Bureau International des Poids et Mesures

Guide on

Secondary Thermometry

Thermistor Thermometry

Consultative Committee for Thermometry
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Thermistor Thermometry

CONTENTS

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ABSTRACT

This guide summarises the typical performance characteristics and sources of uncertainty for thermometers based on Negative Temperature Coefficient (NTC) thermistors. The discussion includes a brief summary of the principles of operation, typical resistance-temperature characteristics, instrumentation, limitations in performance, stability, temperature range, and calibration equations. The document concludes with two calibration examples demonstrating uncertainty calculations and giving an indication of the potential accuracy of thermistors.

1. Introduction

Thermistors (thermal resistors) are temperature-sensitive semiconducting ceramic devices. While there are several types used in a very wide variety of applications, in the discussion that follows we are primarily interested in the negative temperature coefficient (NTC) thermistors designed and manufactured specifically for temperature measurement in the range –80 ºC to +300 ºC. For advice on thermistors used at low temperatures refer to the technical literature on cryogenic sensors, e.g. BIPM [1990].

The main advantages of thermistors are very high sensitivity (typically ten times that of platinum resistance thermometers), small size (some smaller than 0.2 mm), and fast time constants (some as short as a few milliseconds). Disadvantages include very high non-linearity, a limited temperature range, and a risk of high self-heating due to the sensing current. Traditionally, thermistors have had a reputation for instability, but thermistors are now readily available for temperature ranges within –20 °C to 60 °C with stabilities of a few tenths of a millikelvin per year. For applications in this range, their stability, high sensitivity and simple instrumentation enable a short-term measurement accuracy approaching that of standard platinum resistance thermometers (SPRT), but at a much lower cost.

Reviews of the properties and applications of NTC and PTC (positive temperature coefficient) thermistors can be found in Sachse [1975] and Hyde [1971], with simpler overviews in McGee [1988], Michalski *et al.* [1991], and White and Sappoff [2014]. The physics of semiconductors is described in Sze [1981].

2. Principle of operation

NTC thermistors are manufactured from mixtures of metal oxides heated to high temperatures to form a polycrystalline ceramic. Ordinarily, the individual oxides have a large energy gap between the full conduction bands and the empty valence bands, so the electrons are unable to move, and the oxides are electrical insulators. The mixture, however, has intermediate electronic states that make the ceramic a semiconductor. As the temperature increases, increasing numbers of electrons gain sufficient thermal energy to reach the higher energy states, thereby creating larger numbers of mobile holes and electrons, in the valence band and the conduction band, respectively. The total number of charge carriers, n, depends on the energies of the electrons as determined by the Boltzmann distribution, and on the density of electronic states near the band edges:

$$n \propto T^{3/2} \exp(-E_g / 2kT)\,, \tag{1}$$

where E_g is the energy required for the carriers to jump to the higher-energy states, k is Boltzmann’s constant and T is temperature (in kelvin). The leading $T^{3/2}$ term in Equation (1) is due to the electronic density of states at the band edge, and tends to be compensated by a similar $T^{-3/2}$ term in the carrier mobilities (due to the density of states for phonons that scatter the carriers). The combination of the effects gives an overall resistance-temperature behaviour that is well approximated by

$$R(T) = R_0 \exp(E_g / 2kT) , \tag{2}$$

where R_0 is a constant. Usually Equation (2) is rewritten in the form

$$R(T) = R(T_0)\exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right], \tag{3}$$

where T_0 is some convenient reference temperature, often 298.15 K (25 °C). The parameter β is a characteristic of the thermistor material with typical values in the range 2000 K to 6000 K. Figure 1 plots the resistance-temperature characteristic for a range of commercially available thermistors. Figure 2 plots the β values for the same set of thermistors and shows the typical variation in β with temperature.

