# *g* factor of highly charged ions

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# Outline

# Free electron $g$ factor

$$g_{\text{free}} = 2\left(1 + \frac{\alpha}{\pi} A^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A^{(6)} + \cdots + \Delta\right)$$

| | $A^{(2i)}$ | |
|---|---|---|
| | | 2.000 000 000 000 |
| $\alpha/\pi$ | 0.5 | 0.002 322 819 466 |
| $(\alpha/\pi)^2$ | $-0.328\ldots$ | $-0.000\,003\,544\,610$ |
| $(\alpha/\pi)^3$ | $1.181\ldots$ | 0.000 000 029 608 |
| $(\alpha/\pi)^4$ | $-1.911(2)$ | $-0.000\,000\,000\,111$ |
| $(\alpha/\pi)^5$ | 9.16(58) | 0.000 000 000 001 |
| $\Delta$ | | 0.000 000 000 002 |
| $g_{\text{free}}$ | | 2.002 319 304 361 |

T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, PRL, 2012.

# Free electron $g$ factor

$$g_{\rm free} = 2\left(1 + \frac{\alpha}{\pi}A^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A^{(6)} + \cdots + \Delta\right)$$

Experiment: $g_{\rm free} = 2.002\,319\,304\,361\,46(56)$

D. Hanneke, S. Fogwell, and G. Gabrielse, PRL, 2008.

Theory: $A^{(i)}, \Delta$

T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, PRL (2012)

$$\rightarrow \alpha = 1/137.035\,999\,173\,(35)$$

$$\text{vs } \alpha = 1/137.035\,999\,044\,(90)$$

R. Bouchendira, P. Cladé, S. Guellati-Khélifa, F. Nez, and F. Biraben, PRL, 2011.

# Highly charged ions

Nucleus + few electrons: $N_e \ll Z$

Nuclear potential: $V_{\rm nuc}(r) = -\dfrac{\alpha Z}{r}$

Electron velocity: $v/c \sim \alpha Z$

Low-$Z$ systems: $\alpha Z \ll 1 \to \alpha Z$-expansion

High-$Z$ systems: $\alpha Z \sim 1 \to$ all orders in $\alpha Z$

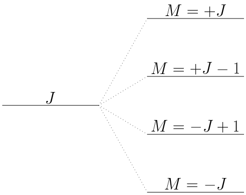
# Highly charged ions

Simple systems: (closed shell(s)) + 1 electron

- H-like: $1s$
- Li-like: $(1s)^2\,2s$
- B-like: $(1s)^2\,(2s)^2\,2p$

Zeeman splitting:

$$\Delta E_{\rm mag} = \langle -\boldsymbol{\mu}\mathbf{B}\rangle = g\,M\,\mu_0\,B$$



# Experiments: past and future

| Z | H-like: 1$s$ | Li-like: $(1s)^2 2s$ | B-like: $(1s)^2(2s)^2 2p$ |
|---|---|---|---|
| 6 | 2000<br>2014 | | |
| 8 | 2004 | | |
| 14 | 2011, 2013 | 2013 | |
| 18 | | | *in progress*<br>GSI |
| 20 | *planned*<br>Uni Mainz | *in progress*<br>Uni Mainz | |
| …<br>up to 92 | MPIK, FAIR | | |

# The $g$ factor of H-like ions

Theory:

$$g = g_{\rm Dirac} + \Delta g_{\rm QED} + \Delta g_{\rm nuc.rec.} + \Delta g_{\rm nuc.size} + \Delta g_{\rm nuc.pol.}\,,$$

where for an $ns$ state

$$\begin{aligned} g_{\rm Dirac} &= 2 + \frac{4}{3}\frac{E-m}{m} \\ &= 2 - \frac{2}{3}\frac{(\alpha Z)^2}{n^2} + \left(\frac{1}{2n} - \frac{2}{3}\right)\frac{(\alpha Z)^4}{n^3} + \cdots\,, \end{aligned}$$

where $E$ is the Dirac energy,

$$\Delta g_{\rm QED} = \Delta g_{\rm free-QED} + \Delta g_{\rm binding-QED}\,.$$

Free-electron g-factor:

$$g_{\rm free} = 2 + \Delta g_{\rm free-QED}\,.$$

Binding-QED correction to the lowest order in $\alpha Z$ and to all orders in $\alpha$ for an *ns* state *(H. Grotch, 1970; R.N. Faustov, 1970; F.E. Close and H. Osborn, 1971; M.I. Eides and H. Grotch,1997; S.G. Karshenboim, 2001; A.Czarnecki et al., 2001)*:

