

Approved by the CCTF in September 2025, active on March 27, 2026

RECOMMENDED VALUES OF STANDARD FREQUENCIES FOR APPLICATIONS INCLUDING THE PRACTICAL REALIZATION OF THE METRE AND SECONDARY REPRESENTATIONS OF THE DEFINITION OF THE SECOND

CALCIUM 40 ION ($f \approx 411$ THz)

$^{40}\text{Ca}^+$, $4s\ ^2S_{1/2} - 3d\ ^2D_{5/2}$ unperturbed optical transition

1. Recommended value [1] of the frequency in the CIPM List of Frequencies

$$\nu(^{40}\text{Ca}^+) = 411\,042\,129\,776\,400.30\text{ Hz}$$

equivalent to

$$\lambda(^{40}\text{Ca}^+) = 729\,347\,276.793\,942\,83\text{ fm},$$

with a relative standard uncertainty of 5.5×10^{-16} .

This radiation was endorsed by the CCTF as a secondary representation of the definition of the second [2].

2. Method to establish the recommended value

A global analysis of measurements of frequency ratios published in peer-reviewed publications and available to the CCL-CCTF WGFS was carried out following the methods presented in [3-9]. This analysis, or least-squares adjustment, determines the frequency of 14 transitions (see Figure 1) which are either already adopted as secondary representations of the second (SRS) [9] or considered as candidates for SRS.

The 2025 analysis took into account 146 measurements, including 56 frequency ratios and 90 absolute frequency measurements (i.e. ratios to the ^{133}Cs frequency). The input data file and a complete list of references for the published data considered are available in the supplementary material [10].

A total of 1168 correlations between these input measurements were estimated and considered in the adjustment. 927 coefficients account for the correlation between absolute measurements due to using the same primary or secondary frequency standards to access the SI second. 241 coefficients arise from a variety of other sources and were determined in a series of different analyses [10].

An initial adjustment indicated that there were inconsistencies in the global body of published data. In the final adjustment, a random effect uncertainty equal to 5.3×10^{-17} was added in quadrature to all fractional uncertainties of input ratios so as to recover a Birge ratio of 1. Further details are provided in the supplementary material [10].

The results of the adjustment consist of the values of recommended frequencies and a full covariance matrix (available in the supplementary material). The recommended value of the standard is the direct result from the adjustment,

rounded as deemed adequate with respect to the recommended uncertainty. The recommended standard uncertainty is 2.35 times the square root of the corresponding diagonal entry in the covariance matrix of the adjustment. This multiplication factor was recommended by the CCL-CCTF WGFS to allow for as-yet-unidentified sources of uncertainty or correlation, in keeping with the purpose of the recommended values [8], and is separate from any coverage factor used to calculate expanded uncertainties or confidence intervals, as described in [11].

The adjustment also provides optimized values for frequency ratios between all pairs of transitions in the input data set, which can be found in the supplementary material [10]. These ratio values are consistent with the recommended frequency values within rounding, but are reported with a higher number of digits.

While the results are from a global adjustment, it is of interest to note (see Figure 1) that the $^{40}\text{Ca}^+$ transition is involved in 8 measurements relative to ^{133}Cs , and in 1 frequency ratio with an optical transition.

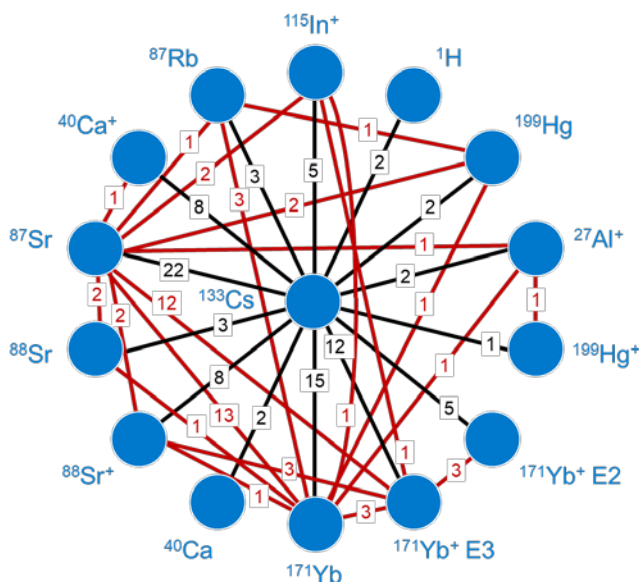


Figure 1: Representation of the 146 measurements linking 14 transitions on the circle and ^{133}Cs at the centre.

3. References

- [1] Consultative Committee for Time and Frequency (CCTF), 24th meeting (session II): Recommendation 24-2 (<https://www.bipm.org/en/committees/cc/cctf/24- 2-2025>)
- [2] Consultative Committee for Time and Frequency (CCTF), 22nd meeting (session II – online), Recommendation PSFS-2 available at <https://www.bipm.org/en/committees/cc/cctf/22- 2-2021>
- [3] H. S. Margolis, P. Gill: Least-squares analysis of clock frequency comparison data to deduce optimized frequency and frequency ratio values; Metrologia 52, 628-34 (2015)
- [4] L. Robertsson: On the evaluation of ultra-high-precision frequency ratio measurements: examining closed loops in a graph theory framework; Metrologia 53, 1272-80 (2016)
- [5] G. Panfilov: communication to the CCL-CCTF WGFS. A new implementation of [3] was realized in MatLab at the BIPM (2020)

- [6] Ch. Oates: communication to the CCL-CCTF WGFS. An independent program was developed in Mathematica at NIST (2017)
- [7] M. Pizzocaro: communication to the CCL-CCTF WGFS. An independent program was developed in Python at INRIM (2025) (available at: <https://github.com/INRIM/GaussMarkovSRS>)
- [8] F. Riehle, P. Gill, F. Arias, L. Robertsson: The CIPM list of recommended frequency standard values: guidelines and procedures; Metrologia 55, 188-200 (2018)
- [9] H. S. Margolis, G. Panfilo, G. Petit, C. Oates, T. Ido and S. Bize: The CIPM list ‘Recommended values of standard frequencies’: 2021 update; Metrologia 61, 035005 (2024)
- [10] H. S. Margolis, G. Panfilo, F. Collini, M. Pizzocaro, A. Ludow, R. Siadat, T. Ido and S. Bize: Data and supplementary material associated with the 2025 adjustment of standard frequencies (2026) (available at: <https://www.doi.org/10.59161/StdFreq2026>)
- [11] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and IOML: Evaluation of measurement data – guide to the expression of uncertainty in measurement; Joint Committee for Guides in Metrology, JCGM vol. 100 (available at: <https://www.bipm.org/en/publications/guides/gum.html>)