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EURAMET.M.T-S6

Torque measurements

SUPPLEMENTARY COMPARISON

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EURAMET.M.T-S6: supplementary comparison of torque realizations from 10 N·m to 500 N·m (Euramet project 1529)

Final Report
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1 Foreword

This report describes the Euramet bilateral comparison M.T-S6, for torque values from 10 N·m to 500 N·m.

2 General information about the Euramet.M.T-S6

This is a bilateral inter-laboratory comparison between the PTB – the German national metrology institute – and the LNE – the national metrology institute of France. The purpose of the comparison is to give support to the uncertainty claims of the LNE's 500 N·m deadweight torque standard machine by comparing the units of measurement as realised at PTB and LNE.

3 Principles of the comparison

The comparison was realised in a star format. The transducers came back to the first laboratory after the second laboratory measurements. The complete measurement cycle (LNE – PTB – LNE) is called a loop. Table 1 summarize the format of the comparison.

Participants	LNE 1st measurements	PTB measurements	LNE 2nd measurements
Period of measurements	02-09/07/2021	12-24/01/2022	08-17/03/2022

Table 1 - Summary of the comparison's dates of measurements

The comparison was done by using five torque transducers of high quality and a high precision frequency-carrier amplifier. The torque transducers were subject to similar loading schemes in the torque calibration machines of the participants following a strict measurement protocol agreed beforehand and similar amplifiers. Figure 1 shows the loading scheme applied in the comparison only the torque steps may be different depending on the transducer.

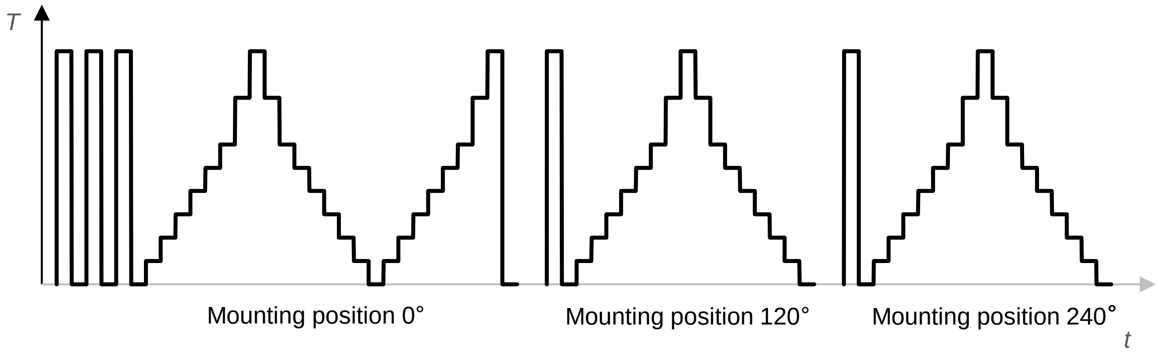

Figure 1 - Loading scheme of the measurement sequence used in M.T-S6

The measurements were performed with clockwise and anticlockwise torque in three mounting positions of 0°, 120° and 240°.
The torque steps realised are:

For 50 N·m transducers: (10, 20, 30, 40, 50) N·m.
For the 100 N·m transducer: (10, 20, 30, 40, 50, 100) N·m
For 500 N·m transducers: (50, 100, 150, 200, 250, 300, 400, 500) N·m.

<u>LNE machines:</u>

The LNE's primary 500 N·m deadweight torque standard machine (TSM) uses the same supported lever arm as the 5 kN·m TSM, only the masses are different. The LNE's TSM has been subject to a bilateral comparison with CEM – the national metrology institute of Spain – for its metrological qualification [1]. The satisfying results drove the launch of the supplementary Euramet comparison, subject of this report. The stated expanded uncertainty (k = 2) of torque (T) applied by the LNE 500 N·m deadweight machine is estimated equal to 3.0 mN·m + 5.0 × 10^{-5} ×T. The measurement range is 10 N·m to 500 N·m.

The BIPM Calibration and Measurement Capabilities (CMC) of LNE currently in the database are (expanded uncertainty in k = 2):

From 5 N·m to 300 N·m : 15 mN·m + 2.0 × 10^{-4} ×T.
From 5 N·m to 2000 N·m : 40 mN·m + 2.0 × 10^{-4} ×T.

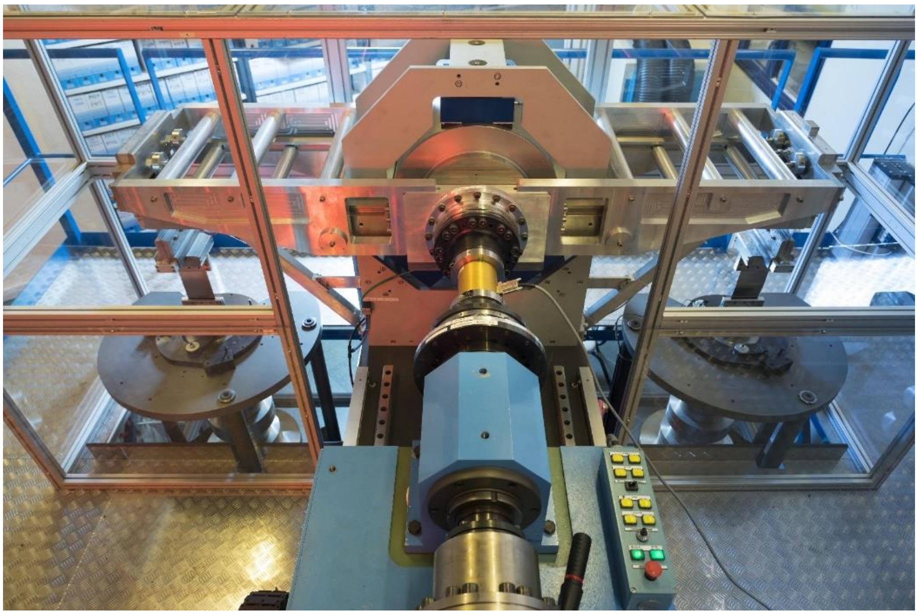

Figure 2 - LNE 500 N·m deadweight torque standard machine

PTB machines

The German primary 1 kN·m Torque Standard Machine (TSM) that was used for PTB's measurements within the CIPM Torque Key Comparison CCM.T-K1. The results of this comparison are published on the BIPM website [2].

At the time of the comparison:
The stated expanded uncertainty (k = 2) of the torque (T) applied by the PTB 1 kN·m deadweight machine is equal to: $2 \times 10^{-5} \times T$ the measurement range is 1 N·m to 1 kN·m.

Figure 3 – PTB 1 kN·m deadweight torque standard machine

4 Instrumentation used in the comparison

Transfer standards:

Five torque transducers owned by LNE were chosen for this comparison. Table 1 presents the details of the transducers.

Capacity	Loop	Manufacturer	Type	Serial number
50 N·m	1	Raute	TT1	36737-03
50 N·m	2	GTM	Dm-TN	68547
100 N·m	3	Raute	TT1	36741-04
500 N·m	4	Raute	TT1	36765-03
500 N·m	5	GTM	Dm-TN	68518

Table 2 - Transducers used in the comparison

Indicator:

Each participating laboratory measured the indication of their DMP40 against the signal of their BN100, at a number of representative voltage ratios. The instrumentation used is:

LNE: DMP40 No 32220008, BN100 No 1899

PTB: DMP40 No 122820045, BN100 No 31320

The interchangeability of the BN100s used should have been confirmed by comparing their readings at a number of representative voltage ratio on the same DMP40. Unfortunately, the logistics of the comparison was disturbed by the COVID situation and this could not be done.
However, given the order of magnitude of the LNE's uncertainty claims for their torque standard machine and the accuracy usually observed for the DMP40s and BN100s, it is assumed that the possible deviations of the amplifiers were low enough to be neglected.

5 Characteristics of the transducers

5.1 Environmental conditions

The temperature and humidity changes are susceptible to have an effect on the sensitivity of the transfer transducers [2]. The measurements were carried out under stable laboratory conditions and the temperature and relative humidity were recorded.

