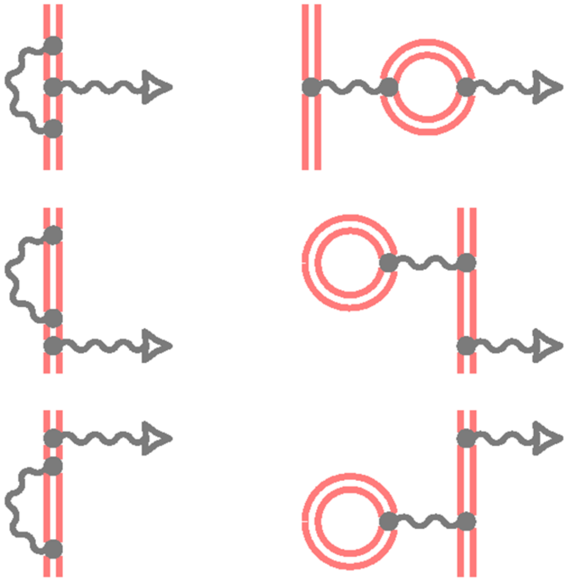
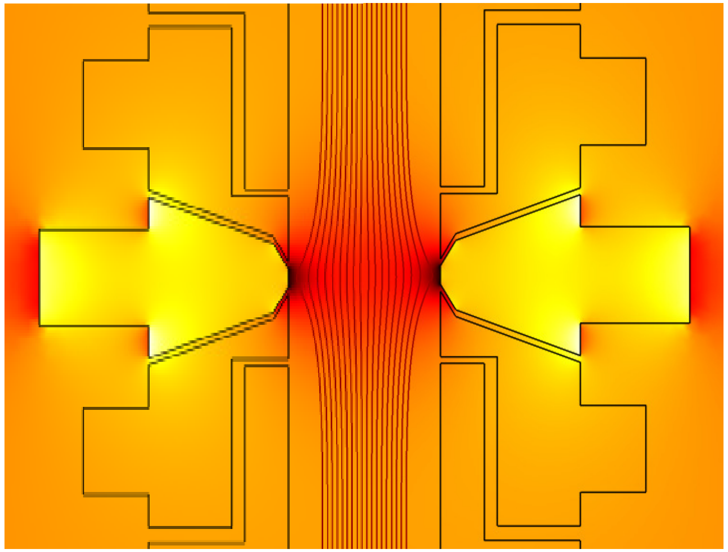
Improved Determination of the Electron Mass in Atomic Mass Units



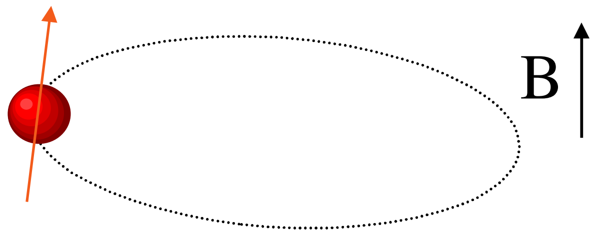


Wolfgang Quint
GSI Darmstadt and Univ. Heidelberg

You know, it would be sufficient to really understand the electron.

Albert Einstein

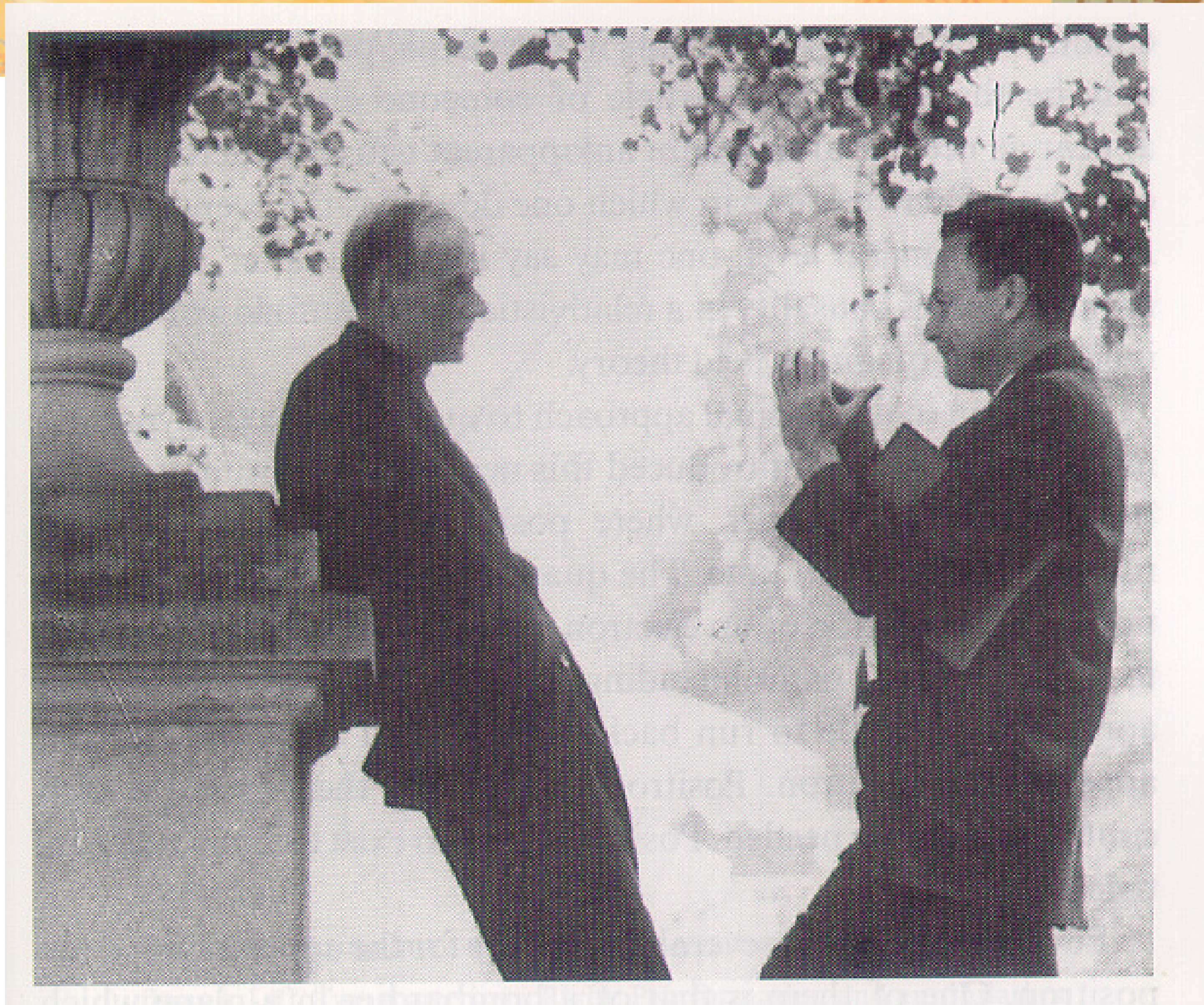
g-Factor of the electron



$$\frac{|\bar{\mu}|}{\mu_B} = g \cdot \frac{|\bar{s}|}{\hbar}$$

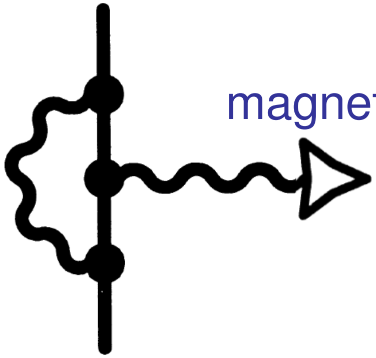
μ: magnetic moment
g: g-factor
s: spin
μ_B: Bohr magneton

g = 2 + α / π

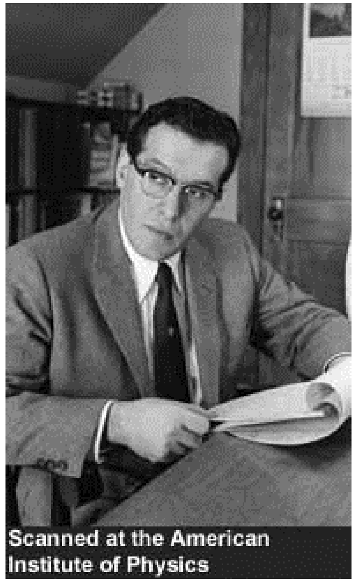


QED contributions to the g-factor of the free electron

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots$$



1st order in α:
Schwinger term
$C_1 = ½$



The theory of quantum electrodynamics is,
I would say, the jewel of physics
- our proudest possession.

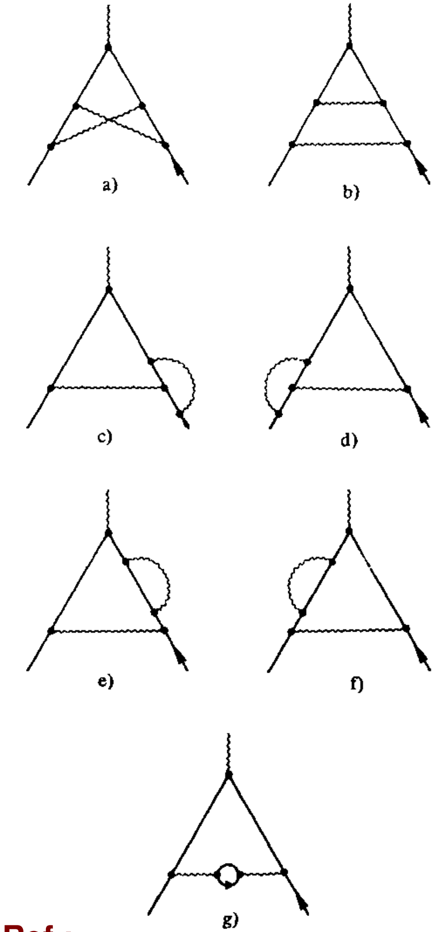
R. Feynman

Ref.:
J. Schwinger, Phys. Rev. 73, 416 (1948); Hanneke et al., PRL 100, 120801 (2008)

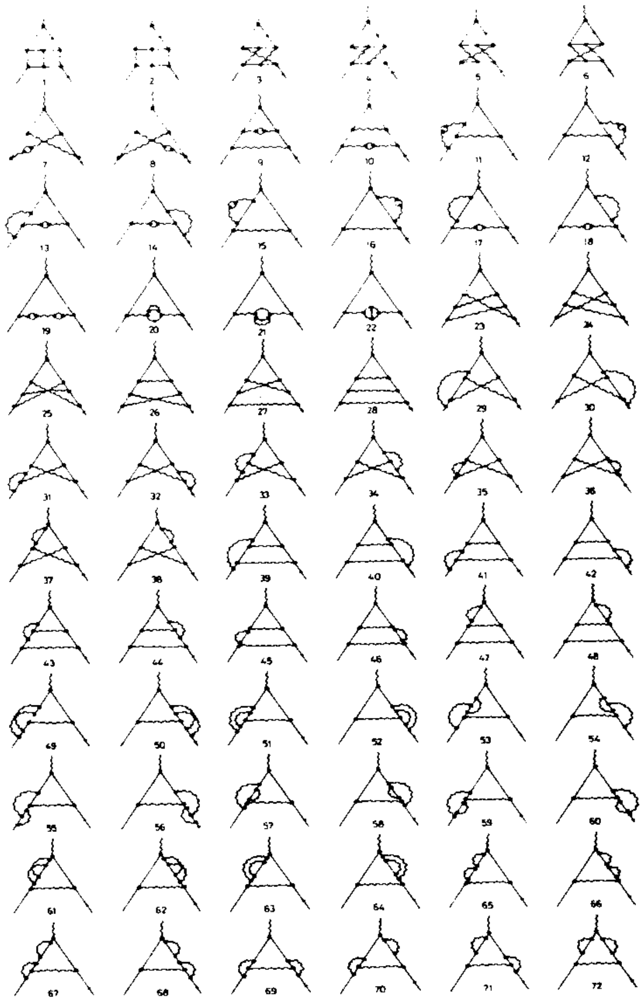
GSI

Free electron: QED contributions of 2nd and 3rd order

$$g_{free} = 2\,(1 + C_1\alpha/\pi + C_2(\alpha/\pi)^2 + C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4 + C_5(\alpha/\pi)^5 + \ldots.$$