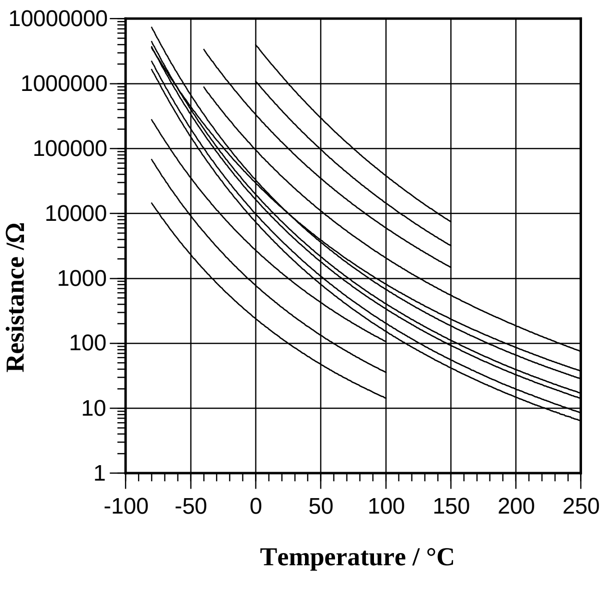

Figure 1. The resistance-temperature characteristics for a range of thermistors showing the typical resistance and temperature ranges.

The sensitivity, S, of a thermistor is the fractional change in resistance for a 1 °C temperature change, and is given by

$$S = \frac{1}{R}\frac{dR}{dT} = \frac{d\ln(R)}{dT} = -\frac{\beta}{T^2} . \tag{4}$$

This constant occurs frequently in error and uncertainty analyses for thermistors. Figure 3 shows the sensitivities for the thermistors of Figure 1. At room temperature they range between about –0.03/ºC and –0.05/ºC. That is, thermistors are about 10 times more sensitive to temperature than platinum resistance thermometers (S for PRTs is approximately equal to the alpha value, $\alpha = 3.85\times10^{-3}$/°C).

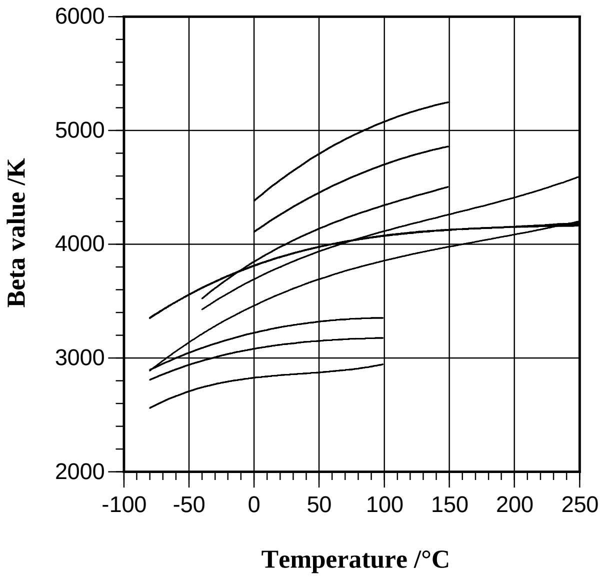

Figure 2. The β values for the thermistors of Figure 1. Some of the thermistors have the same β value.

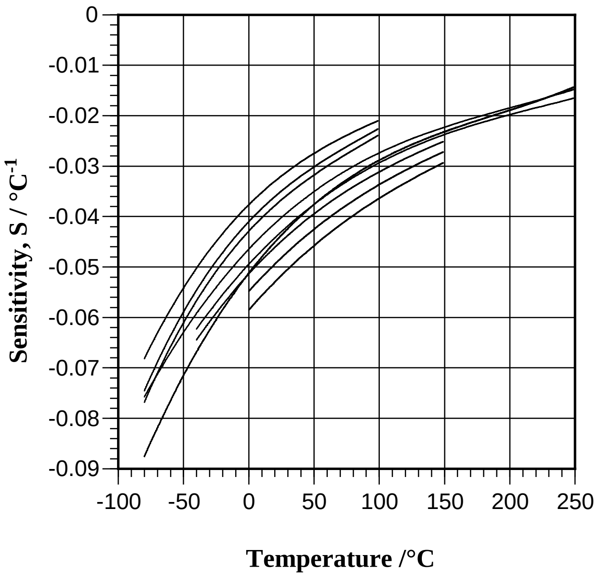

Figure 3. The sensitivities, Equation (4), as a function of temperature for the thermistors of Figure 1.

3. Instrumentation

High sensitivity and high resistance are the main advantages of thermistors. The high resistance enables circuits operating at higher voltages and, in combination with the high sensitivity, ensures the errors that normally affect dc resistance measurements, such as thermoelectric effects and offset voltages, are much reduced. By exploiting these properties, it is relatively simple to make thermistor measurements yielding accuracies approaching 1 mK without, for example, the need for the expensive ac bridges normally associated with high-accuracy platinum resistance thermometry. Thermistors are also available with selected performance characteristics, including interchangeability to 0.05 °C, aged and selected for stability, as well as qualification for military and space applications.

