Photobiological/-chemical actinic functions expressed in terms of photon flux

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Let's start with the actinic function as defined in the present SI-Brochure Appendix 3, part 2:

$$\Phi_{\mathrm{act}} = \int \Phi_{\mathrm{e},\lambda}(\lambda)\, A_{\mathrm{act}}(\lambda)\, \mathrm{d}\lambda \qquad (1)$$

where

- $\Phi_{\mathrm{e},\lambda} = \dfrac{\mathrm{d}\Phi_{\mathrm{e}}(\lambda)}{\mathrm{d}\lambda}$ is the spectral radiant flux, $[\Phi_{\mathrm{e},\lambda}] = \mathrm{W}\cdot\mathrm{nm}^{-1}$
- Φ_{e} is the radiant flux, $[\Phi_{\mathrm{e}}] = \mathrm{W}$
- $A_{\mathrm{act}}(\lambda)$ is the (spectral) actinic function, $[A_{\mathrm{act}}] = 1$
- Φ_{act} is the effective radiant flux $[\Phi_{\mathrm{act}}] = \mathrm{W}$

The actinic function is usually normalized to unit at wavelength $\lambda_{\mathrm{m,act}}$ of highest efficacy.

Let's develop equation (1) in terms of the photon flux Φ_{p}

$$\Phi_{\mathrm{p}} = \frac{\mathrm{d}N_{\mathrm{p}}}{\mathrm{d}t}\,, \qquad (2)$$

where $\mathrm{d}N_{\mathrm{p}}$ number of photons with the time interval $\mathrm{d}t$, NB $[\Phi_{\mathrm{p}}] = \mathrm{s}^{-1}$.

The energy associated with one photon Q_{p} of wavelength λ (in vacuum) is

$$Q_{\mathrm{p}}(\lambda) = \frac{h\,c_0}{\lambda}\ . \qquad (3)$$

The spectral radiant flux of $\Phi_{\mathrm{e},\lambda}$ relates therefore to the spectral photon flux $\Phi_{\mathrm{p},\lambda}$ by

$$\Phi_{\mathrm{e},\lambda} = \Phi_{\mathrm{p},\lambda}\, Q_{\mathrm{p}}(\lambda) = \Phi_{\mathrm{p},\lambda}\, \frac{h\,c_0}{\lambda}\ . \qquad (4)$$

Hence equation (1) can be rewritten as:

$$\Phi_{\mathrm{act}} = \int \Phi_{\mathrm{p},\lambda}(\lambda)\, A_{\mathrm{act}}(\lambda)\, \frac{h\,c_0}{\lambda}\, \mathrm{d}\lambda\,. \qquad (5)$$

Let's define a function

$$a_{\mathrm{p,act}}(\lambda) = A_{\mathrm{act}}(\lambda)\, \frac{h\,c_0}{\lambda} \qquad (6)$$

N.B. $[a_{\mathrm{p,act}}(\lambda)] = \mathrm{J}$.

Equation (6) rewrites now as

$$\Phi_{\mathrm{act}} = \int \Phi_{\mathrm{p},\lambda}(\lambda)\, a_{\mathrm{p,act}}(\lambda)\, \mathrm{d}\lambda\,. \qquad (7)$$

Equation (7) could be considered as if an actinic function is applied to the spectral photon flux. However it is important to note that this is not the case as $a_{\mathrm{p,act}}$ is *not* dimensionless and relates two quantities of different quantity dimension (photon flux with unit s^{-1} and flux with unit W).

Let's define an (SI-compatible) "photon actinic function" which is applied to the photon flux and is dimensionless. This can be done by normalizing $a_{\mathrm{p,act}}(\lambda)$ to 1 at the wavelength of highest efficacy $\lambda_{\mathrm{m,p,act}}$:

$$A_{\mathrm{p,act}}(\lambda) = a_{\mathrm{p,act}}(\lambda) / a_{\mathrm{p,act}}(\lambda_{\mathrm{m,p,act}}) . \qquad (8)$$

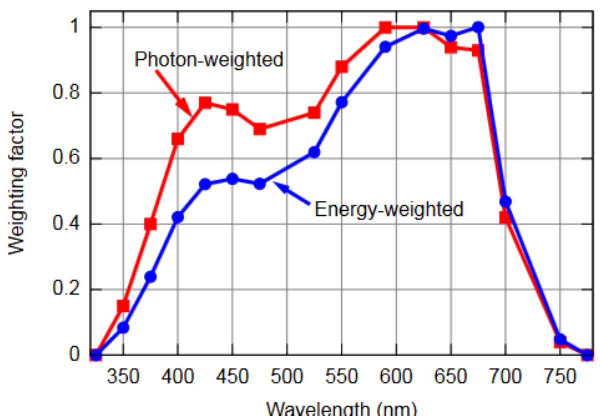
Equation (7) becomes now

$$\Phi_{\mathrm{p,act}} = \int \Phi_{\mathrm{p},\lambda}(\lambda)\, A_{\mathrm{p,act}}(\lambda)\, \mathrm{d}\lambda \qquad (9)$$

where

$\Phi_{\mathrm{p,act}}$ is the effective photon flux, i.e. $\Phi_{\mathrm{p,act}} = \dfrac{\Phi_{\mathrm{act}}}{a_{\mathrm{p,act}}(\lambda_{\mathrm{m,p,act}})}$.