	LNE 1				LNE 2			
	Temperature (°C)		Relative humidity (%)		Temperature (°C)		Relative humidity (%)	
TT1 50 N·m	22.1	± 0.3	58.0	± 2.5	20.9	± 0.5	18.0	± 2.0
Dm-TN 50 N·m	21.3	± 0.2	52.0	± 4.0	21.0	± 0.4	29.0	± 3.0
TT1 100 N·m	21.4	± 0.2	55.0	± 3.0	21.1	± 0.4	32.0	± 3.0
TT1 500 N·m	21.5	± 0.3	52.0	± 2.0	21.2	± 0.3	42.0	± 3.0
Dm-TN 500 N·m	21.4	± 0.2	55.0	± 3.0	21.4	± 0.3	32.5	± 2.5

Table 3 - Environmental conditions at LNE and their maximum associated variations

	LNE (mean)				PTB			
	Temperature (°C)		Relative humidity (%)		Temperature (°C)		Relative humidity (%)	
TT1 50 N·m	21.5	± 0.3	38.0	± 1.6	21.0	± 1.0	38.7	± 2.0
Dm-TN 50 N·m	21.2	± 0.2	40.5	± 2.5	20.9	± 1.0	39.3	± 2.0
TT1 100 N·m	21.2	± 0.2	43.5	± 2.1	20.9	± 1.0	39.3	± 2.0
TT1 500 N·m	21.3	± 0.2	47.0	± 1.8	20.9	± 1.0	38.1	± 2.0
Dm-TN 500 N·m	21.4	± 0.1	43.8	± 2.0	21.0	± 1.0	38.1	± 2.0

Table 4 - Mean environmental conditions at LNE and PTB and their associated maximum variations

The humidity and temperature effect on the sensitivity of the transducers is usually taken into account using a linear regression such as described in equation (1):

$$x' = x \times (1 + e_{rH} \cdot \Delta rH_i + e_T \cdot \Delta T_i) \qquad (1)$$

With x' being the corrected deflection, e_{rH} and e_{T} being respectively the relative humidity and relative temperature sensitivity coefficient of a transducer. The deviations terms ΔrH_i and ΔT_i are the deviations from the reference environmental conditions chosen equal to *T* = 21 °C, *rH* = 40 %.

In this comparison, the humidity and temperature sensitivity coefficients have been assumed to be less than maximum values. These maximum values were estimated using published data for high precision torque transducers: $e_T \leq 1\times10^{-5}$ / K and $e_{rH} \leq 1\times10^{-5}$ / %. These values are consistent with the temperature and humidity sensitivity coefficients used in the key comparison CCM.T-K1 [2] for transducers of similar type.

5.2 Stability of the transducers

The stability of the transfer transducers over the course of the comparison was investigated by determining the relative deviations between LNE's two measurements. The relative deviation is determined as the difference between LNE's second and first measurement relative to their mean value. Figure 4 to 8 show the relative deviations of each transducers used in the comparison.

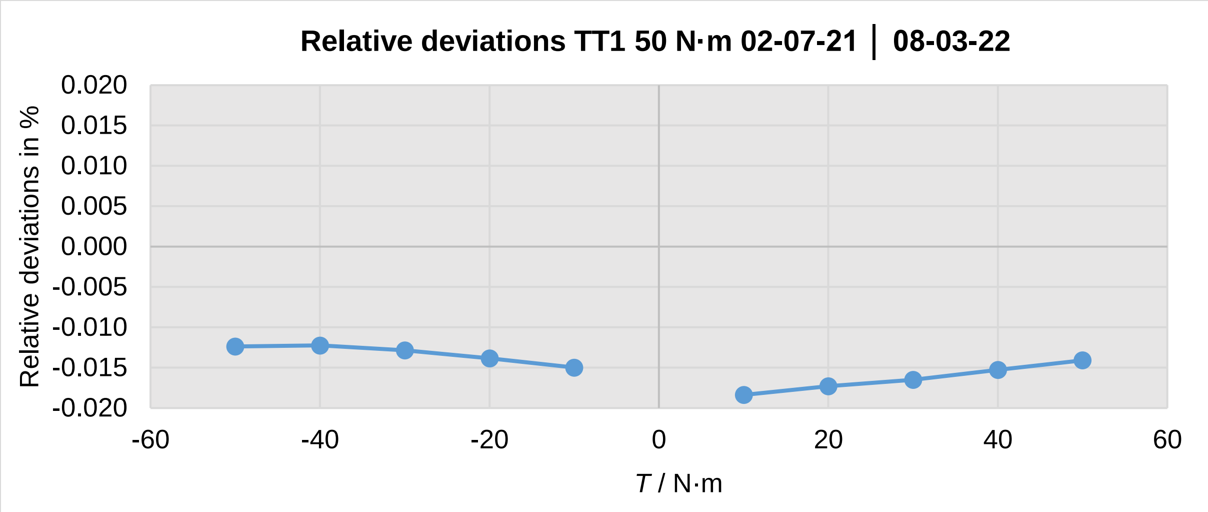

Figure 4 - Relative deviation of LNE's measurement in % of Raute TT1 50 N·m No 36737-03 (loop 1)

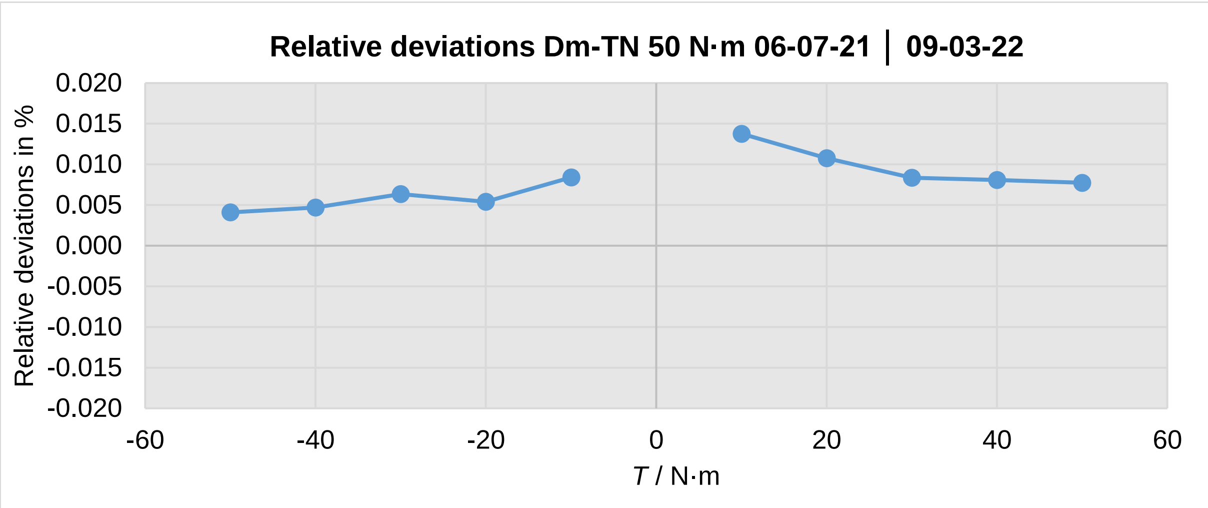

Figure 5 - Relative deviation of LNE's measurement in % of GTM Dm-TN 50 N·m No 68547 (loop 2)

