The uncertainty for the KC in frequency CCTF-K002.FREQ

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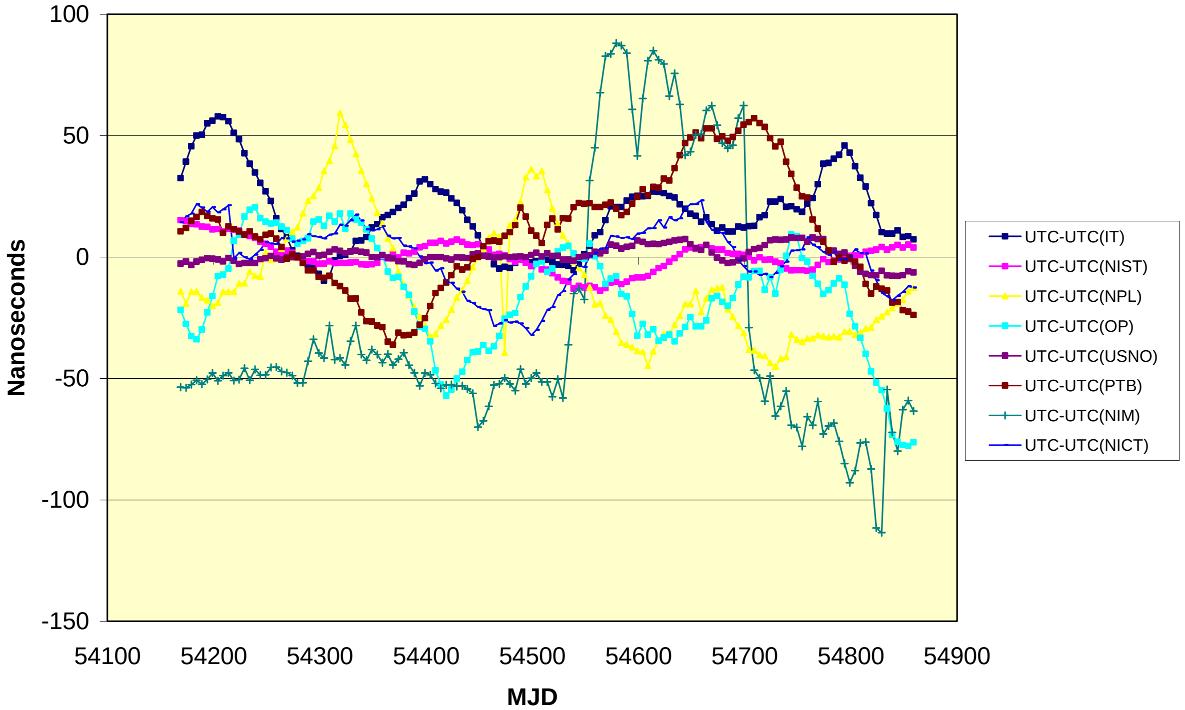
The CCTF has declared [*UTC-UTC*(*k*)] as published in monthly *BIPM Circular T* as a Key Comparison in the Time and Frequency field. *Circular T* gives the deviation for each contributing laboratory in the form of differences [*UTC-UTC*(*k*)] at five-day intervals with their respective combined uncertainty values constant in a month. From this, the corresponding deviation for frequency and its uncertainty are therefore available at time intervals of 5 days. If the laboratories need interval and averaging time shorter than 5 days extrapolation can be done following the guidelines of the WGMRA (CCTF Working group on MRA).

The aim of this note is to explain the evaluation of the key comparison in frequency for supporting laboratories CMC declarations. This key comparison, as established by the CCTF in 2006 is designated as CCTF-K002.FREQ.

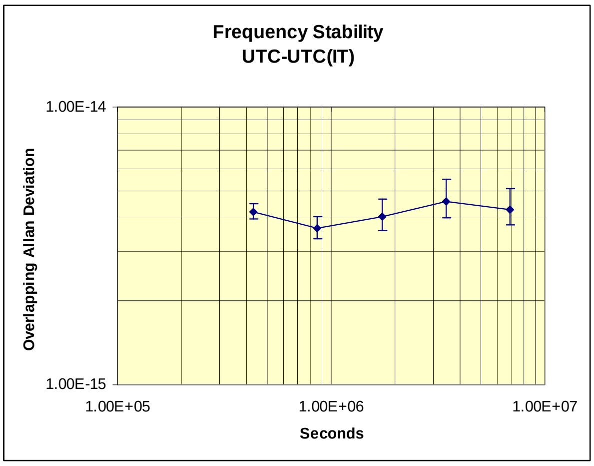
The uncertainty analysis based on the Allan Variance