2nd order in α:
$C_2 = -0.328\,478\,966$
7 graphs



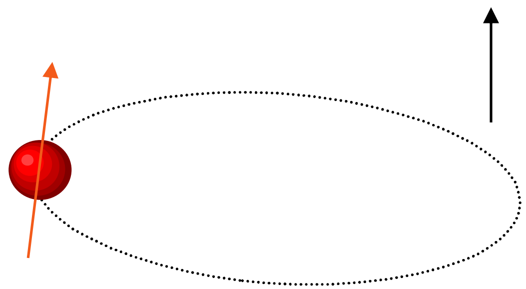
3rd order in α:
$C_3 = 1.1765$
72 graphs

not shown:
4th order in α:
$C_4 = -1.9108$
891 graphs

Ref.:
B. Lautrup et al., Phys. Rep. 3, 193 (1972)

g-Factor of the free electron

B: magnetic field in Penning trap



Larmor precession frequency:

$$\omega_L^e = \frac{g}{2}\frac{e}{m_e}B$$

cyclotron frequency:

$$\omega_c^e = \frac{e}{m_e}B$$

$$g_e = 2 \cdot \frac{\omega_L^e}{\omega_c^e}$$

or rather

$$g_e - 2 = 2 \cdot \frac{\omega_a^e}{\omega_c^e}$$

(get 3 orders of magnitude in accuracy by Nature)

Folie 7

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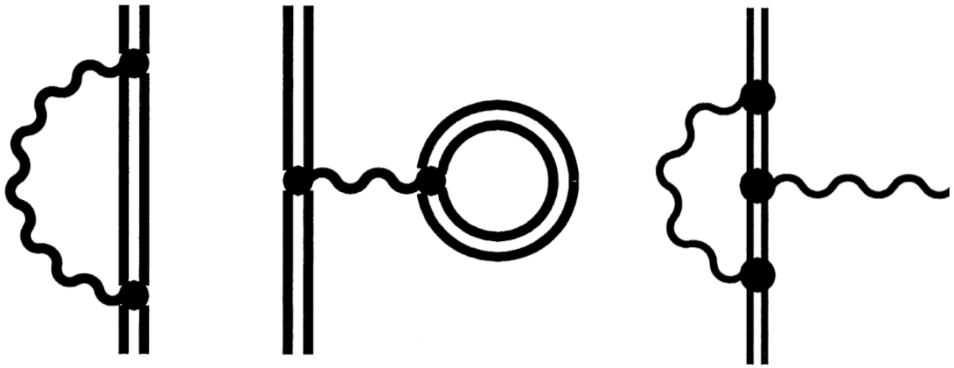
q1 quint_local; 18.05.2003

QED and highly charged ions

bound-state QED: quantum physics in strong fields

basic processes in bound-state QED:



self energy   vacuum polarization vertex correction

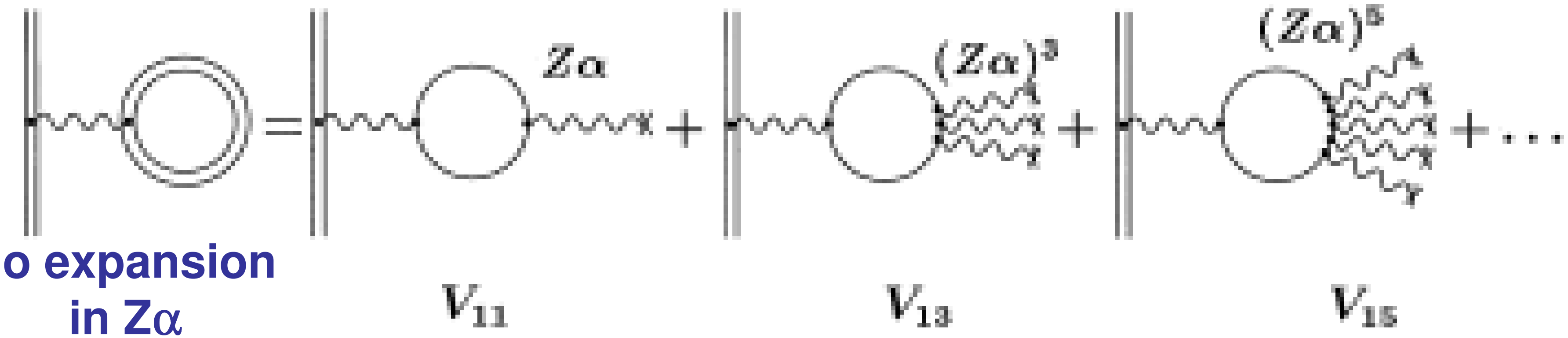
bound-state QED coupling parameter for U^{91+}: $Z\alpha \approx 0.67$

Ref.:
T. Beier, Physics Reports 339, 79 (2000)

Bound-state QED: treatment non-perturbative in Zα

non-perturbative treatment

perturbative treatment: expansion in Zα



no expansion in Zα

Non-perturbative bound-state QED was developed in the last 20 years by excellent theoreticians like:

Beier, Blundell, Breit, Czarnecki, Glazov, Jentschura, Johnson, Karshenboim, Lindgren, Lee, Milstein, Mohr, Pachucki, Persson, Plunien, Salomonson, Sapirstein, Shabaev, Soff, Sunnergreen, Terekhov, Tupitsyn, Volotka, Yerokhin, and others.

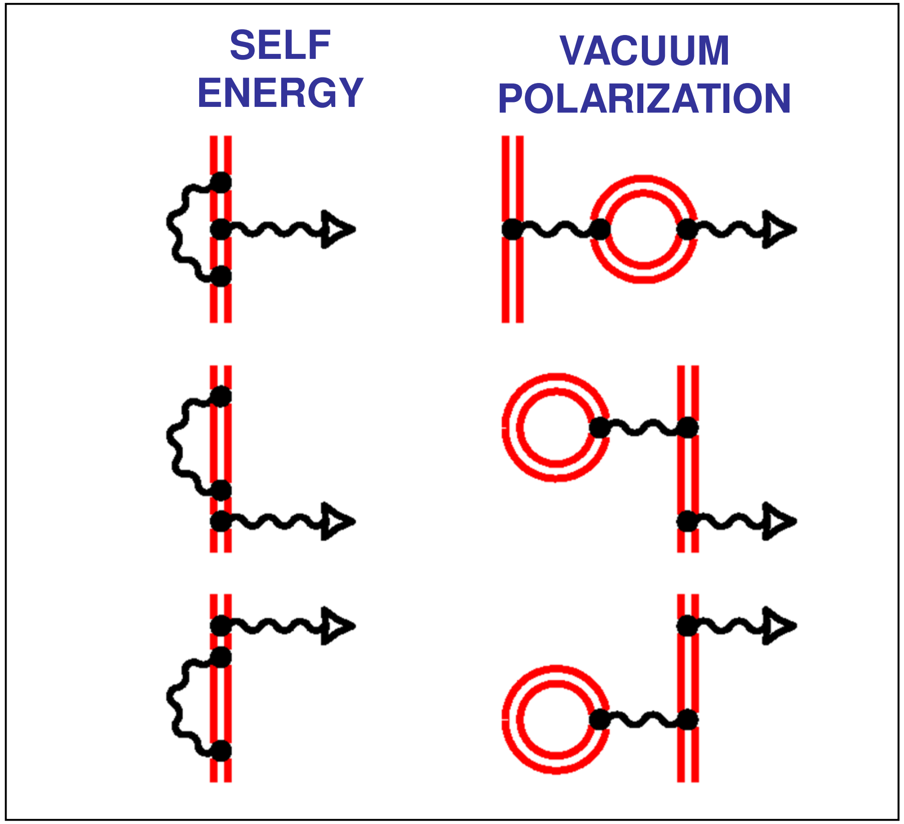
Ref.:
U.D. Jentschura, P.J. Mohr, G. Soff, PRL 82, 53 (1999); T. Beier, Physics Reports 339, 79 (2000)

Bound-electron g-factor: Feynman graphs 1st order in α/π

$$g_{bound}/g_{free} \approx 1 - (Z\alpha)^2/3 + \alpha(Z\alpha)^2/4\pi + \ldots.$$

Dirac theory → $(Z\alpha)^2/3$

bound-state QED → $\alpha(Z\alpha)^2/4\pi$

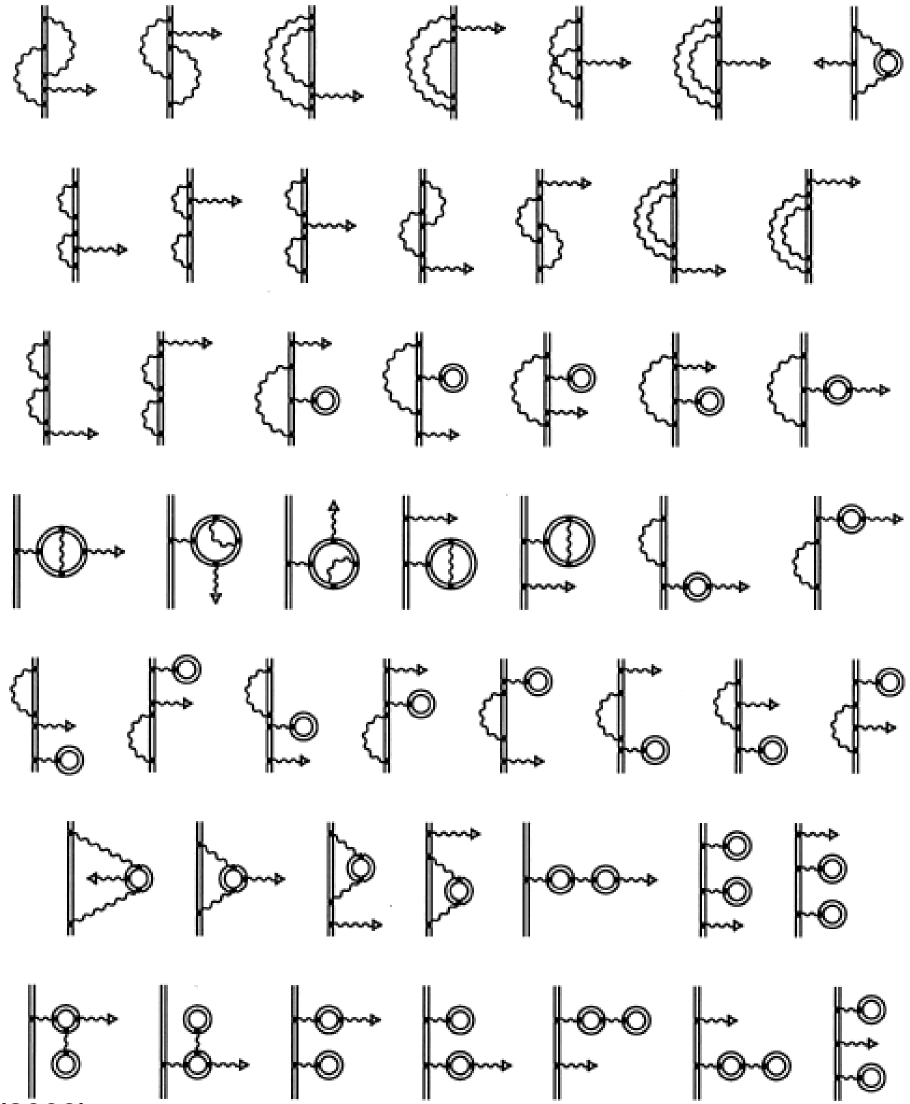


Ref.:

T. Beier, Physics Reports 339, 79 (2000)

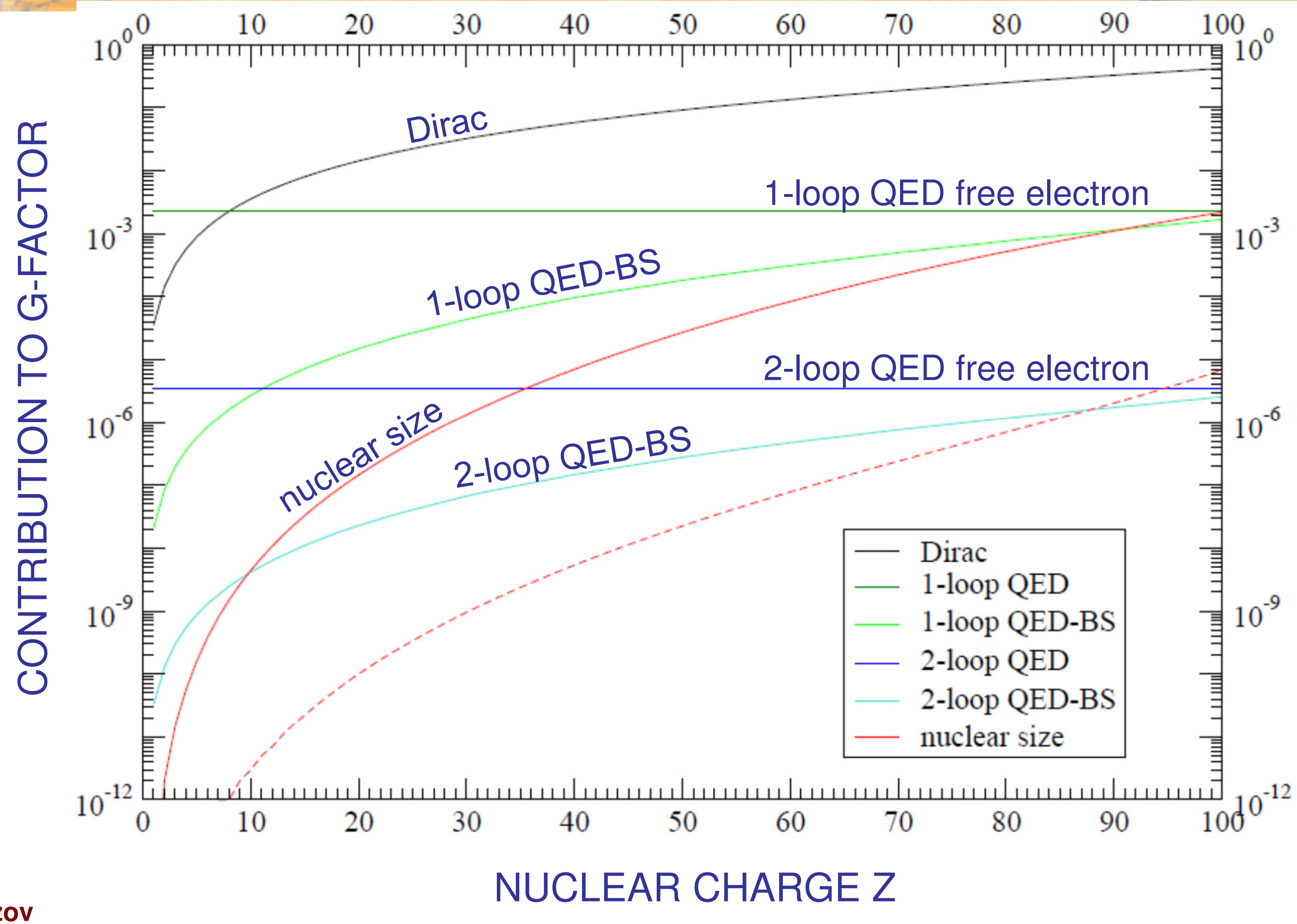
Bound-electron g-factor: Feynman graphs 2nd order in α/π

50 graphs



Ref.:
T. Beier, Physics Reports 339, 79 (2000)

Bound-electron g-factor

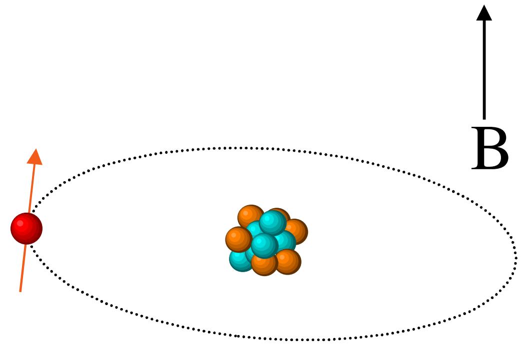


Ref.:
D. Glazov

g-Factor of the bound electron in a hydrogen-like ion (nucleus has no spin, e.g. $^{12}C^{5+}$, $^{16}O^{7+}$, $^{28}Si^{13+}$, $^{40}Ca^{19+}$)

Larmor precession frequency of the bound electron:

$$\omega_L^e = \frac{g_J}{2}\frac{e}{m_e}B$$



Ion cyclotron frequency:

$$\omega_c^{ion} = \frac{Q}{M_{ion}}B$$

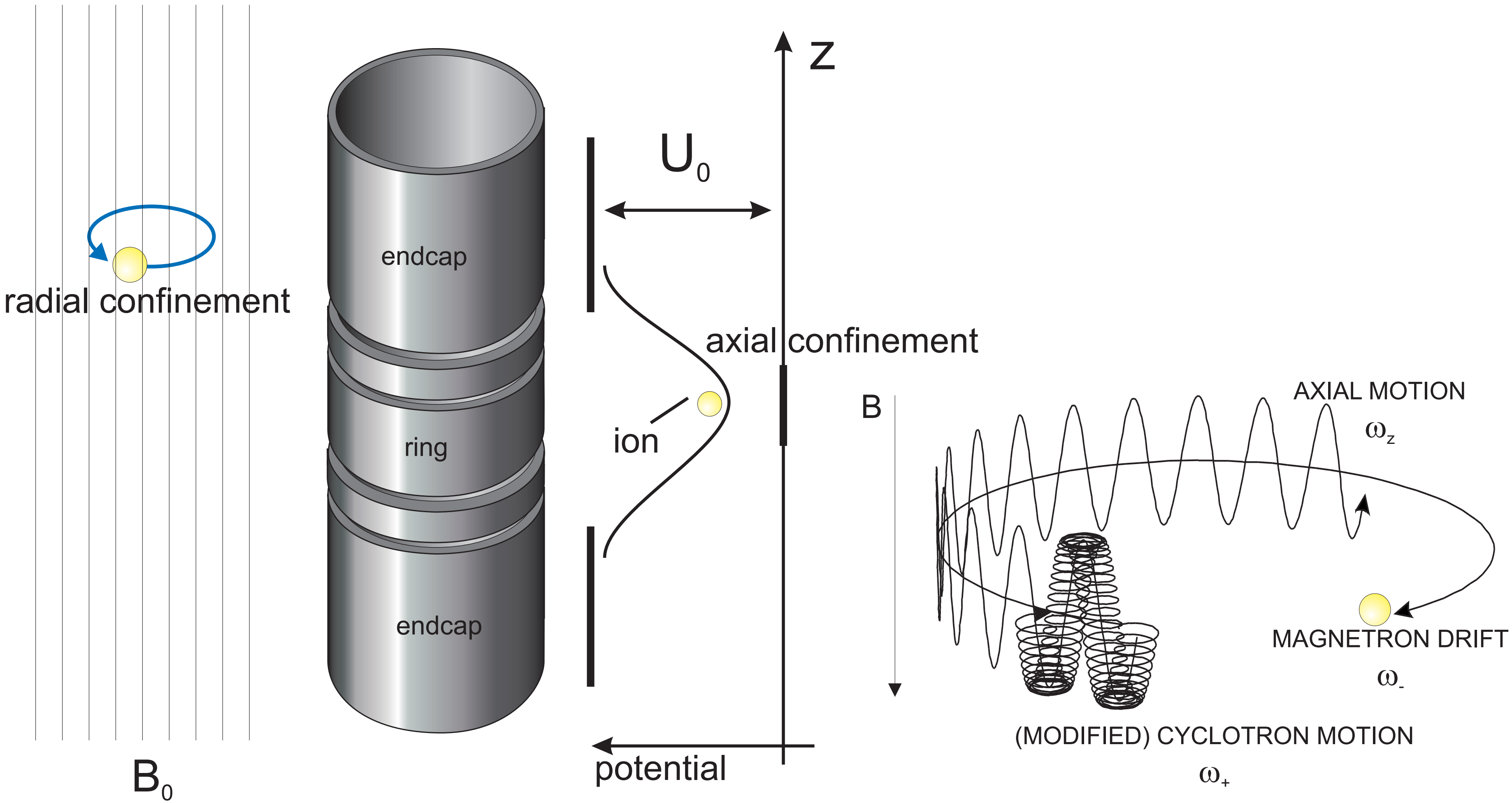
$$g_J = 2 \cdot \frac{\omega_L^e}{\omega_c^{ion}} \cdot \frac{m_e}{M_{ion}} \cdot \frac{Q^{ion}}{e}$$

