Muonic hydrogen and the Proton Radius Puzzle

Nuclear structure from Laser spectroscopy of light muonic atoms

Randolf Pohl
for the
CREMA collaboration

CREMA collaboration

Charge Radius Experiment with Muonic Atoms

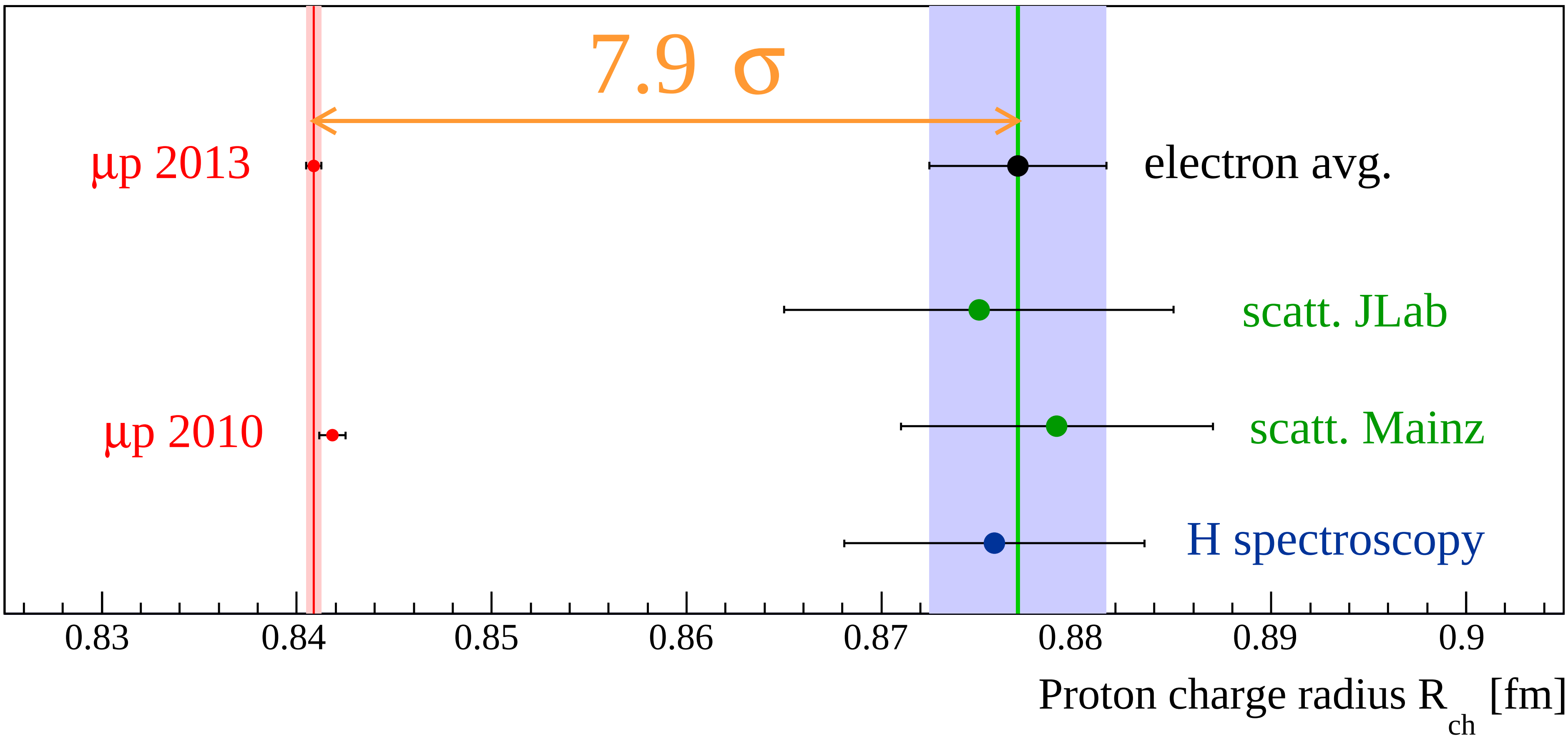
M. Diepold, B. Franke, J. Götzfried, T.W. Hänsch, J. Krauth, F. Mulhauser, T. Nebel, R. Pohl	MPQ, Garching, Germany
A. Antognini, K. Kirch, F. Kottmann, B. Naar, K. Schuhmann, D. Taqqu	ETH Zürich, Switzerland
M. Hildebrandt, A. Knecht, A. Dax	PSI, Switzerland
F. Biraben, P. Indelicato, E.-O. Le Bigot, S. Galtier, L. Julien, F. Nez, C. Szabo-Foster	Labor. Kastler Brossel, Paris, France
F.D. Amaro, J.M.R. Cardoso, L.M.P. Fernandes, A.L. Gouvea, J.A.M. Lopez, C.M.B. Monteiro, J.M.F. dos Santos	Uni Coimbra, Portugal
D.S. Covita, J.F.C.A. Veloso	Uni Aveiro, Portugal
M. Abdou Ahmed, T. Graf, A. Voss, B. Weichelt	IFSW, Uni Stuttgart, Germany
T.-L. Chen, C.-Y. Kao, Y.-W. Liu	Nat. Tsing Hua Uni, Hsinchu, Taiwan
P. Amaro, J.F.D.C. Machado, J.P. Santos	Uni Lisbon, Portugal
L. Ludhova, P.E. Knowles, L.A. Schaller	Uni Fribourg, Switzerland
A. Giesen	Dausinger & Giesen GmbH, Stuttgart, Germany
P. Rabinowitz	Uni Princeton, USA

The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm

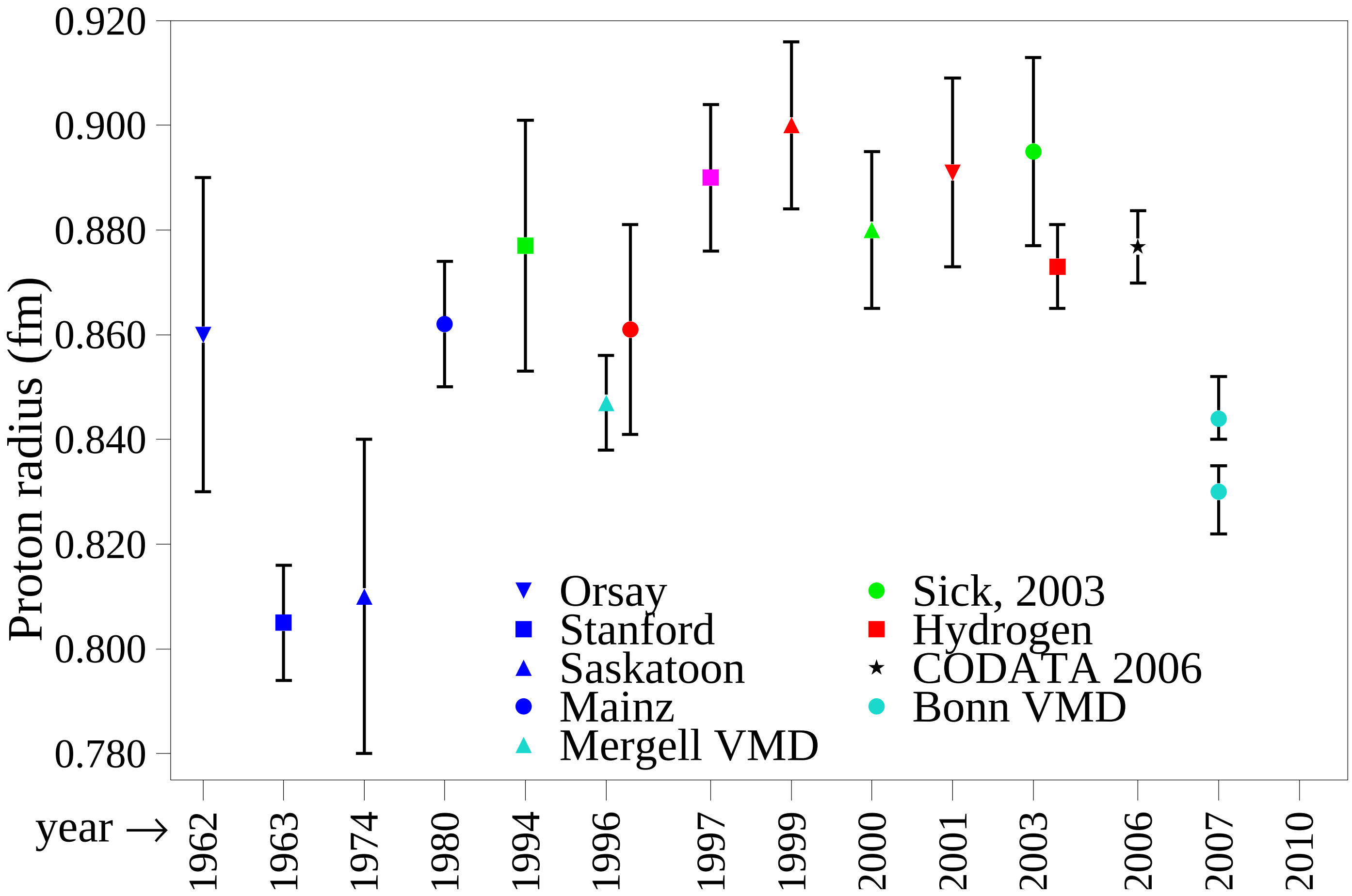


Outline

- Muonic **hydrogen**
  - 2 resonances measured in 2009
  - #1: Pohl et al., Nature (2010) $\longrightarrow$ proton charge radius
  - #2: Antognini et al., Science (2013) $\longrightarrow$ Zemach radius
  - theory in $\mu\mathrm{p}$: Antognini et al. Ann. Phys. (2013)
- Muonic **deuterium**
  - 3 resonances measured in 2009. Analysis (nearly) done
  - Theory in $\mu\mathrm{d}$
  - Results (still somewhat prel.)
- Muonic **helium-4** and **helium-3**
  - 5 transitions measured in 2013/4:
  - Both transitions measured in in $(\mu^4\mathrm{He})^+$
    $\Longrightarrow$ Lamb shift and Fine structure
  - 3 transitions measured in in $(\mu^3\mathrm{He})^+$
    $\Longrightarrow$ Lamb shift, 2S-HFS, 2P-FS/HFS

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.



electron scattering

slope of G_E at $Q^2 = 0$

hydrogen spectr.

Lamb shift (S-states)

e-p scattering: r_p = 0.895(18) fm $(u_r = 2\,\%)$

Hydrogen: r_p = 0.8760(78) fm $(u_r = 0.9\,\%)$

Proton radius vs. time

The proton rms charge radius is not the most accurate quantity in the universe.

Proton radius (fm)

0.92

0.90

0.88

0.86

0.84

0.82

0.80

0.78

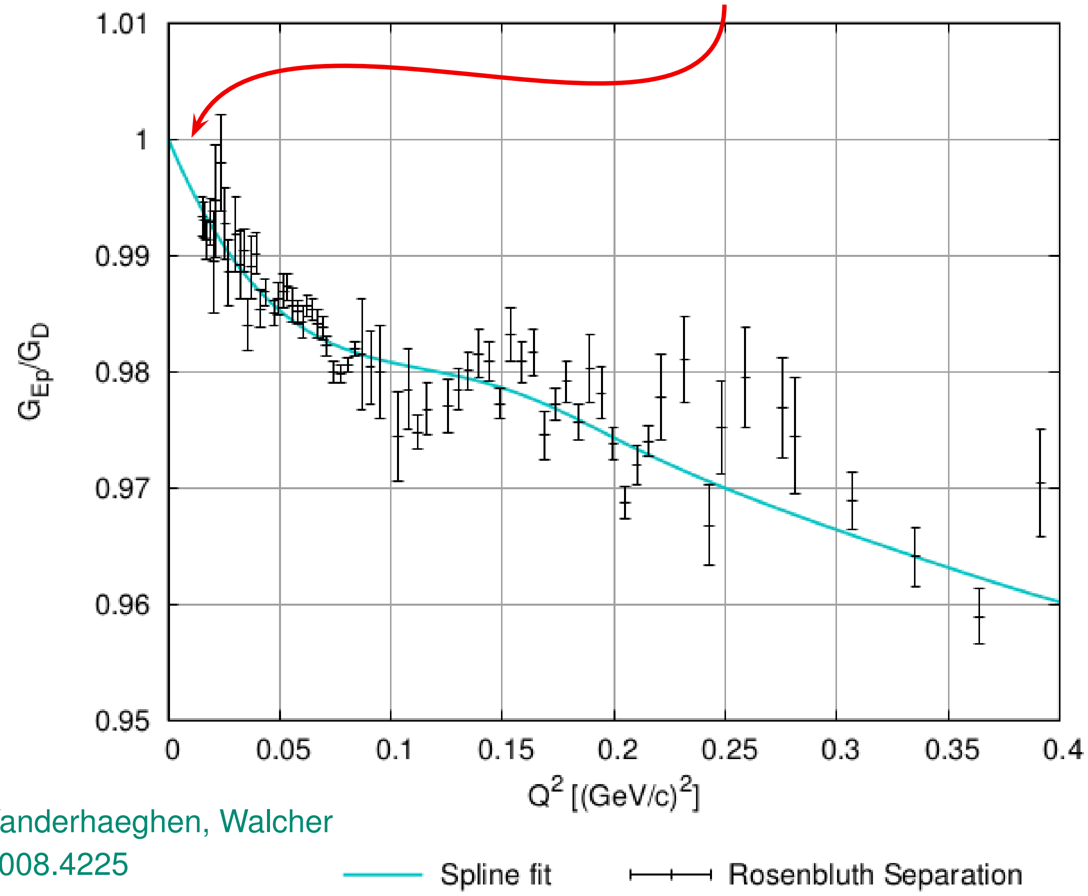
year

e-p

Hy

Electron scattering:

$$\langle r_p^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0} \Rightarrow \boxed{\text{slope of } G_E \text{ at } Q^2 = 0}$$



Vanderhaeghen, Walcher
1008.4225

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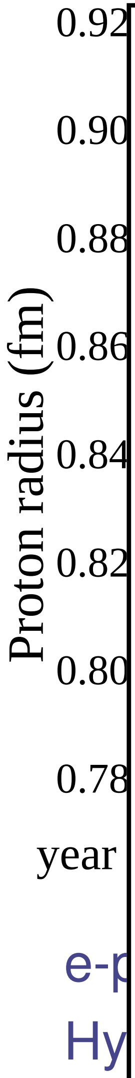
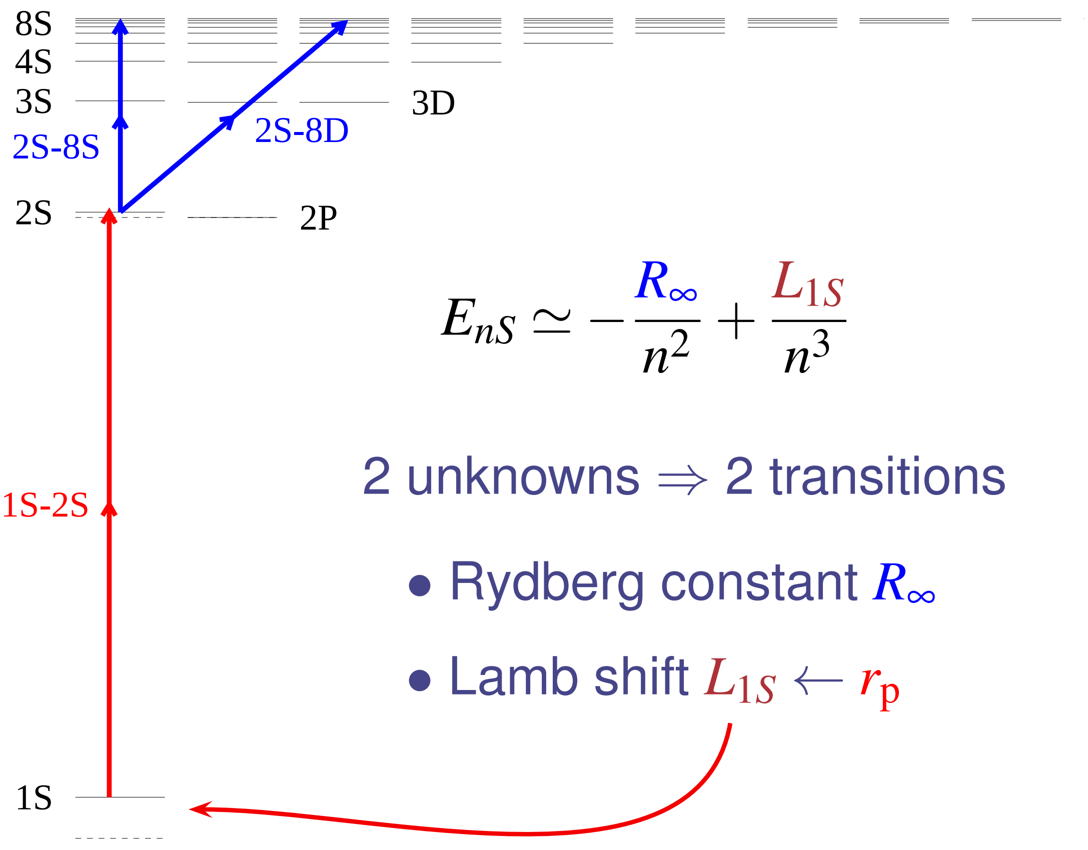


Hydrogen spectroscopy (Lamb shift):

$$\boxed{L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \quad \text{MHz}}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns $\Rightarrow$ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

- electron scattering

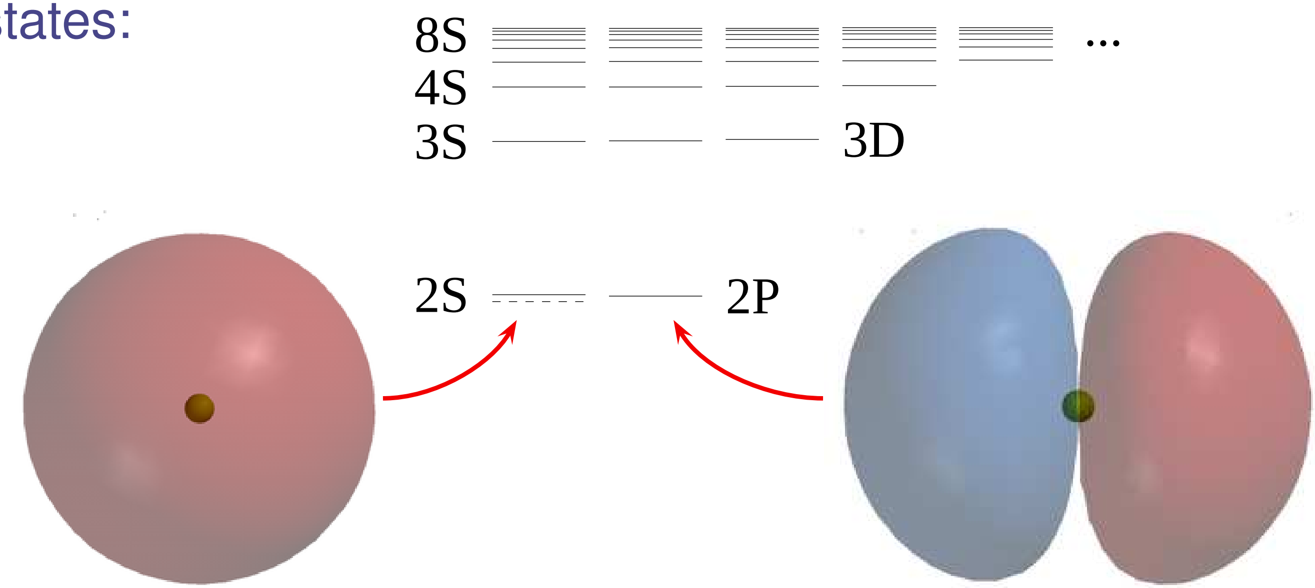
slope of G_E at $Q^2 = 0$

- hydrogen spectr.

Lamb shift (S-states)

Atomic physics

Wave functions of S and P states:



S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the

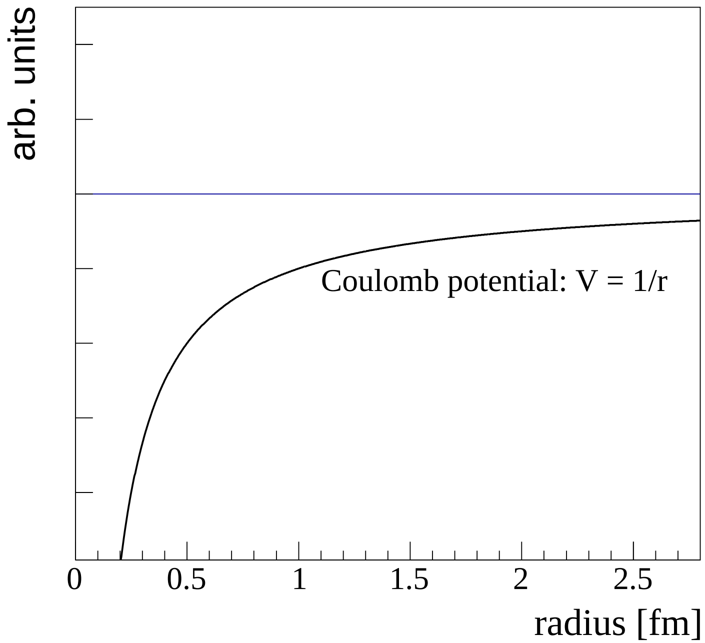
size of the proton

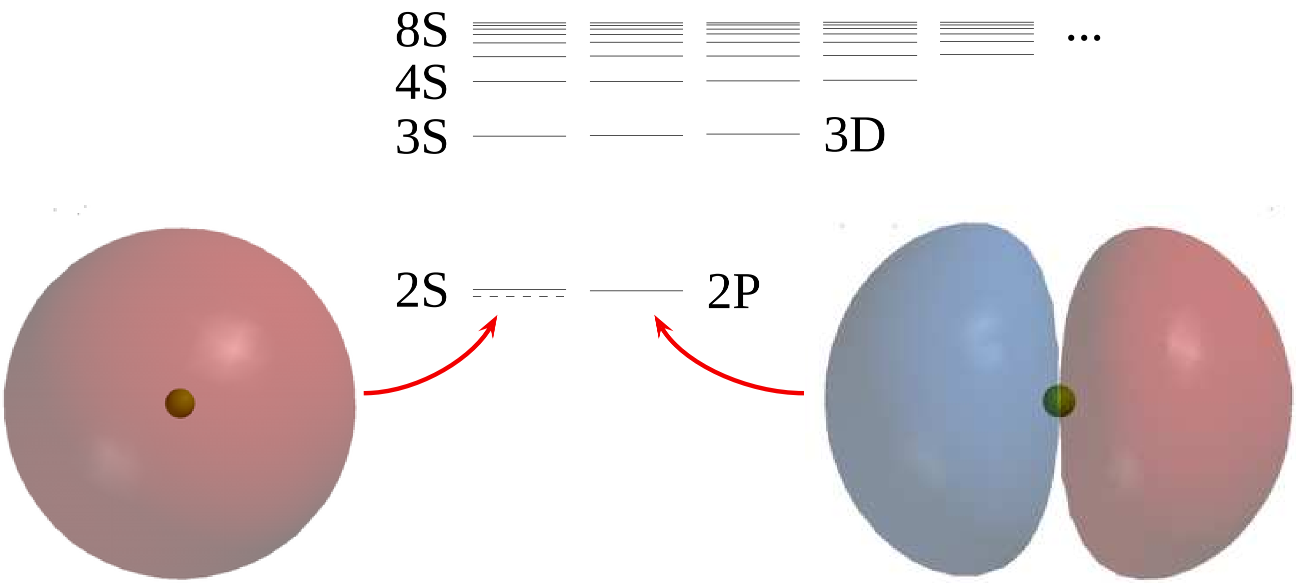
P states: zero at r=0

Electron is not inside the proton.

Orbital pictures from Wikipedia

Atomic physics





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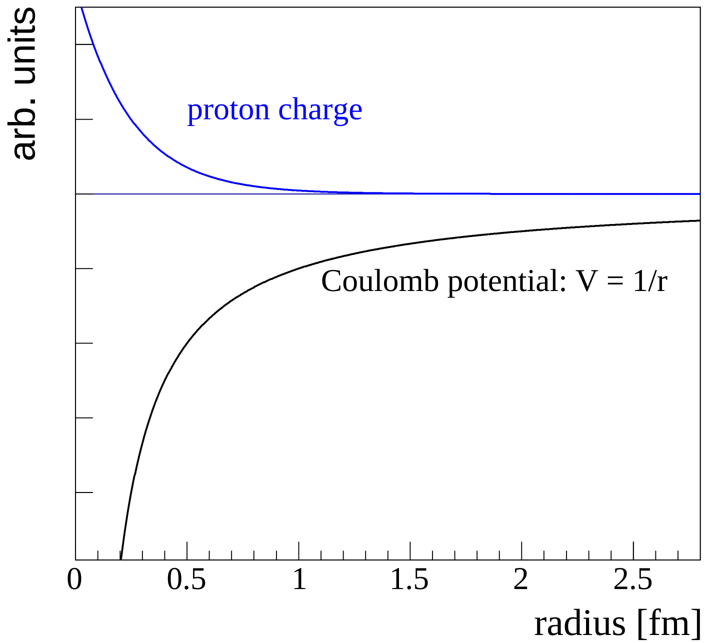
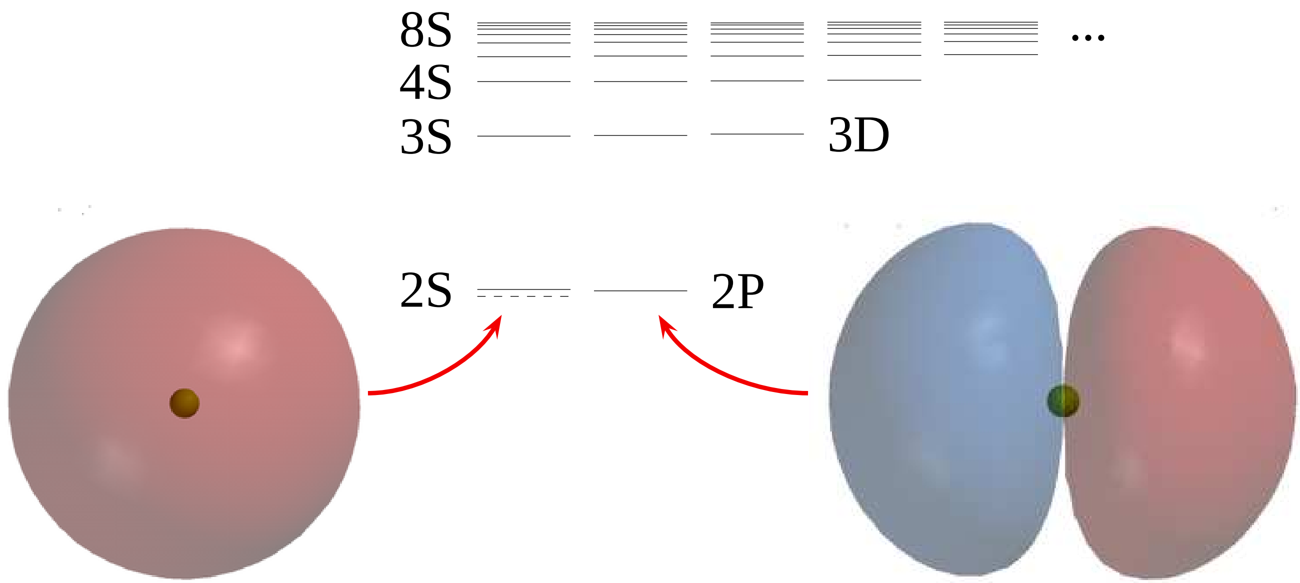
Atomic and nuclear physics





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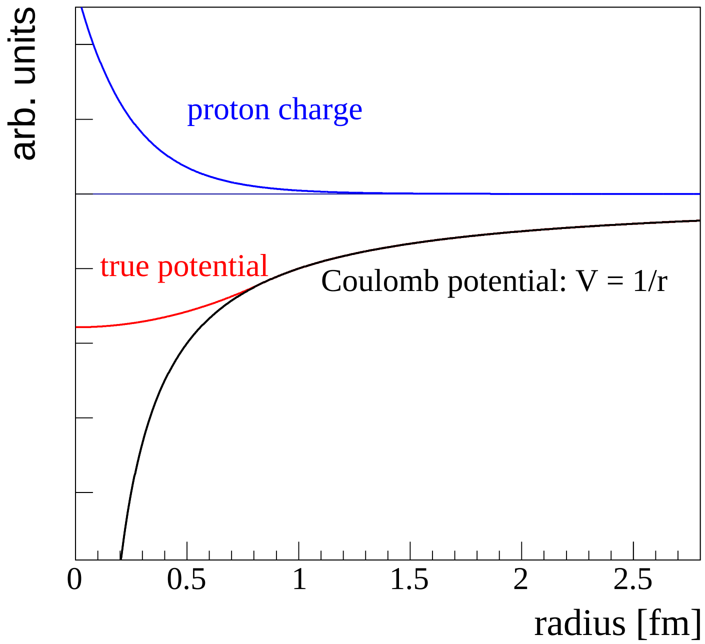

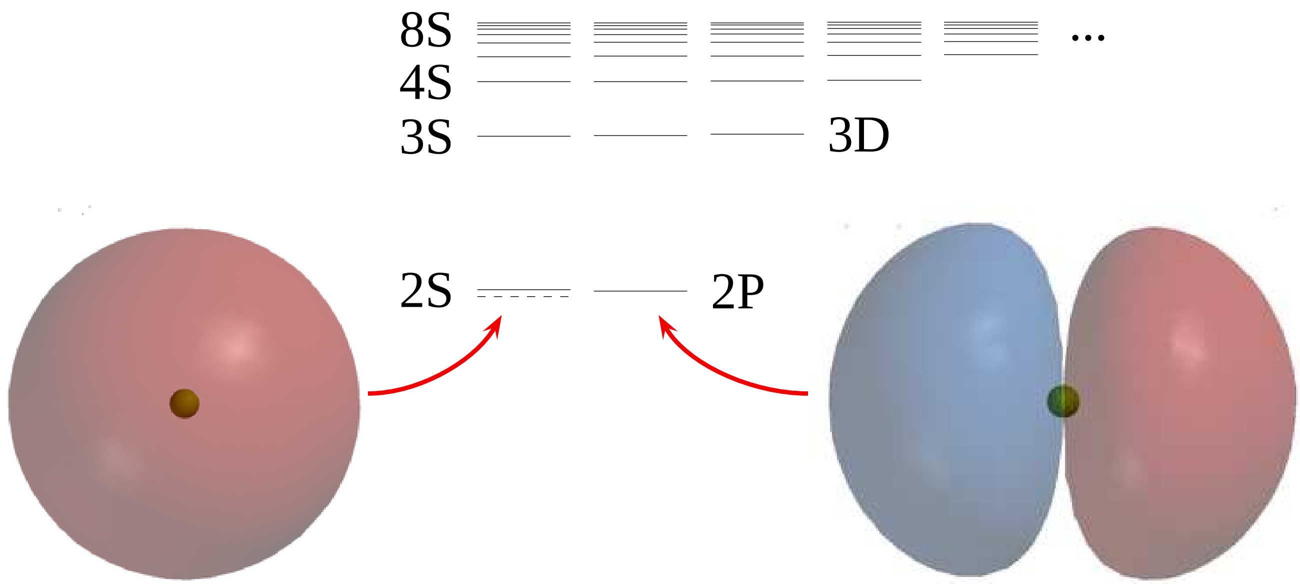

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Orbital pictures from Wikipedia

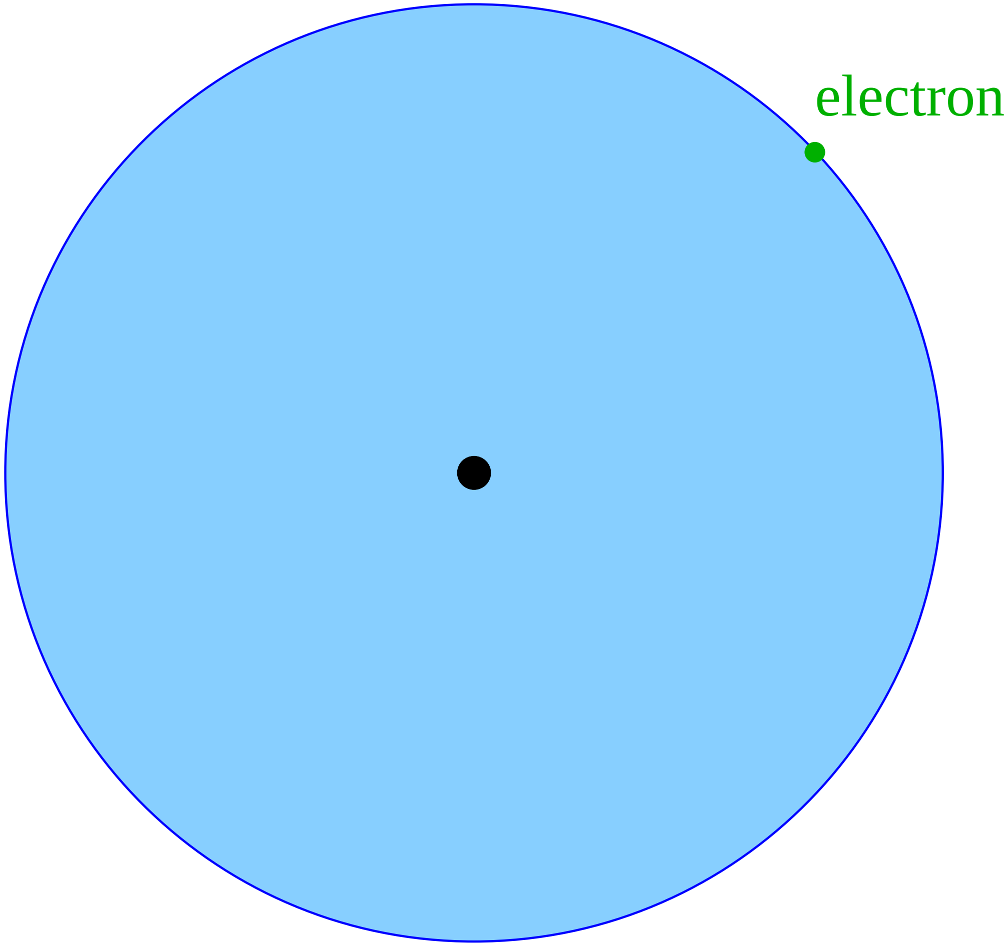
Muonic hydrogen

Regular hydrogen:

electron e^- + proton p

Muonic hydrogen:

muon μ^- + proton p



Muonic hydrogen

Regular hydrogen:

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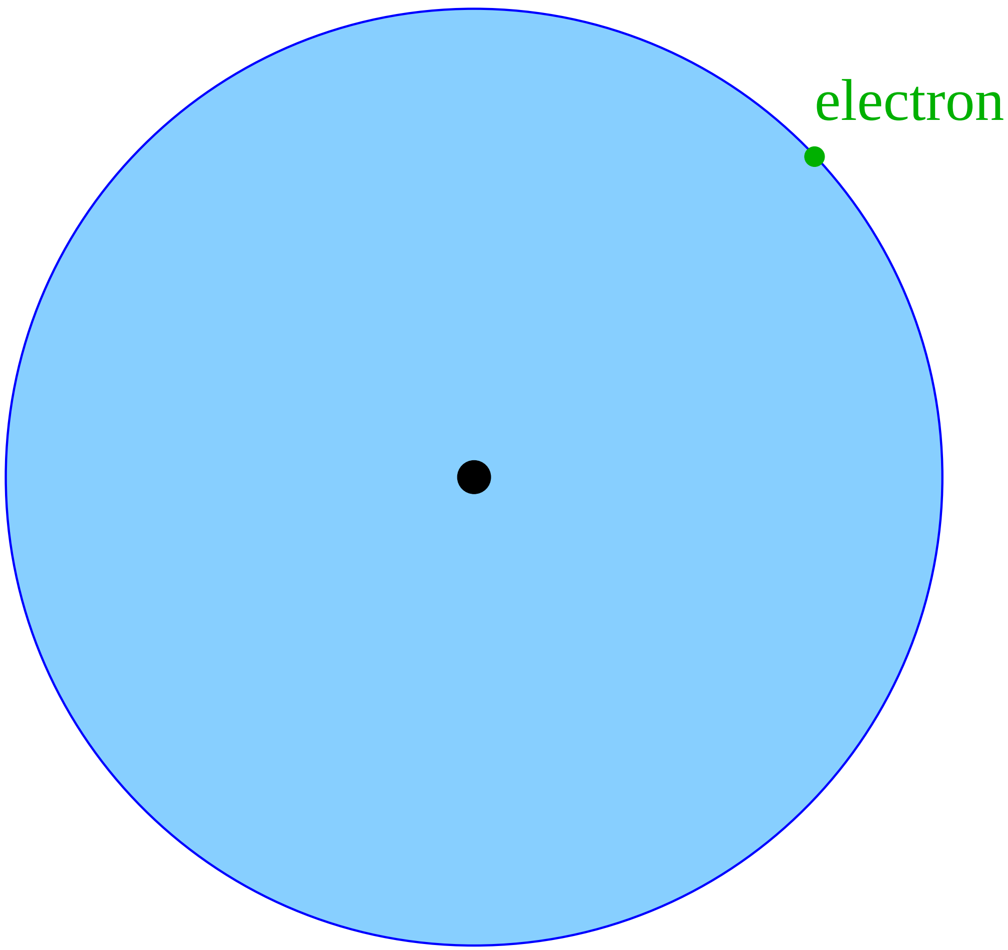

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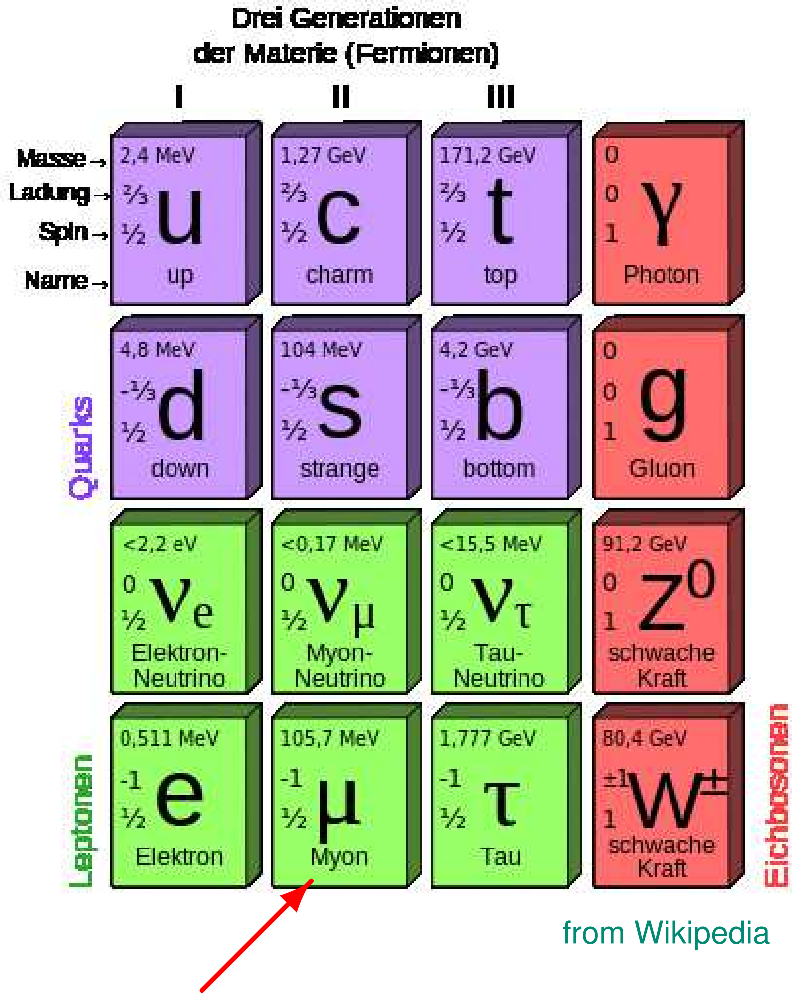
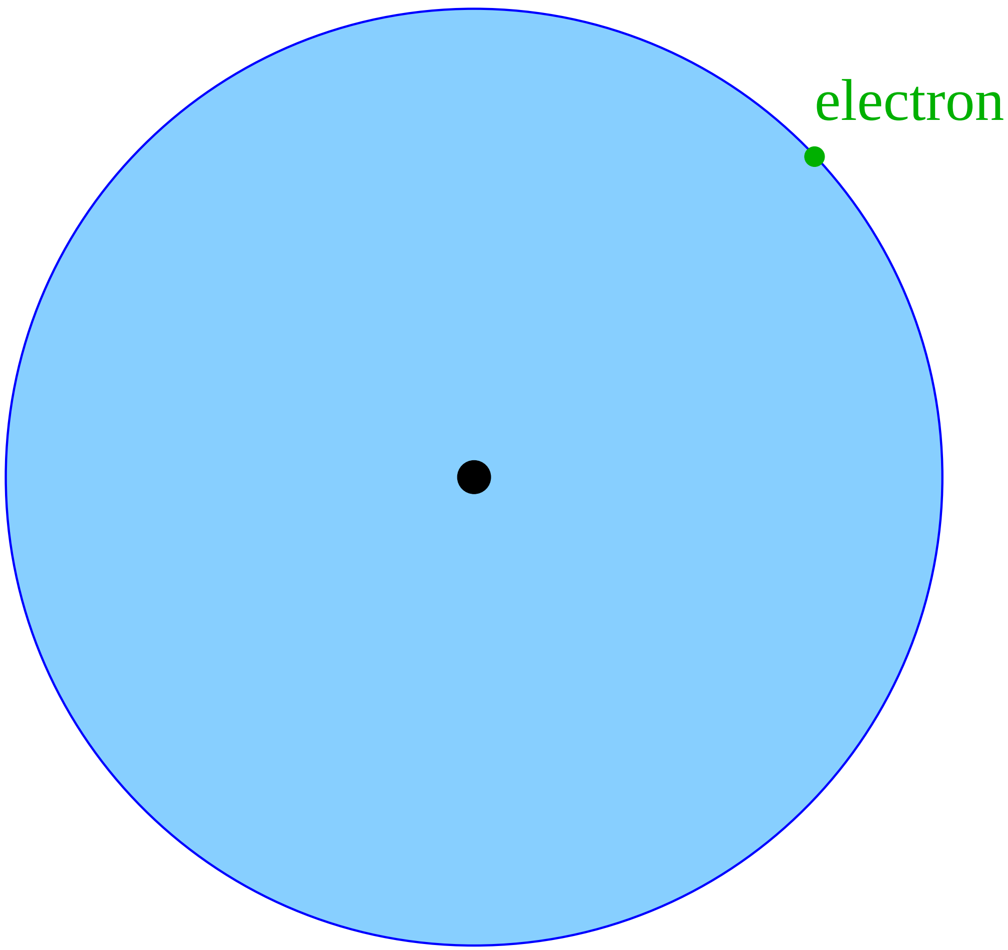


from Wikipedia

Muonic hydrogen

Regular hydrogen:

electron e^- + proton p



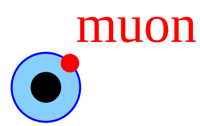
Muonic hydrogen:

muon μ^- + proton p

muon mass $m_\mu \approx$ 200 $\times m_e$

Bohr radius $r_\mu \approx$ 1/200 $\times r_e$

μ inside the proton: $200^3 \approx 10^7$



muon **much** is more sensitive to r_p

Proton charge radius and muonic hydrogen

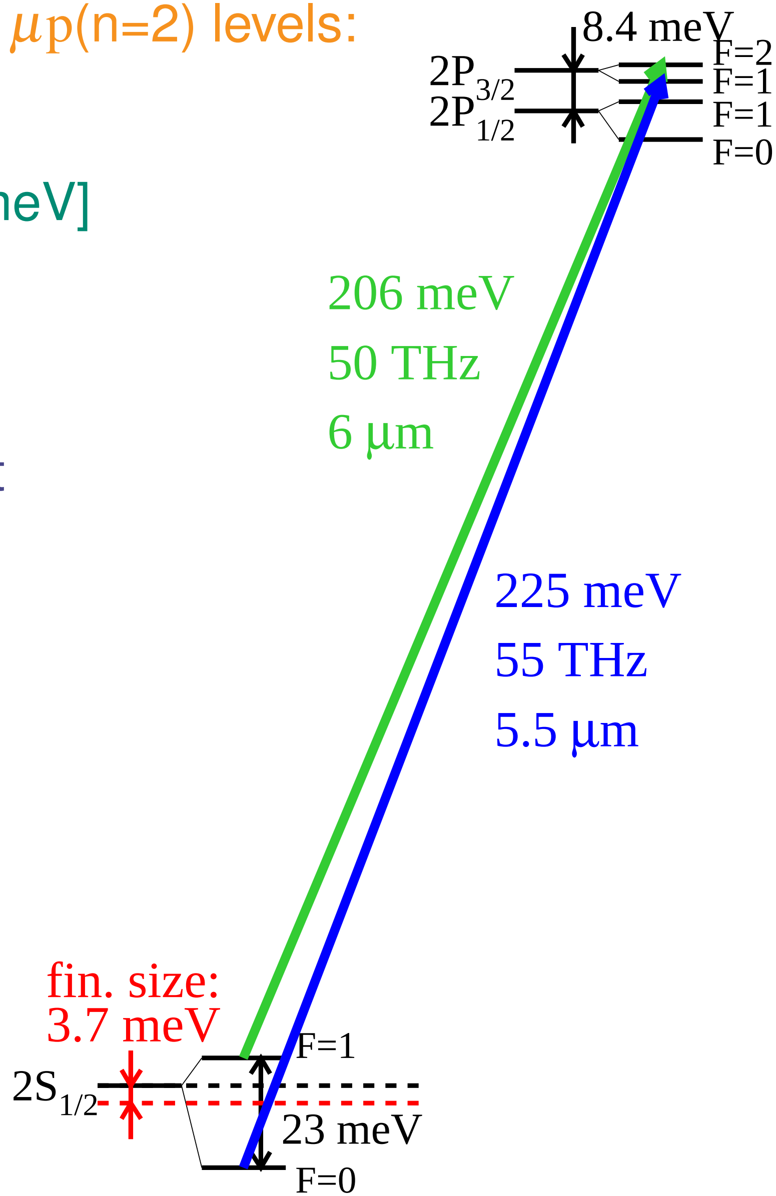
Lamb shift in μp [meV]:

$$\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \quad \text{[meV]}$$

Proton size effect is 2% of the μp Lamb shift

Measure to 10^{-5} $\Rightarrow$ r_p to 0.05 %

μp(n=2) levels:



Experiment:

R. Pohl *et al.*, Nature 466, 213 (2010).

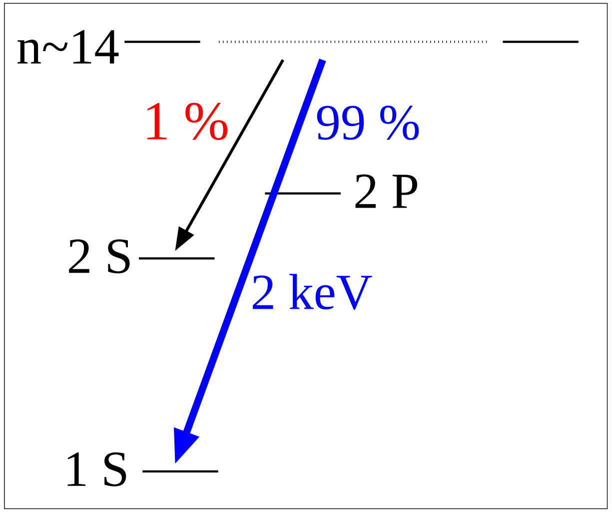
A. Antognini, RP *et al.*, Science 339, 417 (2013).

Theory summary:

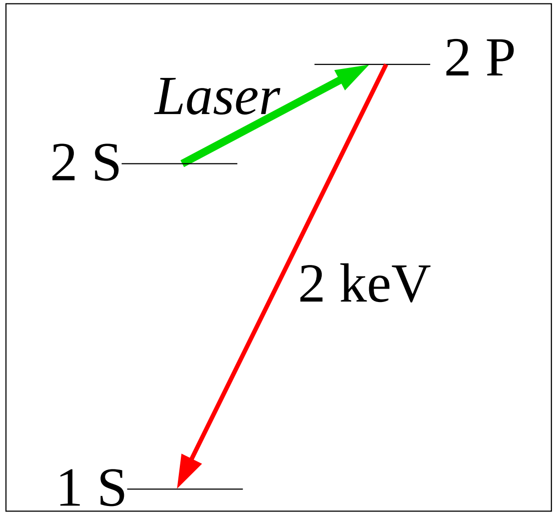
A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013).

μp Lamb shift experiment: Principle

"prompt" ($t \sim 0$)



"delayed" ($t \sim 1\,\mu$s)



μ^- stop in H_2 gas
$\Rightarrow$ μp* atoms formed ($n \sim 14$)

99%: cascade to μp(1S),
emitting prompt K_α, K_β ...