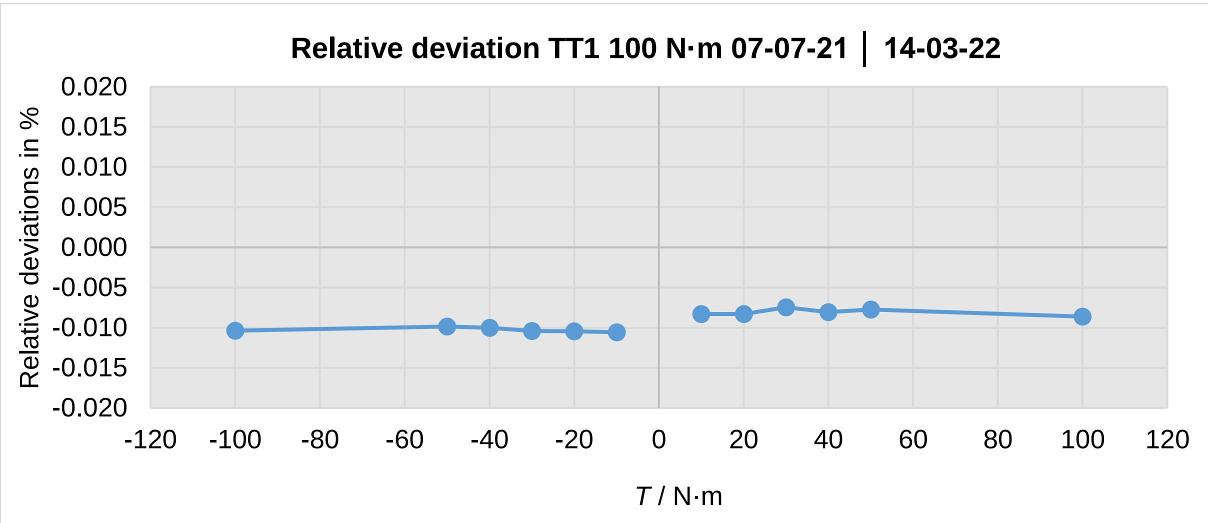

Figure 6 - Relative deviation of LNE's measurement in % of Raute TT1 100 N·m No 36741-04 (loop 3)

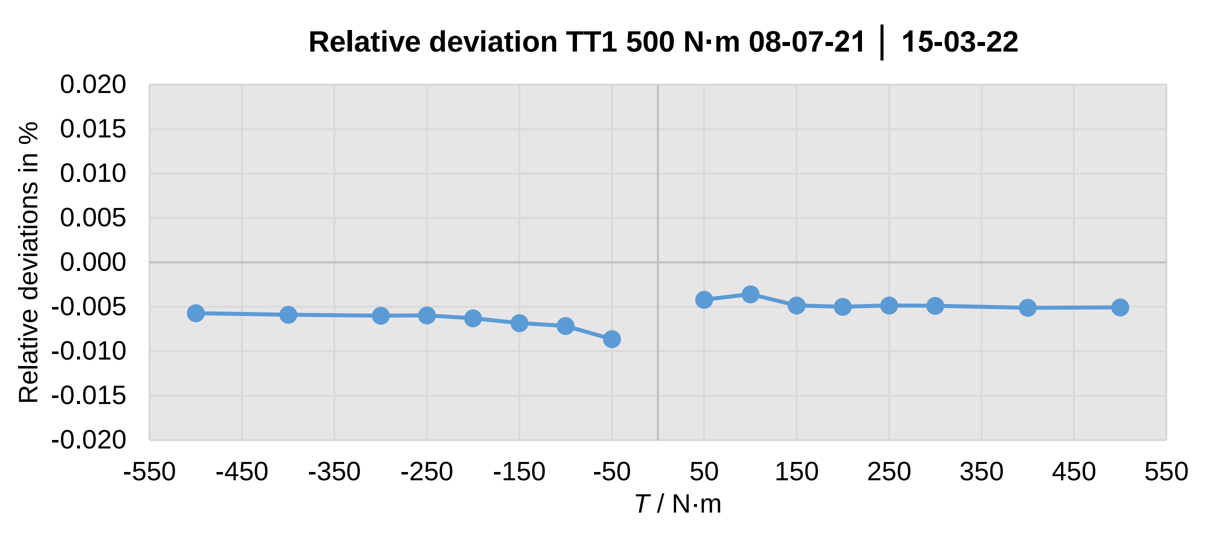

Figure 7 - Relative deviation of LNE's measurement in % of Raute TT1 500 N·m No 36765-03 (loop 4)

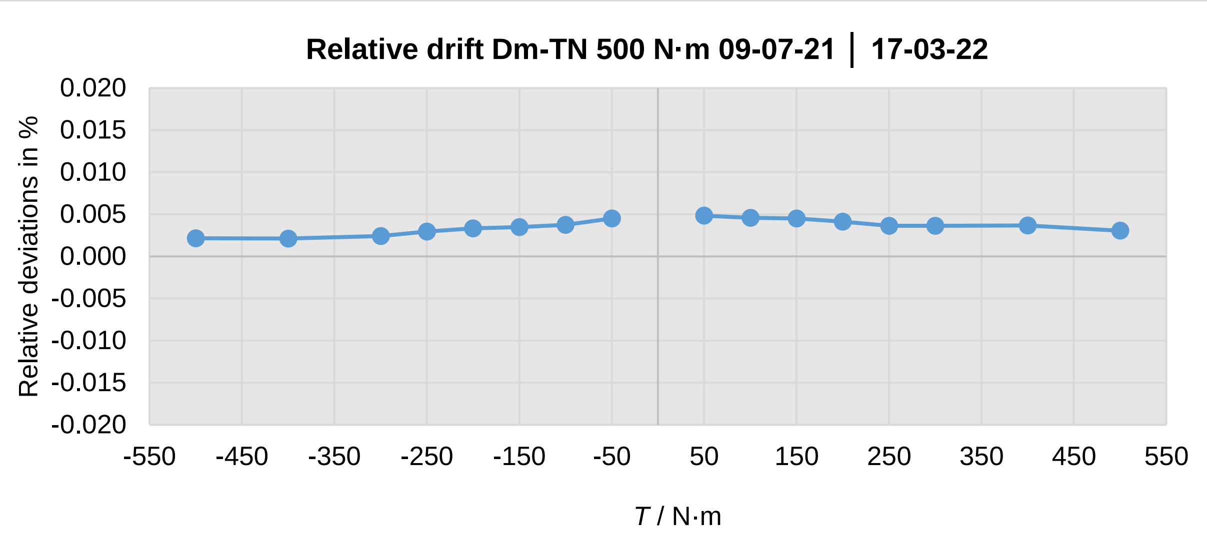

Figure 8 - Relative deviation of LNE's measurement in % of GTM Dm-TN 500 N·m No 68518 (loop 5)

For each transducer, as no correction for the environmental conditions is made, and considering the differences in relative humidity between the LNE measurements (see Table 3), no significant drift of the transducer sensitivity is observed.

In light of these considerations, the stability of the transfer standards related to LNE capabilities seems reasonable. No correction is then applied and the drift is not taken as an uncertainty component.

5.3 Creep effect

The creep of the transfer transducer during the measurements is neglected as both laboratories followed the same measurement protocol with the same loading regime.

6 Results of the measurements

For each torque step, the results of the measurements x_{Lab} are calculated as the mean of the first reported upward deflections from each mounting position, corrected from their zero value. The measurement value at LNE x_{LNE} for each loop i is defined as the mean of the two LNE measurements.

$$x_{i,\text{LNE}} = \frac{x_{i,\text{LNE},1} + x_{i,\text{LNE},2}}{2} \quad (2)$$

With $x_{i,\text{LNE},1}$ and $x_{i,\text{LNE},2}$ being the result of LNE first and second measurement respectively. The relative deviations, expressed in percentage terms, between the deflection values obtained at LNE and the deflection values obtained at PTB for loop i are given by:

$$d_i = 100 \times \frac{x_{i,\text{LNE}} - x_{i,\text{PTB}}}{x_{i,\text{PTB}}} \quad (3)$$

Table 5 to 9 summarise the results of the measurements and their relative deviations all five transducers used in the comparison.