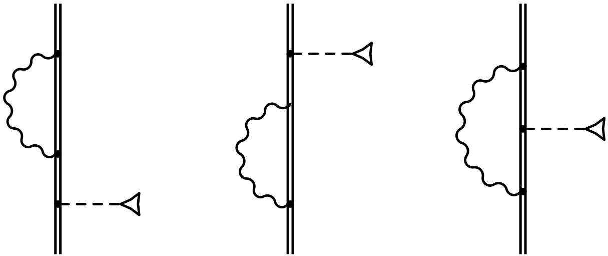
$$\Delta g_{\rm binding-QED} = \Delta g_{\rm free-QED} \frac{(\alpha Z)^2}{6n^2} \,,$$

where

$$\Delta g_{free-QED} = 2 \left( \frac{\alpha}{\pi} A^{(2)} + \left(\frac{\alpha}{\pi}\right)^2 A^{(4)} + \left(\frac{\alpha}{\pi}\right)^3 A^{(6)} + \dots \right) .$$

The corresponding correction for an $np_J$ state: *(V.A. Yerokhin and U. Jentschura, PRA, 2010.)*

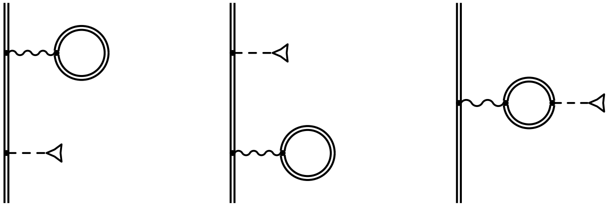
One-loop QED corrections to all orders in $\alpha Z$: self energy



First evaluations: *S.A. Blundell et al., PRA, 1997; H. Persson et al., PRA 1997.*
Most precise evaluations: *V.A. Yerokhin et al., PRL, 2002; PRA, 2004; V.A. Yerokhin and U. Jentschura, PRL, 2008, PRA, 2010.*
Evaluation for arbitrary potential: *D.A. Glazov et al., PLA, 2006.*

# The $g$ factor of H-like ions

One-loop QED corrections to all orders in $\alpha Z$: vacuum polarization



Evaluations: *H. Persson et al., PRA, 1997; T. Beier et al., PRA, 2000.*

# The $g$ factor of H-like ions

Magnetic-loop vacuum-polarization correction to the lowest order in $\alpha Z$ for an $ns$ state
*(S.G. Karshenboim and A.I. Milstein, PLB, 2002)*:

$$\Delta g_{\rm VP}^{\rm magn} = \frac{7}{216}\frac{\alpha(\alpha Z)^5}{n^3}\,.$$

# The $g$ factor of H-like ions

Two-loop QED correction to the order $\alpha^2(\alpha Z)^4$ for an $ns$ state *(K. Pachucki, A. Czarnecki, U. Jentschura, and V.A. Yerokhin, PRA, 2005)*:

$$\Delta g_{\rm two-loop} = \left(\frac{\alpha}{\pi}\right)^2 \frac{(\alpha Z)^4}{n^3}\left\{\frac{28}{9}\ln[(\alpha Z)^{-2}] + \frac{258917}{19440} - \frac{4}{9}\ln k_0 - \frac{8}{3}\ln k_3 + \frac{113}{810}\pi^2 - \frac{379}{90}\pi^2\ln 2 + \frac{379}{60}\zeta(3) + \frac{1}{n}\left[-\frac{985}{1728} - \frac{5}{144}\pi^2 + \frac{5}{24}\pi^2\ln 2 - \frac{5}{16}\zeta(3)\right]\right\},$$

where

$\ln k_0(1s) = 2.984128556,$ $\ln k_3(1s) = 3.272806545,$

$\ln k_0(2s) = 2.811769893,$ $\ln k_3(2s) = 3.546018666.$

# The $g$ factor of H-like ions

Recent progress on calculations of the QED corrections:
Two-loop QED corrections with closed fermion loops
*(U. Jentschura, PRA, 2009; V.A. Yerokhin and Z. Harman, PRA, 2013)*

Two-loop QED corrections with the closed fermion loops for the bound-electron g-factor are evaluated for the 1$s$ state. The calculations are performed within the free-fermion-loop (Uehling) approximation.