→ 'experimental g-factor'
→ comparison with theory

our measurement

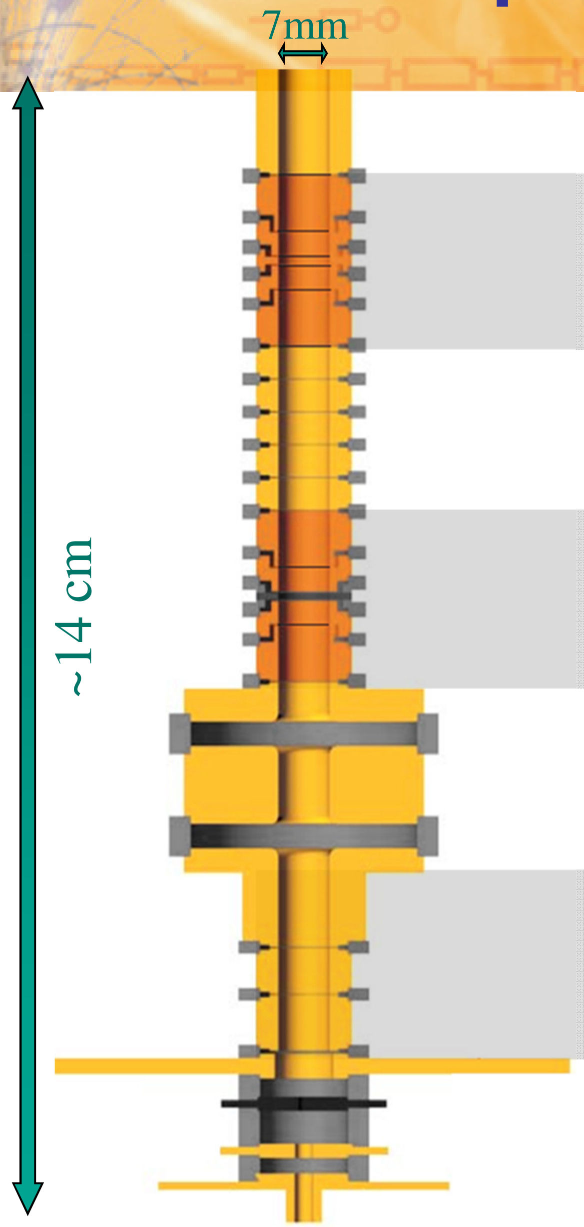
external input parameter

A single highly charged ion stored in a Penning trap



magnetic physical electric combined ion motion

Triple Penning Trap System



Precision trap (PT)
- Very homogeneous magnetic field

Analysis trap (AT)
- Magnetic bottle for spin detection

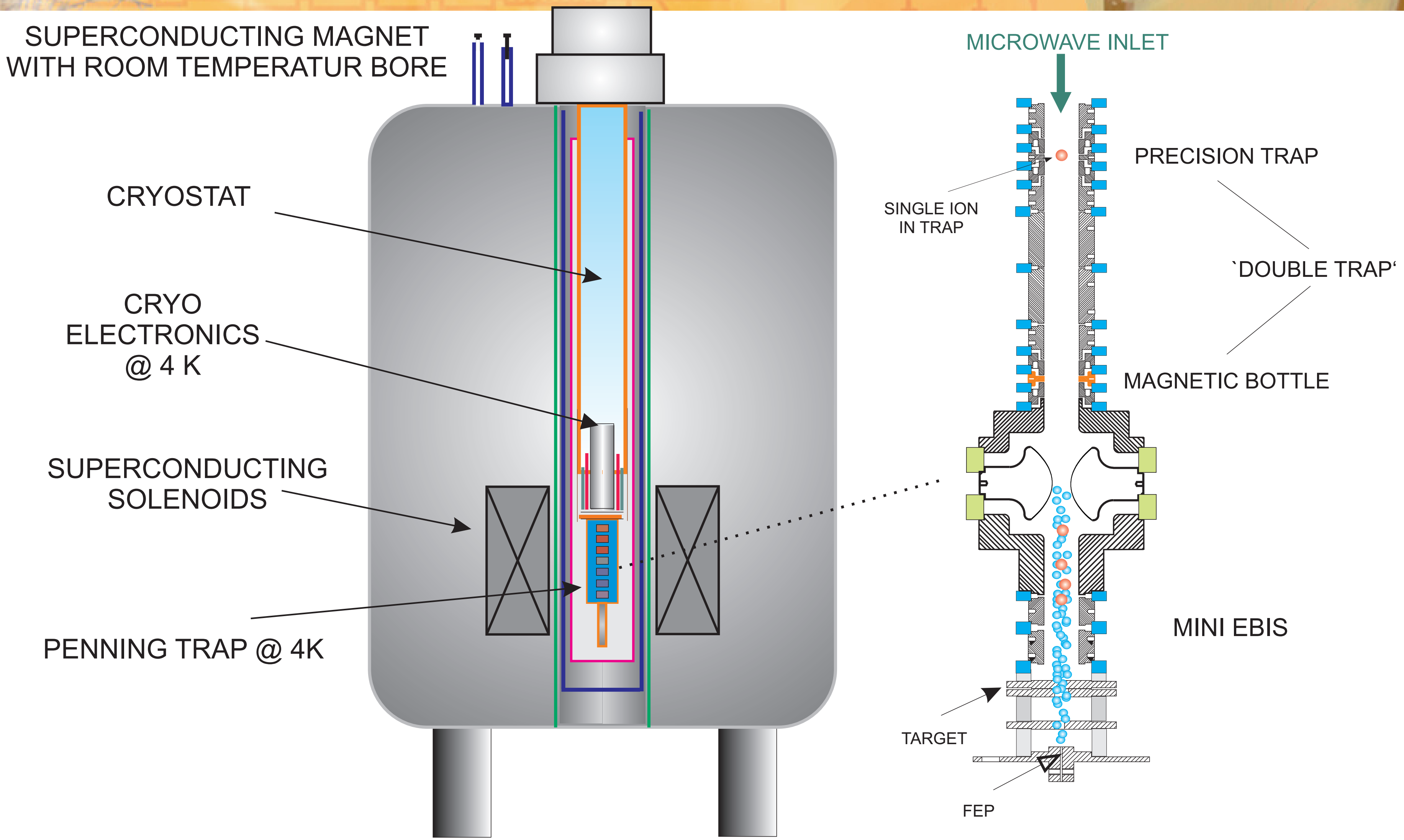
Creation trap (CT)
- In-trap ion creation of highly-charged ions

Triple Penning Trap System

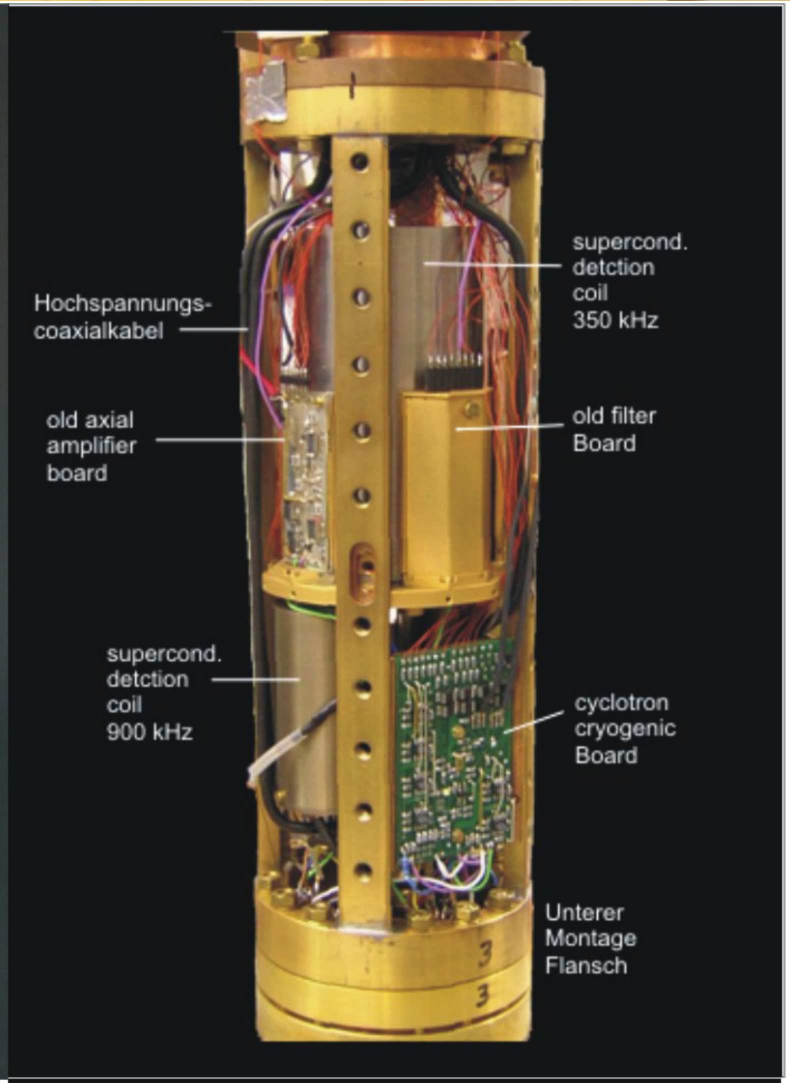
Precision trap (PT)
- Very homogeneous magnetic field

Analysis trap (AT)
- Magnetic bottle for spin detection

HCl g-factor apparatus

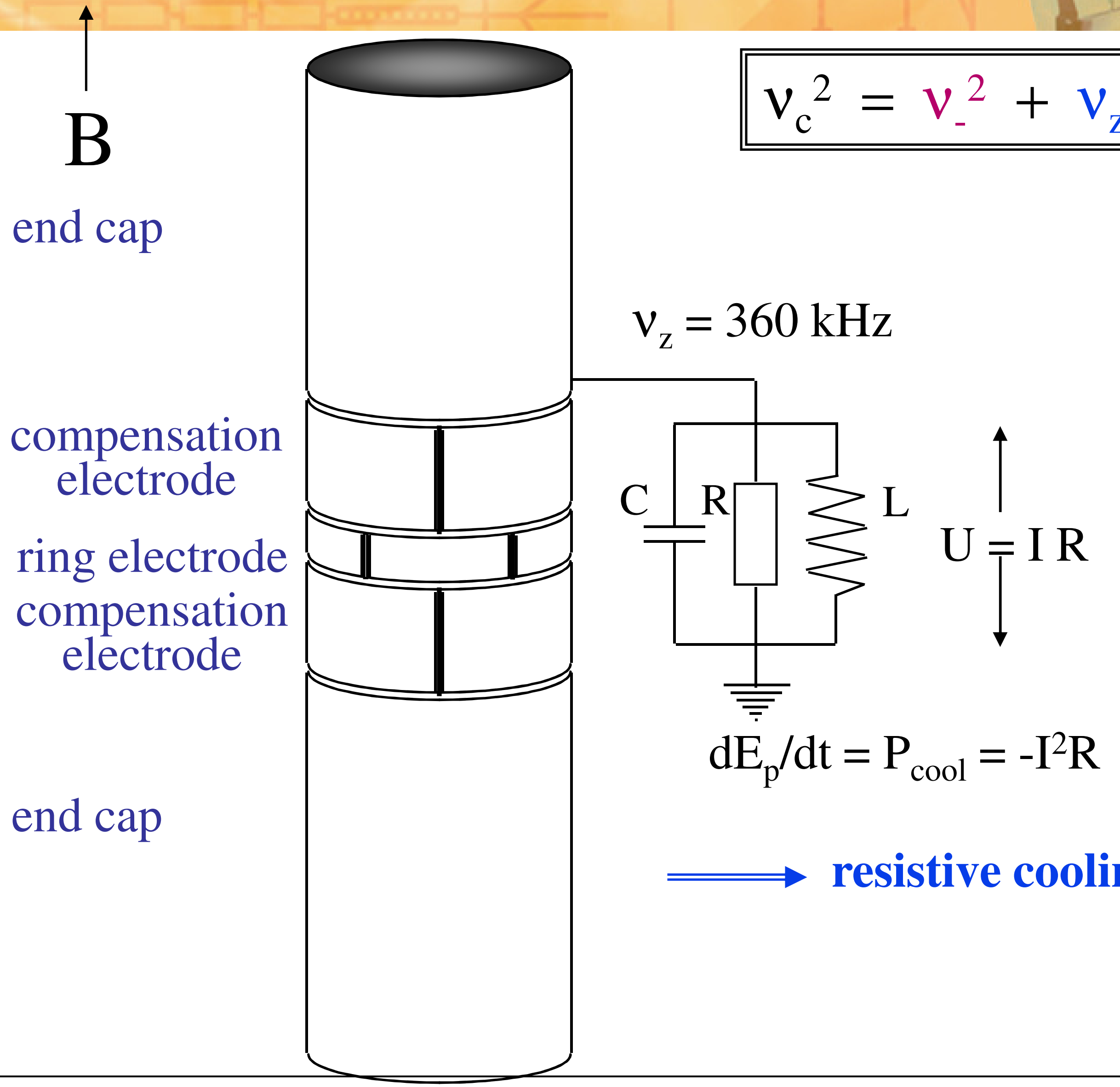


g-Factor trap in Mainz (GSI/Heidelberg collaboration)



Electronic detection and resistive cooling of trapped ions

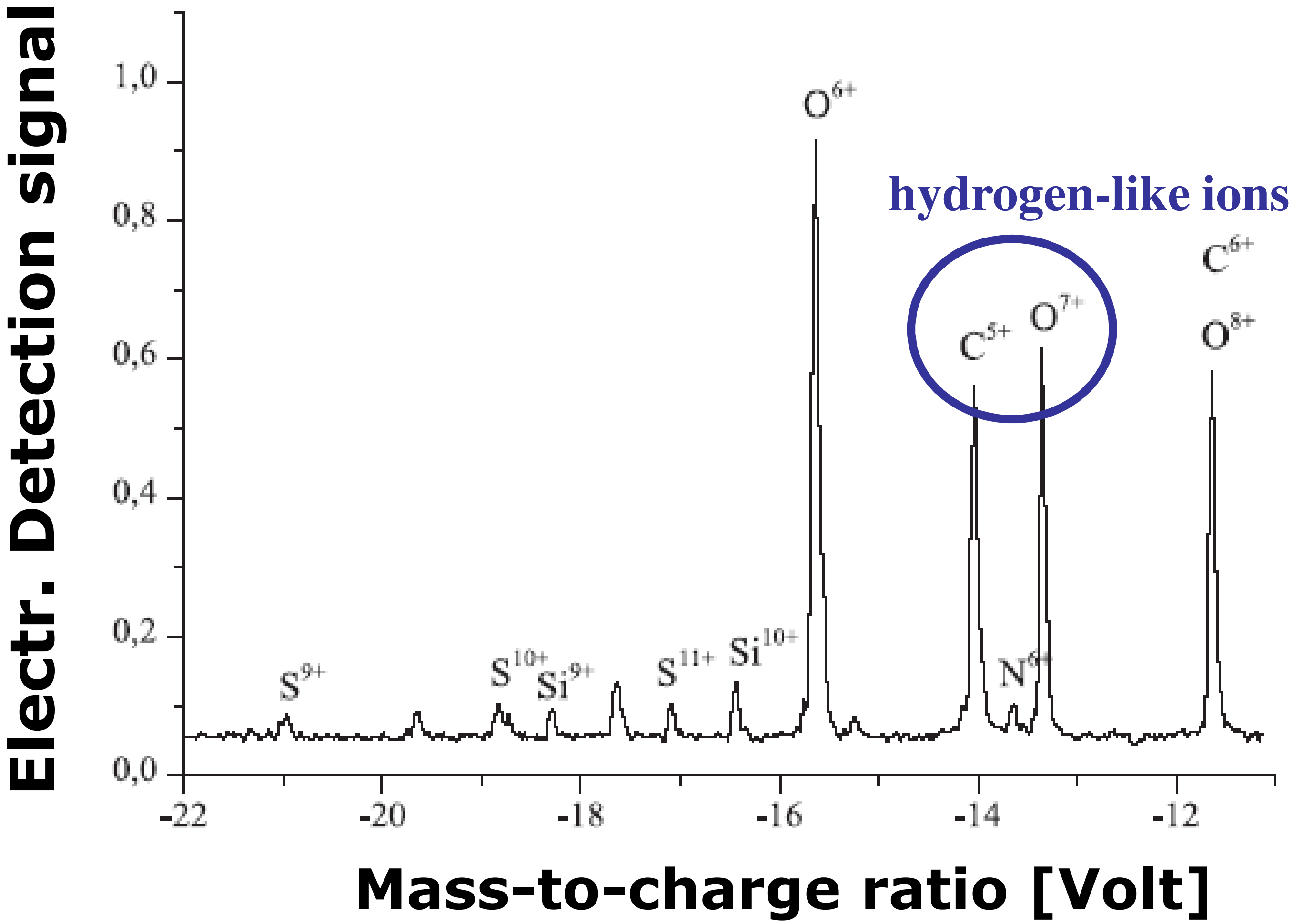
$$\nu_c^2 = \nu_-^2 + \nu_z^2 + \nu_+^2$$