T / (N·m)	x_{Lab} / (mV/V)			$x_{1,\text{LNE}}$ / (mV/V)	Diff LNE - PTB in mV/V	d_1 / %
	LNE1	PTB	LNE2			
10	0.264 853	0.264 884	0.264 902	0.264 877	-0.000 007	-0.003
20	0.529 766	0.529 806	0.529 858	0.529 812	0.000 006	0.001
30	0.794 712	0.794 755	0.794 843	0.794 778	0.000 023	0.003
40	1.059 685	1.059 738	1.059 847	1.059 766	0.000 028	0,003
50	1.324 677	1.324 742	1.324 863	1.324 770	0.000 028	0.002
-10	-0.264 858	-0.264 871	-0.264 898	-0.264 878	-0.000 007	0.003
-20	-0.529 769	-0.529 778	-0.529 842	-0.529 806	-0.000 028	0.005
-30	-0.794 694	-0.794 719	-0.794 796	-0.794 745	-0.000 026	0.003
-40	-1.059 653	-1.059 681	-1.059 783	-1.059 718	-0.000 038	0.004
-50	-1.324 621	-1.324 652	-1.324 785	-1.324 703	-0.000 051	0.004

Table 5 - Deflections in mV/V and relative differences of the Raute TT1 50 N·m No 36737-03 (loop 1)

T / (N·m)	x_{Lab} / (mV/V)			$x_{2,\text{LNE}}$ / (mV/V)	Diff LNE-PTB in mV/V	d_2 / %
	LNE1	PTB	LNE2			
10	0.397 039	0.397 005	0.396 984	0.397 012	0.000 006	0.002
20	0.794 083	0.794 020	0.793 998	0.794 041	0.000 020	0.003
30	1.191 088	1.191 023	1.190 988	1.191 038	0.000 015	0.001
40	1.588 079	1.588 008	1.587 951	1.588 015	0.000 008	0.000
50	1.985 052	1.984 981	1.984 898	1.984 975	-0.000 006	0.000
-10	-0.397 043	-0.396 996	-0.397 009	-0.397 026	-0.000 030	0.007
-20	-0.794 094	-0.794 024	-0.794 051	-0.794 072	-0.000 048	0.006
-30	-1.191 138	-1.191 051	-1.191 063	-1.191 101	-0.000 049	0.004
-40	-1.588 163	-1.588 077	-1.588 088	-1.588 125	-0.000 048	0.003
-50	-1.985 172	-1.985 089	-1.985 091	-1.985 131	-0.000 043	0.002

Table 6 - Deflections in mV/V and relative deviations of the GTM Dm-TN 50 N·m No 68547 (loop 2)

T / (N·m)	x_{Lab} / (mV/V)			$x_{3,\mathrm{LNE}}$ / (mV/V)	Diff LNE-PTB in mV/V	d_3 / %
	LNE1	PTB	LNE2			
10	0.132 468	0.132 487	0.132 479	0.132 473	-0.000 014	-0.011
20	0.264 948	0.264 972	0.264 970	0.264 959	-0.000 014	-0.005
30	0.397 435	0.397 466	0.397 464	0.397 450	-0.000 017	-0.004
40	0.529 924	0.529 971	0.529 967	0.529 946	-0.000 025	-0.005
50	0.662 419	0.662 473	0.662 471	0.662 445	-0.000 028	-0.004
100	1.324 949	1.325 068	1.325 064	1.325 007	-0.000 062	-0.005
-10	-0.132 465	-0.132 479	-0.132 479	-0.132 472	0.000 007	-0.005
-20	-0.264 945	-0.264 965	-0.264 973	-0.264 959	0.000 006	-0.002
-30	-0.397 429	-0.397 466	-0.397 470	-0.397 450	0.000 016	-0.004
-40	-0.529 921	-0.529 965	-0.529 974	-0.529 948	0.000 017	-0.003
-50	-0.662 415	-0.662 468	-0.662 480	-0.662 447	0.000 021	-0.003
-100	-1.324 957	-1.325 071	-1.325 094	-1.325 025	0.000 046	-0.003

Table 7 - Deflections in mV/V and relative deviations of the Raute TT1 100 N·m No 36741-04 (loop 3)

T / (N·m)	x_{Lab} / (mV/V)			$x_{4,\mathrm{LNE}}$ / (mV/V)	Diff LNE-PTB in mV/V	d_4 / %
	LNE1	PTB	LNE2			
50	0.135 122	0.135 139	0.135 128	0.135 125	-0.000 014	-0.010
100	0.270 253	0.270 275	0.270 263	0.270 258	-0.000 017	-0.006
150	0.405 388	0.405 428	0.405 408	0.405 398	-0.000 030	-0.008
200	0.540 530	0.540 576	0.540 557	0.540 543	-0.000 033	-0.006
250	0.675 678	0.675 735	0.675 711	0.675 694	-0.000 041	-0.006
300	0.810 830	0.810 896	0.810 870	0.810 850	-0.000 046	-0.006
400	1.081 145	1.081 231	1.081 200	1.081 172	-0.000 059	-0.005
500	1.351 450	1.351 578	1.351 519	1.351 484	-0.000 094	-0.007
-50	-0.135 118	-0.135 140	-0.135 130	-0.135 124	0.000 016	-0.012
-100	-0.270 248	-0.270 277	-0.270 268	-0.270 258	0.000 019	-0.007
-150	-0.405 385	-0.405 429	-0.405 412	-0.405 399	0.000 030	-0.007
-200	-0.540 528	-0.540 578	-0.540 562	-0.540 545	0.000 033	-0.006
-250	-0.675 678	-0.675 739	-0.675 718	-0.675 698	0.000 041	-0.006
-300	-0.810 831	-0.810 902	-0.810 880	-0.810 856	0.000 046	-0.006
-400	-1.081 152	-1.081 241	-1.081 215	-1.081 184	0.000 057	-0.005
-500	-1.351 491	-1.351 590	-1.351 568	-1.351 530	0.000 061	-0.004

Table 8 - Deflections in mV/V and relative deviations of the Raute TT1 500 N·m No 36765-03 (loop 4)

T / (N·m)	x_{Lab} / (mV/V)			$x_{5,\text{LNE}}$ / (mV/V)	Diff LNE-PTB in mV/V	d_5 / %
	LNE1	PTB	LNE2			
50	0.199 952	0.199 944	0.199 942	0.199 947	0.000 004	0.002
100	0.399 940	0.399 919	0.399 922	0.399 931	0.000 012	0.003
150	0.599 952	0.599 926	0.599 925	0.599 939	0.000 013	0.002
200	0.799 990	0.799 953	0.799 957	0.799 973	0.000 021	0.003
250	1.000 050	0.999 998	1.000 014	1.000 032	0.000 033	0.003
300	1.200 114	1.200 058	1.200 070	1.200 092	0.000 034	0.003
400	1.600 284	1.600 202	1.600 225	1.600 254	0.000 052	0.003
500	2.000 404	2.000 368	2.000 343	2.000 374	0.000 006	0.000
-50	-0.199 951	-0.199 939	-0.199 942	-0.199 947	-0.000 007	0.004
-100	-0.399 938	-0.399 921	-0.399 923	-0.399 931	-0.000 010	0.002
-150	-0.599 954	-0.599 930	-0.599 933	-0.599 944	-0.000 014	0.002
-200	-0.799 995	-0.799 963	-0.799 968	-0.799 981	-0.000 018	0.002
-250	-1.000 060	-1.000 019	-1.000 030	-1.000 045	-0.000 026	0.003
-300	-1.200 137	-1.200 094	-1.200 108	-1.200 123	-0.000 028	0.002
-400	-1.600 325	-1.600 271	-1.600 291	-1.600 308	-0.000 038	0.002
-500	-2.000 528	-2.000 466	-2.000 485	-2.000 507	-0.000 040	0.002

Table 9 - Deflections in mV/V and relative deviations of the GTM Dm-TN 500 N·m No 68518 (loop 5)

7 Uncertainty budget

7.1 Uncertainty of deflections

The uncertainty of the mean deflections obtained for each torque step in each laboratory is calculated. The input variables are supposed uncorrelated. The relative standard calibration uncertainty w_{cal} (*k* = 1) for each laboratory's measurements is given by:

$$w_{\text{cal}} = \sqrt{w_{\text{TM}}^2 + 2 \times w_r^2 + w_b^2 + w_{b'}^2 + w_0^2} \tag{4}$$

Where:

- w_{TM} is the relative standard uncertainty due to the torque calibration machine.
- w_r is the relative standard uncertainty due to the resolution r of the display unit.
- w_b is the relative standard uncertainty due to reproducibility b.
- $w_{b'}$ is the relative standard uncertainty due to repeatability b'.
- w_0 is the relative standard uncertainty due to zero point deviation f_0.