The high sensitivity of thermistors comes at the price of high non-linearity and a wide range of resistance values for modest changes in temperature. A wide variety of measurement circuits have been developed to linearise thermistor responses, including: simple one-resistor circuits, which yield a linearity within 0.1 ºC over ranges of 20 ºC or so [Beakley 1951]; multiple thermistor and resistor circuits, which are linear to within 0.02 °C for ranges up to 100 ºC [Trolander *et al.* 1972, Renneberg and Lehman 2007]; as well as solutions based on antilog-amplifiers or threshold oscillator circuits (for an example incorporating several features see [Kaliyugavaradan *et al.* 1993]). In general, however, the most accurate and wide-range measurements involve direct measurements of the thermistor resistance, e.g. using digital multimeters.

4. Limitations in performance

This section summarises the most significant factors limiting the thermometric performance of NTC thermistors. For some of the subsections we provide a numerical example based on a thermistor with:

- a nominal resistance of R(25 ºC) = 10 kΩ;
- a nominal beta value of β = 3600 K;
- a thermal resistance in stirred oil of ρ = 125 ºC/W
  (dissipation constant = 8 mW/ ºC);
- a constant sensing current of I = 10 µA;
- an operating temperature range of 0 ºC to 50 ºC.

Table 1 summarises the sensitivity (Equation (4)) at 0 ºC, 16.67 ºC, 33.33 ºC, and 50 ºC, as well as the equivalent resistance and voltage sensitivities for this thermistor. Note, particularly, the wide range of values for the sensitivities that is characteristic of thermistor circuits.

Table 1. The various sensitivity coefficients for the example thermistor, where *t* indicates the temperature in °C.

t **ºC**	***R*(*t*)** **Ω**	**Sensitivity *S*** **ºC^{-1}**	**S_R = d*R*/d*T*** **Ω/ ºC**	**S_V = *I* d*R*/d*T*** **mV/ ºC**
0.00	30 196	−0.0483	−1450	−14.5
16.67	14 149	−0.0429	−607	−6.07
33.33	7 202	−0.0383	−276	−2.76
50.00	3 929	−0.0345	−135	−1.35

4.1. Voltage resolution

Typically, a thermistor-resistance measurement involves a voltage measurement, the accuracy of which may be limited by the resolution of a voltmeter, or the input-offset voltage and input bias currents of an operational amplifier. If the uncertainty in the voltage measurement is u_V, then the uncertainty in the temperature measurement is

$$u_T = \frac{u_V}{|S_V|} = \frac{T^2}{\beta}\frac{u_V}{IR}, \qquad (5)$$

where S_V is the voltage sensitivity (Column 5 of Table 1). For the example thermistor and a standard uncertainty in the voltage measurement of 10 µV, the temperature uncertainties at 0 ºC and 50 ºC are 0.68 mK and 7.4 mK, respectively. The uncertainty in the temperature measurement increases very rapidly with increasing temperature because of the combination of the T^2 term and the falling thermistor resistance.

4.2. Self heating

In order to measure a resistance, a current must be passed through the thermistor, which dissipates heat and results in a small temperature increase called the self-heating error. The self-heating error is proportional to the power dissipated and the thermal resistance between the thermistor and its environment:

$$\Delta T_{sh} = I^2 R(T)\left(\rho_{\text{int}} + \rho_{\text{ext}}\right) = \frac{V^2}{R(T)}\left(\rho_{\text{int}} + \rho_{\text{ext}}\right), \qquad (6)$$

where I and V are the sensing current and the voltage across the thermistor, respectively, and ρ_{int} and ρ_{ext} are the internal and external thermal resistances associated with the thermistor and its surroundings. The internal thermal resistance depends on the dimensions of the thermistor and the material from which it is made, while the external thermal resistance depends on the thermal conductivity (and velocity and viscosity if a fluid) of the medium in which the thermistor is immersed.

Self-heating can be a serious problem for measurements made over a wide temperature range. If a constant sensing current is used, the power dissipated (I^2R) at low temperatures becomes large, and if constant voltage excitation is used, the dissipated power at high temperatures (V^2/R) becomes a problem.