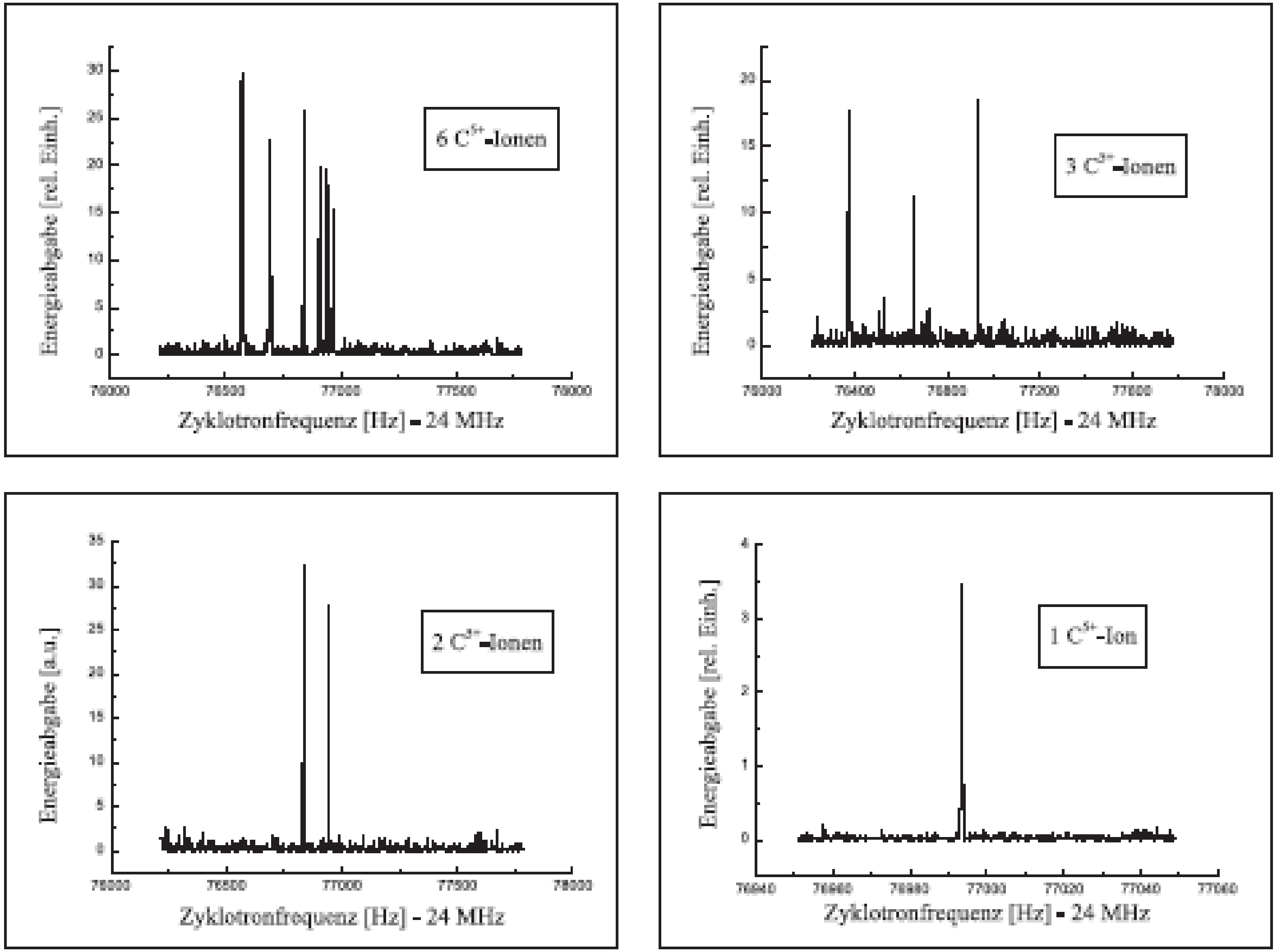
$$dE_p/dt = P_{cool} = -I^2R$$

→ **resistive cooling**

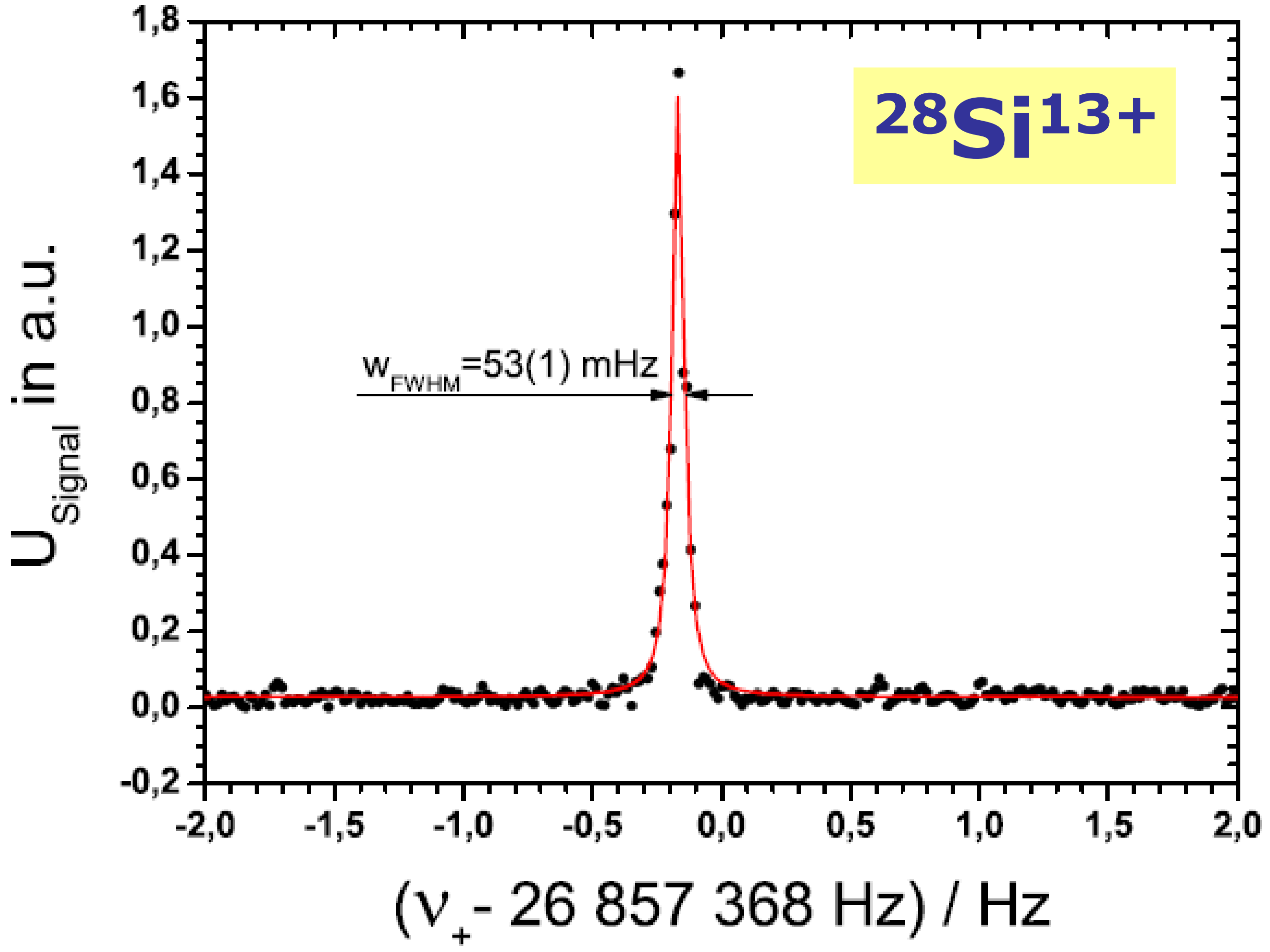
Charge breeding of carbon and oxygen ions in cryogenic EBIS (electron-beam ion source)



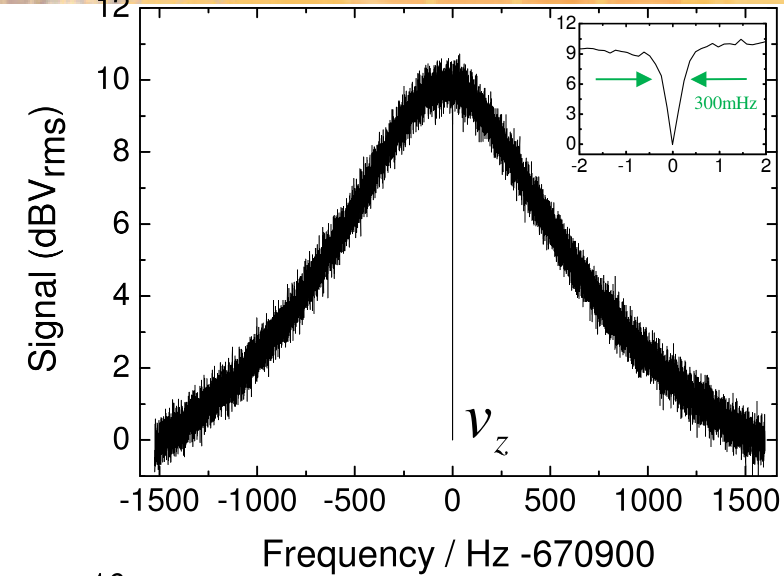
Isolating a single highly charged ion



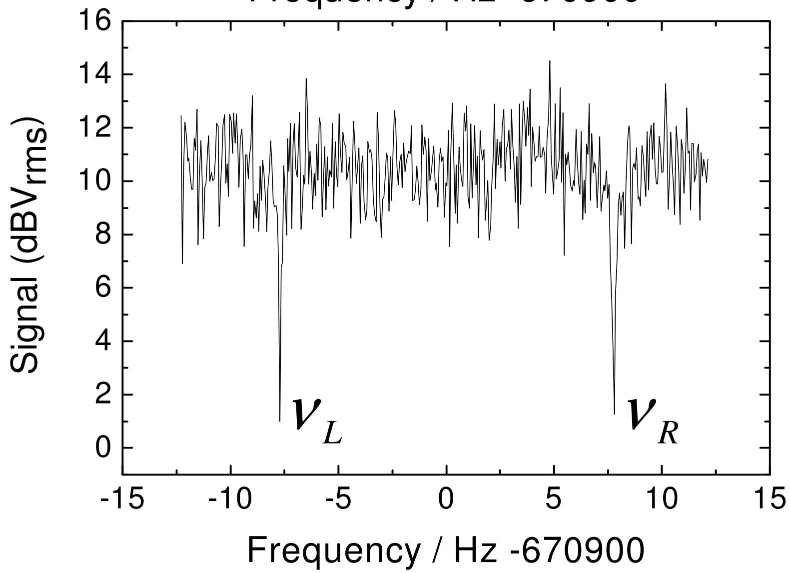
High-resolution cyclotron frequency measurement of a single highly charged silicon ion



