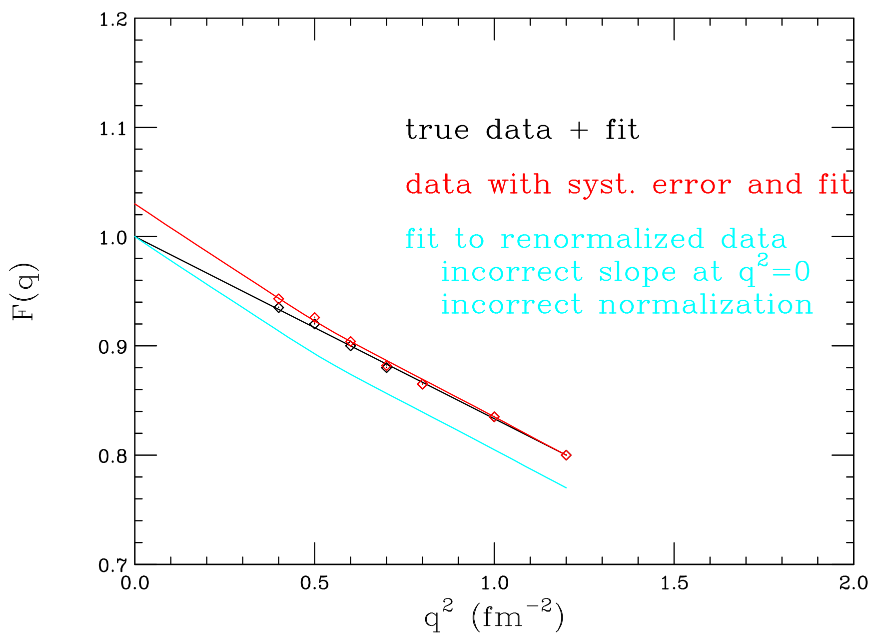
Danger of floating

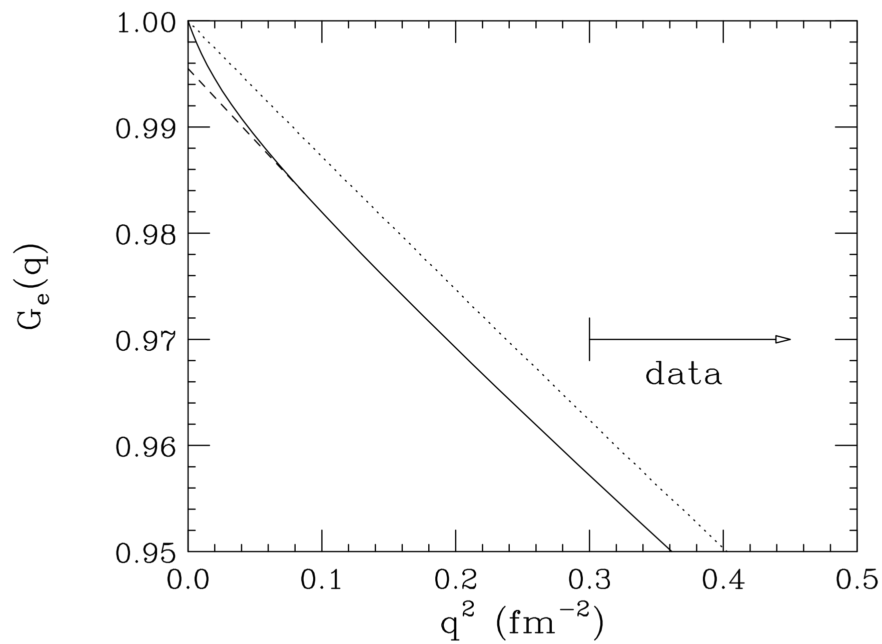
systematic errors increase toward edge of data set