Typical values of the total thermal resistance may vary from 50 °C/W to 2000 °C/W in stirred oil, but are very dependent on the environment so may vary more than 100 times between still air and stirred water. The thermal resistance for the thermistor is commonly expressed as the dissipation constant, which is the power required to raise the thermistor temperature 1 °C, and is often expressed in units of mW/°C. The dissipation constant is the reciprocal of the thermal resistance, so the above range of thermal resistances corresponds to dissipation constants of 0.5 mW/°C to 20 mW/°C. For the example thermistor, which is driven by a constant current, the self-heating error is a maximum at low temperatures. For the example thermistor, at 0 °C and in a stirred oil bath, the error is 0.4 mK, which is quite low. However, if the sensing current is ten times to 100 mA, then the self-heating would be 40 mK. Note that where thermistors are calibrated and used in similar thermal environment so that the self-heating is similar, then there effect of the error is much less.

In many applications, the part of the thermal resistance due to the environment may fluctuate due to turbulence in the air or stirred fluids. This causes the self-heating error to fluctuate and can limit the resolution of thermistors, for example, when used in precision temperature controllers or differential thermometers.

4.3. Stray thermal influences

The generally high thermal resistance between the thermistor and the environment means that thermistors are prone to stray thermal influences from either infrared radiation or heat conducted along the lead wires to the thermistor. This may be exacerbated by the poor thermal design of many commercial thermistor probes, and is a particular problem when thermistors are used to measure air or surface temperatures. In these cases, some care should be taken to thermally anchor the lead wires. For air temperature measurements, the lead wires should be long enough or thermally anchored to another object at the same temperature as the air, so that the leads adjacent to the thermistor come to thermal equilibrium with the air. With surface temperatures, a sufficient length of the leads must be thermally anchored to the surface.

4.4. Lead resistance

The high sensitivity and high resistance of thermistors means that for many measurements, a 2-wire resistance measurement is satisfactory and provides a useful simplification. However, the lead resistances, when neglected, can become a problem at higher temperatures when the thermistor resistance is low. The error due to lead resistance R_L is

$$\Delta T_L = \frac{R_L}{S_R} = -\frac{T^2}{\beta}\frac{R_L}{R}, \quad (7)$$

where S_R is the resistance sensitivity for the thermistor (Column 4 of Table 1). The error is largest at high temperatures. For the numerical example with a 1 Ω lead resistance, the error at 50 °C is 7.4 mK.

4.5. Insulation resistance

At low temperatures, the thermistor resistance becomes very large, often greater than 10 MΩ. In such applications, care should be given to the insulation on the connecting leads to ensure that the insulation resistance does not shunt the measuring current. The temperature error due to an insulation resistance R_{ins} is

$$\Delta T_{\text{ins}} \approx -\frac{1}{S}\frac{R}{R_{\text{ins}}} = \frac{T^2}{\beta}\frac{R}{R_{\text{ins}}}, \quad (8)$$

which increases in proportion to the thermistor resistance (decreasing temperature). For the example thermistor, an insulation resistance of 100 MΩ at 0 °C causes an error of 6.2 mK.

4.6. Stability

There are several intrinsic effects causing instability in thermistors [Zurbuchen and Case 1982], including: mechanical cracking of the thermistor body with temperature cycling, drift at high temperatures due to ingress of atmospheric gases, changes in the crystallographic structure, and changing contact resistance between the leads and the thermistor body. The most stable thermistors are bead thermistors encapsulated in glass. Within the range −20 °C to 60 °C, selected and pre-aged thermistors may be stable to better than a few tenths of a millikelvin per year. Thermistors with resistances in the range 2 kΩ to 10 kΩ also appear to be the most stable. Glass-encapsulated disc thermistors and epoxy-encapsulated bead thermistors are also very good but not quite as stable as the glass beads. (See stability studies by Siwek *et al.* [1992a], Wise [1992], Edwards [1983], La Mers *et al.* [1982], Wood *et al.* [1978], and Strouse *et al.* [2012].)

The extrinsic sources of instability relate to the instrumentation: insulation resistances, stability of the measuring current and stability of the electronic components used in the measurement.

4.7. Temperature range

The temperature range of commercially available thermistors varies from 1 K to over 1000 °C, and depends on the particular metal oxides used, and on the encapsulation. The most stable thermistors for temperature measurement have a much restricted range: glass-encapsulated thermistors have a range of about −80 °C to 300 °C, while epoxy-encapsulated thermistors have an upper temperature limit of about 150 °C (see Figure 1 for examples of the temperature ranges).