1%: long-lived μp(2S) atoms

lifetime $\boxed{\tau_{2S} \approx 1\,\mu s}$ at 1 mbar H_2

RP *et. al.*, Phys. Rev. Lett. 97, 193402 (2006).

fire laser ($\lambda \approx 6\,\mu$m, $\Delta E \approx 0.2$ eV)

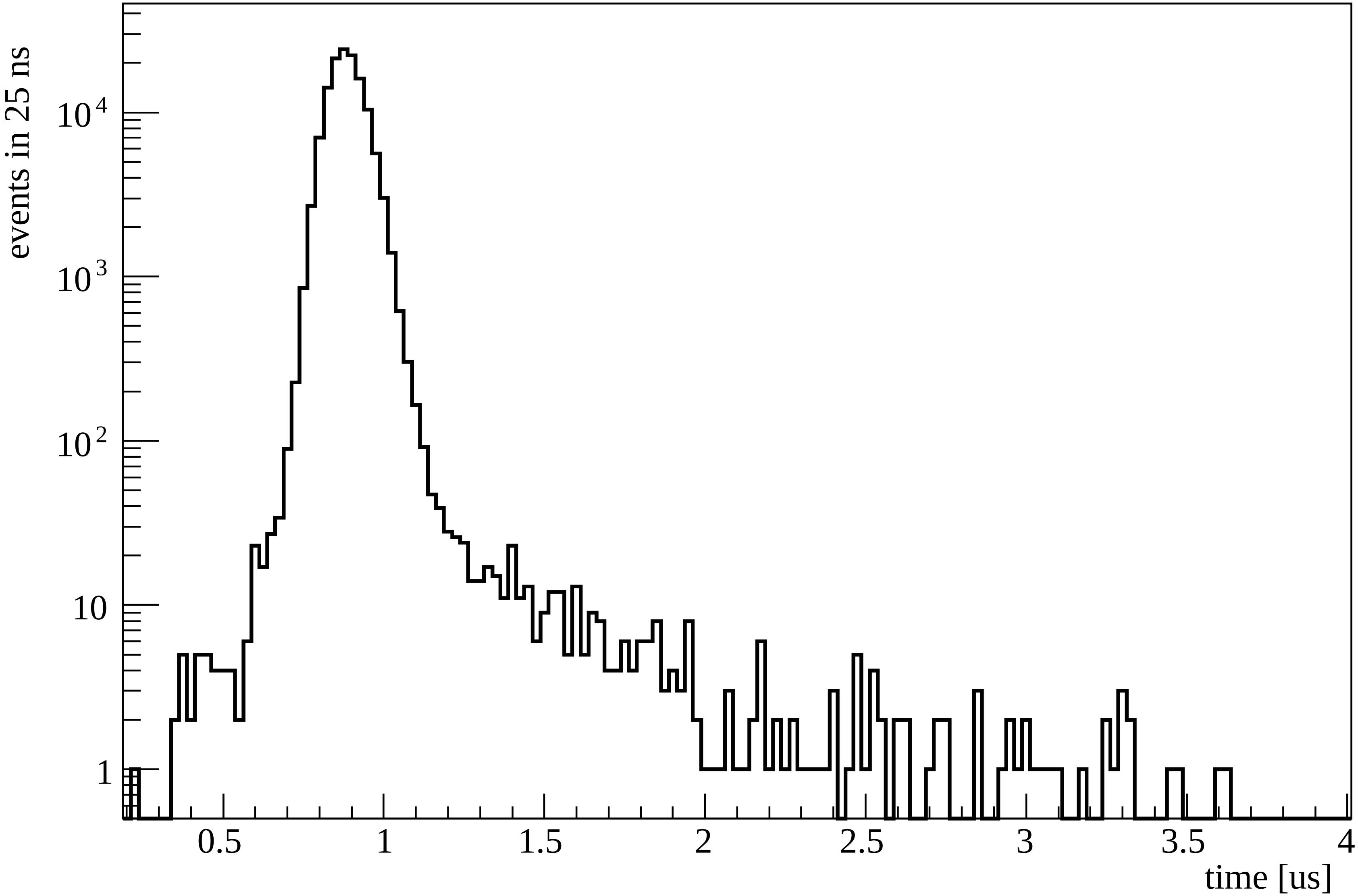
$\Rightarrow$ induce μp(2S) $\rightarrow$ μp(2P)

$\Rightarrow$ observe delayed K_α x-rays

$\Rightarrow$ normalize $\dfrac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha}$ x-rays

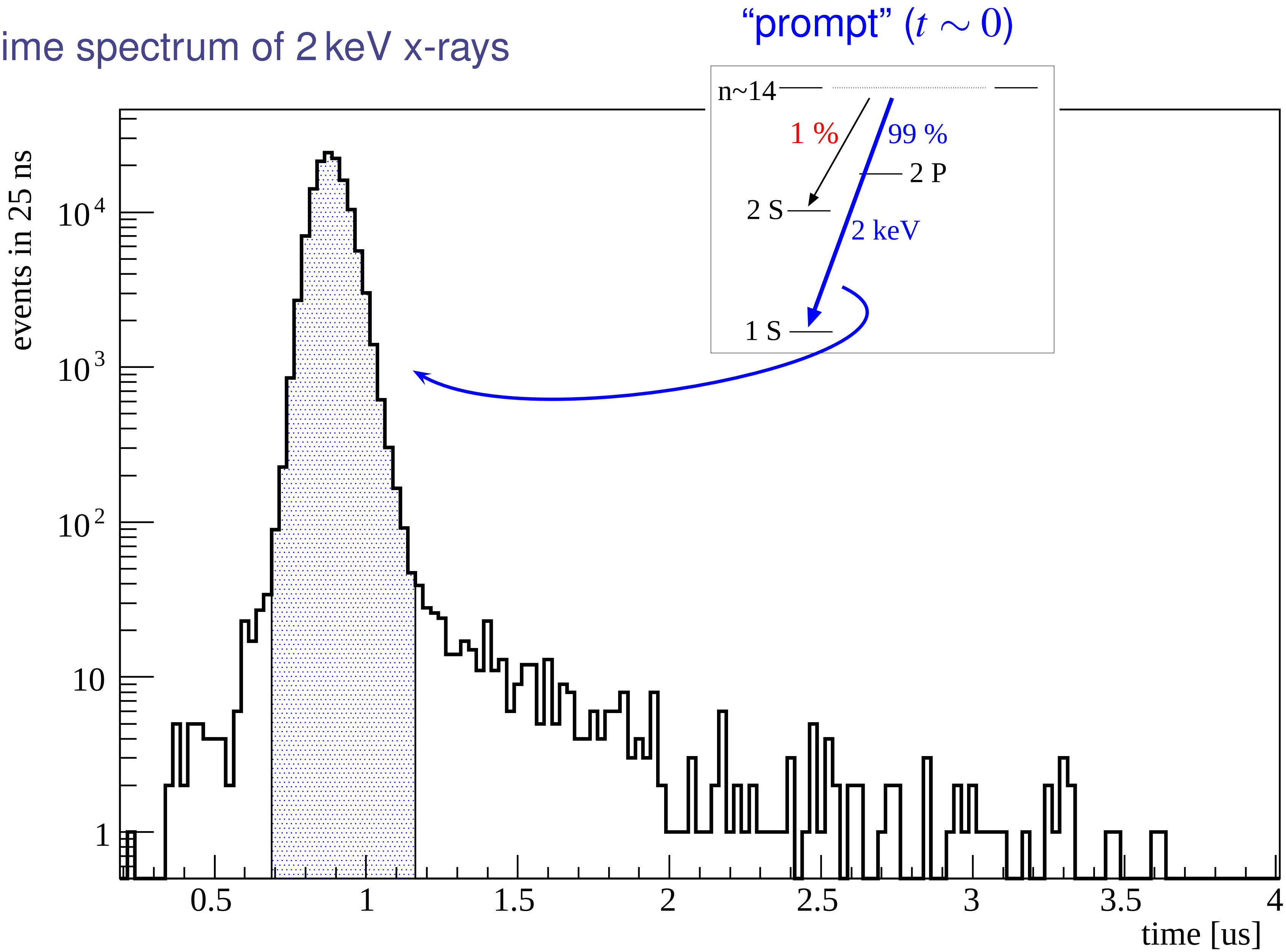
μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays (∼ 13 hours of data @ 1 laser wavelength)



μp Lamb shift experiment: Principle

time spectrum of 2 keV x-rays



μp Lamb shift experiment: Principle

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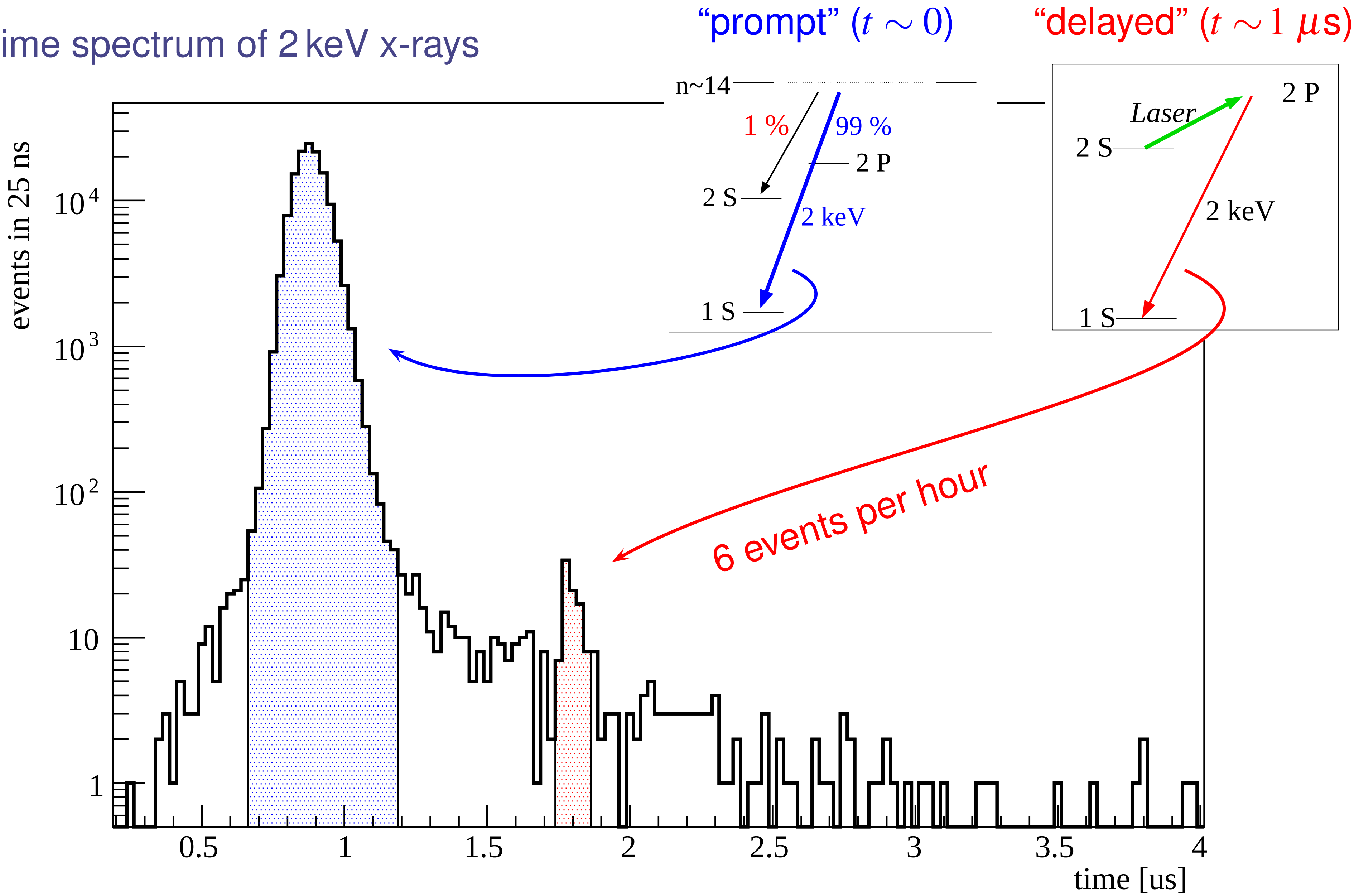

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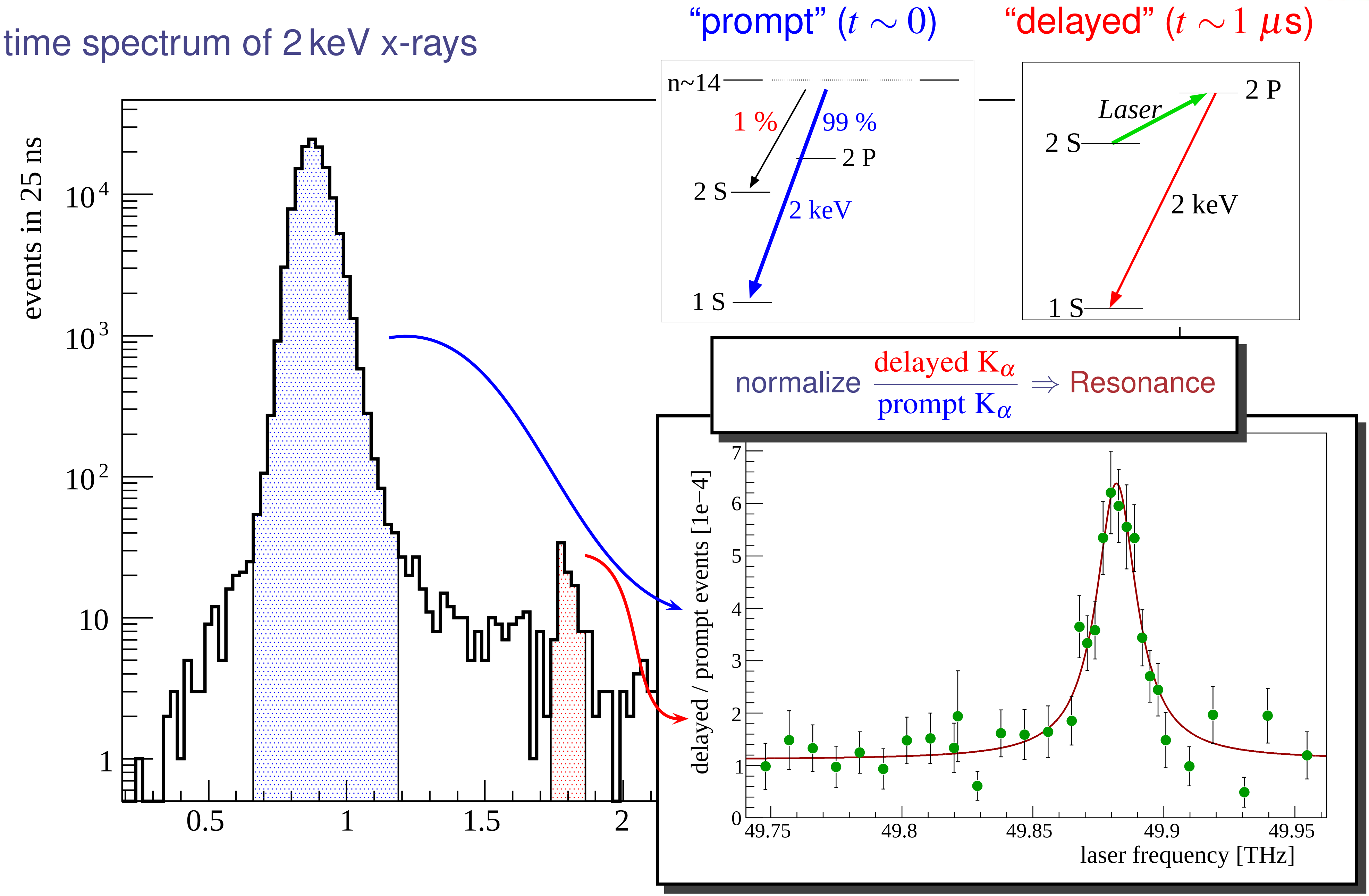
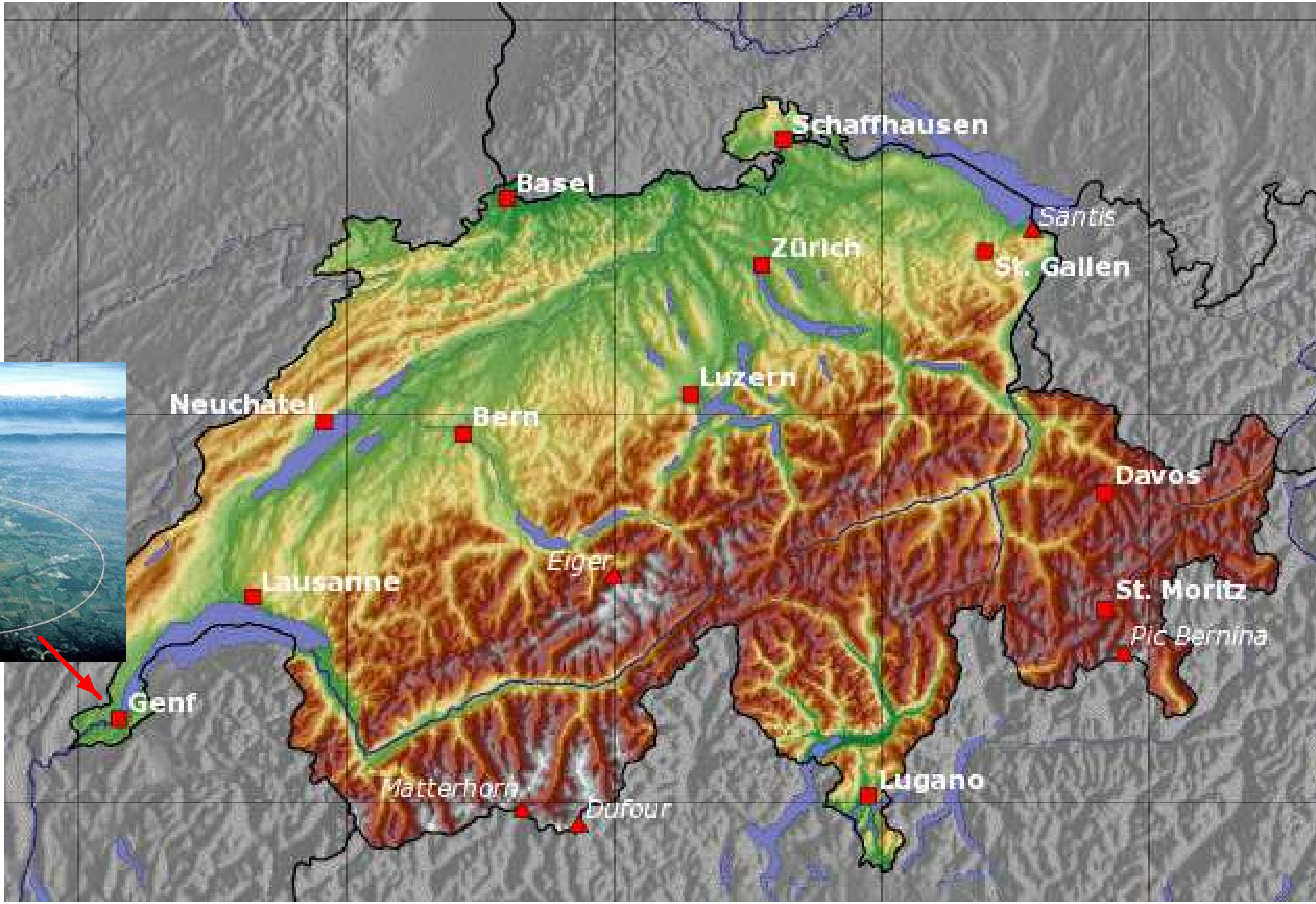
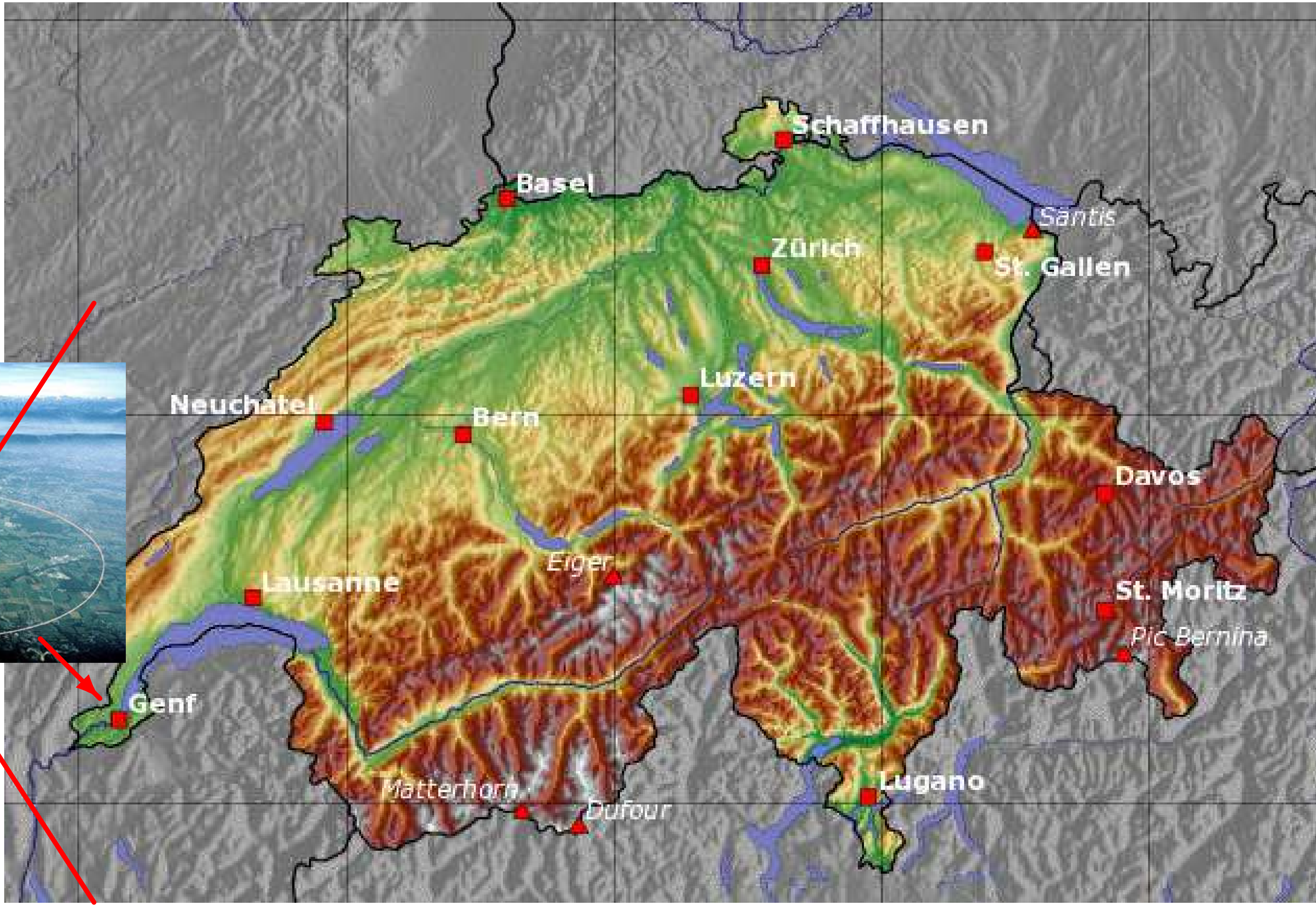


Swiss muons



Schaffhausen
Basel
Säntis
Zürich
St. Gallen
Neuchâtel
Bern
Luzern
Davos
Eiger
Lausanne
St. Moritz
Pic Bernina
Genf
Matterhorn
Dufour
Lugano

Swiss muons



Schaffhausen
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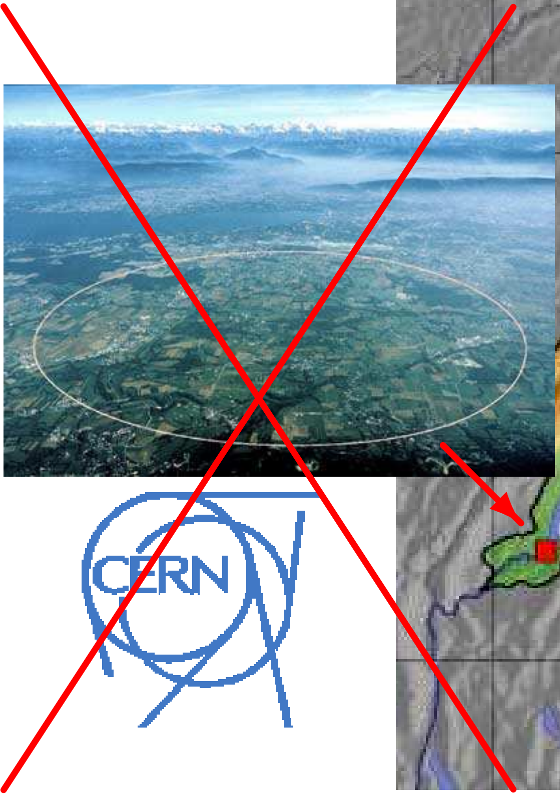


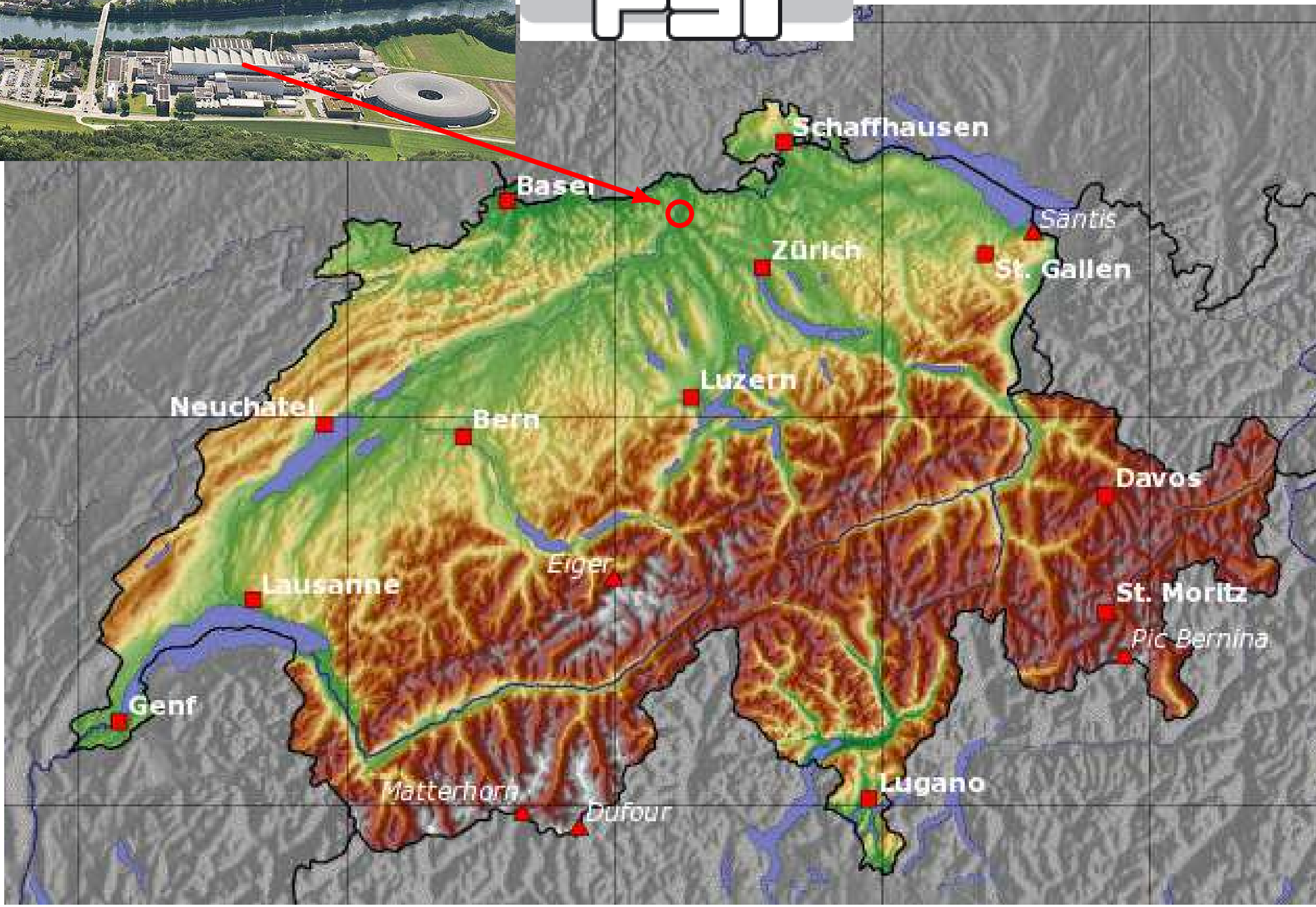
CERN

Swiss muons

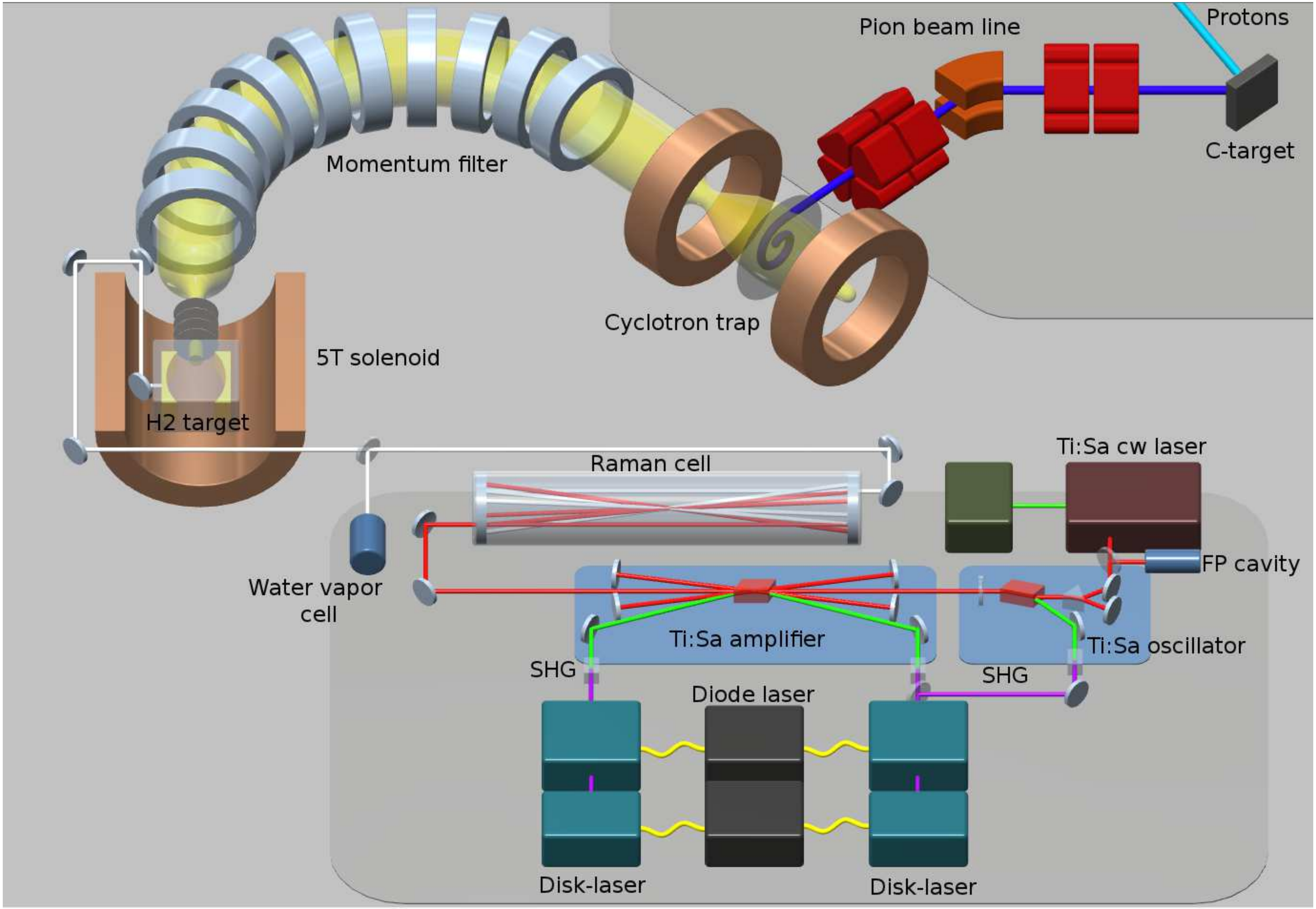
MPQ

PAUL SCHERRER INSTITUT

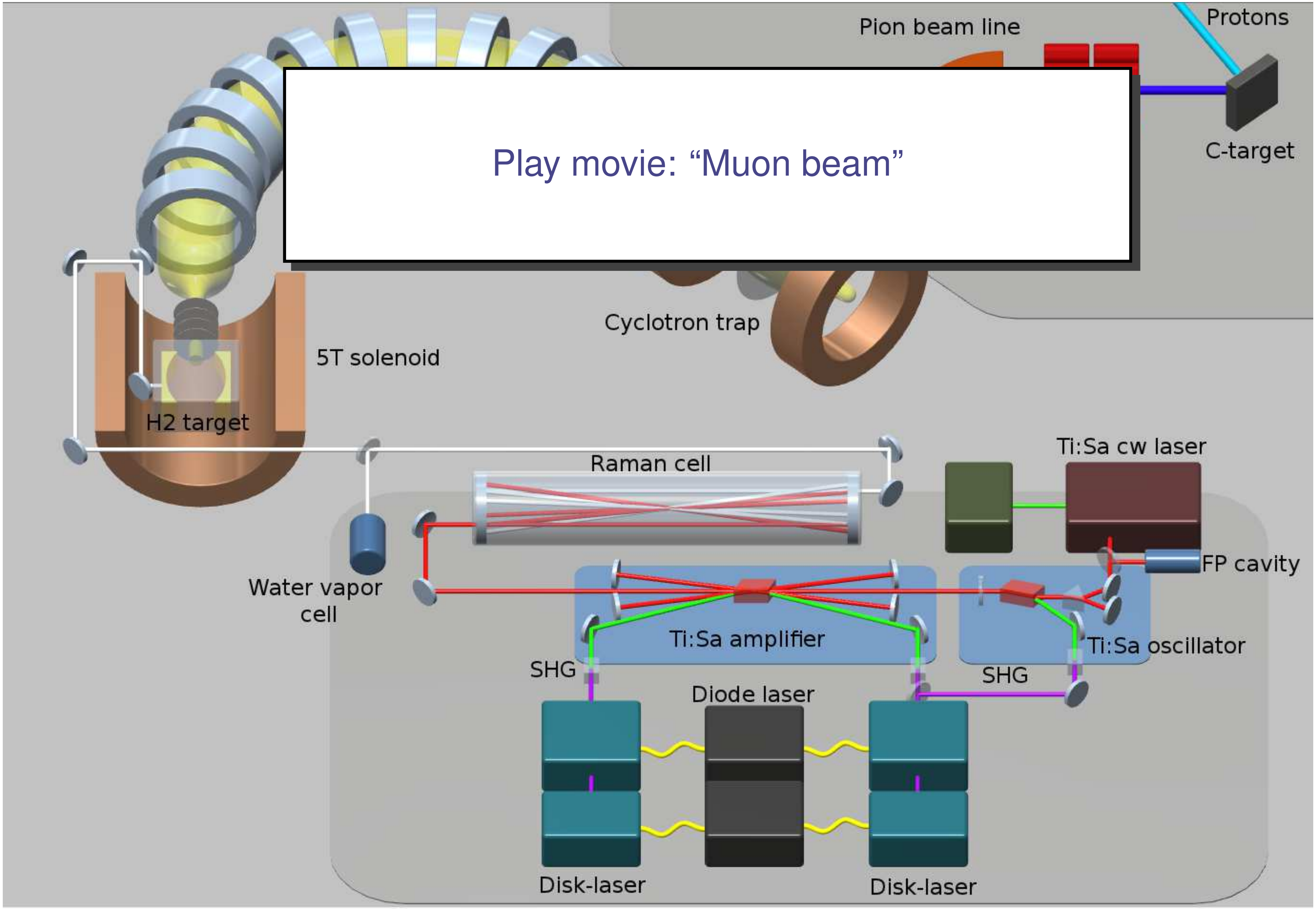
PSI



Setup


Pion beam line
Protons
C-target
Momentum filter
Cyclotron trap
5T solenoid
H2 target
Raman cell
Ti:Sa cw laser
FP cavity
Water vapor
cell
Ti:Sa amplifier
Ti:Sa oscillator
SHG
SHG
Diode laser
Disk-laser
Disk-laser

Setup


Protons
Pion beam line
C-target
Play movie: “Muon beam”
Cyclotron trap
5T solenoid
H2 target
Raman cell
Ti:Sa cw laser
FP cavity
Water vapor
cell
Ti:Sa amplifier
Ti:Sa oscillator
SHG
SHG
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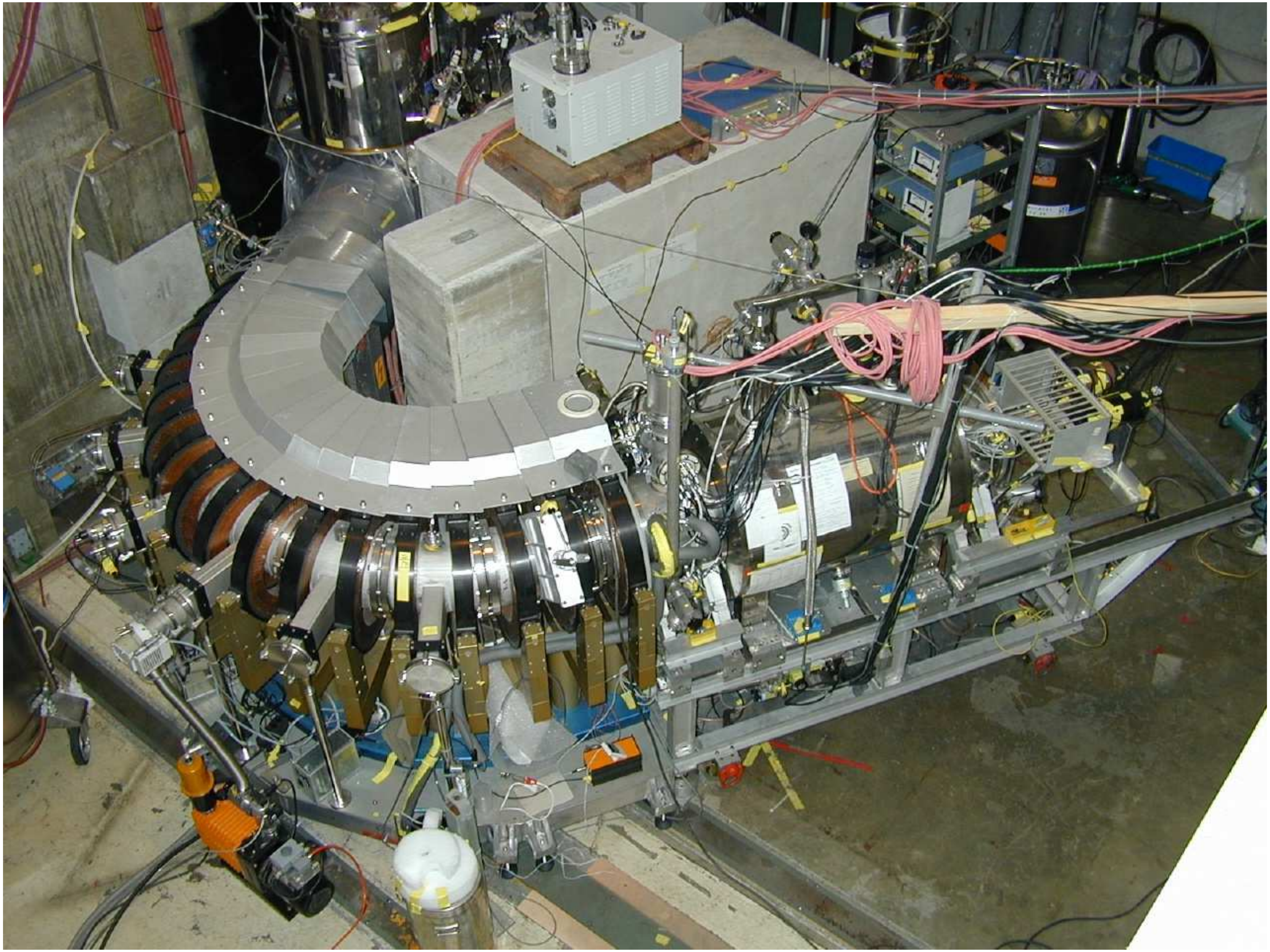
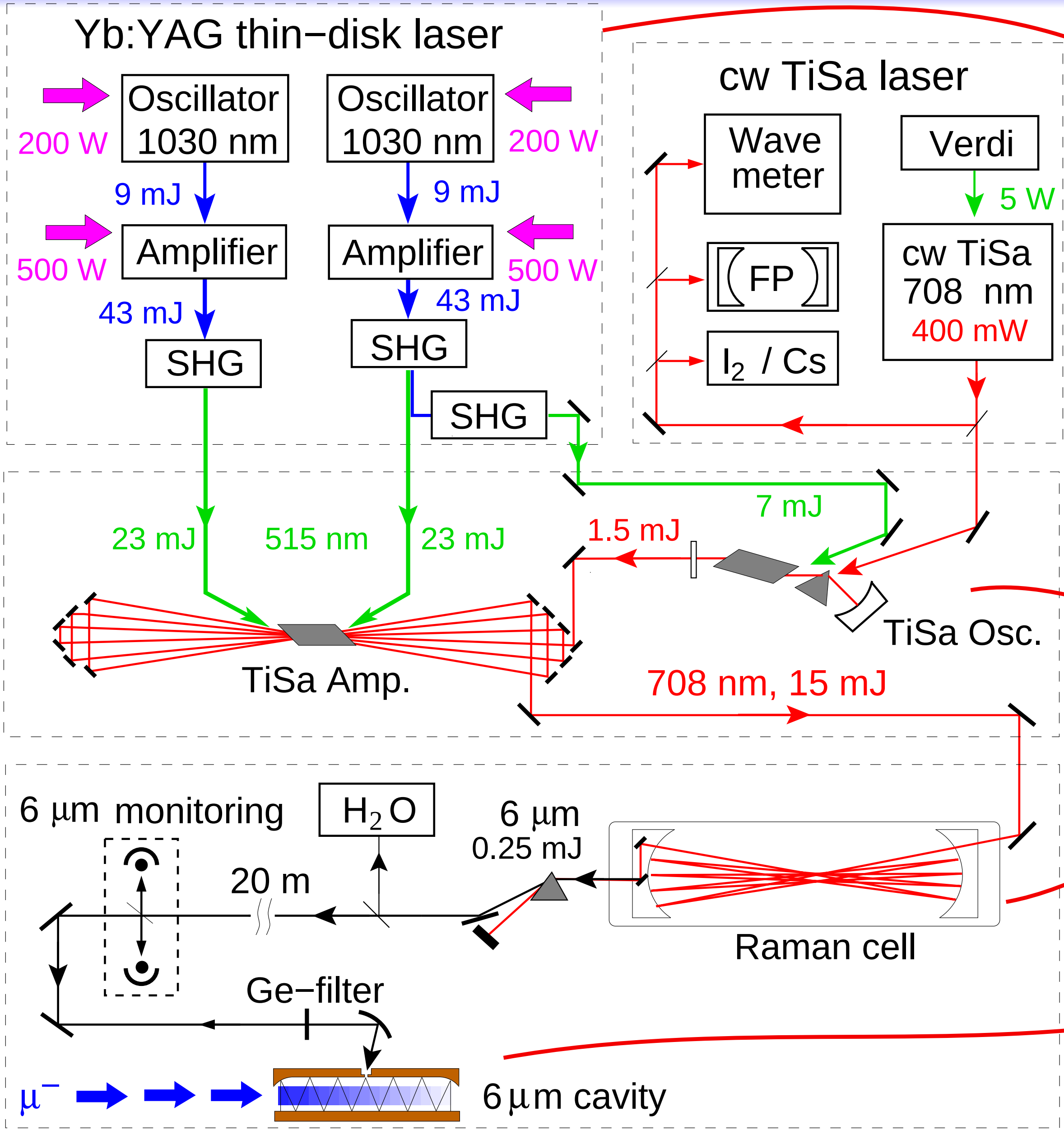
Disk-laser
Disk-laser

Muon beam line



The laser system



Main components:

- Thin-disk laser

  fast response to detected μ^-

- Frequency doubling

- TiSa laser:

  frequency stabilized cw laser

  injection seeded oscillator

  multipass amplifier

- Raman cell

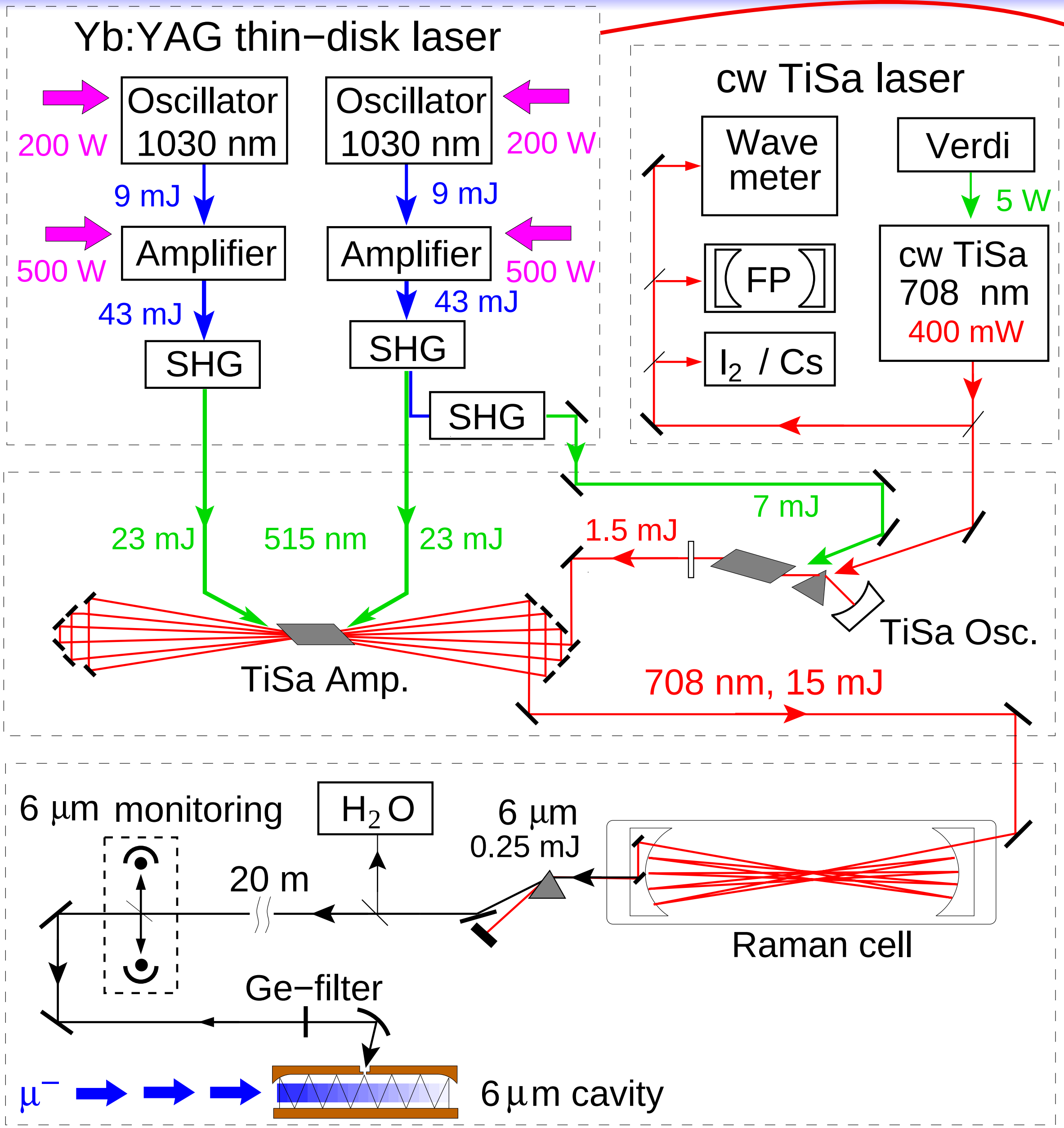
  3 Stokes: 708 nm $\rightarrow$ 6 μm

  λ calibration @ 6 μm

- Target cavity

A. Antognini, RP *et. al.*, Opt. Comm. 253, 362 (2005).

The laser system



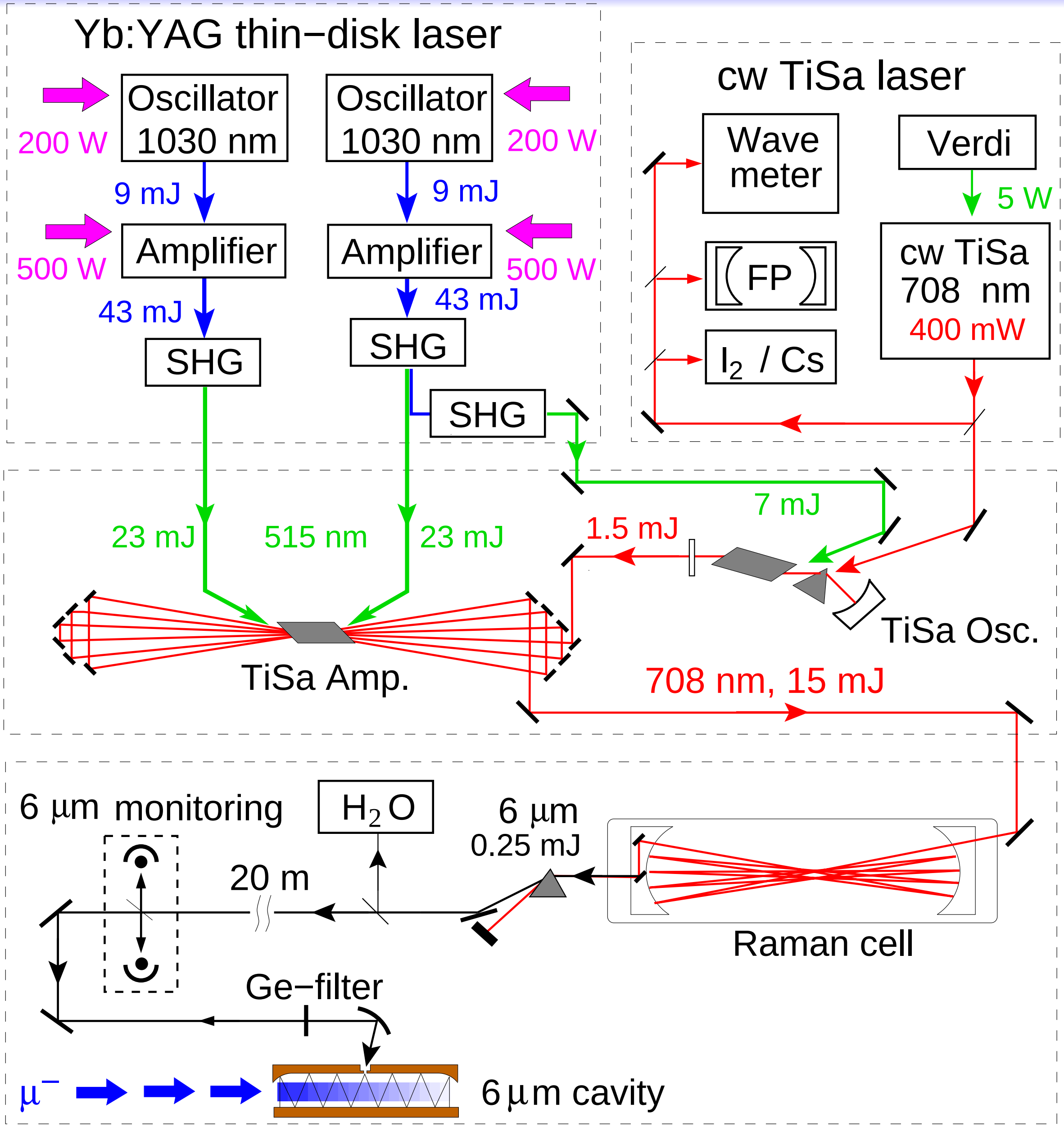
Thin-disk laser

- Large pulse energy: 85 (160) mJ
- Short trigger-to-pulse delay: $\lesssim 400$ ns
- Random trigger
- Pulse-to-pulse delays down to 2 ms (rep. rate $\gtrsim 500$ Hz)

- Each single μ^- triggers the laser system
- $2S$ lifetime $\approx 1\ \mu s \rightarrow$ short laser delay

A. Antognini, RP *et. al.*, IEEE J. Quant. Electr. 45, 993 (2009).

The laser system



MOPA TiSa laser:

cw laser, frequency stabilized

- referenced to a stable FP cavity
- FP cavity calibrated with I_2, Rb, Cs lines

$\nu_{\rm FP} = N \cdot FSR$

$FSR = 1497.344(6)$ MHz

$\nu^{\rm cw}_{\rm TiSa}$ absolutely known to 30 MHz

$\Gamma_{\rm 2P-2S} = 18.6$ GHz

Seeded oscillator

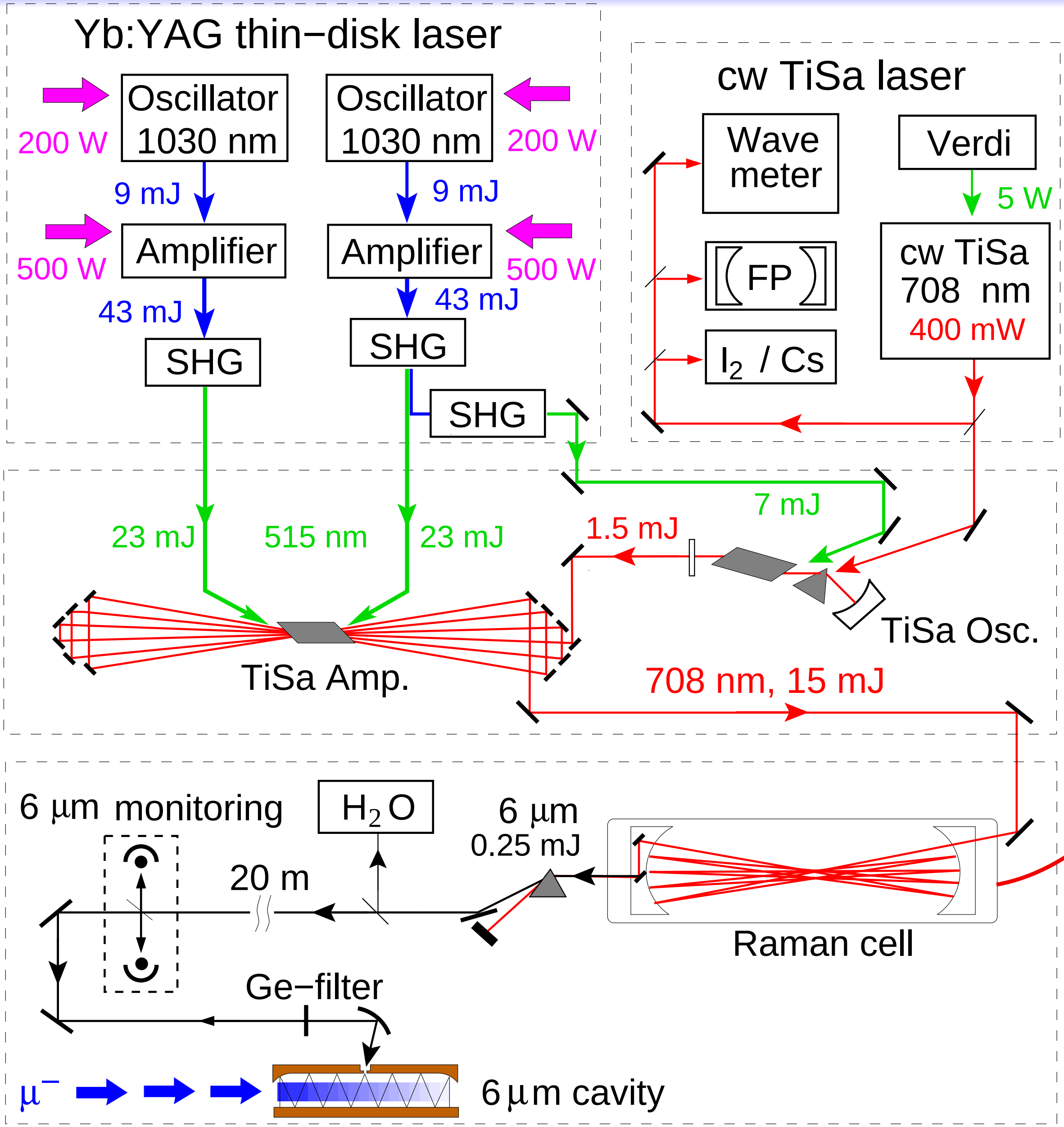
$\rightarrow \nu^{\rm pulsed}_{\rm TiSa} = \nu^{\rm cw}_{\rm TiSa}$