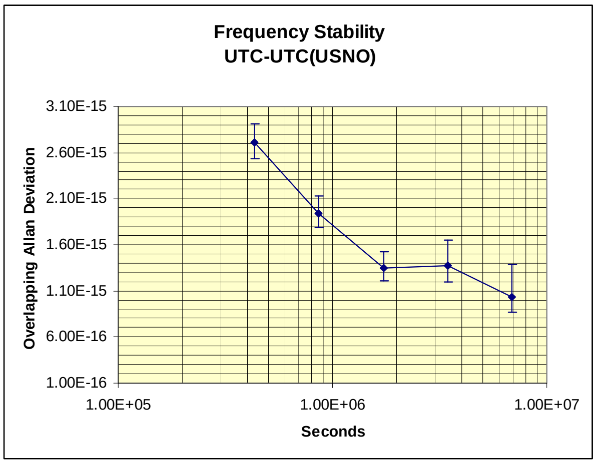
A first analysis is given by the Allan variance of different time scales.
We have considered the following time scales, as compared to the UTC reference for the period between MJD 54169 to 54859: UTC-UTC(USNO), UTC-UTC(NIST), UTC-UTC(IT), UTC-UTC(OP), UTC-UTC(NPL), UTC-UTC(PTB), UTC-UTC(NIM) and UTC-UTC(NICT).

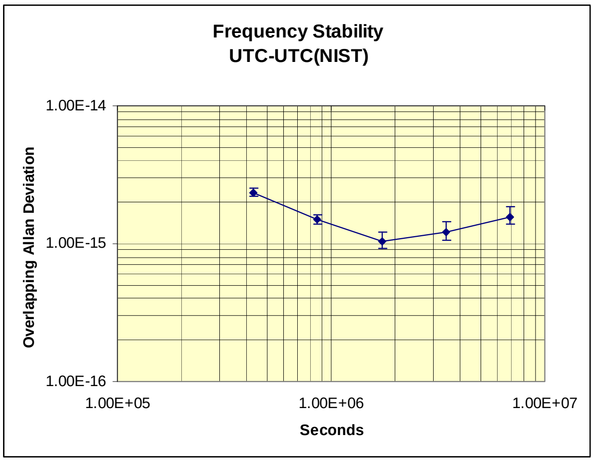


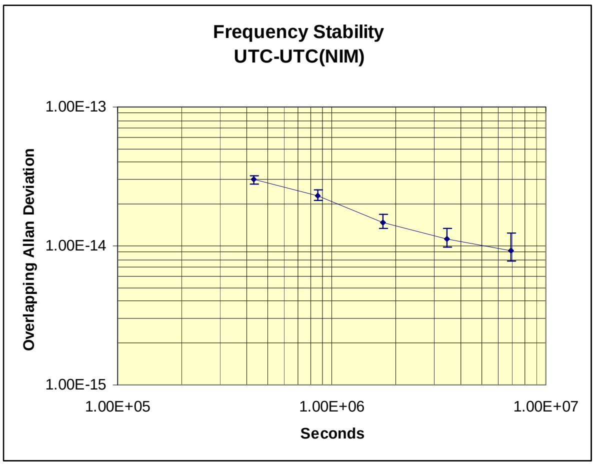
These time scales are of different nature; some are obtained by averaging clock data with an algorithm, other are obtained from a steered atomic clock (hydrogen maser, caesium clock or primary frequency standard).

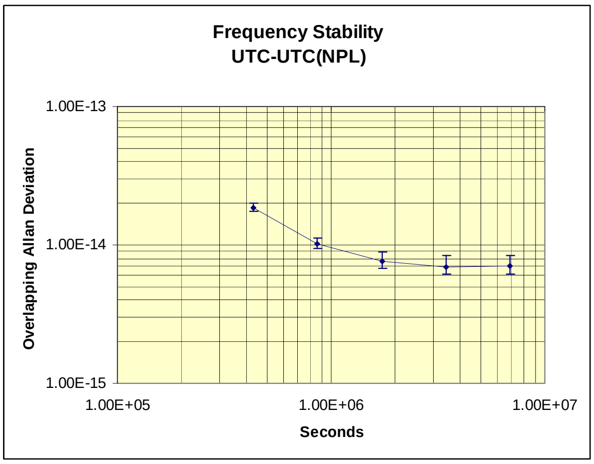
The Allan deviations for the different time scales are reported in the following figures.