# The $g$ factor of H-like ions

Nuclear recoil corrections to the lowest orders in $(\alpha Z)^2$, $(\alpha/\pi)$, and $(m/M)^2$ for an $ns$ state
*(R.N. Faustov, PLB, 1970; H. Grotch and R.A. Hegstrom, PRA, 1971; F.E. Close and H. Osborn, PLB, 1971; M.I. Eides and H. Grotch, Ann. Phys., 1997; K. Pachucki, PRA, 2008; M.I. Eides and T.J.S Martin, PRL. 2010)*:

$$\Delta g^{\rm (LO)}_{\rm nuc.rec.} = \frac{(\alpha Z)^2}{n^2}\Big[\frac{m}{M} - (1+Z)\Big(\frac{m}{M}\Big)^2\Big] + \frac{\alpha}{\pi}\frac{(\alpha Z)^2}{n^2}\Big[-\frac{1}{3}\frac{m}{M} + \frac{3-2Z}{6}\Big(\frac{m}{M}\Big)^2\Big].$$

Formula for the nuclear recoil effect on the $g$-factor of an H-like ion to first order in $m/M$ and to all orders in $\alpha Z$ *(V.M. Shabaev, PRA, 2001)*:

$$\Delta g_{\rm nuc.rec.} = \frac{1}{\mu_0 m_a} \frac{i}{2\pi M} \int_{-\infty}^{\infty} d\omega \left[ \frac{\partial}{\partial B} \langle a | [\vec{p} - \vec{D}(\omega) + e\vec{A}_{\rm cl}] \right.$$
$$\left. \times G(\omega + E_a) [\vec{p} - \vec{D}(\omega) + e\vec{A}_{\rm cl}] | a \rangle \right]_{B=0} .$$

Here $\vec{p}$ is the momentum operator, $G(\omega)$ is the Coulomb Green function, $D_m(\omega) = -4\pi\alpha Z \alpha_l D_{lm}(\omega)$, and $D_{ik}(\omega, r)$ is the transverse part of the photon propagator in the Coulomb gauge. It is implied that all quantities are calculated in the presence of the magnetic field $B$.

# The $g$ factor of H-like ions

For the practical calculations, the nuclear recoil effect can be represented by a sum of a lower-order term and a higher-order term, $\Delta g_{\rm nuc.rec.} = \Delta g^{\rm (L)}_{\rm nuc.rec.} + \Delta g^{\rm (H)}_{\rm nuc.rec.}$, where

$$\begin{aligned}
\Delta g^{\rm (L)}_{\rm nuc.rec.} &= \frac{1}{\mu_0 m_a}\frac{1}{2M}\left[\frac{\partial}{\partial B}\langle a|\Big\{\mathbf{p}^2 - \mathbf{p}\cdot\mathbf{D}(0)\Big\}|a\rangle\right]_{B=0} \\
&\quad -\frac{1}{m_a}\frac{m}{M}\langle a|\Big([\mathbf{r}\times\mathbf{p}]_z - \frac{\alpha Z}{2r}[\mathbf{r}\times\boldsymbol{\alpha}]_z\Big)|a\rangle\,, \\
\Delta g^{\rm (H)}_{\rm nuc.rec.} &= \frac{1}{\mu_0 m_a}\frac{i}{2\pi M}\int_{-\infty}^{\infty} d\omega\,\left[\frac{\partial}{\partial B}\langle a|\Big(D^k(\omega) - \frac{[p^k,V]}{\omega + i0}\Big)\right. \\
&\quad \left.\times G(\omega+E_a)\Big(D^k(\omega) + \frac{[p^k,V]}{\omega+i0}\Big)|a\rangle\right]_{B=0}\,.
\end{aligned}$$

