Newton's Gravitational Constant ‚Big' G – A proposed Free-fall Measurement

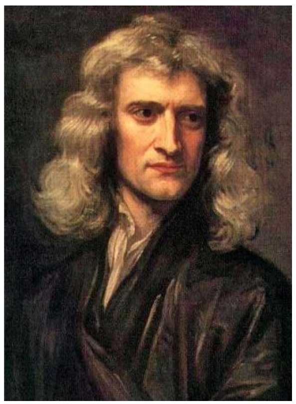
projected and presented by

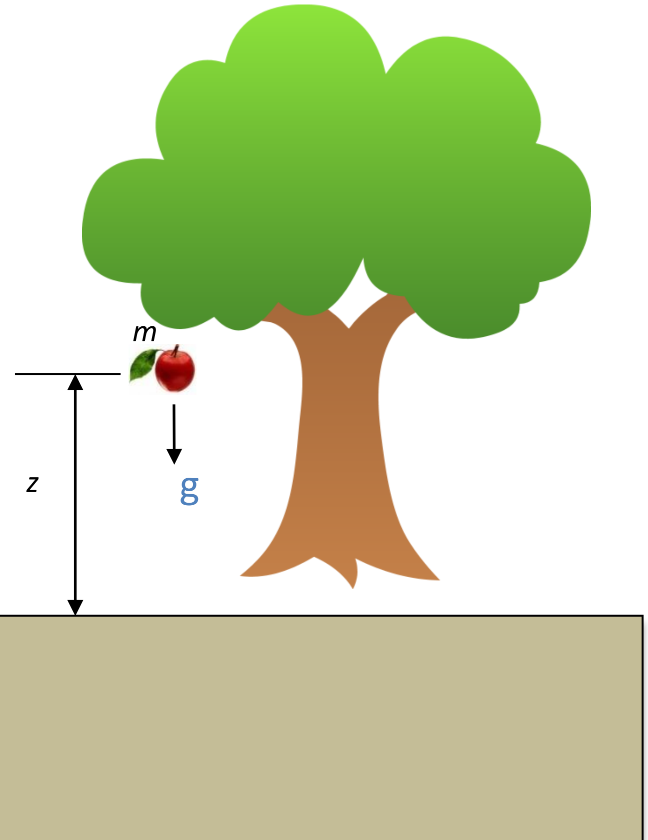
Christian Rothleitner

(currently working at PTB)

CODATA Fundamental Constants Meeting
Eltville – 5 February, 2015

Free-fall experiments to determine ‚Big‘ G





Newtonian constant of gravitation:

$G = 6.673\ 84 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

$$F_G = \frac{G m_1 m_2}{r^2}$$

g

Acceleration due to gravity:

$g = 9.806\ 65\ m\ s^{-2}$

Free-fall absolute gravimeter

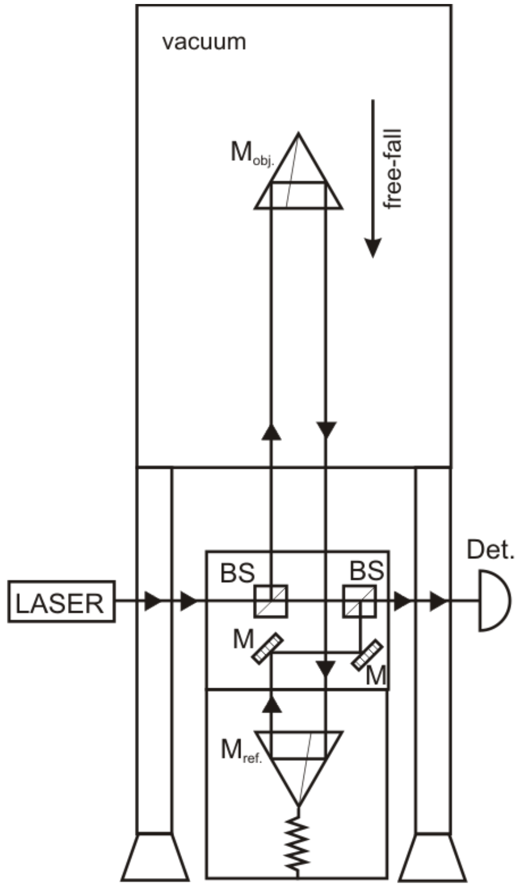
FG5-X

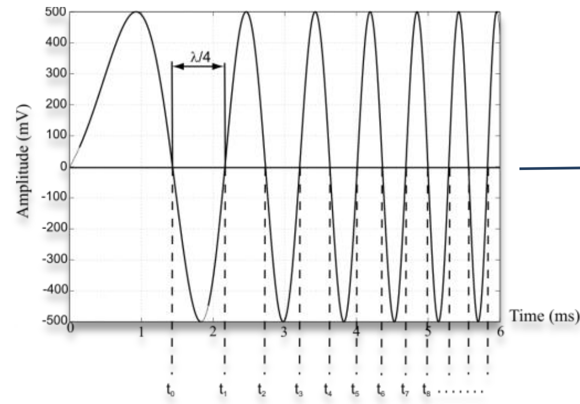
Microg-LaCoste
(Lafayette, CO, USA)

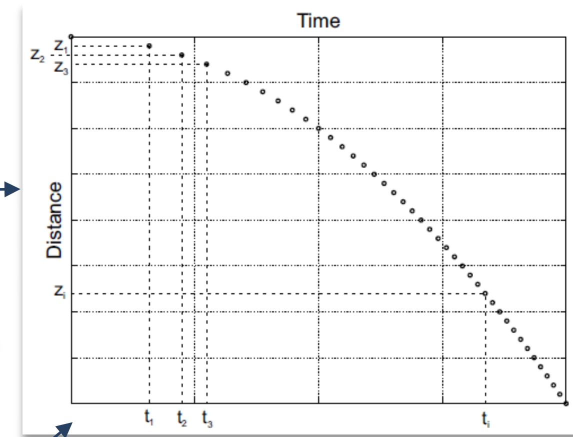
FG5-X : Precision : 15 µGal/√(Hz)*
Accuracy: 2 µGal*

(*from http://www.microglacoste.com/absolutemeters.php)