The details of the uncertainty contributions of w_{cal} are not detailed in this report as the uncertainty contributions of the TSM are heavily predominant, especially for the LNE measurements.
The relative standard calibration uncertainty of LNE is calculated as the mean of the relative standard uncertainty of its two calibrations because these two calibrations are correlated as they are executed on the same standard machine with the same transducer.
The relative standard measurement uncertainty of each laboratory for loop i is then calculated as:

$$w(x_{i,\text{LNE}}) = \sqrt{w_{i,\text{LNE}_{\text{cal}}}^2 + w_{i,\text{LNE}_{\text{envir}}}^2} \tag{5}$$

And:

$$w(x_{i,\text{PTB}}) = \sqrt{w_{i,\text{PTB}_{\text{cal}}}^2 + w_{i,\text{PTB}_{\text{envir}}}^2} \tag{6}$$

Where:

- $w_{i,\mathrm{Lab}_{\mathrm{envir}}}$ is the relative standard uncertainty contribution of the environmental deviations during the measurements. As no correction is applied to the deflections ($e_T = e_{rH} = 0$), the law of propagation of uncertainties is applied to equation (1) considering worst case scenarios for each parameter. So for the temperature at LNE or PTB, the associated standard uncertainty is evaluated as follows:

$$w_T = \sqrt{\left(w(e_T) \times \left(\Delta T_{\mathrm{Lab}i} + u(\Delta T_{\mathrm{Lab}i})\right)\right)^2 + \left(\left(e_T + w(e_T)\right) \times u(\Delta T_{\mathrm{Lab}i})\right)^2} \quad (7)$$

  The same equation is applied for the humidity contributions. The standard uncertainties of the coefficients were estimated using a type B evaluation, with the relative maximal sensitivity coefficients defined in 5.1, assuming a rectangular distribution.

Tables 10 to 14 summarize the relative expanded measurements uncertainties of the comparison.

T / (N·m)	Expanded relative uncertainties (LNE) (*k* = 2)			Expanded relative uncertainties (PTB) (*k* = 2)		
	$W_{1,\mathrm{LNE}_{\mathrm{cal}}}$ / %	$W_{1,\mathrm{LNE}_{\mathrm{envir}}}$ / %	$W(x_{1,\mathrm{LNE}})$ / %	$W_{1,\mathrm{PTB}_{\mathrm{cal}}}$ / %	$W_{1,\mathrm{PTB}_{\mathrm{envir}}}$ / %	$W(x_{1,\mathrm{PTB}})$ / %
10	0.035	0.004	0.035	0.003	0.004	0.005
20	0.020	0.004	0.021	0.003	0.004	0.005
30	0.015	0.004	0.016	0.003	0.004	0.005
40	0.013	0.004	0.013	0.003	0.004	0.005
50	0.011	0.004	0.012	0.003	0.004	0.005
-10	0.035	0.004	0.035	0.003	0.004	0.005
-20	0.020	0.004	0.021	0.003	0.004	0.005
-30	0.015	0.004	0.016	0.003	0.004	0.005
-40	0.013	0.004	0.013	0.003	0.004	0.005
-50	0.011	0.004	0.012	0.003	0.004	0.005

Table 10 - Relative expanded measurement uncertainties (*k* = 2) in % for Raute TT1 50 N·m No 36737 - 03 (loop 1)

T / (N·m)	Relative expanded uncertainties (LNE) (*k* = 2)			Relative expanded uncertainties (PTB) (*k* = 2)		
	$W_{2,\mathrm{LNE}_{\mathrm{cal}}}$ / %	$W_{2,\mathrm{LNE}_{\mathrm{envir}}}$ / %	$W(x_{2,\mathrm{LNE}})$ / %	$W_{2,\mathrm{PTB}_{\mathrm{cal}}}$ / %	$W_{2,\mathrm{PTB}_{\mathrm{envir}}}$ / %	$W(x_{2,\mathrm{PTB}})$ / %
10	0.035	0.003	0.035	0.005	0.003	0.006
20	0.020	0.003	0.020	0.003	0.003	0.005
30	0.015	0.003	0.016	0.003	0.003	0.004
40	0.013	0.003	0.013	0.003	0.003	0.004
50	0.011	0.003	0.012	0.003	0.003	0.004
-10	0.035	0.003	0.035	0.003	0.003	0.004
-20	0.020	0.003	0.020	0.003	0.003	0.004
-30	0.015	0.003	0.015	0.002	0.003	0.004
-40	0.013	0.003	0.013	0.002	0.003	0.004
-50	0.011	0.003	0.012	0.002	0.003	0.004

Table 11 - Relative expanded measurement uncertainties (*k* = 2) in % for GTM Dm-TN 50 N·m No 68547 (loop 2)

T / (N·m)	Relative expanded uncertainties (LNE) (k = 2)			Relative expanded uncertainties (PTB) (k = 2)		
	$W_{3,\mathrm{LNE_{cal}}}$ / %	$W_{3,\mathrm{LNE_{envir}}}$ / %	$W(x_{3,\mathrm{LNE}})$ / %	$W_{3,\mathrm{PTB_{cal}}}$ / %	$W_{3,\mathrm{PTB_{envir}}}$ / %	$\mathrm{W}(x_{3,\mathrm{PTB}})$ / %
10	0.035	0.006	0.036	0.004	0.003	0.005
20	0.020	0.006	0.021	0.003	0.003	0.005
30	0.015	0.006	0.016	0.003	0.003	0.004
40	0.013	0.006	0.014	0.003	0.003	0.005
50	0.011	0.006	0.013	0.003	0.003	0.004
100	0.008	0.006	0.010	0.003	0.003	0.004
-10	0.035	0.006	0.036	0.004	0.003	0.005
-20	0.020	0.006	0.021	0.003	0.003	0.004
-30	0.015	0.006	0.016	0.003	0.003	0.004
-40	0.013	0.006	0.014	0.003	0.003	0.004
-50	0.011	0.006	0.013	0.003	0,003	0.004
-100	0.008	0.006	0.010	0.003	0.003	0.004

Table 12 - Relative expanded measurement uncertainties (k = 2) in % of Raute TT1 100 N·m No 36741-04 (loop 3)

T / (N·m)	Relative expanded uncertainties (LNE) (k = 2)			Relative expanded uncertainties (PTB) (k = 2)		
	$W_{4,\mathrm{LNE_{cal}}}$ / %	$W_{4,\mathrm{LNE_{envir}}}$ / %	$W(x_{4,\mathrm{LNE}})$ / %	$W_{4,\mathrm{PTB_{cal}}}$ / %	$W_{4,\mathrm{PTB_{envir}}}$ / %	$\mathrm{W}(x_{4,\mathrm{PTB}})$ / %
50	0.011	0.010	0.015	0.003	0.004	0.005
100	0.008	0.010	0.013	0.003	0.004	0.005
150	0.007	0.010	0.012	0.002	0.004	0.005
200	0.007	0.010	0.012	0.002	0.004	0.005
250	0.006	0.010	0.012	0.002	0.004	0.005
300	0.006	0.010	0.011	0.002	0.004	0.005
400	0.006	0.010	0.011	0.002	0.004	0.005
500	0.006	0.010	0.011	0.002	0.004	0.005
-50	0.011	0.010	0.015	0.002	0.004	0.005
-100	0.008	0.010	0.013	0.002	0.004	0.005
-150	0.007	0.010	0.012	0.002	0.004	0.005
-200	0.007	0.010	0.012	0.002	0.004	0.005
-250	0.006	0.010	0.012	0.002	0.004	0.005
-300	0.006	0.010	0.011	0.002	0.004	0.005
-400	0.006	0.010	0.011	0.002	0.004	0.005
-500	0.006	0.010	0.011	0.002	0.004	0.005

Table 13 - Relative expanded measurement uncertainties (k = 2) in % of Raute TT1 500 N·m No 36765-03 (loop 4)

