What if μ_0, ε_0, and Z_0 were not exact?

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The purpose of this note is to pose the following question to the CCEM: **Would there be any problems of a significant sort in the fields of electrical or electromagnetic metrology if μ_0, and hence also ε_0 and Z_0, had a relative standard uncertainty $u_r \approx 10^{-9}$?** [Here, as usual, μ_0 is the magnetic constant (also called the permeability of vacuum), ε_0 is the electric constant (also called the permittivity of vacuum), and Z_0 is the characteristic impedance of vacuum.]

Background

In a recent paper, I. M. Mills, P. J. Mohr, T. J. Quinn, B. N. Taylor, and E. R. Williams [*Metrologia* **42**(2), 71-80 (2005)] proposed that the kilogram be redefined so as to fix the value of either the Planck constant h or Avogadro constant N_A, and offered a number of wordings for definitions that would do so.[1] One such definition that fixes h reads "The kilogram is the mass of a body at rest whose equivalent energy is equal to that of 299 792 458 $\times 10^{27}$ optical photons of wavelength in vacuum of 662.606 931 1 nanometers."

One of the arguments put forth by Mills *et al*. in favor of a definition that fixes the value of h is that if the ampere were subsequently redefined so as to fix the value of the elementary charge e, there would be quite significant additional benefits for electrical metrology. Such a definition might read "The ampere is the electric current corresponding to the flow of 6.241 509 468 3 $\times 10^{18}$ elementary charges per second."

The benefits that would accrue to electrical metrology if both h and e were exactly known constants follows from the fact that the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ would themselves become exactly known constants. This would mean that the Josephson effect could be used to directly realize the International System of Units (SI) volt, the quantum Hall effect (QHE) could be used to directly realize the SI ohm, and the two effects together could be used to directly realize the SI ampere and SI watt, the only

[1] We put aside the question of whether or not the redefinition of the kilogram should proceed without delay.

uncertainty in each case being the experimental uncertainty associated with the realization. This would also be the case for the realization of the SI farad and SI henry from the SI ohm as realized from the QHE. Thus, the current practical system of conventional electrical units based on the Josephson effect and the QHE and the conventional values K_{J-90} and R_{K-90} could be replaced with the SI units themselves, obviously a quite major advance.

Although the benefits of fixing both h and e are significant, one must also recognize that if implemented and the present definition of the meter is retained, the magnetic constant μ_0, the electric constant ε_0, and the impedance of vacuum Z_0, which in the current SI are all exactly known constants, would become quantities that must be determined by experiment.

To see this we first recall that at present

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N\ A^{-2}}\ \text{exactly}, \quad \varepsilon_0 = 1/\mu_0 c^2, \quad Z_0 = (\mu_0/\varepsilon_0)^{1/2} = \mu_0 c, \tag{1}$$

where $c = 299\ 792\ 458\ \mathrm{m\ s^{-1}}$ exactly is the speed of light in vacuum. (The value of c is, of course, fixed by the present definition of the SI meter, and the value of μ_0 is fixed by the present definition of the SI ampere.) We then recall the equation

$$R_K = \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}, \tag{2}$$

where $\alpha \approx 1/137$ is the fine-structure constant, a dimensionless quantity that is the coupling constant of the electromagnet force. Thus, because α is simply a number determined by nature, it follows from this relation that if c, h, and e have exact values, μ_0 is a quantity that must be determined by experiment. In fact, Eq. (2) shows that if c, h, e are fixed (we assume by the definitions of the meter, kilogram, and ampere, respectively), then a determination of α—for example, by equating the experimental value of and theoretical expression for the magnetic moment anomaly of the electron a_e—is also a determination of μ_0.

The present relative standard uncertainty u_r of the 2002 CODATA recommended value of α is 3.3×10^{-9} [P. J. Mohr and B. N Taylor, *Rev. Mod. Phys*. **77**(1), 1-107 (2005)], and it is expected that experimental and theoretical work currently underway will reduce $u_r(\alpha)$ to about 1×10^{-9} within the next 1 to 2 years. This implies that if the kilogram and ampere were to be redefined so as to fix the values of h and e, and the definition of the meter is unchanged, μ_0, ε_0, and Z_0 would be known with a relative standard uncertainty of 1×10^{-9}. It is these considerations that have led to the question posed to the CCEM at the beginning of this note.