Eigenfrequency Measurement



- Axial frequency (ν_z) directly measured as narrow "dip"



- Other modes (ν_+, ν_-) can be coupled to axial motion via *rf*-sideband coupling

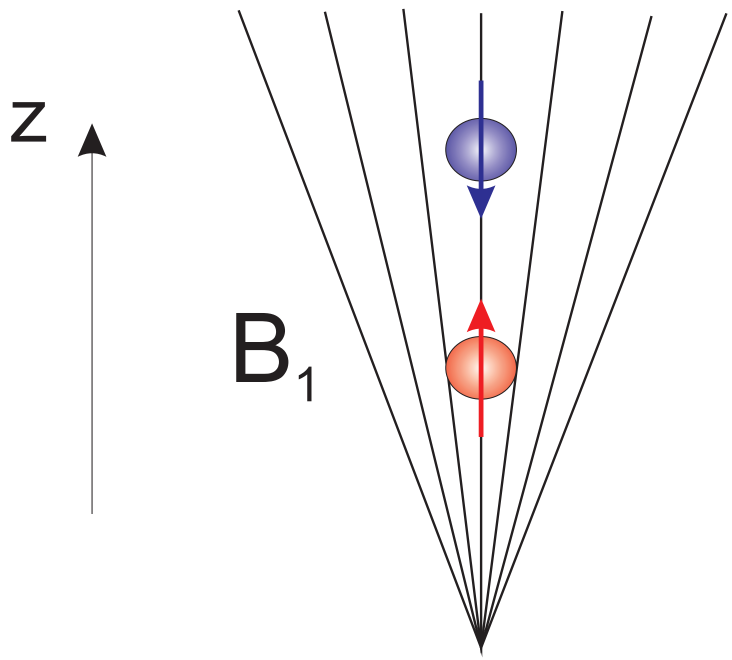
$$\nu_+ = \nu_{rf} - \nu_z + \nu_L + \nu_R$$

$$\nu_- = \nu_{rf} + \nu_z - \nu_L - \nu_R$$

Continuous Stern-Gerlach effect: Determination of spin direction

CLASSICAL STERN-GERLACH

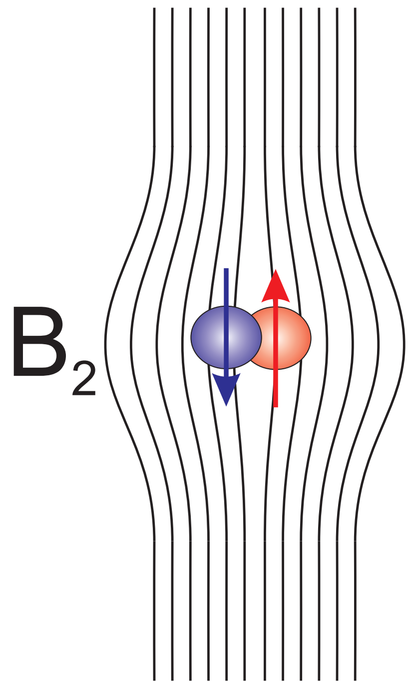
SEPARATION IN POSITION SPACE



$$\Delta z = \frac{\mu L^2}{2KE} B_1$$

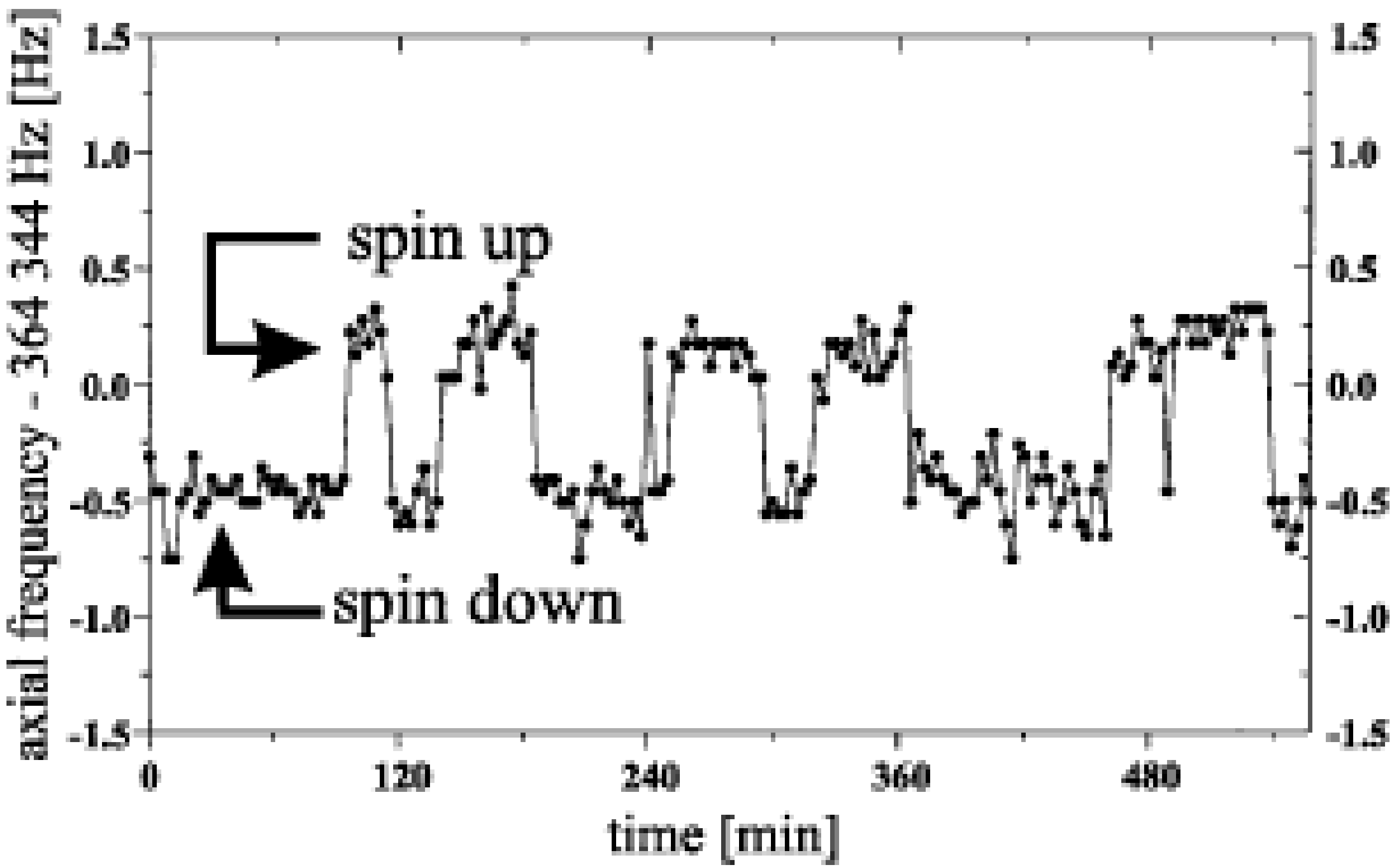
CONTINUOUS STERN-GERLACH

SEPARATION IN FREQUENCY SPACE



$$\Delta\omega_z = \frac{\mu}{m\omega_z} B_2$$

Quantum jump spectroscopy:
Spin-flip transitions in the analysis trap



The lab team (before spinflip)

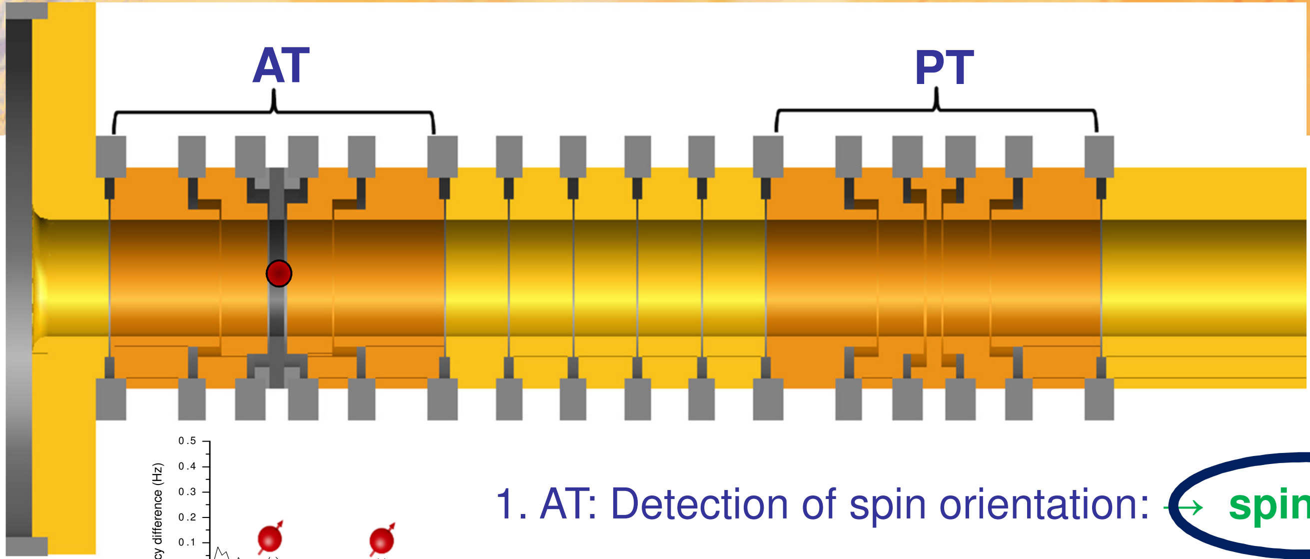
Birgit Schabinger

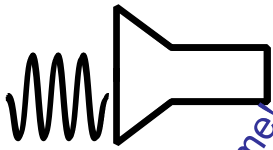
Sven Sturm

Anke Wagner

The lab team (after spinflip)

Measurement Cycle

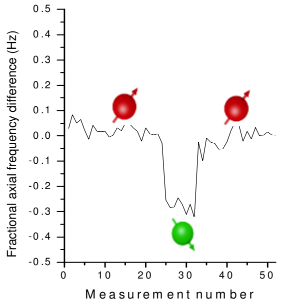




ν_L

$$\Gamma = \frac{\nu_L}{\nu_c}$$

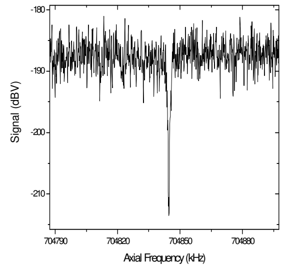
At the same time!



