CCTF
WGMRA Guideline 5
(Rev. 201509)
The prediction uncertainty of [UTC-UTC(k)]

Introduction

Participating time laboratories provide the evaluation for the prediction uncertainty of [UTC-UTC(*k*)] within the framework of Calibration and Measurement Capabilities (CMCs) and CCTF key comparisons for the key comparison database (KCDB). The time interval for declaring the prediction uncertainty is 20 days and the declared values show a large variation; from 20 ns to 200 ns. A study was initiated as a result of this large variation among the declared values to determine reasonable values for the prediction uncertainty of [UTC-UTC(*k*)] depending on the clock used and the time transfer method. The laboratories should exercise caution when evaluating the prediction uncertainty and to avoid misunderstandings due to the deviation between the values of [UTC-UTC(*k*)] over 20 days.

The purpose of the guideline

The aim of this guideline is that, in the case of revision of CMCs, the value reported for the prediction uncertainty should be coherent with the values reported in the guideline. This guideline is a support for the CMCs revision requested by the CIPM MRA.

In fact, every laboratory should already have a procedure for predicting its timescale and the corresponding uncertainty. This document is a useful guide to check if the uncertainty values are reasonable. If there is a significant different from the values given in this document, the laboratory should be able to provide an explanation.

UTC contributing laboratories should consider this document not a replacement for their procedure to estimate the uncertainty in their prediction of [UTC-UTC(k)].

The basic assumptions

The prediction uncertainty depends on:

- The uncertainty on [UTC-UTC(*k*)] declared in Section 1 of *Circular T*. The total uncertainty $u=\mathrm{sqrt}(u_A{}^2+u_B{}^2)$ is composed by two components, u_A is the standard uncertainty accounting the instability and u_B is the estimated uncertainty of the calibration of the difference between UTC and UTC(*k*).
- The stability property of the atomic clock generating UTC(*k*).

The uncertainties reported in Section 1 of *Circular T* are linked to the link uncertainties reported in Section 6. There are different cases depending on the time transfer method used and the calibration status:

- TWSTFT - calibrated time transfer; global uncertainty between 1 ns to 5 ns
- GPSPPP, P3, MC, SC - calibrated time transfer; global uncertainty between 5 ns to 10 ns

- GPSPPP, P3, MC, SC - un-calibrated time transfer; global uncertainty of ~20 ns.
- Combined techniques - TWGPPP (TW and GPSPPP) and GPSGLN (GPS and GLONASS) calibrated and un-calibrated time transfer; global uncertainty between 1 ns to 20 ns

The instability component of the uncertainty u_A can spread from 0.3 ns to 10 ns depending on the time transfer technique.

There are many papers dedicated to the study of the prediction [1-13] and the related uncertainty. The results reported in this note were obtained following [3].

In this note we consider a time scale generated by using a free running caesium clock and H-masers with typical values for the Allan deviation to give consistent values to the uncertainty prediction. In this note the value of the sampling time τ is 1 day.

First example – Time scale generated by a free running caesium clock

We consider the caesium clock stability with a White Frequency Noise (WFN) from $\sigma_y(\tau)=1\times10^{-14}$ to $\sigma_y(\tau)=7\times10^{-14}$. In such a way we aim to consider two extreme cases. Depending on the stability of the clock used to generate the internal realization of UTC, the declared prediction uncertainty should be consistent with the values declared in this guideline. In addition, for the hydrogen masers we consider two possible ranges of values for the WFN (from $\sigma_y(\tau)=1\times10^{-15}$ to $\sigma_y(\tau)=1\times10^{-14}$) and for the Random Walk Frequency Noise (RWFN) (from $\sigma_y(\tau)=1\times10^{-16}$ to $\sigma_y(\tau)=1\times10^{-15}$).
In Figure 1 as example, the prediction uncertainty (2 sigma) is reported in the case of a free running caesium clock with $\sigma_y(\tau)=4\times10^{-14}$. The results are depending on the prediction time expressed in days, on u and u_A with the reported values in Figure 1.