(frequency chirp ≤ 200 MHz)

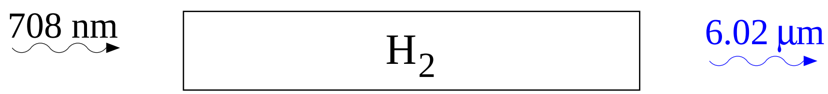
Multipass amplifier (2f- configuration)

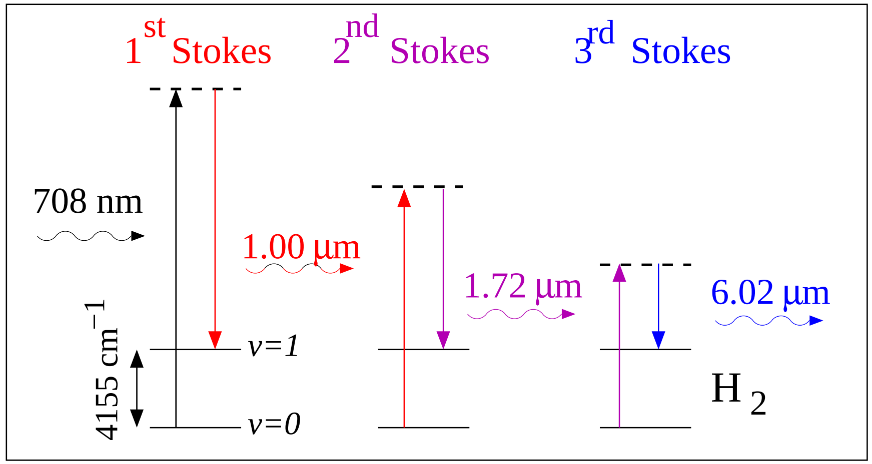
gain=10

The laser system



Raman cell:



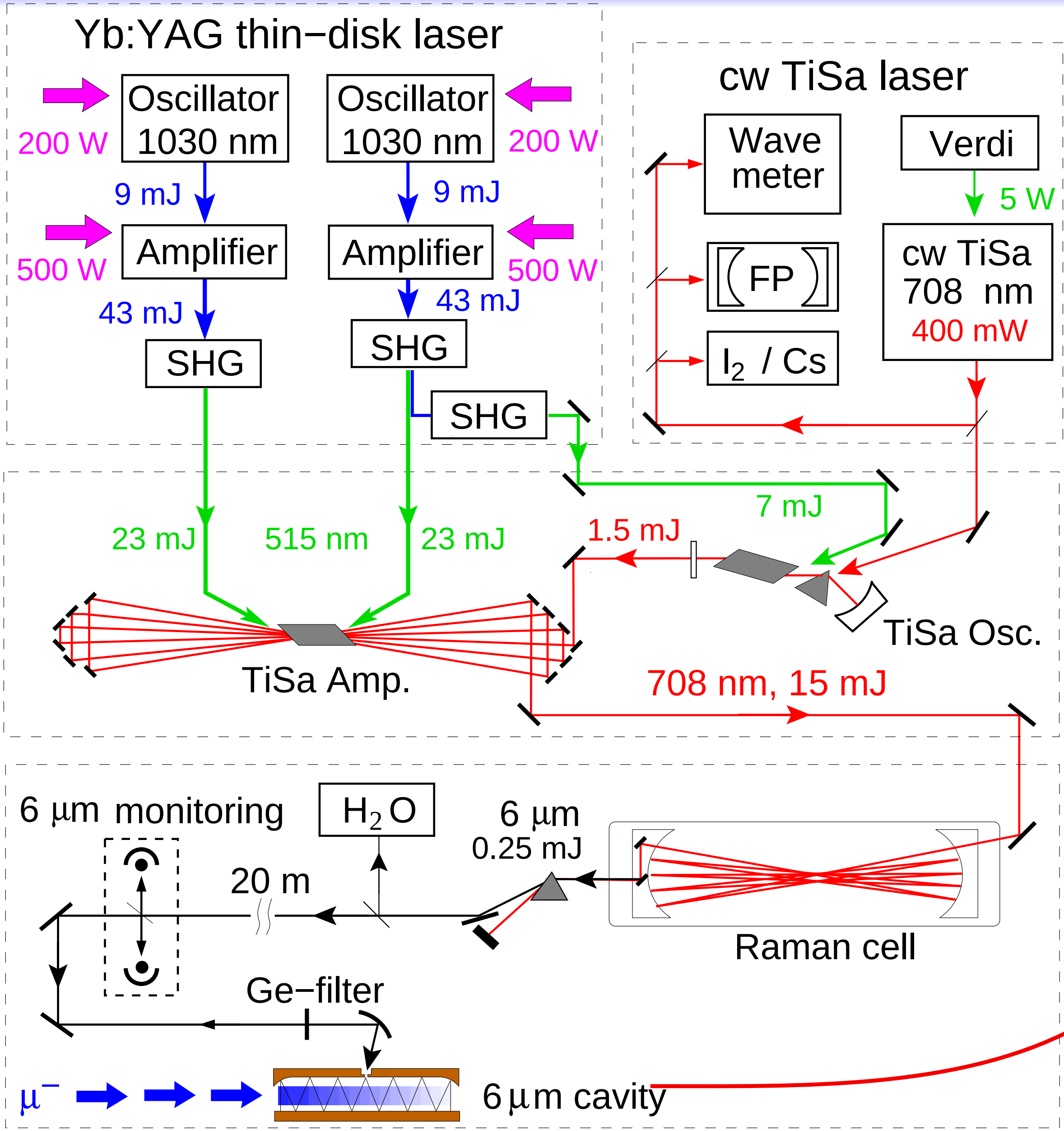


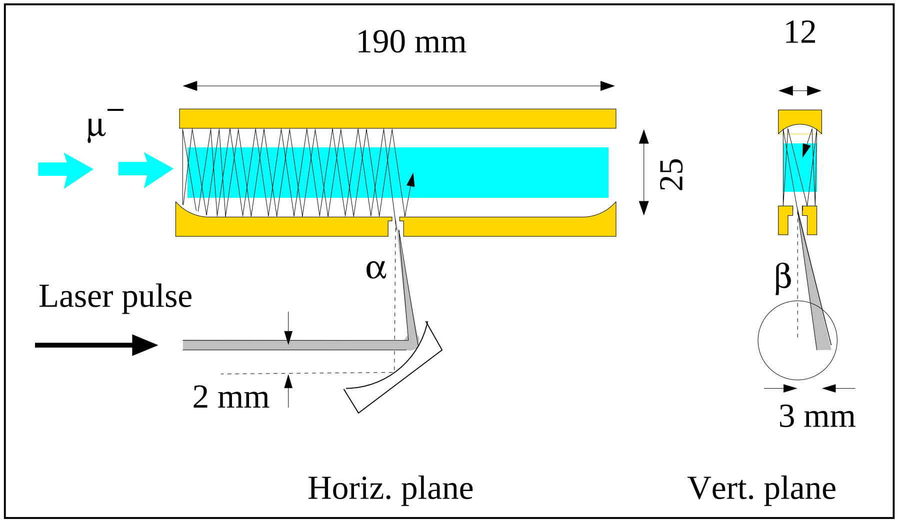
$$\boxed{\nu^{6\mu m} = \nu^{708\text{nm}} - 3 \cdot \hbar\omega_{\text{vib}}}$$

tunable $\quad \omega_{\text{vib}}(p,T) = \text{const}$

P. Rabinowitz *et. al.*, IEEE J. QE **22**, 797 (1986)

The laser system





Design: insensitive to misalignment

Transverse illumination

Large volume

Dielectric coating with $R \geq 99.9\%$ (at 6 μm)

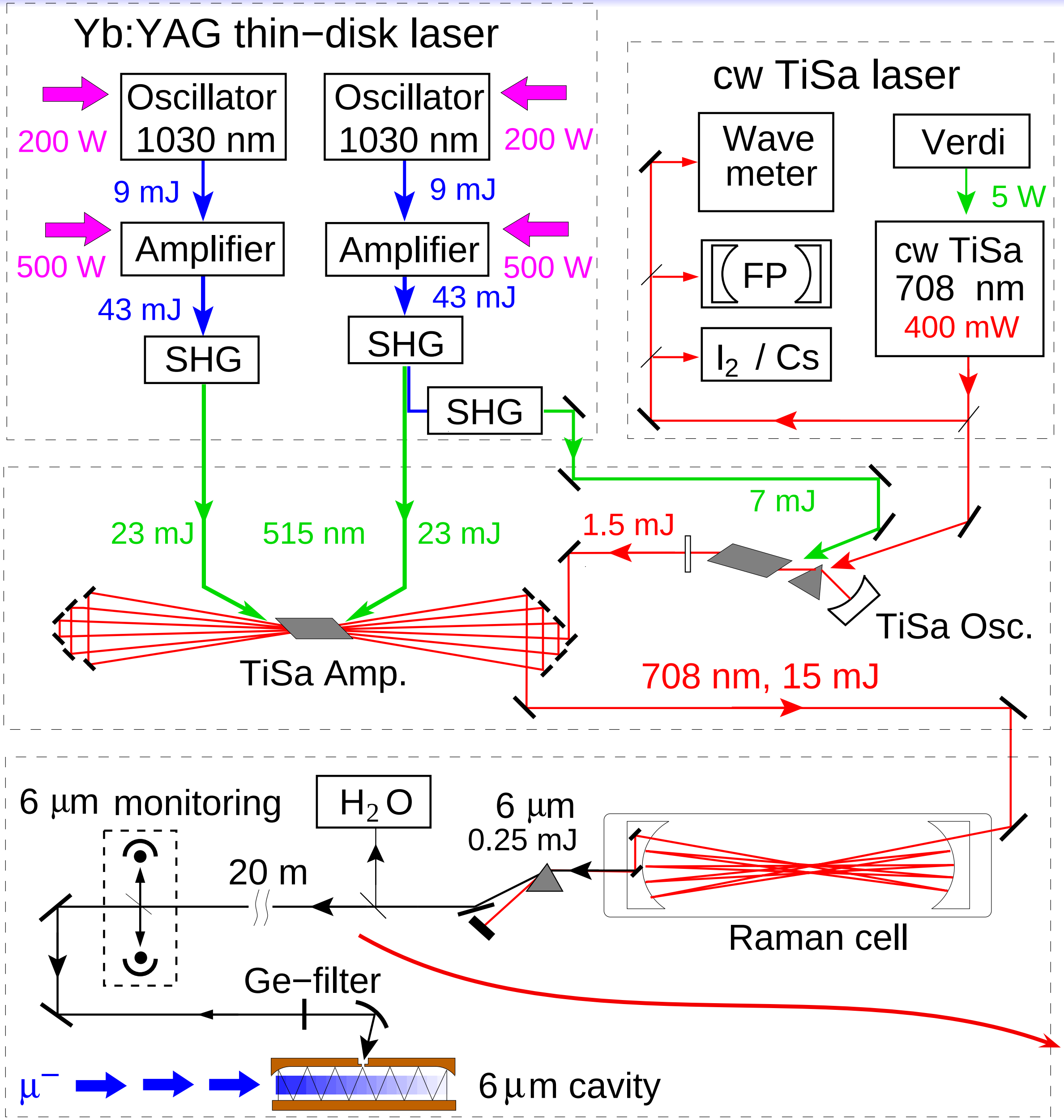
$\rightarrow$ Light makes 1000 reflections

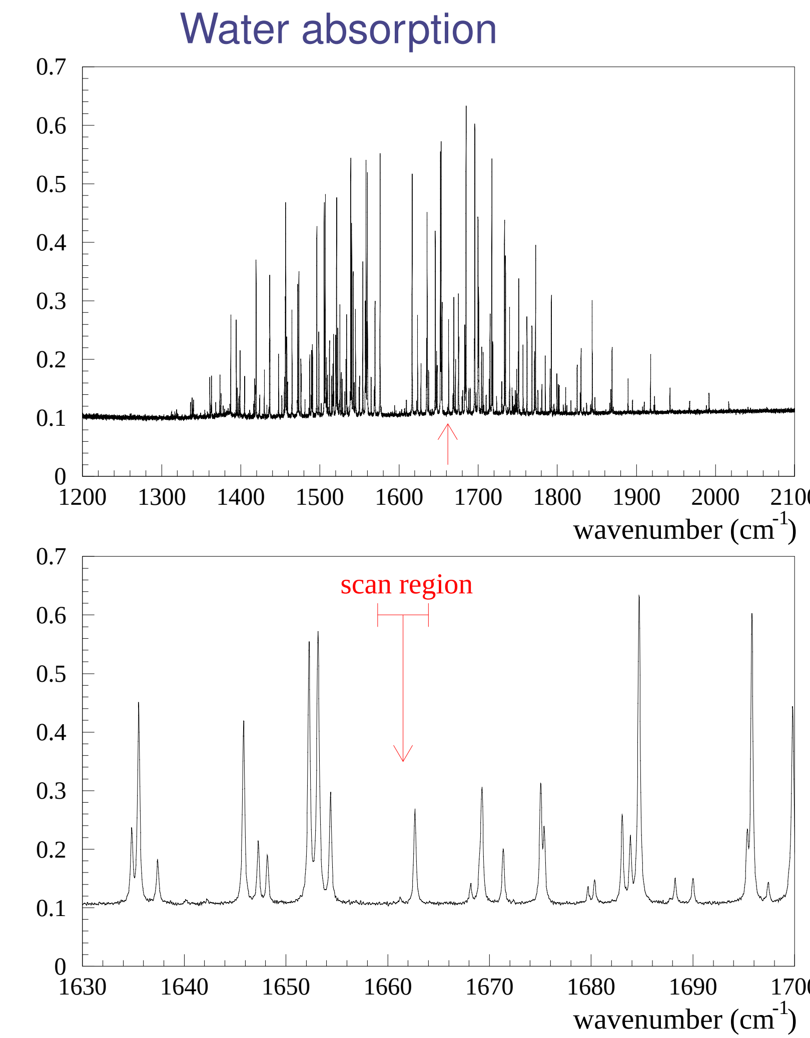
$\rightarrow$ Light is confined for τ=50 ns

$\rightarrow$ 0.15 mJ saturates the $2S - 2P$ transition

J. Vogelsang, RP *et. al.*, Opt. Expr. **22**, 13050 (2014)

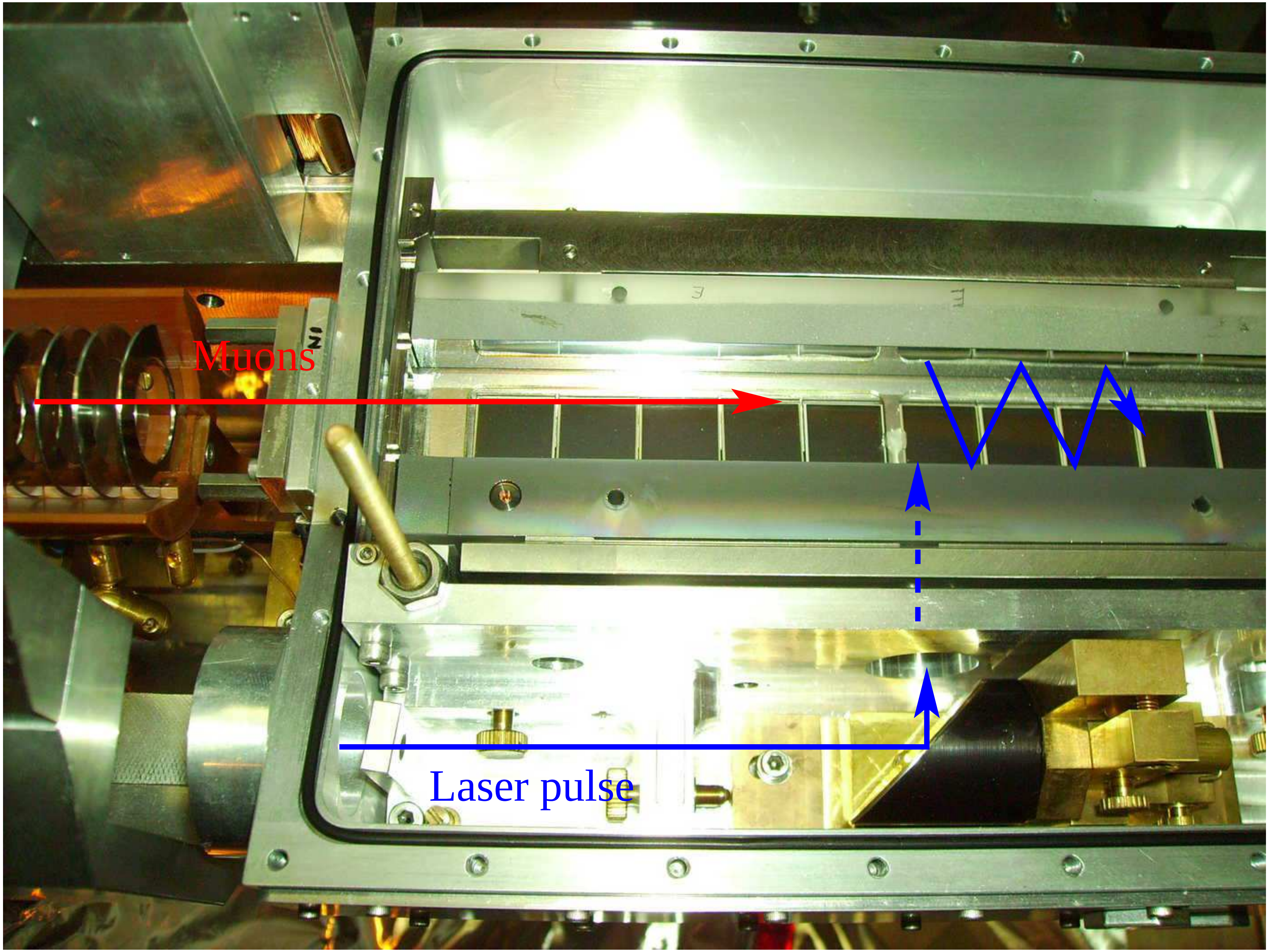
The laser system





- Vacuum tube for 6 μm beam transport.
- Direct frequency calibration at 6 μm.

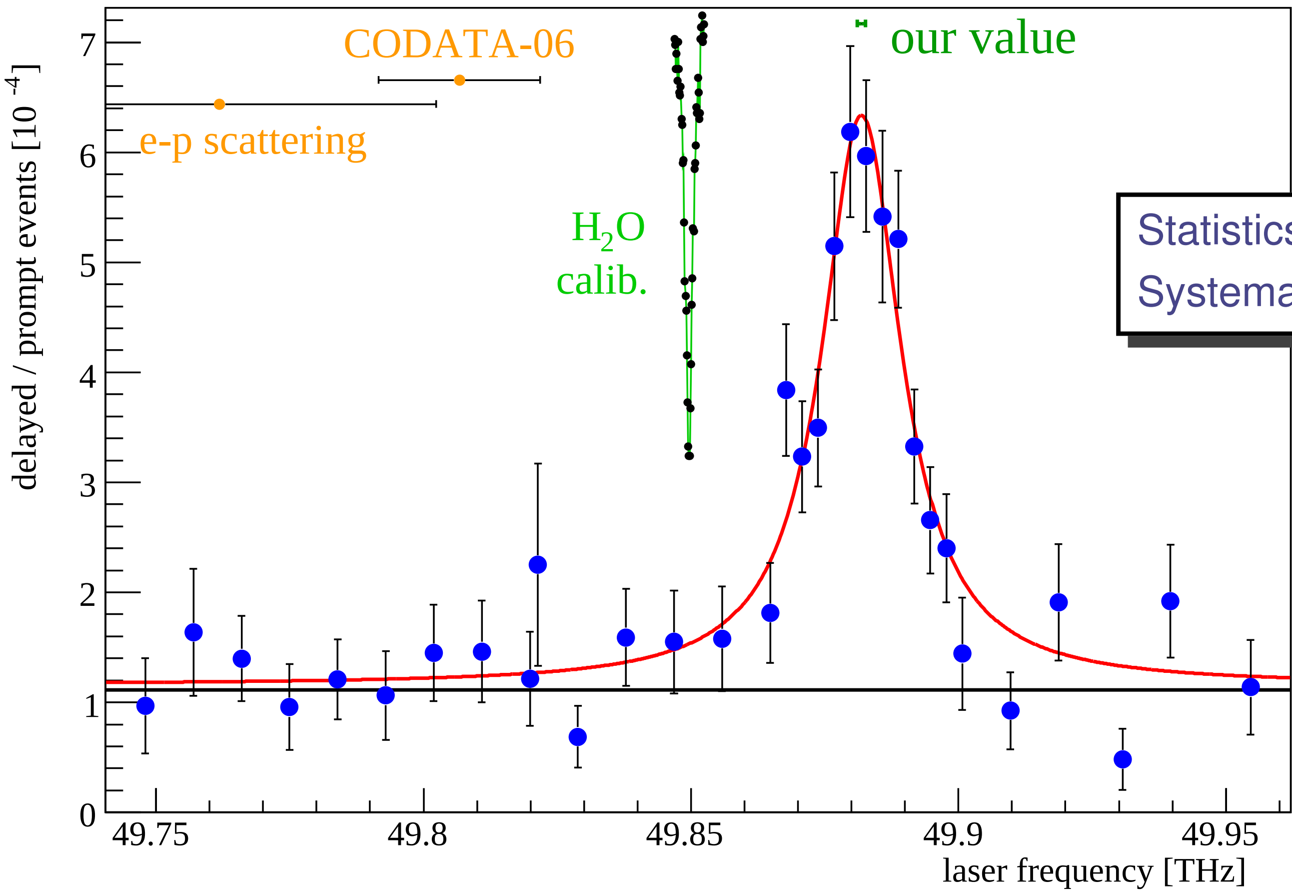
Target, cavity and detectors



The resonance: discrepancy, sys., stat.

Water-line/laser wavelength:
300 MHz uncertainty

$\Delta\nu$ water-line to resonance:
200 kHz uncertainty



Statistics: 700 MHz
Systematics: 300 MHz

Discrepancy:
$5.0\,\sigma \leftrightarrow$ **75 GHz** $\leftrightarrow \delta\nu/\nu = 1.5\times10^{-3}$

R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*,Science 339, 417 (2013).

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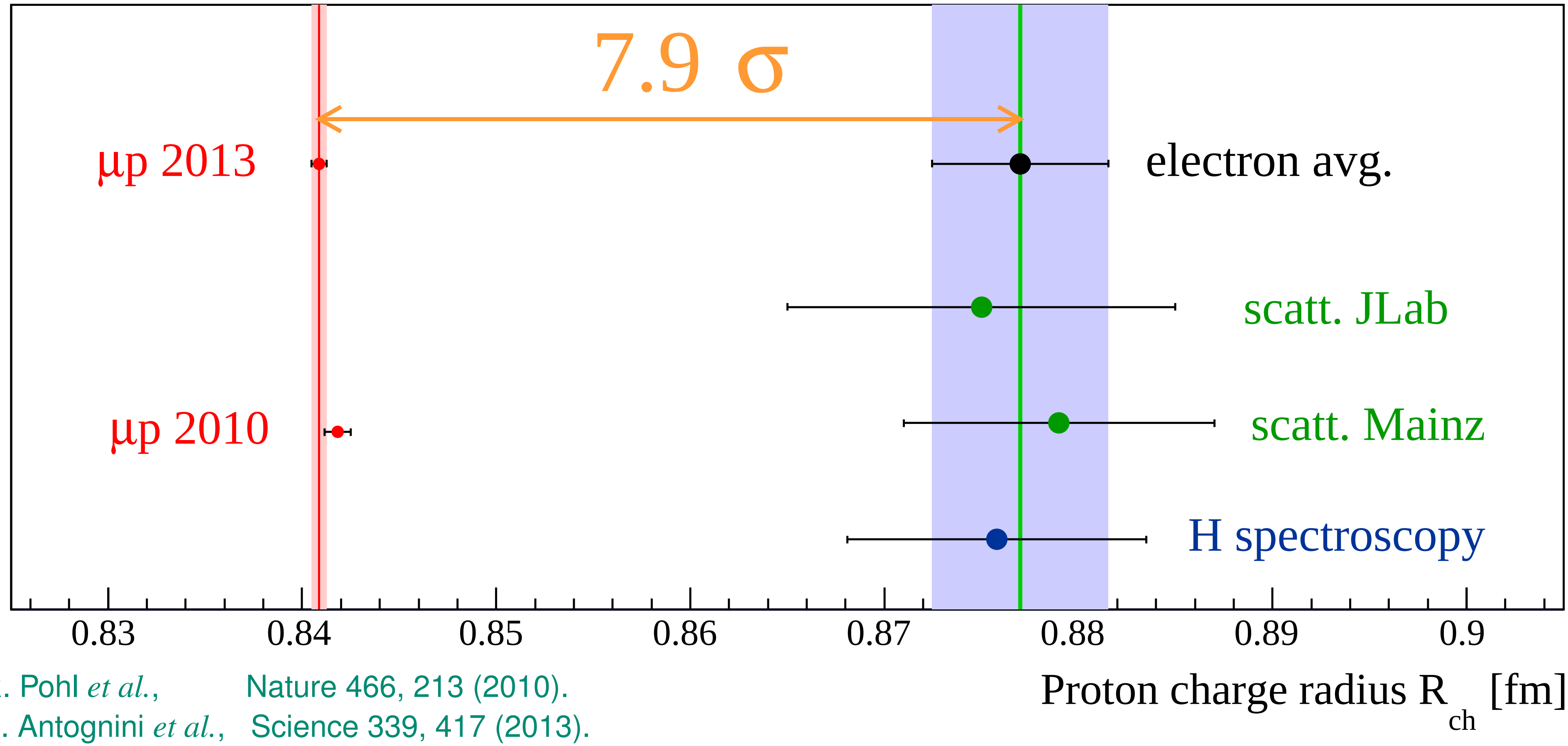

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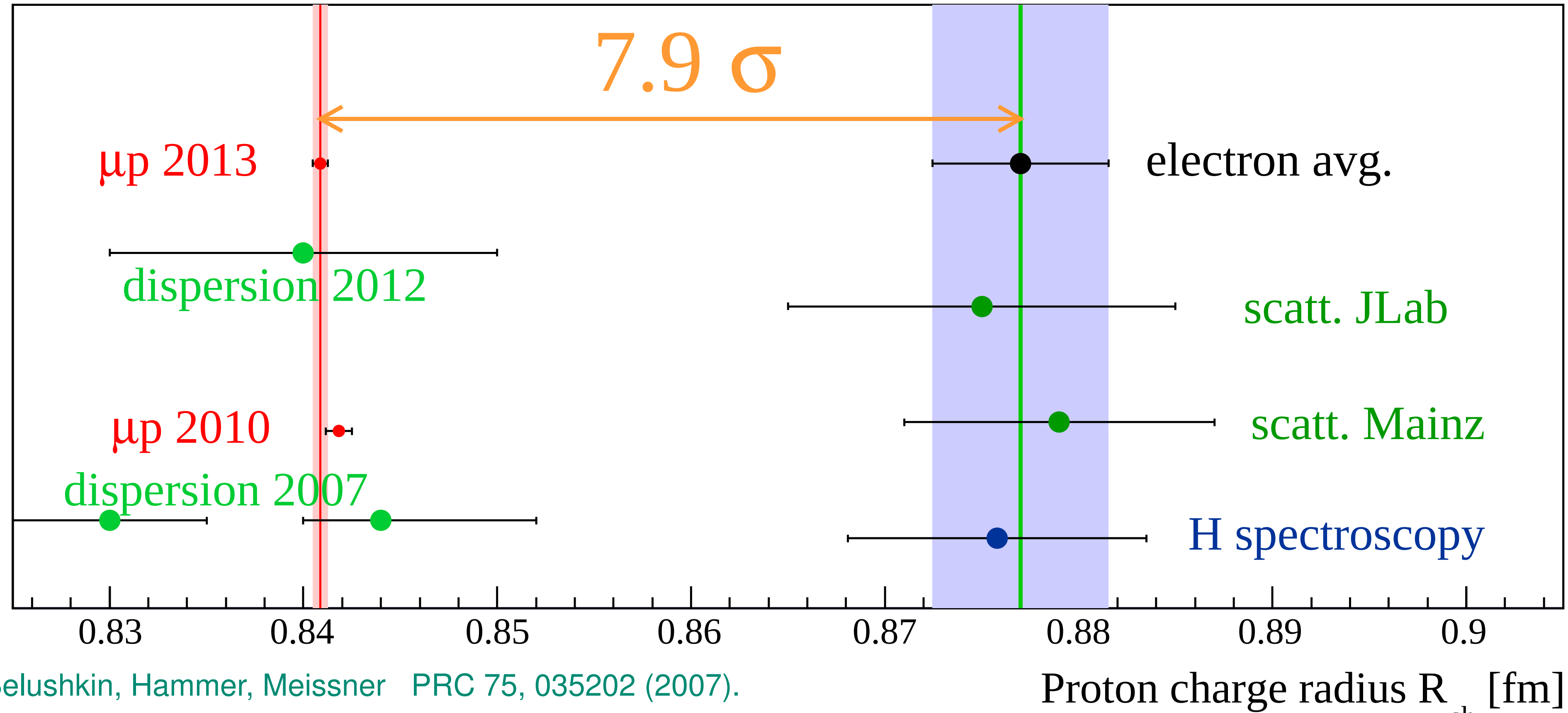


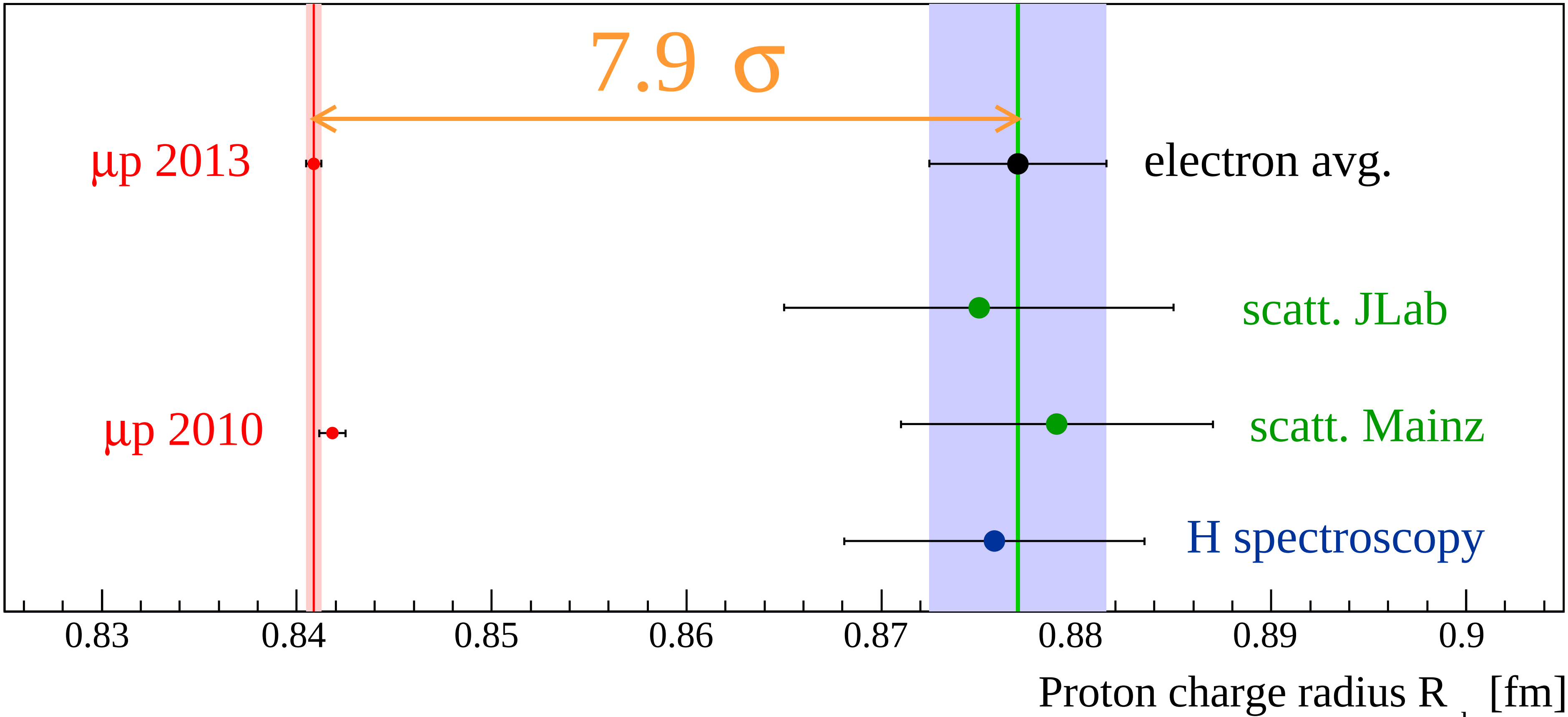
Belushkin, Hammer, Meissner PRC 75, 035202 (2007).
Lorenz, Hammer, Meissner EPJ A 48, 151 (2012).

The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8770 ± 0.0045 fm

muons: 0.8409 ± 0.0004 fm



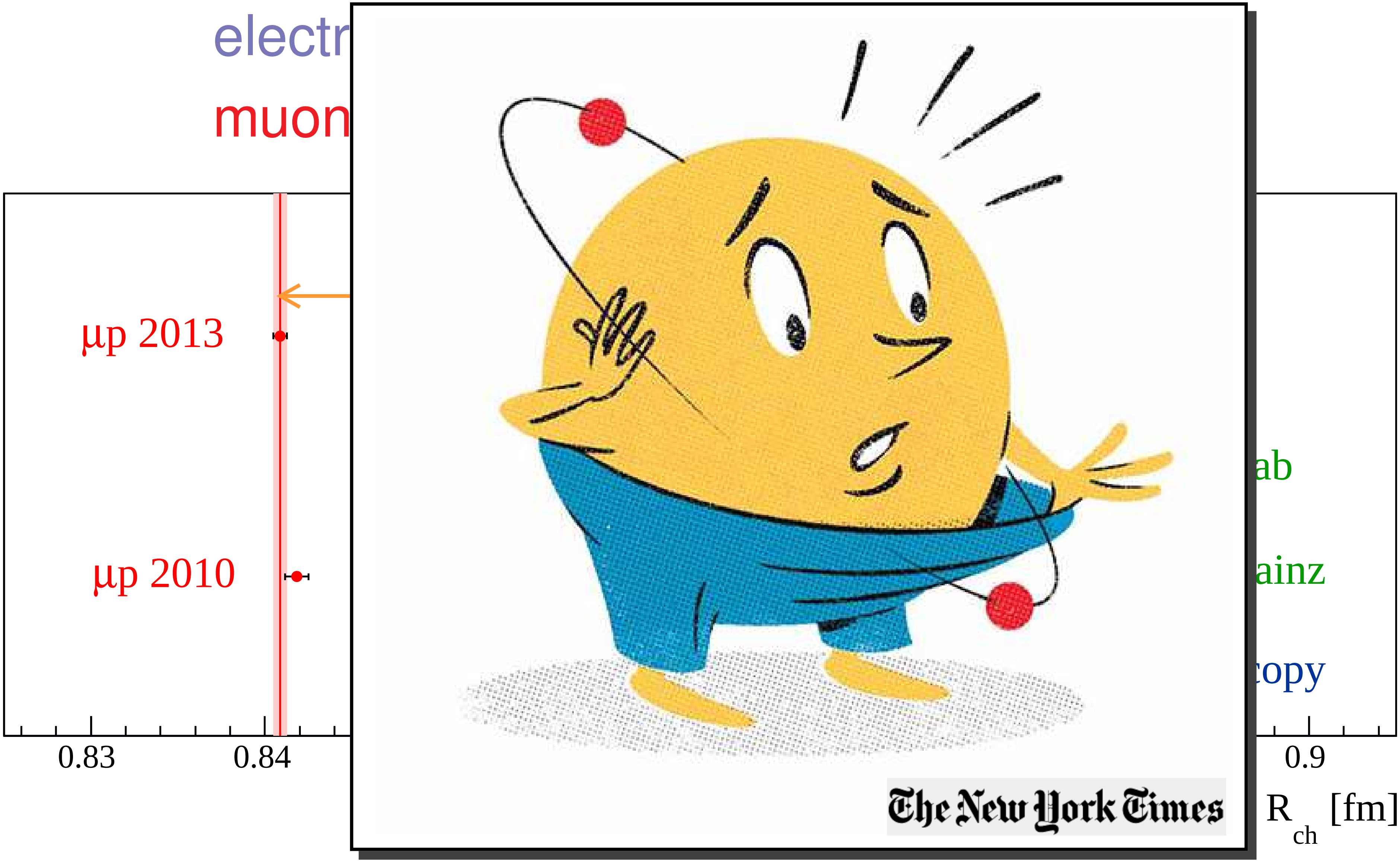
RP, Gilman, Miller, Pachucki, Annu. Rev. Nucl. Part. Sci. 63, 175 (2013).

The proton radius puzzle

The proton rms charge radius measured with

electr

muon



Muons in the news

Muons in the news

Muons in the news

Muons in the news

J. Bernauer, RP

Muons in the news

J. Bernauer, RP

What may be wrong?

$$\widetilde{L}_{\mu p}^{\text{theo.}}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75\text{ GHz} \\ 0.31\text{ meV} \\ 0.15\ \% \end{cases}$$

μp theory wrong?

μp experiment wrong?

H theory wrong?

H experiments wrong? $\rightarrow R_\infty$ wrong?

AND e-p scattering exp. wrong?

Standard Model wrong?!?

RP, R. Gilman, G.A. Miller, K. Pachucki, "Muonic hydrogen and the proton radius puzzle", Annu. Rev. Nucl. Part. Sci. **63**, 175 (2013) (arXiv 1301.0905)

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

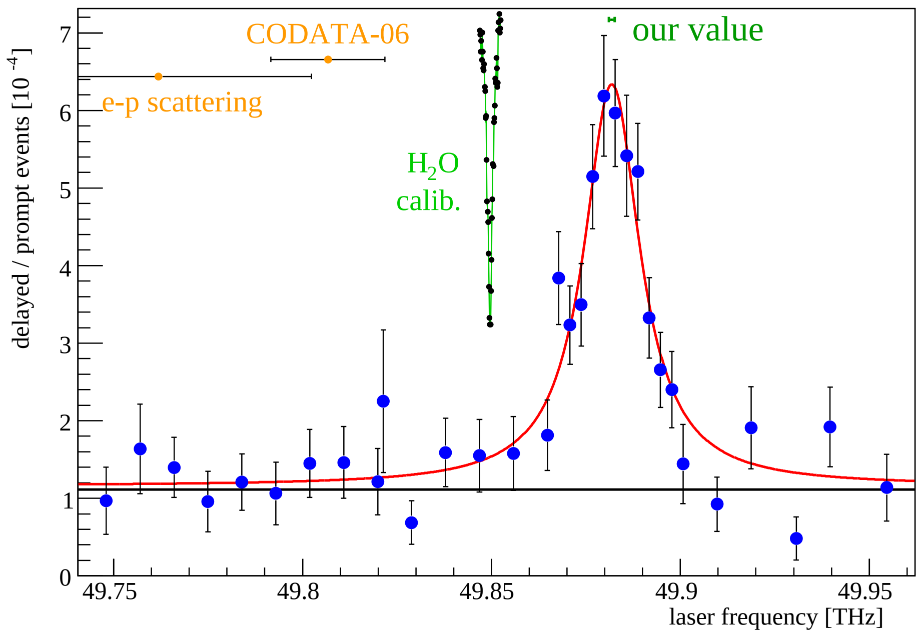
μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?

That is $> 100\ \delta(\mu\text{p})$! σ_{tot} = 650 MHz, [570 MHz$_{\text{stat}}$, 300 MHz$_{\text{syst}}$]

4 line widths ! Γ = 19 GHz

2 resonances in μp give the same r_{p}



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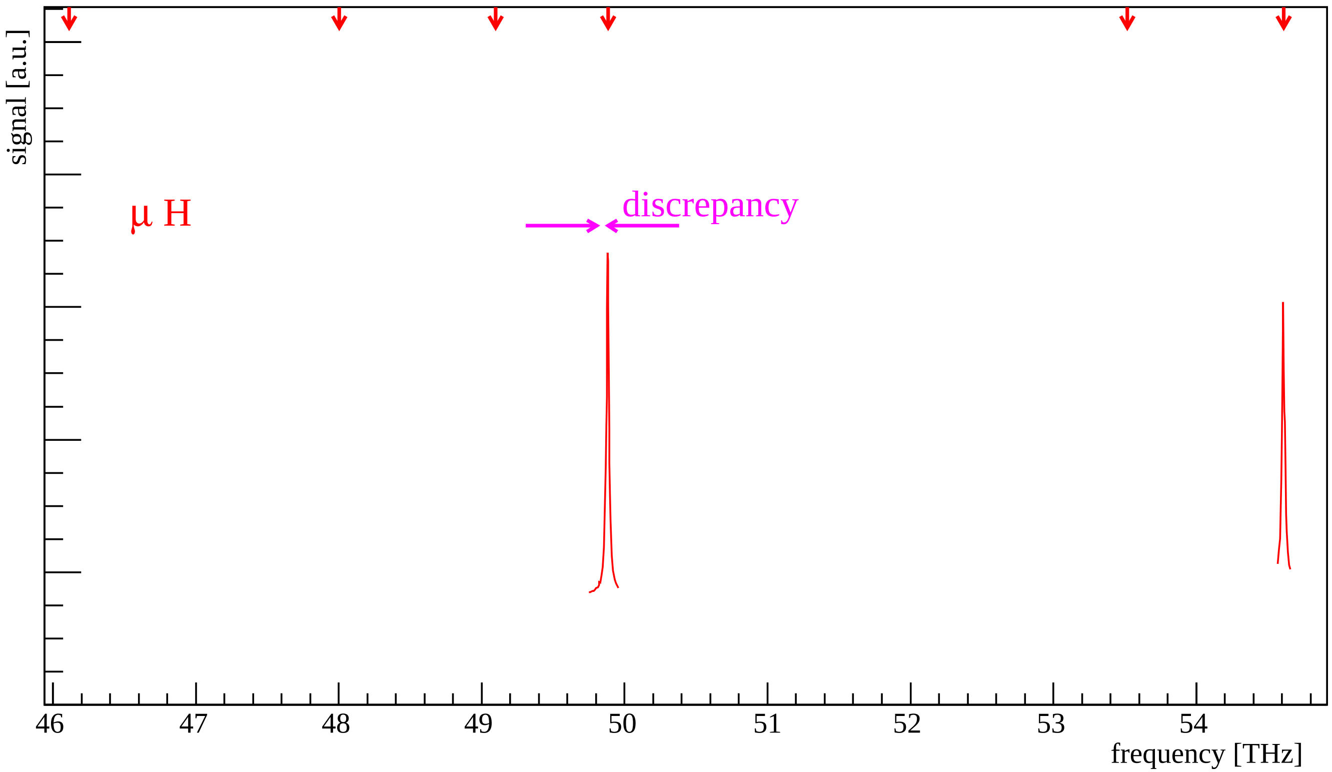
μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?

  FS, HFS huge.

  Next transition:
  ~ 1 THz away.



What may be wrong?

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μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?

Laser frequency (H_20 calibration)	300 MHz
intrinsic H_2O uncertainty	2 MHz
AC and DC stark shift	< 1 MHz
Zeeman shift (5 Tesla)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz
	300 MHz

μp atom is small and not easily perturbed by external fields.

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?
- Molecular effects?

$p\ \mu\ e$ molecular ion? U.D. Jentschura, Annals of Physics 326, 516 (2011).

Does not exist! J.-P. Karr, L. Hilico, PRL 109, 103401 (2012).
M. Umair, S. Jonsell, J. Phys. B 47, 175003 (2014).

Experimentally:

- only 1 line observed ($> 80\,\%$ population)
- expected width
- $pp\mu$ ion short-lived R. Pohl *et al.*, PRL 97, 193402 (2006).

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\rm p}^{\rm CODATA}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\text{ GHz} \\ 0.31\text{ meV} \\ 0.15\ \% \end{cases}$$

μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas imputities? M. Diepold, RP *et al.*, PRA 88, 042520 (2013).

  Target gas contained 0.55(5) % air (leak).

  Back-of-the-envelope calculation:

  collision rate $\lambda \approx 6 \cdot 10^3 s^{-1}$
  2S lifetime $\tau(2S) = 1\,\mu$s

  $\Rightarrow$ Less than 1% of all μp(2S) atoms see any N_2

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} \quad = \quad \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

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- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas imputities?

μp experiment probably not wrong by 100 σ

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_p^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp theory wrong?

$$\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \quad [\text{meV}]$$

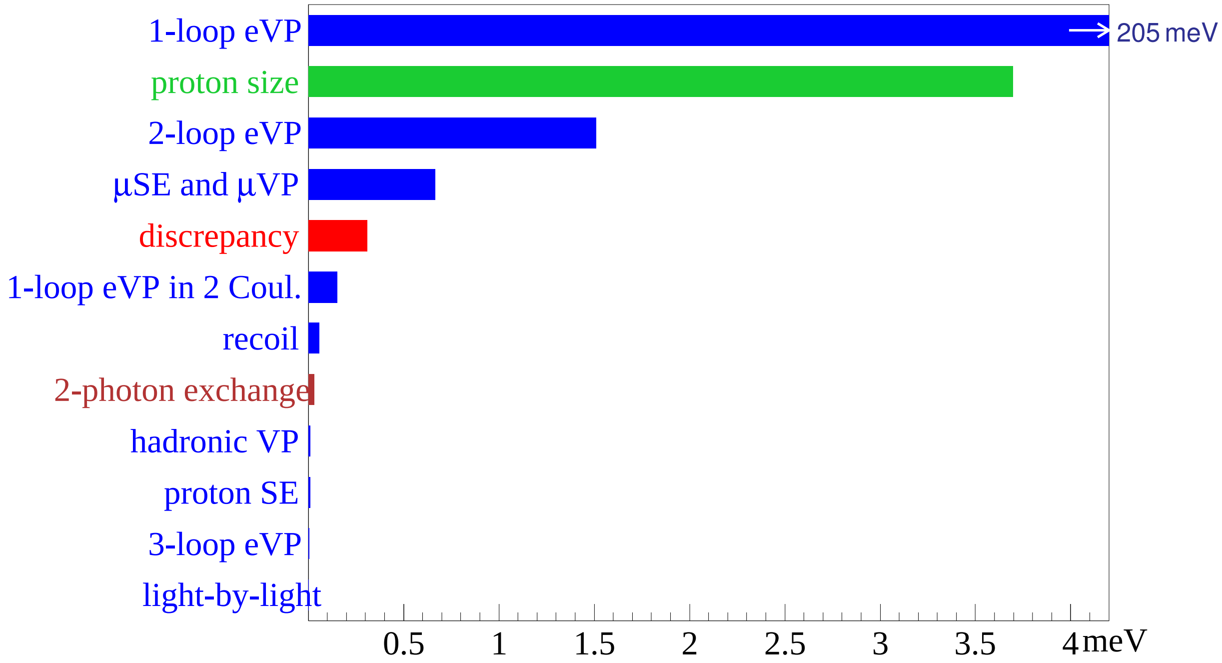
Discrepancy = 0.31 meV
Theory uncert. = 0.0025 meV

$\Longrightarrow$ 120δ(theory) deviation

double-checked by many groups

5th largest term!

Some contributions to the μp Lamb shift



Theory summary:

A. Antognini, RP *et al.*
Annals of Physics 331, 127 (2013)

What may be wrong?

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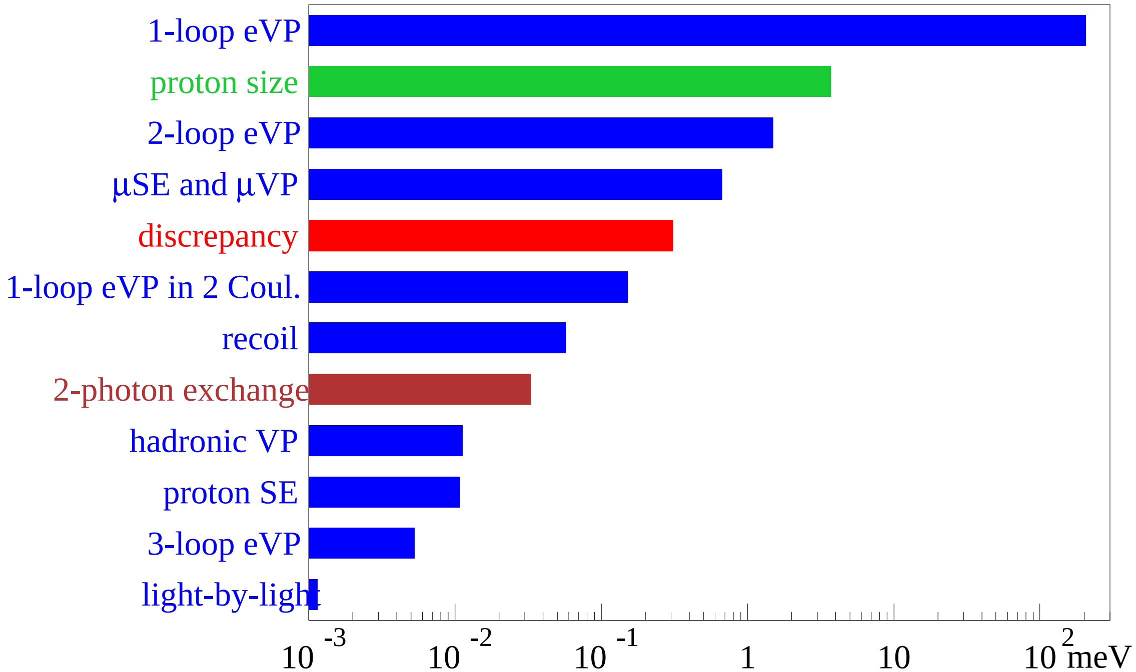

double-checked by many groups

5$^{\text{th}}$ largest term!

Theory summary:

A. Antognini, RP *et al.*
Annals of Physics 331, 127 (2013)

What may be wrong?

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μp theory wrong?

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Discrepancy = 0.31 meV
Theory uncert. = 0.0025 meV

$\Longrightarrow$ $120\delta(\text{theory})$ deviation

μp theory probably not wrong by 100 σ

Lamb shift in μp 1: r_p independent

Table 1
All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The "finite-size to relativistic recoil correction" (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, "1:3" LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, "2:2" LBL		0.00135	0.00115		0.00115	[74,89]
New	"3:1" LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 1: r_p independent

Table 1 (*continued*)

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

[a] This value has been recalculated to be 0.018759 meV [77].
[b] This correction is not necessary here because in #2 the Breit–Pauli contribution has been calculated using a Coulomb potential modified by eVP.
[c] Difference between Eqs. (6) and (4) in [78]: $E_{\mathrm{VP}}^{(\mathrm{rel})}(2\mathrm{P}_{1/2}\text{–}2\mathrm{S}_{1/2}) - E_{\mathrm{VP}}^{(0)}(2\mathrm{P}_{1/2}\text{–}2\mathrm{S}_{1/2}) = 0.018759 - 0.020843 = -0.002084$ meV (see also Table IV). Using these corrected values, the various approaches are consistent. Pachucki becomes $205.0074 + 0.018759 = 205.0262$ meV and Borie $205.0282 - 0.0020843 = 205.0261$ meV.
[d] In Appendix C, incomplete.
[e] Eq. (27) in [85] includes contributions beyond the logarithmic term with modification of the Bethe logarithm to the Uehling potential. The factor 10/9 should be replaced by 5/6.
[f] This term is part of #22, see Fig. 22 in [86].
[g] Borie includes wave-function corrections calculated in [87]. The actual difference between Ref. [13] and Borie-v6 [79] is given by the inclusion of the Källén–Sabry correction with muon loop.
[h] This was calculated in the framework of NRQED. It is related to the definition of the proton radius.