particularly dangerous for low-q edge

consequence of sys.err. enhanced by extrapolation to q=0

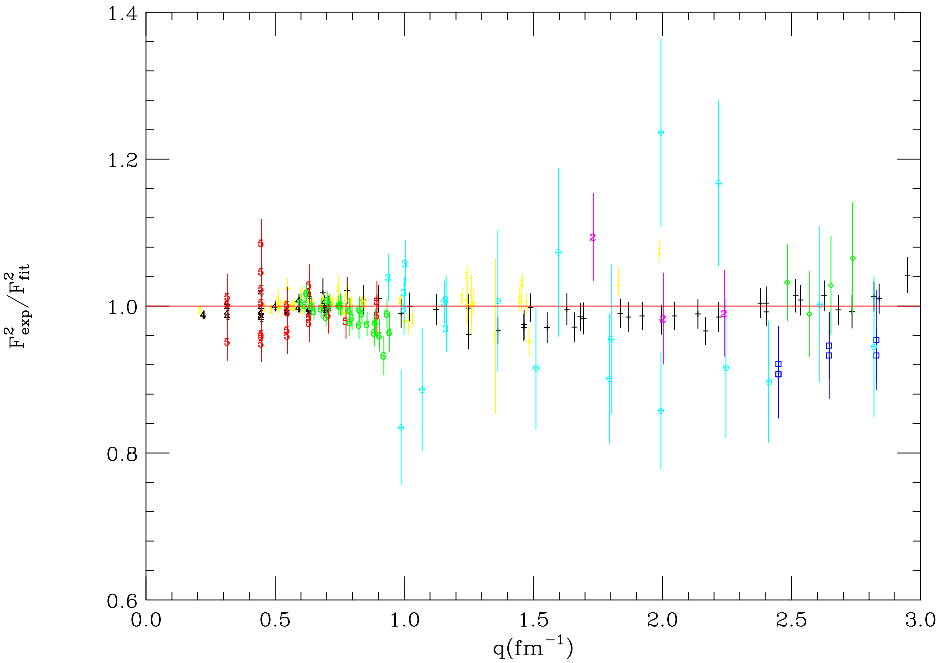
this extrapolation determines overall normalization





Better: do not float, accept poorer χ^2. Much safer!

Results for deuteron radius



electron scattering	**2.130 ±0.010 fm**
muonic ^{2}H (prelim!)	**2.1289±0.0012 fm**
n-p scattering length	**2.131 fm**

good agreement
much better than for proton

Tritium

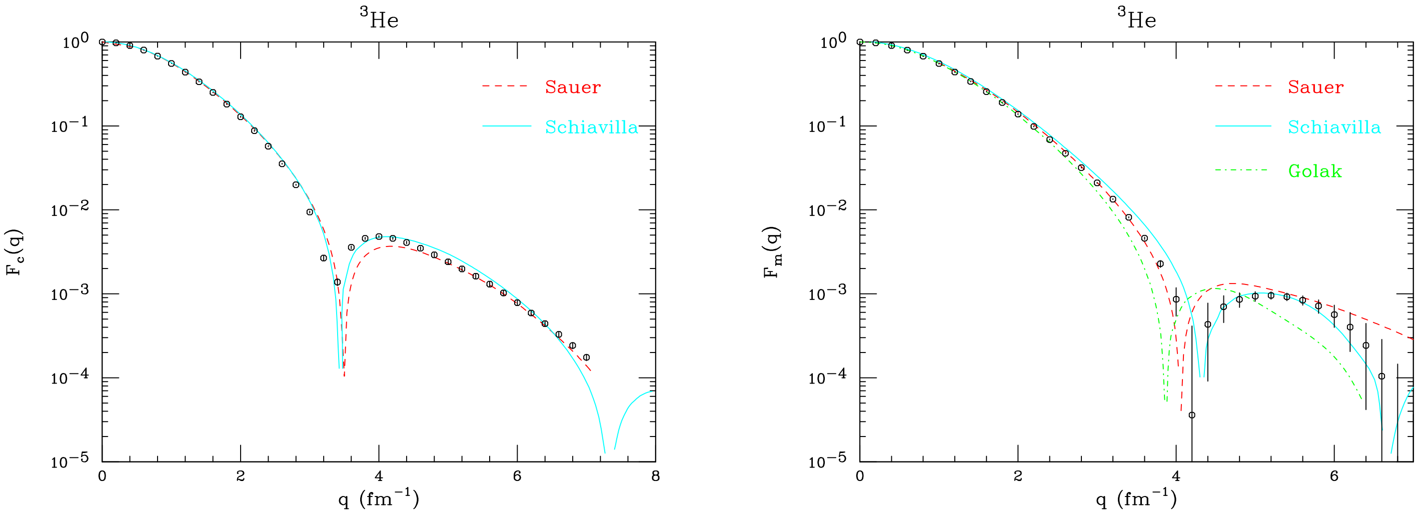
(e,e) data more limited than for ^{3}He
comparison to theory for ^3He more instructive
isotope shifts for ^{3}H poorly known; requires more attention!