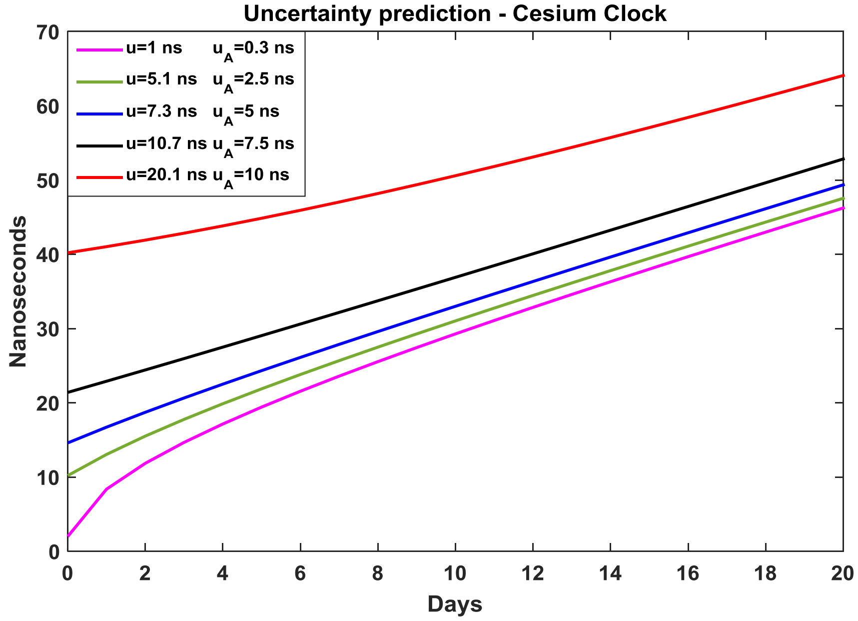

Figure1. Prediction uncertainty (2 sigma) in the case of a free running caesium clock with $\sigma_y(\tau)=4\times10^{-14}$.

Table 1 shows the prediction uncertainties (2 sigma) at 20 days for a time scale obtained by using a free running caesium clock with the stabilities equal to $\sigma_y(\tau)=1\times10^{-14}$ and $\sigma_y(\tau)=7\times10^{-14}$. The results are depending on u and u_A as indicated in Table 1. In the case of $\sigma_y(\tau)=7\times10^{-14}$ only the case of u_A=0.3 ns is reported considering that the results are dominated by the noise of the clock.

	Prediction uncertainty (2 sigma) at 20 days / [ns]					
	$\sigma_y(\tau)=1\times10^{-14}$					$\sigma_y(\tau)=7\times10^{-14}$
u_A[ns] / **u [ns]**	**0.3**	**2.5**	**5**	**7.5**	**10**	**0.3**
1	12					81
5.1	16	16				82
7.3	19	19	21			83
10.7	25	25	26	29		84
20.1	42	42	43	45	46	91

Table 1. The prediction uncertainty (2 sigma) depending on u and u_A at 20 days obtained by using a free running caesium clock with $\sigma_y(\tau)=1\times10^{-14}$ and $\sigma_y(\tau)=7\times10^{-14}$.

By analysing the values reported in this table we can observe the role of the clock generating the internal realization of UTC. Good clock stability has a predominant role in the uncertainty budget. When UTC(*k*) is realized by a clock with good stability the prediction uncertainty is dependent on the time transfer performance, as can be seen in the first part of Table 1. The contribution to the uncertainty of the time transfer is almost negligible when the clock is characterized by a larger instability (last column in table 1).

Second example – Time scale generated by a free running H-Masers

Figure 2 shows as example the results for H-masers with WFN equal to $\sigma_y(\tau)=1\times10^{-15}$, RWFN equal to $\sigma_y(\tau)=1\times10^{-16}$. The results are depending on the prediction time expressed in days, on u and u_A with the reported values in Figure 2.

