Index of Refraction Effects in Blackbody Temperature Measurements

A.A. Gaertner
NRC Canada

Introduction:

Present techniques for the measurement of the operating temperature of high-temperature black-bodies (HTBB) are based upon the absolute measurement of the output of the HTBB, either in the radiance mode or the irradiance mode [1]. As the accuracy of these measurements has progressed over the years, many uncertainty components have become increasingly important. One of these is the effect of the difference in operating medium between the cavity of the HTBB, usually hot Ar, that produces the radiation, and the medium in the laboratory, usually air at room temperature, where this radiation is measured. The importance of this effect has recently been discussed by Hartmann [2].

The purpose of this paper is to discuss the effects of this change in propagation medium and resolve some of the omissions in the earlier paper [2].

Since the measurements are performed spectrally, using a well-defined geometrical configuration, both of these aspects must be considered in arriving at the correct relationship between the output (spectral radiance) of the HTBB and the flux measured by the room temperature device.

Spectral Considerations:

There are two important spectral considerations: the meaning of a spectral distribution [3], and the variable that we use to describe the spectral distribution.

1. The adjective spectral has two possible meanings [3], both of which are necessary in our application:

   i. If X is any quantity pertaining to electromagnetic radiation, X can be a function of the spectral variable, such as frequency ν. This is denoted as $X(\nu)$.

   ii. The spectral quantity X is the spectral concentration of X, denoted as X_ν, where $X_\nu \equiv \frac{dX}{d\nu}$.

   To be pedantic, we should then write our spectral radiation quantities as $X_\nu(\nu)$. Since we rarely do, it is easy to forget the two aspects of this definition. However, both of these aspects must be taken care of when we change the variable in the description of our spectral quantity.

2. For the purposes of moving radiation between different media, the spectral variable of frequency ν is particularly useful since the spectral distribution will not change with index of refraction. Therefore the spectral $X_\nu(\nu)$ measured in the laboratory in air at room temperature will be the same as the spectral $X_\nu(\nu)$ generated in the HTBB in hot Ar. However, most of our measurements made in the laboratory in air at room temperature are performed with equipment that is used, and has been calibrated, in terms of wavelength λ_2, where the subscript 2 indicates, for our purposes, the wavelength in air at room temperature. Therefore the spectral $X_{\lambda_2}(\lambda_2)$ that we measure may be obtained from the $X_\nu(\nu)$ generated in the HTBB by:

$$X_{\lambda_2}(\lambda_2) = X_\nu(\nu)\left|\frac{d\nu}{d\lambda_2}\right| \tag{1}$$

In medium 2 we know that $$\nu = \frac{c}{n_2 \lambda_2} \tag{2}$$

So that $$\left|\frac{d\nu}{d\lambda_2}\right| = \frac{c}{n_2 \lambda_2^2} \tag{3}$$

and $$X_{\lambda_2}\left(\lambda_2\right) = \frac{c}{n_2 \lambda_2^2} X_\nu(\nu) \tag{4}$$

and we can replace any occurrence of ν in the quantity $X(\nu)$ in equation (4) with its equivalent from equation (2). In the above equations, c is the speed of light in vacuum and n_2 is the index of refraction of medium 2, air at room temperature.

Geometrical Considerations:

The geometrical aspects have to do with the specific geometric quantity radiance. Due to the change in the refractive index of the medium in which the radiation moves, from n_1 inside the HTBB to n_2 outside the HTBB where the radiation is measured, the spatial size of the beam of radiation changes due to refraction (Snell's Law). This results [4] in what is known as the conservation of basic radiance, where basic radiance is the quantity L/n^2. In our case this gives:

$$\begin{aligned} \frac{L_\nu^{HTBB}(\nu)}{n_1^2} &= \frac{L_\nu^{air}(\nu)}{n_2^2} \\ L_\nu^{air}(\nu) &= \frac{n_2^2}{n_1^2} L_\nu^{HTBB}(\nu) \end{aligned} \tag{5}$$

where $L_\nu^{air}\left(\nu\right)$ is the radiance of the beam of radiation, from the HTBB, as seen in the air at room temperature.