T / (N·m)	Relative expanded uncertainties (LNE) (k = 2)			Relative expanded uncertainties (PTB) (k = 2)		
	$W_{5,\mathrm{LNE}_{\mathrm{cal}}}$ / %	$W_{5,\mathrm{LNE}_{\mathrm{envir}}}$ / %	$W(x_{5,\mathrm{LNE}})$ / %	$W_{5,\mathrm{PTB}_{\mathrm{cal}}}$ / %	$W_{5,\mathrm{PTB}_{\mathrm{envir}}}$ / %	$W(x_{5,\mathrm{PTB}})$ / %
50	0.011	0.006	0.013	0.002	0.004	0.005
100	0.008	0.006	0.010	0.002	0.004	0.005
150	0.007	0.006	0.009	0.002	0.004	0.005
200	0.007	0.006	0.009	0.002	0.004	0.005
250	0.006	0.006	0.009	0.002	0.004	0.005
300	0.006	0.006	0.009	0.002	0.004	0.005
400	0.006	0.006	0.008	0.002	0.004	0.005
500	0.006	0.006	0.008	0.002	0.004	0.005
-50	0.011	0.006	0.013	0.003	0.004	0.005
-100	0.008	0.006	0.010	0.002	0.004	0.005
-150	0.007	0.006	0.009	0.002	0.004	0.005
-200	0.007	0.006	0.009	0.002	0.004	0.005
-250	0.006	0.006	0.009	0.002	0.004	0.005
-300	0.006	0.006	0.009	0.002	0.004	0.005
-400	0.006	0.006	0.008	0.002	0.004	0.005
-500	0.006	0.006	0.008	0.002	0.004	0.005

Table 14 - Relative expanded measurement uncertainties (k = 2) in % of GTM Dm-Tn 500 N·m No 68518 (loop 5)

At this point of the analysis, we see that at the transducer level, considering only the uncertainty of the LNE torque standard machine, the results of the comparison are very satisfactory for loop 1, 2, 3 and 5 (i.e. E_n values are smaller than one). For loop 4, most E_n values are smaller than one except for four torque steps, and they remain close to one. This could be explained by the difference in relative humidity between the PTB and LNE measurements. The relative deviations between the two LNE measurements for loop 4 reported in figure 7 support this idea. As sensitivity coefficients for temperature and relative humidity are not known the environmental component of uncertainty can become quite large. While this is not ideal, the combination of the results in the following section was proved very satisfactory. Investigations on sensitivity coefficients for temperature and humidity should still be performed for future comparison exercises.

7.2 Uncertainty of deviations and generalised weighted mean

For each loop i, the relative standard uncertainty of the deviations is calculated as:

$$w(d_i) = \sqrt{w^2(x_{i,\mathrm{LNE}}) + w^2(x_{i,\mathrm{PTB}})} \quad (8)$$

Table 15 summarises the relative deviations between LNE and PTB for each loop at each torque step with their associated relative expanded uncertainty (k = 2).

T / (N·m)	$d_i \pm W(d_i)$ (*k* = 2) in %				
	Loop 1 (50 N·m)	Loop 2 (50 N·m)	Loop3 (100 N·m)	Loop 4 (500 N·m)	Loop 5 (500 N·m)
10	-0.003 ± 0.036	0.002 ± 0.036	-0.011 ± 0.036		
20	0.001 ± 0.021	0.003 ± 0.021	-0.005 ± 0.022		
30	0.003 ± 0.016	0.001 ± 0.016	-0.004 ± 0.017		
40	0.003 ± 0.014	0.000 ± 0.014	-0.005 ± 0.015		
50	0.002 ± 0.013	0.000 ± 0.012	-0.004 ± 0.013	-0.010 ± 0.016	0.002 ± 0.014
100			-0.005 ± 0.011	-0.006 ± 0.014	0.003 ± 0.011
150				-0.008 ± 0.013	0.002 ± 0.011
200				-0.006 ± 0.013	0.003 ± 0.010
250				-0.006 ± 0.013	0.003 ± 0.010
300				-0.006 ± 0.012	0.003 ± 0.010
400				-0.005 ± 0.012	0.003 ± 0.010
500				-0.007 ± 0.012	0.000 ± 0.010
-10	0.003 ± 0.036	0.007 ± 0.036	-0.005 ± 0.036		
-20	0.005 ± 0.021	0.006 ± 0.021	-0.002 ± 0.021		
-30	0.003 ± 0.016	0.004 ± 0.016	-0.004 ± 0.017		
-40	0.004 ± 0.014	0.003 ± 0.014	-0.003 ± 0.015		
-50	0.004 ± 0.013	0.002 ± 0.012	-0.003 ± 0.013	-0.012 ± 0.016	0.004 ± 0.014
-100			-0.003 ± 0.011	-0.007 ± 0.014	0.002 ± 0.011
-150				-0.007 ± 0.013	0.002 ± 0.011
-200				-0.006 ± 0.013	0.002 ± 0.010
-250				-0.006 ± 0.013	0.003 ± 0.010
-300				-0.006 ± 0.012	0.002 ± 0.010
-400				-0.005 ± 0.012	0.002 ± 0.010
-500				-0.004 ± 0.012	0.002 ± 0.010

Table 15 – Relative deviations and their associated relative expanded uncertainty in % (*k* = 2) for each loop.

In order to establish the mean relative deviation between LNE and PTB at a given torque step, the generalised weighted mean was calculated. This model is chosen in order to give less weight to the loops with higher uncertainties associated with the deviations. Two evaluations are performed: the first with the deviations supposed uncorrelated and the second with the deviations supposed correlated.

For the first evaluation, the weighted mean of the relative deviations obtained from each loop is calculated according to equation (9):

$$\bar{d} = \frac{\sum_{i=1}^{5} d_i / w^2(d_i)}{\sum_{i=1}^{5} \frac{1}{w^2(d_i)}} \quad (9)$$

Note that equation (9) is not applied literally for every torque step as for example, only three transducers out of five that are measured at the first torque step. If no relative deviation is calculated at a certain torque step for a certain loop i, its value $d_i \pm w(d_i)$ is simply not considered in equation (9).

For uncorrelated quantities, the uncertainty of the weighted mean is given by:

$$w(\bar{d}) = \sqrt{\frac{1}{\sum_{i=1}^{5} \frac{1}{w^2(d_i)}}} \qquad (10)$$

For the second evaluation, the deviation are supposed correlated. It is reasonable to make this assumption as the transducer are calibrated on the same TSM for each loop. According to M G Cox and P M Harris [3] (section 5.1), the covariance associated with any pair of measurements measured on the same standard machine would be the variance $u^2(t)$ associated with the best estimate t of the systematic error of the machine.
In the case of a pair of deviations of the same transducer measured on the same standard machines, the covariance can be assumed equal to the combined variances of the standard machines. Note that in this comparison, transducers of different capacities and different manufacturers are used, the systematic error of the deviations then may not be the same for all deviations d_i. Still, it is assumed for this evaluation that the covariance for any pair of deviations at each torque step is equal to the combined variances of the standard machine:

$$w(d_i, d_j) = w_{\mathrm{TM,LNE}}^2 + w_{\mathrm{TM,PTB}}^2 \qquad (11)$$

The generalised weighted mean is a form of average that accounts for the covariance of the quantities. As shown in [4] by M G Cox and al. the effect of a strong positive correlation can result in reducing the variance associated with the generalised weighted mean. However, as the correlation is traced back to a common effect in the measurement process, no paradoxical mean and uncertainty values can be observed [4]. In this evaluation, it is then expected that the uncertainty associated with the generalised weighted mean increases as the correlation increases.

Before applying the generalised weighted mean to the deviations d_i, it is necessary to perform a consistency check. As described in [4], the deviation d_i must be regarded as inconsistent if the following equation is not satisfied:

$$\frac{|d_i - d_j|}{2 \times w(d_i - d_j)} = \frac{|d_i - d_j|}{2 \times \sqrt{w^2(d_i) + w^2(d_j) - w(d_i, d_j)}} < 1 \text{ with } j > i \qquad (12)$$

Data consistency was confirmed by applying equation (12) to the pairs of deviations. No value was greater than one. The generalised weighted mean of N correlated quantities is given in [4]:

$$\bar{d} = w^2(\bar{d}) W^T C^{-1} D \qquad (13)$$

With:

- $w^2(\bar{d})$: the variance of the weighted mean of N deviations d_i at a given torque step
- $W = [1 \quad 1]^T$ the design matrix for the weighted mean of N = 2 deviations d_i
- C : the covariance matrix and C^{-1} its inverse
- $D = [d_i \quad d_j]^T$ the deviation matrix for N = 2.