Principle of measurement





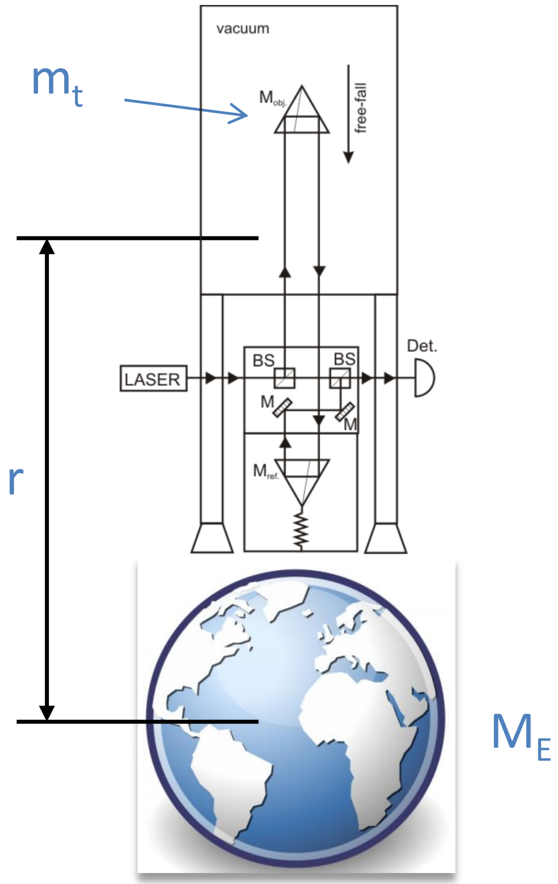


$$z(t) = z_0 + v_0 t + \frac{g}{2} t^2$$

acceleration due to gravity

Measurement of G with a gravimeter

Earth as source mass M_E



g

$$F_G = \frac{G m_1 m_2}{r^2}$$

$$G = 6.7x10^{-11} \frac{Nm^2}{kg^2}$$

$M_E = m_1$

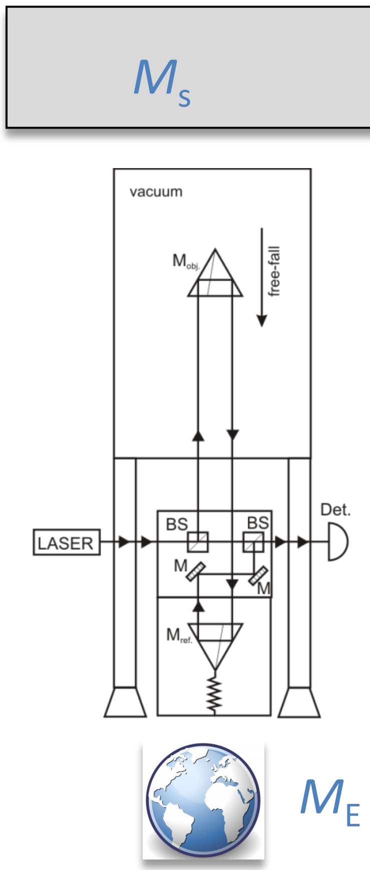
$m_t = m_2$

- Calculate g with theoretical model
- Measure g
- Determine G

Problem: Mass, geometry and density distribution of Earth are not well known!

Use of well defined source mass, M_S

configuration 1



M_s produces perturbing acceleration *P(z,G)*

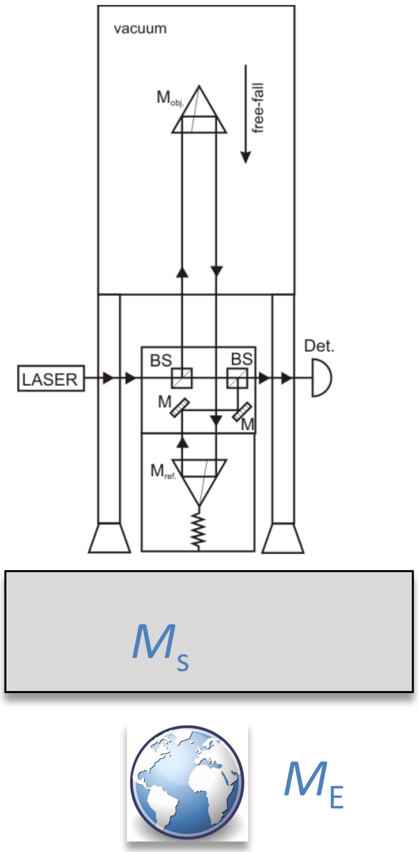
Total signal is

$$g_1 = \underbrace{g_0 + \gamma z}_{Earth} - \underbrace{P(z,G)}_{sourcemass}$$

gravity gradient

Use of well defined source mass M_S

configuration 2



M_s produces perturbing acceleration *P(z,G)*

Total signal is

$$g_2 = \underbrace{g_0 + \gamma z}_{Earth} + \underbrace{P(z,G)}_{sourcemass}$$

Differential signal

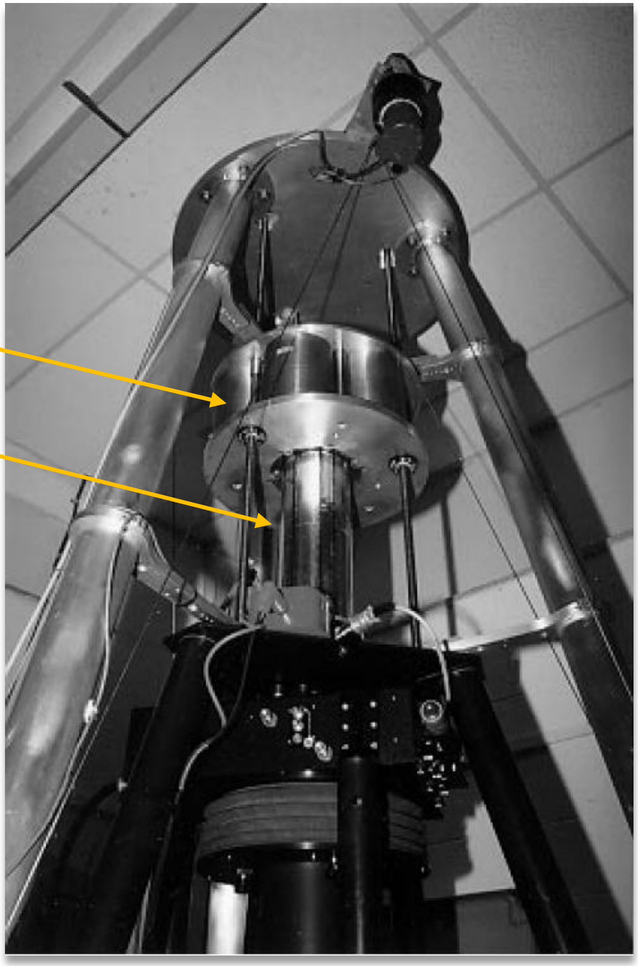
$$g_2 - g_1 = 2\underbrace{P(z,G)}_{sourcemass}$$