Combining equations (4) and (5) we have:

$$\begin{aligned} L_{\lambda_2}^{air}\left(\lambda_2\right) &= \frac{c}{n_2 \lambda_2^2} L_\nu^{air}(\nu) \\ &= \frac{c}{n_2 \lambda_2^2} \cdot \frac{n_2^2}{n_1^2} \cdot L_\nu^{HTBB}(\nu) \end{aligned} \tag{6}$$

Planckian Radiators:

The spectral radiance $L_\nu^{HTBB}\left(\nu\right)$ of the HTBB is given by:

$$\begin{aligned} L_\nu^{HTBB}(\nu) &= \varepsilon \cdot \frac{2h\nu^3}{c_{n_1}^2} \cdot \frac{1}{(e^{\frac{h\nu}{kT}} - 1)} \\ &= \varepsilon \cdot 2h\nu^3 \cdot \left(\frac{n_1}{c}\right)^2 \cdot \frac{1}{(e^{\frac{h\nu}{kT}} - 1)} \end{aligned} \tag{7}$$

where ε is the emissivity of the HTBB, h is Planck's constant, k is Boltzmann's constant, n_1 is the index of refraction of the medium (hot Argon) inside the HTBB, c_{n_1} is the speed of light inside the HTBB ($c = n_1 c_{n_1}$), c is the speed of light in vacuum, and T is the temperature inside the HTBB.

This spectral distribution is generated inside the HTBB, and does not change when it moves out of the HTBB since it is in units of frequency, as discussed above.

Therefore, combining equations (2), (6) and (7) we have:

$$\begin{aligned} L_{\lambda_2}^{air}(\lambda_2) &= \frac{c}{n_2\lambda_2^2} \cdot \frac{n_2^2}{n_1^2} \cdot \varepsilon \cdot 2h\left(\frac{c}{n_2\lambda_2}\right)^3 \cdot \left(\frac{n_1}{c}\right)^2 \cdot \frac{1}{(e^{\frac{hc}{n_2\lambda_2 kT}} - 1)} \\ &= \varepsilon \cdot \frac{2hc^2}{\lambda_2^5} \cdot \frac{1}{n_2^2} \cdot \frac{1}{(e^{\frac{hc}{n_2\lambda_2 kT}} - 1)} \end{aligned} \tag{8}$$

Note that this only involves variables for medium 2, the air at room temperature. For this special case of a Planckian radiator, the spatial and spectral effects have cancelled out any dependence on the index of refraction of the HTBB medium.

The above analysis has neglected the reflection loss at the Argon-air interface as given by Fresnel's Equations and discussed by Hartmann [1,2]. As discussed in [2], this is a very small correction.

Comparison with previous results:

1. Hartmann [2]:

The results in [2] differ from equation (8) by a factor of $\left(n_1/n_2\right)^3$. This is due to two factors:

i. The analysis in [2] has not taken account of the effect of Snell's law on the spatial beam size as described in equation (5) above.

ii. The analysis in [2] has not taken into account the change in the size of the spectral unit from λ_1 to λ_0 to λ_2 in a manner similar to that discussed at equation (1) above. i.e., since $\lambda_0 = n_1\lambda_1 = n_2\lambda_2$, we require the terms:

$$\left|\frac{d\lambda_1}{d\lambda_0}\right| = \frac{1}{n_1} \text{ and } \left|\frac{d\lambda_0}{d\lambda_2}\right| = n_2 \tag{9}$$

When these two factors are considered in the analysis in [2], the results are the same as that given in equation (8) above.

2. Nicodemus [5,6]:

The equations in this Self Study Manual have been developed for a general radiance. If the specific form of a Planckian radiator, as in equation (7) above, are introduced into their equations, the results are the same as developed in this paper.

References:

[1] Jürgen Hartmann, "High-temperature measurement techniques for the application in photometry, radiometry and thermometry", Physics Reports **469**, p 205-269, (2009).

[2] Jürgen Hartmann, “Correct consideration of the index of refraction using blackbody radiation”, Optics Express **14**, p 8121-8126, (2006).
[3] “International Lighting Vocabulary”, CIE Publication Number 17.4, Sections 845-01-16 and 845-01-17, Bureau Central de la Commission Electrotechnique Internationale, Gèneve, Suisse, 1987.
[4] Franc Grum and Richard J. Becherer, “Optical Radiation Measurements, Volume 1, Radiometry”, Academic Press, New York, 1979.
[5] “Self-Study Manual on Optical Radiation Measurements: Part 1—Concepts, Chapters1 to 3”, Fred E. Nicodemus, Editor, U.S. Department of Commerce, Washington, 1976.
[6] I am grateful to Dr. Emma Woolliams of NPL for bringing this reference to my attention.