43 R.J. Hill, G. Paz, Phys. Rev. Lett. 107, 160402 (2011)
74 M.I. Eides, H. Grotch, V.A. Shelyuto, Phys. Rep. 342, 63 (2001)
77 U.D. Jentschura, Phys. Rev. A 84, 012505 (2011)
78 S.G. Karshenboim, V.G. Ivanov, E.Y. Korzinin, Phys. Rev. A 85, 032509 (2012)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
85 U.D. Jentschura, B.J. Wundt, Eur. Phys. J. D 65, 357 (2011)
86 E. Borie, G.A. Rinker, Rev. Mod. Phys. 54, 67 (1982)
87 V.G. Ivanov, E.Y. Korzinin, S.G. Karshenboim, Phys. Rev. D 80, 027702 (2009)
88 T. Kinoshita, M. Nio, Phys. Rev. Lett. 82, 3240 (1999)
89 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, V.A. Shelyuto, JETP Lett. 92, 8 (2010)
90 R. Barbieri, M. Caffo, E. Remiddi, Lett. Nuovo Cimento 7, 60 (1963)
91 H. Suura, E.H. Wichmann, Phys. Rev. 105, 1930 (1957)
92 A. Petermann, Phys. Rev. 105, 1931 (1957)
93 J. Friar, J. Martorell, D. Sprung, Phys. Rev. A 59, 4061 (1999)
94 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 63, 845 (2000)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 2: r_p-dependent

Table 2

Proton-structure-dependent contributions to the Lamb shift in μp from different authors and the one we selected. Values are in meV, $\langle r^2 \rangle$ in fm^2. The entry # in the first column refers to Table 1 in Ref. [13] supplementary information [9]. Entry # 18 is under debate. TPE: two-photon exchange, VP: vacuum polarization, SE: self-energy, Rel: relativistic.

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
	Non-rel. finite-size	$-5.1973\ \langle r^2 \rangle$	$-5.1975\ \langle r^2 \rangle$	$-5.1975\ \langle r^2 \rangle$			
	Rel. corr. to non-rel. finite size	$-0.0018\ \langle r^2 \rangle$		-0.0009 meV[a]			
	Rel. finite-size						
	Exponential				$-5.1994\ \langle r^2 \rangle$	$-5.2001\ \langle r^2 \rangle$	$-5.1994\ \langle r^2 \rangle$
	Yukawa					$-5.2000\ \langle r^2 \rangle$	
	Gaussian					$-5.2001\ \langle r^2 \rangle$	
	Finite size corr. to one-loop eVP	$-0.0110\ \langle r^2 \rangle$	$-0.0110\ \langle r^2 \rangle$	$-0.010\ \langle r^2 \rangle$	$-0.0282\ \langle r^2 \rangle$		$-0.0282\ \langle r^2 \rangle$
	Finite size to one-loop eVP-it.	$-0.0165\ \langle r^2 \rangle$	$-0.0170\ \langle r^2 \rangle$	$-0.017\ \langle r^2 \rangle$	(incl. in -0.0282)		
	Finite-size corr. to Källén–Sabry	[b]			$-0.0002\ \langle r^2 \rangle$		$-0.0002\ \langle r^2 \rangle$
New	Finite size corr. to μ self-energy	(0.00699)[c]			$0.0008\ \langle r^2 \rangle$		$0.0009(3)\ \langle r^2 \rangle$[d]
	ΔE_{TPE} [46]						0.0332(20) meV
	Elastic (third Zemach)[e]						
	Measured $R^3_{(2)}$	$0.0365(18)\ \langle r^2 \rangle^{3/2}$					(incl. above)
	Exponential			$0.0363\ \langle r^2 \rangle^{3/2}$	$0.0353\ \langle r^2 \rangle^{3/2}$ [f]	$0.0353\ \langle r^2 \rangle^{3/2}$	
	Yukawa					$0.0378\ \langle r^2 \rangle^{3/2}$	
	Gaussian					$0.0323\ \langle r^2 \rangle^{3/2}$	
25	Inelastic (polarizability)	0.0129(5) meV [101]		0.012(2) meV			(incl. above)
New	Rad. corr. to TPE	$-0.00062\ \langle r^2 \rangle$					$-0.00062\ \langle r^2 \rangle$
26	eVP corr. to polarizability						0.00019 meV [95]

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

Lamb shift in μp 2: r_p-dependent

Table 2 (*continued*)

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
27	SE corr. to polarizability						−0.00001 meV [95]
18	Finite-size to rel. recoil corr.	(0.013 meV)[g]		[h]			(incl. in ΔE_{TPE})
	Higher order finite-size corr.	−0.000123 meV			0.00001(10) meV		0.00001(10) meV
	$2P_{1/2}$ finite-size corr.	$-0.0000519\langle r^2\rangle$[i]			(incl. above)	(incl. above)	(incl. above)

[a] Corresponds to Eq. (6) in [11] which accounts only for the main terms in F_{REL} and F_{NREL}.
[b] This contribution has been accounted already in both the −0.0110 meV/fm^2 and −0.0165 meV/fm^2 coefficients.
[c] Given only in Appendix C. Bethe logarithm is not included.
[d] This uncertainty accounts for the difference between all-order in $Z\alpha$ and perturbative approaches [82].
[e] Corresponds to Eq. (20).
[f] This value is slightly different from Eq. (22) because here an all-order in finite-size and an all-order in eVP approaches were used.
[g] See Appendix F of [96]. This term is under debate.
[h] Included in ΔE_{TPE}. This correction of $0.018 - 0.021 = -0.003$ meV is given by Eq. (64) in [10] and Eq. (25) in [11]. This correction is also discussed in [76] where the 6/7 factor results from 0.018/0.021.
[i] Eq. (6a) in [79].

46 M.C. Birse, J.A. McGovern, Eur. Phys. J. A 48, 120 (2012); arXiv:1206.3030
76 U.D. Jentschura, Ann. Phys. 326, 500 (2011)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
82 P. Indelicato, P.J. Mohr, 2012 (in preparation)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)
96 J.L. Friar, Ann. Phys. 122, 151 (1979)
101 C.E. Carlson, M. Vanderhaeghen, Phys. Rev. A 84, 020102 (2011)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

HFS in μp

Table 3
All known contributions to the 2S-HFS in μp from different authors and the one we selected. Values are in meV, radii in fm. SE: self-energy, VP: vacuum polarization, Rel: relativistic, RC: recoil correction, PT: perturbation theory, p: proton, int: interaction, AMM: anomalous magnetic moment.

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h1	Fermi energy, $(Z\alpha)^4$	22.8054	22.8054			
h2	Breit corr., $(Z\alpha)^6$	0.0026	0.00258			
h3	Dirac energy (+ Breit corr. in all-order)			22.807995	22.807995	Eq. (107) in [80]
h4	μ AMM corr., $\alpha(Z\alpha)^4$, $\alpha(Z\alpha)^4$	0.0266	0.02659		0.02659	
h5	eVP in 2nd-order PT, $\alpha(Z\alpha)^5$ (ϵ_{VP2})	0.0746	0.07443			
h6	All-order eVP corr.			0.07437	0.07437	Eq. (109) in [80]
h7	Two-loop corr. to Fermi-energy (ϵ_{VP2})		0.00056		0.00056	
h8	One-loop eVP in 1γ int., $\alpha(Z\alpha)^4$ (ϵ_{VP1})	0.0482	0.04818		0.04818	
h9	Two-loop eVP in 1γ int., $\alpha^2(Z\alpha)^4$ (ϵ_{VP1})	0.0003	0.00037		0.00037	
h10	Further two-loop eVP corr.		0.00037		0.00037	[113,114]
h11	μVP (similar to ϵ_{VP2})		0.00091		0.00091	
h12	μVP (similar to ϵ_{VP1})	0.0004	(incl. in h13)		(incl. in h13)	
h13	Vertex, $\alpha(Z\alpha)^5$		−0.00311		−0.00311	[a]
h14	Higher order corr. of (h13), (part with $\ln(\alpha)$)		−0.00017		−0.00017	[115]
h15	μ SE with p structure, $\alpha(Z\alpha)^5$	0.0010				
h16	Vertex corr. with proton structure, $\alpha(Z\alpha)^5$	−0.0018				
h17	"Jellyfish" corr. with p structure, $\alpha(Z\alpha)^5$	0.0005				
h18	Hadron VP, α^6	0.0005(1)	0.00060(10)		0.00060(10)	
h19	Weak interaction contribution	0.0003	0.00027		0.00027	[116]
h20	Finite-size (Zemach) corr. to $\Delta E_{\mathrm{Fermi}}$, $(Z\alpha)^5$	−0.1518[b]	$-0.16037\, r_Z$	$-0.16034\, r_Z$	$-0.16034\, r_Z$	Eq. (107) in [80]

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

Table 3 (*continued*)

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h21	Higher order finite-size corr. to $\Delta E_{\mathrm{Fermi}}$			$-0.0022\, r_E{}^2 + 0.0009$	$-0.0022\, r_E{}^2 + 0.0009$	Eq. (107) in [80]
h22	Proton polarizability, $(Z\alpha)^5$, $\Delta E^{\mathrm{pol}}_{\mathrm{HFS}}$	0.0105(18)	0.0080(26)		0.00801(260)	[117,118]
h23	Recoil corr.	(incl. in h20)	0.02123		0.02123	[112]
h24	eVP + proton structure corr., α^6	-0.0026				
h25	eVP corr. to finite-size (similar to ϵ_{VP2})		-0.00114	$-0.0018\, r_Z - 0.0001$	$-0.0018\, r_Z - 0.0001$	Eq. (109) in [80]
h26	eVP corr. to finite-size (similar to ϵ_{VP1})		-0.00114		$-0.00114(20)$	
h27	Proton structure corr., $\alpha(Z\alpha)^5$	-0.0017				
h28	Rel. + radiative RC with p AMM, α^6	0.0018				
	Sum	22.8148(20)[c]	22.9839(26) $- 0.1604\, r_Z$		22.9858(26) $-$ 0.1621(10) $r_Z - 0.0022(5)\, r_E^2$	
	Sum with $r_E = 0.841$ fm, $r_Z = 1.045$ fm [28]	22.8148 meV	22.8163 meV		22.8149 meV	

[a] Includes a correction $\alpha(Z\alpha)^5$ due to μVP.
[b] Calculated using the Simon et al. form factor.
[c] The uncertainty is 0.0078 meV if the uncertainty of the Zemach term (h20) is included (see Table II of [72]).

28 M.O. Distler, J.C. Bernauer, T. Walcher, Phys. Lett. B 696, 343 (2011)
80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
112 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 78, 022517 (2008)
113 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 88, 641 (2008)
114 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 89, 216 (2009)
115 S.J. Brodsky, G.W. Erickson, Phys. Rev. 148, 26 (1966)
116 M.I. Eides, Phys. Rev. A 85, 034503 (2012)
117 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 83, 042509 (2011)
118 E. Cherednikova, R. Faustov, A. Martynenko, Nuclear Phys. A 703, 365 (2002)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

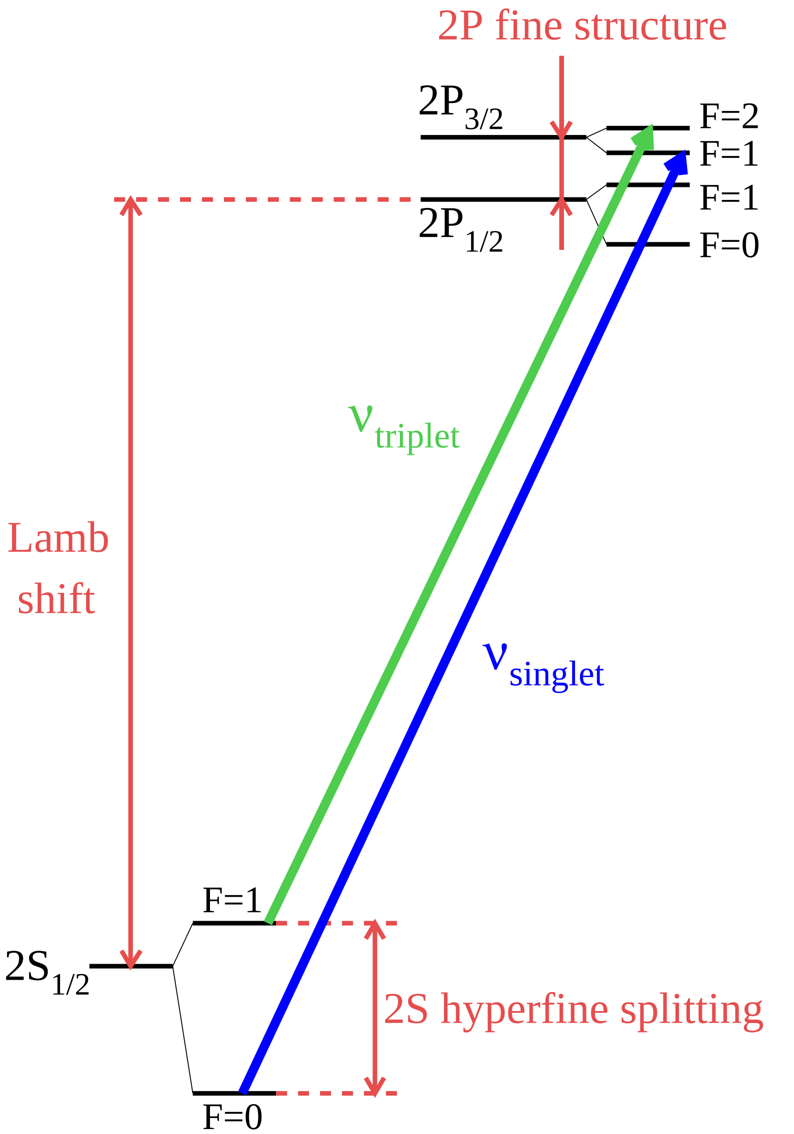
Theory in μp: new papers

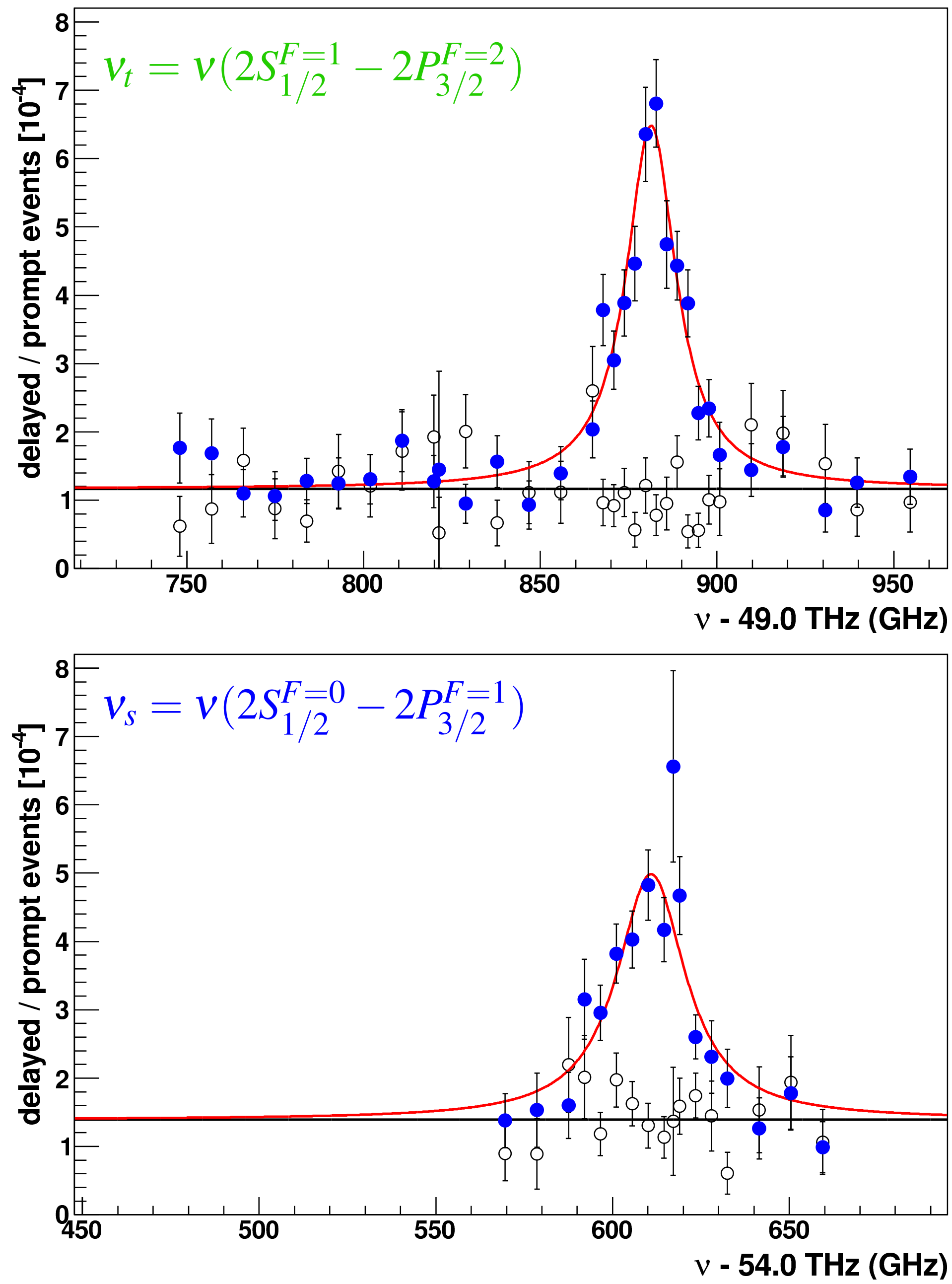
Papers that have appeared after we wrote A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013):

- Mohr, Griffith, Sapirstein, PRA 87, 052511 (2013), [1304.2076], "Bound-state field-theory approach to proton-structure effects in muonic hydrogen"
- Korzinin, Ivanov Karshenboim, PRD 88, 125019 (2013) [1311.5784]: "The $\alpha^2(Z\alpha)^4m$ contributions to the Lamb shift and fine structure in light muonic atoms"
- Karshenboim, Ivanov, Korzinin, PRA 89, 022102 (2014) [1311.5789]: "Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. I. General approach"
- Ivanov, Korzinin, Karshenboim, PRA 90, 022103 (2014) [1311.5790]: "Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. II. An application to the one-loop electronic vacuum polarization"
- Alarcon, Lensky, Pascalutsa, EPJ C 74, 2852 (2014) [1312.1219]: "Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution"
- Indelicato, Mohr, Sapirstein, PRA 89, 054017 (2014) [1402.0439]: "Coordinate-space approach to vacuum polarization"
- Peset, Pineda, arXiv 1403.3408 [hep-ph]: "Model independent determination of the muonic hydrogen Lamb shift and proton radius"
- Peset, Pineda, Nucl. Phys. B 887, 69 (2014) [1406.4524]: "The two-photon exchange contribution to muonic hydrogen from chiral perturbation theory"

No big changes.
Polarizability terms confirmed a couple of times.

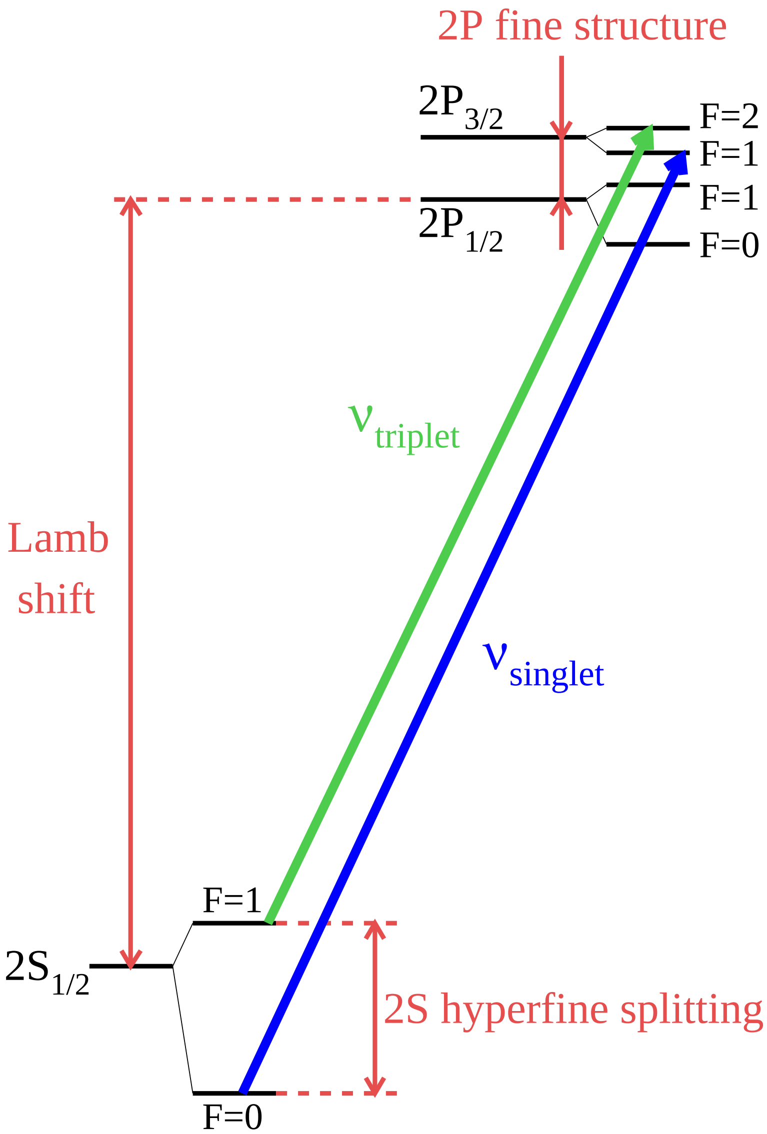
We have measured two transitions in μp





R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).

We have measured two transitions in μp



- Consider the two measurements **separately**

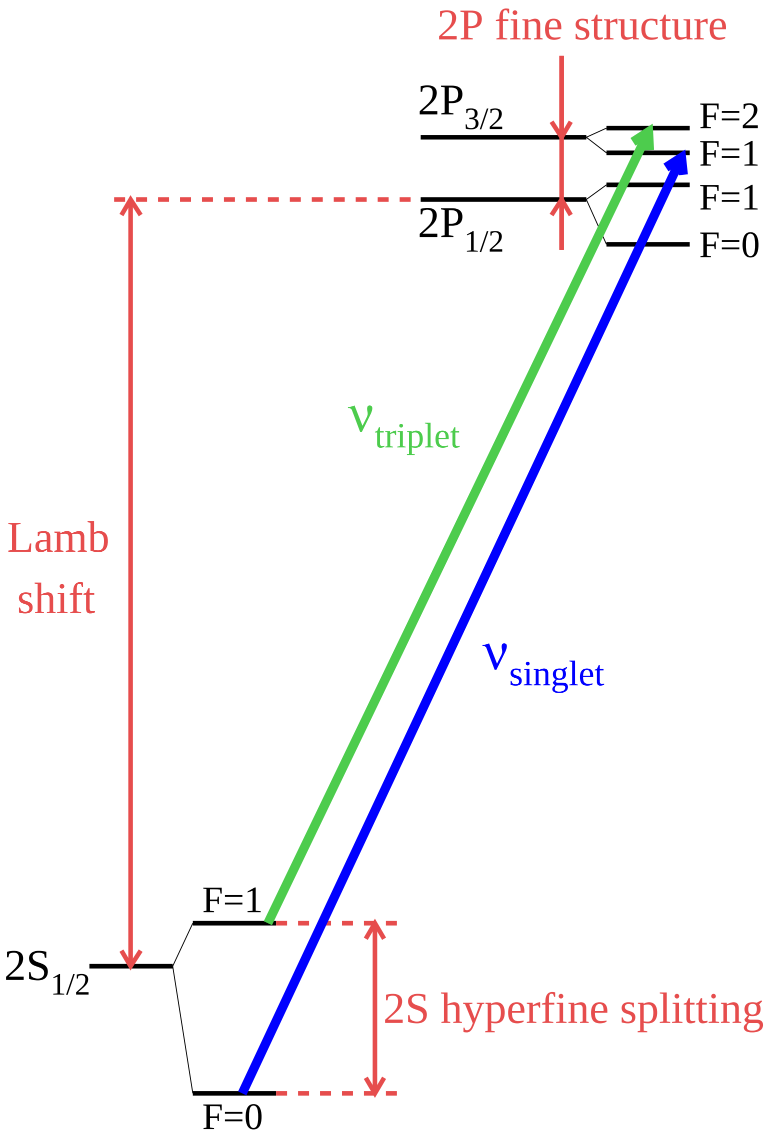
Two independent determinations of r_p

$(\nu_t \rightarrow r_p \ , \ \nu_s \rightarrow r_p)$

Consistent results!

R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).

We have measured two transitions in μp



- <u>Consider the two measurements **separately**</u>

  Two independent determinations of r_p

  $(\nu_t \rightarrow r_p\ ,\ \nu_s \rightarrow r_p)$ **Consistent results!**

- <u>Combine the two measurements</u>

  Two measurements $\rightarrow$ determine two parameters

  $\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{HFS} \rightarrow$ $\boxed{r_p,\ r_Z}$

$$
\begin{array}{rcll}
\frac{3}{4}\nu_t + \frac{1}{4}\nu_s & = & \Delta E_L(r_p) & +8.8123\ \text{meV} \\
\nu_s - \nu_t & = & \Delta E_{HFS}(r_Z) & -3.2480\ \text{meV}
\end{array}
$$

R. Pohl *et al.*, Nature 466, 213 (2010).
A. Antognini, RP *et al.*, Science 339, 417 (2013).

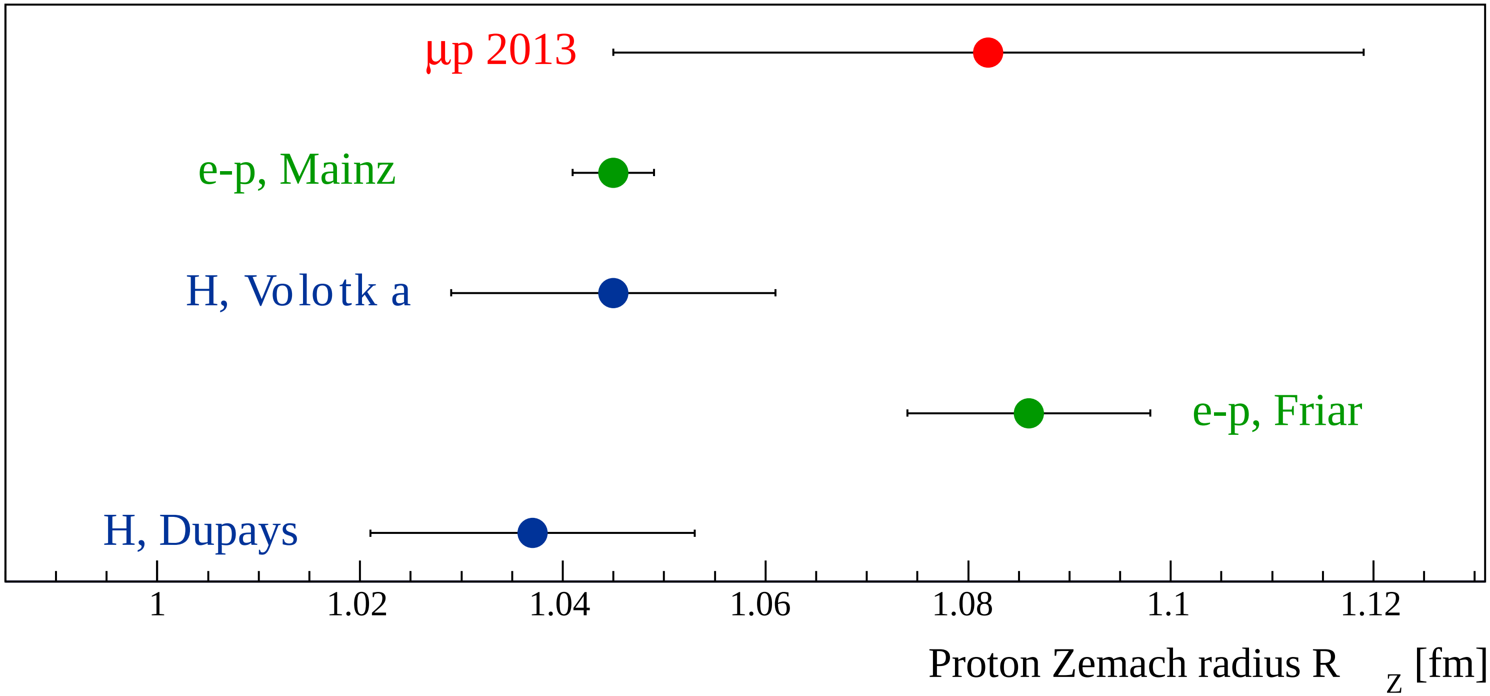
Proton Zemach radius

2S hyperfine splitting in μp is: $\Delta E_{\mathrm{HFS}} = 22.9843(30) - 0.1621(10)\, r_Z$ [fm] meV

with $r_Z = \int d^3r \int d^3r'\, r\, \rho_E(r)\, \rho_M(r - r')$

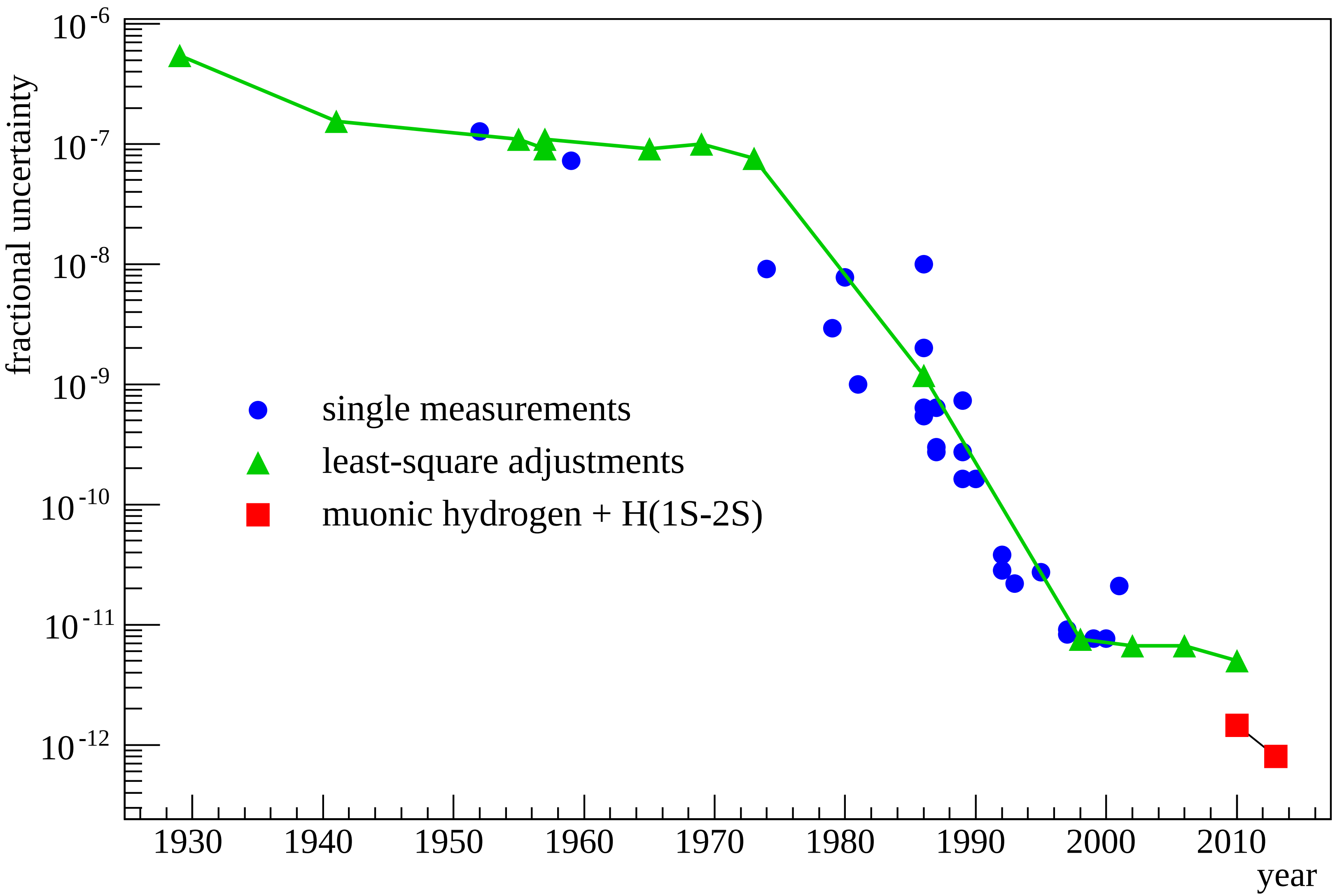
We measured $\Delta E_{\mathrm{HFS}} = 22.8089(51)$ meV

This gives a proton Zemach radius r_Z = 1.082 $(31)_{\mathrm{exp}}$ $(20)_{\mathrm{th}}$ = 1.082 (37) fm



A. Antognini, RP *et al.*, Science 339, 417 (2013)

Rydberg constant

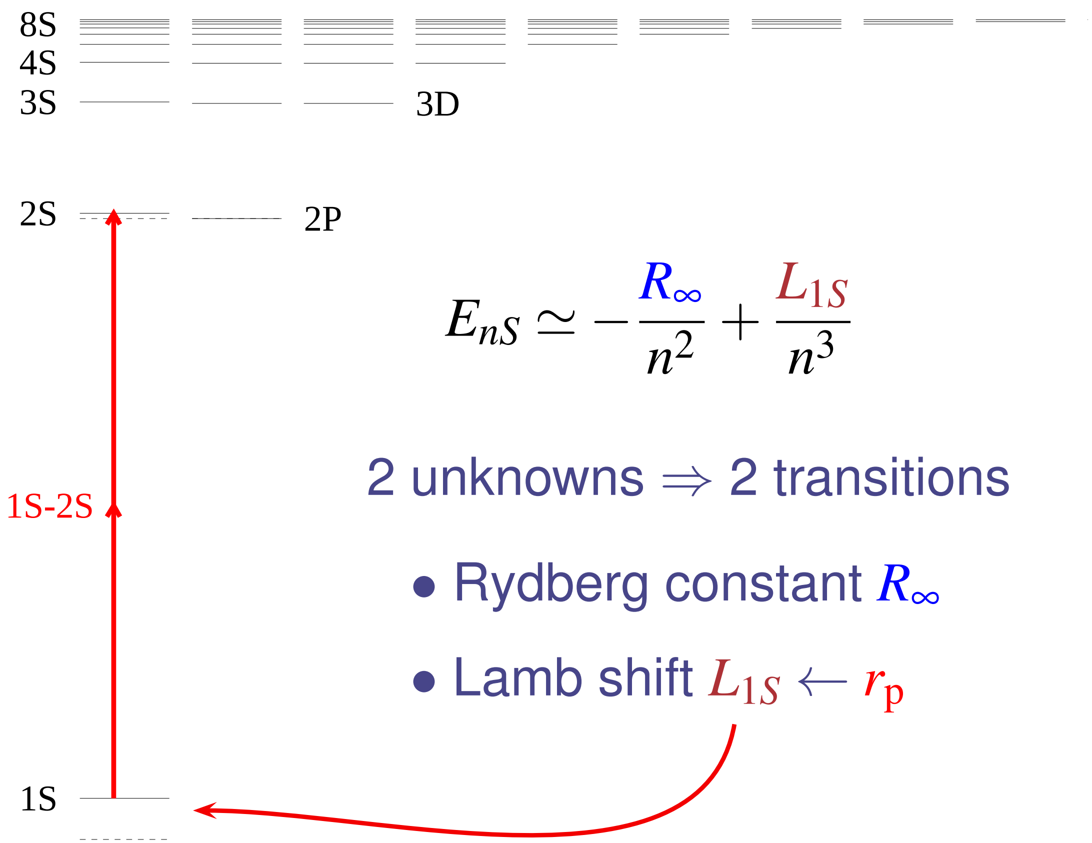


H(1S-2S): C.G. Parthey, RP *et al.*, PRL 107, 203001 (2011). r_p: A. Antognini, RP *et al.*, Science 339, 417 (2013).

Rydberg constant

Hydrogen spectroscopy (Lamb shift):

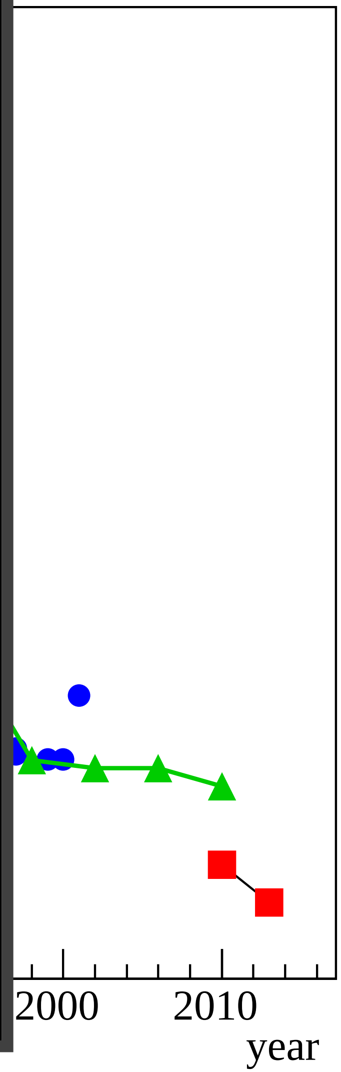
$$\boxed{L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \quad \text{MHz}}$$



$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns $\Rightarrow$ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

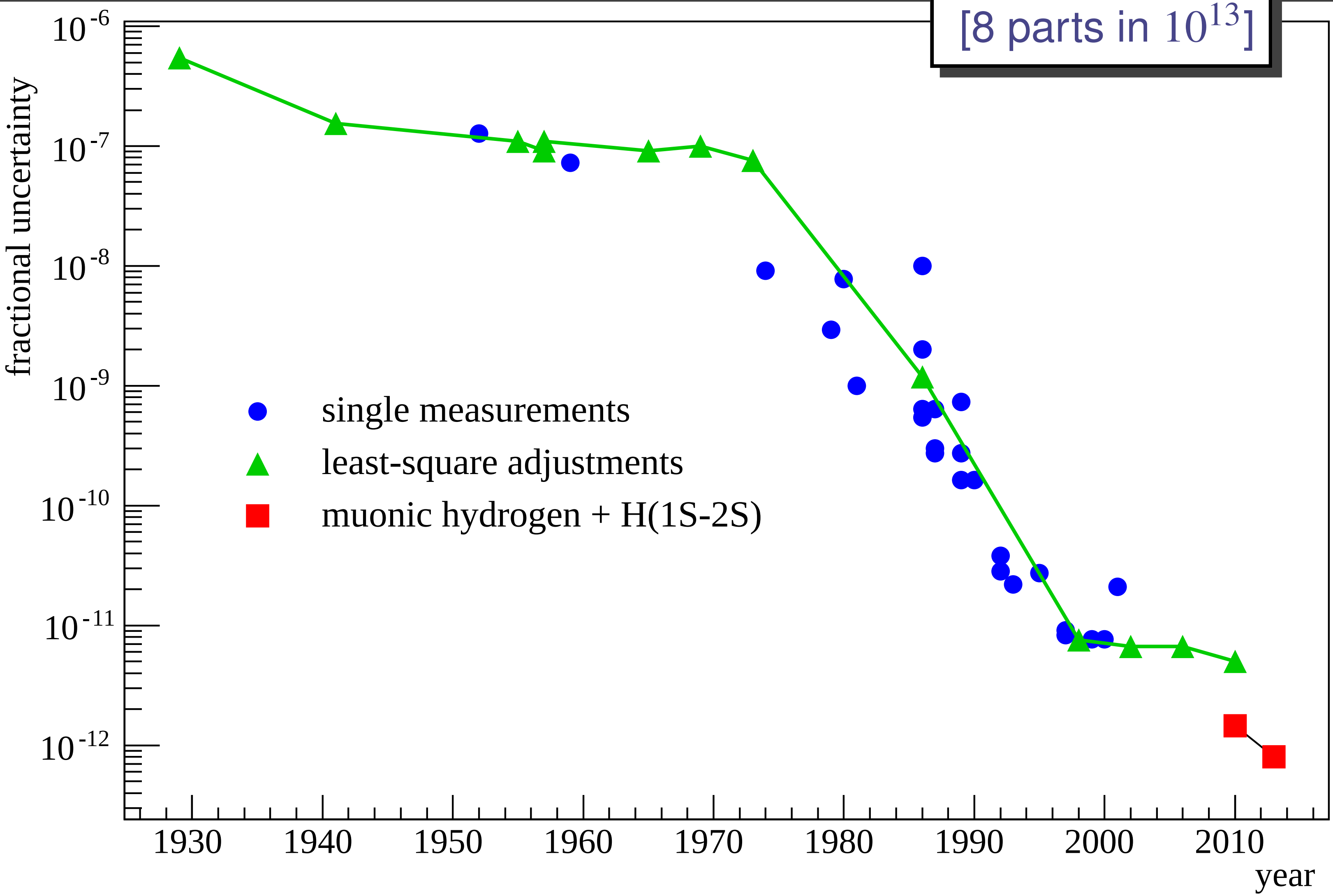


H(1S-2S): C.G. Parthey, RP *et al.*, PRL 107, 203001 (2011). r_p: A. Antognini, RP *et al.*, Science 339, 417 (2013).

Rydberg constant

$$R_\infty = 3.289\,841\,960\,249\,5\,(10)^{r_p}\,(25)^{\mathrm{QED}} \times 10^{15}\ \mathrm{Hz/c}$$

[8 parts in 10^{13}]



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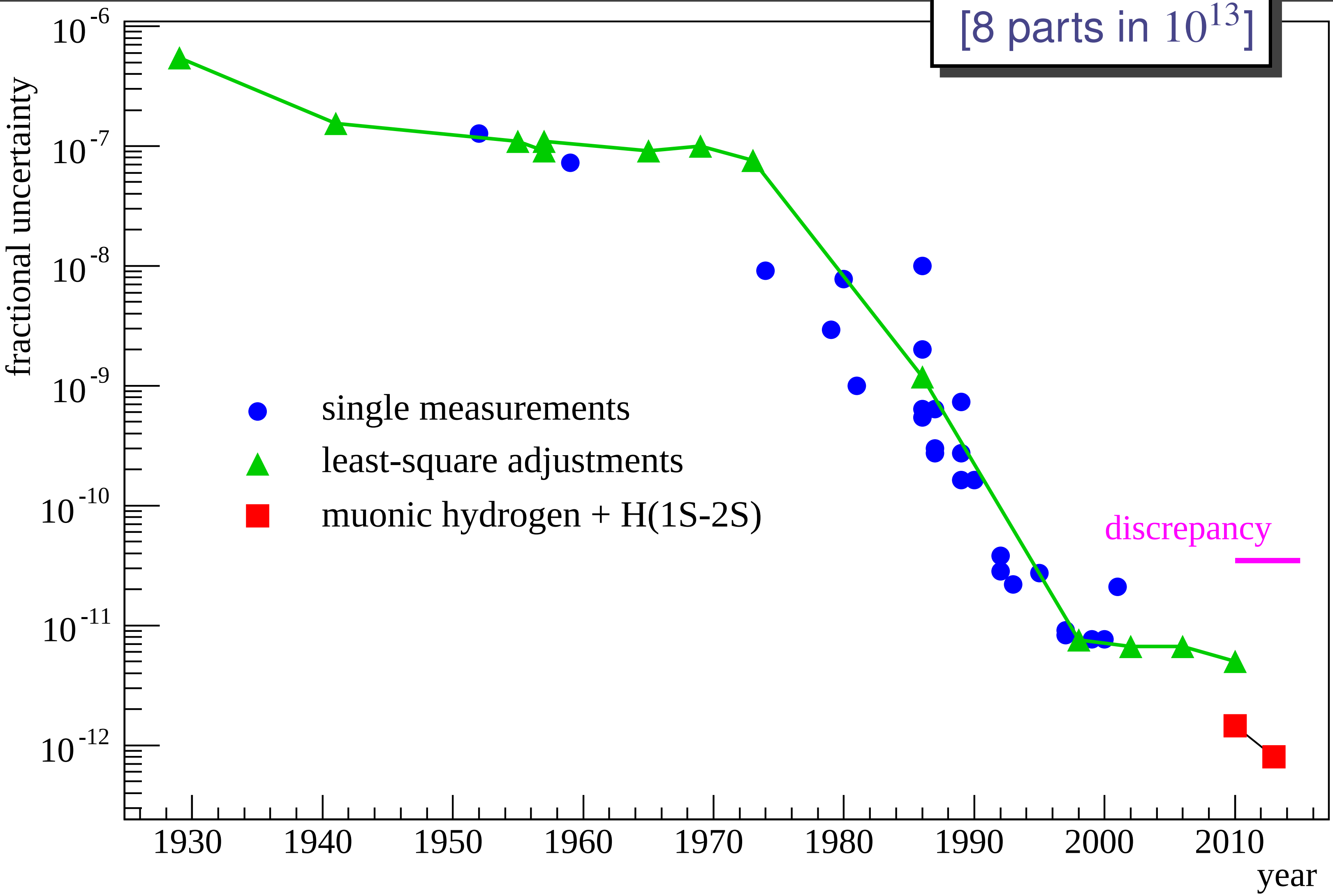
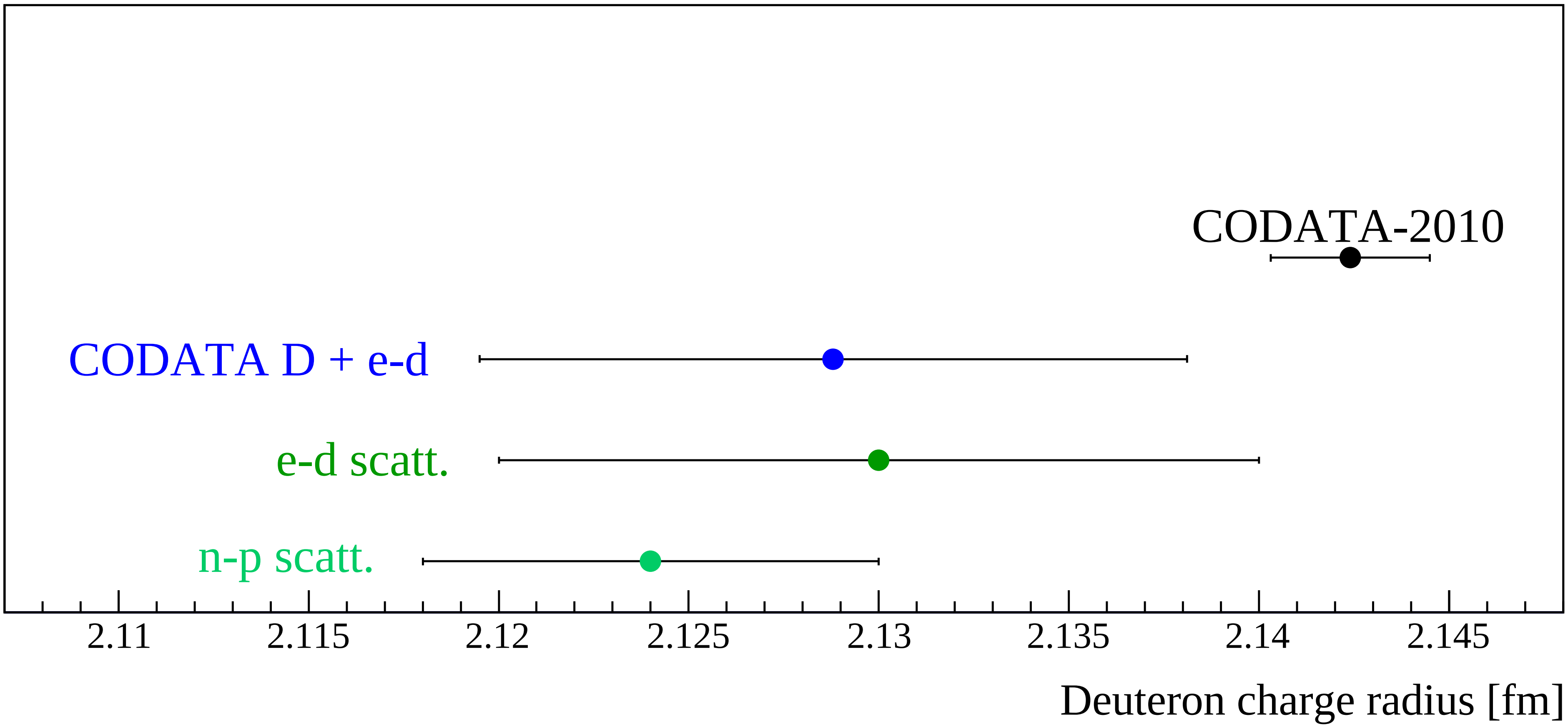


H(1S-2S): C.G. Parthey, RP *et al.*, PRL 107, 203001 (2011). r_p: A. Antognini, RP *et al.*, Science 339, 417 (2013).