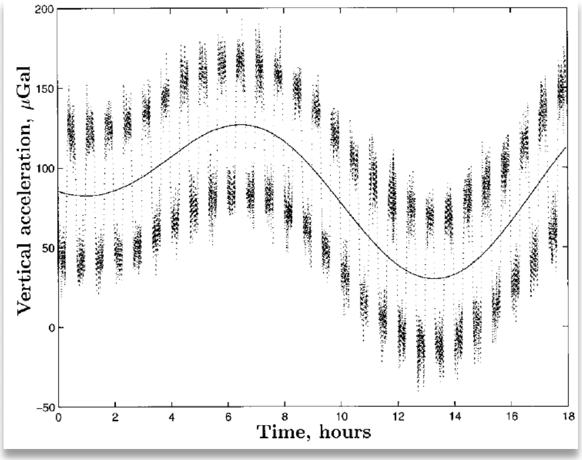
Schwarz et al., 1998, University of Colorado – Free-Fall Gravimeter

source mass:

~500 kg

FG5 gravimeter

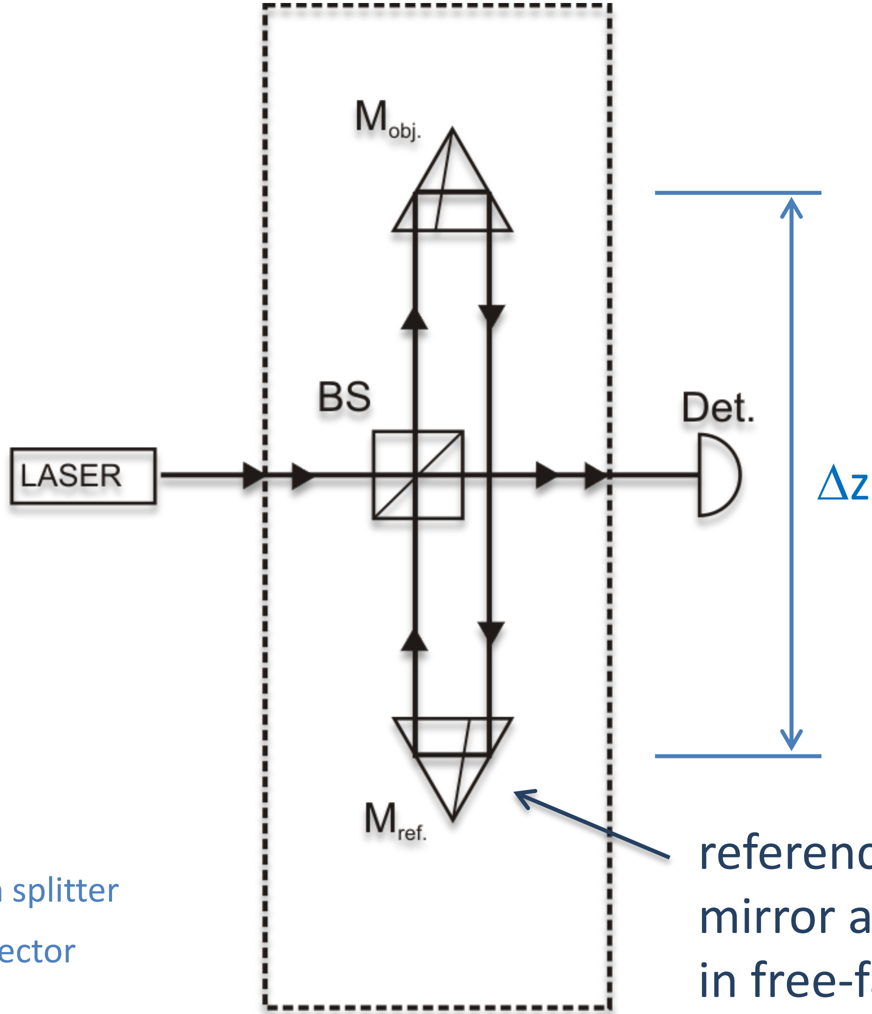




Result:
$\Delta G/G = 1.4 \cdot 10^{-3}$

JP Schwarz, DS Robertson, TM Niebauer & JE Faller (1998). *A free-fall determination of the universal constant of gravity*. Science, **18**, 2230-2234