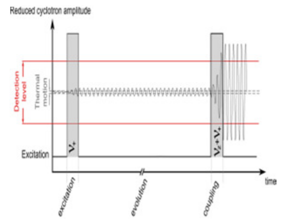
1. AT: Detection of spin orientation: → **spin up**

2. PT: Measurement of eigenfrequencies and simultaneous irradiation with microwaves

ν_z

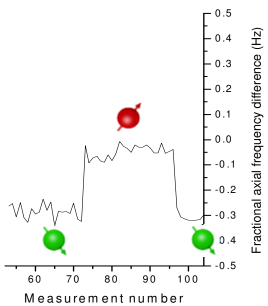


ν_+



ν_- (offline)

ν_c



3. AT: Detection of spin orientation → **spin down**

4. Spin flip in PT? → **spin has flipped!**

GSI

Precision measurement of bound electron g-factor in hydrogen-like silicon

Bound electron magnetic moment measurement on hydrogen-like silicon $^{28}Si^{13+}$

PRL 107, 023002 (2011) PHYSICAL REVIEW LETTERS week ending 8 JULY 2011

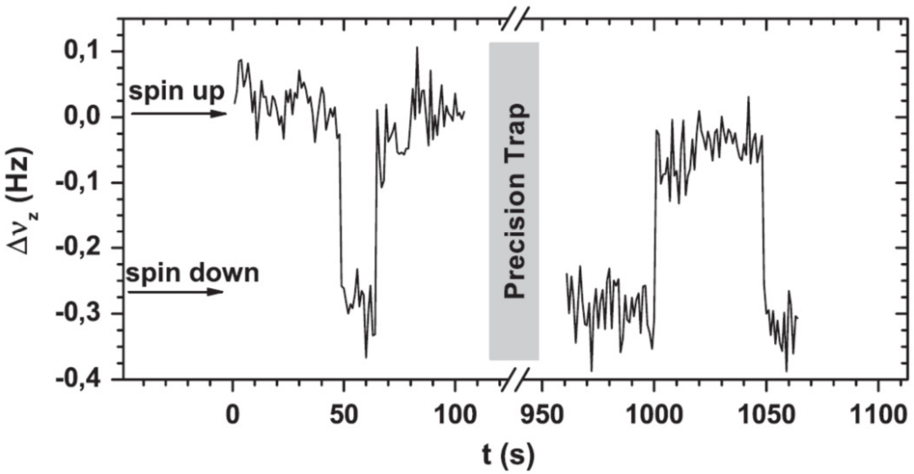
g Factor of Hydrogenlike $^{28}Si^{13+}$

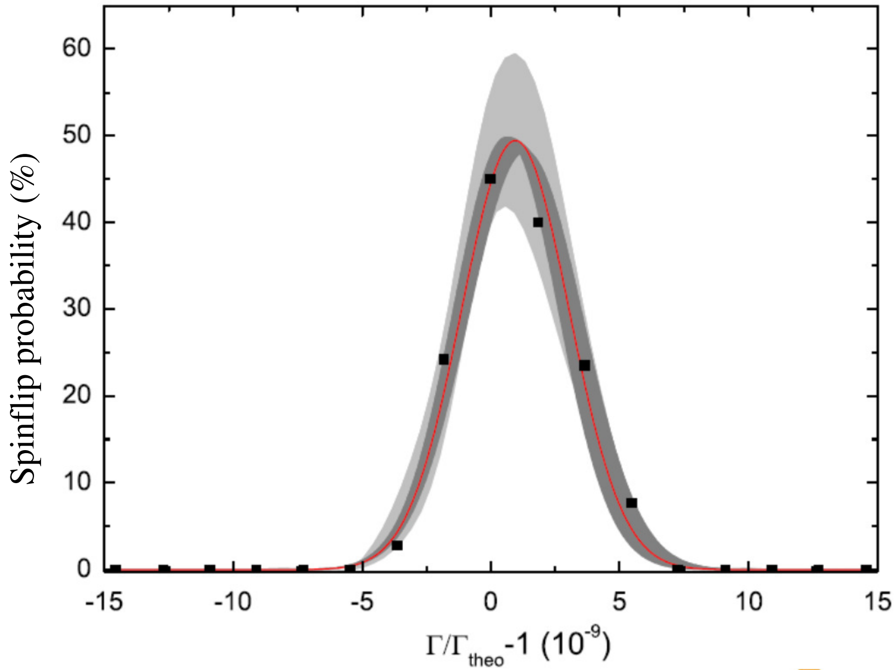
S. Sturm,[1,2] A. Wagner,[1] B. Schabinger,[1,2] J. Zatorski,[1] Z. Harman,[1,3] W. Quint,[4] G. Werth,[2] C. H. Keitel,[1] and K. Blaum[1]

[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany*
[2]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[3]*ExtreMe Matter Institute EMMI, Planckstraße 1, 64291 Darmstadt, Germany*
[4]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany*

(Received 6 May 2011; published 7 July 2011)

We determined the experimental value of the g factor of the electron bound in hydrogenlike $^{28}Si^{13+}$ by using a single ion confined in a cylindrical Penning trap. From the ratio of the ion's cyclotron frequency and the induced spin flip frequency, we obtain $g = 1.995\,348\,958\,7(5)(3)(8)$. It is in excellent agreement with the state-of-the-art theoretical value of $1.995\,348\,958\,0(17)$, which includes QED contributions up to the two-loop level of the order of $(Z\alpha)^2$ and $(Z\alpha)^4$ and represents a stringent test of bound-state quantum electrodynamics calculations.





Comparison of theory and experiment: g-Factor of the bound electron in H-like carbon $^{12}C^{5+}$, oxygen $^{16}O^{7+}$ and silicon $^{28}Si^{13+}$

$g_J(^{12}C^{5+})$ = 2.001 041 590 18 (3) theoretical value
$g_J(^{12}C^{5+})$ = 2.001 041 596 4 (10)(44) our measurement

$g_J(^{16}O^{7+})$ = 2.000 047 020 32 (11) theoretical value
$g_J(^{16}O^{7+})$ = 2.000 047 025 4 (15)(44) our measurement

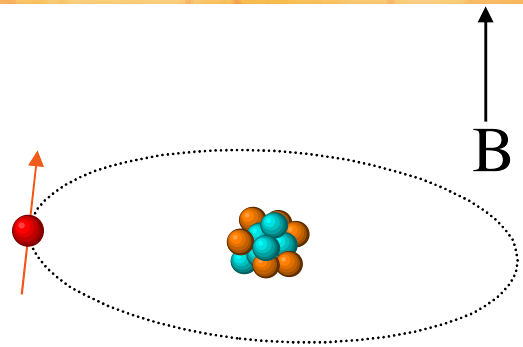
$g_J(^{28}S^{13+})$ = 1.995 348 958 0 (17) theoretical value
$g_J(^{28}S^{13+})$ = 1.995 348 958 7 (5)(3)(8) our measurement

Lit.:
T. Beier et al., PRL 88, 011603 (2002)
V. Shabaev et al., PRL 88, 091801 (2002)
V. Yerokhin et al., PRL 89, 143001 (2002)
K. Pachucki, V. Yerokhin et al., PRA 72, 022108 (2005)
S. Sturm et al., PRL 107, 023002 (2011)

Electron mass

Larmor precession frequency of the bound electron:

$$\omega_L^e = \frac{g_J}{2}\frac{e}{m_e}B$$



Ion cyclotron frequency:

$$\omega_c^{ion} = \frac{Q}{M_{ion}}B$$

$$\frac{m_e}{M_{ion}} = \frac{g_J}{2}\cdot\frac{\omega_c^{ion}}{\omega_L^e}\cdot\frac{e}{Q}$$

→ determination of electron mass

theory as input parameter

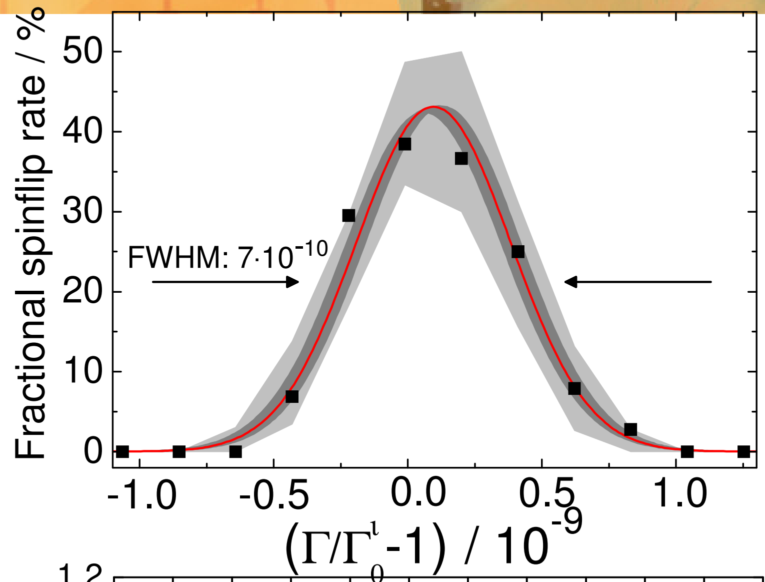
our measurement

Experimental Result

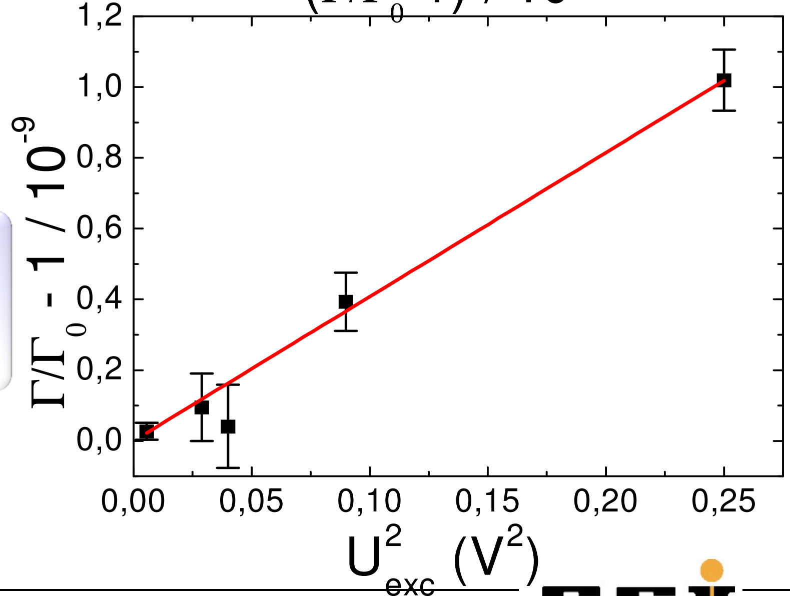
- Probing Zeeman transition at different Γ's several 100 times:

  → ***Γ*-resonance**

  - Γ_0' is extracted from fit
- Resonance width and thus statistical error limited by magnetic field fluctuations



- Several *Γ*-resonances at different cyclotron energies to check systematics
  → extrapolation to zero energy



(stat.) (syst.)

$$\Gamma_0 = 4\,736.\,210\,500\,89\,(11)(7)$$

- <u>Dominant systematics</u>:
- image charge shift: -282(14) ppt

Result

Experiment

$\Gamma_0 = 4\,736.\,210\,500\,89\,(11)(7)$
(stat.) (syst.)

Theory

$g_{theo} = 2.\,001\,041\,590\,176\,(6)$

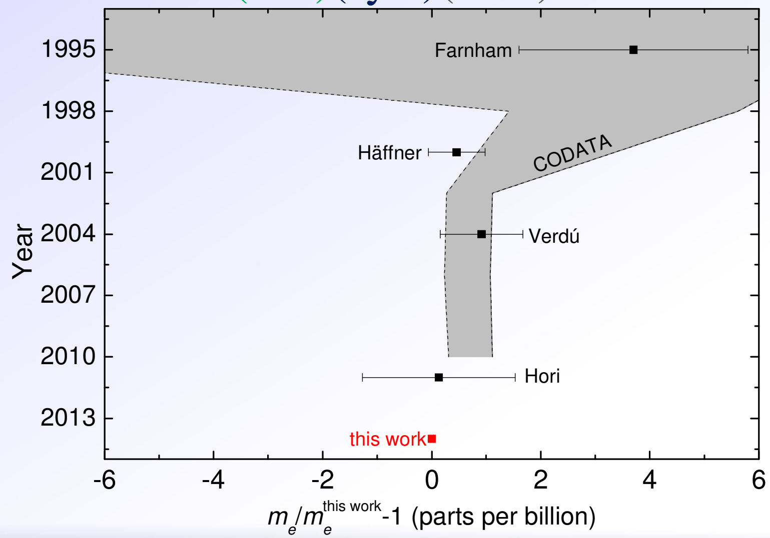
$$m_e = m_{ion} \frac{g_{theo}}{2} \frac{e}{q_{ion}} \frac{1}{\Gamma_0}$$

