# **Updated report on the evaluation of**

# **degrees of equivalence in regional dosimetry comparisons**

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This report is a revision of that presented in 2007 as CCRI(I)/07-04, with two main changes: (i) removal of the evaluation of pairs of degrees of equivalence, as these are no longer published in the KCDB, and (ii) advice on the evaluation of *u*tr (formerly *u*stab) and *u*LINK, including the treatment of data for which the observed variations are larger than predicted from the relevant uncertainties.

The analysis assumes in the first instance a single linking laboratory, LINK, and a regional comparison of a quantity, which we take to be air-kerma rate, by measurement of the calibration coefficient *NK* for a single transfer chamber. The link to the BIPM is by means of an existing BIPM international comparison by the LINK for this quantity. The analysis is then generalized to multiple transfer chambers and linking laboratories.

The aim is a simplified analysis resembling the analysis that would be made for a comparison with the BIPM rather than linked through a regional comparison. To this uncertainty two components are added, *u*tr arising from the transfer chambers and *u*LINK arising from the linking mechanism. The result is a relatively straightforward analysis that is appropriate for the level of uncertainty of the dosimetric input data.

# **1. Evaluation of degrees of equivalence**

The degree of equivalence for each of *n* participating laboratories *i*  = 1 to *n* (excluding the LINK, which by definition cannot obtain a new degree of equivalence) is defined as the difference *Di* = *Ri* – 1 and its expanded (*k* = 2) uncertainty *Ui* = 2*uR*,*i*, where[[1]](#footnote-1)

 (1)

and we take[[2]](#footnote-2)

 (2)

Here, *NK,i* is the transfer chamber calibration coefficient for laboratory *i* (expanded in terms of air-kerma rate and ionization current as *Ki*/*Ii*) and *ui* is its combined standard uncertainty. Similarly, for the LINK result in the regional comparison we have *NK*,LINK. The ratio *R*LINK,BIPM represents the result of the LINK in the corresponding BIPM international comparison (available in the KCDB), in which *u*BIPM is the combined standard uncertainty of the BIPM standard as described in the relevant comparison report.

The two expansions on the right-hand side of equation (1) express, respectively, a direct or indirect comparison with the BIPM, from which we anticipate cancellation of the non-statistical uncertainties for *K*LINK in both cases and, for an indirect comparison, cancellation also of those for *I*LINK at the expense of introducing those for *I*BIPM[[3]](#footnote-3). The superscripts ‘reg’ and ‘inter’ for the LINK identify the regional and BIPM international comparison, respectively. The uncertainties *u*tr and *u*LINK are discussed in Section 2.

The summation in equation (2) contains those components *fjui,j* and *fj u*BIPM,*j* that are correlated between laboratory *i* and the BIPM, with correlation factor *fj*. The physical constants that enter in the air-kerma determinations are fully correlated (*fj* = 1); certain correction factors, for example *k*wall, might be considered partially correlated (0 < *fj* <1). When laboratory *i* is traceable to the BIPM, the summation contains all of the non-statistical components of *u*BIPM, each with correlation factor *fj* = 1. The case of a laboratory *i* traceable to the LINK might be noted; the LINK Type B uncertainties will then enter three times in equation (5). As always, they will cancel for *NK*,LINK and *R*LINK,BIPM, which means that they will remain for *NK*,*i* so that in the end this case is no different.

For a comparison of absorbed dose to water standards the equations are the same, but the correlated components *j* and the correlation factors *fj* are different, depending on whether a laboratory is traceable to a graphite calorimeter, to a water calorimeter, or to an ionometric standard. An example analysis, with choices for *fj* relating to mass-energy absorption coefficients and to the heat defect in water calorimetry, can be found in [1].

# **2. The uncertainties *u*tr and *u*LINK**

The uncertainty *ui* for a given laboratory *i* will normally already include statistical components consistent with the repeatability of transfer chamber calibrations, assuming a ‘well-behaved’ transfer chamber. Under these conditions, the additional component *u*tr in equation (2) is not required.

The behaviour of the transfer chamber should be characterized by the pilot laboratory through the ‘adjusted’ standard deviation of its *m* repeat calibrations *NK*,pilot,*j* (*j* = 1 to *m*);

(3)



where *NK*,pilot is the mean of the *NK*,pilot,*j*[[4]](#footnote-4). If there is reason to believe that the transfer chamber is not ‘well-behaved’ (notably, if equation (3) yields a value that is greater than the normal repeatability at the pilot laboratory) then the value of *u*tr from equation (3) should be included in equation (2). Furthermore, if the pilot laboratory measurements indicate a statistically-significant jump or drift in the response of the transfer chamber, then the ratios *NK*,*i* / *NK*,LINK entering in equation (1) must be corrected for this effect by a case-specific analysis (or the transfer chamber rejected if others have been used).

Regarding *u*LINK, as noted above the nature of the linking mechanism is such that many of the non-statistical components for the LINK cancel in the linking equation (1). However, the remaining non-statistical components are not always straightforward to estimate. This is among the reasons for using more than one LINK when possible, as this allows an alternative estimate of *u*LINK to be made from the difference between the results for different LINKs. Even so, the number of LINKs will rarely exceed two and therefore the latter estimate will never be robust.