Deuteron charge radius

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65)$ fm^2 C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)

CODATA 2010 r_d = 2.1424(21) fm



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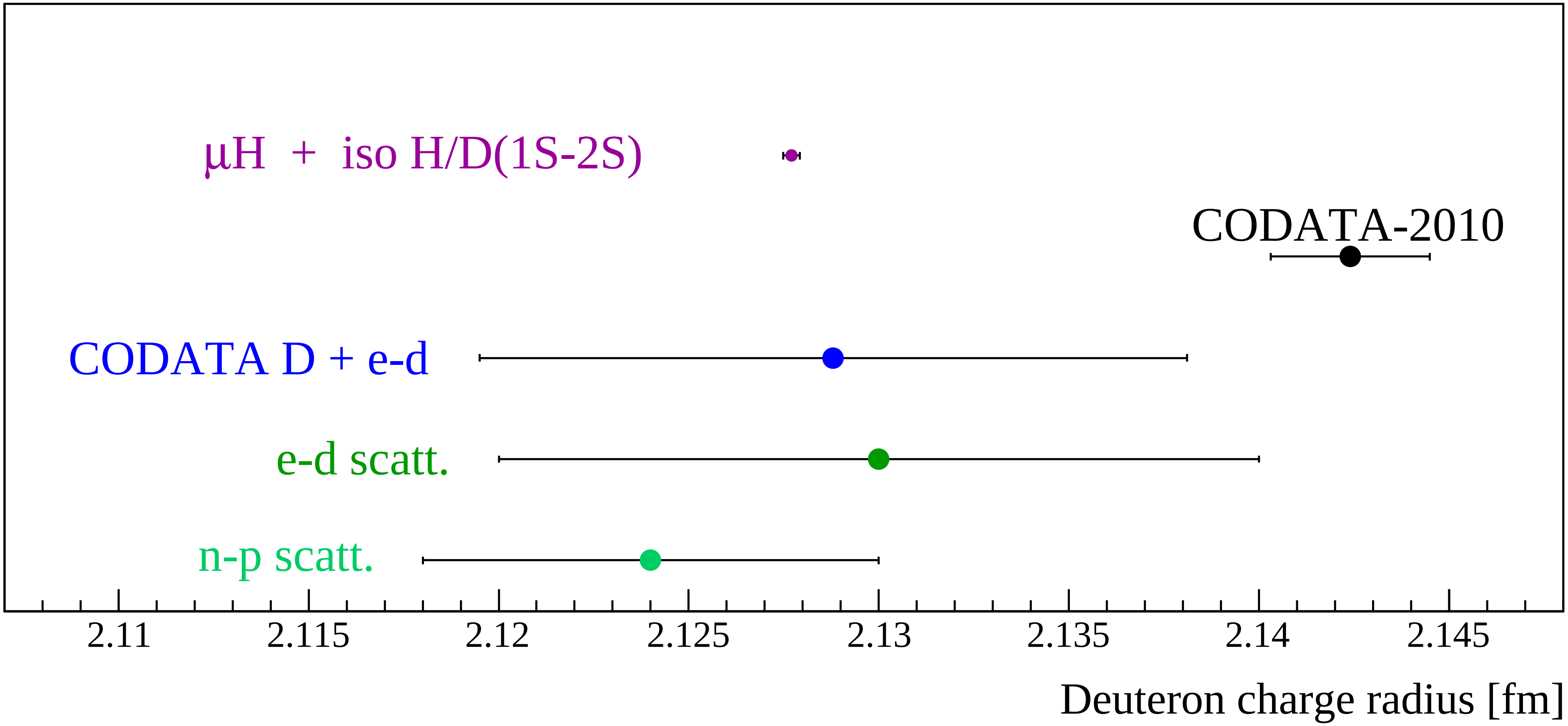
r_p= 0.84087(39) fm from μH gives r_d = 2.12771(22) fm



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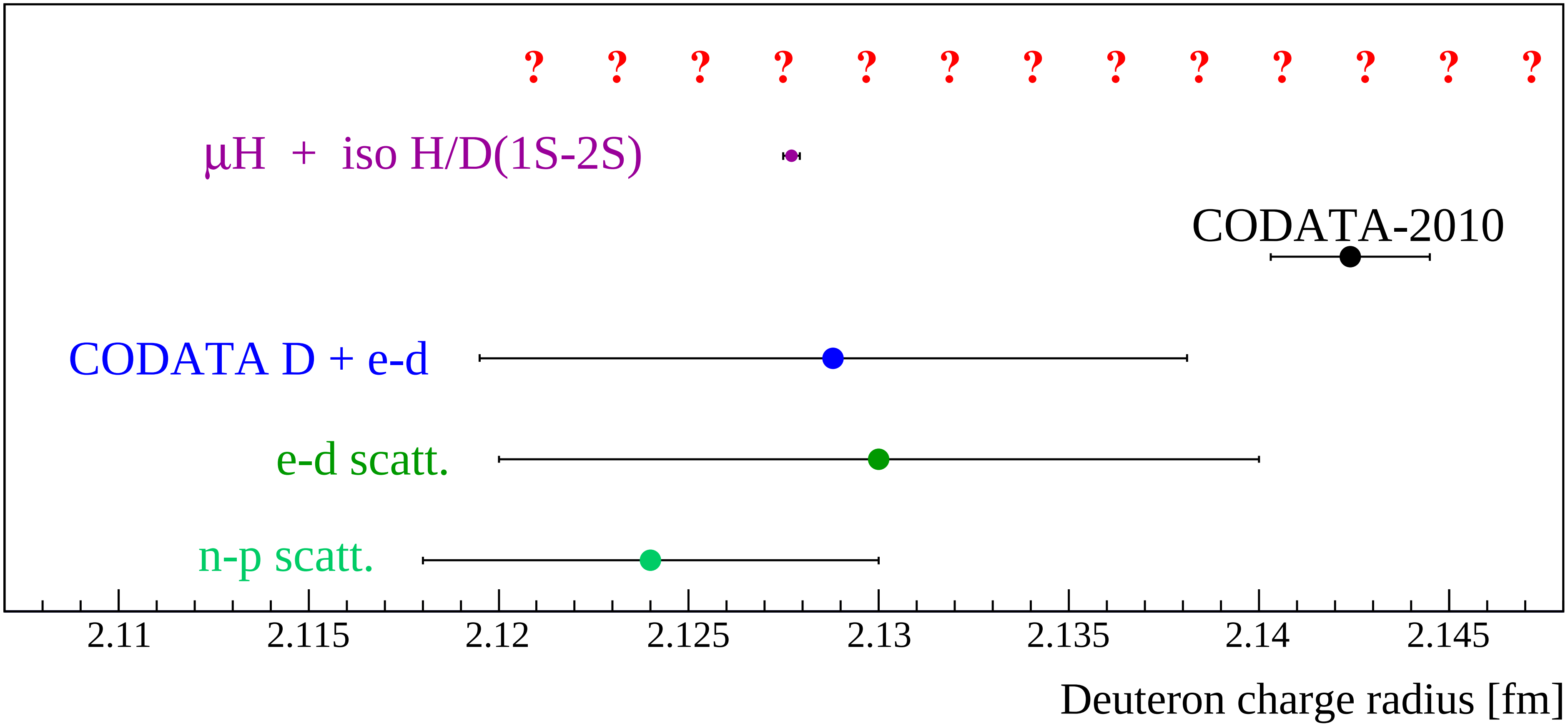
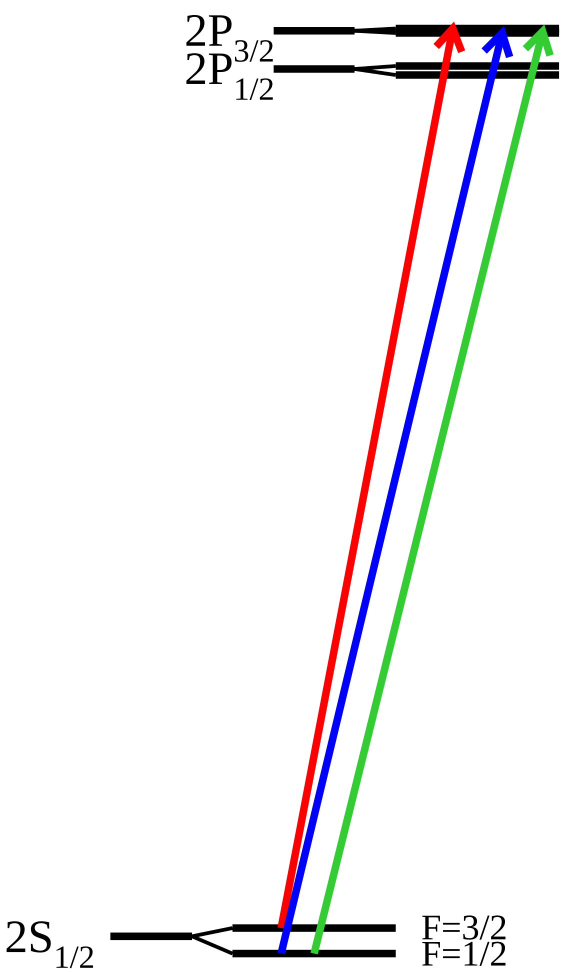
Lamb shift in muonic DEUTERIUM

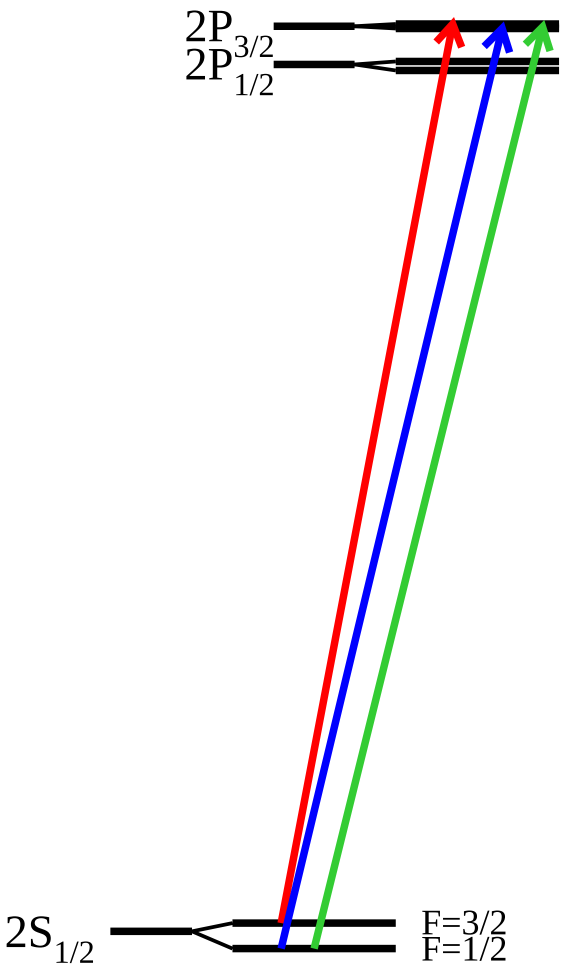


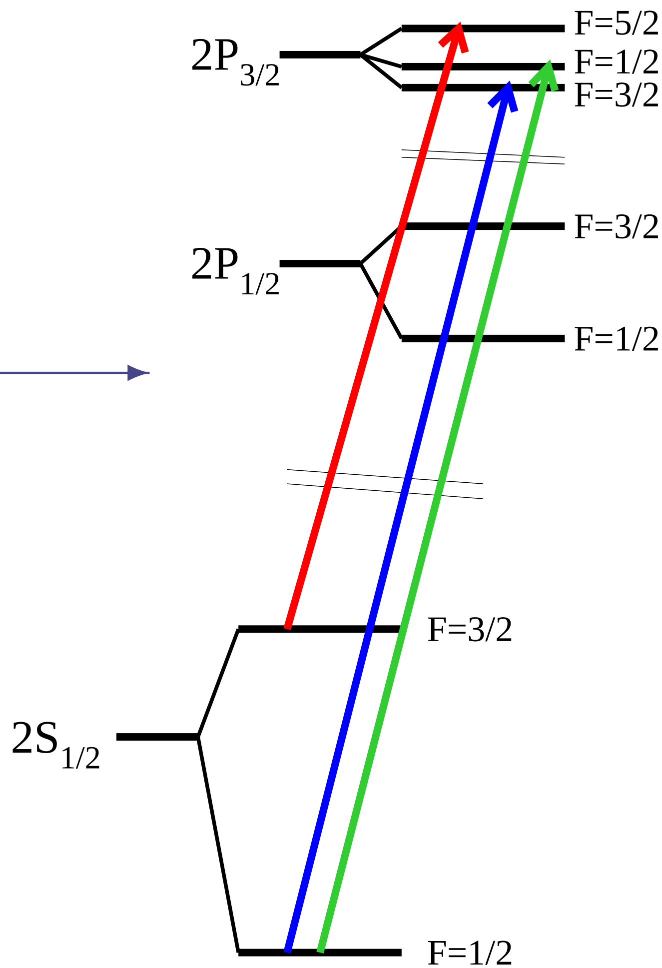
Muonic deuterium

muonic deuterium

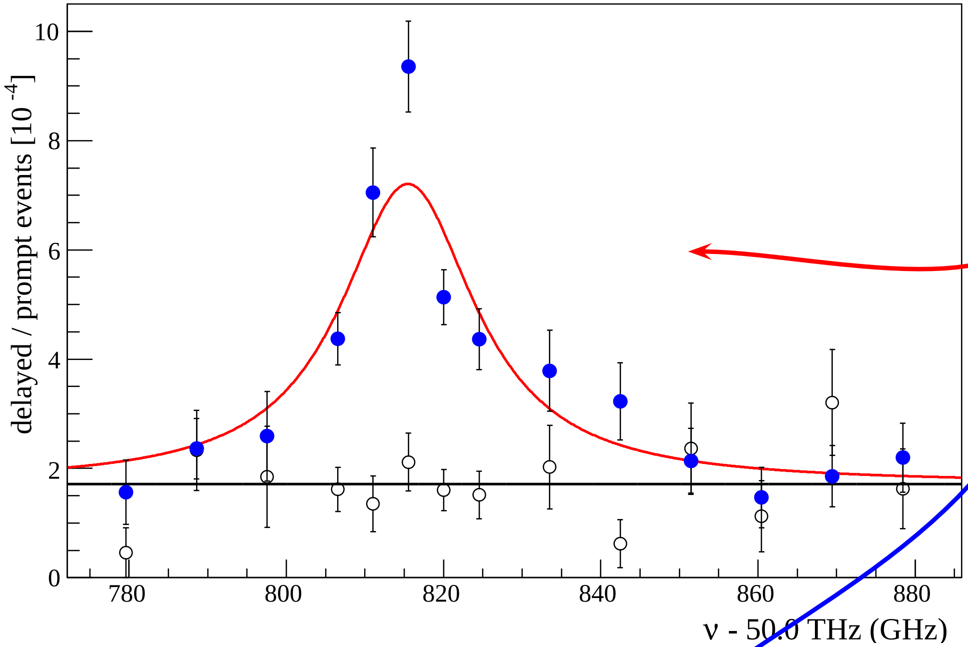


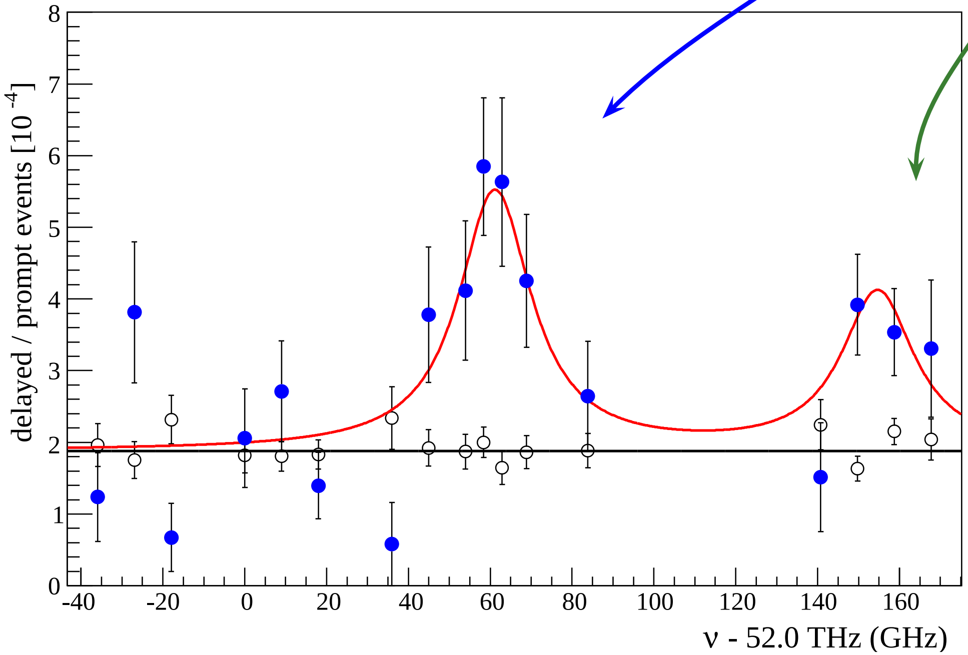
muonic deuterium





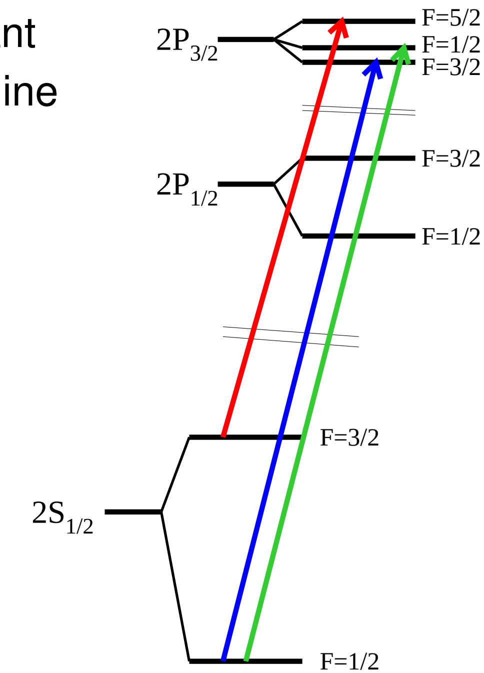
Muonic DEUTERIUM





2.5 resonances in muonic **deuterium**

- μd [$2S_{1/2}$(F=3/2) $\rightarrow$ $2P_{3/2}$(F=5/2)]
  20 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=3/2)]
  45 ppm (stat., online)
- μd [$2S_{1/2}$(F=1/2) $\rightarrow$ $2P_{3/2}$(F=1/2)]
  70 ppm (stat., online)
  only 5σ significant
  identifies F=3/2 line



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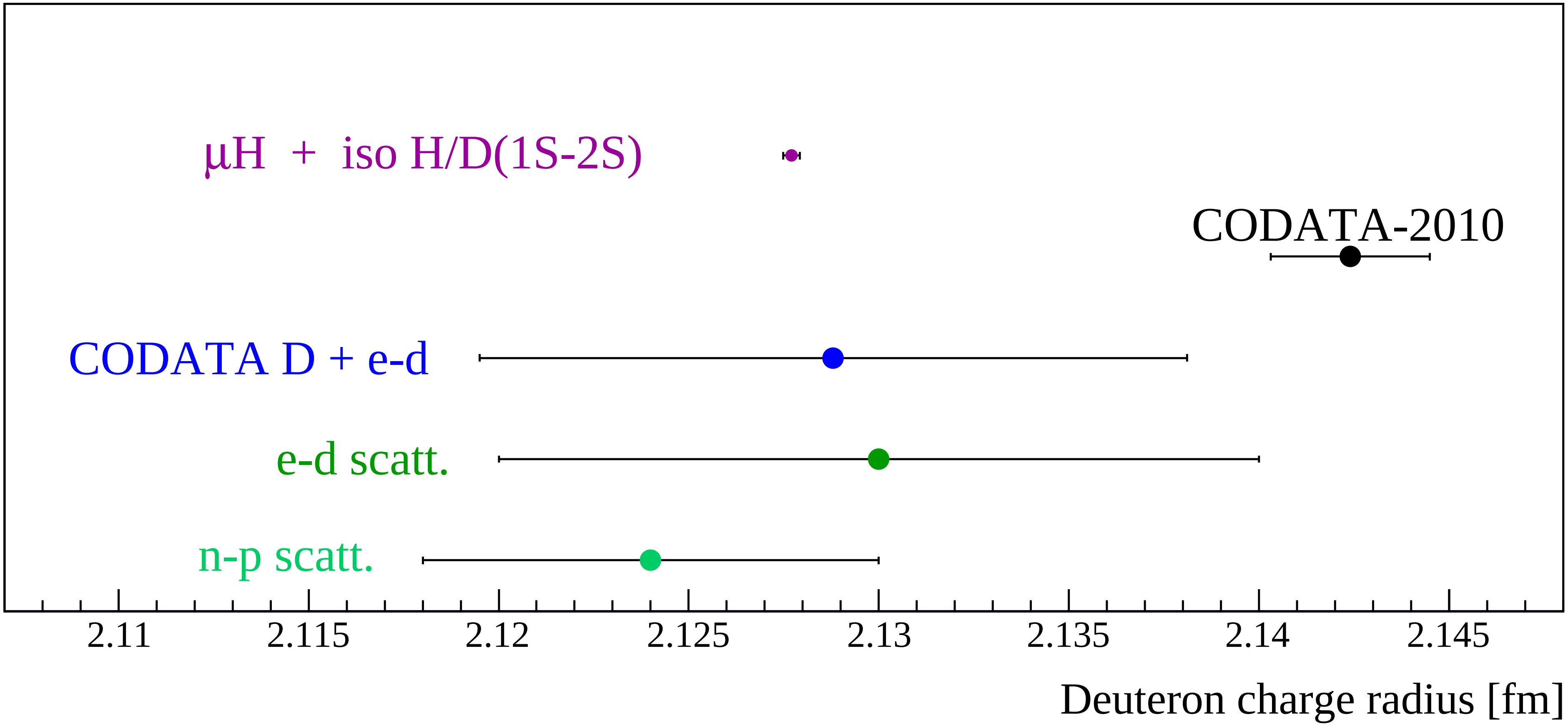
CODATA 2010 r_d = 2.1424(21) fm

r_p= 0.84087(39) fm from μH gives r_d = 2.1277(2) fm



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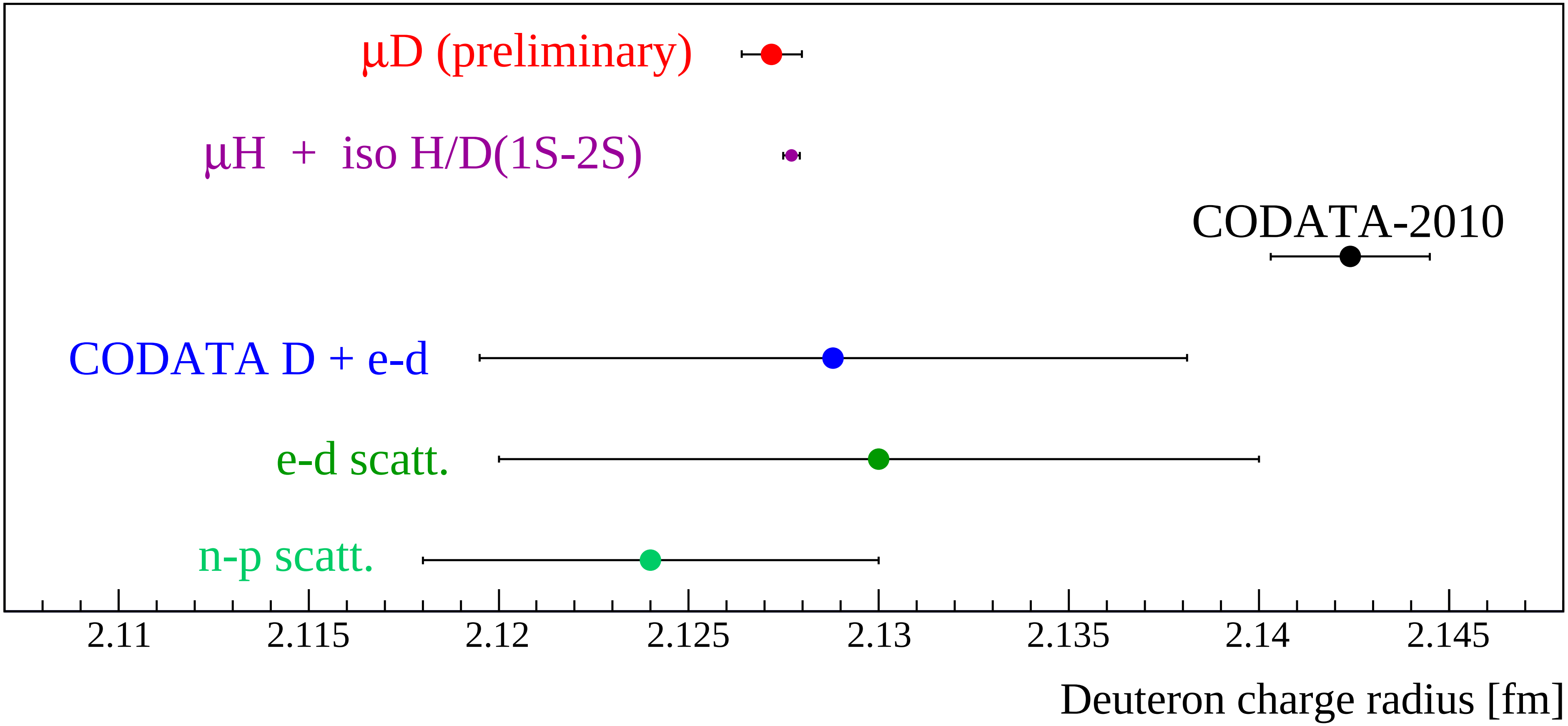
CODATA 2010	r_d = 2.1424(21) fm
r_p= 0.84087(39) fm from μH gives	r_d = 2.1277(2) fm
Lamb shift in muonic DEUTERIUM	r_d = 2.1272(12) fm PRELIMINARY!

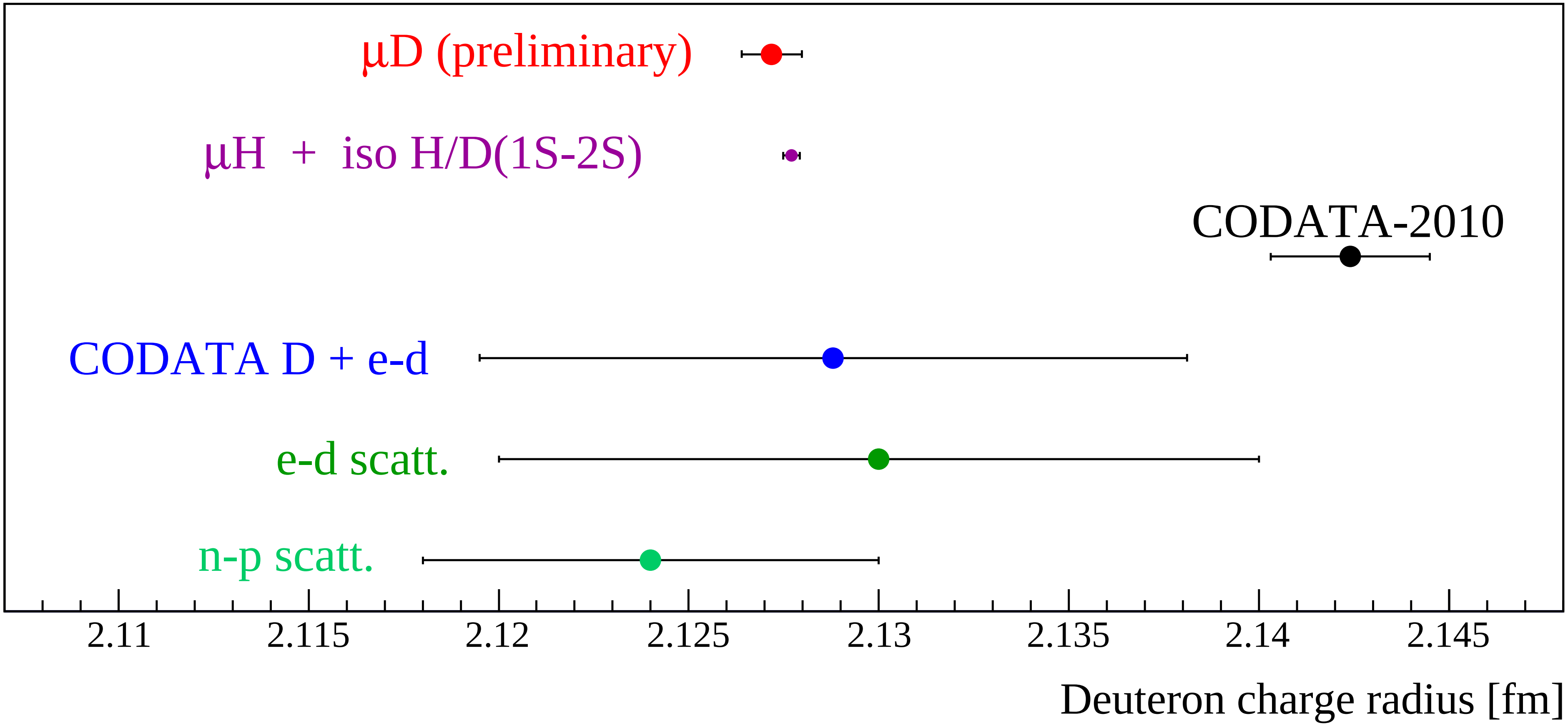


Deuteron charge radius

- μH and μD are **consistent!**
- proton discrepancy μH vs. H: 4.5σ

  deuteron discrepancy μD vs. D: 4.4σ

  combined atomic physics discr.: 6.3σ



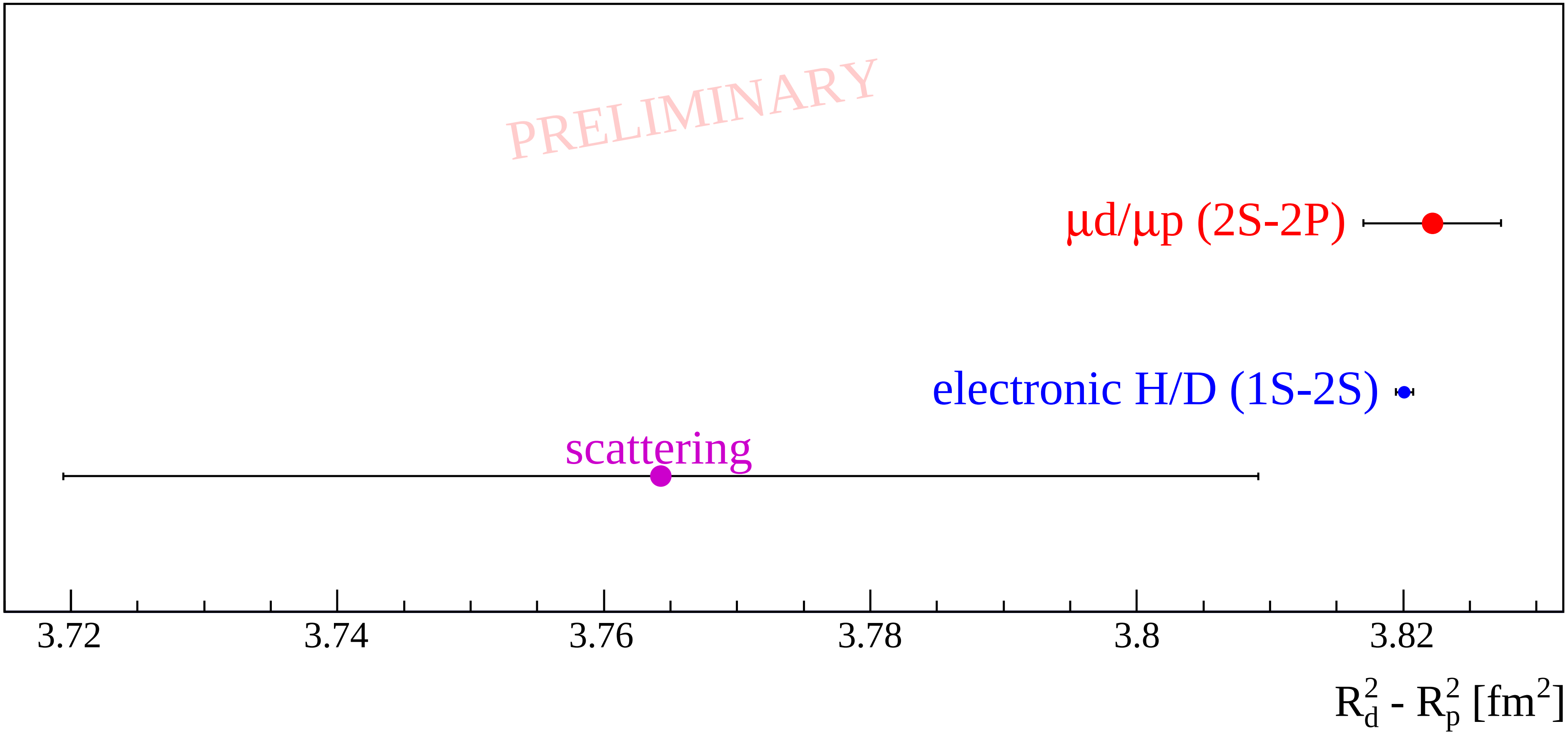
Proton-deuteron isotope shift

In other words: The muonic isotope shift agrees with the electronic one!

$r_d^2 - r_p^2$:

H/D isotope shift	$3.82007 \pm 0.00065\,\mathrm{fm}^2$		
muonic Lamb shift	$3.8221 \pm 0.0052\,\mathrm{fm}^2$	PRELIMINARY!	
scattering	$3.764 \pm 0.045\,\mathrm{fm}^2$		

The muonic error is conservative (nucl. structure terms).



Theory in μd

Aim: A compilation for muD, similar to A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013):

- Borie, 1103.1772v7: “Lamb Shift in Light Muonic Atoms - Revisited” update Aug 21, 2014
- Krutov, Martynenko, PRA 84, 052514 (2011): “Lamb shift in the muonic deuterium atom”
- several papers by Jentschura
- several papers by Karshenboim et al.

- Pachucki, PRL 106, 193007 (2011): “Nuclear Structure Corrections in Muonic Deuterium”
- Hernandez, Ji, Bacca, Nevo Dinur, Barnea, PLB 736, 344 (2014) “Improved estimates of the nuclear structure corrections in muD”
- Friar, PRC 88, 034003 (2013): “Nuclear polarization corrections to mu-d atoms in zero-range approximation”
- ...

	Tab. 1		our choice			Borie				Martynenko			
#	Contribution		value	err.	who?	v7	v6			v4 [A04]		Tab I	Eq.
1	NR one-loop eVP		--				--			227.6347	227.6347	1	(6)
											-0.0353	7	(27)-(30)
											0.0530	10	(43)
											-0.0002	8	(32)
											0.0004	11	(44)-(45)
2	rel. corr (Breit-Pauli)		--			0.02178	0.0204	Tab. I (p9)		0.0179		7+10+8+11	
3	Rel. one-loop eVP: 2S – 2P_1/2					227.6577	227.6577	p.3 Tab b					
3b	same, 2P_3/2		--			227.6635	227.6635	p.3 Tab b					
19	Rel. RC to eVP, a(Za)⁴	"Reco‣	-		[B]	-0.00093	-0.00246	Tab.1+4	(b)				
	SUM of the above		227.65555	0.00208	A	227.65677				227.6526			
											1.6660	2	(9), (14)
											0.0001	3	(11)
4	Two-loop eVP (Kallen-Sabry): 2S-2P_1/2					1.66622	1.66622	p.3 Tab b		1.6661		2+3	
4b	same: 2P_3/2						1.66626	p.3 Tab b					
5	One-loop eVP in 2-Coul. lines a2(Za)5	VP it.	0.17194	0.00011		0.1718	0.1718	p.3		0.1720		9	(39)-(41)
4+5													
	SUM of the above		1.83805	0.00004	A	1.83802				1.8381			
7	eVP corr. to Kallen-Sabry		--										
6	NR three-loop eVP		--										
											0.0060	4	(17), (18)
											0.0025	12	(46)-(47)
											0.0001	13	(48)
6+7	Karsh., PRA81 (2010) Tab. 1: “third order VP”		0.00842	0.00007	K	0.00842	0.00842	p.4		0.00842	0.0086	4+12+13	p. 5 bot.
9	Wichmann-Kroll “1:3” LBL		--			-0.00111	-0.00111	p.4		-0.0011		5	(21)
10	Virtual Delbruck “2:2” LBL		--										
New	“3:1” LBL		--										
	Total LBL		-0.00096	0.00002	K	-0.00096	-0.00096(2)	p.5 Tab		-0.00096	0.0001	6	sign wrong!
20	muSE and muVP (2S-2P_1/2)		-0.77466	0.00004	A	-0.774616	-0.774616	p11 Tab 2		-0.7747		28	(72)-(73)
20b	same (Muon Lamb shift) to 2P_3/2		--			-0.755125	-0.755125	p11 Tab 2					
											-0.0018	29	(78) = [10] (39

Muonic helium ions.

Lamb shift in muonic helium

- CREMA collaboration: Charge Radius Experiment with Muonic Atoms
- PSI Experiment R10-01
- ERC Starting Grant for RP, 2011-2016
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- ⇒ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)

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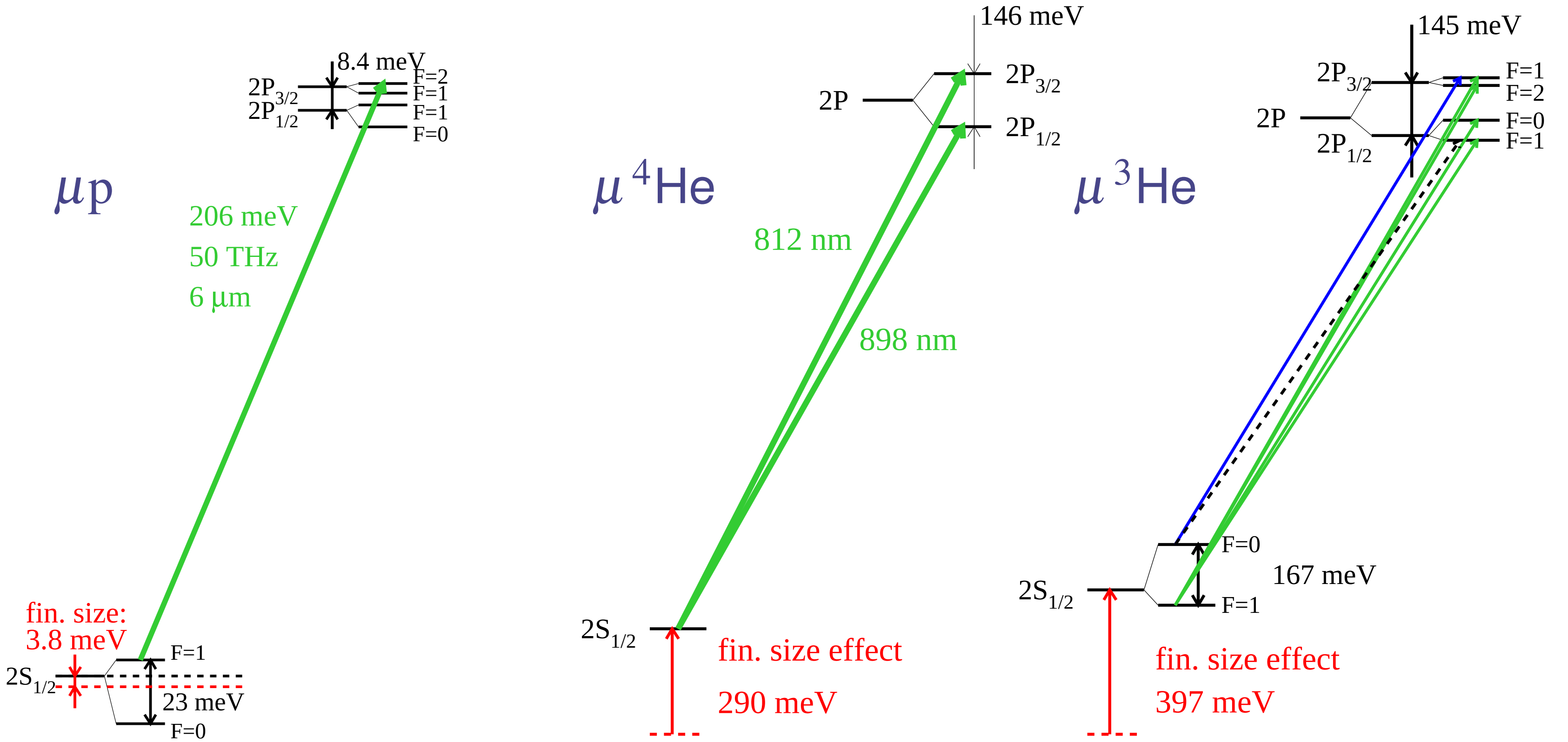

Lamb shift in muonic helium

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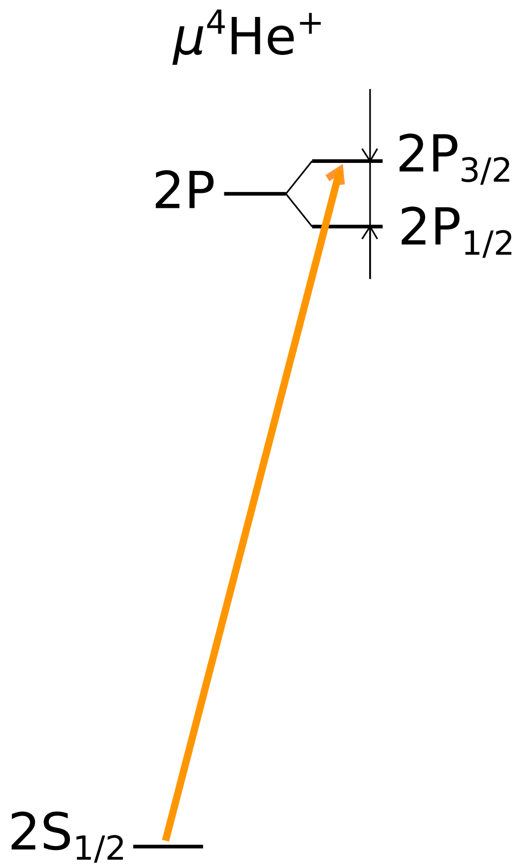
- Goal: Measure ΔE(2S-2P) in $\mu\,^4$He, $\mu\,^3$He
- $\Rightarrow$ alpha particle and helion charge radius to 3×10^{-4} (0.0005 fm)
- aims:
  - help to solve the proton size puzzle
  - absolute charge radii of helion, alpha
  - low-energy effective nuclear models: ^{1}H, ^{2}D, ^{3}He, ^{4}He
  - QED test with He$^+$(1S-2S) [Udem @ MPQ, Eikema @ Amsterdam]

A. Antognini, RP *et al.*, Can. J. Phys. **89**, 47 (2011)

Lamb shift in muonic helium

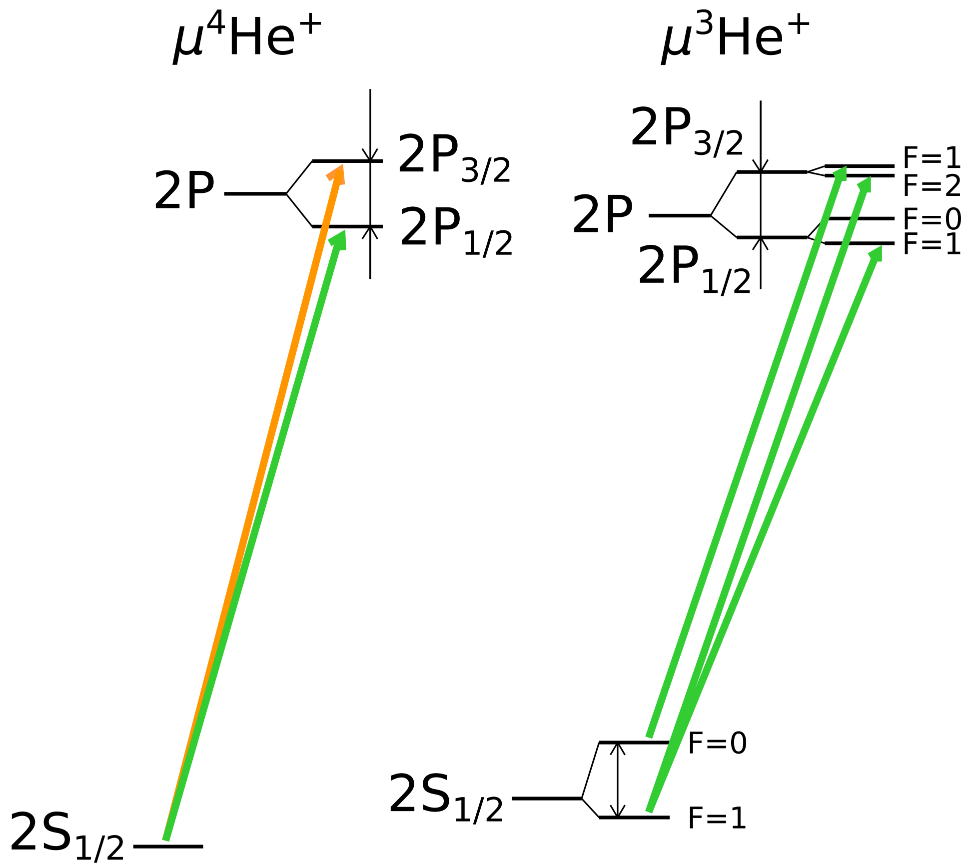
Measured transitions:



- Sept. 23 – Dec. 23, 2013

Lamb shift in muonic helium

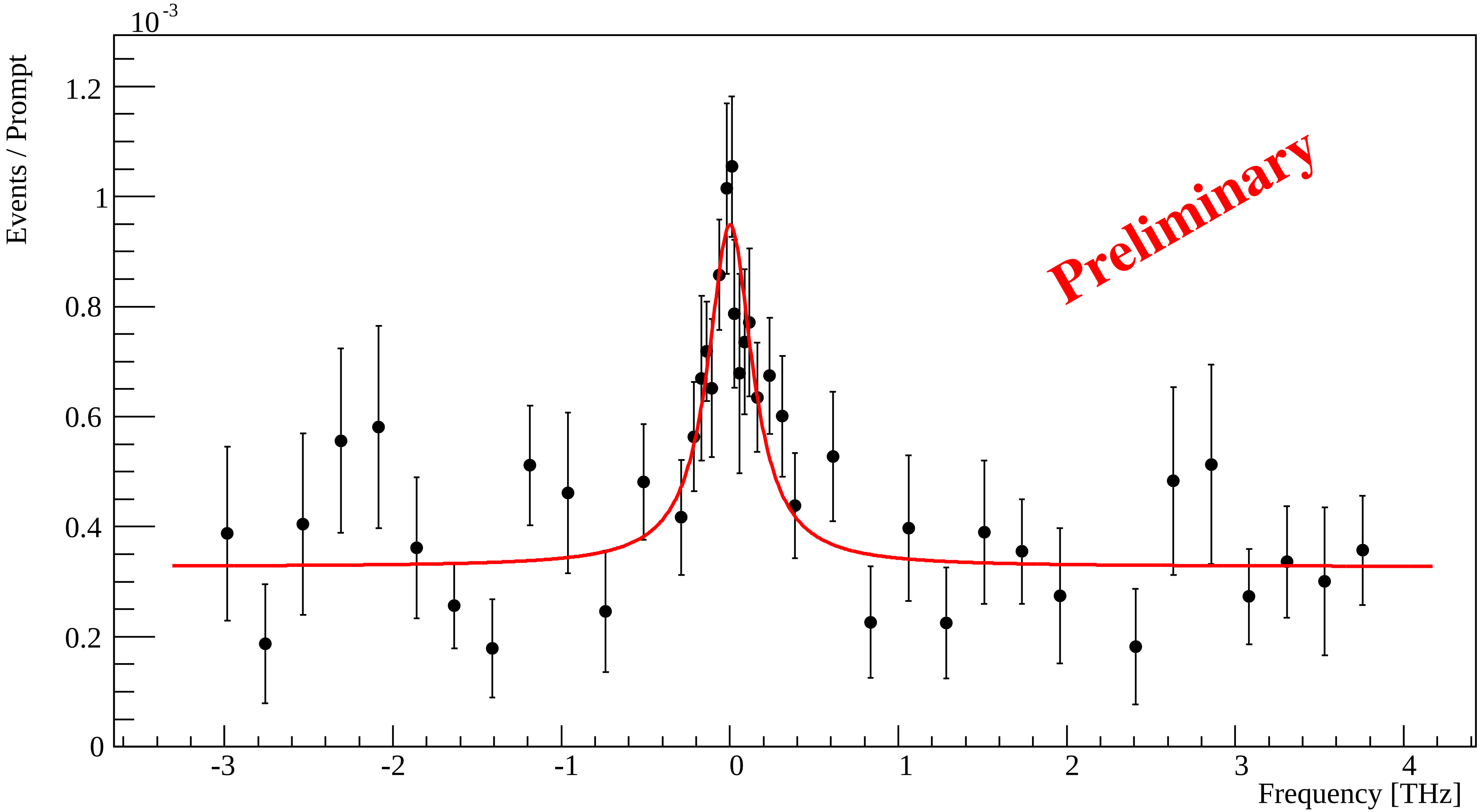
Measured transitions:



- Sept. 23 – Dec. 23, 2013
- May 15 – Aug. 6, 2014

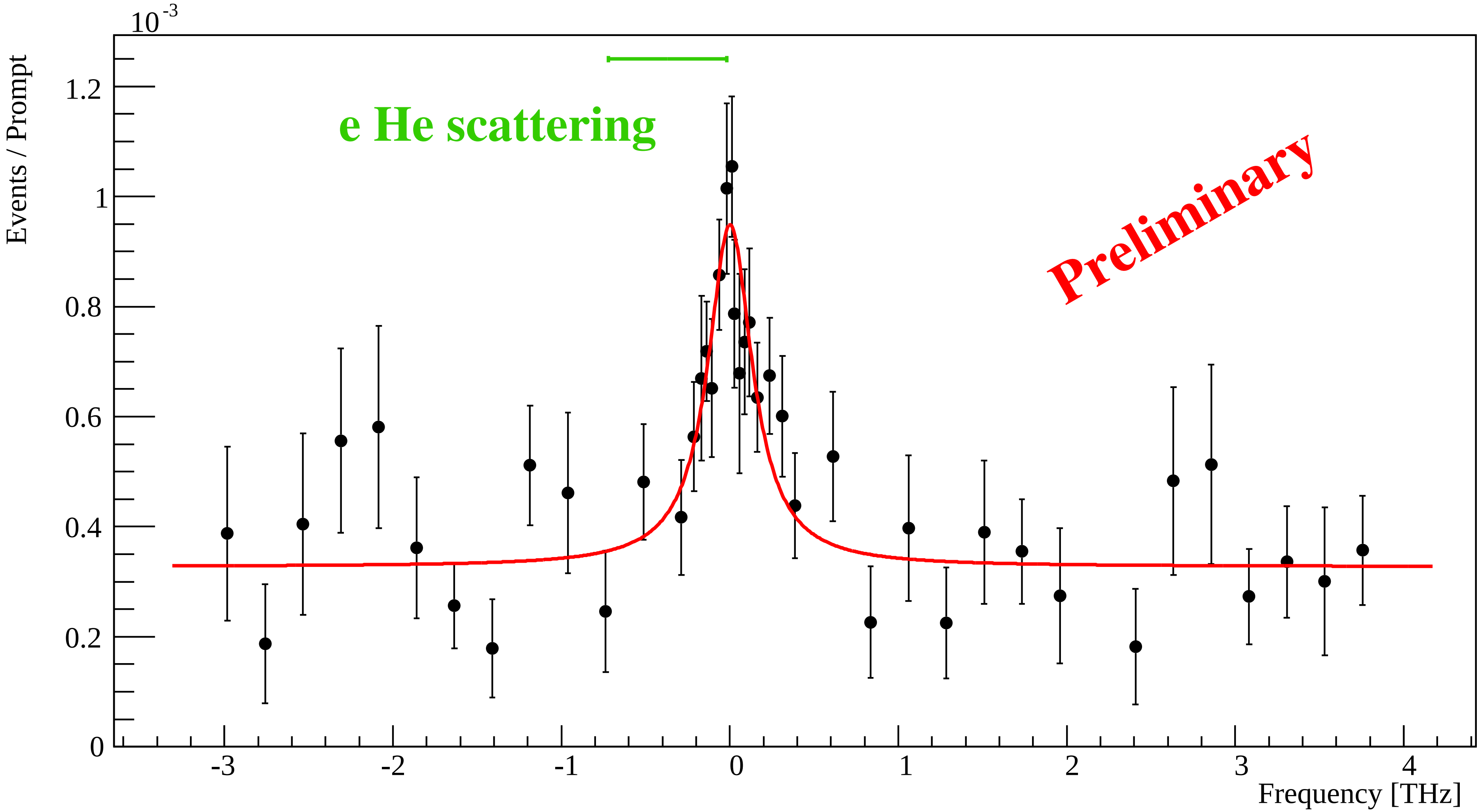
1st resonance in muonic He-4

$$\mu^4\mathrm{He}(2S_{1/2} \rightarrow 2P_{3/2})$$ at ~ 813 nm wavelength



1st resonance in muonic He-4

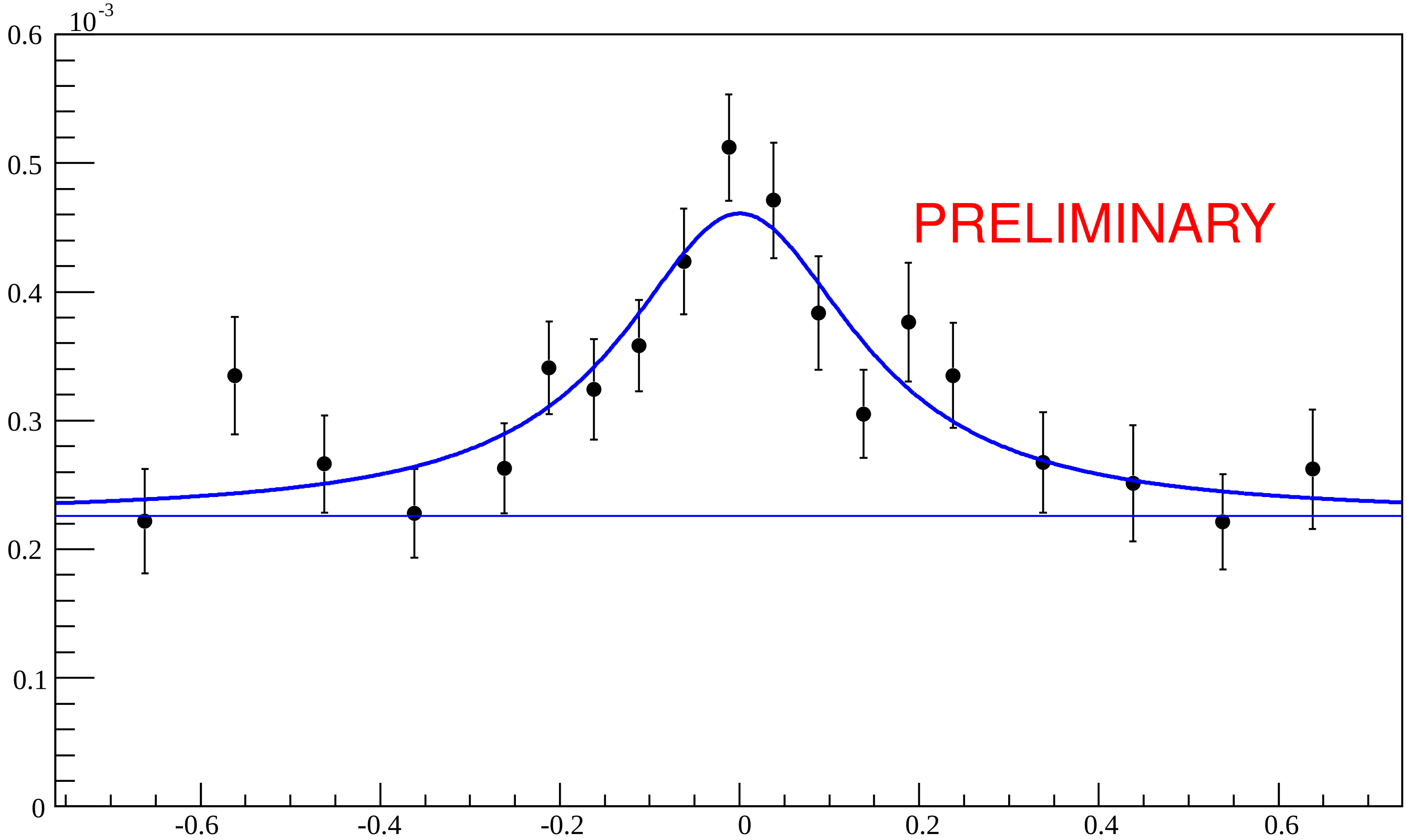
$\mu\,{}^4\mathrm{He}(2S_{1/2} \rightarrow 2P_{3/2})$ at $\sim$ 813 nm wavelength



Sick, PRD 77, 040302(R) (2008)   Borie, Ann. Phys. 327, 733 (2012)

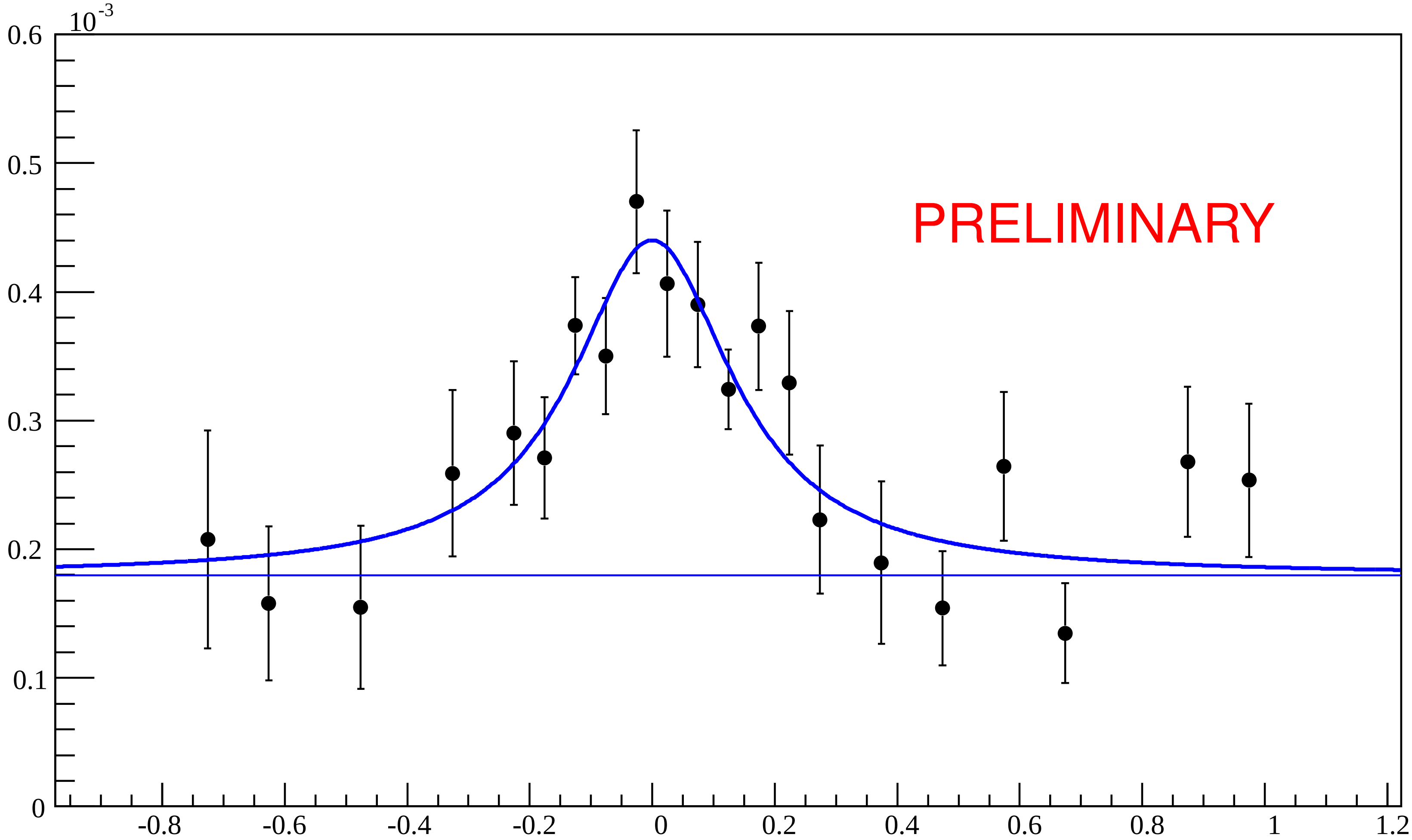
2nd resonance in muonic He-4

$\mu^4\mathrm{He}(2S_{1/2} \rightarrow 2P_{1/2})$ at $\sim$ 899 nm wavelength



1st resonance in muonic He-3

$\mu\,^3\mathrm{He}(2\mathrm{S}^{F=1}_{1/2} \rightarrow 2\mathrm{P}^{F=2}_{3/2})$ at $\sim$ 864 nm wavelength



Muonic summary

- Muonic hydrogen gives:
  - Proton charge radius: r_{p}= 0.84087 (39) fm
    7.9σ away from electronic average (H, e-p scatt.)
  - Deuteron charge radius: r_d = 2.12771 (22) fm from μH + H/D(1S-2S)
- Muonic deuterium:
  - Deuteron charge radius: r_d = 2.1272 (12) fm (PRELIMINARY!)
    consistent with muonic proton radius, but
    6.3σ away from CODATA value (H & D, e-p/d scatt.)
- "Proton" Radius Puzzle is in fact "Z=1 Radius Puzzle":
  combined $> \sim 8\sigma$ discrepancy
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- muonic helium-3 and -4 ions: No big discrepancy (PRELIMINARY)
- Could **ALL** be solved if the **Rydberg constant** [and hence the (electronic) proton radius] was wrong.