Helium isotopes: interest

form factors
comparison to theory
isotope shifts (charge, matter)
comparison electron scattering – muonic ^{4}He

^{3}He form factors

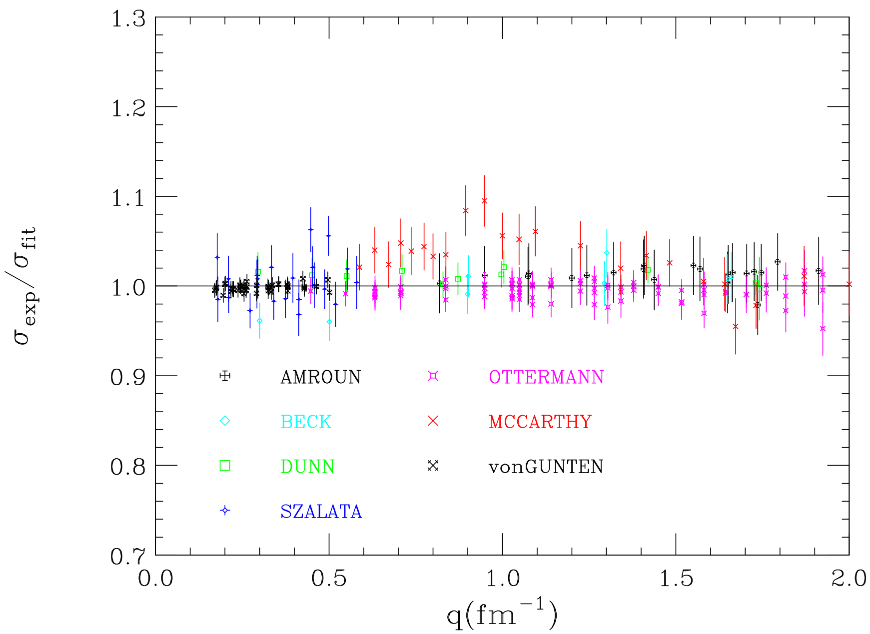
fairly compete set of σ measured, $\sim$ 275 data points
F_{ch} and F_m determined from global fit



quite good agreement theory-experiment
also for C0, despite importance of MEC

^{3}He rms-radius and Zemach moments

Zemach moments needed for analysis of muonic ^{3}He (CREMA)
recently determined (PRC90(14)064002)
fit of *world* data
including constraint on tail-shape