Table 1 presents the components to be included in *u*LINK, depending on whether the BIPM comparison with the LINK was direct (both primary standards at one laboratory) or indirect (exchange of transfer instruments). Note that the statistical uncertainties for *I*LINK include chamber positioning and any other aspect of repeatability stated by the LINK. A value of 2 in the table indicates that the uncertainty should be included twice because the instrument was measured in both comparisons.

Not included in the table are uncertainties that might arise if reference conditions are not maintained for the LINK measurements during the two comparisons, for example if the LINK uses different values for distance or field size in the two comparisons. These can be difficult to estimate and are best avoided by using the same conditions in both.

**Table 1. Uncertainty components to be included in *u*LINK. A value of 2 indicates that the component should be included twice.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Direct BIPM comparison | | Indirect BIPM comparison | |
| Parameter from equation (1) | statistical | non- statistical | statistical | non- statistical |
| *K*LINK | 2a | - | 2a | - |
| *I*LINK | 1a | 1 | 2a | - |
| *I*BIPM | - | - | 1b | 1b |

a In the unlikely event that *u*tr is derived using equation (3) *and* the pilot laboratory is the LINK, each of these components should be reduced by 1 because they are already included in *u*tr.

b These are required because *u*BIPM in equation (2) is for the air-kerma rate determination *K*BIPM only. Typically, the combined value will be around 0.03 % and can be found in the relevant comparison report for the LINK.

**3. Multiple transfer chambers**

We consider a comparison using *p* transfer chambers (*j* = 1 to *p*), for which each of the *n* laboratories (*i* = 1 to *n*) has the *p* results *NK,i,j* and the LINK has the results *NK*,LINK,*j*. Relation (1) gives rise to the *np* values *Ri,j* with mean value for each laboratory

(4)



The standard uncertainty of each mean *Ri* gives information on the uncertainty arising from the transfer chambers,

(5)



This value for *u*tr can be used in equation (2) for the case of multiple transfer chambers. Equation (5) reflects the improvement in uncertainty that can be achieved by the use of multiple transfer chambers (as well as the advantage of redundancy in the event of failure of a transfer chamber). However, this improvement is only realized if the spread of the transfer chamber results is consistent with the reproducibilities of the individual chambers.

**4. Multiple linking laboratories**

A comparison can be made more robust, and in particular *u*LINK can be more reliably determined, by including more than one LINK. We consider a comparison with *q* linking laboratories (*k* = 1 to *q*) and a single transfer chamber. In this case relation (1) gives rise to the *nq* values *Ri,k* and Section 2 will yield the *q* estimates *u*LINK,*k*. Given the difficulty sometimes encountered in making a robust estimate of *u*LINK,*k*, an alternative estimate can be made from the spread of the *Ri,k* for different linking laboratories.

We evaluate *Ri* as the unweighted mean of the *Ri,k*

(6)



and make a first estimate of *u*LINK as

(7)



The best estimate of *u*LINK to be used in equation (2) is that derived from equation (7) or from

 (8)

whichever is the larger. This approach prevents fortuitous agreement from resulting in an unrealistically low value for *u*LINK.

**5. General case**

We consider the general case of *n* laboratories (*i* = 1 to *n*), *p* transfer chambers (*j* = 1 to *p*) and *q* linking laboratories (*k* = 1 to *q*), yielding the *npq* values *Ri,j,k* with *q* linking uncertainties *u*LINK,*k*. Treatment of these data can follow sequentially the analyses presented above. The method described in Section 3 can be used to derive the mean value

(9)



for each laboratory *i*, as determined using each linking laboratory *k*, along with the combined *u*tr derived using

(10)



Following Section 4, we can then use these *Ri*,*k* in equation (6) to evaluate the *Ri* and use the *u*LINK,*k* to obtain the combined linking uncertainty *u*LINK as the maximum value from equations (7) and (8). Finally, *u*tr and *u*LINK are inserted in equation (2) to yield the combined *uR,i* for each *Ri*.

## References

[1] P. J. Allisy-Roberts and D. T. Burns, Summary of the BIPM.RI(I)-K4 comparison for absorbed dose to water in 60Co gamma radiation, *Metrologia* **42** (2005) *Technical Supplement* 06002.

1. To simplify the notation, air-kerma rate is denoted by *K* without a dot above. [↑](#footnote-ref-1)
2. For ease of presentation, equations are expressed as variances while the text refers to standard uncertainties. [↑](#footnote-ref-2)
3. The term ‘statistical’ is used to refer to those uncertainty components that result in variations for repeat measurements using a transfer chamber, and ‘non-statistical’ to those that are not revealed by repeat measurements. This classification does not necessarily correspond to that supplied by each laboratory as Type A and Type B components. [↑](#footnote-ref-3)
4. It can be shown empirically that the term (*m*–1.4) in the denominator is a better choice than (*m*–1) to estimate the standard deviation for low values of *m*, including *m* = 2. [↑](#footnote-ref-4)