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp theory wrong?

μp experiment wrong?

H theory wrong?
H experiments wrong? $\rightarrow R_\infty$ wrong?
AND e-p scattering exp. wrong?

Standard Model wrong?

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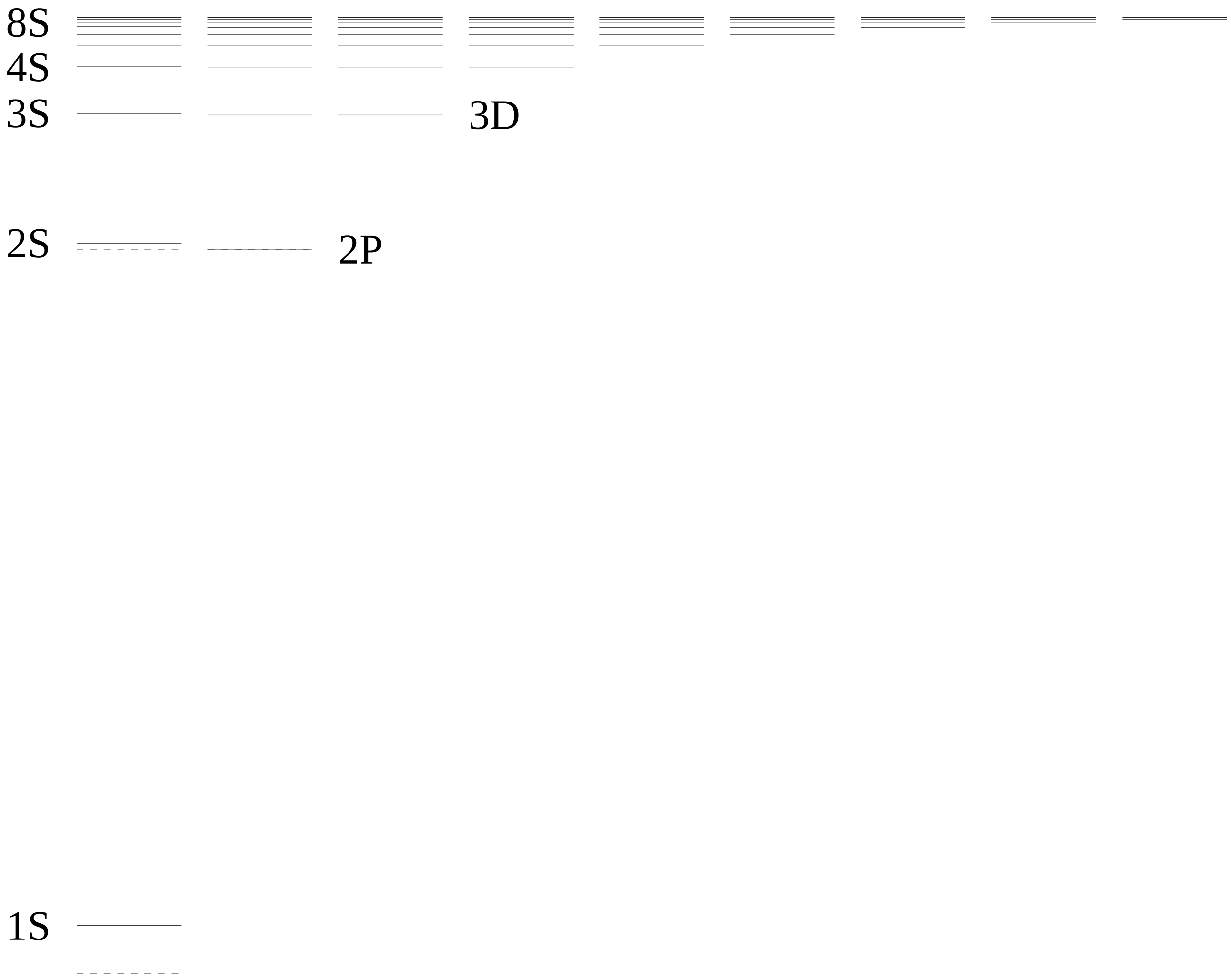
(Electronic) hydrogen.

Hydrogen spectroscopy

$$\text{Lamb shift}: \quad L_{1S}(r_p) = 8171.636(4) + 1.5645\,\langle r_p^2 \rangle \text{ MHz}$$

$$L_{nS} \simeq \frac{L_{1S}}{n^3}$$



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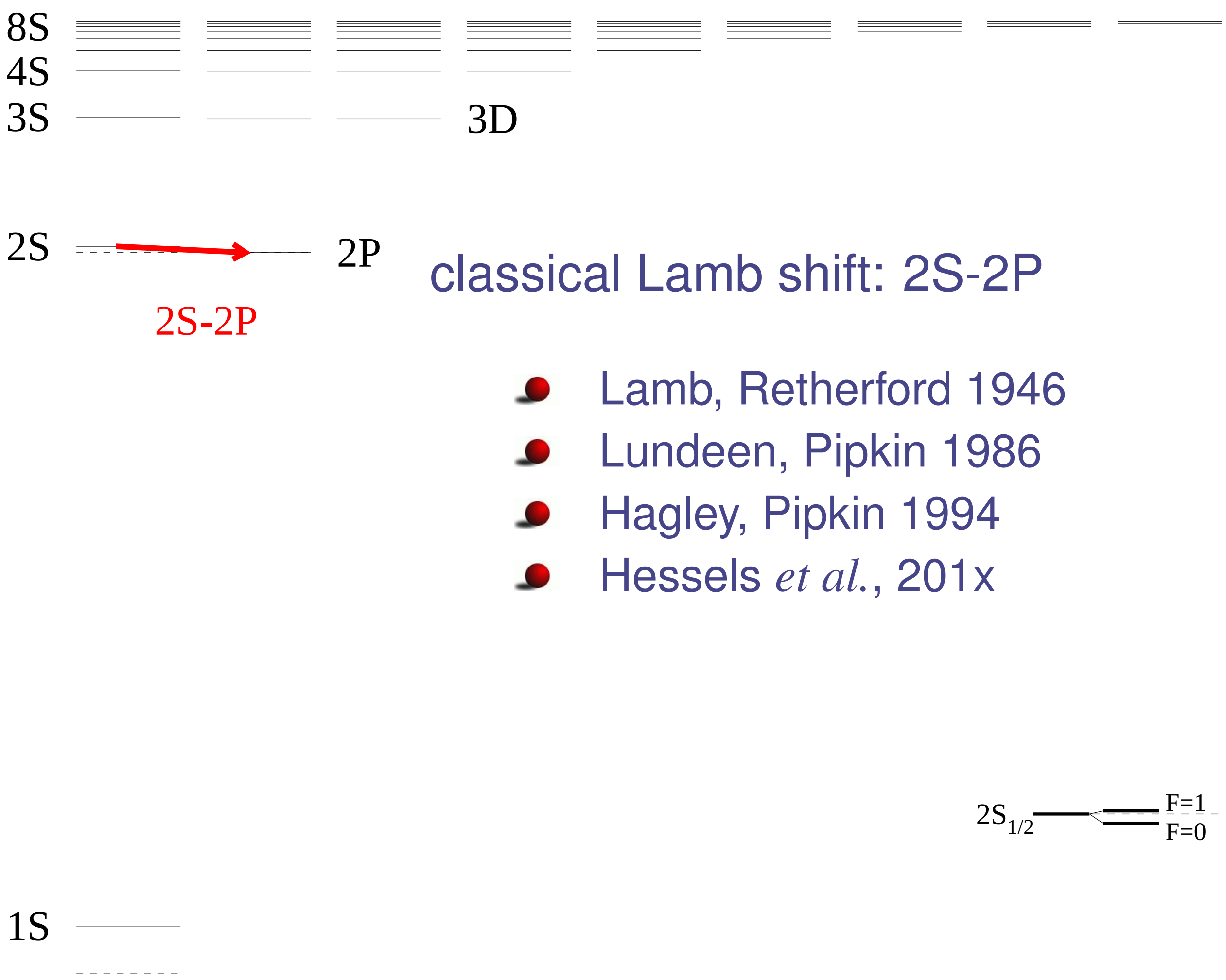
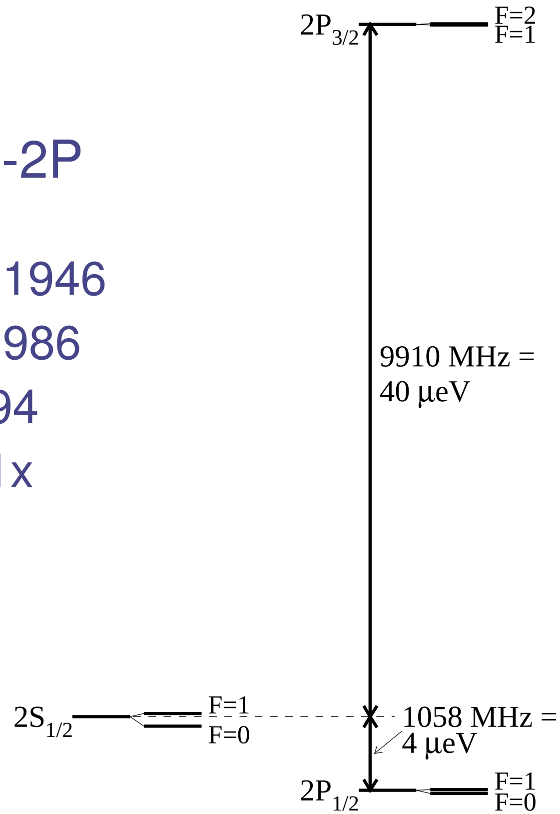


classical Lamb shift: 2S-2P

- Lamb, Retherford 1946
- Lundeen, Pipkin 1986
- Hagley, Pipkin 1994
- Hessels *et al.*, 201x



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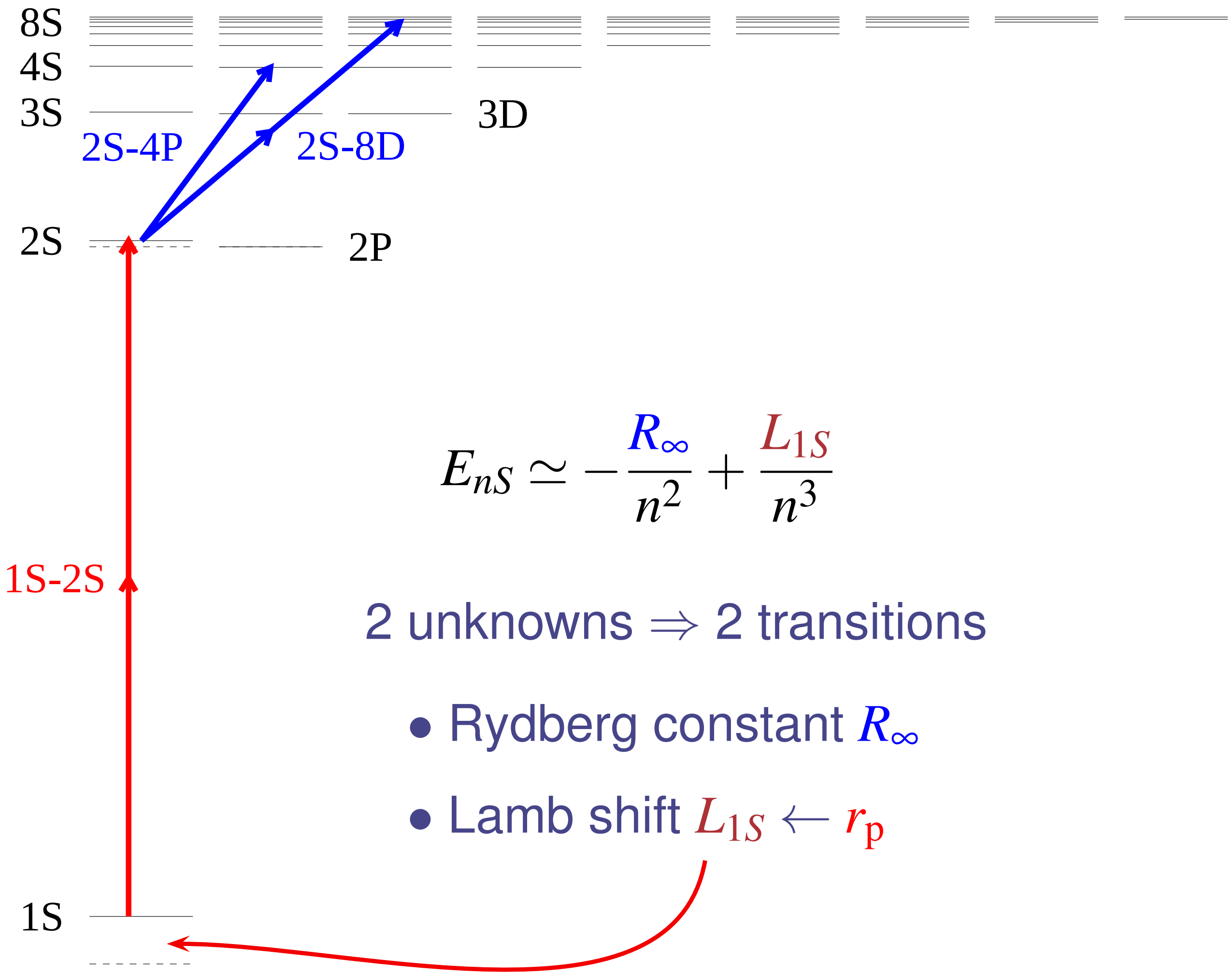
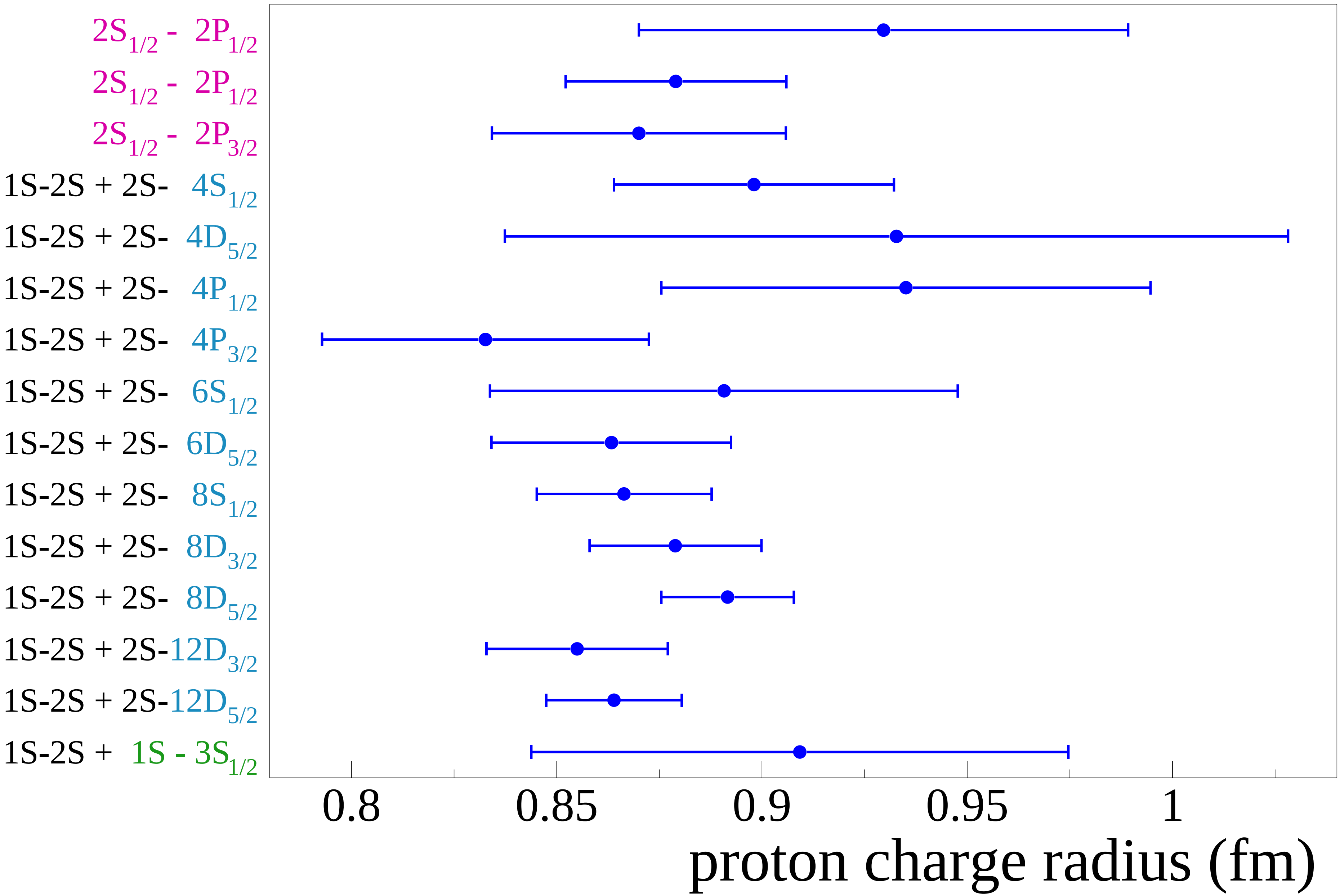


$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

2 unknowns $\Rightarrow$ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

Hydrogen spectroscopy



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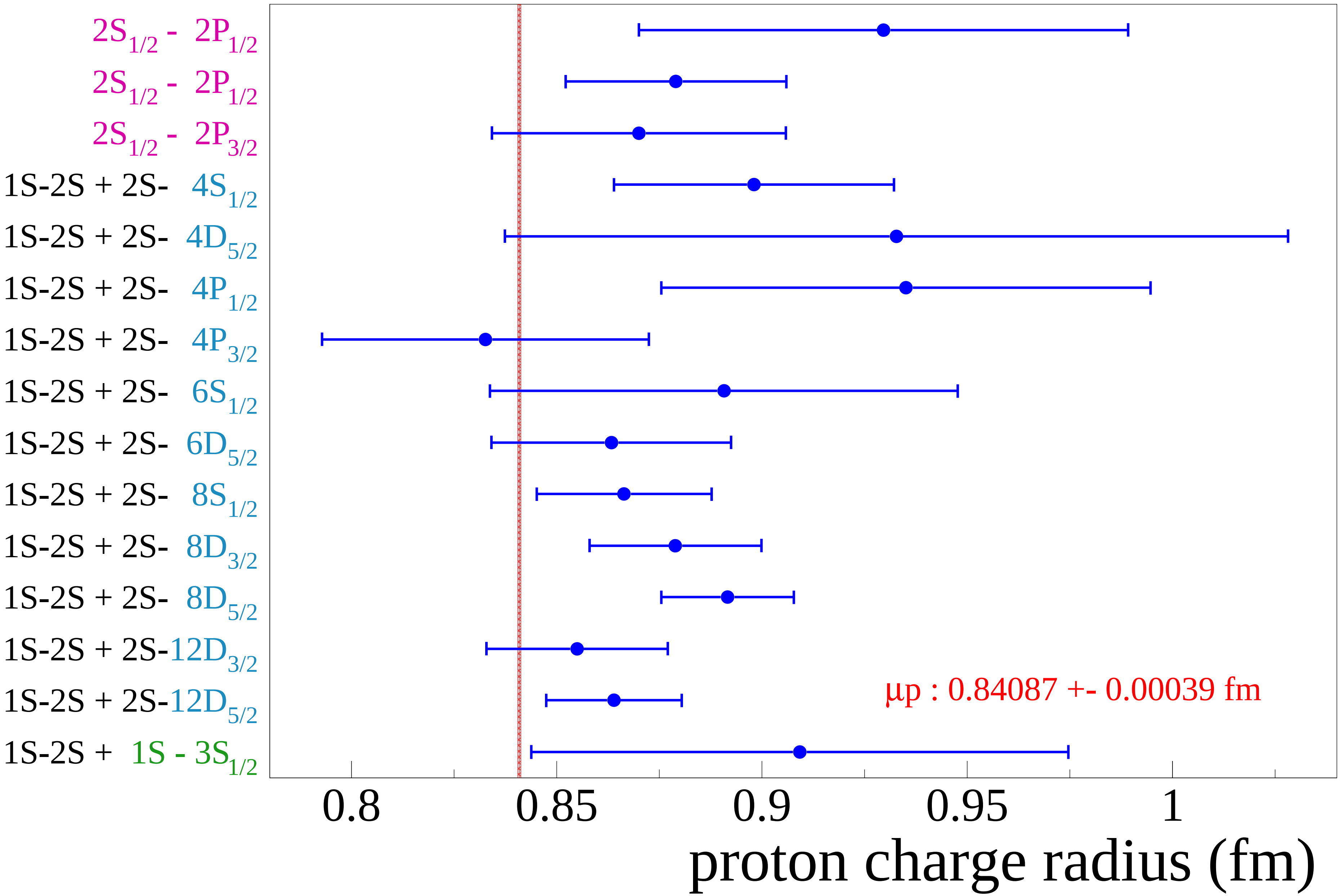
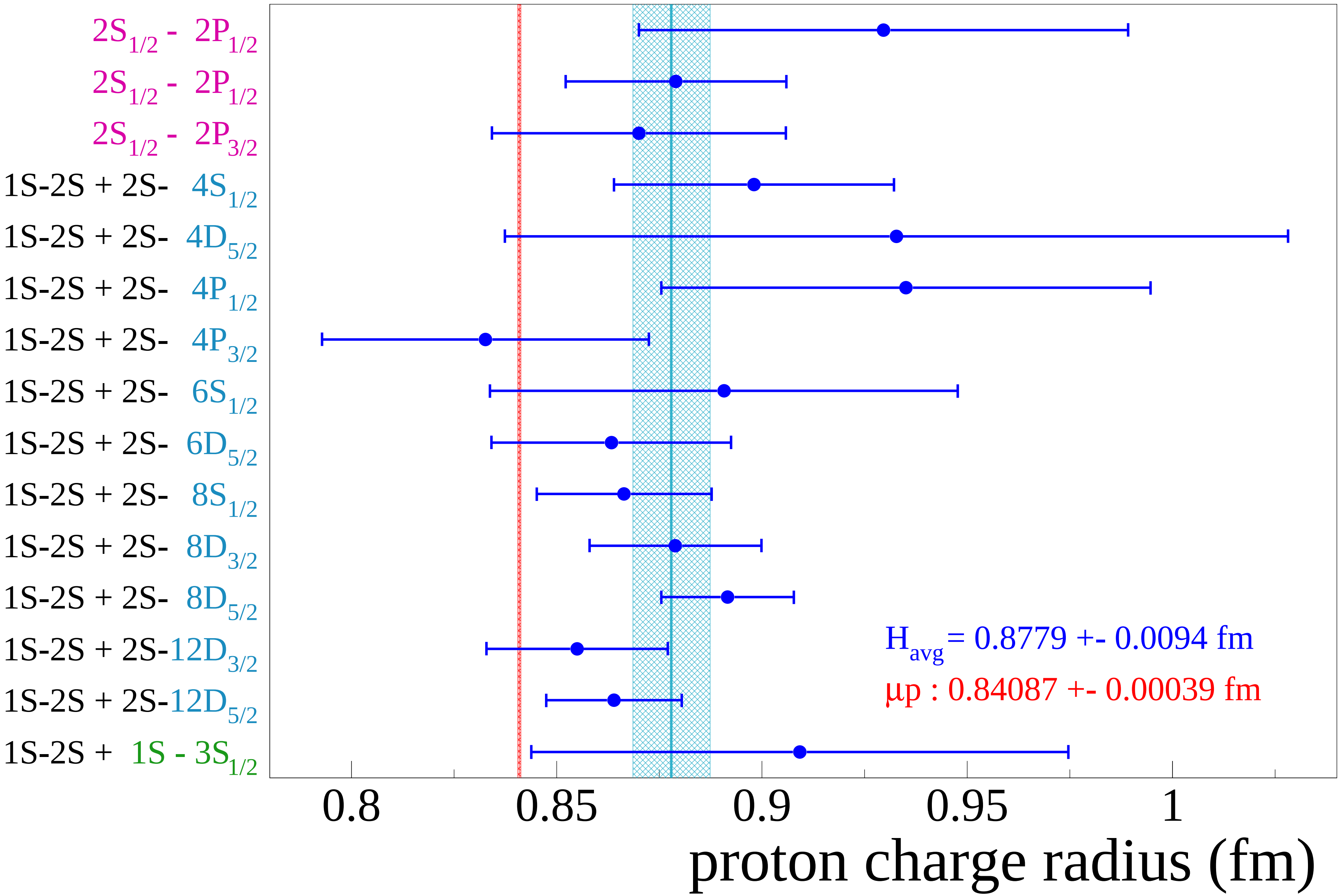
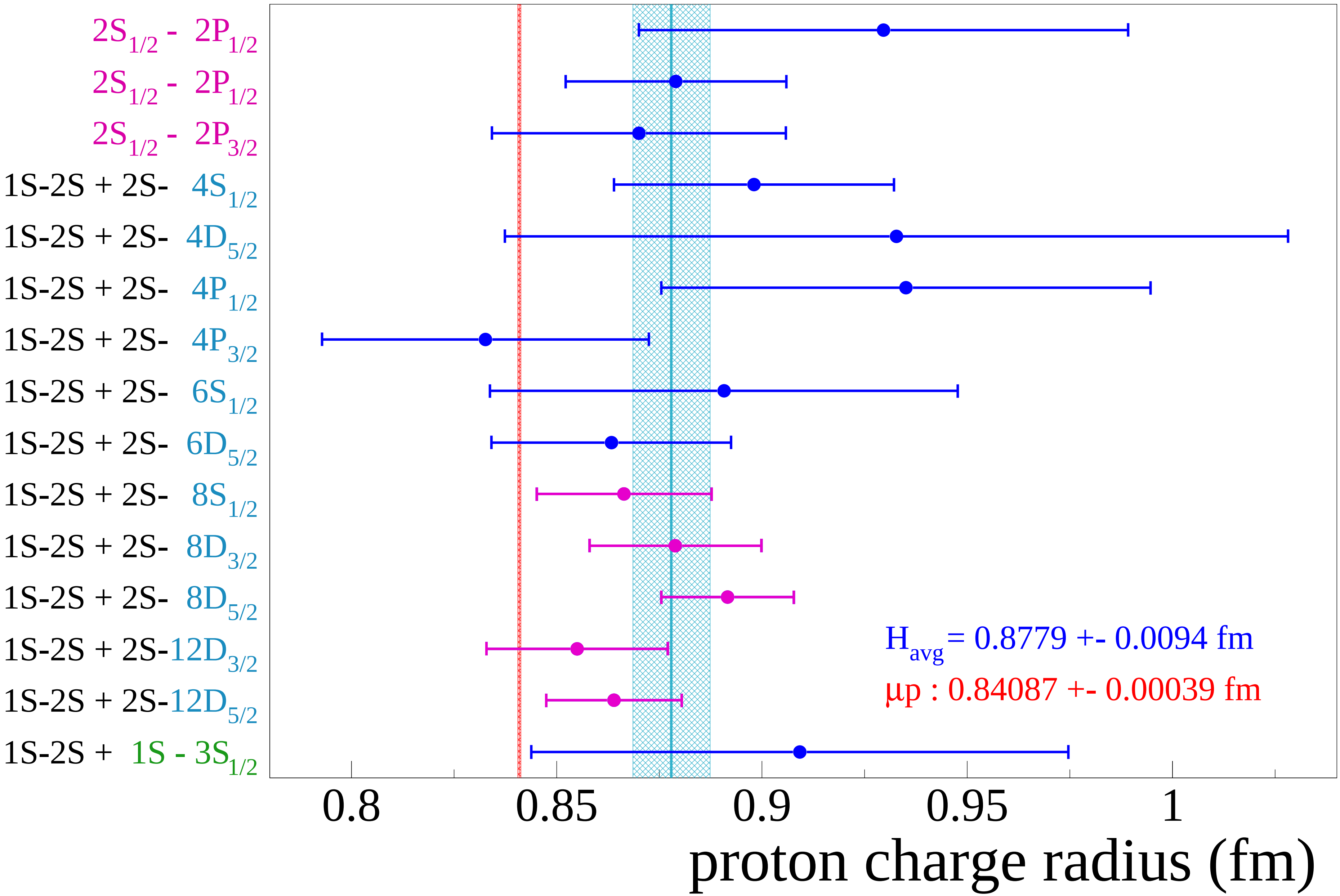


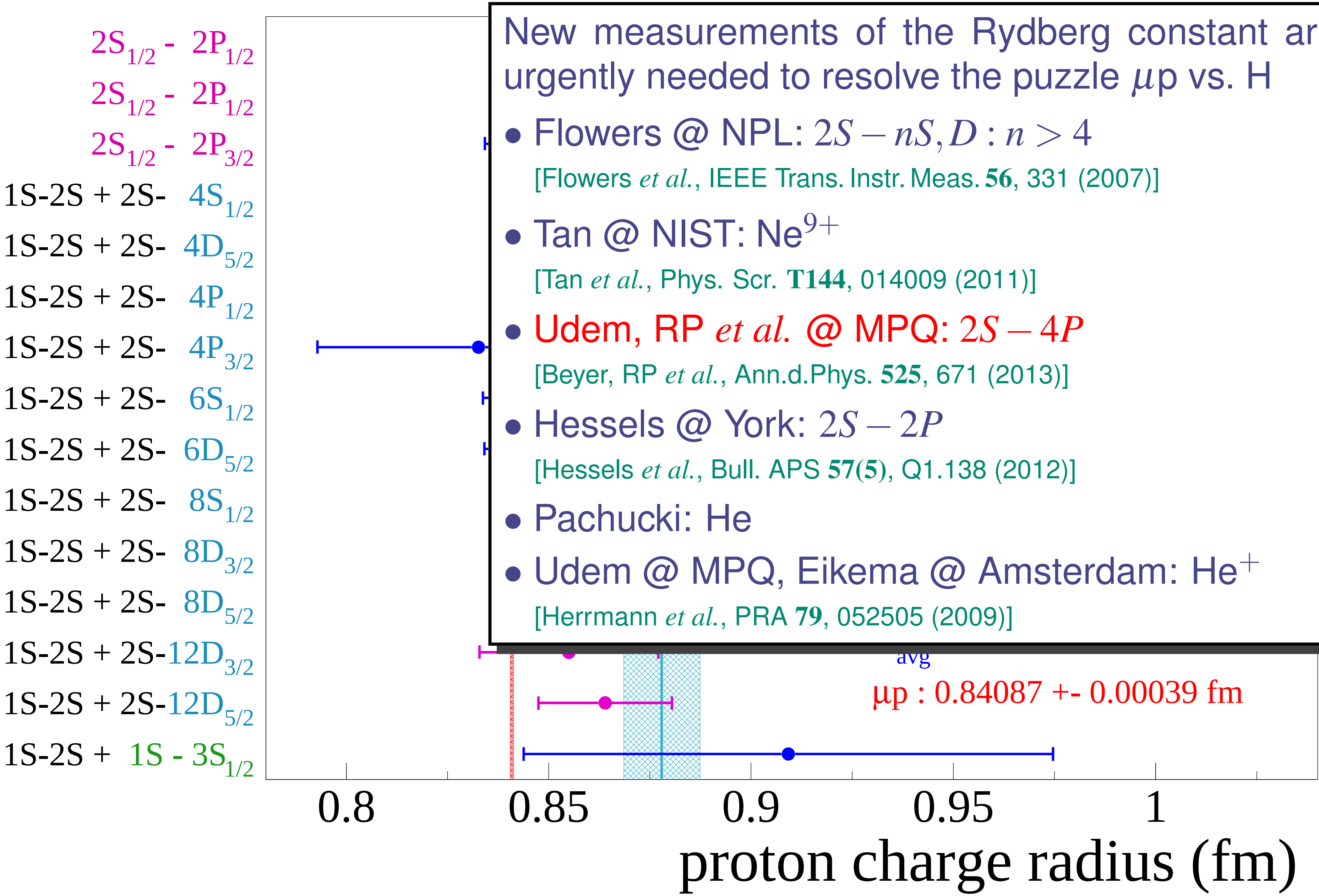
Hydrogen spectroscopy



Hydrogen spectroscopy



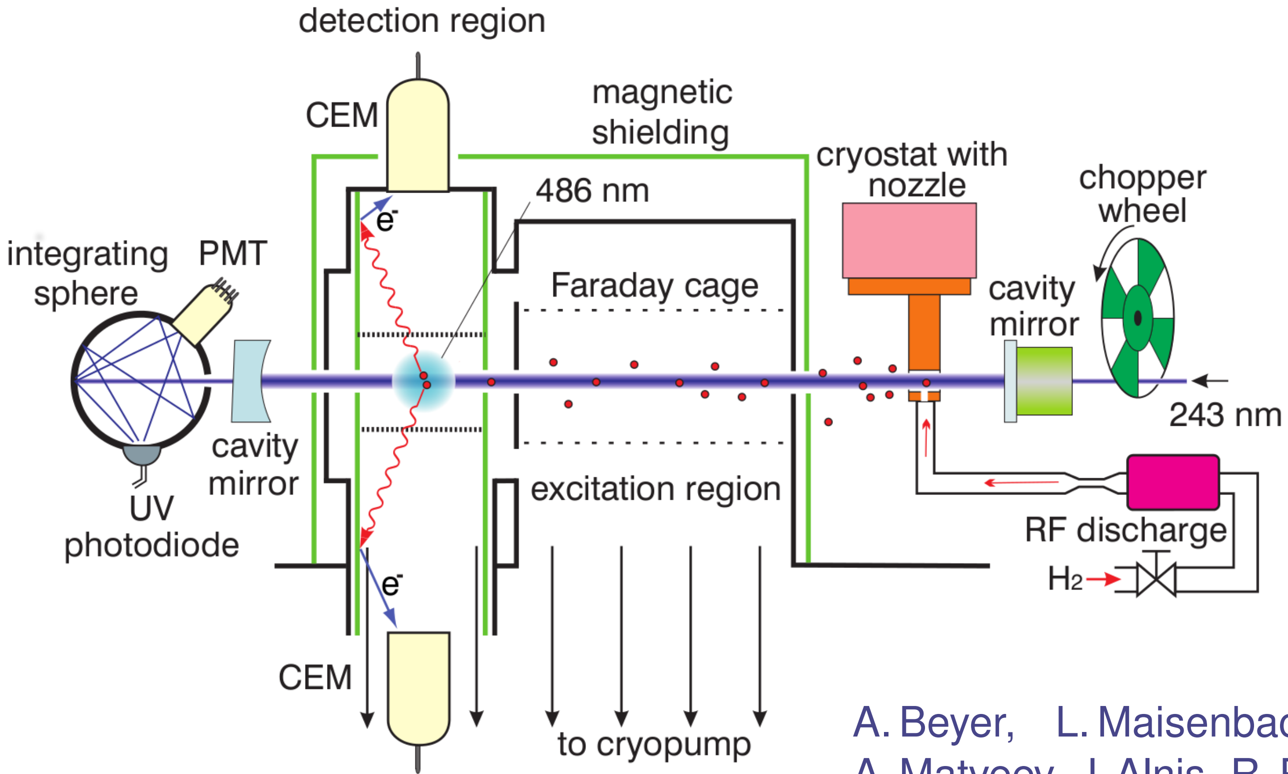
Hydrogen spectroscopy

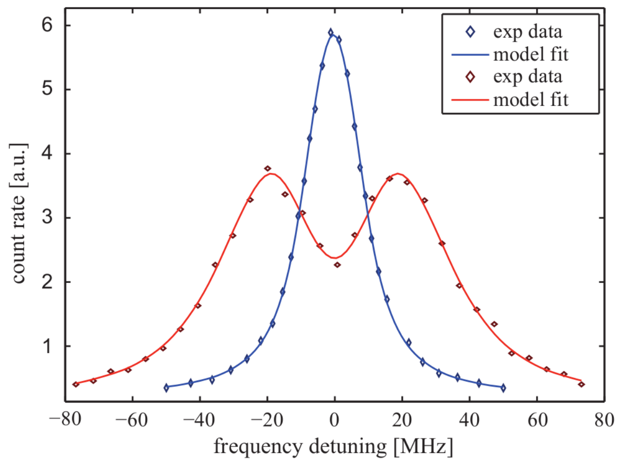


New measurements of the Rydberg constant are urgently needed to resolve the puzzle μp vs. H

- Flowers @ NPL: $2S - nS, D : n > 4$
  [Flowers *et al.*, IEEE Trans. Instr. Meas. **56**, 331 (2007)]
- Tan @ NIST: Ne^{9+}
  [Tan *et al.*, Phys. Scr. **T144**, 014009 (2011)]
- Udem, RP *et al.* @ MPQ: $2S - 4P$
  [Beyer, RP *et al.*, Ann.d.Phys. **525**, 671 (2013)]
- Hessels @ York: $2S - 2P$
  [Hessels *et al.*, Bull. APS **57(5)**, Q1.138 (2012)]
- Pachucki: He
- Udem @ MPQ, Eikema @ Amsterdam: He^{+}
  [Herrmann *et al.*, PRA **79**, 052505 (2009)]

Rydberg constant from hydrogen





2S – 4P resonance at
$88 \pm 0.5\,^\circ$ and $90 \pm 0.08\,^\circ$

A. Beyer, L. Maisenbacher, K. Khabarova, C.G. Parthey, A. Matveev, J. Alnis, R. Pohl, N. Kolachevsky, Th. Udem and T.W. Hänsch

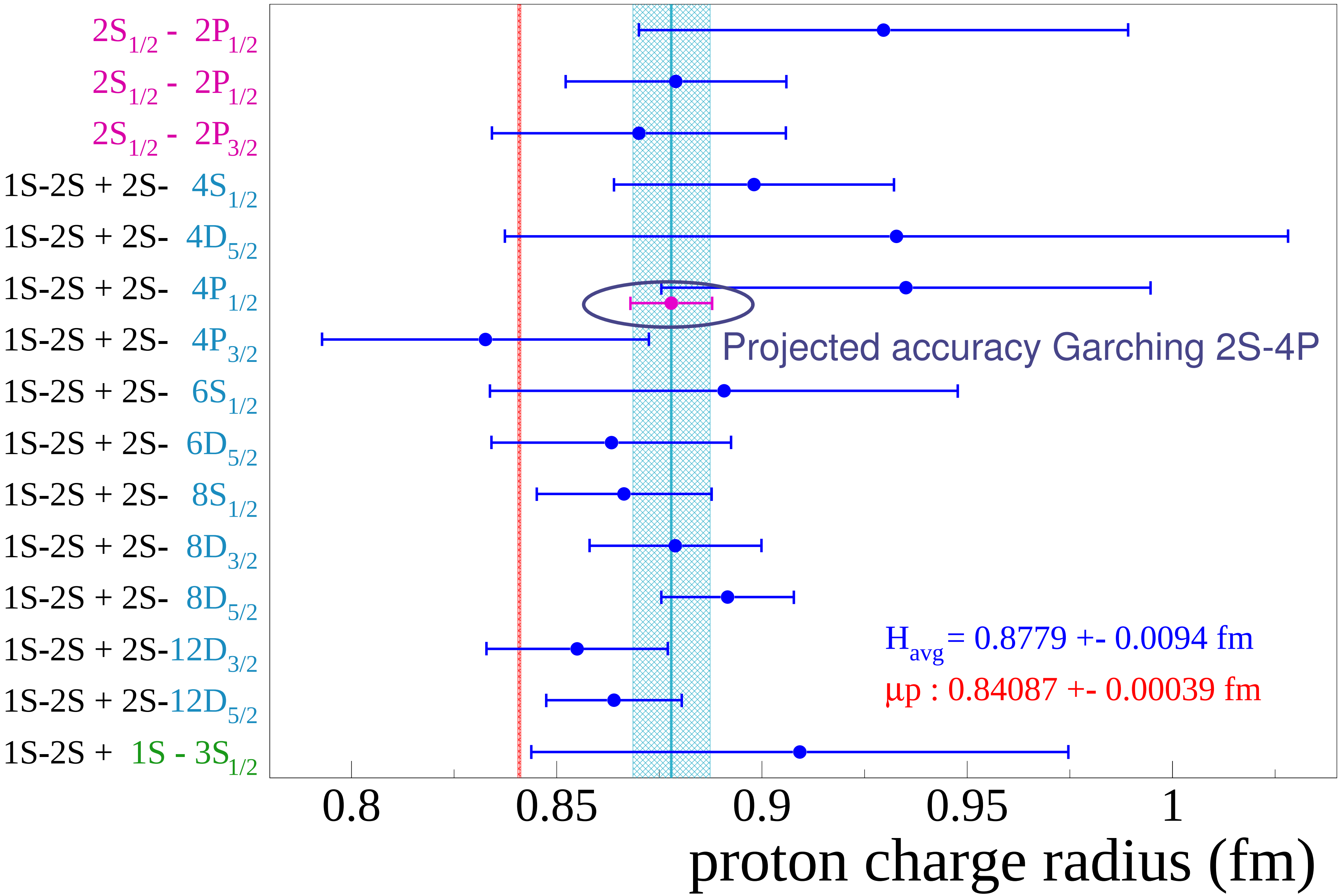
- Apparatus used for H/D(1S-2S)

C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)
C.G. Parthey, RP *et al.*, PRL **107**, 203001 (2011)

- 486 nm at 90° + Retroreflector $\Rightarrow$ Doppler-free 2S-4P excitation
- 1st oder Doppler vs. ac-Stark shift
- ~ 2.5 kHz accuracy (vs. 15 kHz Yale, 1995)
- **cryogenic** H beam, **optical** excitation to 2S

A. Beyer, RP *et al.*, Ann. d. Phys. **525**, 671 (2013)

Hydrogen spectroscopy



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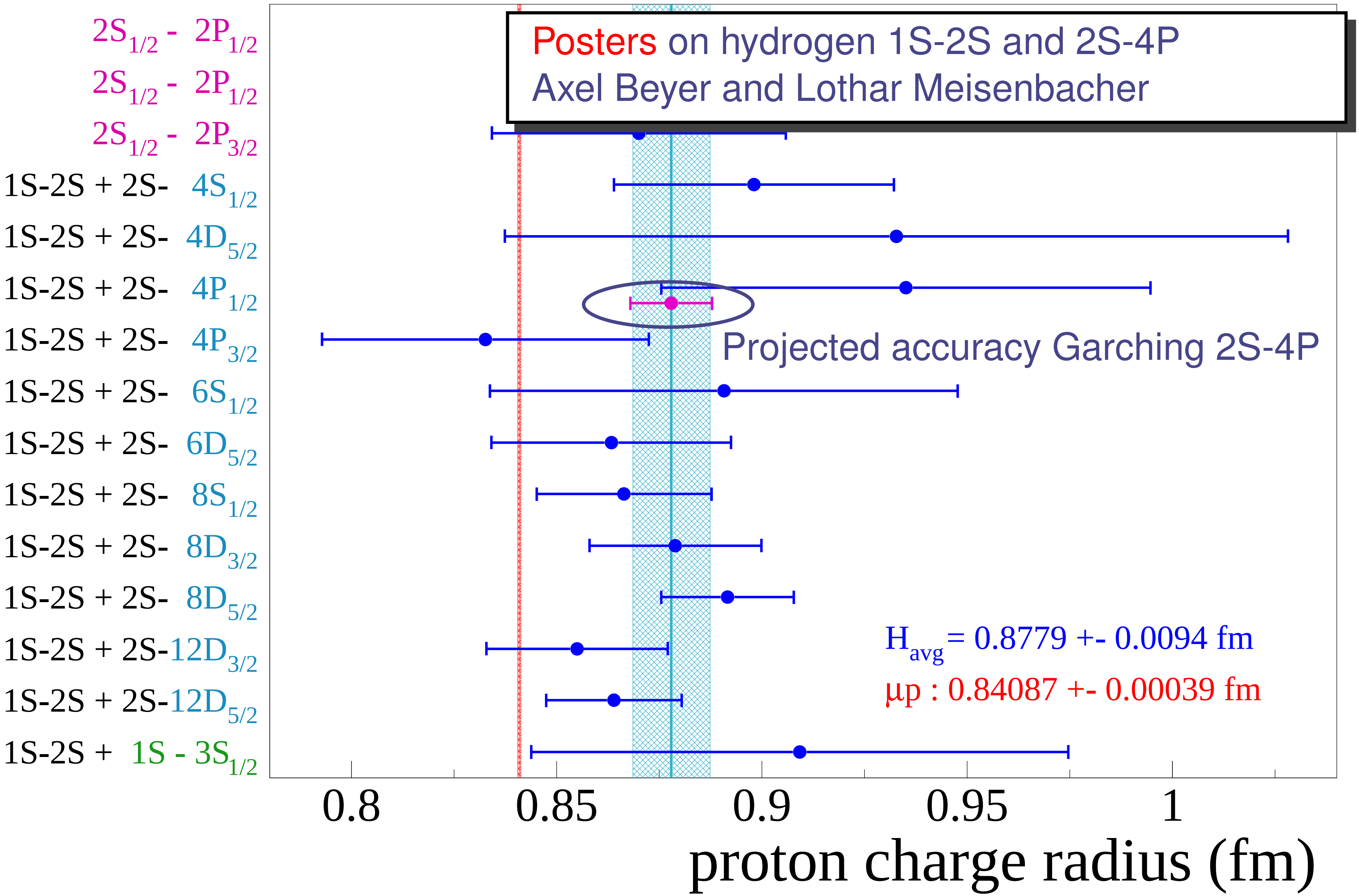
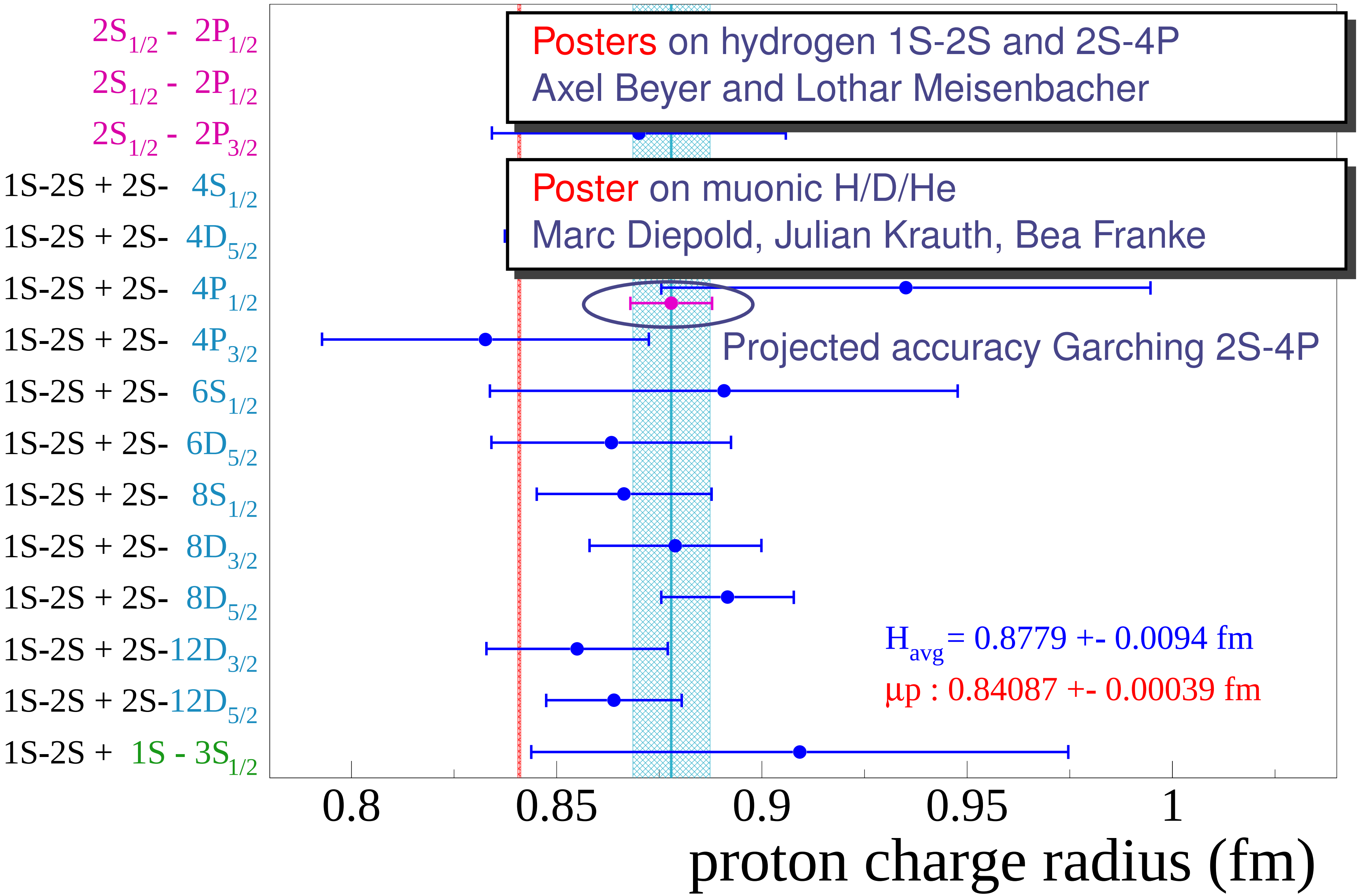


Hydrogen spectroscopy



Future muonic experiments

- Z=1:
  - Muonic hydrogen: HFS
  - Muonic deuterium: Lamb shift, HFS
  - Muonic tritium
- Z=2:
  - Muonic ^{4}He: Fine structure
  - Muonic ^{3}He: Lamb shift, fine and hyperfine structure
- Z=3, 4, 5:

Lamb shift:	absolute charge radius
	nuclear polarizability
2S-HFS:	Zemach / magnetic radius
	nuclear polarizability

PHYSICAL REVIEW A VOLUME 32, NUMBER 2 AUGUST 1985

Lamb shifts and fine-structure splittings for the muonic ions μ^--Li, μ^--Be, and μ^--B: A proposed experiment

G. W. F. Drake and Louis L. Byer*
Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4
(Received 28 February 1985)

Detailed calculations are presented for the energy splittings of the states $2s_{1/2}$-$2p_{1/2}$ and $2s_{1/2}$-$2p_{3/2}$ for the muonic ions μ^--Li, μ^--Be, and μ^--B obtained by numerical integration of the Dirac equation. It is shown that there is severe cancellation between the vacuum polarization and finite nuclear size contributions to the energy differences, leading to transition frequencies which lie in the visible region of the spectrum. As a consequence of the cancellation, a measurement of the transition frequency would provide a sensitive probe of nuclear size and structure. The system μ^--^{7}Li appears to offer particularly good possibilities for performing such an experiment.