Free-fall Gradiometer



reference mirror also in free-fall

height 1: g_T

$$\gamma := \frac{\Delta g}{\Delta z} = \frac{g_B - g_T}{\Delta z}$$

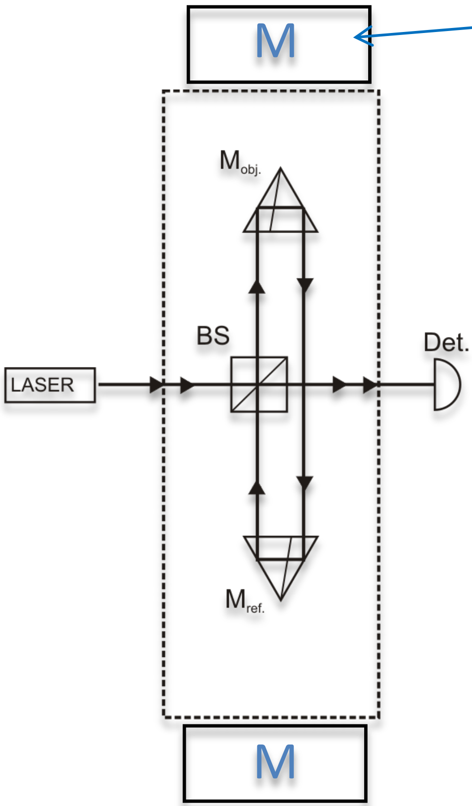
height 2: g_B

BS: Beam splitter

Det.: Detector

Measurement of G with a gravity gradiometer

Configuration 1



acceleration due to external mass

$$g_T = {}^0g_T - {}^M\Delta g$$ top

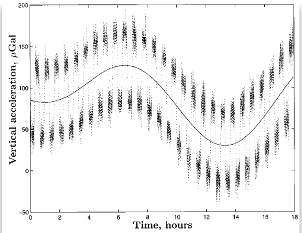
acceleration due to Earth

$$\Rightarrow g_B - g_T = \Delta z \cdot \gamma + 2\,{}^M\Delta g$$

$$g_B = {}^0g_B + {}^M\Delta g$$ bottom

second source mass to increase signal

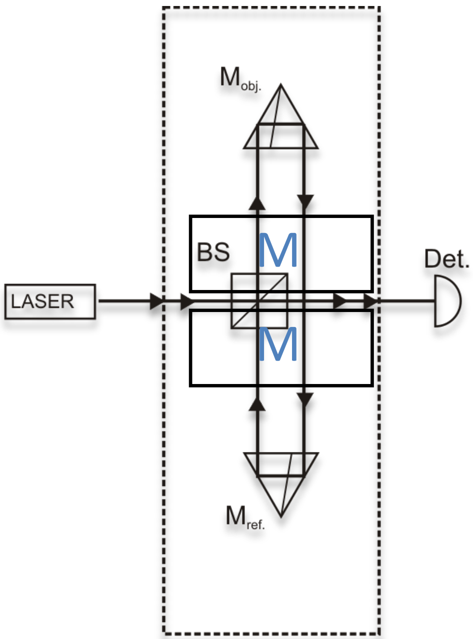
equivalent to



however, cancels tides, ocean loading, etc.

Measurement of G with a gravity gradiometer

Configuration 2



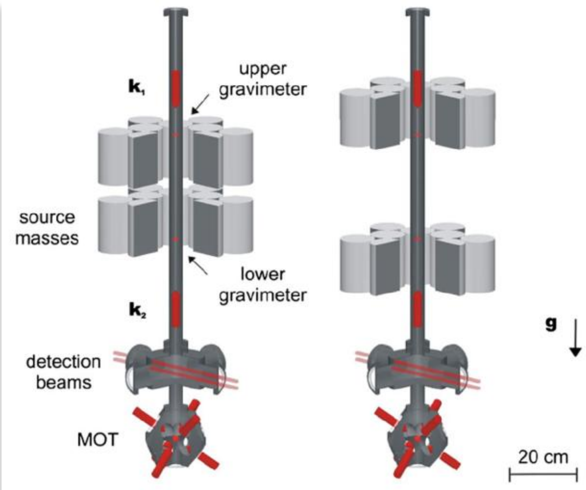
$$g_T = {}^0g_T + {}^M\Delta g$$ top

$$\Rightarrow g_B - g_T = \Delta z \cdot \gamma - 2\, {}^M\Delta g$$

$$g_B = {}^0g_B - {}^M\Delta g$$ bottom

Config. 2 – Config. 1: $$(g_B - g_T)_{,2} - (g_B - g_T)_{,1} = -4\, {}^M\Delta g$$