# The $g$ factor of H-like ions

Analytical evaluation of the low-order term yields *(V.M. Shabaev, PRA, 2001)*:

$$\Delta g^{(\mathrm{L})}_{\mathrm{nuc.rec.}} = -\frac{m}{M}\,\frac{2\kappa^2 E^2 + \kappa m E - m^2}{2m^2 j(j+1)}\,,$$

where $\kappa = (-1)^{j+l+1/2}(j+1/2)$. To the two lowest orders in $\alpha Z$, we have

$$\Delta g^{(\mathrm{L})}_{\mathrm{nuc.rec.}} = -\frac{m}{M}\,\frac{1}{j(j+1)}\left[\kappa^2 + \frac{\kappa}{2} - \frac{1}{2} - \left(\kappa^2 + \frac{\kappa}{4}\right)\frac{(\alpha Z)^2}{n^2}\right].$$

# The $g$ factor of H-like ions

For the 1$s$ state, the exact formula for the low-order term takes the form:

$$\Delta g^{(\mathrm{L})}_{\mathrm{nuc.rec.}} = \frac{m}{M}(\alpha Z)^2 - \frac{m}{M}\frac{(\alpha Z)^4}{3[1+\sqrt{1-(\alpha Z)^2}]^2}\,.$$

The higher-order term can be represented as

$$\Delta g^{(\mathrm{H})}_{\mathrm{nuc.rec.}} = \frac{m}{M}\frac{(\alpha Z)^5}{n^3}P(\alpha Z)\,.$$

The numerical evaluation of the function $P(\alpha Z)$ for the 1$s$ state: *(V.M. Shabaev and V.A. Yerokhin, PRL, 2002).*

Finite nuclear size correction for an $ns$ state to the lowest order in $\alpha Z$ *(S.G. Karshenboim, PLA, 2000)*:

$$\Delta g_{\rm nuc.size} = \frac{8}{3n^3}(\alpha Z)^4 m^2 \langle r^2 \rangle_{\rm nuc} \,.$$

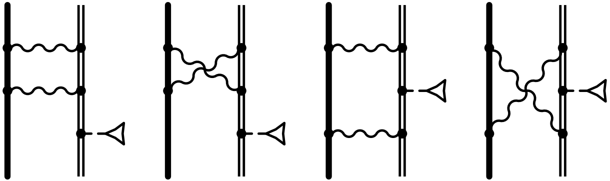
To two lowest orders in $\alpha Z$ *(D.A. Glazov and V.M. Shabaev, PLA, 2002)*:

$$\begin{aligned}\Delta g_{\rm nuc.size} &= \frac{8}{3n^3}(\alpha Z)^4 m^2 \langle r^2 \rangle_{\rm nuc} \Biggl[1 + (\alpha Z)^2 \biggl(\frac{1}{4} + \frac{12n^2 - n - 9}{4n^2(n+1)} \\ &\quad + 2\psi(3) - \psi(2+n) - \frac{\langle r^2 \ln(2\alpha Z m r/n)\rangle_{\rm nuc}}{\langle r^2 \rangle_{\rm nuc}}\biggr)\Biggr] \,.\end{aligned}$$

where $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$.

# The *g* factor of H-like ions

Nuclear-polarization corrections to the bound-electron *g* factor



Evaluations: *A.V. Nefiodov et al., PLB, 2003; A.V. Volotka and G. Plunien, PRL, 2014.*

Nuclear magnetic susceptibility correction to the bound-electron *g* factor: *U.D. Jentschura, A. Czarnecki, K. Pachucki, and V.A. Yerokhin, IJMS, 2006.*

# The $g$ factor of H-like ions

First high-precision measurement of the g-factor of $^{12}\mathrm{C}^{5+}$ using a single ion confined in a Penning ion trap *(H. Häffner et al., PRL, 2000)*:

$$g_{\mathrm{exp}} = 2(\omega_L/\omega_c)(m_e/M)(q/|e|) = 2.001\,041\,596\,3\,(10)(44)\,.$$

Here $\omega_c = (q/M)B$ is the cyclotron frequency, $\omega_L = \Delta E/\hbar$ is the Larmor precession frequency, $M$ is the ion mass, and $q$ is the ion charge. The second uncertainty (44) was due to the uncertainty of the $(m_e/M)$ ratio.

This experiment strongly stimulated high-precision calculations of the nuclear recoil and QED corrections.