It is evident that the actinic function applied to the radiant flux $A_{\mathrm{act}}(\lambda)$ is of different form and peaks at different wavelength than the actinic function applied to the photon flux $A_{\mathrm{p,act}}(\lambda)$. This is nicely shown in the Wikipedia entry on "Photosynthetically active radiation" https://en.wikipedia.org/wiki/Photosynthetically_active_radiation#Yield_photon_flux :



Relation to photometric quantities

Photometric quantities are related to radiometric quantities by

$$\Phi_{\mathrm{v}} = \frac{K_{\mathrm{cd}}}{V(\lambda_a)} \int \Phi_{\mathrm{e},\lambda}(\lambda)\, V(\lambda)\, \mathrm{d}\lambda = K_{\mathrm{m}} \int \Phi_{\mathrm{e},\lambda}(\lambda)\, V(\lambda)\, \mathrm{d}\lambda \tag{10}$$

where $\lambda_a = 555.017\ \mathrm{nm}$ is the wavelength in standard air at the frequency of 540 x 10^{12} Hz, $K_{\mathrm{cd}} = 683\ \mathrm{lm} \cdot \mathrm{W}^{-1}$ is the luminous efficacy at λ_a and $K_{\mathrm{m}} = 683.002\ \mathrm{lm} \cdot \mathrm{W}^{-1} \approx 683\ \mathrm{lm} \cdot \mathrm{W}^{-1}$ the maximum luminous efficacy (for photopic vision).

$V(\lambda)$ is the spectral luminous efficiency of a monochromatic radiation of wavelength λ (for photopic vision), which is normalized to one at $\lambda_{\mathrm{m}} = 555\ \mathrm{nm}$, i.e. at the peak of efficacy for the photopic observer.

Substituting the spectral flux with the spectral photon flux according (4) equation (10) becomes

$$\Phi_{\mathrm{v}} = K_{\mathrm{m}} \int \Phi_{\mathrm{p},\lambda} \frac{h\, c_0}{\lambda} V(\lambda)\, \mathrm{d}\lambda \tag{11}$$

Let's define

$$v_{\mathrm{p}}(\lambda) = \frac{h\, c_0}{\lambda} V(\lambda) \tag{12}$$

$$V_{\mathrm{p}}(\lambda) = v_{\mathrm{p}}(\lambda) / v_{\mathrm{p}}(\lambda_{\mathrm{m,p}}) \tag{13}$$

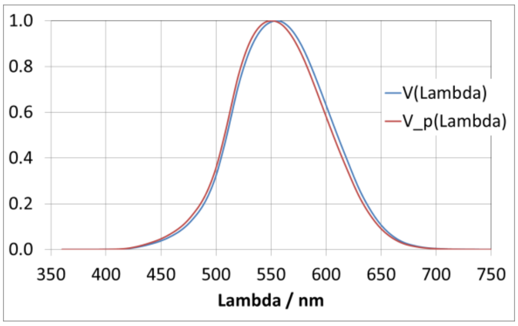
and

$$K_{\mathrm{m,p}} = K_{\mathrm{m}} \cdot v(\lambda_{\mathrm{m,p}}) \tag{14}$$

Equation (11) is then rewritten as

$$\Phi_{\mathrm{v}} = K_{\mathrm{m,p}} \int \Phi_{\mathrm{p},\lambda}\, V_{\mathrm{p}}(\lambda)\, \mathrm{d}\lambda \tag{15}$$

The following figure show both functions $V(\lambda)$ and $V_{\mathrm{p}}(\lambda)$



It was found that $\lambda_{\mathrm{m,p}} = 550.37\,\mathrm{nm}$ and $K_{\mathrm{m,p}} = 2.454411 \cdot 10^{-16}\ \mathrm{lm} \cdot \mathrm{s}$.

Note: In the present *mise-en-pratique* for the realization of the unit Candela (Appendix 2 of the SI Brochure, http://www.bipm.org/en/publications/mises-en-pratique) equation (15) is formulated slightly differently.