University of Florence
– Atom interferometer (Raman interferometry)

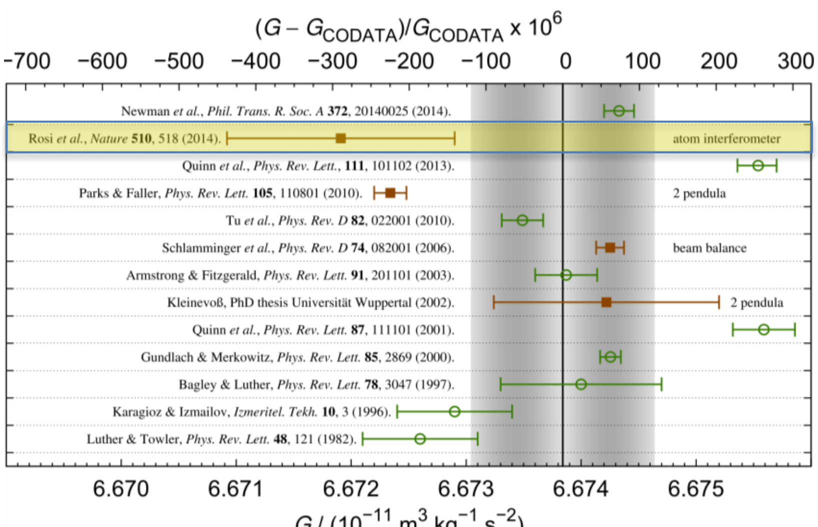


Test mass:

Rubidium atoms

Source mass:

~516 kg tungsten

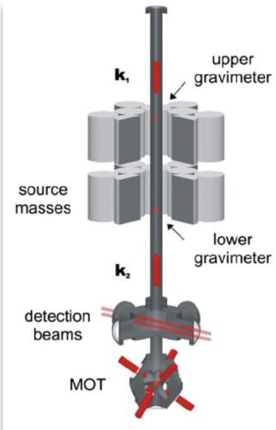


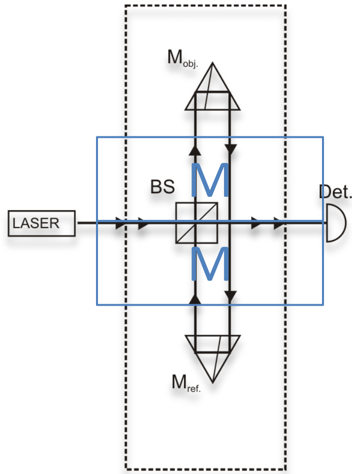
Result: $\Delta G/G = 1.5 \cdot 10^{-4}$

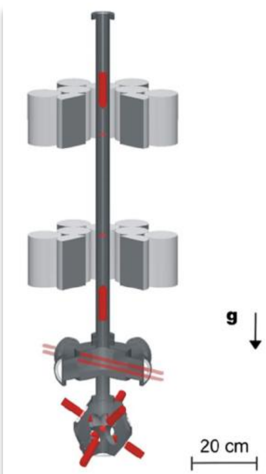
source: http://www.nist.gov/pml/div684/fcdc/newtonian-constant.cfm

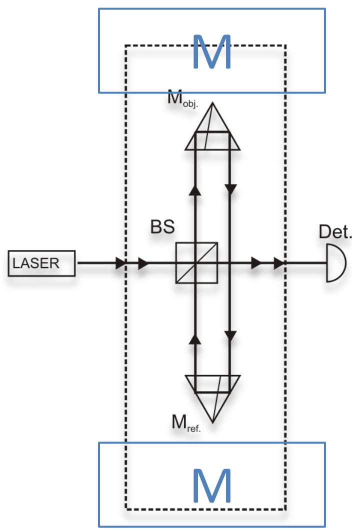
G Rosi et al. (2014). Precision measurement of the Newtonian gravitational constant using cold atoms. *Nature*, 510, 518–521.