Future muonic experiments

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- Z=3, 4, 5:

Lamb shift:	absolute charge radius
	nuclear polarizability
2S-HFS:	Zemach / magnetic radius
	nuclear polarizability

TABLE VII. Calculated absorption wavelengths (in Å) for transitions in muonic ions. The first uncertainty listed for the wavelengths is that due to nuclear polarization and the second is that due to the rms nuclear radius R.

Ion	R (fm)	$\lambda(2s_{1/2}$-$2p_{1/2})$	$\lambda(2s_{1/2}$-$2p_{3/2})$
^{4}He	1.674±0.012	8978.0± 4±27	8118.0± 3±22
^{6}Li	2.56 ±0.05	10 097.0± 33±1072	6275.0± 13±414
^{7}Li	2.39 ±0.03	7473.0± 18±334	5147.0± 9±159
^{9}Be	2.520±0.012	−9520.0±116±703	11 512.0±173±1048
^{10}B	2.45 ±0.12	−1393.0± 3±354	−4033.0± 27±2947
^{11}B	2.42 ±0.12	−1481.0± 4±397	−4887.0± 46±4286

Drake, Byer, PRA 32, 713 (1985)

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  - Muonic ^{3}He: Lamb shift, fine and hyperfine structure
- Z=3, 4, 5:
  - (Electronic) isotope shifts have been measured very accurately.
    $\Rightarrow$ (squared) charge radius **differences** are very well known.
  - Muonic Lamb shifts provide **absolute** charge radii.

  - Test of few-electron (QED) calculations.
  - *Ab initio* nuclear structure calculations.

Lamb shift:	absolute charge radius
	nuclear polarizability
2S-HFS:	Zemach / magnetic radius
	nuclear polarizability

- Also: 1S-2S in (electronic) tritium. “Missing link” at A=3

The cost for LHC

According to **Forbes** (Jul. 2012), the Higgs discovery cost

13.25 billion USD.

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Cost increase for Higgs discovery: 1.06 billion USD.

The cost for LHC

My aplogies.

:-)

Proton Size Investigators thank you for your attention

Backup slides.

Lamb shift in μp 1: r_p independent

Table 1
All known radius-*independent* contributions to the Lamb shift in μp from different authors, and the one we selected. Values are in meV. The entry # in the first column refers to Table 1 in Ref. [13]. The "finite-size to relativistic recoil correction" (entry #18 in [13]), which depends on the proton structure, has been shifted to Table 2, together with the small terms #26 and #27, and the proton polarizability term #25. SE: self-energy, VP: vacuum polarization, LBL: light-by-light scattering, Rel: relativistic, NR: non-relativistic, RC: recoil correction.

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit–Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[80] Eq. (54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	−0.0041	−0.0041		−0.00208[c]	[77,78]
4	Two-loop eVP (Källén–Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[80] Eq. (57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[80] Eq. (60)
7	eVP corr. to Källén–Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[80] Eq. (62), [87]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[87,88]
9	Wichmann–Kroll, "1:3" LBL		−0.00103	−0.00102	−0.00102	−0.00102	[80] Eq. (64), [89]
10	Virtual Delbrück, "2:2" LBL		0.00135	0.00115		0.00115	[74,89]
New	"3:1" LBL			−0.00102		−0.00102	[89]
20	μSE and μVP	−0.6677	−0.66770	−0.66788	−0.66761	−0.66761	[80] Eqs. (72) + (76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	−0.005(1)	−0.00500	−0.004924[d]		−0.00254	[85] Eq. (29a)[e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	−0.001	−0.00150			[f]	[74,90–92]
21	Higher order corr. to μSE and μVP		−0.00169	−0.00171[g]		−0.00171	[86] Eq. (177)
13	Mixed eVP + μVP		0.00007	0.00007		0.00007	[74]
New	eVP and μVP in two Coulomb lines				0.00005	0.00005	[80] Eq. (78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[93–95]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[94,95]
16	Rad corr. to hadronic VP		−0.000015			−0.000015	[94,95]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[80] Eq. (88)
22	Rel. RC $(Z\alpha)^5$	−0.045	−0.04497	−0.04497	−0.04497	−0.04497	[80] Eq. (88), [74]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[80] Eq. (86)+Tab.II

(*continued on next page*)

A. Antognini, RP *et al*., Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 1: r_p independent

Table 1 (*continued*)

#	Contribution	Pachucki [10,11]	Nature [13]	Borie-v6 [79]	Indelicato [80]	Our choice	Ref.
New	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[85] Eq. (64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	−0.0099	−0.00960	−0.0100		−0.01080(100)	[43][h] [74]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

[a] This value has been recalculated to be 0.018759 meV [77].
[b] This correction is not necessary here because in #2 the Breit–Pauli contribution has been calculated using a Coulomb potential modified by eVP.
[c] Difference between Eqs. (6) and (4) in [78]: $E_{\rm VP}^{\rm (rel)}(2P_{1/2}\text{–}2S_{1/2}) - E_{\rm VP}^{(0)}(2P_{1/2}\text{–}2S_{1/2}) = 0.018759 - 0.020843 = -0.002084$ meV (see also Table IV). Using these corrected values, the various approaches are consistent. Pachucki becomes $205.0074 + 0.018759 = 205.0262$ meV and Borie $205.0282 - 0.0020843 = 205.0261$ meV.
[d] In Appendix C, incomplete.
[e] Eq. (27) in [85] includes contributions beyond the logarithmic term with modification of the Bethe logarithm to the Uehling potential. The factor 10/9 should be replaced by 5/6.
[f] This term is part of #22, see Fig. 22 in [86].
[g] Borie includes wave-function corrections calculated in [87]. The actual difference between Ref. [13] and Borie-v6 [79] is given by the inclusion of the Källén–Sabry correction with muon loop.
[h] This was calculated in the framework of NRQED. It is related to the definition of the proton radius.

43 R.J. Hill, G. Paz, Phys. Rev. Lett. 107, 160402 (2011)
74 M.I. Eides, H. Grotch, V.A. Shelyuto, Phys. Rep. 342, 63 (2001)
77 U.D. Jentschura, Phys. Rev. A 84, 012505 (2011)
78 S.G. Karshenboim, V.G. Ivanov, E.Y. Korzinin, Phys. Rev. A 85, 032509 (2012)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
85 U.D. Jentschura, B.J. Wundt, Eur. Phys. J. D 65, 357 (2011)
86 E. Borie, G.A. Rinker, Rev. Mod. Phys. 54, 67 (1982)
87 V.G. Ivanov, E.Y. Korzinin, S.G. Karshenboim, Phys. Rev. D 80, 027702 (2009)
88 T. Kinoshita, M. Nio, Phys. Rev. Lett. 82, 3240 (1999)
89 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, V.A. Shelyuto, JETP Lett. 92, 8 (2010)
90 R. Barbieri, M. Caffo, E. Remiddi, Lett. Nuovo Cimento 7, 60 (1963)
91 H. Suura, E.H. Wichmann, Phys. Rev. 105, 1930 (1957)
92 A. Petermann, Phys. Rev. 105, 1931 (1957)
93 J. Friar, J. Martorell, D. Sprung, Phys. Rev. A 59, 4061 (1999)
94 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 63, 845 (2000)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 1

Lamb shift in μp 2: r_p-dependent

Table 2

Proton-structure-dependent contributions to the Lamb shift in μp from different authors and the one we selected. Values are in meV, $\langle r^2 \rangle$ in fm^2. The entry # in the first column refers to Table 1 in Ref. [13] supplementary information [9]. Entry # 18 is under debate. TPE: two-photon exchange, VP: vacuum polarization, SE: self-energy, Rel: relativistic.

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
	Non-rel. finite-size	$-5.1973 \langle r^2 \rangle$	$-5.1975 \langle r^2 \rangle$	$-5.1975 \langle r^2 \rangle$			
	Rel. corr. to non-rel. finite size	$-0.0018 \langle r^2 \rangle$		-0.0009 meV[a]			
	Rel. finite-size						
	Exponential				$-5.1994 \langle r^2 \rangle$	$-5.2001 \langle r^2 \rangle$	$-5.1994 \langle r^2 \rangle$
	Yukawa					$-5.2000 \langle r^2 \rangle$	
	Gaussian					$-5.2001 \langle r^2 \rangle$	
	Finite size corr. to one-loop eVP	$-0.0110 \langle r^2 \rangle$	$-0.0110 \langle r^2 \rangle$	$-0.010 \langle r^2 \rangle$	$-0.0282 \langle r^2 \rangle$		$-0.0282 \langle r^2 \rangle$
	Finite size to one-loop eVP-it.	$-0.0165 \langle r^2 \rangle$	$-0.0170 \langle r^2 \rangle$	$-0.017 \langle r^2 \rangle$	(incl. in -0.0282)		
	Finite-size corr. to Källén–Sabry	[b]			$-0.0002 \langle r^2 \rangle$		$-0.0002 \langle r^2 \rangle$
New	Finite size corr. to μ self-energy	(0.00699)[c]			$0.0008 \langle r^2 \rangle$		$0.0009(3) \langle r^2 \rangle$[d]
	ΔE_{TPE} [46]						0.0332(20) meV
	Elastic (third Zemach)[e]						
	Measured $R^3_{(2)}$	$0.0365(18) \langle r^2 \rangle^{3/2}$					(incl. above)
	Exponential			$0.0363 \langle r^2 \rangle^{3/2}$	$0.0353 \langle r^2 \rangle^{3/2}$ [f]	$0.0353 \langle r^2 \rangle^{3/2}$	
	Yukawa					$0.0378 \langle r^2 \rangle^{3/2}$	
	Gaussian					$0.0323 \langle r^2 \rangle^{3/2}$	
25	Inelastic (polarizability)	0.0129(5) meV [101]		0.012(2) meV			(incl. above)
New	Rad. corr. to TPE	$-0.00062 \langle r^2 \rangle$					$-0.00062 \langle r^2 \rangle$
26	eVP corr. to polarizability						0.00019 meV [95]

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

Lamb shift in μp 2: r_p-dependent

Table 2 (*continued*)

#	Contribution	Borie-v6 [79]	Karshenboim [78]	Pachucki [10,11]	Indelicato [80]	Carroll [84]	Our choice
27	SE corr. to polarizability						-0.00001 meV [95]
18	Finite-size to rel. recoil corr.	(0.013 meV)[g]		[h]			(incl. in ΔE_{TPE})
	Higher order finite-size corr.	-0.000123 meV			0.00001(10) meV		0.00001(10) meV
	$2P_{1/2}$ finite-size corr.	$-0.0000519\langle r^2\rangle$[i]			(incl. above)	(incl. above)	(incl. above)

[a] Corresponds to Eq. (6) in [11] which accounts only for the main terms in F_{REL} and F_{NREL}.
[b] This contribution has been accounted already in both the -0.0110 meV/fm^2 and -0.0165 meV/fm^2 coefficients.
[c] Given only in Appendix C. Bethe logarithm is not included.
[d] This uncertainty accounts for the difference between all-order in $Z\alpha$ and perturbative approaches [82].
[e] Corresponds to Eq. (20).
[f] This value is slightly different from Eq. (22) because here an all-order in finite-size and an all-order in eVP approaches were used.
[g] See Appendix F of [96]. This term is under debate.
[h] Included in ΔE_{TPE}. This correction of $0.018-0.021=-0.003$ meV is given by Eq. (64) in [10] and Eq. (25) in [11]. This correction is also discussed in [76] where the 6/7 factor results from 0.018/0.021.
[i] Eq. (6a) in [79].

46 M.C. Birse, J.A. McGovern, Eur. Phys. J. A 48, 120 (2012); arXiv:1206.3030
76 U.D. Jentschura, Ann. Phys. 326, 500 (2011)
79 E. Borie, Ann. Phys. 327, 733 (2012); arXiv:1103.1772-v6
82 P. Indelicato, P.J. Mohr, 2012 (in preparation)
95 A.P. Martynenko, R. Faustov, Phys. Atomic Nuclei 64, 1282 (2001)
96 J.L. Friar, Ann. Phys. 122, 151 (1979)
101 C.E. Carlson, M. Vanderhaeghen, Phys. Rev. A 84, 020102 (2011)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 2

Table 3

All known contributions to the 2S-HFS in μp from different authors and the one we selected. Values are in meV, radii in fm. SE: self-energy, VP: vacuum polarization, Rel: relativistic, RC: recoil correction, PT: perturbation theory, p: proton, int: interaction, AMM: anomalous magnetic moment.

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h1	Fermi energy, $(Z\alpha)^4$	22.8054	22.8054			
h2	Breit corr., $(Z\alpha)^6$	0.0026	0.00258			
h3	Dirac energy (+ Breit corr. in all-order)			22.807995	22.807995	Eq. (107) in [80]
h4	μ AMM corr., $\alpha(Z\alpha)^4$, $\alpha(Z\alpha)^4$	0.0266	0.02659		0.02659	
h5	eVP in 2nd-order PT, $\alpha(Z\alpha)^5$ (ϵ_{VP2})	0.0746	0.07443			
h6	All-order eVP corr.			0.07437	0.07437	Eq. (109) in [80]
h7	Two-loop corr. to Fermi-energy (ϵ_{VP2})		0.00056		0.00056	
h8	One-loop eVP in 1γ int., $\alpha(Z\alpha)^4$ (ϵ_{VP1})	0.0482	0.04818		0.04818	
h9	Two-loop eVP in 1γ int., $\alpha^2(Z\alpha)^4$ (ϵ_{VP1})	0.0003	0.00037		0.00037	
h10	Further two-loop eVP corr.		0.00037		0.00037	[113,114]
h11	μVP (similar to ϵ_{VP2})		0.00091		0.00091	
h12	μVP (similar to ϵ_{VP1})	0.0004	(incl. in h13)		(incl. in h13)	
h13	Vertex, $\alpha(Z\alpha)^5$		−0.00311		−0.00311	[a]
h14	Higher order corr. of (h13), (part with $\ln(\alpha)$)		−0.00017		−0.00017	[115]
h15	μ SE with p structure, $\alpha(Z\alpha)^5$	0.0010				
h16	Vertex corr. with proton structure, $\alpha(Z\alpha)^5$	−0.0018				
h17	"Jellyfish" corr. with p structure, $\alpha(Z\alpha)^5$	0.0005				
h18	Hadron VP, α^6	0.0005(1)	0.00060(10)		0.00060(10)	
h19	Weak interaction contribution	0.0003	0.00027		0.00027	[116]
h20	Finite-size (Zemach) corr. to ΔE_{Fermi}, $(Z\alpha)^5$	−0.1518[b]	−0.16037 r_Z	−0.16034 r_Z	−0.16034 r_Z	Eq. (107) in [80]

(*continued on next page*)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

HFS in μp

Table 3 (*continued*)

	Contribution	Martynenko [72]	Borie-v6 [79]	Indelicato	Our choice [80]	Ref.
h21	Higher order finite-size corr. to $\Delta E_{\mathrm{Fermi}}$			$-0.0022\, r_E{}^2 + 0.0009$	$-0.0022\, r_E{}^2 + 0.0009$	Eq. (107) in [80]
h22	Proton polarizability, $(Z\alpha)^5$, $\Delta E^{\mathrm{pol}}_{\mathrm{HFS}}$	0.0105(18)	0.0080(26)		0.00801(260)	[117,118]
h23	Recoil corr.	(incl. in h20)	0.02123		0.02123	[112]
h24	eVP + proton structure corr., α^6	-0.0026				
h25	eVP corr. to finite-size (similar to ϵ_{VP2})		-0.00114	$-0.0018\, r_Z - 0.0001$	$-0.0018\, r_Z - 0.0001$	Eq. (109) in [80]
h26	eVP corr. to finite-size (similar to ϵ_{VP1})		-0.00114		$-0.00114(20)$	
h27	Proton structure corr., $\alpha(Z\alpha)^5$	-0.0017				
h28	Rel. + radiative RC with p AMM, α^6	0.0018				
	Sum	22.8148(20)[c]	22.9839(26) $- 0.1604\, r_Z$		22.9858(26) $-$ $0.1621(10)\, r_Z - 0.0022(5)\, r_E^2$	
	Sum with $r_E = 0.841$ fm, $r_Z = 1.045$ fm [28]	22.8148 meV	22.8163 meV		22.8149 meV	

[a] Includes a correction $\alpha(Z\alpha)^5$ due to μVP.
[b] Calculated using the Simon et al. form factor.
[c] The uncertainty is 0.0078 meV if the uncertainty of the Zemach term (h20) is included (see Table II of [72]).

28 M.O. Distler, J.C. Bernauer, T. Walcher, Phys. Lett. B 696, 343 (2011)
80 P. Indelicato, arXiv:1210.5828v2 [PRA 87, 022501 (2013)]
112 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 78, 022517 (2008)
113 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 88, 641 (2008)
114 S.G. Karshenboim, E.Y. Korzinin, V.G. Ivanov, JETP Lett. 89, 216 (2009)
115 S.J. Brodsky, G.W. Erickson, Phys. Rev. 148, 26 (1966)
116 M.I. Eides, Phys. Rev. A 85, 034503 (2012)
117 C.E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A 83, 042509 (2011)
118 E. Cherednikova, R. Faustov, A. Martynenko, Nuclear Phys. A 703, 365 (2002)

A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013), Tab 3

Theory in μp: new papers

Papers that have appeared after we wrote A. Antognini, RP *et al.*, Ann. Phys. 331, 127 (2013):

- Mohr, Griffith, Sapirstein, PRA 87, 052511 (2013), [1304.2076], "Bound-state field-theory approach to proton-structure effects in muonic hydrogen"
- Korzinin, Ivanov Karshenboim, PRD 88, 125019 (2013) [1311.5784]: "The $\alpha^2(Z\alpha)^4 m$ contributions to the Lamb shift and fine structure in light muonic atoms"
- Karshenboim, Ivanov, Korzinin, PRA 89, 022102 (2014) [1311.5789]: "Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. I. General approach"
- Ivanov, Korzinin, Karshenboim, PRA 90, 022103 (2014) [1311.5790]: "Relativistic recoil effects for energy levels in a muonic atom within a Grotch-type approach. II. An application to the one-loop electronic vacuum polarization"
- Alarcon, Lensky, Pascalutsa, EPJ C 74, 2852 (2014) [1312.1219]: "Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution"
- Indelicato, Mohr, Sapirstein, PRA 89, 054017 (2014) [1402.0439]: "Coordinate-space approach to vacuum polarization"
- Peset, Pineda, arXiv 1403.3408 [hep-ph]: "Model independent determination of the muonic hydrogen Lamb shift and proton radius"
- Peset, Pineda, Nucl. Phys. B 887, 69 (2014) [1406.4524]: "The two-photon exchange contribution to muonic hydrogen from chiral perturbation theory"

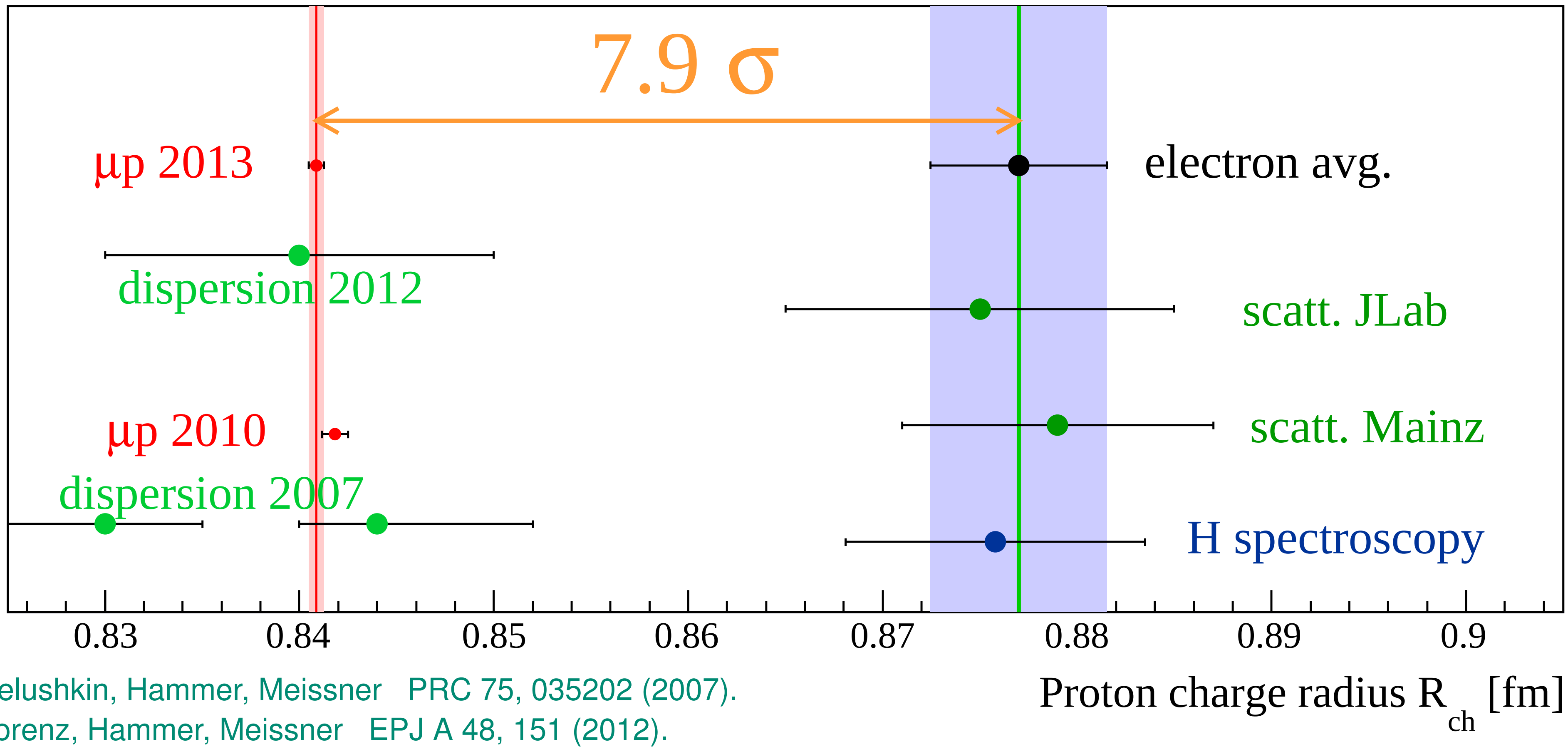
No big changes.
Polarizability terms confirmed a couple of times.

The proton radius puzzle

The proton rms charge radius measured with

electrons: 0.8770 $\pm$ 0.0045 fm

muons: 0.8409 $\pm$ 0.0004 fm



Belushkin, Hammer, Meissner PRC 75, 035202 (2007).
Lorenz, Hammer, Meissner EPJ A 48, 151 (2012).

What may be wrong?

$$\widetilde{L}^{\text{theo.}}_{\mu p}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}^{\text{exp.}}_{\mu p} = \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp theory wrong?

μp experiment wrong?

H theory wrong?

H experiments wrong? $\rightarrow R_\infty$ wrong?

AND e-p scattering exp. wrong?

Standard Model wrong?!?

RP, R. Gilman, G.A. Miller, K. Pachucki, "Muonic hydrogen and the proton radius puzzle", Annu. Rev. Nucl. Part. Sci. **63**, 175 (2013) (arXiv 1301.0905)

What may be wrong?

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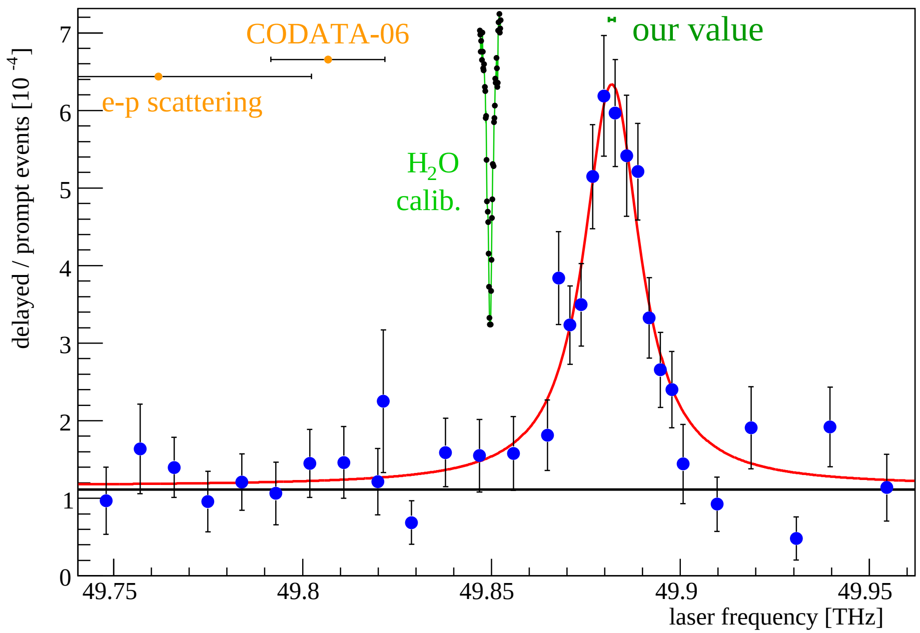
μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?

That is $> 100\ \delta(\mu\text{p})$! σ_{tot} = 650 MHz, [570 MHz$_{stat}$, 300 MHz$_{syst}$]

4 line widths ! Γ = 19 GHz

2 resonances in μp give the same r_p



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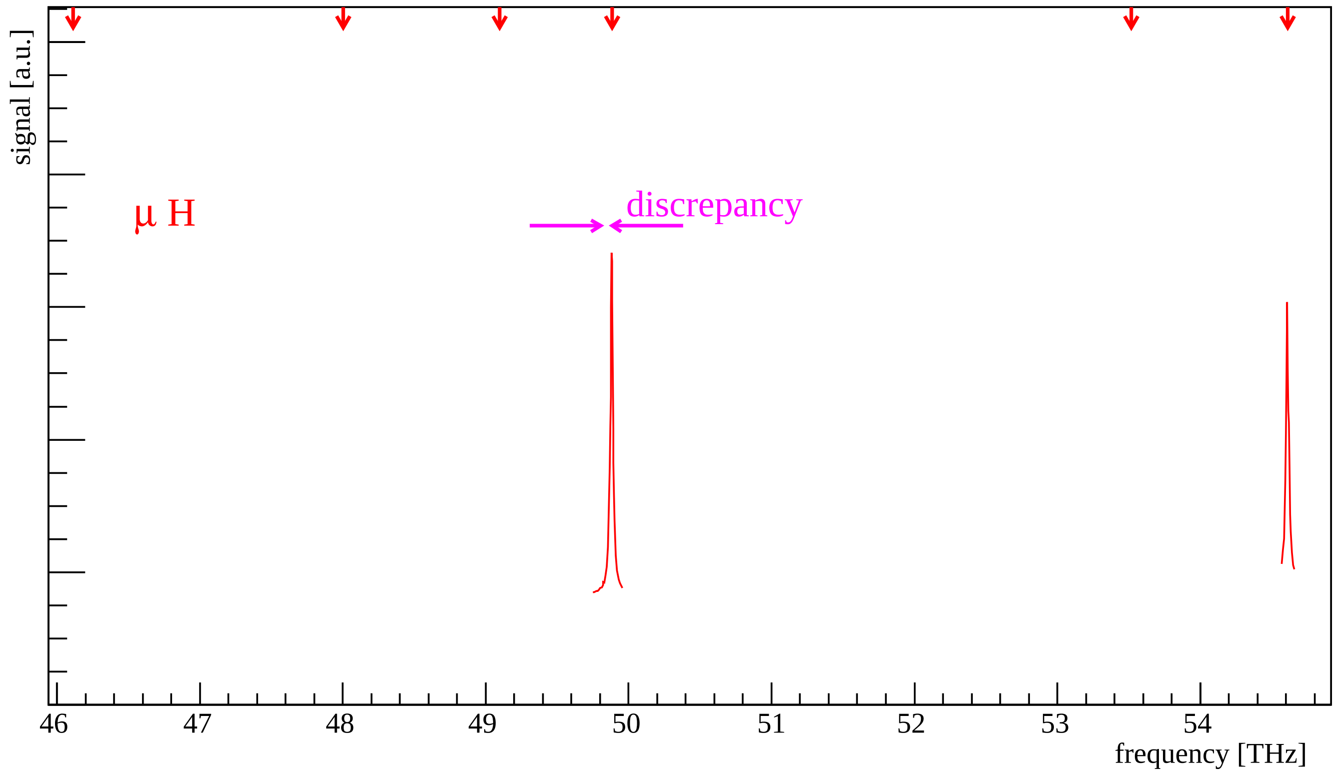
μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?

  FS, HFS huge.

  Next transition: ~ 1 THz away.



What may be wrong?

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μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?

Laser frequency (H_20 calibration)	300 MHz
intrinsic H_2O uncertainty	2 MHz
AC and DC stark shift	< 1 MHz
Zeeman shift (5 Tesla)	< 30 MHz
Doppler shift	< 1 MHz
Collisional shift	2 MHz
	300 MHz

μp atom is small and not easily perturbed by external fields.

What may be wrong?

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μp experiment wrong?

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- Wrong transition?
- Systematic error?
- Molecular effects?

$p\ \mu\ e$ molecular ion? U.D. Jentschura, Annals of Physics 326, 516 (2011).

Does not exist! J.-P. Karr, L. Hilico, PRL 109, 103401 (2012).
M. Umair, S. Jonsell, J. Phys. B 47, 175003 (2014).

Experimentally:

- only 1 line observed ($> 80\,\%$ population)
- expected width
- $pp\mu$ ion short-lived R. Pohl *et al.*, PRL 97, 193402 (2006).

What may be wrong?

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μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
- Wrong transition?
- Systematic error?
- Molecular effects?
- Gas imputities? M. Diepold, RP *et al.*, PRA 88, 042520 (2013).

  Target gas contained 0.55(5) % air (leak).

  Back-of-the-envelope calculation:

  collision rate $\lambda \approx 6 \cdot 10^3 s^{-1}$
  2S lifetime $\tau(2S) = 1\,\mu$s

  $\Rightarrow$ Less than 1% of all μp(2S) atoms see any N_2

What may be wrong?

$$\widetilde{L}_{\mu p}^{\text{theo.}}(r_{\text{p}}^{\text{CODATA}}) - \widetilde{L}_{\mu p}^{\text{exp.}} \quad = \quad \begin{cases} 75\ \text{GHz} \\ 0.31\ \text{meV} \\ 0.15\ \% \end{cases}$$

μp experiment wrong?

- Frequency mistake by 75 GHz ($\Leftrightarrow 0.15\%$)?
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- Molecular effects?
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μp experiment probably not wrong by 100 σ

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μp theory wrong?

Discrepancy = 0.31 meV
Theory uncert. = 0.0025 meV

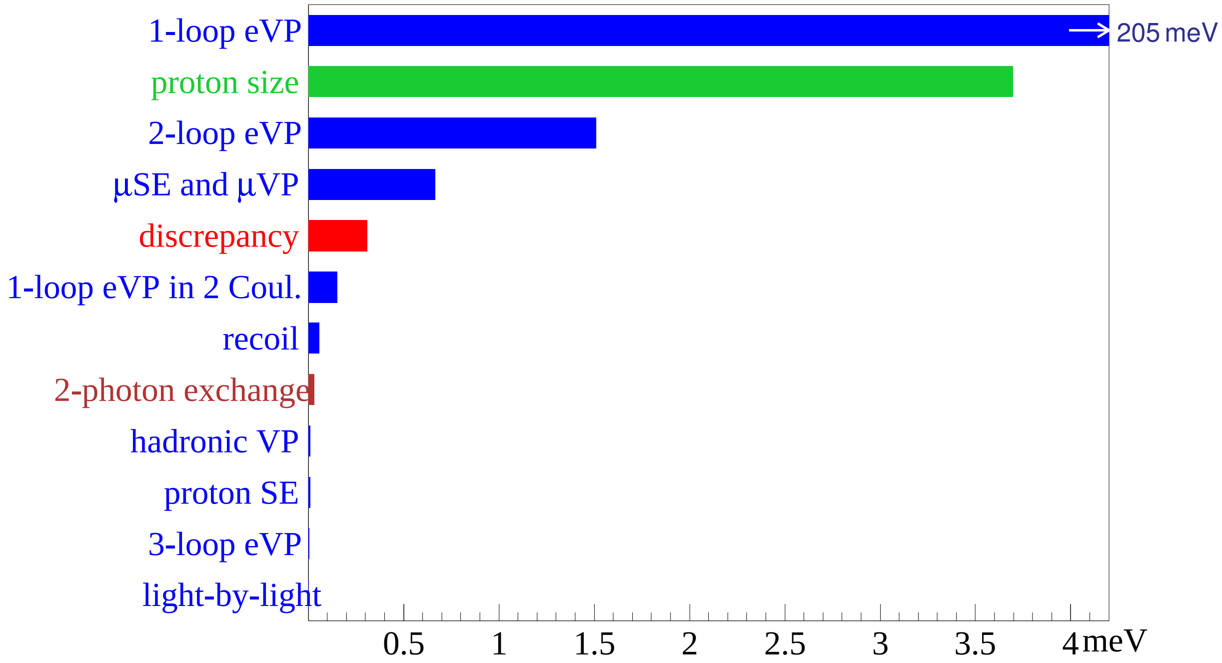
$\Longrightarrow$ 120δ(theory) deviation

$$\Delta E = 206.0668(25) - 5.2275(10)\, r_p^2 \ \text{[meV]}$$

double-checked by many groups

5th largest term!

Some contributions to the μp Lamb shift



Theory summary:

A. Antognini, RP *et al.*
Annals of Physics 331, 127 (2013)

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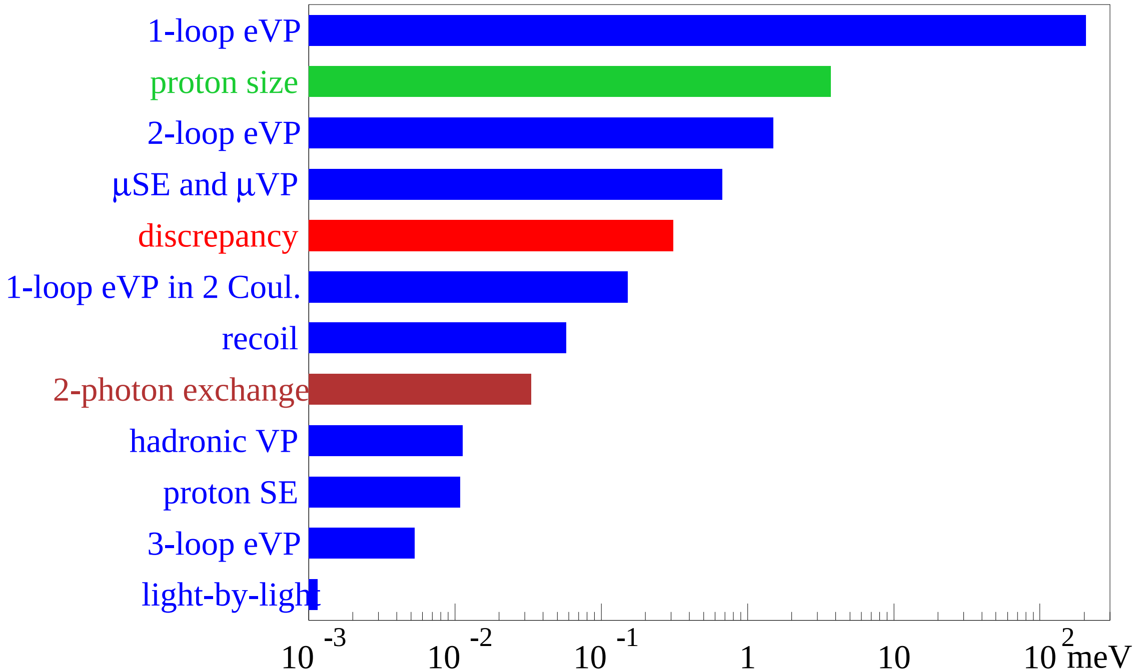

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5$^{\text{th}}$ largest term!

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Discussions: 3rd Zemach moment

- PLB **693**, 555 De Rujula: *"QED is not endangered by the proton's size"* *(1008.3861)*

  A large third Zemach moment $\langle r_p^3 \rangle_{(2)} = \int d^3 r_1\, d^3 r_2\, \rho(r1)\, \rho(r_2)\, |r_1 - r_2|^3$

  of the proton can explain all three measurements: μp, H, e-p

  $\rho(r)$ is not a simple Dipole, but has "core" and "tail"

- PRC **83**, 012201 Cloet, Miller: *"Third Zemach moment of the proton"* *(1008.4345)*

  Such a large third Zemach moment is impossible.

  $\langle r_p^3 \rangle_{(2)}$ (De Rujula) = 36.6 ± 6.9 fm^3

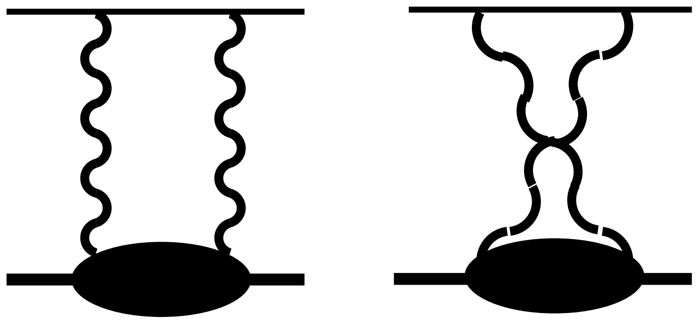
  $\langle r_p^3 \rangle_{(2)}$ (Sick) = 2.71 ± 0.13 fm^3

- PLB **696**, 343 Distler *et al*: *"The RMS radius of the proton and Zemach moments"* (1011.1861)

  $\langle r_p^3 \rangle_{(2)}$ (Mainz 2010) = 2.85 ± 0.08 fm^3

Discussions: Proton polarizability

proton polarizability aka. two-photon exchange



Seems to be the only contribution which *might* be able to solve the proton size puzzle by changing theory in μp.

Keep in mind:

Discrepancy:	0.31 meV	
Polarizability:	0.015(4) meV	20 times smaller!

Discussions: Proton polarizability

- G.A. Miller *et al.*, PRA **84**, 020101(R) (2011)
  "Toward a resolution of the proton size puzzle"
  - New off-mass-shell effect $\sim \alpha \frac{m^4}{M^3}$ solves puzzle.
- R.J. Hill, G. Paz, PRL, **107**, 160402 (2011)
  "Model independent analysis of proton structure for hydrogenic bound states"
  - forward Compton amplitude's $W_1(0, Q^2)$ is now well known
  - "Crazy" functional behaviour can give any correction.
  - No numbers given.
- C.E. Carlson, M. Vanderhaeghen, PRA **84**, 020102(R) (2011)
  "Higher-order proton structure corrections to the Lamb shift in muonic hydrogen"
  - All off-shell effects are automatically included in standard treatment.
- C.E. Carlson, M. Vanderhaeghen, arXiv 1109.3779 (atom-ph)
  "Constraining off-shell effects using low-energy Compton scattering"
  - Off-shell effects are 100 times smaller than needed to explain the puzzle.

Discussions: Proton polarizability

- M.C. Birse, J.A. McGovern, Eur. Phys. J. A 48, 120 (2012)
  "Proton polarisability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory"
  - Calculate $T_1(0, Q^2)$ in heavy-baryon chiral pert. theory.
  - Proton polarizability is **not** responsible for the radius puzzle.

- Gorchtein, Llanes-Estrada, Szczepaniak, PRA **87**, 052501 (2013)
  "μ-H Lamb shift: dispersing the nucleon-excitation uncertainty with a finite energy sum rule"
  - Sum rule + virtual photoabsorption data.
  - *"We conclude that nucleon structure-dependent uncertainty by itself is unlikely to resolve the large discrepancy..."*

- Karshenboim, McKeen, Pospelov, arXiv 1401.6156 [hep-ph]
  "Constraints on muon-specific dark forces"
  - *"These estimates show that if indeed large muon-proton interactions are responsible for the r_p discrepancy, one can no longer insist that theoretical calculations of the muon $g-2$ are under control. Thus, a resolution of the r_p problem is urgently needed in light of the new significant investments made in the continuation of the experimental $g-2$ program."*

Discussions: Proton polarizability

- Proton off-shell effects can in principle shift the μp value.
- Evil subtraction function.

- Evidence is growing, that this effect can **NOT** solve the puzzle.

- Clarification needed $[(g-2)_\mu!]$

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μp experiment wrong?

H theory wrong?
H experiments wrong? $\rightarrow R_\infty$ wrong?
AND e-p scattering exp. wrong?

Standard Model wrong?!?

Discussions: New Physics

- Jaeckel, Roy, PRD **82**, 125020 (2010)
  "Spectroscopy as a test of Coulomb's law - A probe of the hidden sector"

  hidden photons, minicharged particles $\rightarrow$ deviations from Coulomb's law.

  μp transition can NOT be explained this. (contradicts Lamb shift in H)

- U.D. Jentschura, Ann. Phys. **326**, 516 (2011)
  "Lamb shift in muonic hydrogen – II. Analysis of the discrepancy of theory and experiment"

  no millicharged particles, no unstable neutral vector boson.

- Barger, Chiang, Keung, Marfatia, PRL **106**, 153001 (2011)
  "Proton size anomaly"

  decay of Υ, J/ψ, π^0, η, neutron scattering, muon g-2, μ^{24}Mg, μ^{28}Si

  $\Rightarrow$ It's NOT a new flavor-conserving spin-0, 1 or 2 particle

- Tucker-Smith, Yavin, PRD **83**, 101702 (2011)
  "Muonic hydrogen and MeV forces"

  MeV force carrier can explain discrepancies for r_p and (g-2)$_\mu$
  IF coupling to e, n is suppressed relative to coupling to μ, p
  prediction for μHe$^+$, $\mu^+\mu^-$

Discussions: New Physics

- Batell, McKeen, Pospelov, PRL **107**, 011803 (2011)
  "New Parity-violating muonic forces and the proton charge radius"
  10...100 MeV heavy photon ("light Higgs") can explain r_{p} and (g-2)$_\mu$
  prediction for μHe$^+$, enhanced PNC in muonic systems

- Barger, Chiang, Keung, Marfatia, PRL **108**, 081802 (2011)
  "Constraint on Parity-violating muonic forces"
  No missing mass events observed in leptonic Kaon decay.
  $\Rightarrow$ contraints on light Higgs.

- Pospelov (private comm.)
  Lack of missing mass events in leptonic Kaon decays no problem.
  Light Higgs is short-lived (decays inside the detector).

- C.E. Carlson, B.C. Rislow, PRD **86**, 035013 (2012)
  "New physics and the proton radius problem"
  "New physics with fine-tuned couplings may be entertained as a possible explanation for the Lamb shift discrepancy."

Discussions: New Physics

- Wang, Ni, Mod. Phys. Lett. A **28**, 1350094 (2013)
  "Proton puzzle and large extra dimensions" (arXiv 1303.4885)

  "Extra gravitational force between the proton and the muon at very short range provides an energy shift which accounts for the discrepancy..."

- Li, Chen, arXiv 1303.5146
  "Can large extra dimensions solve the proton radius puzzle?"

  "We find that such effect could be produced by four or more large extra dimensions which are allowed by the current constraints from low energy physics."

- R. Onofrio, Eur. Phys. Lett. **104**, 20002 (2013)
  "Proton radius puzzle and quantum gravity at the Fermi scale" (1312.3469)

  "We show how the proton radius puzzle ... may be solved by means of ... an effective Yukawian gravitational potential related to charged weak interactions. [...] Muonic hydrogen plays a crucial role to test possible scenarios for a gravitoweak unification, with weak interactions seen as manifestations of quantum gravity effects at the Fermi scale.

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μp theory wrong?

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H theory wrong?

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Standard Model wrong?

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AND e-p scattering exp. wrong?

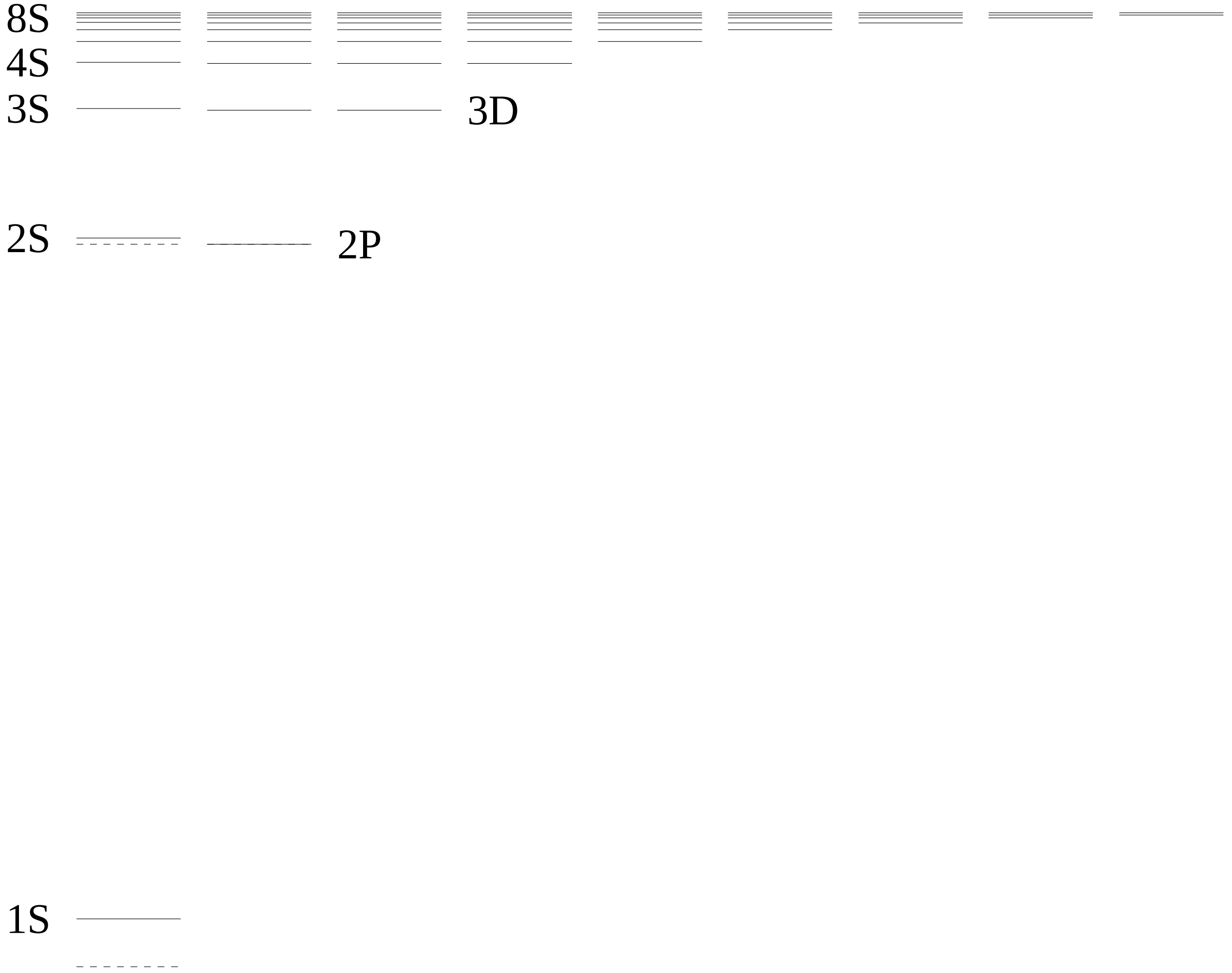
Standard Model wrong?

(Electronic) hydrogen.