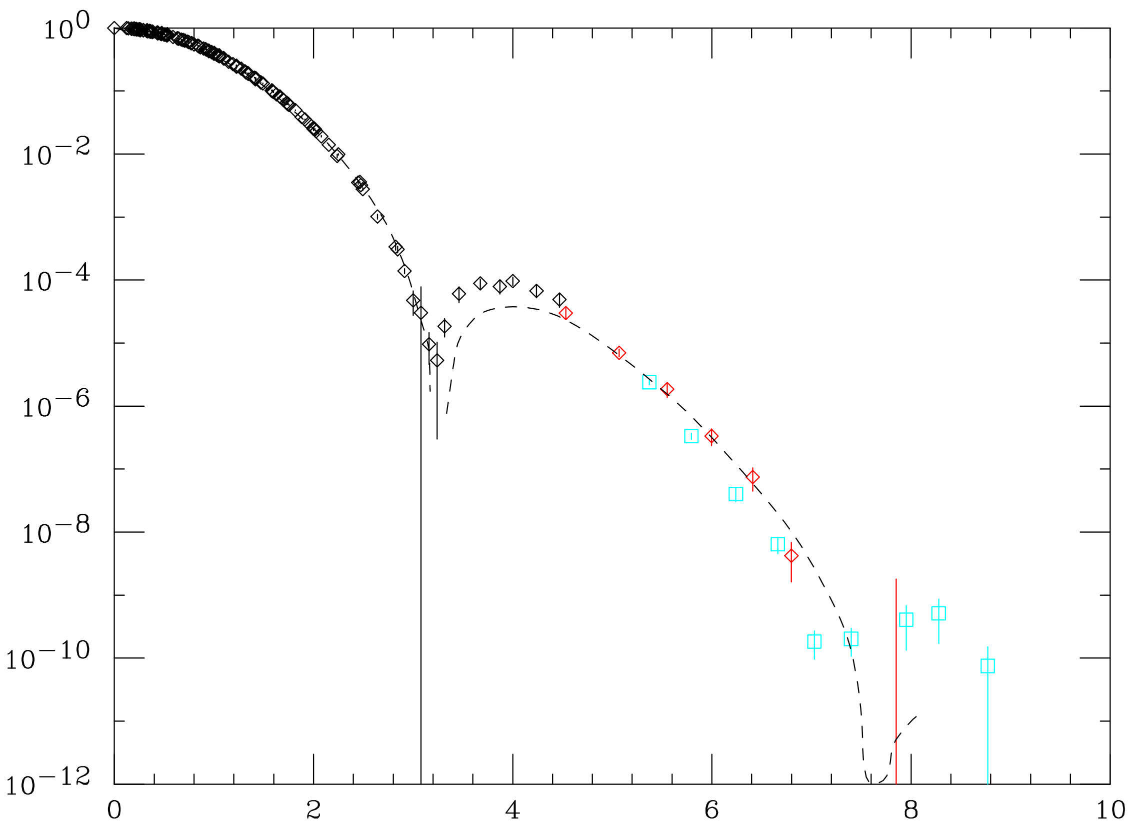


$\langle r \rangle_{(2)}$	$2.528 \pm 0.016 fm$
$\langle r^3 \rangle_{(2)}$	$28.15 \pm 0.70 fm^3$
$\langle r^2_{ch} \rangle^{1/2}$	$1.973 \pm 0.014 fm$
$\langle r^2_{m} \rangle^{1/2}$	$1.976 \pm 0.047 fm$
$\langle r^4_{ch} \rangle$	$32.9 \pm 1.60 fm^4$

for Gauss (Exp) 26.68 (29.10) fm^3

^{4}He

fairly complete data set, up to 8 fm^{-1}
recent high-q data from JLab
confirm 2. minimum
disagree somewhat with previous data
total of 192 data points
decent agreement with VMC V14 Schiavilla *et al.*



Moments

for rms-radius see below

Zemach moment and $\langle r^4 \rangle \to$ atomic data

Recent determination

analysis of world data

with constraint on tail of ρ (as for d)

PRC 90(14)064002

Interesting question: which q-region important?

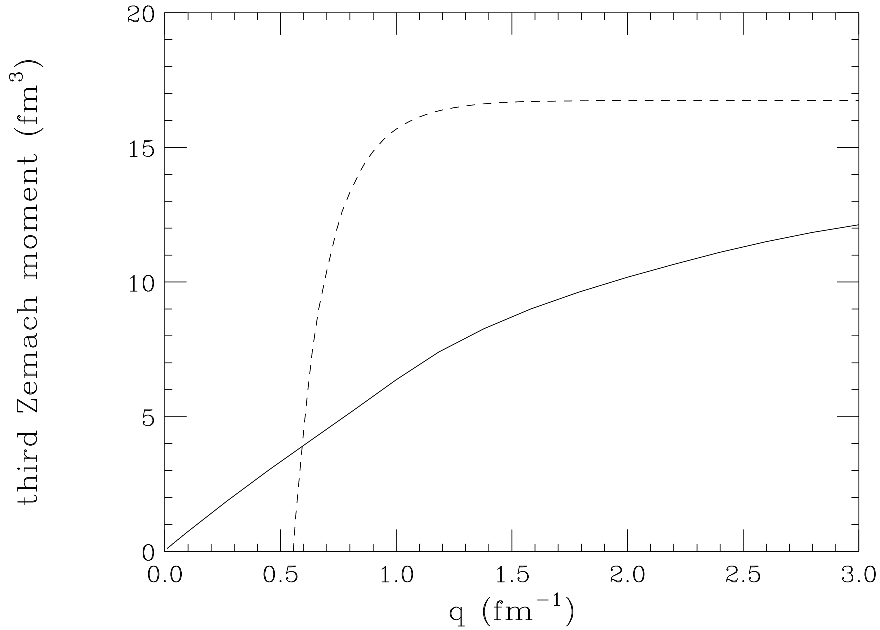
$$\langle r^3 \rangle_{(2)} = \frac{48}{\pi} \int_0^\infty \frac{dq}{q^4} \; (G_e^2(q) - 1 + q^2 R^2/3)$$

dominated by extremely low q?

Sensitivity similar to the one of R

$\langle r^3 \rangle_{(2)}$ differs significantly from standard values for gaussian/exponential ρ's

$\langle r^3 \rangle_{(2)}$	$16.73 \pm 0.10 fm^3$	16.50(17.99)
$\langle r^2 \rangle^{1/2}$	$1.681 \pm 0.004 fm$	
$\langle r^4 \rangle$	$14.35 \pm 0.11 fm^4$	