The temperature range over which a thermistor is used has a significant effect on the non-linearity in the temperature measurements. Whether using linearising instrumentation or direct measurement and calibration equations, there is a residual error that increases in proportion to ΔT^N, where ΔT is the temperature range and N is the number of adjustable parameters in the instrumentation or the number of parameters in the calibration equation. Beakley [1951] shows an example of a linear circuit with a residual non-linearity scaling as ΔT^3, and Figure 4 below plots the interpolating errors in calibration equations, which scale as ΔT^N.

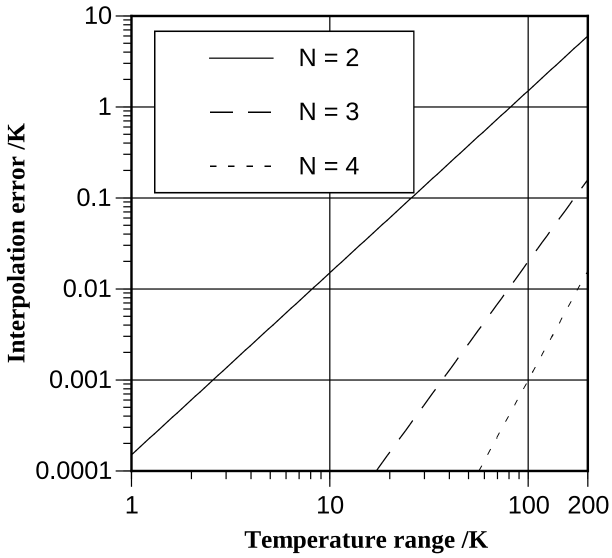

Figure 4. Summary of interpolation errors, expressed as standard uncertainty, versus the number of terms used in Equation (9). The lines are indicative only and are based on a thermistor with a β value of 3800 K. The error will tend to be greater for temperatures below 0 °C, for thermistors with a high β value, and may vary by a factor of 2 or more for different thermistors.

5. Calibration equations

A variety of calibration equations have been used for thermistors, including Equation (3). For high-accuracy applications, there are two extended series expansions of Equation (3) in common use:

$$\frac{1}{T} = A + B\ln(R/R_0) + C\left[\ln(R/R_0)\right]^2 + D\left[\ln(R/R_0)\right]^3 + \ldots, \quad (9)$$

and

$$\ln(R/R_0) = A + \frac{B}{T} + \frac{C}{T^2} + \frac{D}{T^3} + \dots , \tag{10}$$

where R_0 is a convenient reference resistance, e.g. 1 Ω or 1 kΩ, depending on the measurement units. The number of terms in the equations, N, is chosen according to the temperature range and the accuracy required. Most commonly, the equations are used with two or four terms, but sometimes up to five terms.

Equation (9) has the advantage over most other calibration equations of giving temperature directly from the measured resistance. Both Equations (9) and (10) are linear in the coefficients and hence amenable to least-squares fits and uncertainty analysis. Figure 4 summarises the performance of the various versions of Equation (9). Note that the $N = 2$ case corresponds to Equation (3).

Steinhart and Hart [1968] recommended Equation (9) with $N = 3$ but with the second-order term omitted ($C = 0$, $D \neq 0$), and this equation is commonly recommended by thermistor manufacturers. However, the original recommendation for the equation was based on a numerical error, see Bennett [1971], and in practice, the performance of the Steinhart-Hart equation is only sometimes better than the normal 3-term equation with $D = 0$. The four-term version of Equation (9) is always more accurate than the Hart-Steinhart equation. Additionally, the accuracy of the Steinhart-Hart equation depends on the R(25 ºC) value of the thermistor and on the choice of R_0, so its performance is much less predictable than the versions of Equation (9) summarised in Figure 4. The performance of the various calibration equations has been investigated by Bennett [1971], Siwek *et al.* [1992b], Sapoff *et al.* [1982], and Chen [2009].