# The $g$ factor of H-like ions

g factor of $^{12}\mathrm{C}^{5+}$

| | |
|---|---|
| Dirac value (point nucleus) | 1.998 721 354 39 |
| Free QED | 0.002 319 304 36 |
| Binding QED | 0.000 000 843 40(3) |
| Nuclear recoil | 0.000 000 087 62 |
| Nuclear size | 0.000 000 000 41 |
| Total theory | 2.001 041 590 17(3) |
| Experiment [1] | 2.001 041 596 3(10)(44) |

[1] *H. Häffner et al., PRL, 2000.*

Outcome 2002: four-times improvement of the accuracy of the electron mass.

# The $g$ factor of H-like ions

g factor of $^{16}O^{7+}$

| | |
|---|---|
| Dirac value (point nucleus) | 1.997 726 003 06 |
| Free QED | 0.002 319 304 36 |
| Binding QED | 0.000 001 594 38(11) |
| Nuclear recoil | 0.000 000 116 97 |
| Nuclear size | 0.000 000 001 55(1) |
| Total theory | 2.000 047 020 32(11) |
| Experiment [1] | 2.000 047 025 4 (15) (44) |

[1] *J. Verdu et al., PRL, 2004.*

The value of the electron mass derived from the $O^{7+}$ experiment agrees with the value derived from the $C^{5+}$ experiment.

# The $g$ factor of H-like ions

g factor of $^{28}\mathrm{Si}^{13+}$

| | |
|---|---|
| Dirac value (point nucleus) | 1.993 023 571 6 |
| Free QED | 0.002 319 304 4 |
| Binding QED | 0.000 005 855 8(17) |
| Nuclear recoil | 0.000 000 205 8(1) |
| Nuclear size | 0.000 000 020 5 |
| Total theory | 1.995 348 958 0(17) |
| Experiment [1] | 1.995 348 959 10(7)(7)(80) |

[1] *S. Sturm et al., PRL, 2011; PRA, 2013.*

These experiment and theory provide to date the most accurate test of bound-state QED with middle-Z ions.

High-precision measurement of the atomic mass of the electron (S. Sturm et al., Nature, 2014):

$$g_{\rm exp} = 2(Z-1)(m_e/M_{\rm ion})(\omega_L/\omega_c) = g^*_{\rm theor} + (\alpha/\pi)^2(\alpha Z)^5 b_{50}$$

This equation is considered for $^{12}C^{5+}$ and $^{28}Si^{13+}$:

$$\left(\frac{\alpha}{\pi}\right)^2(6\alpha)^5 b_{50} = 2(6-1)\frac{m_e}{M_{^{12}{\rm C}^{5+}}}\left(\frac{\omega_L}{\omega_c}\right)_{^{12}{\rm C}^{5+}} - g^*_{\rm theor}(6)\,,$$

$$\left(\frac{\alpha}{\pi}\right)^2(14\alpha)^5 b_{50} = 2(14-1)\frac{m_e}{M_{^{28}{\rm Si}^{13+}}}\left(\frac{\omega_L}{\omega_c}\right)_{^{28}{\rm Si}^{13+}} - g^*_{\rm theor}(14)\,.$$

This yields:

$m_e = 0.000548579909067(14)(9)(2)u\,,$ $\qquad b_{50} = -4.0(5.1).$

# The $g$ factor of Li-like ions

The theoretical uncertainty of the g-factor for high-Z ions is defined by the nuclear effects. This uncertainty can be significantly reduced in the difference: *(V.M. Shabaev et al., PRA, 2002)*:

$$g' = g_{(1s)^2 2s} - \xi g_{1s},$$

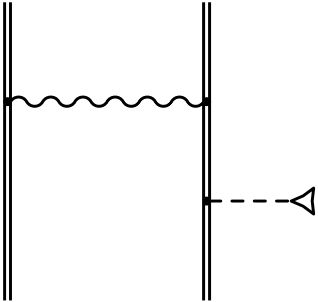
where $\xi$ is chosen to cancel the nuclear size effect.

Theoretical value of the g-factor of a Li-like ion:

$$g = g_{\rm one-elec.} + \Delta g_{\rm int.} + \Delta g_{\rm scr.QED} + \cdots .$$

The interelectronic interaction and screened QED corrections are evaluated using the perturbarion theory in $1/Z$.