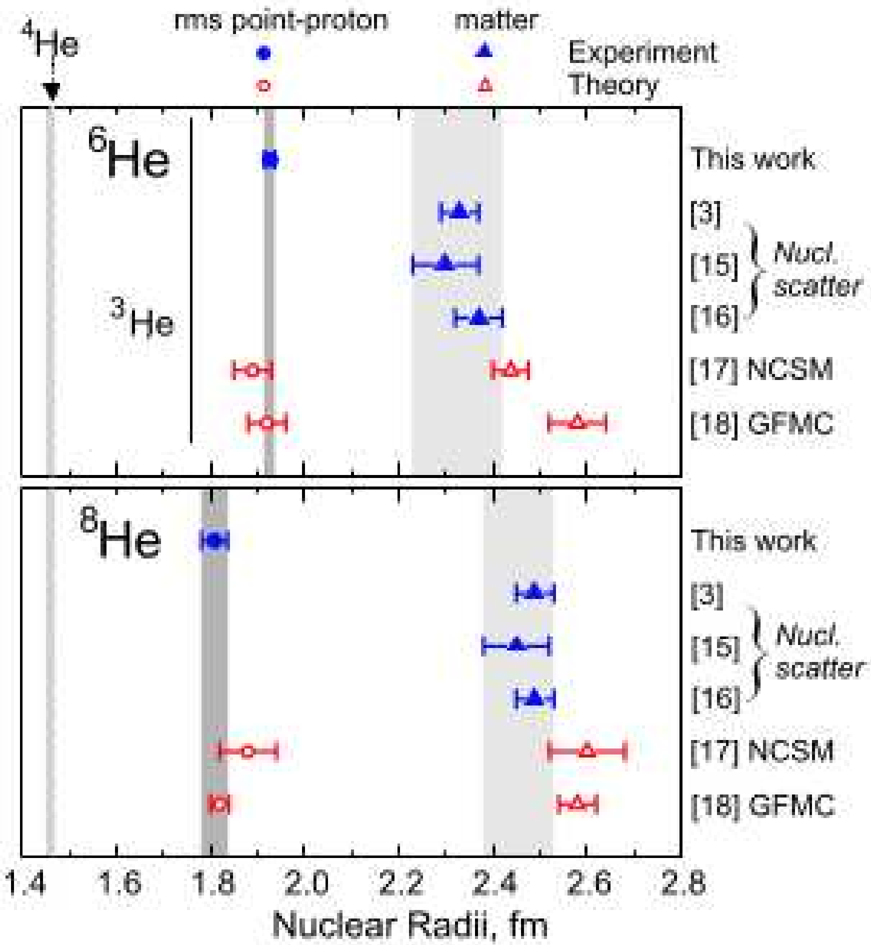


Isotope shifts

3-4 measured via (e,e)

4-6-8 measured via atomic transitions at ANL (PRL 99(2007)252501)

matter radii from scattering in inverse kinematics at GSI (EPJ A15(02)27)

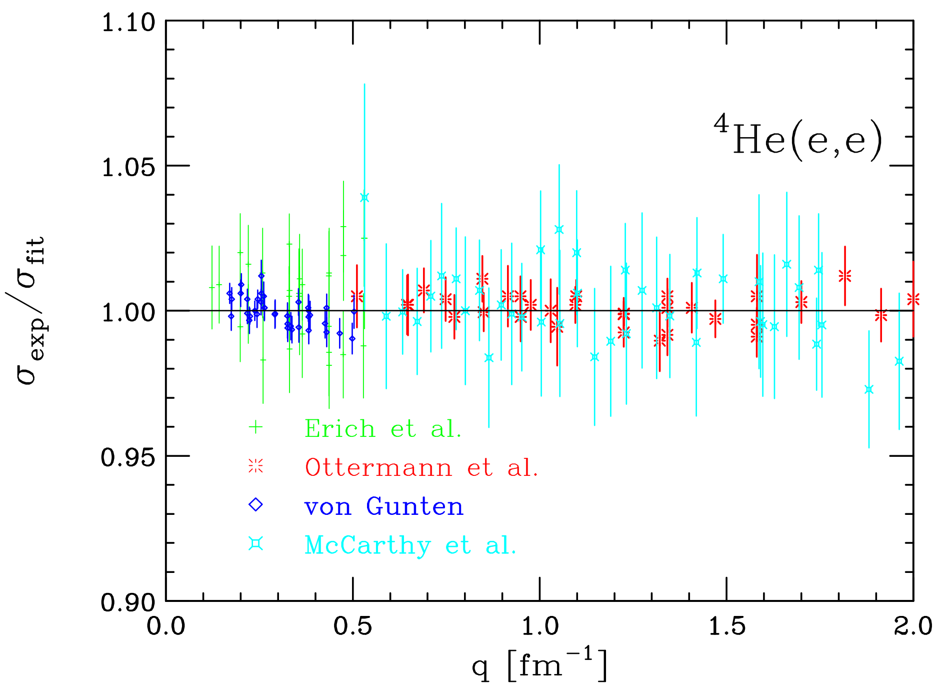


3-4 shift from (e,e) agrees with shifts from atomic helium (1.066,1.028,1.074fm^2)

cannot resolve discrepancies

RMS-radius: (e,e) $\Leftrightarrow$ μX

for ^{4}He low-q data base excellent, data with small syst. errors
not only *shape* of large-r tail known
absolute value of density in tail also known
  world data on p-^{4}He scattering + Forward Dispersion Relation analysis
  yields residuum of closest singularity
  this gives absolute normalization of tail density
tail steeply falling as SE$\sim$19.8MeV
together with (e,e) produces more accurate value for rms-radius



rms-radius = $1.681 \pm 0.004 fm$, smallest relative error of *all* nuclei
  relevant with regards to proton radius puzzle