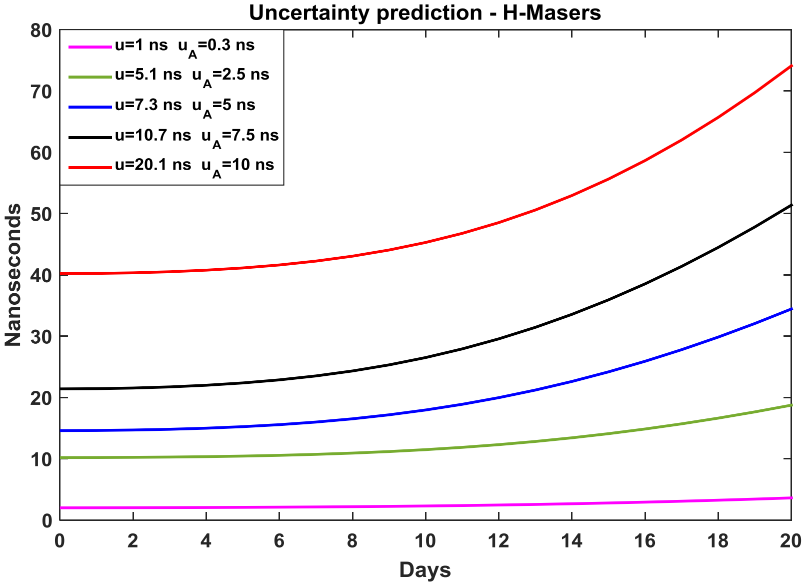

Figure2. Prediction uncertainty (2 sigma) of a free running H-maser with WFN equal to $\sigma_y(\tau)=1\times10^{-15}$, RWFN equal to $\sigma_y(\tau)=1\times10^{-16}$.

Tables 2 and 3 give the prediction uncertainties (2 sigma) at 20 days in the case of a time scale obtained by using a free running H-maser with the stabilities characterized by WFN (from $\sigma_y(\tau)=1\times10^{-15}$ to $\sigma_y(\tau)=1\times10^{-14}$) and RWFN (from $\sigma_y(\tau)=1\times10^{-16}$ to $\sigma_y(\tau)=1\times10^{-15}$). In these tables the results are depending on u and u_A.

Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_{WFN}(\tau)=1\times10^{-14}$ $\sigma_{RWFN}(\tau)=1\times10^{-15}$					
u_A [ns] / **u [ns]**	**0.3**	**2.5**	**5**	**7**	**10**
1	24				
5.1	26	31			
7.3	28	32	42		
10.7	32	36	45	57	
20.1	47	50	57	66	78

Table 2. The prediction uncertainties (2 sigma), depending on u and u_A at 20 days by using an H-maser characterized by the stated frequency stability are reported.

Prediction uncertainty (2 sigma) at 20 days / [ns] $\sigma_{WFN}(\tau)=1\times10^{-15}$ $\sigma_{RWFN}(\tau)=1\times10^{-16}$					
u_A [ns] / **u [ns]**	**0.3**	**2.5**	**5**	**7**	**10**
1	4				
5	11	19			
7	15	22	35		
10	22	27	38	52	
20	41	44	51	62	75

Table 3. The prediction uncertainties (2 sigma), depending on u and u_A at 20 days by using an H-maser characterized by the stated frequency stability are reported.

By analysing the results reported in tables 2 and 3 we can conclude that when a good quality time transfer technique is used, the noise affecting the atomic clocks has a significant impact on the uncertainty prediction but with an un-calibrated technique the results do not depend on the stability of the clock. This is evident by observing the last columns of both the tables reporting almost the same results obtained with very different clocks.

Final remarks and limitation of the method

In this guideline only two examples are considered (free running caesium clock and H-masers) to simplify the explication but a generalization of the mathematical model can be done in case of a time scale generated by an ensemble of atomic clocks. For example in [3] the ensemble of cesium clocks is considered.

Some problems linked to the use of the model were observed [3]. For example:
• the frequency drift, considered constant in the model, can change with time,
• the random noises can be not stationary while they should be stationary in the mathematical model.

For this reason even if, in theory, it would be optimal to consider all the available past data to estimate the deterministic and random parameters, in the real case the use of a not too long previous period of data is advisable. From the experimental data 1 month of past data observation seems a good compromise.

References

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