Differential gradiometer

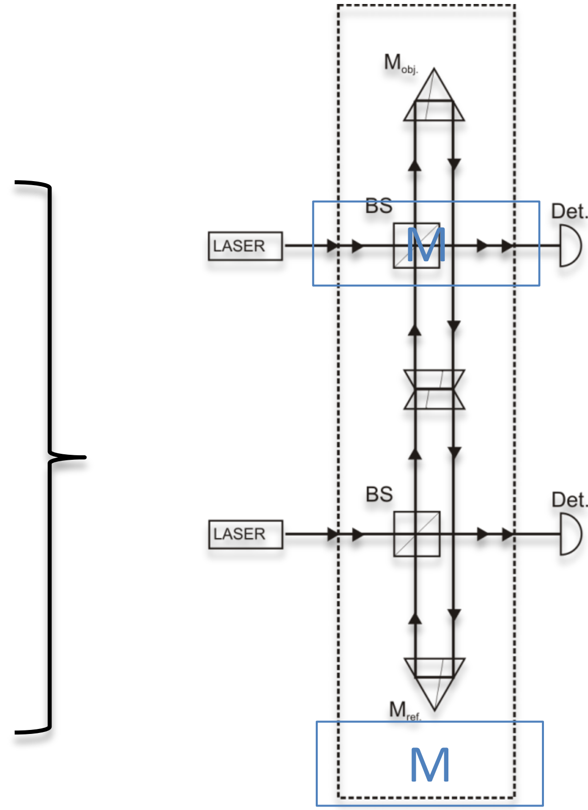






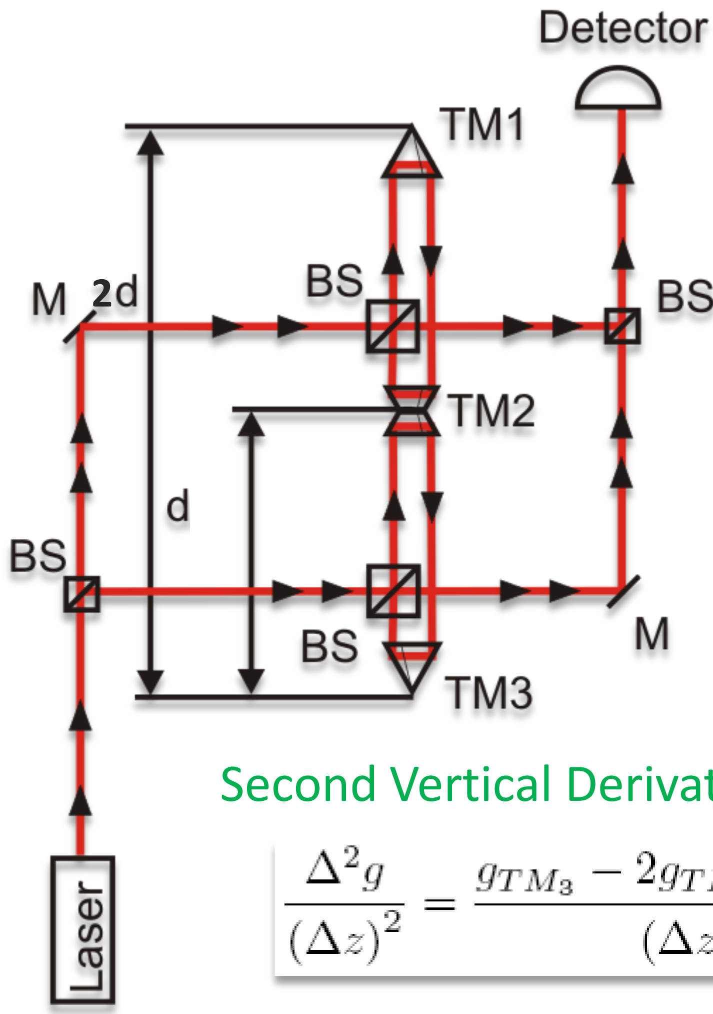


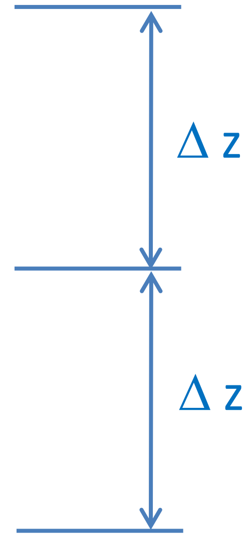
merge



3 simultaneously dropped masses

Differential gradiometer





height 1: g_T

$$\frac{\Delta g_T}{\Delta z} = \frac{g_{TM_2} - g_{TM_1}}{\Delta z}$$

height 2: g_M

$$\frac{\Delta g_B}{\Delta z} = \frac{g_{TM_3} - g_{TM_2}}{\Delta z}$$

height 3: g_B

Second Vertical Derivative of g (SVD)

$$\frac{\Delta^2 g}{(\Delta z)^2} = \frac{g_{TM_3} - 2g_{TM_2} + g_{TM_1}}{(\Delta z)^2}$$