## One-photon exchange

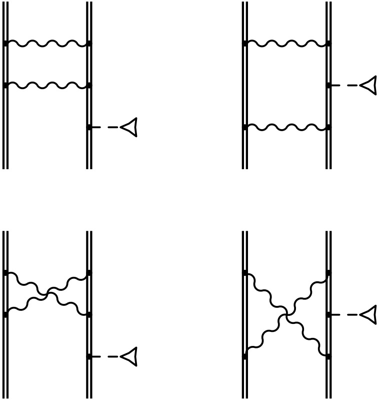


$$\Delta g_{\rm int} = \Delta g_{\rm int}^{(1)} + \Delta g_{\rm int}^{(2)} + \dots$$

$$\Delta g_{\rm int}^{(1)} = \frac{1}{Z}(\alpha Z)^2 B(\alpha Z)$$

V.M. Shabaev et al., PRA, 2002.

## Two-photon exchange: 2-electron diagrams

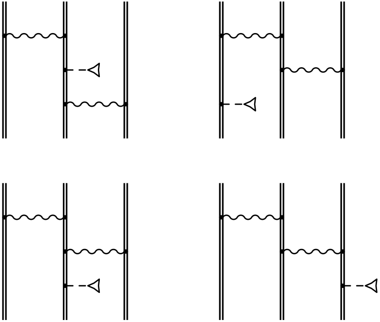


$$\Delta g_{\mathrm{int}} = \Delta g_{\mathrm{int}}^{(1)} + \Delta g_{\mathrm{int}}^{(2)} + \dots$$

$$\Delta g_{\mathrm{int}}^{(2)} = \frac{1}{Z^2} (\alpha Z)^2 \, C(\alpha Z)$$

A.V. Volotka et al., PRL, 2014.

## Two-photon exchange: 3-electron diagrams

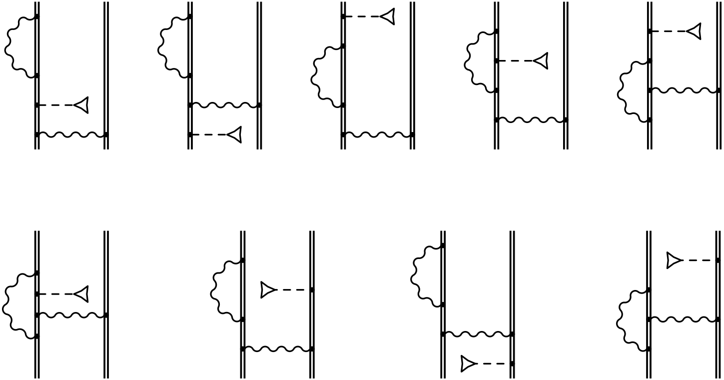


$$\Delta g_{\rm int} = \Delta g_{\rm int}^{(1)} + \Delta g_{\rm int}^{(2)} + \ldots$$

$$\Delta g_{\rm int}^{(2)} = \frac{1}{Z^2} (\alpha Z)^2 C(\alpha Z)$$

A.V. Volotka et al., PRL, 2014.

## Two-electron self-energy



A.V. Volotka et al., PRL, 2009; D.A. Glazov et al., PRA, 2010.

## $g$ factor of Li-like silicon (Z=14)

| | | |
|---|---|---|
| Free electron | Dirac + QED | 2.002 319 304 |
| Binding | Dirac | −0.001 395 613 |
| | QED $\sim \alpha$ | 0.000 001 224 (3) |
| | QED $\sim \alpha^2$ | −0.000 000 001 |
| $e^-$–$e^-$ interaction | $\sim 1/Z$ | −0.000 033 846 |
| | $\sim 1/Z^2$ | −0.000 000 976 |
| | $\sim 1/Z^{3+}$ | −0.000 000 005 (6) |
| | $\sim \alpha/Z^{+}$ | −0.000 000 236 (5) |
| Nuclear effects | Recoil | 0.000 000 039 (1) |
| | Finite size | 0.000 000 003 |
| | Total theory | 2.000 889 892 (8) |
| | Experiment | 2.000 889 890 (2) |

A. Wagner et al., PRL, 2013; A.V. Volotka et al., PRL, 2014.

# Isotope shift of the $g$ factor: H-like Ca

Isotope shift of the g factor: $^{40}\mathrm{Ca}^{19+} - {}^{48}\mathrm{Ca}^{19+}$

| | |
|---|---|
| Nuclear recoil: non-QED $\sim m/M$ | 0.000000048657 |
| Nuclear recoil: non-QED $\sim (m/M)^2$ | $-$0.000000000026(2) |
| Nuclear recoil: QED $\sim m/M$ | 0.000000000904 |
| Nuclear recoil: QED $\sim \alpha(m/M)$ | $-$0.000000000038(3) |
| Finite nuclear size | 0.000000000032(75) |
| Total theory | 0.000000049529(75) |

The current theoretical uncertainty is about 8% of the QED nuclear recoil contribution.