Note that not all torque step are measured with the same number of transducers so the dimensions of the matrices vary depending on the torque step evaluated. For example the covariance matrix at the torque step of 100 N·m with N = 3 transducers is:

$$C_{100} = \begin{bmatrix} w^2(d_3) & w(d_3, d_4) & w(d_3, d_5) \\ w(d_4, d_3) & w^2(d_4) & w(d_4, d_5) \\ w(d_5, d_3) & w(d_5, d_4) & w^2(d_5) \end{bmatrix} \qquad (14)$$

The variance of the generalised weighted mean is given by:

$$w^2(\bar{d}) = (W^T C^{-1} W)^{-1} \qquad (15)$$

Note that equation (13) and (15) are equivalent to equation (9) and (10) respectively for a correlation coefficient equal to zero.

In order to evaluate the results of the comparison, we can calculate the normalized errors E_n. This statistical evaluation is calculated as the difference of the two participant's value divided by their combined expanded uncertainty. Given that we work with relative quantities in this comparison, the E_n values are then calculated for each torque step according to equation (16).

$$E_\mathrm{n} = \frac{\bar{d}}{2 \times w(\bar{d})} \tag{16}$$

Table 16 summarise the value of the generalised weighted mean of the deviations, its associated expanded uncertainty as well as the E_n values for both evaluations.

T / (N·m)	Evaluation 1: Uncorrelated deviations			Evaluation 2: Correlated deviations		
	$\bar{d}$ / %	$W(\bar{d})$ / % (*k* = 2)	E_n	$\bar{d}$ / %	$W(\bar{d})$ / % (*k* = 2)	E_n
10	-0.004	0.021	-0.2	-0.003	0.035	-0.1
20	0.000	0.012	0.0	0.000	0.020	0.0
30	0.000	0.009	0.0	0.000	0.016	0.0
40	0.000	0.008	0.0	0.000	0.013	0.0
50	-0.002	0.006	-0.3	-0.001	0.012	-0.1
100	-0.002	0.007	-0.3	-0.002	0.010	-0.2
150	-0.002	0.008	-0.2	-0.001	0.010	-0.1
200	-0.001	0.008	-0.1	0.000	0.009	0.0
250	0.000	0.008	0.0	0.000	0.009	0.0
300	0.000	0.008	-0.1	0.000	0.009	0.0
400	0.000	0.008	0.0	0.000	0.009	0.0
500	-0.002	0.008	-0.3	-0.002	0.009	-0.2
-10	0.002	0.021	0.1	0.003	0.035	0.0
-20	0.003	0.012	0.3	0.004	0.020	0.2
-30	0.001	0.009	0.1	0.002	0.016	0.1
-40	0.001	0.008	0.2	0.002	0.013	0.1
-50	0.000	0.006	-0.1	-0.001	0.012	0.0
-100	-0.002	0.007	-0.3	-0.003	0.010	-0.2
-150	-0.002	0.008	-0.2	-0.001	0.010	-0.2
-200	-0.001	0.008	-0.1	-0.001	0.009	-0.1
-250	-0.001	0.008	-0.1	0.000	0.009	-0.1
-300	-0.001	0.008	-0.1	0.000	0.009	-0.1
-400	-0.001	0.008	-0.1	0.000	0.009	-0.1
-500	0.000	0.008	-0.1	0.000	0.009	-0.1

Table 16 - Evaluations of the results of the comparison

Figure 9 and 10 represent the plotted results of the generalised weighted mean of the relative deviations between LNE and PTB for uncorrelated quantities. Figure 11 and 12 represent the plotted results of the generalised weighted mean of the relative deviations between LNE and PTB for correlated quantities. The vertical bars represent the relative expanded uncertainty (*k* = 2) associated with the deviations.

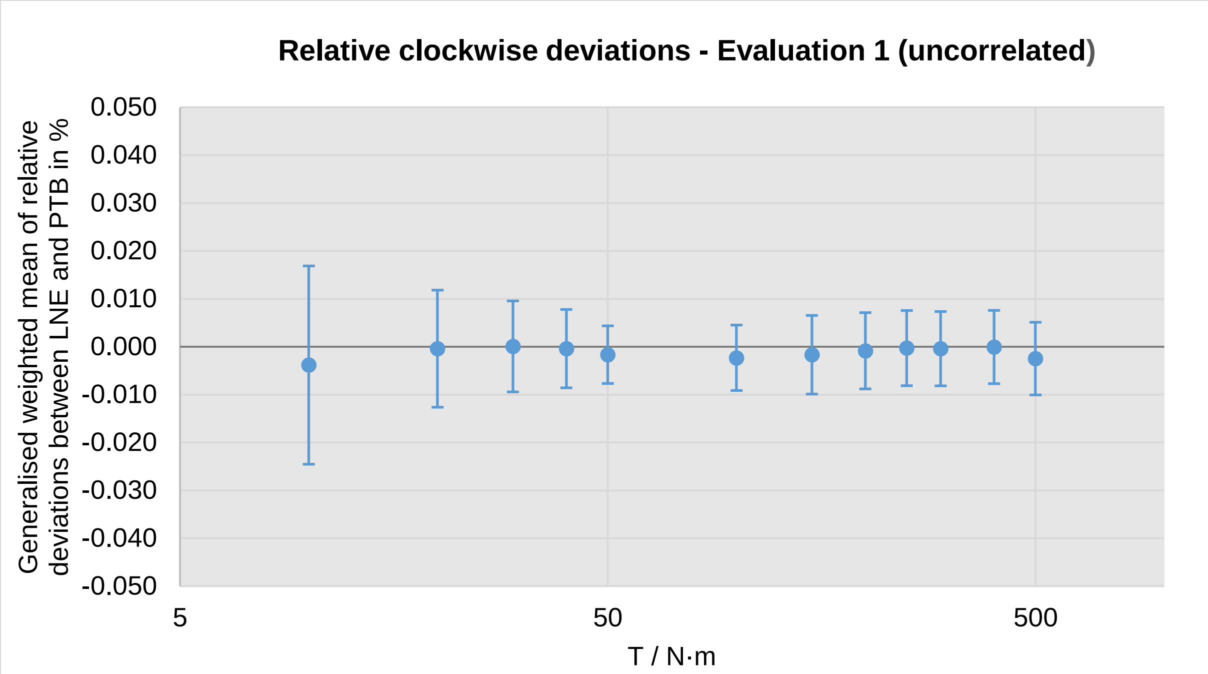

Figure 9 - Generalised weighted mean of the relative clockwise deviations between LNE and PTB for uncorrelated quantities. Vertical bars represent the expanded relative uncertainty (*k* = 2) associated with the mean in %. The scale of the x-axis is logarithmic.