Null instrument
(no *g* or γ)

Rothleitner Ch & Francis O. (2014). Measuring the Newtonian constant of gravitation with a differential free-fall gradiometer: A feasibility study. *Rev. Sci. Instrum,* **85**, 044501.
Rothleitner, C. (2010). Interferometric differential free-fall gradiometer. EP2348338B1, US20130205894.

Separation of inertial from gravitational forces

Measured force in a non-inertial frame:

$$\ddot{x}_k = f_k - 2a_{ik}\dot{a}_{ij}\dot{x}_j - a_{ik}\ddot{a}_{ij}x_j - \ddot{b}_k$$

Gravitat. force | Coriolis force | Euler and centrifugal force | Linear acceleration

$$f_i = \text{gravitational force, derived from potential } f_i = -\frac{\partial V}{\partial x_i}$$

$$a_{ik} = \text{rotation matrix; rel. rotation between inertial and non-inertial frame}$$

$$\ddot{b} = \text{rel. linear acceleration between both frames}$$

$$\textbf{SVD} \quad \frac{\partial^2 \ddot{x}_i}{\partial x_i^2} = \frac{\partial^2 f_i}{\partial x_i^2} = -\frac{\partial^3 V}{\partial x_i^3}$$

- Inertial forces disappear; only gravitational signal measured
- No inertial stabilization necessary
- Applications: fundamental physics, gravimetry, navigation

see e.g. Hofmann-Wellenhof, B & Moritz, H (2005). *Physical Geodesy.* Springer Wien New York.

Proposal

Perform a Big G measurement by

- Using an **atom** and a **classical gradiometer**

↓

- Using the **same source mass**

↓

- **Comparing uncertainty budgets**

↓

- **Identifying and eliminating systematic errors** if present

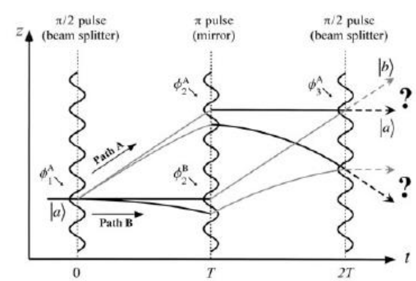
Advantages of free-fall experiments

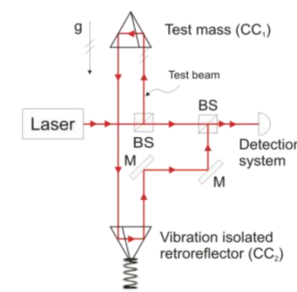
- **Same physical sizes of** classical and atom gravimeter
  - same source masses can be used

Differential gradiometer @ Univ. Luxembourg

Atom gradiometer, Tino group, Florence University**

- **Two different technologies / physical laws for** same experiment
  - Test of physical theories
  - Detection of systematic errors





*See also ‘G Rosi et al. (2015). *Measurement of the Gravity-Field Curvature by Atom Interferometry*. *PRL*, 114.‘ for differential gradiometer.
**Picture from Lamporesi, 2006, PhD thesis

Advantages of free-fall experiments

- Both technologies are under **intense research**
  - good know-how; immediate start possible

- **Benefit for industry**
  - use in other areas of science and technology

- Reflects the popular story about **Newton´s apple**
  - **Attractive for popular readership**

Acknowledgements

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Physikalisch-Technische Bundesanstalt
Braunschweig und Berlin
Bundesallee 100
38116 Braunschweig
Dr. rer. nat. Christian Rothleitner
Arbeitsgruppe 5.34 Multisensor-Koordinatenmesstechnik
(Working Group 5.34 Multisensor Coordinate Metrology)
Telefon: +49 (0)531 592-5348
E-Mail: christian.rothleitner@ptb.de
www.ptb.de

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