# Isotope shift of the $g$ factor: Li-like Ca

Isotope shift of the g-factor: $^{40}\mathrm{Ca}^{17+} - {}^{48}\mathrm{Ca}^{17+}$

| | |
|---|---|
| Nuclear recoil: one-electron non-QED | 0.000000012240(1) |
| Nuclear recoil: interelectronic int. | $-$0.000000002051(22) |
| Nuclear recoil: QED $\sim m/M$ | 0.000000000123(12) |
| Nuclear recoil: QED $\sim \alpha(m/M)$ | $-$0.000000000009(1) |
| Finite nuclear size | 0.000000000004(9) |
| Total theory | 0.000000010305(27) |

This study will provide the first test of QED beyond the Furry picture with highly charged ions.

# Future prospects for the g-factor investigations

1) Tests of bound-state QED within the Furry picture and beyond

2) Determination of the nuclear magnetic moments

$$g_{\rm atom} = g^{(e)} \frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)} - \frac{m_e}{m_p} g^{(N)} \frac{F(F+1)+I(I+1)-J(J+1)}{2F(F+1)} \,.$$

3) Investigations of the higher-order Zeeman effects with highly charged ions *(D. von Lindenfels et al., PRA, 2013)*

4) Determination of the fine structure constant by studying the g-factors of H- and B-like Pb *(V.M. Shabaev et al., PRL, 2006)*

$$g_{1s} = 2 - \frac{2}{3}(\alpha Z)^2 + \cdots \Rightarrow \frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2} \frac{\sqrt{(\delta g_{\rm exp})^2 + (\delta g_{\rm th})^2}}{g} \,.$$

# The $g$ factor: determination of $\alpha$

free electron

$$\frac{\delta\alpha}{\alpha} = \frac{2\pi}{\alpha}\frac{\delta g_{\rm free}}{g_{\rm free}}$$

$$\delta g^{\rm exp}_{free} = 3 \times 10^{-13}$$

$$\rightarrow \frac{\delta\alpha}{\alpha} = 3 \times 10^{-10}$$

bound electron

$$\frac{\delta\alpha}{\alpha} \sim \frac{1}{(\alpha Z)^2}\frac{\delta g_{1s}}{g_{1s}}$$

$$\delta g^{\rm exp}_{1s} = 1.5 \times 10^{-10}$$

$$\rightarrow \frac{\delta\alpha}{\alpha} = 3 \times 10^{-10}$$

(for Pb, $Z = 82$)

# The $g$ factor of heavy ions: a new access to $\alpha$

We consider a specific difference of the g-factors of B- and H-like lead *(V.M. Shabaev et al., PRL, 2006)*:

$$g' = g^{[(1s)^2(2s)^2 2p_{1/2}]} - \xi g^{[1s]} \,,$$

where $\xi = 0.0097416$ is chosen to cancel the nuclear size effect.

The uncertainties of $g' \approx 0.585$ for Pb due to various effects:

| Effect | $\delta g'$ | $\delta g'/g'$ |
|---|---|---|
| $1/\alpha = 137.035999173(35)$ | $0.4 \times 10^{-10}$ | $0.7 \times 10^{-10}$ |
| Nuc. polarization* | $0.3 \times 10^{-10}$ | $0.5 \times 10^{-10}$ |

* *A.V. Volotka and G. Plunien, PRL, 2014*

# Conclusion

The $g$ factor of highly charged ions is available for both

- high-precision measurements
- accurate calculations

$\rightarrow$ Test of theory: bound-state QED
$\rightarrow$ Determination of

- $m_e$ — electron mass (in a.u.)
- $\mu_N$ — nuclear magnetic moment
- $R_N$ — nuclear charge radius
- $\alpha$ — fine structure constant