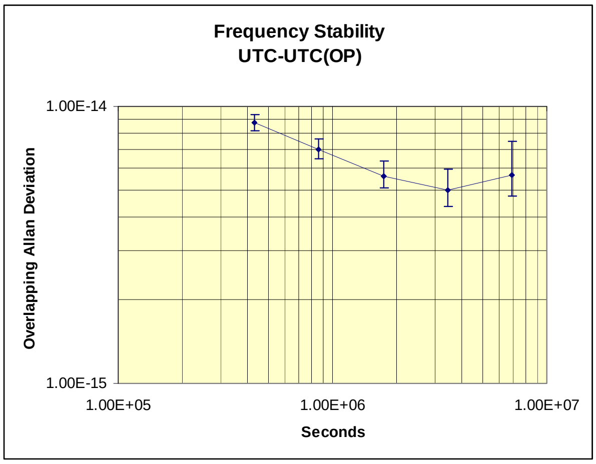


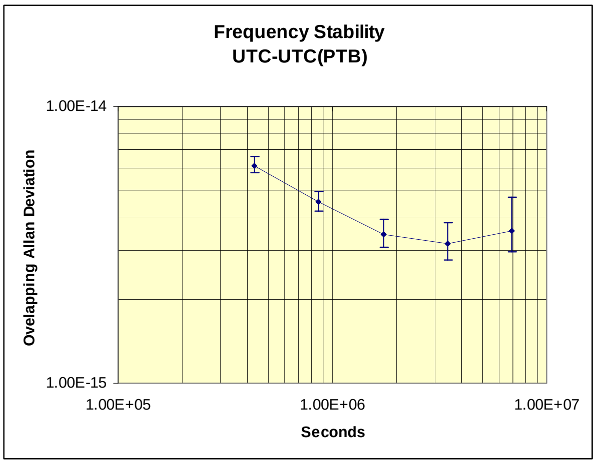


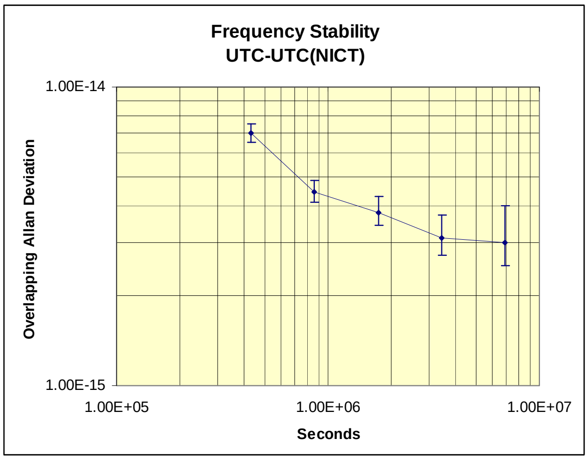








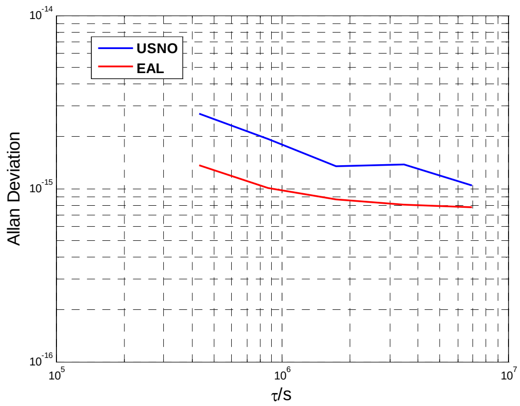


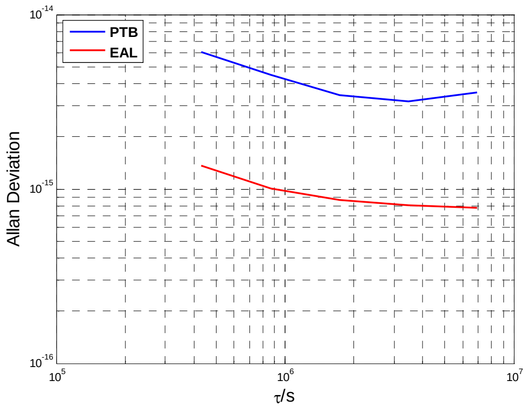


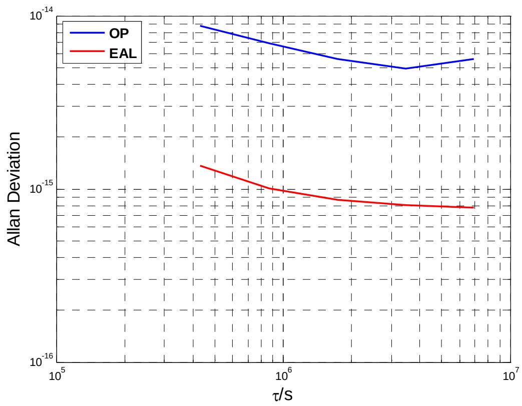
We can make a comparative analysis of stability between these individual time scales and the atomic free scale (EAL). Considering the EAL noise levels as reported in the *Circular T*:

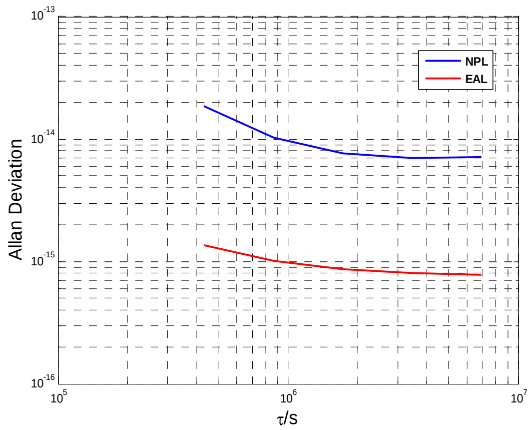
1. White Frequency Noise : $3\times10^{-15}\dfrac{1}{\sqrt{\tau}}$ with τ in days
2. Flicker Frequency Noise : 0.5×10^{-15}
3. Random Walk frequency Noise: $1\times10^{-16}\sqrt{\tau}$ with τ in days

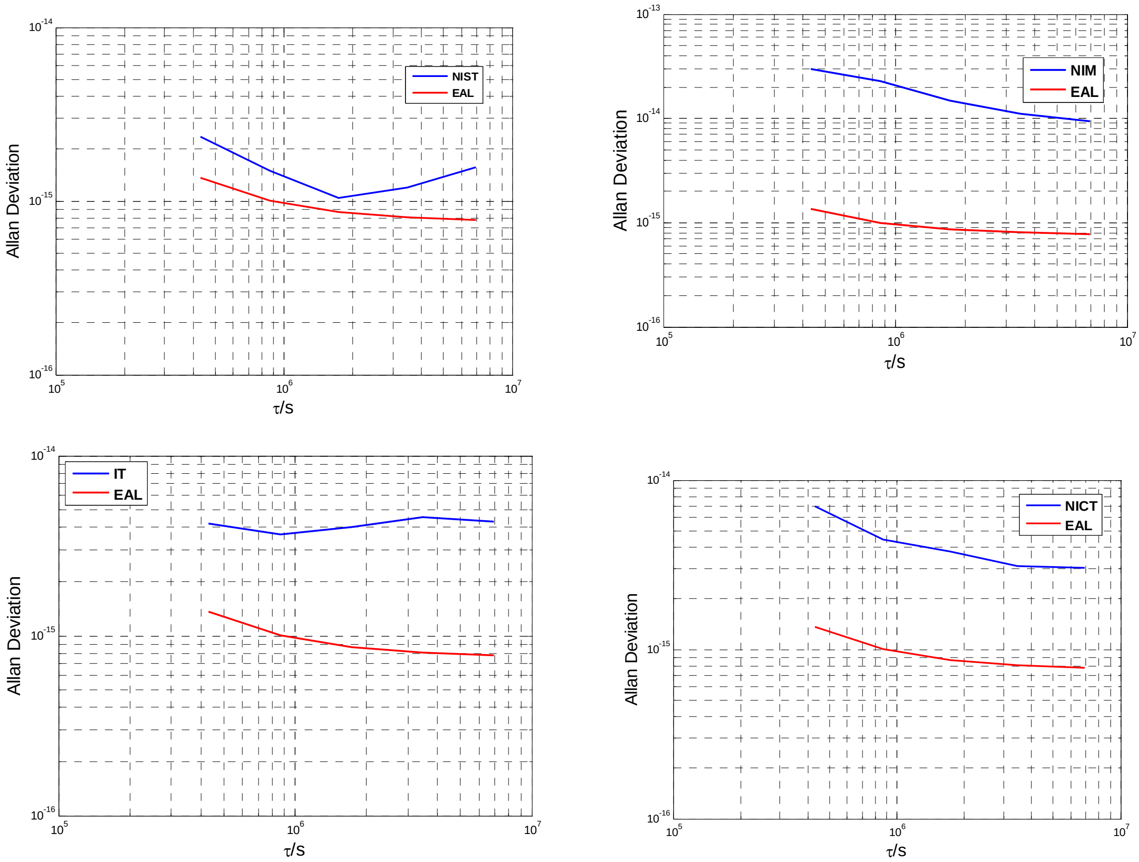
We report here the time scales stability analysis compared with the EAL stability:











By the stability analysis it is clear that the Allan Variance is not a good estimator for the frequency uncertainty due to the steering algorithm as can be observed by the bump present in the stability analysis. In [1] it has been shown that the uncertainty of [*UTC-UTC*(*k*)] is dependent on the link uncertainty; when a time scale is obtained by an hydrogen maser or an algorithm the results reported in the first section of *Circular T* are consistent with the stability analysis at 5 days. This statement is not evident in the case of time scale based on a caesium clock due to the big difference between the performance of the atomic clock and the link. By this consideration we can conclude that the Allan variance is not a good method useful to obtain the uncertainty on the frequency.

In the following section we report on a method based for evaluating the frequency uncertainty based on the uncertainties of [*UTC-UTC*(*k*)].

The uncertainty analysis based on the uncertainty of [*UTC-UTC*(*k*)]

The degree of equivalence for the key comparison in frequency can be derived from the existing key comparison for UTC (or from the values published in *Circular T*).

The BIPM computes the differences between UTC and UTC(k), and based on these values the degrees of equivalence between the reference frequency and that realized in the laboratory *k* can be calculated. Following this statement the frequency uncertainties are linked to the uncertainties of [*UTC-UTC*(*k*)] reported monthly in Section 1 of *Circular T*. These uncertainties as explained in [1] are related to the link uncertainties reported in Section 6 of *Circular T*.

On the other hand the laboratories receive the result of [*UTC-UTC*(*k*)] and their uncertainty with a delay of about 15 days after the last date of data in a month, and in this period (45 days from the beginning of the month of data) they are not aware of the difference between the UTC and the local time scale UTC(k) and they neither know the uncertainty that should be declared. In the case they cannot wait until the publication of *Circular T* to have the final values, they should add a “prediction component” to the global budget of frequency uncertainty. However, if the laboratories wait for the *Circular T* results they know the uncertainty information.

The BIPM can only work based on data received from contributing laboratories and on the information obtained from *Circular T*. The frequency uncertainty will be thus obtained by applying the law of uncertainty propagation [2] to the relation between phase and frequency.

The mean frequency is defined as:

$$\bar{y} = \frac{(UTC - UTC(k))_t - (UTC - UTC(k))_{t-\tau}}{\tau}$$

From this, the uncertainty on the mean frequency is given by:

$$u_{\bar{y}}^2 = \frac{u^2_{(UTC-UTC(k))_t} + u^2_{(UTC-UTC(k))_{t-\tau}} - 2Cov\left((UTC - UTC(k))_t, (UTC - UTC(k))_{t-\tau}\right)}{\tau^2}.$$

Within our hypothesis, the uncertainty of the difference [*UTC-UTC*(*k*)] reported monthly on *Circular T* for laboratory *k* remains constant over the whole period.

Therefore, making use of the uncertainty reported in *Circular T* the previous relation becomes:

$$u_{\bar{y}}^2 = \frac{2(uA)^2}{\tau^2}.$$

where τ is the integration time, minimum 5 days, since this is the step between consecutive results in *Circular T*. Only the statistical component uA of the uncertainty of [*UTC-UTC*(*k*)] should be used considering that the calibration uncertainty (represented by the systematic uncertainty uB) does not affect the frequency measurements.