The propagation of uncertainties with Equation (9) is complicated, but when used as an interpolating equation, i.e., using just N calibration points to determine the N constants $\{A, B, C, \dots\}$, the total uncertainty in the measured temperature can be calculated in terms of the various calibration uncertainties [White and Saunders 2007],

$$u_T^2 = \sum_{i=1}^{N} \left[l_i^2(T) \left(\frac{T}{T_i} \right)^{6-2N} \left(u_{T_i}^2 + \frac{T_i^4}{\beta^2} \frac{u_{R_i}^2}{R_i^2} \right) \right] + \frac{T^4}{\beta^2} \frac{u_R^2}{R^2} , \tag{11}$$

where (T_i, R_i) are the temperature and resistance measurements for each calibration point, (T, R) are the resistance and temperature at the unknown (measured) temperature, and $l_i(T)$ are Lagrange polynomials:

$$l_i(T) = \prod_{j=1, j \neq i}^{N} \frac{(T - T_j)}{(T_i - T_j)} . \tag{12}$$

Equation (11) has the curious property that, independent of the number of calibration points, the uncertainties associated with the calibration measurements always extrapolate to higher temperatures as T^2 (this is not true of extrapolations to lower temperatures). Examples of the application of Equation (11) for uncertainty analysis are given in the next section.

6. Calibration examples

6.1. A narrow-range thermometer (*N* = 2)

This section summarises the calibration and uncertainty analysis for a narrow-range thermistor thermometer operating between 15 °C and 25 °C with an expected accuracy of about 0.1 °C. Figure 4 shows that for a 10 °C range, the standard deviation of the interpolation error with two calibration points ($N = 2$) is expected to be about 0.02 °C, so the error is small enough to be neglected. The appropriate calibration equation, Equation (9) with $N = 2$, is

$$\frac{1}{T} = A + B\ln(R)\,, \tag{13}$$

where the resistance is measured in ohms ($R_0 = 1\ \Omega$). By measuring the resistance at two temperatures we get two points, (T_1, R_1) and (T_2, R_2), to fix the values of A and B. If the calculated values of A and B are substituted back into (13), the calibration equation can be rearranged in the form of a Lagrange interpolation:

$$\frac{1}{T} = \frac{1}{T_1}\left(\frac{\ln(R)-\ln(R_2)}{\ln(R_1)-\ln(R_2)}\right) + \frac{1}{T_2}\left(\frac{\ln(R)-\ln(R_1)}{\ln(R_2)-\ln(R_1)}\right). \tag{14}$$

Although more complicated than Equation (13), this equation explicitly includes all of the calibration data, R_1, R_2, T_1, T_2, and the measured resistance, R, from which an unknown temperature, T, is determined, and is a better starting point for an uncertainty analysis.

Partial differentiation of Equation (14) with respect to all five of the measured variables, followed by replacement of the terms in $\ln(R)$ using the approximation

$$\ln\left(R(T)\right) \approx \frac{\beta}{T} - \frac{\beta}{T_0} + \ln\left(R(T_0)\right), \tag{15}$$

which follows from Equation (3), leads to the propagation-of-error equation

$$dT = \frac{T}{T_1}\left(\frac{T-T_2}{T_1-T_2}\right)\left(dT_1 - \frac{T_1^2}{\beta}\frac{dR_1}{R_1}\right) + \frac{T}{T_2}\left(\frac{T-T_1}{T_2-T_1}\right)\left(dT_2 - \frac{T_2^2}{\beta}\frac{dR_2}{R_2}\right) - \frac{T^2}{\beta}\frac{dR}{R}\,. \tag{16}$$

This equation relates small changes in the five measured variables, dR_1, dR_2, dT_1, dT_2, and dR, to a change in the measured temperature, dT. Note the presence of the sensitivity, S, from Equation (4) associated with all of the uncertainties in resistance. The propagation-of-uncertainty relation is obtained by calculating the sum of the squares of all of the terms in Equation (16) so that the total uncertainty is

$$u_T^2 = \left(\frac{T}{T_1}\right)^2 \left(\frac{T-T_2}{T_1-T_2}\right)^2 \left(u_{T1}^2 + \frac{T_1^4}{\beta^2}\frac{u_{R1}^2}{R_1^2}\right) + \left(\frac{T}{T_2}\right)^2 \left(\frac{T-T_1}{T_2-T_1}\right)^2 \left(u_{T2}^2 + \frac{T_1^4}{\beta^2}\frac{u_{R2}^2}{R_2^2}\right) + \frac{T^4}{\beta^2}\frac{u_R^2}{R^2}, \qquad (17)$$

where u_x indicates the standard uncertainty in x. This equation is the same as Equation (11) for the case when N = 2. Note that the uncertainties in the measured resistances all appear as relative uncertainties, u_R/R. All of the remaining terms in the equation are functions of temperature only, so that the effects of uncertainties can be calculated easily for any measured temperature. Table 2 below summarises an uncertainty calculation for this example, and Figure 5 plots the temperature dependence of the various contributions to the uncertainty.