Proton (charge) rms-radius

from electron scattering (world data w/o Bernauer)

radius = 0.887±0.008 fm

from muonic hydrogen (Pohl *et al.*)

radius = 0.8409±0.0004 fm

from electronic hydrogen 1S–nD

radius = 0.8779±0.0094 fm (Beyer *et al.*)

Unsolved problem, many speculations!

One idea: e- and μ "electromagnetic" interaction different

MUSE experiment at PSI

study of e^+ scattering at DESY, JLab, ...

e $\Longleftrightarrow$ μ for helium

relative error of ^{4}He radius from (e,e) 4 times smaller than for proton

find agreement between (e,e) and μX: 1.681±0.004 $\Longleftrightarrow$ 1.679±0.001fm!

(value from μX still preliminary; A. Antognini, CREMA collaboration)

good agreement only deepens puzzle

Lithium

^{6}Li and ^{7}Li accessible to electron scattering
shifts of A=8, 9, 11 measured by laser spectroscopy (Nörtershäuser et al.)
matter radii for A=6, 8, 9, 11 from proton scattering (Dobrovolsky *et al.*)
  in inverse kinematics
pronounced A-dependent shape changes (clustering)
  interesting comparison to ab-initio calculations

Electron scattering

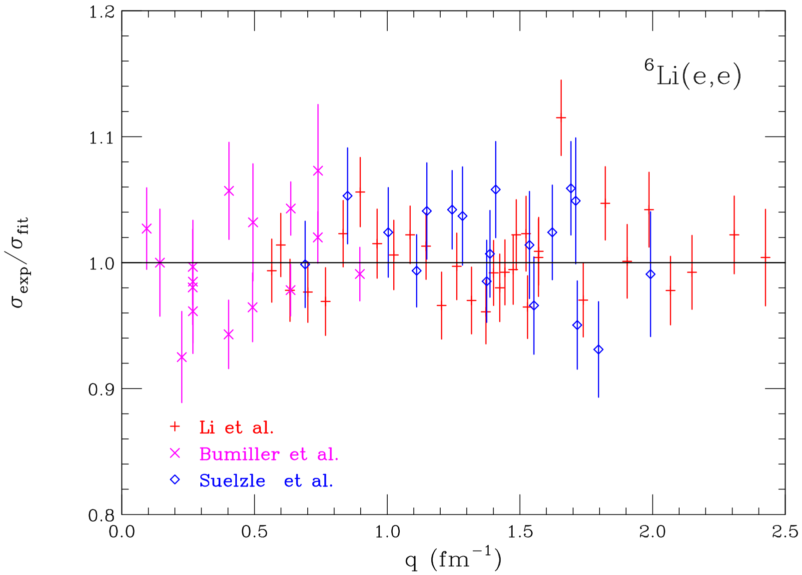
^{7}Li in past standard reference for rms-radius
not a good idea
  data for ^{6}Li more extensive (86 σ), more precise
  ^{7}Li experiments did not resolve 1. excited state
  quadrupole contribution in ^{7} Li much more important, cannot be separated

Analysis of world data for ^{6}Li (PRC84(11)024307)

use tail constraint as well
complication: p-tail or d-tail? (cluster structure of ^{6}Li)
  SE_p=4.6MeV, SE_d=1.5MeV
as GFMC calculation (Pieper *et al.*) gives correct BE: use GFMC

Result

charge rms-radius = 2.589±0.039fm
comparatively large uncertainty due to poor low-q data