We have evaluated the uncertainty for all laboratories participating to the calculation of *Circular T 253* (January 2009). Only those laboratories signatories of the MRA or designated will participate to the key comparison CCTF-K002.FREQ; however we include all in the table to have a complete set of possible cases:

Lab k	uA / ns	D_k = y([UTC - $UTC(k)$])	U_k
AOS	0.6	-2.8E-15	4E-15
APL	1.5	-2.8E-14	9.8E-15
AUS	1.5	-7.1E-14	9.8E-15
BEV	1.5	3.7E-15	9.8E-15
BIM	2	8.5E-14	1.3E-14
BIRM	2	1.0E-13	1.3E-14
BY	7	-1.2E-13	4.6E-14
CAO	1.5	3.4E-14	9.8E-15
CH	0.6	-9.3E-16	4E-15
CNM	2.5	-1.5E-14	1.64E-14
CNMP	3	2.1E-15	1.96E-14
DLR	0.7	2.1E-15	4.6E-15
DTAG	4	5.0E-14	2.6E-14
EIM	3	5.8E-15	1.96E-14
HKO	2.5	8.1E-15	1.64E-14
IFAG	0.7	-4.4E-15	4.6E-15
INTI	4	2.6E-14	2.6E-14
IT	0.6	3.0E-15	4E-15
JATC	1.4	-4.2E-15	9.2E-15
JV	5	-2.2E-13	3.2E-14
KIM	3	6.0E-14	1.96E-14
KRIS	0.7	-7.9E-15	4.6E-15
KZ	2	1.4E-14	1.3E-14
LT	1.5	-5.6E-15	9.8E-15
LV	2	-4.8E-14	1.3E-14
MIKE	5	-5.9E-14	3.2E-14
MKEH	2.5	7.3E-11	1.64E-14
MSL	1	-6.4E-14	6.6E-15
NAO	3	-5.6E-15	1.96E-14
NICT	0.7	1.2E-15	4.6E-15
NIM	1	1.0E-14	6.6E-15
NIMB	2	9.5E-15	1.3E-14
NIMT	1	3.4E-14	6.6E-15
NIS	1.5	-6.3E-15	9.8E-15
NIST	0.5	2.8E-15	3.2E-15
NMIJ	0.7	2.8E-15	4.6E-15
NMLS	2	3.1E-14	1.3E-14
NPL	0.6	-3.2E-15	4E-15
NPLI	2.5	-2.0E-14	1.64E-14
NRC	0.7	-3.9E-15	4.6E-15
NRL	0.7	2.8E-15	4.6E-15
NTSC	1.4	-2.3E-16	9.2E-15
ONBA	2.5	2.1E-14	1.64E-14
ONRJ	3.9	4.1E-14	2.6E-14
OP	0.5	-3.5E-15	3.2E-15
ORB	0.7	3.0E-15	4.6E-15
PL	1.5	-7.9E-15	9.8E-15
PTB	0.3	3.2E-15	1.96E-15
ROA	0.7	-5.3E-15	4.6E-15
SCL	3	9.7E-15	1.96E-14
SG	3	-1.8E-14	1.96E-14
SIQ	5	-3.3E-14	3.2E-14
SMU	5	6.9E-15	3.2E-14
SP	0.5	-1.6E-15	3.2E-15
SU	3	-1.1E-14	1.96E-14
TCC	1.5	2.1E-14	9.8E-15
TL	0.7	8.1E-15	4.6E-15
TP	0.9	7.2E-15	5.8E-15
UA	2.5	1.4E-14	1.64E-14
UME	1.5	1.7E-13	9.8E-15
USNO	0.4	9.3E-16	2.6E-15
VMI	1	-3.5E-14	6.6E-15
VSL	0.7	7.2E-15	4.6E-15
ZMDM	2	-7.2E-14	1.3E-14

The first column lists the laboratory acronyms as in *Circular T*, the second column the value of uA reported in the section 1 of the Circular T, the third the value for the frequency corresponding to 23 January 2009 (MJD 54854) and the last one the expanded uncertainty on the frequency.

References

[1] W. Lewandowski, D. Matsakis, G. Panfilo and P, Tavella, "The evaluation of uncertainties in [UTC-UTC(k)]," *Metrologia, vol 43*, pp. 278-286, 2006.

[2] ISO 1993 Guide to the Expression of Uncertainty in Measurement (Geneva: International Organization for Standardization).