Table 2. Summary of data and uncertainty calculations for temperatures measured using a thermistor calibrated at 2 points. All uncertainties are expressed as standard uncertainties (one-sigma or k = 1values). The shaded cells show input data.

Measured parameters	**at T_1**	**at T_2**
Calibration temperature, T_i	288.15 K (15 °C)	298.15 K (25 °C)
Thermistor resistance, R_i,	15 205 Ω	10 000 Ω
Uncertainty associated with temperature		
Uncertainty in calibration temperature reading	0.05 °C	0.05 °C
Uncertainty in calibration bath spatial uniformity	0.03 °C	0.04 °C
Total uncertainty in temperature terms	**0.058 °C**	**0.064 °C**
Uncertainty associated with resistance		
Relative uncertainty in resistance measurement (u_{Ri}/R_i)	0.2%	0.2%
Sensitivity $\left(\beta / T_i^2\right)$	0.0434 °C^{-1}	0.0405 °C^{-1}
Equivalent temperature uncertainty $\left(\frac{T_i^2}{\beta}\frac{u_{R_i}}{R_i}\right)$	**0.046 °C**	**0.049 °C**
Combined Uncertainty	**0.074 °C**	**0.081 °C**

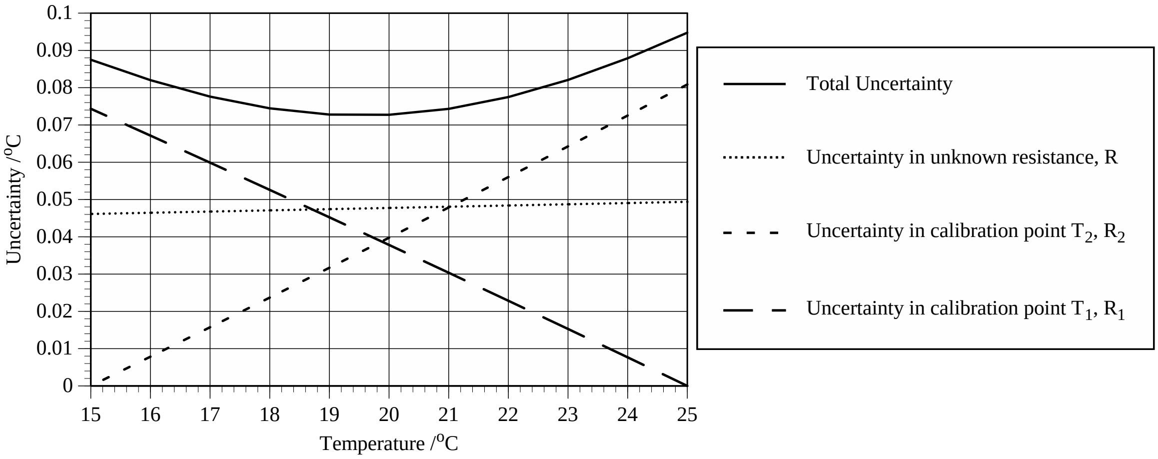

Figure 5. The propagation of uncertainty (k = 1) for a thermistor calibrated at two points. The numerical data is from Table 2, and the propagation equations for the three contributions are from Equation (17).

6.2. A wide-range thermometer (N = 4)

This section summarises the evaluation of Equation (11) for a high-accuracy thermistor operating between 0 ºC and 50 ºC. Figure 4 shows that the four-term version of Equation (9) should be used for the calibration equation. The interpolation errors are then below about 0.1 mK and can be neglected. The three-term equation would lead to interpolation errors of about 2.5 mK.

The thermistor is calibrated using a standard platinum resistance thermometer in a stirred water bath, achieving a standard uncertainty of about 1 mK. The thermistor resistance is measured using a digital voltmeter employing a 4-wire measurement to eliminate lead resistance effects, and achieves a relative uncertainty of about 0.003%. Table 3 summarizes the uncertainty calculation.