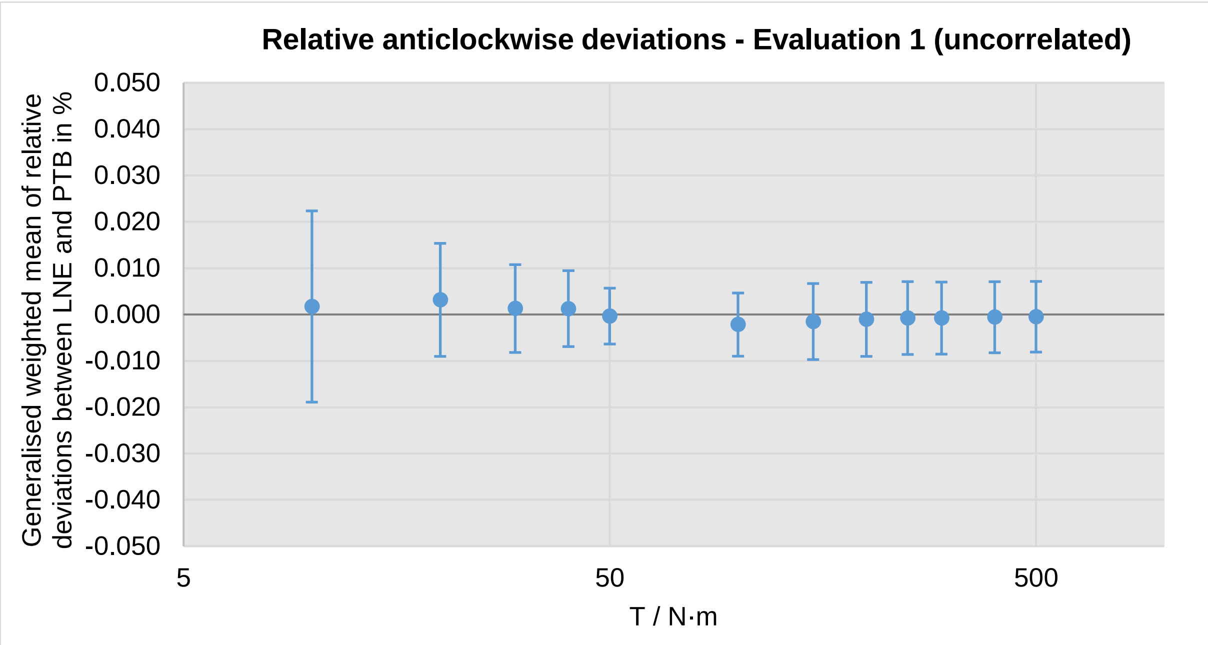

Figure 10 - Generalised weighted mean of the relative anticlockwise deviation between LNE and PTB for uncorrelated quantities. Vertical bars represent the expanded uncertainty (*k* = 2) associated with the mean in %. The scale of the x-axis is logarithmic.

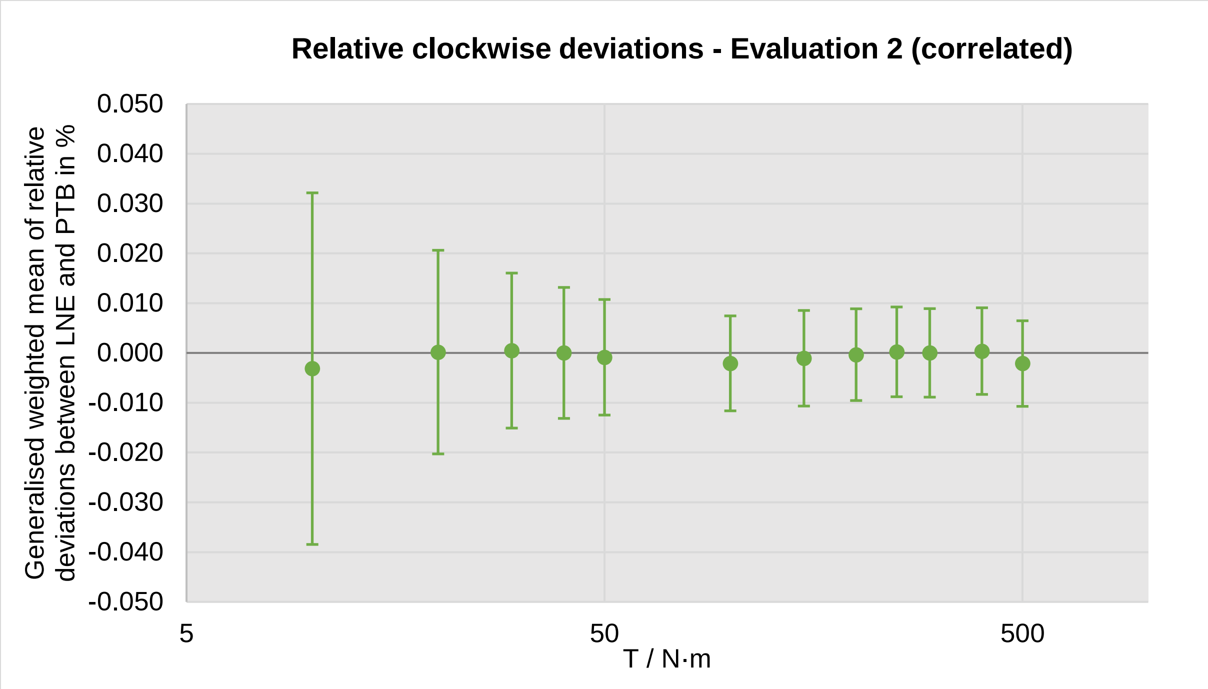

Figure 11 - Generalised weighted mean of the relative clockwise deviations between LNE and PTB for correlated quantities. Vertical bars represent the expanded relative uncertainty (k = 2) associated with the mean in %. The scale of the x-axis is logarithmic.

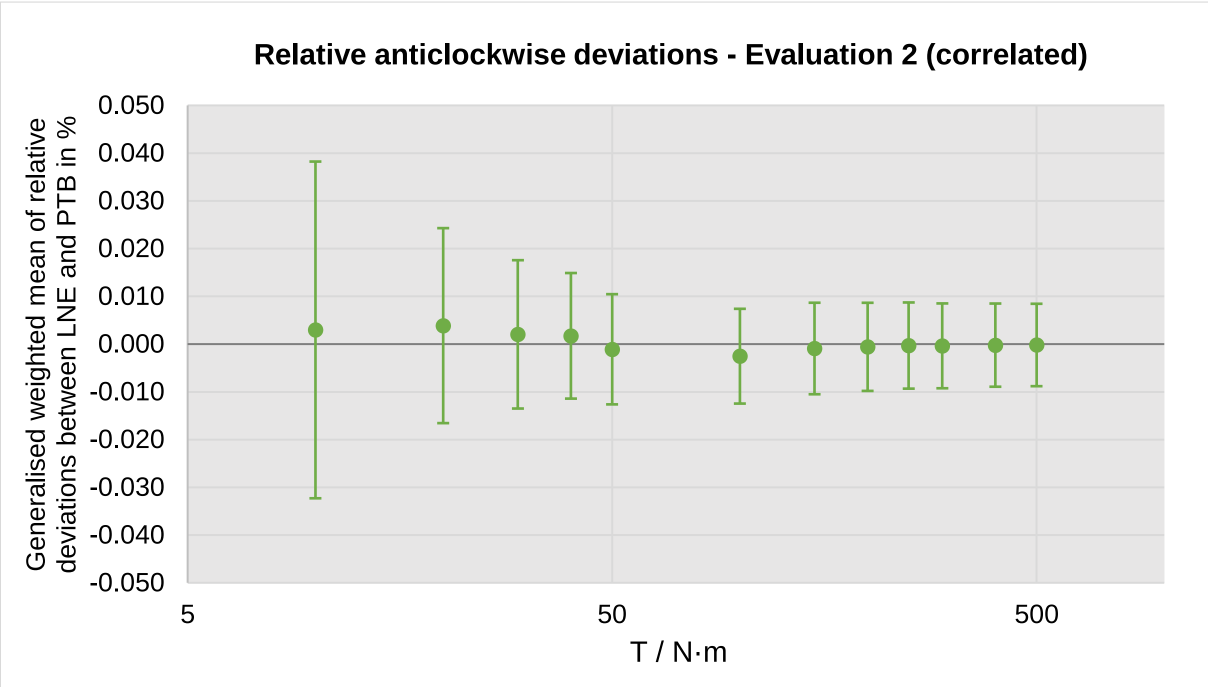

Figure 12 - Generalised weighted mean of the relative anticlockwise deviations between LNE and PTB for correlated quantities. Vertical bars represent the expanded relative uncertainty (k = 2) associated with the mean in %. The scale of the x-axis is logarithmic.

8 Conclusion

The results presented in the previous chapter show that the measurement values obtained at LNE for their 500 N·m deadweight torque standard machine, may not be statistically different from the measurement values obtained at PTB. Two evaluations have been performed using the generalised weighted mean: the first assuming the deviations uncorrelated and the second assuming the deviations correlated. Both evaluations lead to E_n values smaller than one.

These results support the validity of the uncertainty claims of LNE for their 500 N·m deadweight torque standard machine.

9 References

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[2] D. Röske, “Final Report on the Torque Key Comparison CCM.T-K1 Measurand Torque: 0 N·m, 500 N・m, 1000 N・m”, *Metrologia*, volume 46, 2018.

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[4] M G Cox et al, ”The generalized weighted mean of correlated quantities”, *Metrologia*, volume 43, Number 4, 2006.