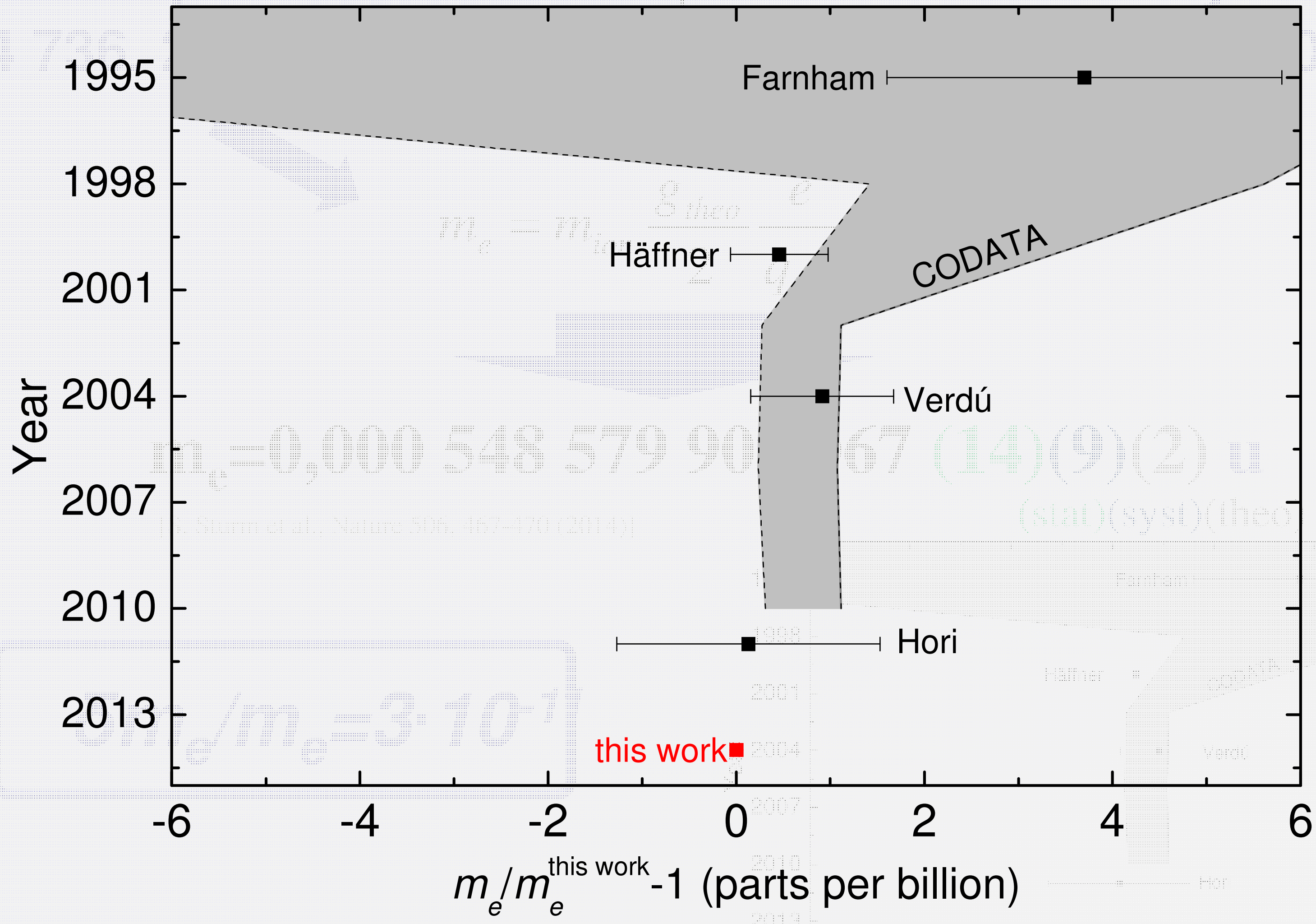
m_e=0,000 548 579 909 067 (14)(9)(2) u

[S. Sturm et al., Nature 506, 467-470 (2014)]

(stat)(syst)(theo)

$\delta m_e / m_e = 3 \cdot 10^{-11}$




Farnham
Häffner
CODATA
Verdú
Hori
this work
1995
1998
2001
2004
2007
2010
2013
Year
-6
-4
-2
0
2
4
6
$m_e/m_e^{\text{this work}}$-1 (parts per billion)

LETTER

doi:10.1038/nature13026

High-precision measurement of the atomic mass of the electron

S. Sturm[1], F. Köhler[1,2], J. Zatorski[1], A. Wagner[1], Z. Harman[1,3], G. Werth[4], W. Quint[2], C. H. Keitel[1] & K. Blaum[1]

Table 1 | Relative systematic corrections and their uncertainties applied to the measured frequency ratio

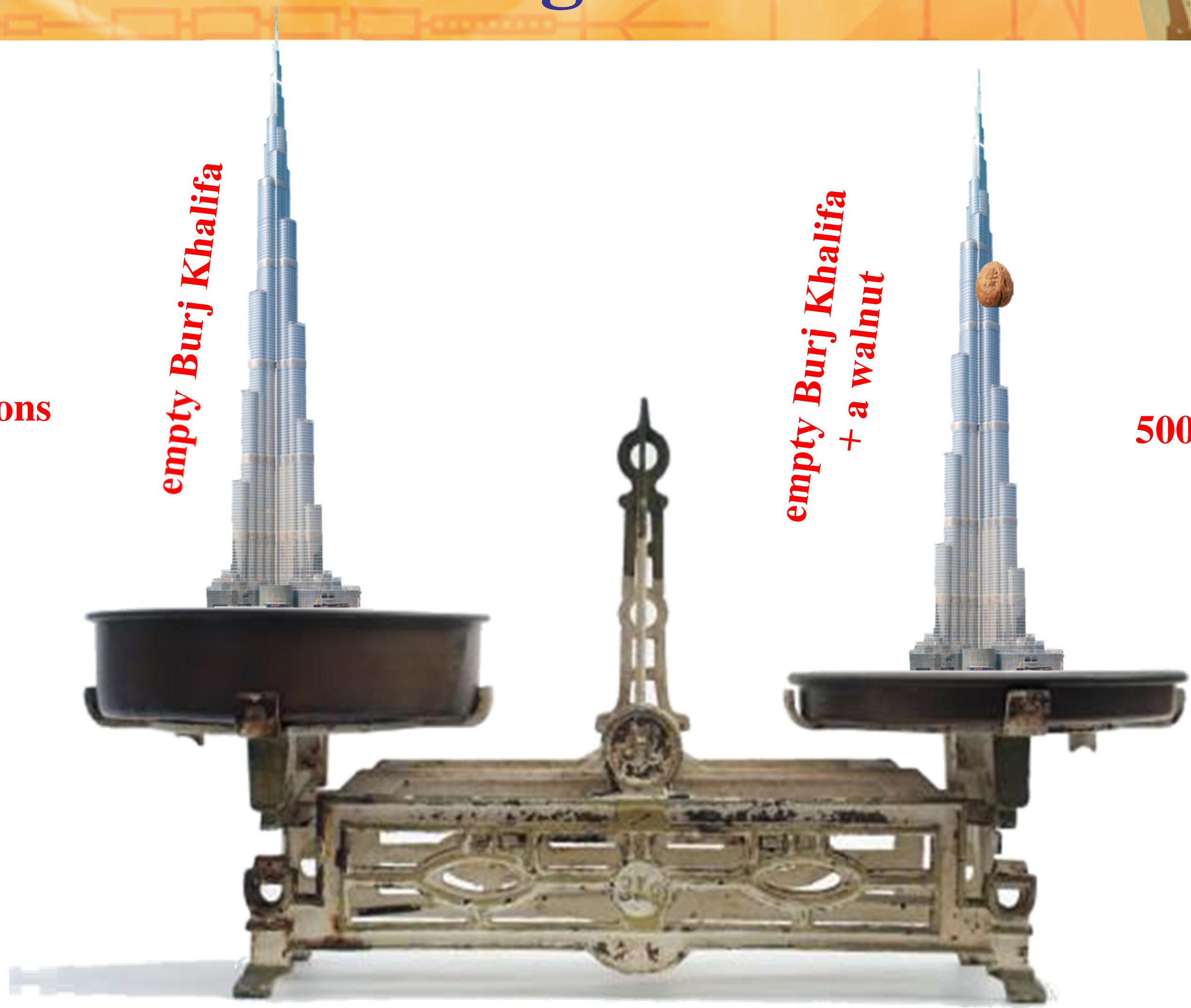
Effect	Correction (parts per trillion)	Uncertainty (parts per trillion)
Image charge	−282.4	14.1
Image current	2.2	0.5
Residual electrostatic anharmonicity	0	0.25
Axial and magnetron temperature	0.04	0.04
Ionic mass $^{12}C^{5+}$	0	0.1

$$m_e = 0.000548579909067(14)(9)(2) \qquad (5)$$

The first two errors are the statistical and systematic uncertainties of the measurement, and the third error represents the uncertainties of the theoretical prediction of the *g*-factor and the electron binding

Who has forgotten the walnut?

500000 tons



500000.000015 tons

Relative Precision: $3 \cdot 10^{-11}$

Profit of an improved electron mass m_e

Important ingredient in fine-structure constant measurement:

$$\alpha^2_{recoil} = \frac{2R_\infty h}{c\, m_e} = \frac{2R_\infty}{c}\frac{M_{Rb}}{m_e}\frac{h}{M_{Rb}}$$

5 ppt, CODATA (T. Hänsch) 2010 — R_∞

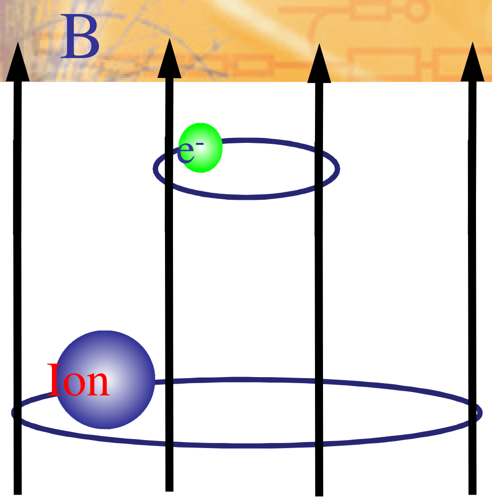
115 ppt, E. Myers 2010 — M_{Rb}

30 ppt, our value — m_e

1241 ppt, F. Biraben 2011 — h/M_{Rb}

$$v_{recoil} = \frac{\hbar k}{M_{Rb}}$$

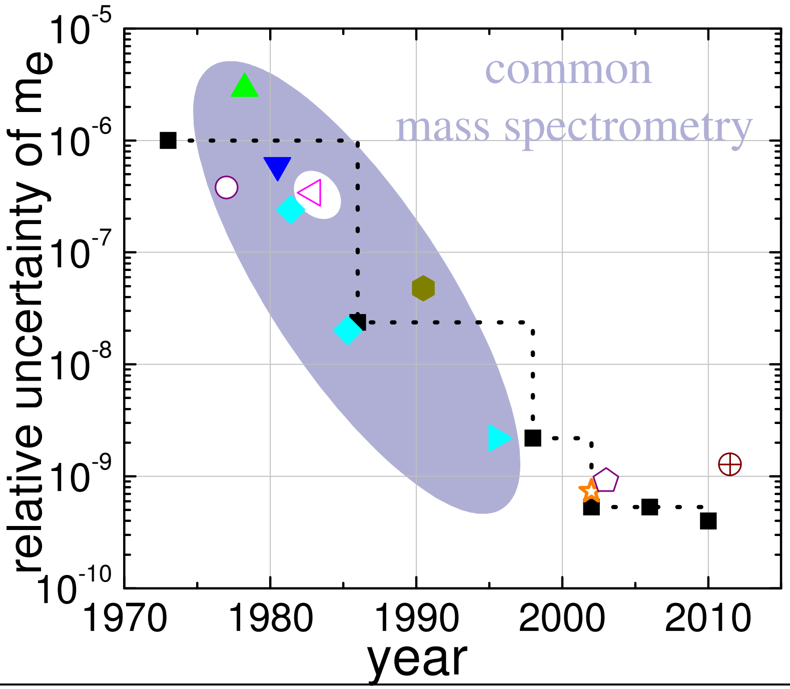
Measurements of the electron mass over the years



$$\nu_c^{electron} = \frac{e}{2\pi m_e} B$$

$$\nu_c^{ion} = \frac{q}{2\pi M} B$$

$$m_e = \frac{e}{q} \frac{\nu_c^{ion}}{\nu_c^{electron}} M$$



- CODATA
- Indirect measurement via μ_B/μ_N
- Gärtner: ν_c via particle loss **(first direct)**
- Gräff: ν_c via ToF
- Van Dyck: ν_c via image currents **(first non-destructive)**
- Wineland: ν_c via laser fluorescence + ν_L + g_{theo} **(first bound electron)**
- Gabrielse: ν_c via image currents
- Farnham: ν_c via image currents **(first single ion)**
- Häffner: ν_c/ν_L of $^{12}C^{5+}$ + g_{theo}
- Verdú: ν_c/ν_L of $^{16}O^{7+}$ + g_{theo}
- Hori: antiprotonic helium

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