The total uncertainty propagated from the calibration is calculated from the sums at the bottom of Table 3, according to Equation (11). Each of the four sums is multiplied, in Equation (11), by one of four Lagrange polynomials. The first Lagrange polynomial is

$$l_1(T) = \frac{(T-T_2)(T-T_3)(T-T_4)}{(T_1-T_2)(T_1-T_3)(T_1-T_4)}, \tag{18}$$

and the other three polynomials l_2(T), l_3(T), and l_4(T) can be found by permuting the indices in Equation (18). Note that $l_1(T)$ is equal to 1 at $T = T_1$, and is zero at the other calibration points (where $T = T_2$, T_3, or T_4), and the other polynomials have similar properties. Figure 6 shows the total uncertainty due to the calibration uncertainties only, i.e., it excludes the additional uncertainty in measured resistance when measuring an unknown temperature (the u_R/R term of Equation (11)). Note too, the rapid increase in uncertainty that occurs when the equation is extrapolated beyond the 0 ºC to 50 ºC calibration range.

Table 3. A summary of the uncertainty contributions for a four-point calibration of a high-accuracy thermistor. All uncertainties are expressed as standard uncertainties (one-sigma values). The thermistor is calibrated at four temperatures equally spaced over the range 0 °C to 50 °C. The shaded cells indicate input data.

Measured parameters	$\boldsymbol{T_1}$	$\boldsymbol{T_2}$	$\boldsymbol{T_3}$	$\boldsymbol{T_4}$
Calibration temperatures, T_i	273.15 K (0.00 °C)	298.81 K (16.66 °C)	306.48 K (33.33 °C)	323.15 K (50.00 °C)
Thermistor resistance, R_i	30 196	14 149	7 202	3 929
Uncertainties associated with temperature				
Standard platinum resistance thermometer including bridge, and standard resistor	0.2 mK	0.5 mK	0.8 mK	1.0 mK
Calibration bath, spatial non-uniformity	0.3 mK	0.3 mK	0.3 mK	0.3 mK
Self-heating	0.2 mK	0.1 mK	0.0 mK	0.0 mK
Stray thermal influences	0.1 mK	0.1 mK	0.1 mK	0.1 mK
Insulation resistance (R_{ins} = 1 GΩ)	0.2 mK	0.1 mK	0.0 mK	0.0 mK
Total of temperature terms $\left(u_{T_i}\right)$	**0.4 mK**	**0.6 mK**	**0.9 mK**	**1.1 mK**
Uncertainties associated with resistance				
Digital multimeter (0.003 % R + 0.1 Ω)	0.9 Ω	0.4 Ω	0.2 Ω	0.2 Ω
Repeatability (includes DMM noise, self-heating fluctuations and calibration bath instability)	0.6 Ω	0.3 Ω	0.15 Ω	0.1 Ω
Total of resistance terms $\left(u_{R_i}\right)$	**1.1 Ω**	**0.5 Ω**	**0.25 Ω**	**0.22 Ω**
Combined Uncertainty $\left(u_{T_i}^2 + \frac{T_i^4}{\beta^2}\frac{u_{R_i}^2}{R_i^2}\right)^{1/2}$	**0.85 mK**	**1.0 mK**	**1.3 mK**	**2.0 mK**

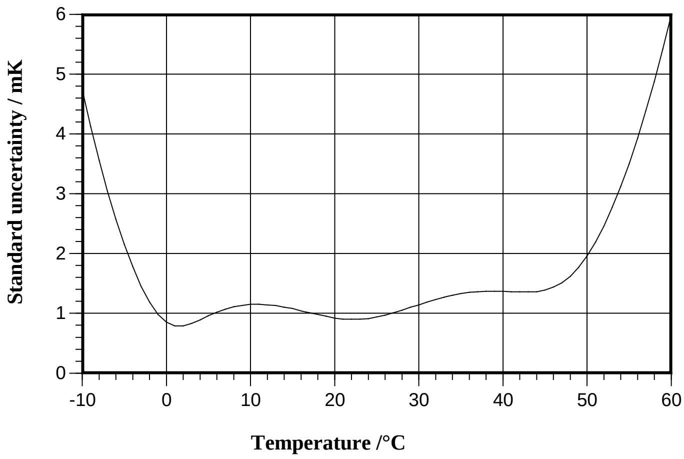

Figure 6. The total standard uncertainty (k = 1) propagated from the calibration measurements of the thermistor using Equation (11), and the data tabulated in Table 3.

In practice, most calibrations are based on a least-squares fit of Equation (9) with to up to 20 calibration points. In that case the total propagated uncertainty may be less than that shown in Figure 6, but the curve will retain the same basic shape; it will be flat over the interpolation range, and rise very quickly where temperature is extrapolated beyond the calibration range. Other examples of thermistor calibrations are given in [Bennet 1971, Chen 2009 and Vaughn *et al.* 2005].

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