Theoretical understanding

GFMC calculation

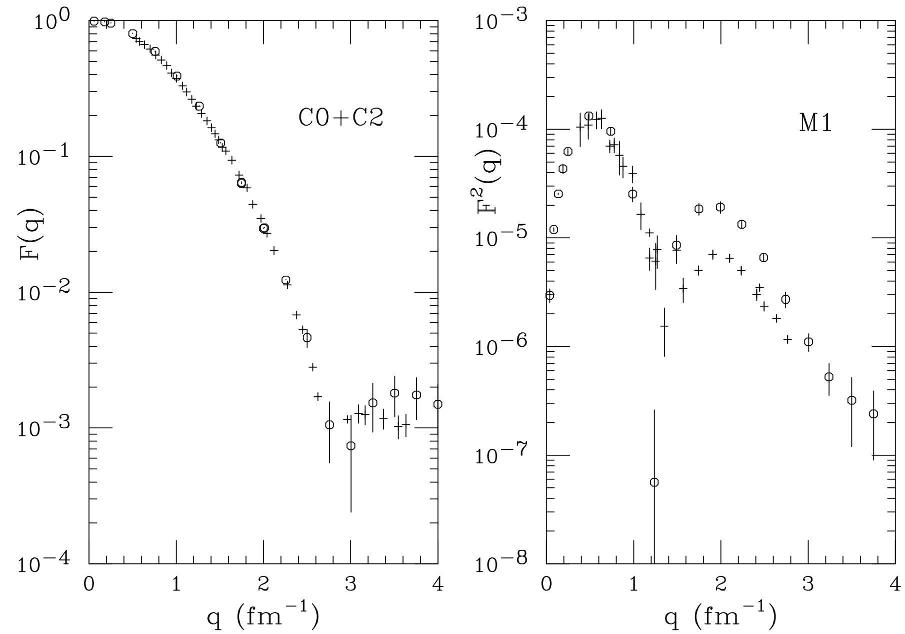
V18+Urbana 3BF

Wiringa *et al.*

MEC included, for C0 small

C0 well understood

M1 problematic



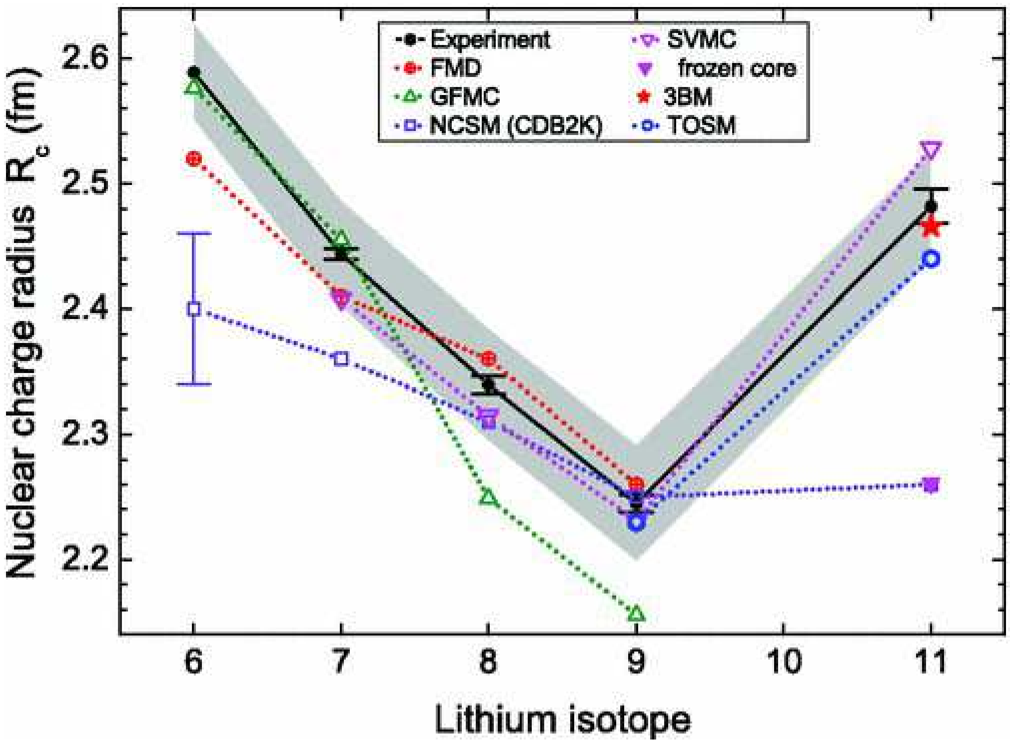
Isotope shifts

measured by laser spectroscopy with stored ions

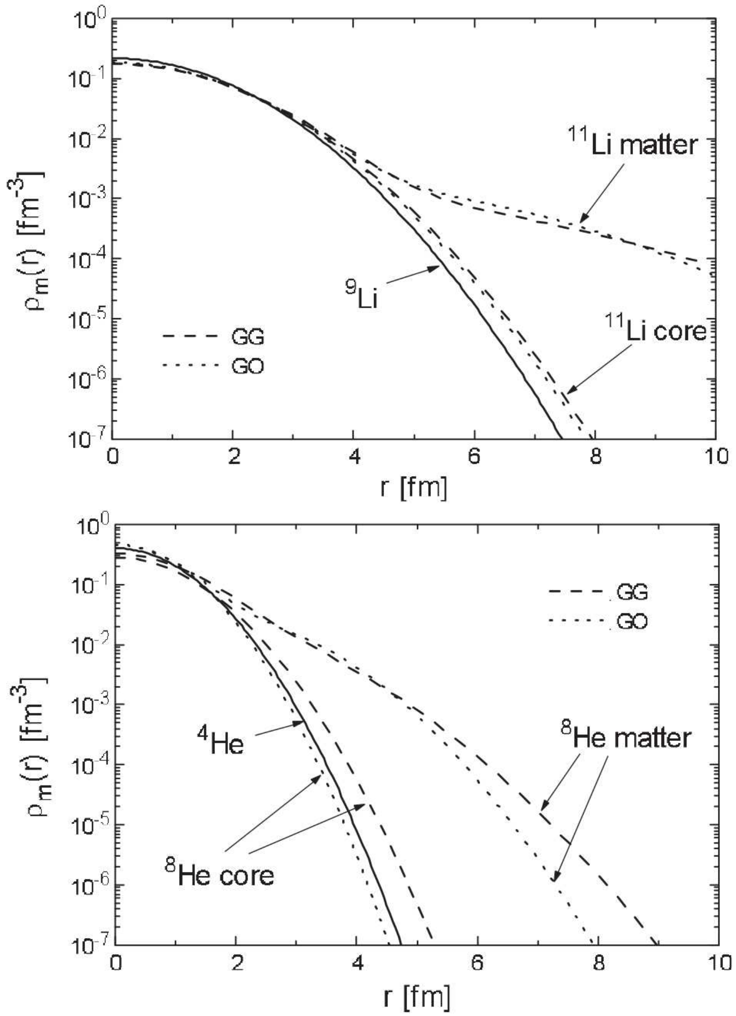
Nörtershäuser *et al.*

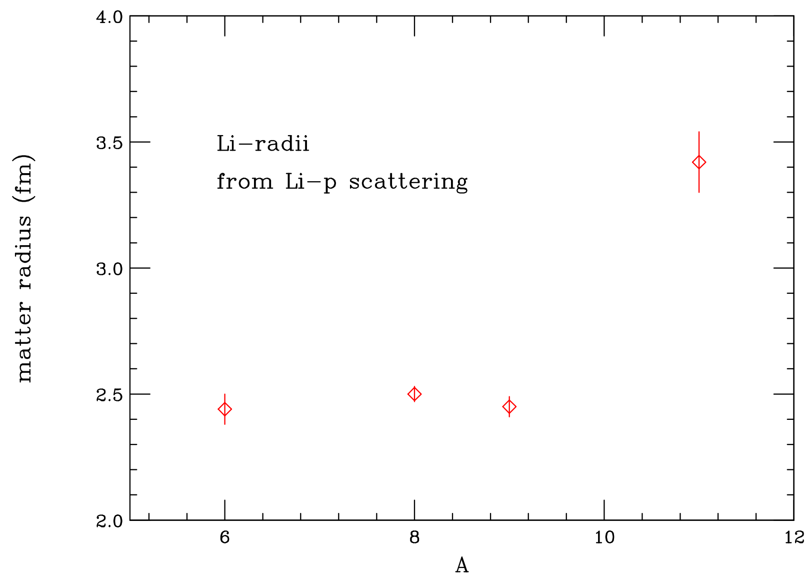
^{11}Li = Borromean nucleus (2n, ^{10}Li unbound)

2n-separation energy only 369KeV



Extreme case of tail-importance: matter radii





Not emphasized: magnetic form factors + radii

- data in general not as good
- understanding more involved (MEC)
- rms-radii even more difficult to measure
  - at low q $\boldsymbol{\sigma}$ is dominated by F_{ch}
  - polarization transfer useful only for p
  - best results from (old) 180° facilities
- small contribution from $\boldsymbol{F_m}$ enhances effect of systematic errors
  - example: proton
- information from HFS limited

Heavier p-shell nuclei

- complication: spin=3/2 (^{9}Be, ^{11}B), =3 (^{10}B)
- little accurate data available
- could do accurate experiment on ^{9}Be
  - despite loss of knowhow
- as accuracy for ^{12}C excellent, could do Be/C ratio measurement
  - produce precise reference radius for isotope shift data

for review: see I.Sick, Prog. Part. Nucl. Phys. 47 (01) 245