Hydrogen spectroscopy

$$\text{Lamb shift}: \quad L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz}$$

$$L_{nS} \simeq \frac{L_{1S}}{n^3}$$



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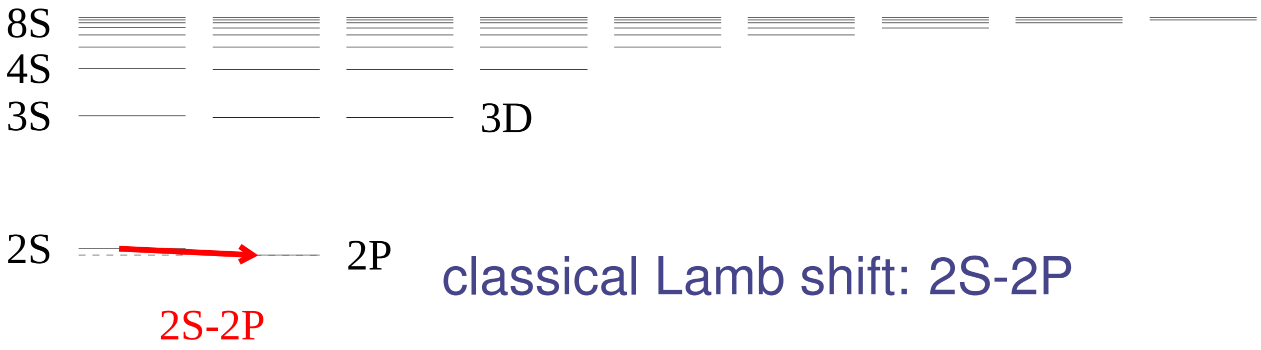
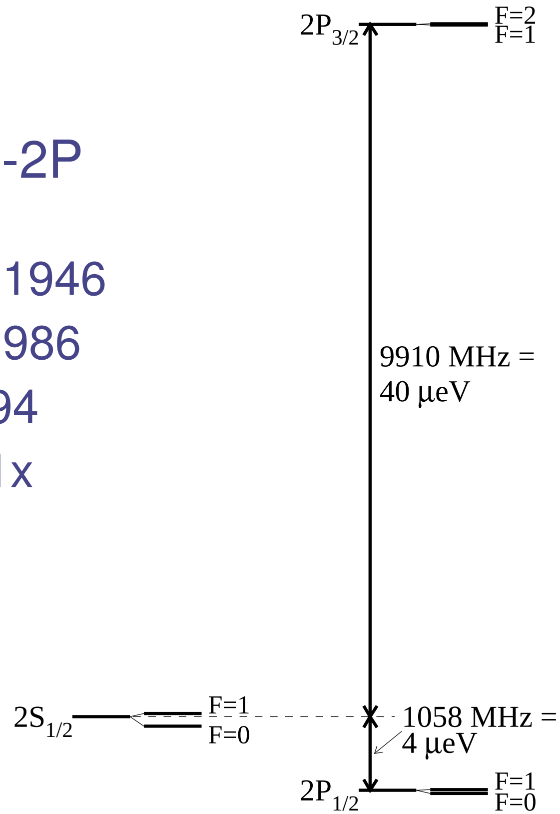


classical Lamb shift: 2S-2P

- Lamb, Retherford 1946
- Lundeen, Pipkin 1986
- Hagley, Pipkin 1994
- Hessels *et al.*, 201x



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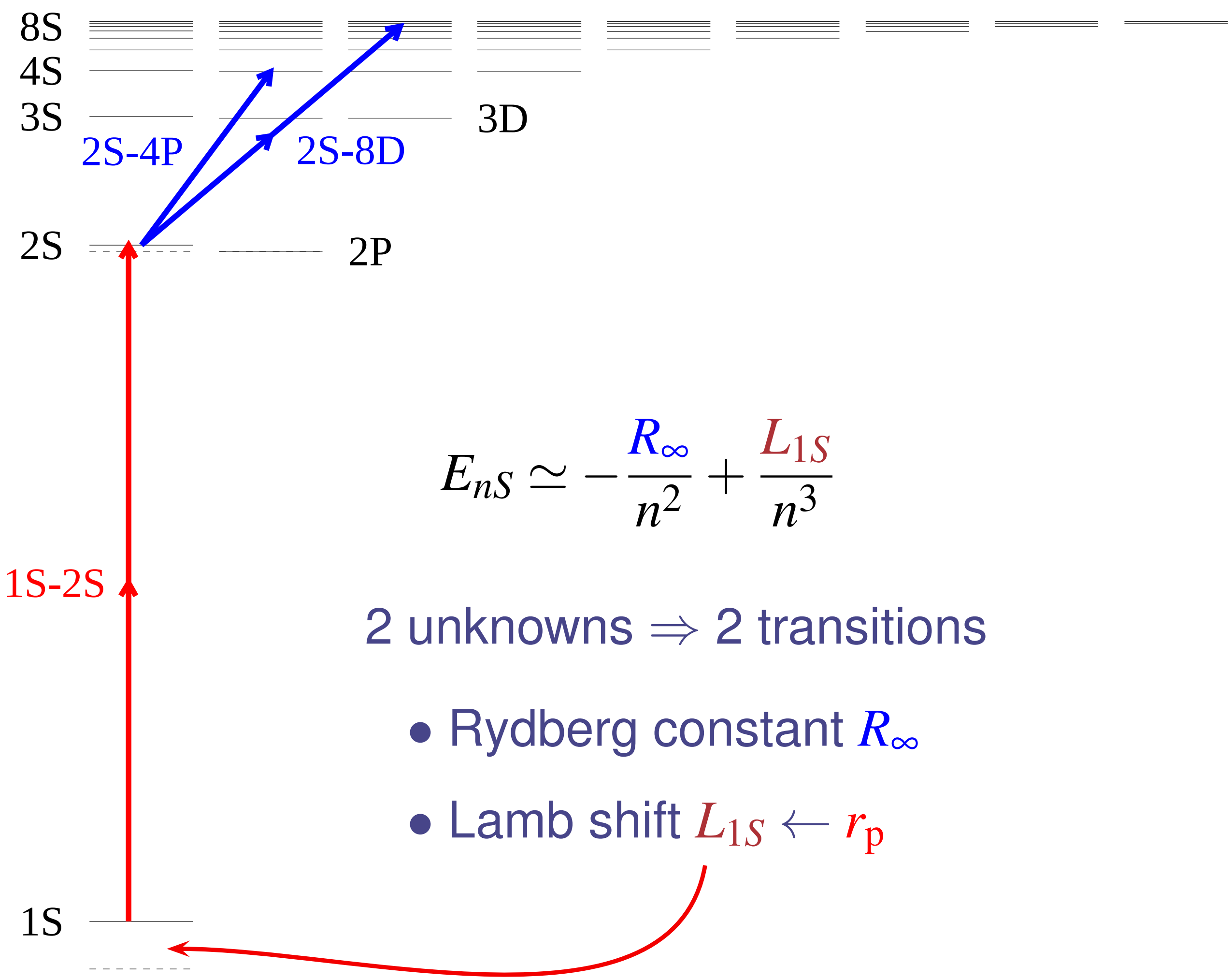
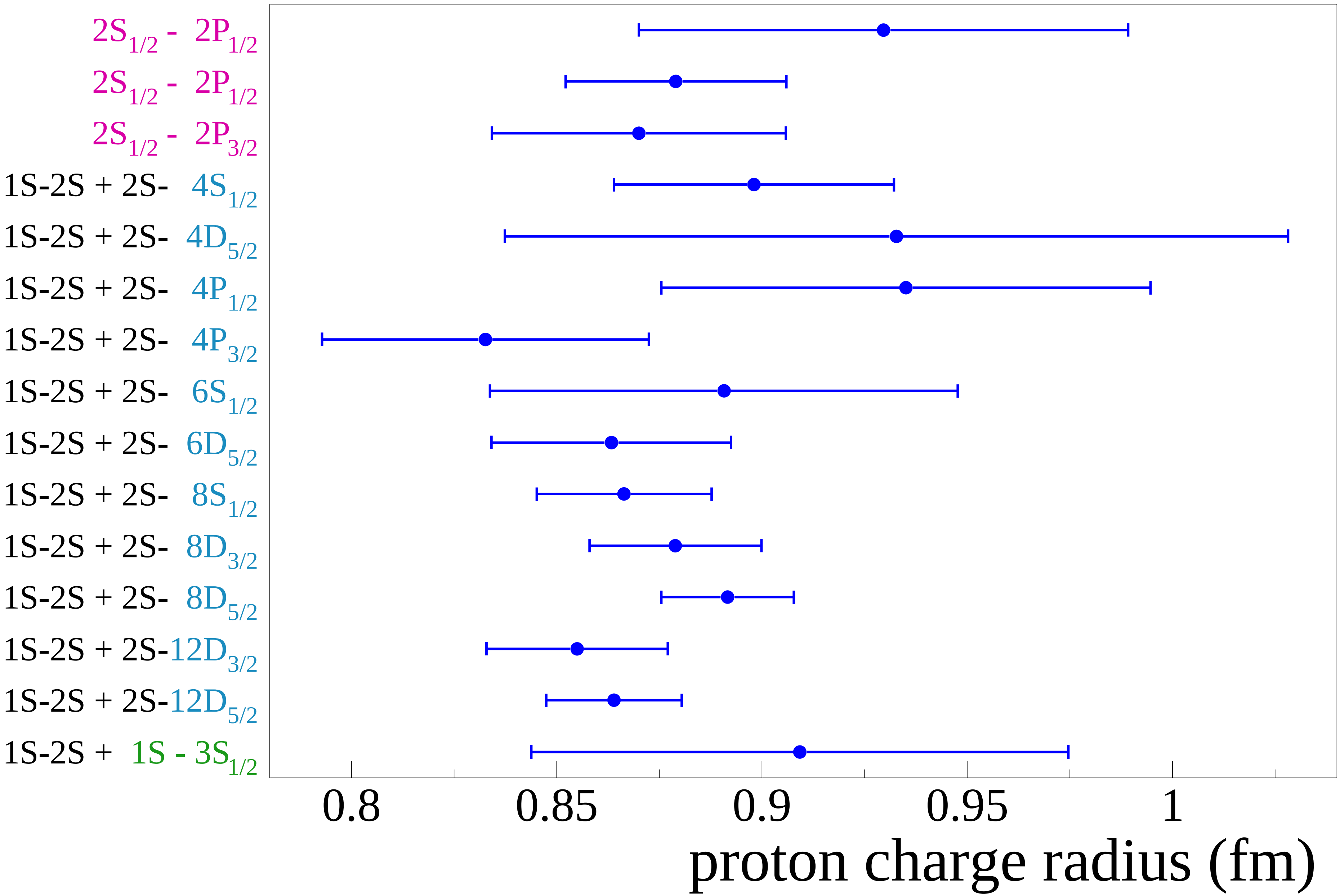


$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

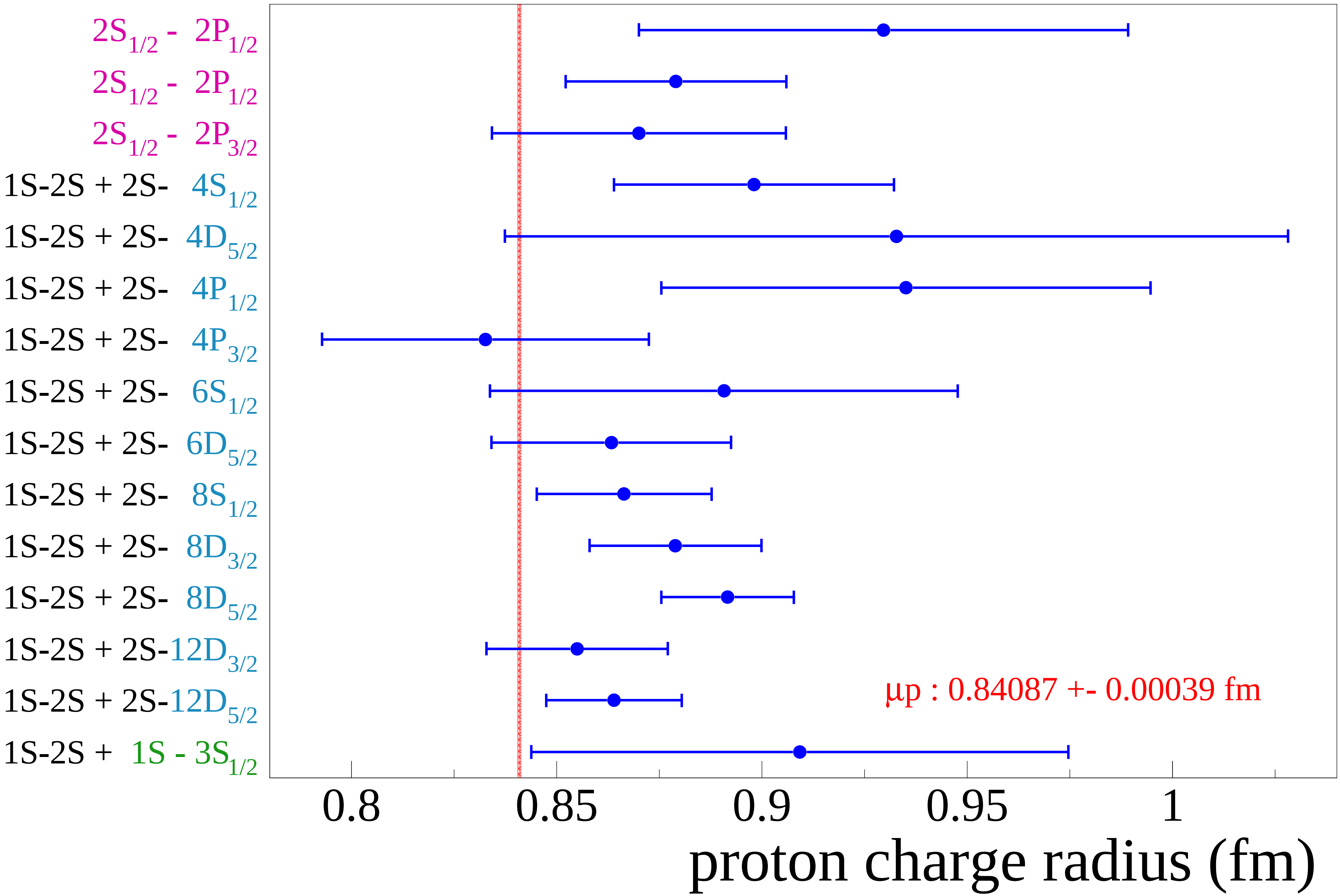
2 unknowns $\Rightarrow$ 2 transitions

- Rydberg constant R_∞
- Lamb shift $L_{1S} \leftarrow r_p$

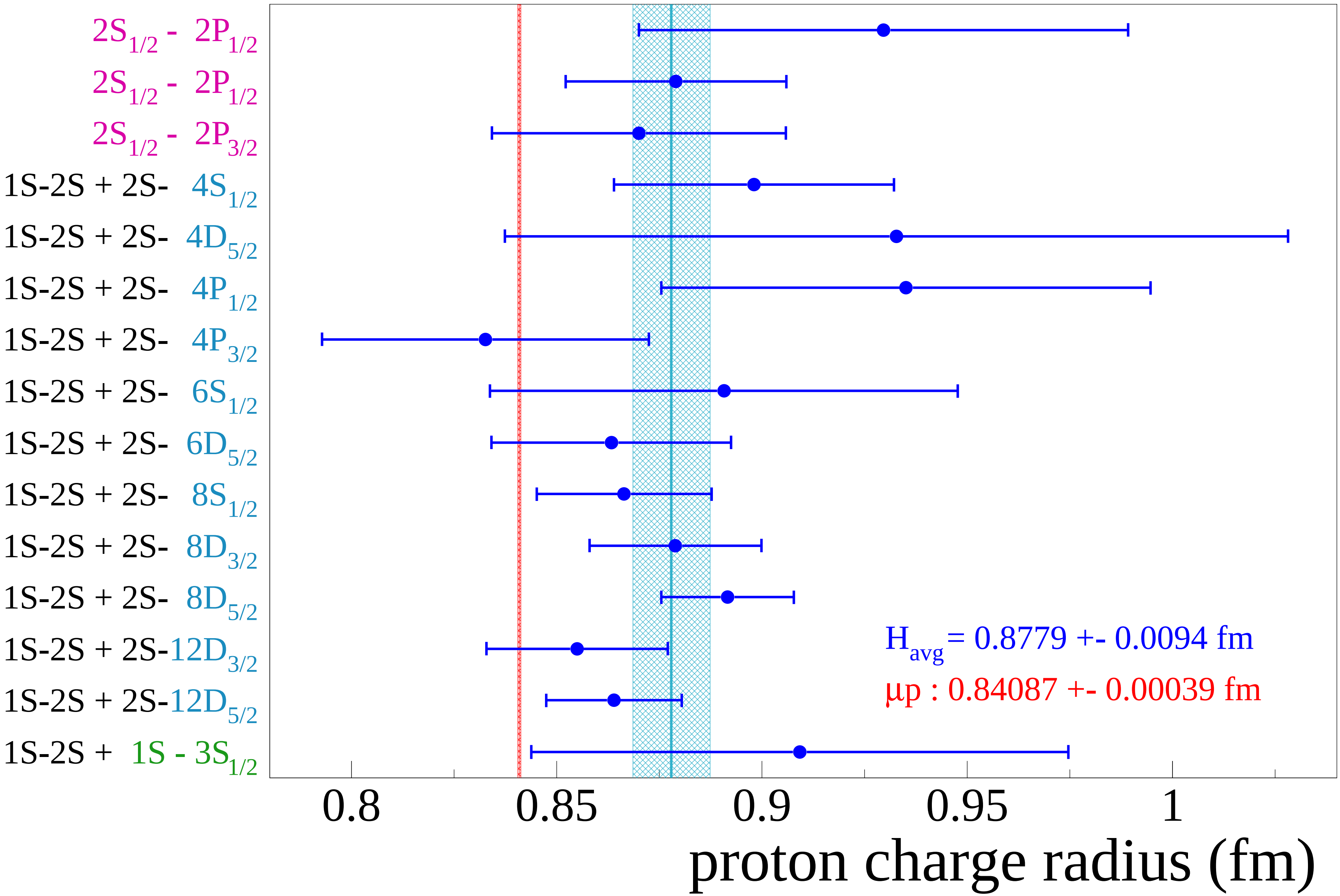
Hydrogen spectroscopy



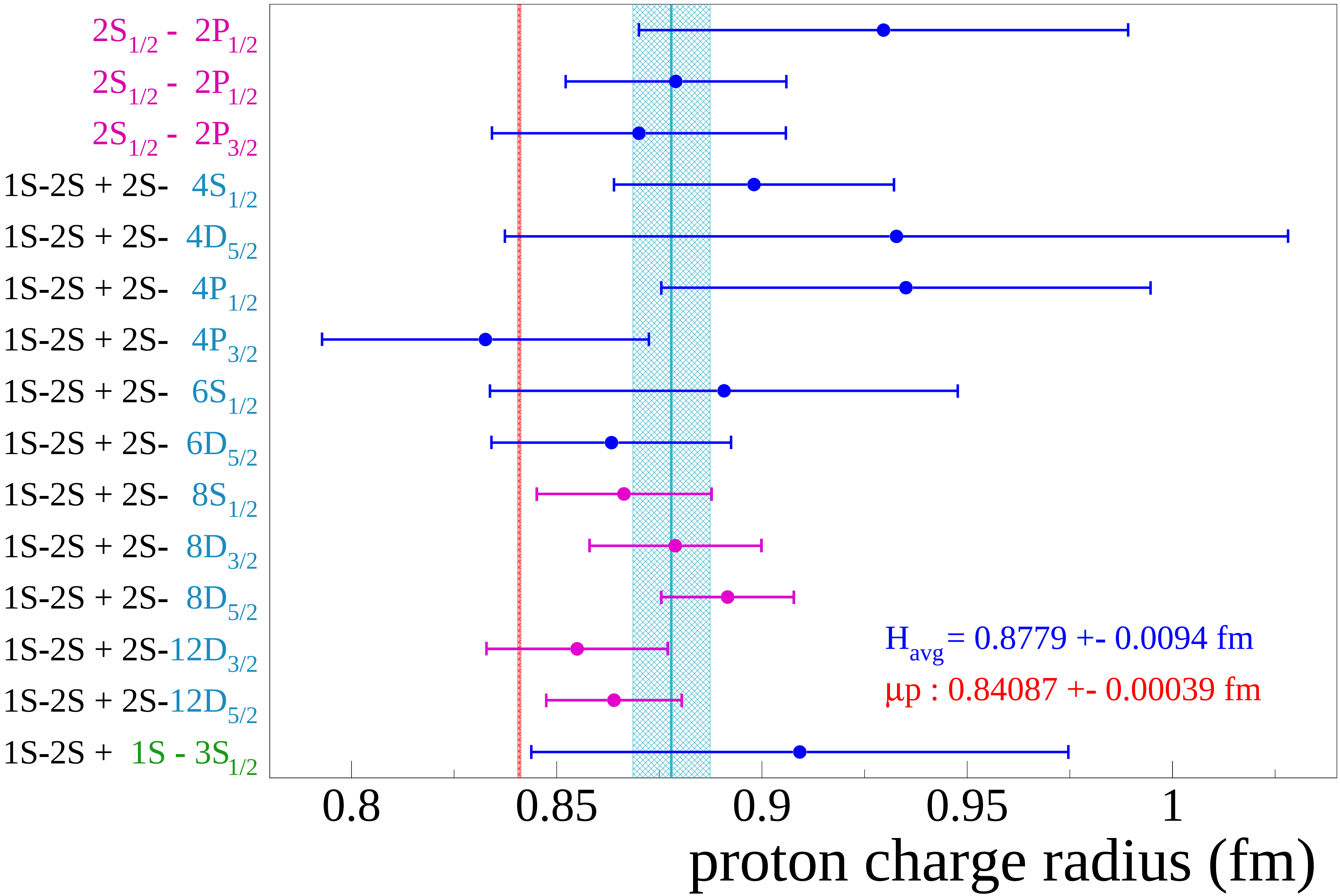
Hydrogen spectroscopy



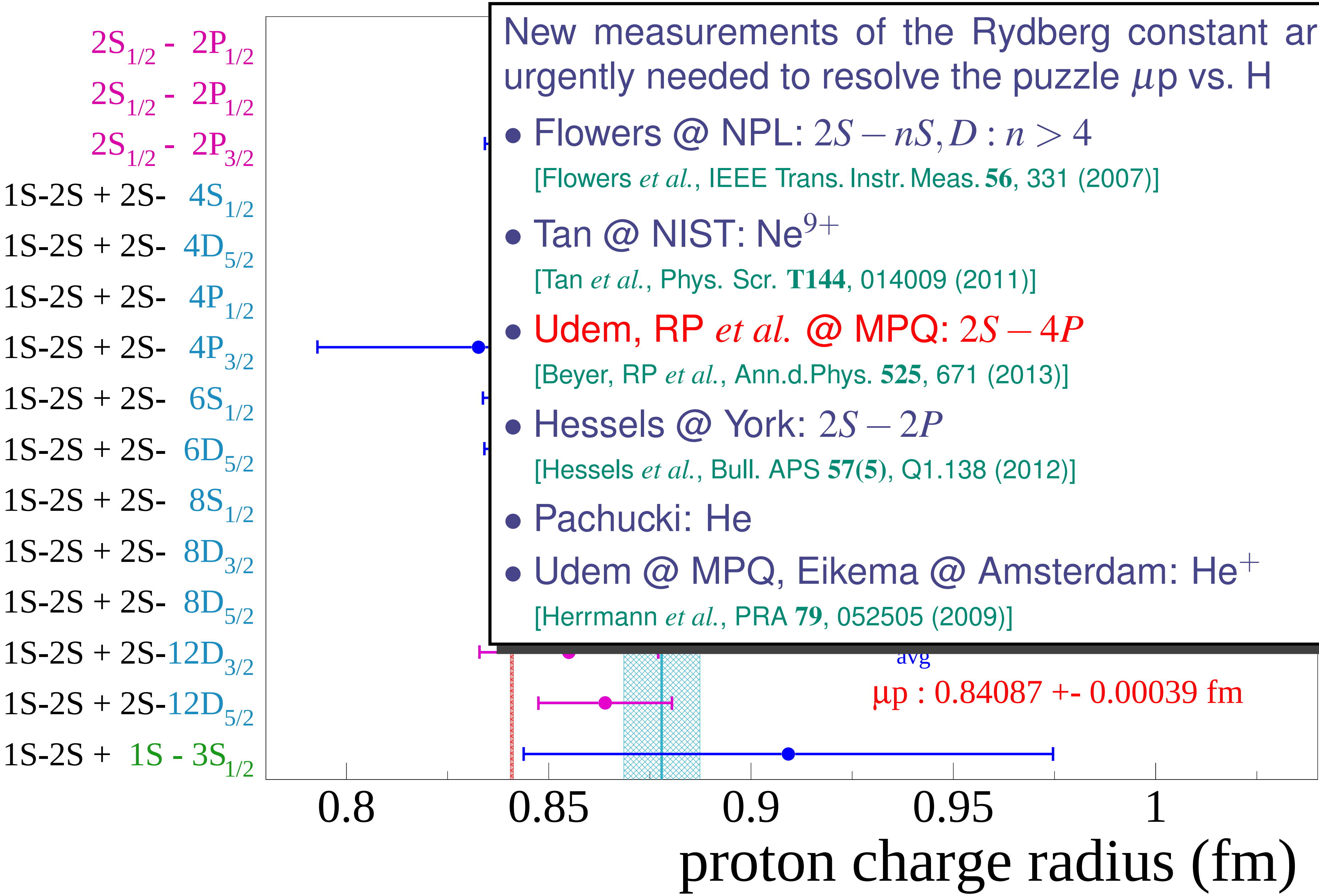
Hydrogen spectroscopy



Hydrogen spectroscopy



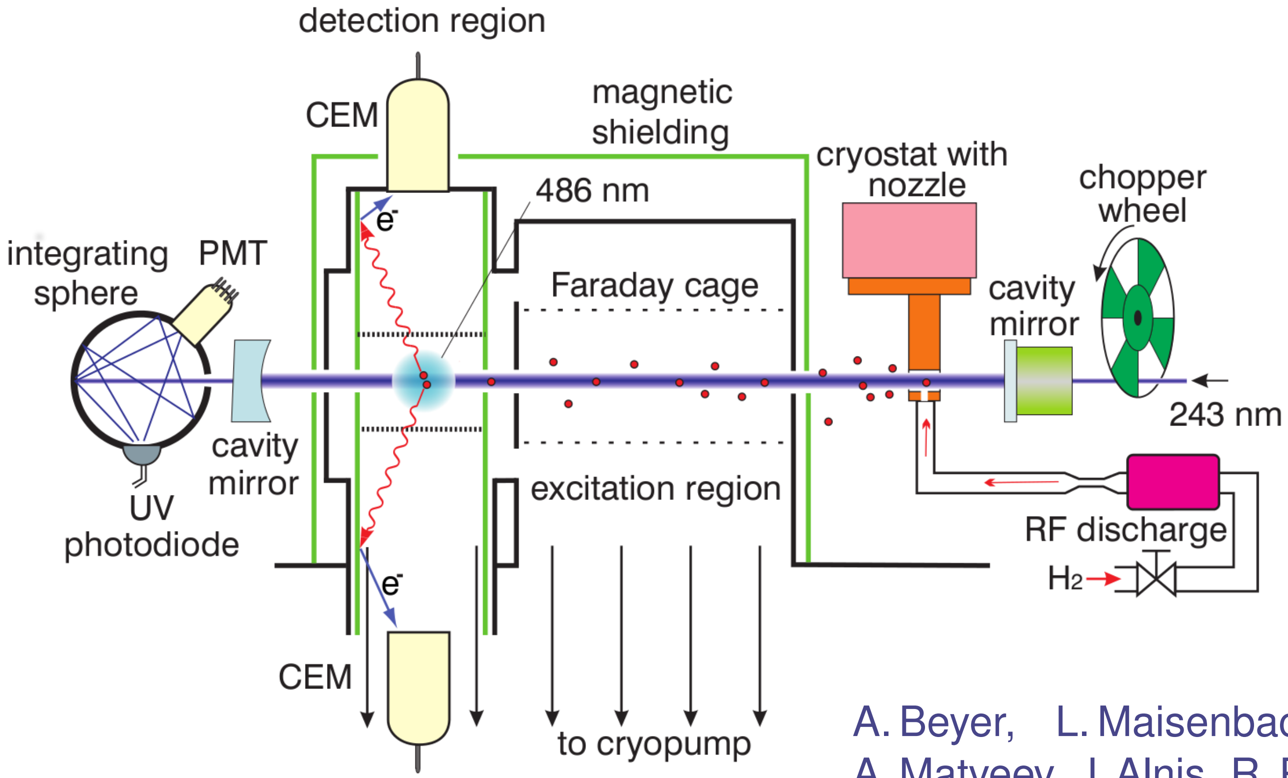
Hydrogen spectroscopy

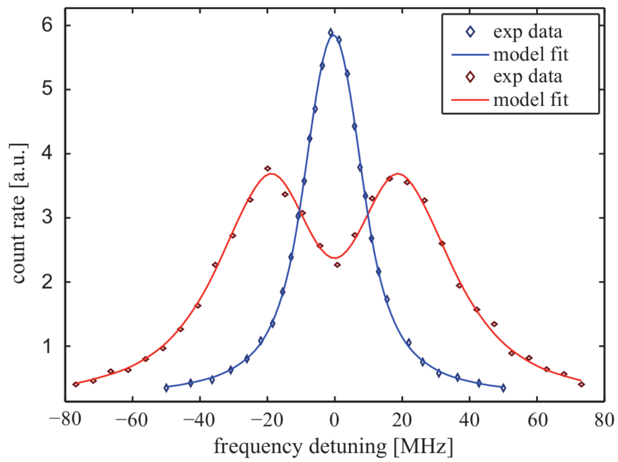


New measurements of the Rydberg constant are urgently needed to resolve the puzzle μp vs. H

- Flowers @ NPL: $2S - nS, D : n > 4$
  [Flowers *et al.*, IEEE Trans. Instr. Meas. **56**, 331 (2007)]
- Tan @ NIST: Ne^{9+}
  [Tan *et al.*, Phys. Scr. **T144**, 014009 (2011)]
- Udem, RP *et al.* @ MPQ: $2S - 4P$
  [Beyer, RP *et al.*, Ann.d.Phys. **525**, 671 (2013)]
- Hessels @ York: $2S - 2P$
  [Hessels *et al.*, Bull. APS **57(5)**, Q1.138 (2012)]
- Pachucki: He
- Udem @ MPQ, Eikema @ Amsterdam: He^{+}
  [Herrmann *et al.*, PRA **79**, 052505 (2009)]

Rydberg constant from hydrogen





2S – 4P resonance at
$88 \pm 0.5\,^\circ$ and $90 \pm 0.08\,^\circ$

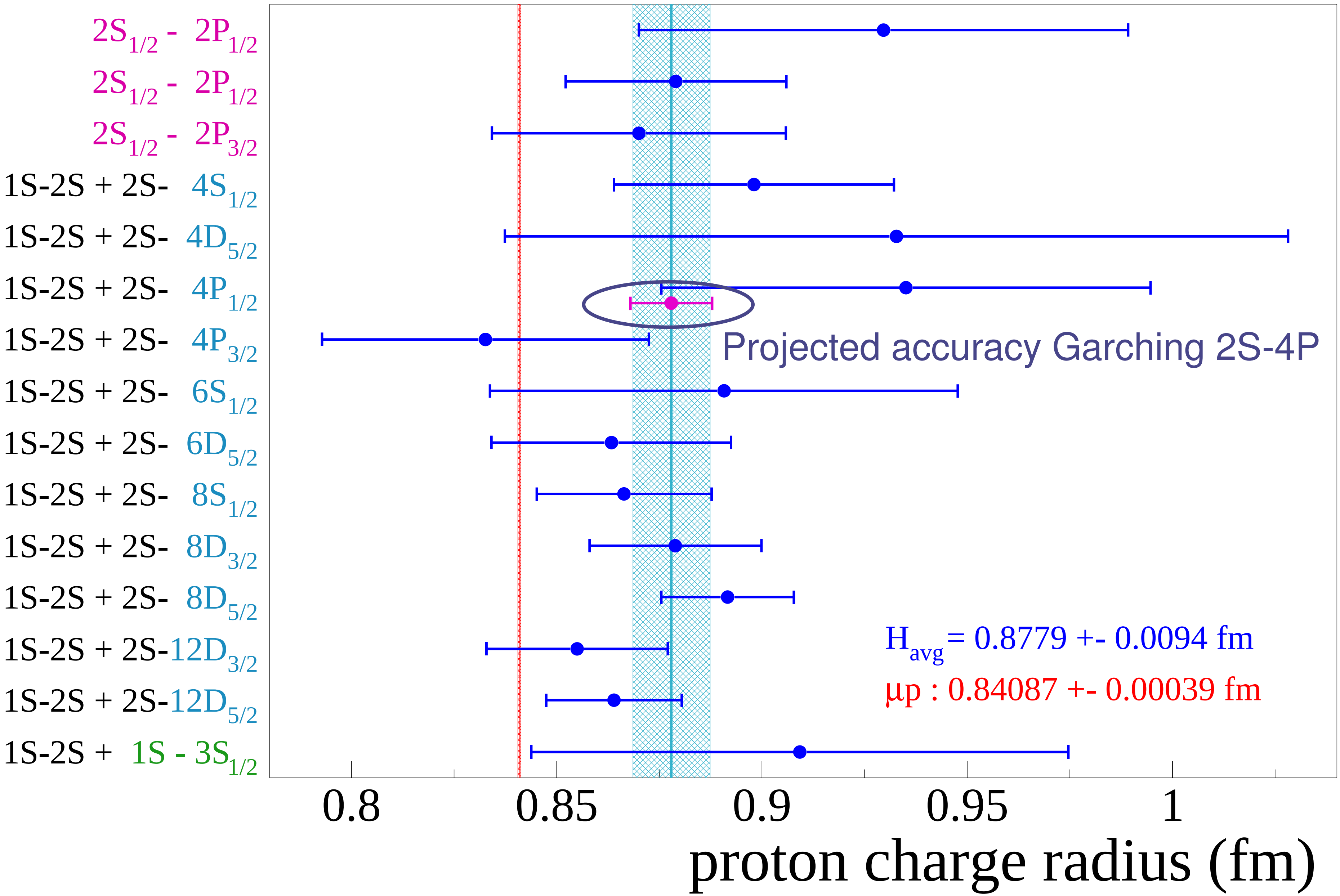
A. Beyer, L. Maisenbacher, K. Khabarova, C.G. Parthey, A. Matveev, J. Alnis, R. Pohl, N. Kolachevsky, Th. Udem and T.W. Hänsch

- Apparatus used for H/D(1S-2S)

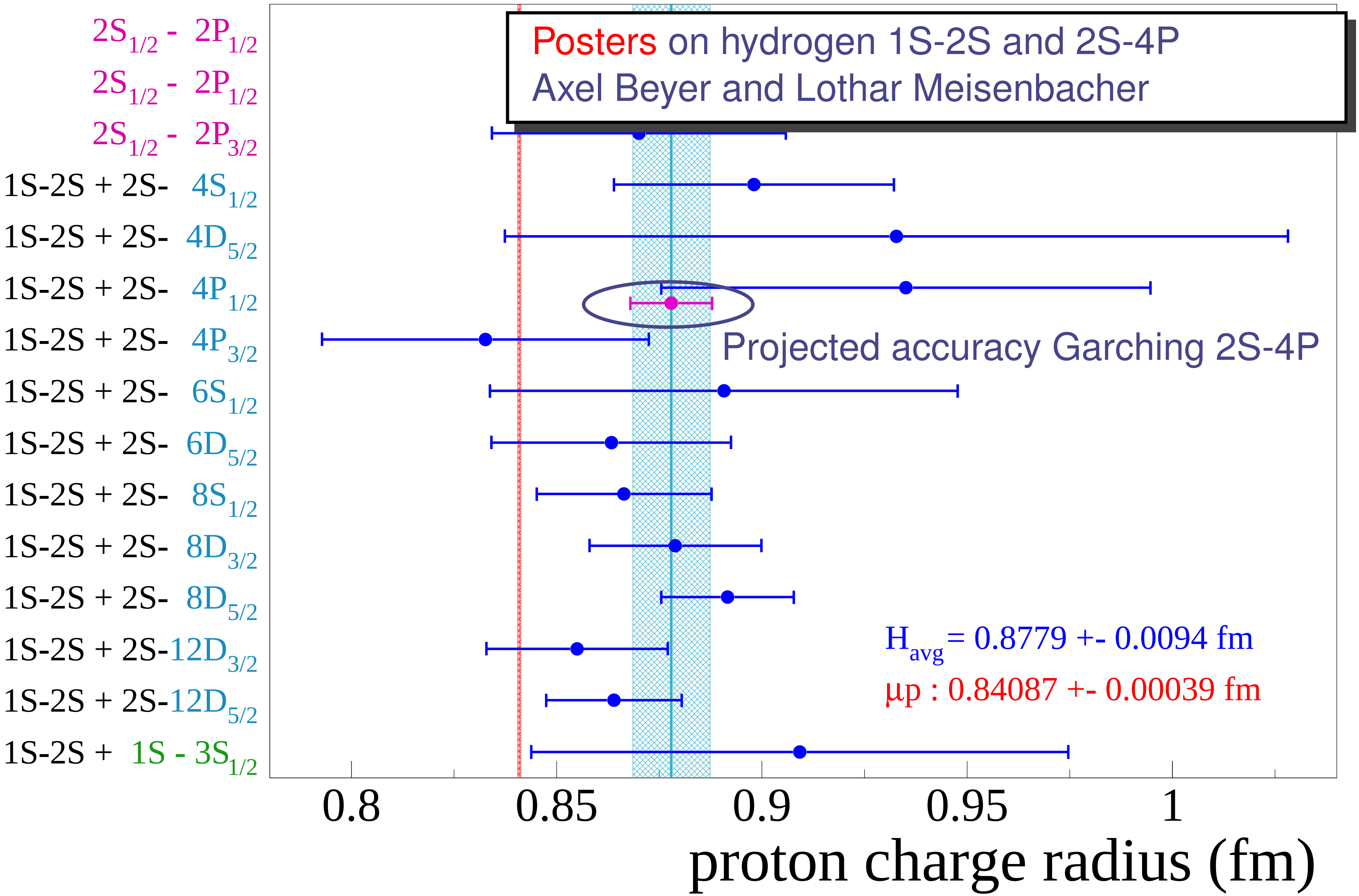
  C.G. Parthey, RP *et al.*, PRL **104**, 233001 (2010)
  C.G. Parthey, RP *et al.*, PRL **107**, 203001 (2011)
- 486 nm at 90° + Retroreflector $\Rightarrow$ Doppler-free 2S-4P excitation
- 1st oder Doppler vs. ac-Stark shift
- ~ 2.5 kHz accuracy (vs. 15 kHz Yale, 1995)
- **cryogenic** H beam, **optical** excitation to 2S

  A. Beyer, RP *et al.*, Ann. d. Phys. **525**, 671 (2013)

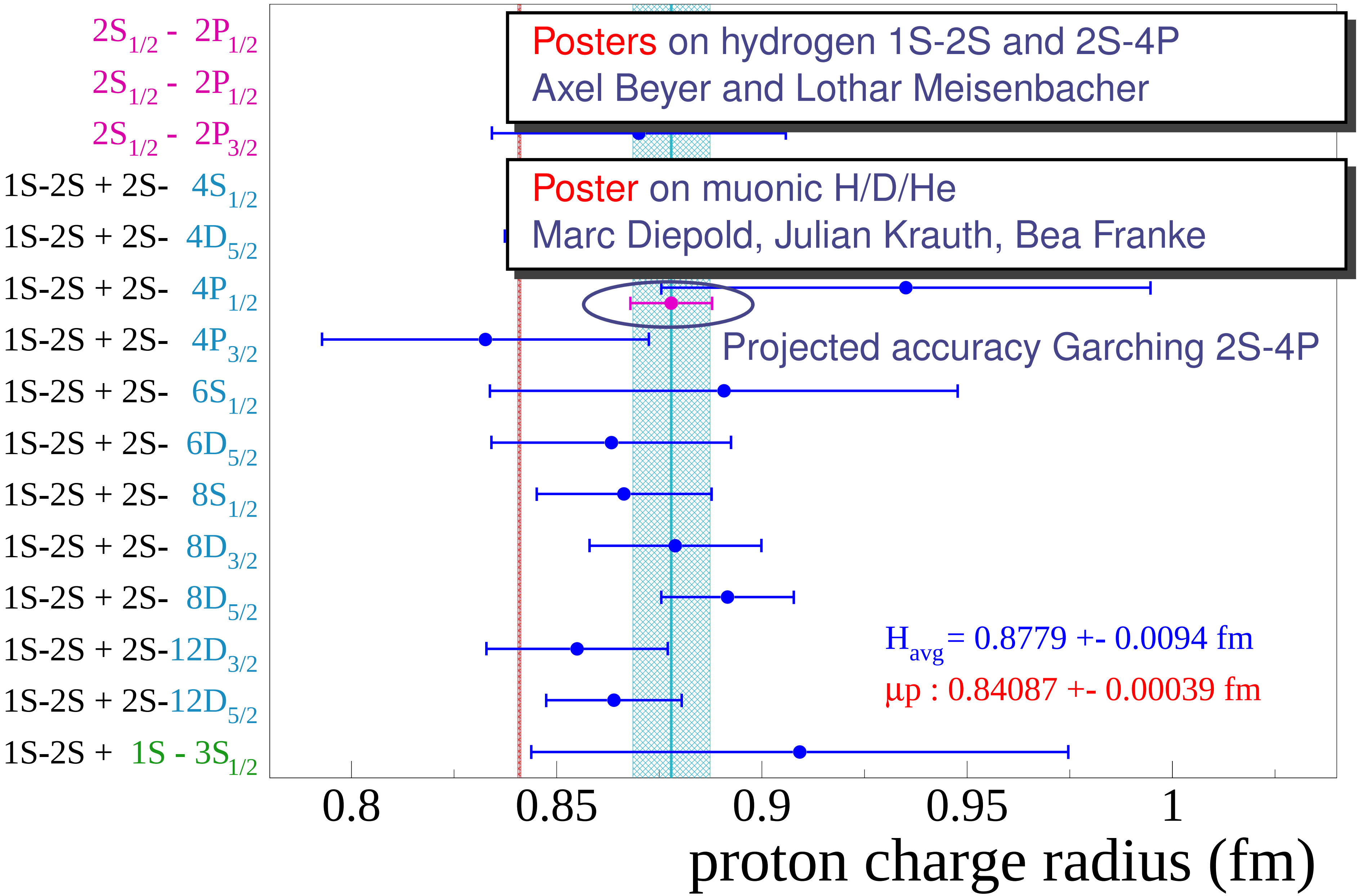
Hydrogen spectroscopy



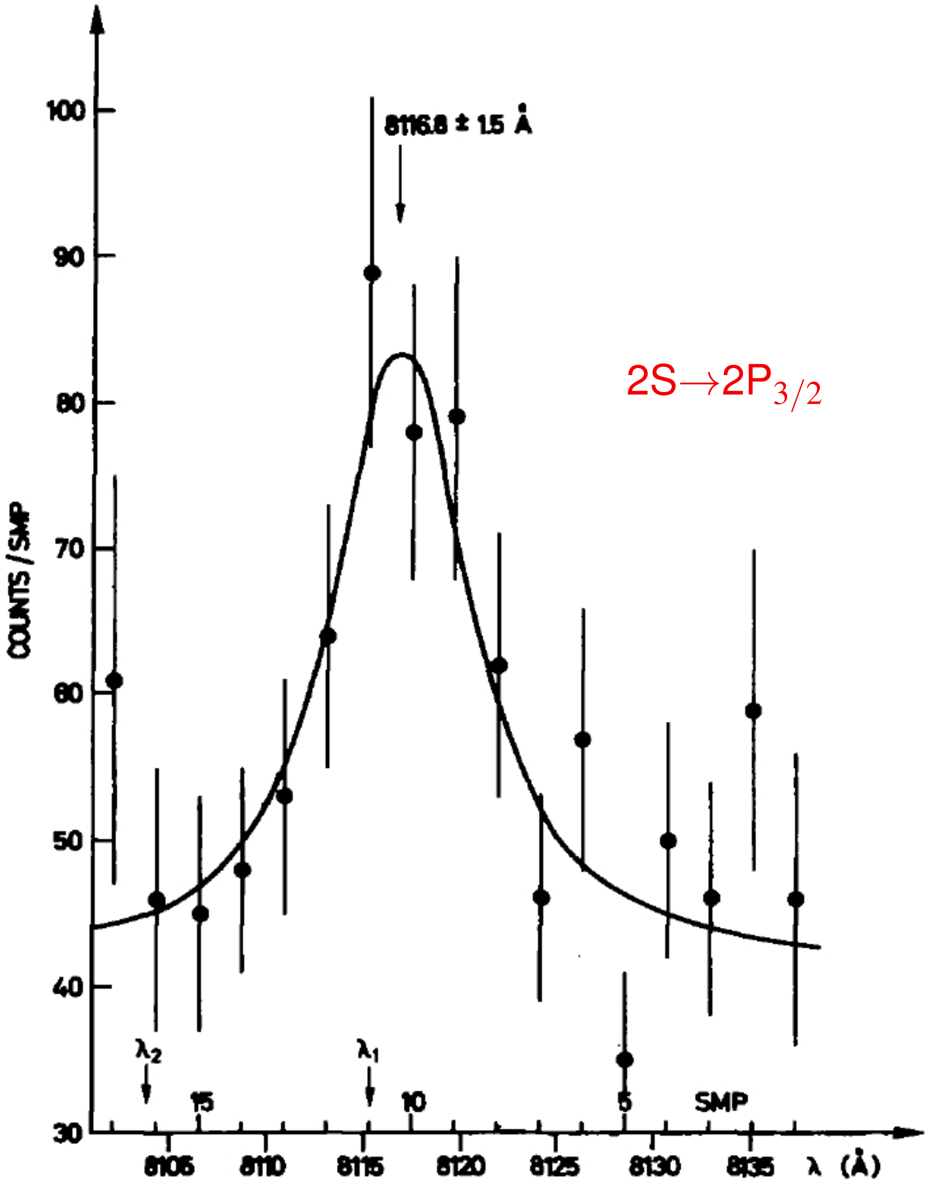
Hydrogen spectroscopy



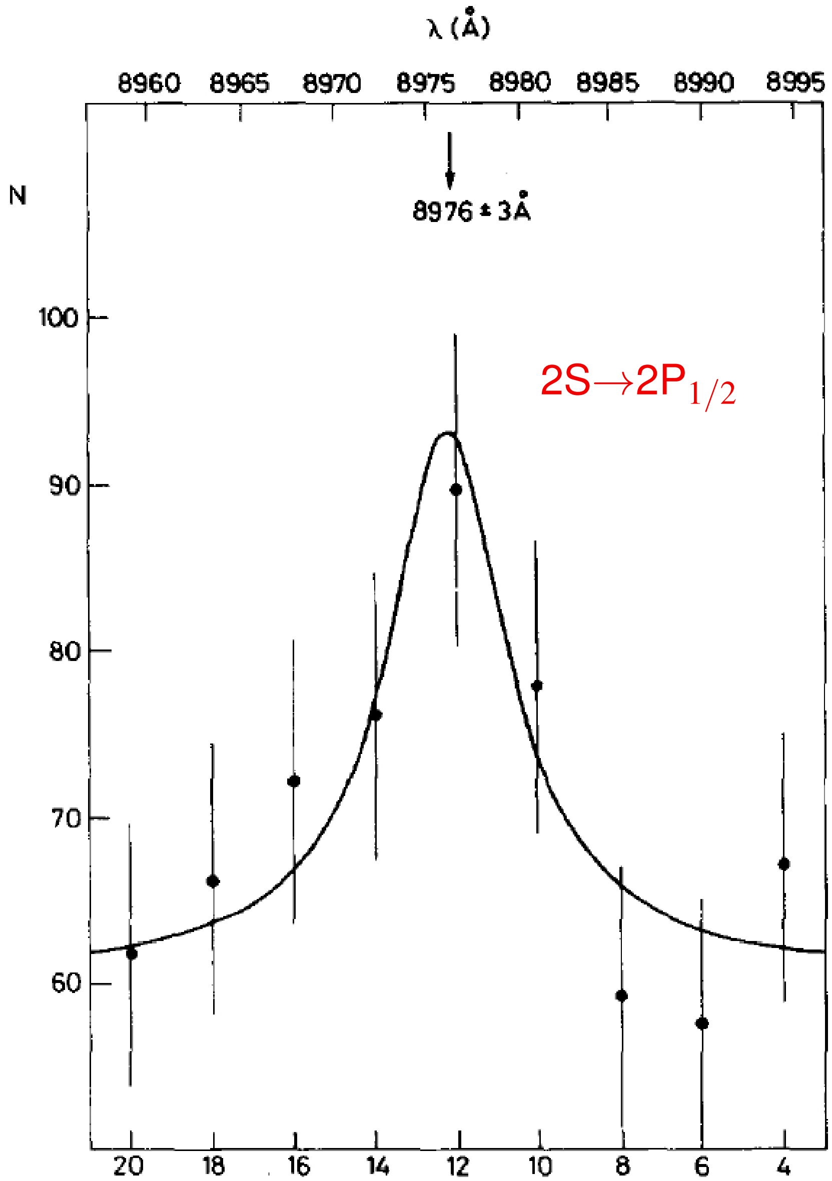
Hydrogen spectroscopy



Old μHe^+ resonances

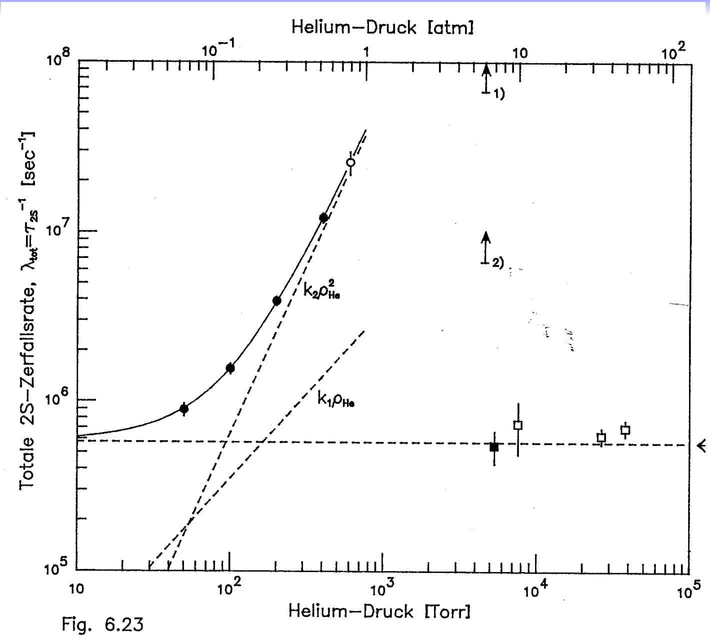


Carboni et al, Nucl. Phys. A273, 381 (1977)



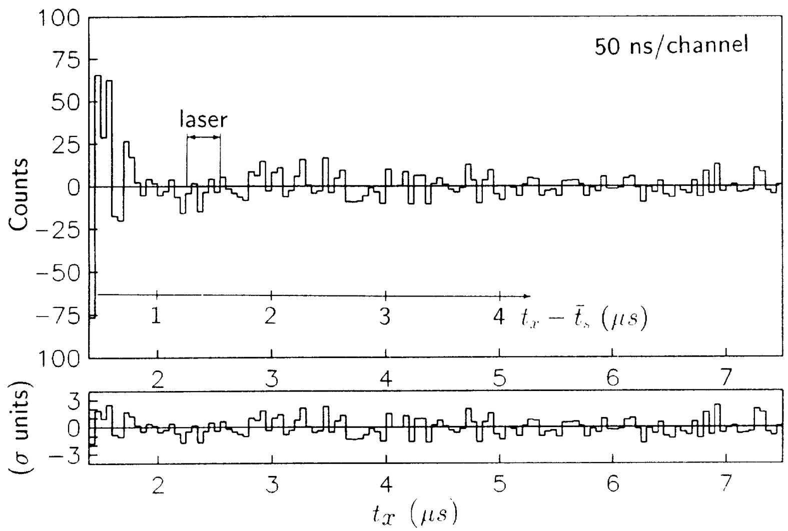
Carboni et al, Phys. Lett. 73B, 229 (1978)

μHe^{+}(2S) lifetime

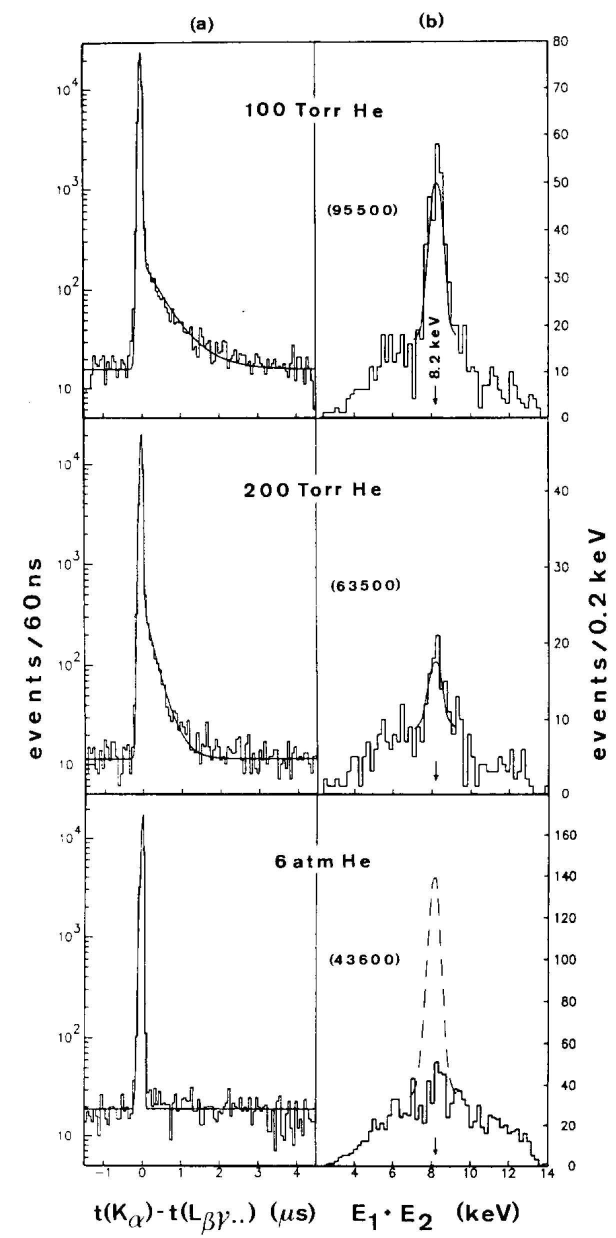


laser exp.:

Dittus, PhD thesis ETH Zurich (1985)



Hauser *et al.*, PRA 46, 2363 (1992)



von Arb *et al.*, PLB 136, 232 (1984)

1st resonance in muonic He-4